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## Performance Analysis of Reduced-rank Beamformers for Estimating Dipole Source Signals Using EEG/MEG\*

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## Abstract

We study the performance of various beamformers for estimating a current dipole source at a known location using electroencephalography (EEG) and magnetoencephalography (MEG). We present our beamformers in the form of the generalized sidelobe canceler (GSC). Under this structure, the beamformer can be solved by finding a filter that achieves the minimum mean-squared error (MMSE) between the mainbeam response and filtered observed signal. We express the MMSE as a function of the filter's rank and use it as a criterion to evaluate the performance of the beamformers. We do not make any assumptions on the rank of the interference-plus-noise covariance matrix. Instead, we treat it as low-rank and derive a general expression for the MMSE. We present numerical examples to compare the MSE performance of beamformers commonly studied in the literature: principal components (PC), cross-spectral metrics (CSM), and eigencanceler beamformers. Our results show that good estimates of the dipole source signals can be achieved using reduced-rank beamformers even for low SNR values.

### Keywords:

beamforming, sensor array processing, low-rank covariance matrix, electroencephalography, magnetoencephalography, dipole source signal.

# I Introduction

Beamforming techniques have been used to solve various problems of analyzing neuroelectric and neuromagnetic signals, such as the localization of brain activity sources using electroencephalography (EEG) and magnetoencephalography (MEG) sensor arrays, as well as source signal reconstruction and interference cancellation [1]. Specifically, methods based on linearly constrained minimum variance (LCMV) beamforming, eigenvalue decomposition, and principal component (PC) selection have been proposed to remove the interference and recover the dipole moments for the case of known source position [2].

In [3] we proposed a beamformer based on the cross-spectral metrics (CSM) for the case when the assumption of sources of neural activity being distinctly characterized in the spectral decomposition of the covariance matrix by a few large eigenvalues does not hold. This may happen under the presence of interference with localized biological origins, such as eye blinking, cardiac sources, or background brain activity (e.g.  $\alpha$  rhythm) [4]-[6]. The CSM beamformer offers a solution to this problem by characterizing the neural activity sources not according to the magnitude of their corresponding eigenvalues but based on their cross-spectral content [7]. Furthermore, the CSM beamformer makes it possible to find a reduced-rank subspace such that the beamformer is approximated by a few eigenvalues without significant loss of performance in terms of the signal to interference-plus-noise ratio (SINR) [8]. Hence, in this paper we revisit the problem of reduced-rank beamformers in order to establish a measure of performance for LCMV spatial filters when estimating a current dipole source at a known location using EEG/MEG data.

The low-rank nature of our problem is not only a result of the distinctly characterized few large eigenvalues, but also because of the interference-plus-noise covariance matrix being unknown. In this case, an estimate of such matrix must be used. Typically, the interference-plus-noise covariance matrix is assumed to be of full rank even when its estimate is singular.

Here, as in [9], we consider a general case where the interference-plus-noise covariance matrix has *arbitrary* rank, thus allowing for low-rank interference. We distinguish two scenarios for which low-rank interference-plus-noise covariance matrix is of interest: (i) available training data is insufficient to obtain a full-rank estimate of the covariance matrix of interference and noise, and (ii) we consider the low-rank covariance matrix of interference only, i.e. the noise term is neglected (as in e.g. [10]). The majority of current methods deal with these problems by using diagonal loading [11], which results in suboptimal solutions. We approach the problem in a different way: instead of forcing the covariance matrix of the interference-plus-noise to be non-singular, we assume that it is singular and generalize the beamforming problem under this low-rank condition.

We first write the constrained beamformer in an equivalent unconstrained form based on the generalized sidelobe canceler (GSC) [12]. This unconstrained structure allows us to derive a general expression for the filter that achieves the minimum mean-squared error (MMSE) between the mainbeam response and filtered observed signal. We use the MMSE as measure of performance because it is directly related to the SINR within the low-rank subspace spanned by the reduced-rank eigen-basis [13].

