An examination of monolingual preservice teachers’ set-up of cognitively demanding mathematics tasks with emergent multilingual students

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Abstract

Implementing challenging mathematics tasks with multilingual students who are not yet fluent in the instructional language is difficult for monolingual teachers because of the linguistic and cultural differences between the teacher and students. In this study, we examined how monolingual preservice teachers set up cognitively demanding mathematics tasks with emergent bilingual/multilingual students (a.k.a. English language learners). Drawing on a situated and socio-cultural perspective, we analysed the strategies enacted by two preservice teachers, who consistently maintained the cognitive demand of tasks with emergent bilinguals in a one-on-one setting, during the set-up phase of problem-solving activities. We found common aspects of their set-up including assessing student's holistic understanding, building a common experience, and empowering students. Our findings help articulate strategies that aid monolingual teachers in effectively enacting cognitively demanding tasks and improving emergent bilinguals' access to and engagement with high-quality mathematics.

Keywords

Emergent bilinguals, English language learner, set-up phase, preservice teacher

Disciplines

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Comments

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An examination of monolingual preservice teachers' set-up of cognitively demanding mathematics tasks with emergent multilingual students

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Implementing challenging mathematics tasks with multilingual students who are not yet fluent in the instructional language is difficult for monolingual teachers because of the linguistic and cultural differences between the teacher and students. In this study, we examined how monolingual preservice teachers set up cognitively demanding mathematics tasks with emergent bilingual/multilingual students (a.k.a. English language learners). Drawing on a situated and socio-cultural perspective, we analyzed the strategies enacted by two preservice teachers, who consistently maintained the cognitive demand of tasks with emergent bilinguals in a one-on-one setting, during the set-up phase of problem-solving activities. We found common aspects of their set-up including assessing student's holistic understanding, building a common experience, and empowering students. Our findings help articulate strategies that aid monolingual teachers in effectively enacting cognitively demanding tasks and improving emergent bilinguals' access to and engagement with high-quality mathematics.

Keywords: emergent bilinguals; English language learner; set-up phase; preservice teacher

Introduction

Emergent Bilinguals (EBs; Garcia, Kleifgen, & Falchi, 2008), commonly referred to as English language learners (ELLs) in the U.S., are the fastest-growing student population (Grantmakers for Education, 2013). The percentage of students labeled ELL in U.S. public schools was 9.4, or 4.6 million students in 2014-15 (National Center for Education Statistics, 2017). Although the number of EBs and the racial and ethnic diversity in the U.S. continues to increase, the majority of teachers in the U.S. remains White, middle class, and monolingual women (Sable & Plotts, 2010). Guerra, Castro-Villarreal, Cheatham and Claeys (2014) stated "this mismatch is seen as problematic for the learner and for the teacher as navigating cultural and linguistic diversity is complex" (p. 78). The complexities of the rapid change of student demographic in conjunction with the sustaining White-dominant teacher group suggest the need for preparing teachers to understand how to effectively teach EBs.

In this study, we examined the strategies two White, monolingual preservice teachers (PSTs) enacted with rigorous mathematics tasks for EBs in the U.S. This focus on PSTs is important; a recent review of the literature on the teaching and learning of mathematics with EBs noted the dearth of literature regarding the preparation of PSTs to teach EBs (de Araujo, Roberts, Willey, & Zahner, 2018). Moreover, we focused on PSTs because teacher preparation courses do not tend to include adequate integration of issues pertaining to EBs (Meskill, 2005) and over 30 states in the U.S. did not yet require EB-relevant training for general classroom teachers beyond federal requirements, which state that school districts must provide research-based professional development to any teachers, administrators, and staff who work with ELLs (Education Commission of the State, 2014). To address this increasing need for teacher preparation specific to teaching EBs, there have been recommendations that emphasize the inclusion of EB-focused courses in teacher preparation programs (Vomvoridi-Ivanovic & Chval, 2014).

The present study draws on data from two larger research studies, one with elementary PSTs and the other with middle school mathematics PSTs. In each study the authors investigated how PSTs implemented cognitively demanding mathematical tasks with EBs. We used cognitively demanding mathematics tasks because although there is ample support for the use of such tasks (e.g., National Council of Teachers of Mathematics, 2014), prior studies have suggested that teachers may not provide EBs with opportunities to engage in cognitively demanding tasks (de Araujo, 2017). In examining data, we found that in each of our studies there was one PST that stood out due to her ability to maintain a high-level cognitive demand of the tasks (Stein, Smith, Henningsen, & Silver, 2009). Thus, we set out to explore aspects of these PSTs' practice that might have contributed to their maintaining the cognitive demand of tasks with EBs.

In examining these two PSTs’ task implementations with EBs, we noted differences between them and the other PSTs in the larger studies with regard to the amount of time and interaction they provided during the task set-up (described further below). This initial inquiry led us to examine the ways in which the PSTs set-up tasks due to the importance of the set-up at a high level (Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013; Stein & Lane, 1996). We argue that while working with EBs, the set-up may hold particular relevance as it is the phase of task enactment through which the teacher is working with students to develop shared understandings of and expectations for their work on the task (Campbell, Adams, & Davis, 2007; Jackson et al., 2013). Thus, the research question guiding this study was, *What common instructional strategies were implemented during task set-up by monolingual preservice teachers who maintained the cognitive demand of mathematical tasks with emergent bilingual/multilingual students?* The findings of this study identify strategies that may aid monolingual PSTs in effectively enacting cognitively demanding tasks so as to improve students’ access to and engagement with high-quality mathematics.

Prior Research of Set-up and Emergent Bilinguals

It is well known that when an instructional language is different from a student’s home language, the student may encounter challenges in academic learning (e.g., Lim & Presmeg, 2011; Setati & Adler, 2000). Hansson (2012) argued that the individualization of mathematics education in Sweden was related with low academic performance in language learners. Similarly, I and Chang (2014)’s study with EBs in monolingual teachers’ classes in South Korea revealed EBs did not receive proper accommodations and support to help them access rigorous mathematics or class discussions. Prediger and Wessel (2013) found that immigrant students in Germany encounter difficulty developing conceptual understandings of mathematics due to linguistic challenges. In order to counter these challenges, the authors provided an intervention to treat these challenges concurrently and the intervention yielded promising results. Together, the findings of these studies suggest the need to enact specific strategies to support EBs' in developing deep understandings of and engagement with mathematics.