In Section III, we pose the beamforming problem of estimating the current source dipole signals at known positions in the form of a GSC. Under these conditions, we derive the MMSE as a function of the rank. In Section III.3 we present different reduced-rank beamformers: principal component, cross-spectral metrics, and eigencanceler (EIG). These beamformers are defined in terms on the structure of the interference-plus-noise covariance matrix. Then, we analyze the robustness and MMSE optimality of these well-known beamformers to establish the conditions under which they can improve the brain source analysis using reduced-rank techniques.

In Section IV we show the applicability of our methods through numerical examples using simulated MEG data. In Section V, we discuss the results, limitations, and future work.

## II Source and Measurement Models

Consider the case of measuring the potentials over the scalp and the magnetic field outside the head produced by  $L$  dipole sources using a bimodal array of  $m_E$  EEG and  $m_B$  MEG sensors. The subscripts E and B refer to the EEG and MEG sensors, respectively. Assume that the sources change in time but remain at the same position  $\boldsymbol{\theta}$  during the measurements period. This assumption holds in practice for evoked response and event-related experiments [14]. Then, EEG/MEG data is collected by the array of  $m = m_E + m_B$  sensors at time samples  $t = 1, 2, \dots, N$ . The  $m \times N$  spatio-temporal data matrix of this array at the  $k$ th trial is

$$Y_k = A(\boldsymbol{\theta})Q + E_k, \quad k = 1, 2, \dots, K, \quad (1)$$

where  $Y_k = [Y_{kB}^T, Y_{kE}^T]^T$ ,  $A(\boldsymbol{\theta})$  is the  $m \times 3L$  array response matrix,  $Q$  is the  $3L \times N$  matrix of dipole moments, and  $E_k = [E_{kB}^T, E_{kE}^T]^T$  is the interference-plus-noise matrix (considered to be arbitrary but constant between trials). The array response matrix is derived using the quasistatic approximation of Maxwell's equations and spherical head model (see [15] and references therein). Using a vector representation, we can rewrite equation (1) as  $\text{vec}(Y_k) = [I_N \otimes A(\boldsymbol{\theta})]\text{vec}(Q) + \text{vec}(E_k)$ . Define  $\mathbf{y}_k = \text{vec}(Y_k)$ ,  $C = C(\boldsymbol{\theta}) = [I_N \otimes A(\boldsymbol{\theta})]$ ,  $\mathbf{q} = \text{vec}(Q)$ , and  $\mathbf{e}_k = \text{vec}(E_k)$ . Then, our measurement model is finally expressed as

$$\mathbf{y}_k = C\mathbf{q} + \mathbf{e}_k. \quad (2)$$

In the previous model, the dimensions of  $\mathbf{y}_k$ ,  $C$ , and  $\mathbf{q}$  are, respectively,  $mN \times 1$ ,  $mN \times 3LN$ , and  $3LN \times 1$ .

Assume that the measurements are taken in the presence of zero mean Gaussian noise uncorrelated in time and space between time samples. Then, we define the covariance matrix

of the interference-plus-noise as  $R = E[\mathbf{e}_k \mathbf{e}_k^T]$ . For the case of unknown  $R$ , we can obtain a consistent estimate of this covariance matrix as

$$\widehat{R} = \left( \frac{1}{K} \sum_{k=1}^K \mathbf{y}_k \mathbf{y}_k^T \right) - \bar{\mathbf{y}} \bar{\mathbf{y}}^T \quad (3)$$

where

$$\bar{\mathbf{y}} = \frac{1}{K} \sum_{k=1}^K \mathbf{y}_k. \quad (4)$$

### III The Proposed Methods

In this section we present various spatial filters whose optimal weights are defined in a reduced-rank space. We use a minimum mean-squared filter with the structure of a GSC to evaluate the performance of these reduced-rank beamformers in the estimation of the dipole signal components at a given location  $\boldsymbol{\theta}$ .

#### III.1 Generalized Sidelobe Canceler

Consider the following LCMV filtering problem

$$\widehat{W} = \min_W W^T R W \quad \text{subject to} \quad C^T W = F, \quad (5)$$

where  $F$  is the desired matrix response (i.e. the one that defines the gain of the signals at the location of interest while nullifying signals from elsewhere), and the optimal weights  $\widehat{W}$  give solution to  $\widehat{\mathbf{q}} = \widehat{W}^T \mathbf{y}_k$ . Equation (5) can be solved using the unconstrained structure of the GSC as follows:

Assume that  $\widehat{W}$  can be decomposed into two orthogonal components  $W_0$  and  $-W_h$ , i.e.  $\widehat{W} =$

$W_0 - W_h$ , where  $W_0$  lies in the range space of  $C$ , and  $W_h$  lies in its null space. Since  $C^T W_h = 0$ , if  $\widehat{W}$  is to satisfy the constraints  $C^T W = F$  we must have

$$W_0 = C(C^T C)^- F, \quad (6)$$

where  $(\cdot)^-$  denotes the generalized inverse of the matrix. Furthermore,  $W_h$  is a linear combination of the columns of an  $mN \times mN - \text{rank}(C)$  matrix  $C_\perp$  whose columns are orthonormal to  $C$ , i.e.

$$W_h = C_\perp W_\perp. \quad (7)$$

The choices of  $W_0$  and  $C_\perp$  imply that  $\widehat{W}$  satisfies the constraints independent of  $W_h$ . Then, the LCMV is reduced to the unconstrained problem

$$\widehat{W}_{\text{GSC}} = \min_{W_\perp} [W_0 - C_\perp W_\perp]^T R [W_0 - C_\perp W_\perp], \quad (8)$$

where the solution is given by

$$\widehat{W}_{\text{GSC}} = \left[ I - C_\perp (C_\perp^T R C_\perp)^- C_\perp^T R \right] W_0. \quad (9)$$

The general structure of the GSC is shown in Fig. 1. There,  $\mathbf{q}_0 = W_0^T \mathbf{y}$  is the mainbeam response,  $\mathbf{y}_\perp = C_\perp^T \mathbf{y}$  is the auxiliary data, and  $\mathbf{q}_h = W_\perp^T \mathbf{y}_\perp$  is a filtered version of the observed signal, where

$$W_\perp = (C_\perp^T R C_\perp)^- C_\perp^T R W_0. \quad (10)$$

### III.2 Minimum Mean-squared Error

The term  $W_{\perp}$  can be written as

$$W_{\perp} = R_{y_{\perp}}^{-} R_{y_{\perp} q_0}, \quad (11)$$

where  $R_{y_{\perp}} = \text{E}[\mathbf{y}_{\perp} \mathbf{y}_{\perp}^T] = C_{\perp}^T R C_{\perp}$ , and  $R_{y_{\perp} q_0} = \text{E}[\mathbf{y}_{\perp} \mathbf{q}_0^T] = C_{\perp}^T R W_0$ . Equation (11) corresponds to a more general form of the Wiener-Hopf solution and therefore, our filtering problem can be seen as that of minimizing the error between  $\mathbf{q}_0$  and  $\mathbf{q}_h$ , i.e.

$$\begin{aligned} \xi &= \text{E}[|\mathbf{q}_0 - \mathbf{q}_h|^2] \\ &= \text{tr} \left\{ P_0 - R_{y_{\perp} q_0}^T R_{y_{\perp}}^{-} R_{y_{\perp} q_0} \right\}, \end{aligned} \quad (12)$$

where  $\xi$  is the minimum mean-squared error (MMSE) between the mainbeam response and filtered observed signal,  $\text{tr}(\cdot)$  is the trace, and  $P_0 = W_0^T R W_0$  is the matrix whose diagonal elements correspond to the expected power of the mainbeam output for one dipole signal component at one particular time. Note that the value of  $\xi$  will be the same regardless of the generalized inverse selected. For this reason, in our following calculations we focus (without loss of generalization) on the Moore-Penrose generalized inverse, which we denote as  $(\cdot)^+$ . Then, we can rewrite the MMSE in (12) as

$$\xi = \text{tr} \left\{ P_0 - R_{y_{\perp} q_0}^T R_{y_{\perp}}^+ R_{y_{\perp} q_0} \right\}. \quad (13)$$