Researchers have also found that teachers may resist using cognitively demanding tasks with EBs due to the language demand typically associated with such tasks as well as deficit beliefs about EBs (e.g., de Araujo, 2017; Reeves, 2006). Acknowledging EBs' historic lack of access to such tasks has led researchers to recommend the provision of cognitively demanding tasks to EBs in effort to afford them an equal opportunity to learn (Celedon-Pattichis & Ramirez, 2012; Moschkovich, 2010). However, as teachers endeavor to meet these calls, the question of how teachers can effectively implement such tasks at a high level of cognitive demand when teaching EBs remains.

When considering the enactment of tasks, Stein and colleagues discussed three phases through which the implementation occurs: tasks as written, tasks as set-up, and tasks as implemented by students. In terms of the first phase, de Araujo (2017) found teachers chose not to select tasks high in cognitive demand when working with EBs due to the increased language demands and the teachers' deficit views of EBs. However, even when teachers select a cognitively demanding task to use, this does not guarantee that students engage in rigorous mathematical activity in the subsequent phases (Stein, Grover, & Henningsen, 1996). For example, teachers might set-up a challenging task at a lower level of cognitive demand by omitting aspects of the task that request explanations or students may draw on resources during implementation that diminish the cognitive demand (e.g., de Araujo, 2012a, 2012b). Indeed, teachers' difficulties maintaining the cognitive demand throughout all phases of enactment is well documented with students in general (e.g., Stein, Grover, & Henningsen, 1996), and EBs in particular (e.g., de Araujo, 2012a; I, 2015). However, in examining the implementation of tasks, Jackson et al. (2013) determined that teachers' actions during the set-up could profoundly influence whether or not the cognitive demand was maintained.

The task set-up phase, also called the launch, is the process through which a teacher introduces the task, or "the teacher's communication to students regarding what they are expected to do, how they are expected to do it, and with what resources" (Stein et al., 2009, p. 25). Effective instructional practices during the set-up phase may be particularly important for EBs because cognitively demanding tasks tend to contain higher language demands. Zahner, Milbourne and Wynn (2018) developed a framework to characterize the mathematical and linguistic complexity of tasks. They include tasks that request a mathematical explanation or generalization as a factor related with higher mathematical and linguistic complexity. As this factor is related with higher cognitive demand tasks, it stands to reason that tasks higher in cognitive demand tend to carry higher linguistic demand. Relatedly, when tasks are situated in a particular context, that context may serve as a lever for student understanding, if it is relevant to students (e.g., Dominguez, 2011), or as a barrier in instances when the context is not familiar (e.g., Wilburne, Marinak, & Strickland, 2011). Hence, it is essential for teachers to provide EBs with scaffolding in this first phase so they can fully understand the problem statement and expectations before they begin the solving process. Research has suggested that EBs are capable of problem solving if they understand the problem statements (Campbell et al., 2007).

Though they did not study EBs in particular, Jackson et al. (2013) identified high-quality set-ups of cognitively demanding tasks as an important factor as to whether the cognitive demand was maintained during enactment. They identified four aspects of high-quality set-ups: (1) key contextual features of the task scenario are explicitly discussed; (2) key mathematical ideas and relationships are explicitly discussed; (3) common language is developed to describe contextual features, mathematical ideas and relationships, and any vocabulary central to the task statement that might be confusing or unfamiliar to students;

and (4) the cognitive demand of the task is maintained over the course of the set-up (Jackson et al., 2013, p. 652). Although a quality set-up is important for all students, it is essential when working with EBs because EBs may have unique challenges in engaging in the common classroom experience due to their linguistic and cultural differences.

These four aspects of high-quality set-ups should be further examined to understand strategies and modifications effective for EBs because the set-up provides an appropriate venue for the enactment of supports that address possible linguistic and cultural differences. For example, when building common language between a teacher and EBs, it is important to bring a familiar context to students, such as the students' everyday experiences (Domínguez, 2011) and funds of knowledge (Moll, Amanti, Neff, & Gonzalez, 1992). In terms of integrating students' previous experiences, this practice is connected to culturally responsive pedagogy (Villegas & Lucas, 2002), and there is wide agreement that ethnic minority students can benefit from this approach to instruction (Brenner, 1998; Hogg, 2011). Therefore, because the set-up is a span of instruction aimed at developing common understandings, there is a need to better understand how teachers might connect their practice during this phase to extant research related to effective instruction for EBs.

Conceptual Framework

This study draws on a situated-sociocultural perspective (Moschkovich, 2002) of mathematics learning. This perspective recognizes the situated nature of learning and considers the ways in which students use resources from multiple registers (e.g., mathematics and everyday languages) and multiple languages to communicate in mathematics classrooms. Through these lenses, learning mathematics occurs not only with performing mathematics but also through participating in a discursive activity and interactions between a teacher and students. Each student brings his or her multiple perspectives constructed from their previous experiences (Vygotsky, 1978). Thus, the differences among learners can be negotiated and understood through conversations and social interactions. Eventually, a common space (Campbell, 2007) will be constructed where they understand each other while their differences still exist.

Researchers have examined different aspects of teachers' instructional practice in order to learn how to support the mathematical learning of EBs (de Araujo et al., 2018) including providing linguistic support (Echevarria, Vogt, & Short, 2010), applying non-verbal activities (Ahn et al., 2015; Fernandes, 2012; Morales, Khisty, & Chval, 2003), utilizing visuals with mathematical information (I & Stanford, 2018) and integrating students' culture and previous experiences (Aguirre et al., 2013; Lipka, Sharp, Adams, & Sharp, 2007). Campbell et al. (2007) proposed a model specific to the use of problem solving with EBs. This model describes how the interactions between the teacher and students construct the classroom experience, especially when a teacher has a different cultural/linguistic background from her/his students.

In Campbell et al. (2007)'s model, the teacher and students build common understandings of mathematical concepts and language through the interactions based on their common "interpretation of the problem-solving situation" (Lesh & Doerr, 2003, p. 9). This "common interpretation" is crucial when solving mathematical problems because a student is unlikely to engage in solving a problem if she/he does not understand—or have a shared interpretation of—key aspects of the task (Jackson et al., 2013). Jackson et al. (2013) emphasized the importance of developing a common language between teachers and students through the set-up phase and found that effective task set-ups have a positive relationship with high-quality discussion.

The importance of developing common language and experiences as a starting point for instruction may be more crucial when working with EBs because of possible misalignments with teachers' and students' linguistic and cultural resources. To address the importance of building common experiences in a classroom of EBs, Campbell et al. (2007) described a model in which shared classroom experiences are created through the interactions between students and a teacher. In the commonly developed classroom experiences, the classroom community build shared understandings of mathematics, language, and culture so that they can effectively engage in the conversation around the task. We adapted Campbell et al.'s model to align with our research design, a one-on-one setting and problem-solving activity (Figure 1).

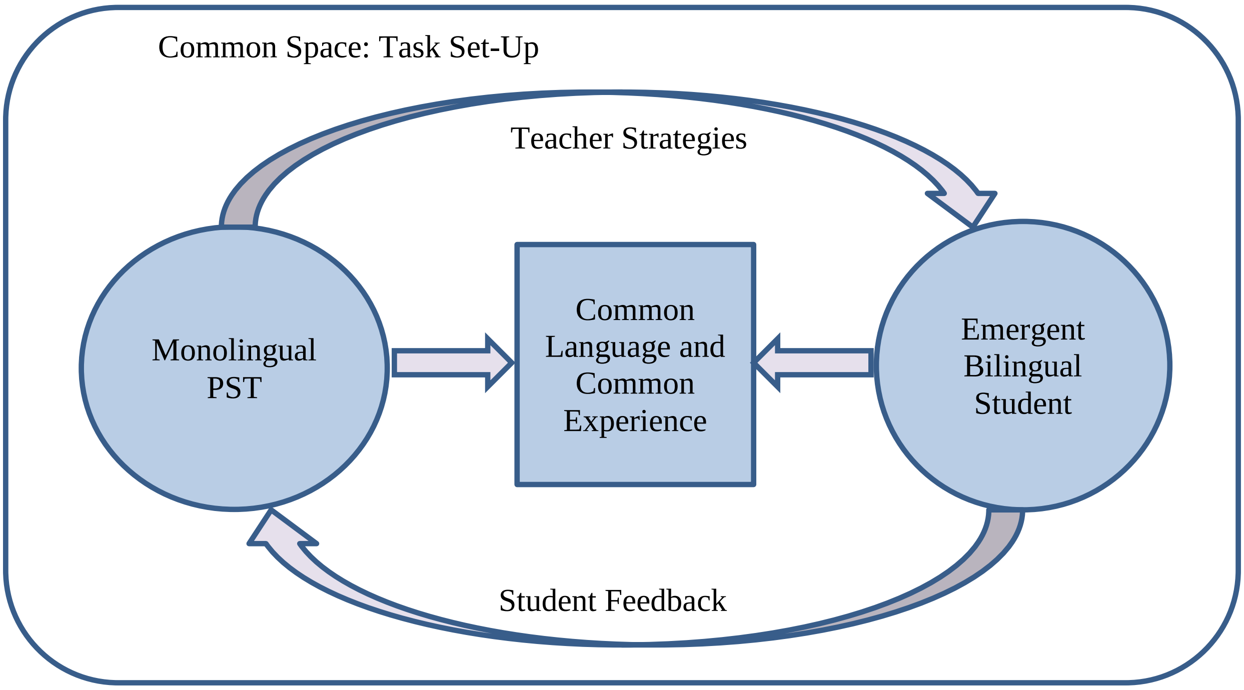


Figure 1. A model of common space between a monolingual teacher and an emergent bilingual student. Adapted from Campbell et al. (2007).

In Figure 1, the two circles represent one monolingual PST and one EB respectively. The curved arrows connecting a PST and an EB represent their interactions during the set-up phase through conversations. For the present study, we were particularly interested in understanding the strategies the PSTs enacted to support the development of a common understanding with the EBs during set-up. The short linear arrows show the input for developing common language and experience from both sides. This input includes the PSTs' and EBs' experiences and knowledge related to language, culture, and mathematics that are influenced through the cycle of interactions developed through the strategies the PSTs employ and the feedback from the EBs. These efforts create a common space that includes both the teacher and the EB. Furthermore, in this common space they can communicate using common languages developed based on common experiences and, consequently, through this communication EBs are empowered as they are positioned as important members who mutually establish the common space with the teacher. Furthermore, EBs' culture and experiences are valued and integrated into this common space as essential components.

Method

In this section, we provide detailed description of the research design, data resources, and data analysis method utilized for this study.

Defining the Set-up Phase

To begin, we decided upon establishing criteria to define the set-up phase so we could determine which data to include. This was important because we found that in one-on-one settings, the separation between phases of enactment were often less clear than in whole-class settings. Thus, it was necessary for us to clearly define what constituted the set-up. Jackson et al. (2013) defined the set-up as the activity before the teacher introduces the main task to students. Stein et al. (1996) defined the set-up as "the task that is announced by the teacher. It can be quite elaborate, including verbal directions, distribution of various materials and tools, and lengthy discussions of what is expected" (p. 460). Considering additional linguistic support for EBs in this study, our definition of the set-up phase was measured from when a pair (of one PST and one EB) began a mathematical conversation (related to the context or mathematics of a given task) to when the student began to solve the given problem independently. This often included an opening conversation a PST initiated related to the task topic and the questions the PSTs used to assess EBs' understanding of words or situations in the problem statement.

Context and Participants

The data used in this study were drawn from two larger research studies. In the studies, we designed a one-on-one setting between a PST and an EB in which each pair worked on a mathematical task for 4 or 5 weeks. The first study, led by Author2, focused on elementary EBs and included four elementary PSTs and four EBs from a local elementary school. In the second study, led by Author1, the focus was on middle school EBs and included four middle school mathematics PSTs and four EBs from a local middle school.

The EBs, who were Korean and Japanese speakers, were purposefully chosen because they were relatively new to the U.S. (less than 2 years) and enrolled in a class for ELLs in their school. We also selected EBs whose mathematical proficiency was at or above grade level by their mathematics class placement to allow the PSTs to focus primarily on strategies to support the EBs' problem solving rather than mathematical remediation. All the PSTs were White women who had no prior teaching experience with EBs and no instruction in EB-specific practices related to mathematics. Each PST had completed one mathematics method course. Among the total of eight PSTs in the larger studies, we purposefully selected one from each, Hannah and Lucy (pseudonyms), for the present study.