The eigenvalue decomposition of  $R_{y_{\perp}}^+$  is given by

$$R_{y_{\perp}}^+ = \sum_{i=1}^n \lambda_i^+ \mathbf{v}_i \mathbf{v}_i^T, \quad (14)$$

where  $\lambda_i^+$  is the reciprocal of the  $i$ th nonzero eigenvalue  $\lambda_i$  of  $R_{y_{\perp}}$  in decreasing order, for

$i = 1, 2, \dots, n$ , and  $\mathbf{v}_i$  are the orthonormal eigenvectors of  $R_{y_\perp}$  corresponding to  $\lambda_i$ . Substituting (14) in (13), we have

$$\xi = \text{tr} \left\{ P_0 - \sum_{i=1}^n \lambda_i^+ \mathbf{r}_i \mathbf{r}_i^T \right\}, \quad (15)$$

where

$$\mathbf{r}_i = R_{y_\perp q_0}^T \mathbf{v}_i. \quad (16)$$

The MMSE in (15) represents the best performance of the beamformer of rank  $n$ . However, we can evaluate the MMSE at a reduced-rank  $n_r$  as

$$\xi_{n_r} = \text{tr} \left\{ P_0 - \sum_{j \in J} \lambda_j^+ \mathbf{r}_j \mathbf{r}_j^T \right\}, \quad (17)$$

where  $J$  is a set containing a selection of  $n_r$  values of the index  $i$ . We describe the procedure to select the indexes in  $J$  below.

### III.3 Reduced-rank LCMV beamformers

In this section we describe different beamformers whose rank is reduced by selecting  $n_r$  eigenvalues to approximate  $W_\perp$  as

$$\widetilde{W}_\perp = \sum_{j \in J} \lambda_j^+ \mathbf{v}_j \mathbf{v}_j^T. \quad (18)$$

The reduced-rank beamformers described here differ between each other on the covariance matrix used in each case and the criterion to select the eigenvalues  $\lambda_j$  and corresponding eigenvectors  $\mathbf{v}_j$  in (18):

A. *Principal Components (PC)*: In this case,  $\widetilde{W}_\perp$  is obtained using the  $n_r$  largest eigenvalues, i.e.  $J = \{i \mid i \leq n_r < n\}$ .

B. *Cross-spectral Metrics (CSM)*: Here, the eigenvalues are arranged in decreasing order

according to their CSM values  $\gamma_i$ , which are obtained as [3]

$$\gamma_i = \lambda_i^+ \text{tr} \{ \mathbf{r}_i \mathbf{r}_i^T \}. \quad (19)$$

Therefore,  $J$  contains the  $n_r$  values of  $i$  corresponding to the eigenvalues with largest  $\gamma_i$ , which not necessarily corresponds to the the largest eigenvalues.

C. *Eigencanceler (EIG)*: While the calculations for the case of PC and CSM beamformers depend on  $R$ , the eigencanceler is based on a modified version of the “classical” LCMV solution, and has the following structure

$$\widehat{W}_P = \Pi_R^\perp C (C^T \Pi_R^\perp C)^{-1} F, \quad (20)$$

where  $\Pi_R^\perp$  replaces  $R$  in the classical solution and corresponds to the projection matrix of the received data onto the null space of the covariance matrix. The projection matrix  $\Pi_R^\perp$  that characterizes the eigencanceler [10] is given by

$$\Pi_R^\perp = U_0 U_0^T, \quad (21)$$

where  $U_0$  is the matrix whose columns are the orthonormal eigenvectors of  $R$  that correspond to its zero eigenvalues.

## IV Numerical Examples

We conduct a series of simulations for MEG measurements using a spherical head model in order to evaluate the performance of our reduced-rank beamformers for different rank and SNR

values. In this simulations, we consider that the covariance matrix of the interference-plus-noise is given by the estimate  $\widehat{R}$  defined in equation (3).