Table 1. Detailed information of the selected PSTs and EBs.

PST Name	Grade level for Teaching	Year	Assigned EB
Hannah	Elementary	Senior	Hwa-Young (female, 4th grade, native in Korean, proficient in Japanese, and intermediate in English)
Lucy	Middle	Senior	Si-Young (female, 8th grade, enrolled in Algebra, native in Korean, intermediate in English)

We selected Hannah and Lucy because they consistently maintained the cognitive demand of tasks while working with EBs whereas other PSTs tended to lower the cognitive demand of tasks. For example, the other PSTs would suggest a specific solution pathway as they set-up a task or removed aspects they anticipated would be challenging for the EBs such as to explain their thinking. In contrast, Hannah and Lucy tended to adhere to the task as written and avoiding suggesting particular strategies when working with their EBs. In addition, we considered the amount of time the PSTs' spent on the set-up phase. As shown in Table 2, Hannah and Lucy had a significantly longer set-up time (averaged across all the tasks) than the other PSTs. Given these factors, we thought it important to further understand Hannah and Lucy's work with their EBs, particularly during the set-up phase.

Table 2. The average time of set-up phase of all eight PSTs from two studies

PST	Average set-up time in percent	PST	Average set-up time in percent
Hannah	30%	Lucy	36%
PST A	13%	PST A	19%
PST B	13%	PST B	3%
PST C	9%	PST C	27%

In both studies, the PSTs were given cognitively demanding mathematics tasks to enact with the EBs (see Appendix for the tasks). We selected and adapted the tasks from various sources using the criteria for high cognitive demand tasks provided by Stein et al. (2009). These criteria include aspects such as requiring considerable cognitive effort, non-algorithmic and unpredictable solution pathways, and analysis of task conditions and constraints. Once given the task, the PSTs were encouraged to adapt the tasks to best meet the needs of their students. The weekly routine consisted of the PSTs developing a lesson plan based on a given task, participating in a pre-interview before teaching, teaching an assigned EB using the lesson plan, participating in a post-interview after teaching, and submitting a written reflection about their lesson. Author1's study additionally included a short intervention after the post-interview. The intervention's design was based on teaching strategies advocated for in the book *Beyond Good Teaching* (Celedon-Pattichis & Ramirez, 2012) and seven key strategies to support EBs' mathematical proficiency (Chval & Chavez, 2011, p. 262). The intervention covered four topics: needs of EBs, connecting mathematics to life experience, visual supports, and building rich environments in mathematics and language.

Data sources and analysis

We used data collected from the larger studies related to the set-up phase for the Hannah and Lucy. The main data sources were the video-recorded teaching sessions and the accompanying transcripts. In addition, written lesson plans and video-recorded pre-/post-interviews were included to examine the rationale for using particular strategies during the set-ups. After designating the set-up phase of each session of each PST, each coder read and watched both PSTs' data, including transcripts and lesson plans during the set-up phase. Throughout this process, we coded the instructional strategies found during set-up phases based on the research question and the conceptual framework. Then, we conducted another iteration of coding in which we noted particular strategies the PSTs employed in working to develop common language and/or experiences. After completing this coding phase, both coders discussed the strategies they noted in order to develop common codes for the common strategies and characterizations of those strategies within and then across cases. An excerpt from our coding manual is shown in Table 3.

Table 3. Part of the developed code manual

Code	Description
Definition check	Assess a student’s understanding of mathematical terms or any unknown words and provide linguistic support (e.g., gesture, sound, picture, or verbal explanation) to help a student understand the meaning of words
Check prior knowledge	Assess a student’s prior mathematical knowledge that is required or related to solve the given problem.
Conversation about contextual topic	Assess a student’s previous experience related to the context of the given problem or have a conversation about the student’s experience related to the problem context
Analogy tasks	Provide a simpler task or a similar context to the main problem to help a student build experience of doing a similar reasoning process and understand the context.
Integrate student’s voice	Have a student decide a part of a problem or create their own problem.

Through multiple processes of merging and categorizing codes using our framework with attention to building a common space between a PST and an EB, we found three common themes; (1) assessing student’s understanding in context, language, and mathematics of the task, (2) building a common understanding in context, language and mathematics with the EBs about the task, and (3) empowering EBs through student-centered approaches through the common space they built. After we found these themes, we rearranged the codes and their related strategies under the three themes.

Findings

This section describes our findings within the three themes of practice the PSTs employed during their set-up phases as stated above.

Assessing Students' Understanding

> "Do you know what it means to round a number?" (Hannah, Week 3)
>
> "Can you think of any patterns in everyday life?" (Lucy, Week 5)

The PSTs established a routine aimed at understanding students' prior knowledge and experiences during the set-up. Both Hannah and Lucy assessed not only mathematical aspects, but also linguistic and contextual aspects. Since the entwinement of language and contextual knowledge makes it difficult to understand a task if students are unclear about a particular word or the meaning of the word in the context, related to registers (Halliday, 1978), both PSTs tried to uncover not only meanings of terms with their students but also whether the students had prior experiences related to the term. For example, in a problem about a zoo, some of the other PSTs in the larger project asked students if they knew what a zoo is. This indicated a focus on zoo as a term or language focus. In contrast, Hannah asked Hwa-Young if she had ever been to a zoo which seemed to be connecting to prior experiences with zoos, but may also be her way of seeing the student's understanding of the word *zoo*.

Hannah and Lucy usually assessed students' understanding of task language and context prior to assessing the EBs' understanding of the mathematical aspects with similar strategies. Most often, Hannah and Lucy questioned students if any words were unfamiliar and probed their responses. Similarly, when seeking to understand students' prior experiences, both PSTs asked questions such as, "Have you done this?" In addition, the PSTs routinely asked questions such as, "Do you understand what the question is asking?" in an effort to gauge the students' broader understanding of the task. The following is an example from Hannah (H)'s second meeting with Hwa-Young (Y) that illustrates this pattern.

> H: So, you know how last time we went through it [the task] line by line and I read it and asked, do you understand what's happening. Today, I'm going to let you read it.
> Y: Okay.
> H: You can read it line by line, you can read the whole thing at one time. It's up to you, but I want you to make sure that in your own brain, you know what's happening. Okay?
> Y: Okay. Is it section 2?
> H: Hmmm.
> Y: Section 2. (read the task, had difficulty reading "creature")
> H: It's creature.
> Y: creature.
> H: You know what that is.
> Y: Nope.

In this example, Hannah begins by referring back to how they started the prior session. As Hwa-Young read, Hannah noted any words that seemed unfamiliar to Hwa-Young and were to support her understanding. We see a similar interaction between Lucy (L) and Si-Young (S) in their third meeting.

> L: Have you ever had to split money with people? (S: (nodding)) When have you had to do that?
> S: (um… [long pause]) I had to share money with my sister when dad gives me a money.
> L: Do you get allowance? like a certain amount of money per week?
> S: Yeah.
> L: Yeah. Do you have to do chores for it? No? (S: (nodding)) [Lucy is nodding, too] I hate the chores. I know a lot of times I have to split my money so my friends and I go out to eat, we get one big bill (use gestures to show a big bill) and we have to split it up (use gestures to show splitting) for putting out what we got. It's lots of annoying because it's really tough with money. We're going to work on this problem, we will work with splitting money, splitting stories of apartment, and all of that sorts, so do you want to read this first statement up here?

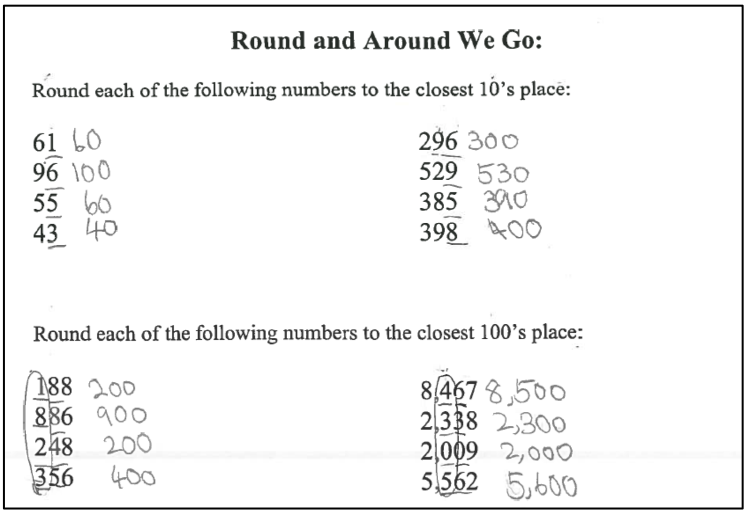
S: (reads) People living in an apartment building decide to buy the building. They will put their money together in such a way that each will pay an amount that is proportional to the size of their apartment.
L: Is there anything in there you don't understand? (S: (shaking her head)) Okay. So, what did it basically say?

As with Hannah's case, we see Lucy checking for understanding throughout this excerpt. Lucy asked questions related to the task context of allowances to assess whether Si-Young had familiarity with the context. Then, Lucy asked Si-Young to read the task and checked for understanding along the way.

Hannah and Lucy sought to understand the students' knowledge about mathematical concepts; however, they tended to do so differently than when assessing understanding of language or context. Rather than simply asking students whether they knew what a particular mathematical term meant, Hannah and Lucy provided skill-based questions related to that mathematical term or concept. The PSTs used supplemental activities as a means of assessing the students' fluency with the key mathematical ideas. Consider, for example, the following excerpt from Hannah's third meeting with Hwa-Young.

H: Do you know what it means round number?
Y: No.
H: No, so have you been taught any sort of, when you have 5, you jump up 1, or when you have 4, you jump down 1, you haven't done anything like that?
Y: Ummm, like?
H: like if I give you, this worksheet (Figure 2), and tell you look at this number, and round it to the nearest tens place, that, would you write that for me?

Figure 2. Excerpt from Hannah's rounding worksheet.



Round and Around We Go:

Round each of the following numbers to the closest 10's place:

61	60	296	300
96	100	529	530
55	60	385	390
43	40	398	400

Round each of the following numbers to the closest 100's place:

188	200	8,467	8,500
886	900	2,338	2,300
248	200	2,009	2,000
356	400	5,562	5,600

In this excerpt Hannah asks whether Hwa-Young knows "what it means to round a number." After Hwa-Young states she is not familiar with rounding, Hannah pulls out the supplemental rounding activity she had prepared. Similarly, when working on the problem of perimeter and area of a house floorplan, Lucy designed a pre-activity to assess Si-Young's understanding of required mathematical concepts and terms, such as dimension, perimeter, and area. In her lesson plan, Lucy wrote, "We will review area and perimeter of rectangles. I will give her a couple of shapes and we will calculate the area and perimeter." The following excerpt shows how Lucy enacted her plan of reviewing the mathematical concepts for Si-Young.

> L: Now we can talk about dimension. Do you know what dimensions are? (S: (shaking her head)) So, by measurement, so my whole apartment, let's say thirty feet long (draws a line and writes 30 ft on the line) then I would say my room is about ten feet (writes 10 ft) so it's all about scaling and things like that. Do you want to put the dimensions inside your room? I would say mine is ten by ten.
>
> S: (writes dimensions on her drawing, see Figure 3)
>
> L: Good. Was it hard to do?
>
> S: No.
>
> L: No. Okay. So, do you know perimeter and area? (S: (nodding)) Do you want to figure these two things out? What's the perimeter?
>
> S: Um... this? (points the surrounding of shape)

L: What's this?

S: Perimeter?

L: Good. Okay. Can you explain it in another way?

S: No.

L: No? Do you know what length is? What would be the length?

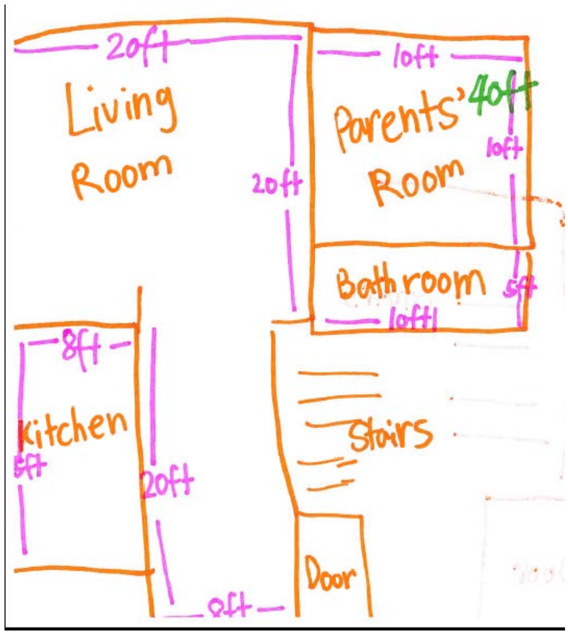
S: Ten?