We generated MEG data using an array of  $m = 37$  sensors located on a sphere of radius 10.5 cm with a single sensor at the top position and 3 rings at elevation angles of  $\pi/12$ ,  $\pi/6$ , and  $\pi/4$  rad, containing, respectively 6, 12, and 18 sensors equally spaced in the azimuthal direction.

To simulate the sources, we used two dipoles located at  $\mathbf{p}_1 = [-1, 1, 8]^T$  cm and  $\mathbf{p}_2 = [-1, 2.34, 8]^T$  cm. The dipole source components are defined as  $\mathbf{q}_1 = [0.7\rho_1, 0.7\rho_2, 0]^T$  and  $\mathbf{q}_2 = \rho_3[0.7, 0, 0.5]^T$ , where the magnitudes  $\rho_1, \rho_2$ , and  $\rho_3$  are allowed to change in time according to

$$\rho_1(t) = 10e^{-(t-100)^2/11^2} - 5e^{-(t-80)^2/17^2}, \quad (22.a)$$

$$\rho_2(t) = 5e^{-(t-80)^2/8^2} - 10e^{-(t-100)^2/11^2}, \quad (22.b)$$

$$\rho_3(t) = 10 \sin(0.07\pi t), \quad (22.c)$$

all with units of [nA·m], and  $t$  in milliseconds. Similar models has been used in previous research (see e.g. [16]-[17]) as they approximate a typical evoked response. Then, we sampled these signals every 2 ms thus obtaining  $N = 100$  samples for our computer simulations.

To generate the measurements, we used the forward solution of the MEG spherical radial field described in [15]. Then, to approximate realistic spatially correlated noise, we generated 400 random dipoles uniformly distributed on a sphere with radius of 5 cm (for a discusson on random dipole modeling of spontaneous brain activity, see [18]). For each noise dipole, we assumed that its components were uncorrelated and distributed as  $\mathcal{N}(0, \sigma^2)$  with  $\sigma$  ranging from 3.6 to 0.36 nA·m in order to achieve mean SNR values between 0 and 10 dB, respectively. Note that we defined the SNR as the ratio (in dB) of the Frobenious norm of the signal data

matrix to that of the noise matrix. Finally, we repeated this process with independent noise realizations to obtain  $K = 100$  trials.

Under these conditions, we developed a series of numerical examples to evaluate  $\xi_{n_r}$  for different rank and SNR values using the different low-rank beamformers described in Section III.3. The results in terms of the normalized MMSE are shown in Figure 2. These results show that, for high SNR values, the CSM represents the lower bound on the MMSE performance, while the PC stays very close to this bound. For low SNR values, the eigencanceler provided the best performance. Also note that in all cases there was not a significant loss of performance due to using reduced-rank beamformers and, since the difference in MMSE between a rank-one and a rank-twenty beamformer is negligible, we can use reduced-rank beamformers to obtain good source estimates even for low SNR scenarios.

## V Concluding Remarks

We proposed a method to analyze the MSE performance of reduced-rank beamformers for the estimation of dipole source signals using EEG/MEG data. In our derivations, we did not make any assumptions on the rank of the covariance matrix of the interference-plus-noise. Therefore, our results hold for the case when this matrix is low-rank, which is usually true in practice when the number of independent experiments is small, or when only the covariance matrix of the interference (without the noise term) is considered.

Using the structure of the GSC, we derived a general expression for the MMSE as a function of rank. The MMSE is an appropriate measure of performance given that its minimization is equivalent to SINR maximization for the subset of reduced-rank processors.

We presented numerical examples demonstrating the performance of the principal components, cross-spectral metrics, and eigencanceler beamformers. Even though all of them showed similar performance for high SNR, our results showed the reliability of the eigencanceler for

the low SNR case. At high SNR values, the CSM acted as a lower bound on the performance, while the eigencanceler provided the best response at low SNR.

Further research in this area will consider a further generalization to other types of beamformers, different interference conditions, unknown source location, as well as including more extensive applications to real EEG/MEG data.