L: So, the length would be here and here. What do you think this would be called?

S: Y, wi, width?

L: Good. So, what would the perimeter be?

Figure 3. Si-Young's drawing of the floorplan of her house.



Before this excerpt, Lucy asked Si-Young to draw a floorplan of her house and Lucy also drew the floorplan of her apartment. In the beginning of this excerpt, Lucy asks Si-Young if she knows what dimensions are prior to providing her with explanation. Moreover, when Si-Young responds she knows about perimeter and area, Lucy does not stop asking but keeps assessing Si-Young's mathematical knowledge using her drawing. This practice of using supplemental items to assess the EBs' mathematical understandings was enacted multiple times by each PST.

It is notable that Hannah and Lucy did not provide the EBs with support prior to assessing the students' understandings. Though each PST had prepared for possible supports ahead of time, these supports were not enacted until the need was evident to the PST. Rather, they sought to understand students' needs in terms of language, context, and mathematics and then provided supports for those particular aspects. In the following section, we further examine the ways in which the PSTs worked to build a common understanding of language, context, and mathematics with the students after this assessing practice.

Build a common understanding

> "Creature...so monsters, aliens, bugs, animals, they are all considered creatures" (Hannah, Week 2)

> "I'm going to draw a picture of my house I live in and why don't you do the same so we can kind of compare them, okay?" (Lucy, Week 4)

Hannah and Lucy both pre-emptively and reactively worked to build a common understanding with students. With regard to pre-emptive strategies, both PSTs talked with the students in an effort to understand their cultural resources and used that knowledge to make tasks more relevant to the students. For example, Hannah researched common Korean names and Japanese desserts and used what she learned to create a task for Hwa-Young who was Korean but had also lived in Japan. Strategies such as this task adaptation were employed prior to the set-up phase and so were not the focus of the present study. Instead, we focused our examination on the reactive strategies that the PSTs enacted after assessing the students' understanding.

As discussed in the prior section, both PSTs assessed the students' understanding of the tasks during the set-up phase. When, through this process, they found the EBs were unfamiliar with some aspects of a task, they worked with the students to build a common understanding of those aspects. The PSTs used two broad strategies to help develop a common understanding with students, verbal explanations and supplemental tasks. The use of these strategies depended upon whether the misunderstood aspect was perceived as stemming from language/culture or mathematics.

For example, when Hannah and Lucy found the students were unfamiliar with a word or context, they engaged in a discussion with them to help build an understanding of that aspect. These discussions typically centred on the use of synonyms or on attempts to relate the word to students' prior knowledge or experiences. Consider the following episode evidencing what Hannah did after she found out Hwa-Young did not know the word *creature*.

> H: Okay. So, a creature is a broad term that covers multiple animals, so it could be, do you know what a monster is?
> Y: Yes.
> H: Do you know what an alien is?
> Y: alien?
> H: Like, UFO, like Ti-Yoong Ti-Yoong (make a sound of UFO and gesture it flying).
> Y: Oh, oh, I know.
> H: Okay, so monsters, aliens, bugs, animals, they are all considered creatures.
> Y: Oh.
> H: So, today, it's going to be the space creatures, so it's going to be aliens. That would say.
> Y: Okay. Aliens.

In this short moment, Hannah explains the unfamiliar word, creature, by providing the meaning of creature, using synonyms, and giving an example with sounds and gestures. Through this support, Hannah and Hwa-Young could have a common interpretation of the word. Similarly, Lucy built a common language with Si-Young as the following dialogue shows.

L: Have you ever lived another building with someone else, or how to share rooms? (S: (nodding)) Do you know what this is? (points a picture of building)

S: Building?

L: Can you think about what kind of building it is or…

S: Apartment?

L: Apartment, could be anything else?

S: Um... villas?

L: Yeah. It could be a villa. We will say it as an apartment, okay?

Before giving the task about apartment, Lucy checked what word Si-Young would use for the picture in the task and made agreement to use a common word so Si-Young could avoid a possible confusion when she read the task.

The second strategy the PSTs reactively applied was to use supplemental materials in which the EBs could build or connect their experiences related to mathematics. As described previously, Hannah prepared a worksheet of rounding numbers to help determine whether Hwa-Young understood rounding (see Figure 2) and had Hwa-Young complete the worksheet when she found Hwa-Young was unsure of the concept. She later found that Hwa-Young had some familiarity with rounding, but through the completion of this supplemental task they came to a shared language related to rounded such as we “round to the hundreds place.” Lucy prepared similar supplemental tasks weekly. Her tasks were similar to the main task in terms of mathematical content or context. Sometimes, the analogical tasks had a very similar structure to the main task and as a result, it reduced the cognitive demand of the main task. At other times, her supplemental tasks served to bridge the student’s life experience and the mathematical concept embedded in the main task, such as with the floor plan task previously described (Figure 3).

Empower students

“...now you’re in charge” (Hannah, Week 1)

"We work together" (Lucy, Week 1)

In examining the PSTs' strategies during their set-ups, we found they employed several approaches that empowered the EBs as competent mathematical collaborators who can build a common language and experience together. Though the PSTs did not mention these strategies on their lesson plan or in their interviews, our analysis of their sessions evidenced the PSTs' inclusion of the students in various decision-making processes. For example, Hannah usually let Hwa-Young have her choice of tools (e.g., drawing, manipulatives) to solve a task, and Lucy often asked Si-Young to participate in designing a task as the following excerpt describes.

> L: I'm going to make a little track. I'll put a little *a* (writes *a* on a post-it and attach it on the desk) So that's where we start. Say, where you want to start, like school or home?
> S: Home.
> L: Home? Okay. We start point *a* and going to point *b*. So, where do you want to end?
> S: Um. A mall?
> L: Mall? Okay. Good idea. Do you like to go to a mall?
> S: Yes.
> L: What's your favourite store?
> S: Um... Pink?
> L: Pink? I like that too. You've decided a point *a* is going to be your home and you're going to the mall to go to the Pink, right?