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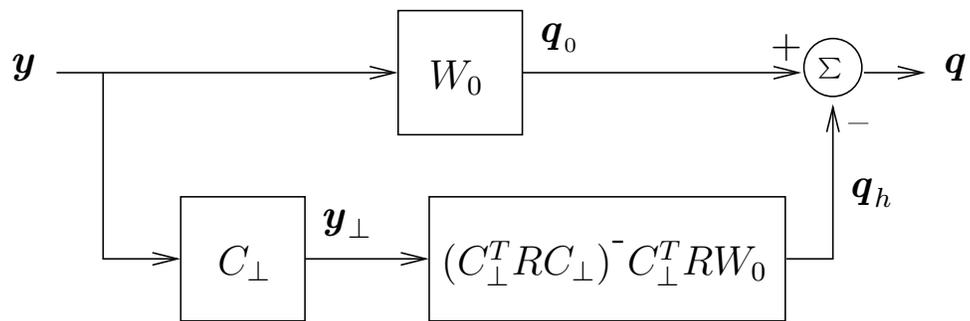


Figure 1.

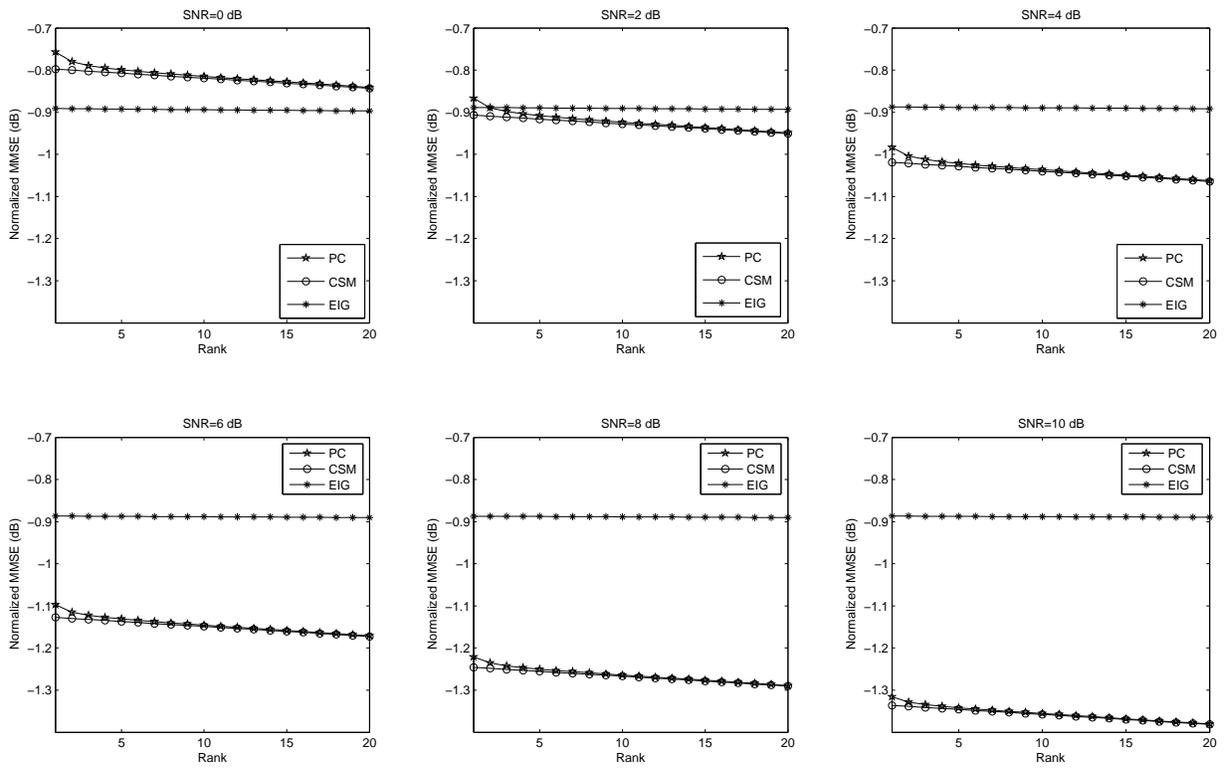


Figure 2.