The above excerpt occurred during the second week after a conversation about transportation and speed limits. In this excerpt, Lucy asked Si-Young to decide the starting point and destination used in the task context. Through this action, Si-Young actively contributed to the design of the contextual features of the task.

Hannah and Lucy regularly positioned the students as knowledgeable others in many other interactions. This was evident in their first week, as the quotes at the start of this section illustrate, in which each PST framed the session in such a way as to empower the students as learners with knowledge and authority. Hannah specifically told Hwa-Young “You’re in charge” suggesting she was responsible for leading the thinking. Lucy noted that they would “work together” implying that it was a cooperative effort in which each of them would contribute to the solution process. In subsequent weeks, we see other examples of these types of statements from both Lucy and Hannah as below.

> H: All right, still I have manipulatives here. If you feel free and you need to use them, I have different pieces of paper. It’s totally up to you how you would like to start.
> Y: Okay… I’m going to use paper.
> H: All right.

In this short dialogue, Hannah explains Hwa-Young has the right to choose her tool for solving the task. Her saying, “It’s totally up to you” emphasizes Hwa-Young has an ownership of the solving process. Similarly, Lucy invites Si-Young to choose her tool to create her own patterns in the following excerpt.

> L: Let’s write down some of our own patterns. We can write out one pattern on here of a bunch of things or put post-its with different patterns. Or we can use the blocks or circles to make our own patterns. So how will you do it?
> S: Use these (points blocks).
> L: Which one do you want to use?
> S: This one (blocks).
> L: Okay. When you create one of your own patterns, you can go by number of blocks or colours of blocks, however you want to do.
> (both move blocks and make patterns).

As Hannah did, Lucy provided several tools from which Si-Young could choose as they both worked on the task. The PSTs explicitly positioned the students during these experiences and also gave students ample choices in approaches and resources that further evidenced their ability to empower students.

Together, the PSTs' provision of choices to allow students to draw on their resources in conjunction with their explicit positioning of students, empowered the EBs as competent mathematical collaborators. It is important to note that this empowerment appeared from the set-up and continued through the whole session; the enactment of these strategies during the set-up established the stage for the EBs to engage in the subsequent mathematical activity.

Discussion and Implication

As described in the previous section, we found that the strategies implemented during the set-up by the PSTs who maintained the task's cognitive demand included: assessing EBs' understanding, building a common understanding with the EBs, and empowering EBs. It should be noted that the term, *understanding*, is used holistically rather than within a specific area, such as mathematics, context, or language. For instance, students may know every word in a problem but still struggle to understand its meaning due to cultural conflicts or mathematical complexities. It is unknown if the PSTs were aware of the holistic aspect, but they consistently assessed students' understanding of language, culture/context, mathematical situations embedded in each task, which intertwined with each other. We believe this is one reason why they could maintain the cognitive demand as Domínguez (2011) found "Students showed that their capacity to make sense of problems as measured by reinvention actions (meaning-making actions that contribute to mathematize an unmathematized situation) is enhanced when the problems to be solved include familiar experiences" (p. 324). It is difficult to distinguish an understanding of mathematics from an understanding of language or vice versa. This is a different view from Jackson et al. (2013) because Jackson and colleagues discussed the key features of mathematics and context separately. The results of Prediger and Wessel (2013) support that the holistic view is effective to EBs because in their study the immigrant students in Germany showed a significant growth in conceptual understanding in mathematics through an intervention that related verbal representations to mathematical concepts.

In comparing the three themes we found to the four aspects of high-quality set-up from Jackson et al. (2013), we note that both emphasize the common language of context and mathematical ideas. However, in our findings, empowering students, a notion not discussed in Jackson et al., had a crucial role in maintaining the cognitive demand and building a common space between a monolingual PST and an EB. This is related to the reason why our conceptual framework includes not only teachers' set-up strategies but also students' input as a required process to build a common space.

Moreover, it is important to distinguish the practice of *assessing students' understandings* from that of *building a common understanding*. The process of assessing students' understandings and prior knowledge is necessary if a teacher is to use that knowledge for the subsequent process of building a common understanding between her/him and her/his students. The core component of the set-up is to interactively build a common understanding and common language between the PST and her EB through multimodal explanation and supplemental tasks (Jackson et al., 2013). This common understanding and language then serves as a shared communicative space (Turner, Dominguez, Empson, & Maldonado, 2013). The third theme, empowering students, was a vehicle that enabled the EBs to provide their input, strategies, and feedback, so the PSTs could integrate those aspects in building common understanding. It also served to support student-driven solution pathways and aided in maintaining the cognitive demand.

Several limitations exist in this study. First, designating the end of set-up phase was sometimes unclear because the PSTs interacted with the EBs very closely and as a result, there were some portions in which the set-up and the implementation phases overlapped. Furthermore, we have only two cases, both of which were female-female pairs while all other participants in the larger projects were female-male pairs. The gender difference may have affected the interactions between the PST and EBs and/or helped them build a common understanding. However, a deeper examination of possible gender implications is outside the scope of this study. Lastly, the model minority myth involving Asian students (Hartlep, 2013) might have had some influence on the PSTs' perspectives and practices to some degree, further examination of that influence could be the subject of further study.

Our findings carry some important implications for classroom teaching, particularly when a monolingual teacher works with EBs on cognitively demanding mathematics tasks. First and most importantly, well-designed set-ups must be enacted with students. It is necessary to invest enough time for assessing student's understanding of language, context, mathematics, and the task itself. Before students solve a task, teachers need to build a common understanding of the task with students. This process should not begin by assumptions about what the students do or don't know, rather it should begin by engaging EBs in sharing their experiences and knowledge related to the task's features. This recommendation applies to not only the countries with one instructional language like the U.S., South Korea (I & Chang, 2014), or Sweden (Hansson, 2012), but also to countries with multiple instructional languages such as South Africa (Setati & Adler, 2000) and Malaysia (Lim & Presmeg, 2011). In methods courses for future teachers, teacher educators can provide explicit instruction related to setting up tasks using the instructional strategies we discuss in this study. Along with selecting and implementing high-quality, rigorous tasks, *setting up* tasks should be given sufficient attention. While focus on a high-quality set-up may benefit all students, we believe it is crucial for EBs because they need the opportunity to develop a common space with monolingual teachers who do not share the dominant culture and language with them (Campbell et al., 2007; Moschkovich, 2010).

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Appendix

Tasks Used in Two Studies

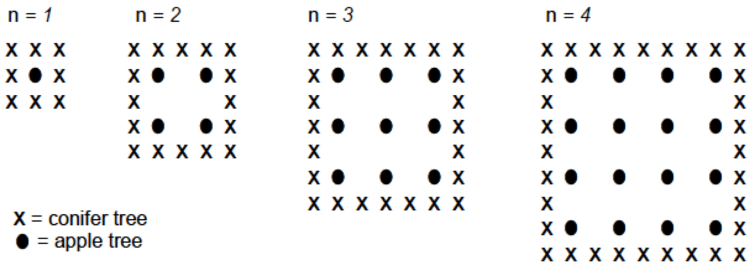
1. Elementary School

Week	Task as Written	Cognitive Demand level (Stein et al. 2009)
1	Three classes at Paxton Keeley Elementary School are going on a fieldtrip to the zoo. Mrs. Ruiz's class has 23 people, Mr. Yang's class has 25, and Mrs. Evans' class has 24 people (all numbers include the teacher). They can choose to use buses, vans, and/or cars. Buses have 20 seats, vans have 16 seats, and cars have 5 seats. You are in charge of deciding how to transport all of the classes to the zoo. Explain how you would choose how many of each type of vehicle to take and why. Write a response and explain your thinking. Extension: 1. If there cannot be any empty seats in a vehicle, how would you choose the vehicles? Explain your strategy. 2. If you can only take less than five vehicles, how many different ways can you choose them? Explain your strategy. (Adapted from PARCC)	Procedures with connections
2	The two-eyed space creatures, three-eyed space creatures, and four-eyed space creatures are having a contest to create a group with 24 total eyes. 1. If you have to include only two-eyed space creatures and three-eyed space creatures, how many of each kind are needed to make a group with 24 total eyes? If it is possible, list all possible combinations and explain your strategy. If it is impossible, explain why. 2. If you have to include only three-eyed space creatures and four-eyed space creatures, how many of each kind are needed to make a group with 24 total eyes? If it is possible, list all possible combinations and explain your strategy. If it is impossible, explain why. 3. If you have to include at least one space creature from each kind, how many space creatures of each kind are needed to make a group with 24 total eyes? If it is possible, list all possible combinations and explain your strategy. If it is impossible, explain why. (Adapted from Smarter Balanced)	Procedures with connections
3	Baseball stadiums have different numbers of seats. Giants' stadium in San Francisco has 41,915 seats and Nationals' stadium in Washington has 41,888 seats. Padres' stadium in San Diego has 42,445 seats. Compare these statements from two students. • Jeff said, " I get the same number when I round all three numbers of seats in these stadiums." • Sara said, "When I round them, I get the same number for two of the stadiums but a different number for the other stadium."	Procedures with connections

	Can Jeff and Sara both be correct? Explain how you know. Extension: Round the three numbers of seats in your own way and build a statement about those rounded numbers as Jeff and Sara did. Compare all three statements and decide which statement best describes the numbers of seats of three stadiums. Explain why you chose the statement. (Adapted from PARCC)	

2. Middle School

Week	Task	Cognitive Demand Level
1	Claire is filling bags with sand. All the bags are the same size. Each bag must weigh less than 50 pounds. One sand bag weighs 58 pounds, another sand bag weighs 41 pounds, and another sand bag weighs 53 pounds. Explain whether Claire can pour sand between sand bags so that the weight of each bag is less than 50 pounds. (Adapted from Smarter Balanced)	Procedures with connections
2	Anne's family is driving to her uncle's house. The family travels 383.5 miles between 10:15 a.m. and 4:45 p.m. (1) Calculate the family's average rate of travel for the day. (2) Anne tells her family, "It's a good thing we traveled as fast as we did. If our rate had been 50 miles per hour, we wouldn't have gotten to his house until about …" Complete her statement. (Adapted from PARCC)	Procedures with connections
3	People living in an apartment building decide to buy the building. They will put their money together in such a way that each will pay an amount that is proportional to the size of their apartment. For example, a man living in an apartment that occupies one fifth of the floor area of all apartments will pay one fifth of the total price of the building. Read the following statements and explain if each statement is correct or incorrect. Write your reasoning. 1. A person living in the largest apartment will pay more money for each square meter of his apartment than the person living in the smallest apartment. 2. If we know the areas of two apartments and the price of one of them we can calculate the price of the second. 3. If we know the price of the building and how much each owner will pay, then the total area of all apartments can be calculated. 4. If the total price of the building were reduced by 10%, each of the owners would pay 10% less. (Adapted from PISA)	Procedures with connections
4	The Morrisons are going to build a new one-story house. The floor of the house will be rectangular with a length of 30 feet and a width of 20 feet. The house will have a living room, a kitchen, two bedrooms, and a bathroom. Create a floor plan that shows these five rooms by dividing the rectangle into rooms. Your floor plan should meet the following conditions.	Doing mathematics

	1) Each one of the five rooms must share at least one side with the rectangle, that is, each room must have at least one outside wall. 2) The floor area of the bathroom should be 50 square feet. 3) Each of the other four rooms (not the bathroom) should have a length of at least 10 feet and a width of at least 10 feet. Be sure to label each room by name (living room, kitchen, bedroom, etc.) and include its length and width, in feet. Draw your floor plan. Remember to label your rooms by name and include the length and width, in feet, for each room. (Adapted from NAEP)	
5	A farmer plants apple trees in a square pattern. In order to protect the apple trees against the wind he plants conifer trees all around the orchard. Here you see a diagram of this situation where you can see the pattern of apple trees and conifer trees for any number (*n*) of rows of apple trees:  (1) Find patterns of the number of apple trees and the number of conifer trees and express them in two equations in terms of *n*. (2) Suppose the farmer wants to make a much larger orchard with many rows of trees. As the farmer makes the orchard bigger, which will increase more quickly: the number of apple trees or the number of conifer trees? Explain how you found your answer. (Adapted from PISA)	Doing mathematics