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Gas Holdup in Opaque Cellulose Fiber Slurries

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Abstract

Three different cellulose fiber types are used to study their effect on gas holdup and flow regime transition in a 10.2 cm semi-batch bubble column. The three natural fiber types include bleached softwood chemical pulp (softwood), bleached hardwood chemical pulp (hardwood), and bleached softwood chemithermomechanical pulp (BCTMP). Gas holdup is recorded over a range of fiber mass fractions ($0 \leq C \leq 1.6\%$) and superficial gas velocities ($U_g \leq 23$ cm/s). Experimental results show that gas holdup decreases with increasing fiber mass fraction. Homogeneous, transitional, and heterogeneous flow is observed for all three fiber types at low fiber mass fractions. All three fiber types produce similar results in the homogeneous flow regime while significant differences are recorded in the heterogeneous flow regime; those being low mass fraction hardwood (softwood) fiber slurries produce the highest (lowest) gas holdup. At higher fiber mass fractions, only pure heterogeneous flow is observed and softwood fiber slurries still produce the lowest gas holdup, although the differences in gas holdup between fiber types are small. The Zuber-Findlay drift flux model is used to describe the gas holdup results in cellulose fiber slurries when the flow conditions are heterogeneous. The Zuber-Findlay drift flux model is also used to identify the superficial gas velocity at which homogeneous flow is no longer observed with some success. Generally, the superficial gas velocity at which the flow deviates from homogeneous flow decreases with increasing fiber mass fraction.

Keywords

bubble columns, fiber slurry, flow regime, gas holdup, hydrodynamics, slurry bubble column

Disciplines

Acoustics, Dynamics, and Controls | Complex Fluids | Fluid Dynamics

Comments

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GAS HOLDUP IN OPAQUE CELLULOSE FIBER SLURRIES

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ABSTRACT

Three different cellulose fiber types are used to study their effect on gas holdup and flow regime transition in a 10.2 cm semi-batch bubble column. The three natural fiber types include bleached softwood chemical pulp (softwood), bleached hardwood chemical pulp (hardwood), and bleached softwood chemithermomechanical pulp (BCTMP). Gas holdup is recorded over a range of fiber mass fractions ($0 \leq C \leq 1.6\%$) and superficial gas velocities ($U_g \leq 23$ cm/s).

Experimental results show that gas holdup decreases with increasing fiber mass fraction. Homogeneous, transitional, and heterogeneous flow is observed for all three fiber types at low fiber mass fractions. All three fiber types produce similar results in the homogeneous flow regime while significant differences are recorded in the heterogeneous flow regime; those being low mass fraction hardwood (softwood) fiber slurries produce the highest (lowest) gas holdup. At higher fiber mass fractions, only pure heterogeneous flow is observed and softwood fiber slurries still produce the lowest gas holdup, although the differences in gas holdup between fiber types are small. The Zuber-Findlay drift flux model is used to describe the gas holdup results in cellulose fiber slurries when the flow conditions are heterogeneous. The Zuber-Findlay drift flux model is also used to identify the superficial gas velocity at which homogeneous flow is no longer observed with some success. Generally, the superficial gas velocity at which the flow deviates from homogeneous flow decreases with increasing fiber mass fraction.

Keywords: Bubble column; Fiber slurry; Flow regime; Gas holdup; Hydrodynamics; Slurry bubble column

NOMENCLATURE

C	fiber mass fraction
C_o	drift flux distribution parameter
GLF	gas-liquid-fiber
GLS	gas-liquid-solid
M_f	fiber mass
M_t	total slurry mass
ΔP	pressure drop with gas flow
ΔP_o	pressure drop without gas flow
U_g	superficial gas velocity
U_∞	drift flux velocity
V	slurry volume

Symbols

ε	gas holdup
ρ_{eff}	effective slurry density
ρ_f	fiber density
ρ_w	water density

INTRODUCTION

Gas-liquid-fiber (GLF) systems are found in the pulp and paper industry in a variety of unit operations including flotation deinking, direct-contact steam heating, gaseous fiber bleaching, and papermaking [1]. GLF systems differ from gas-liquid-solid (GLS) systems only in the fact that the solid material is some type of fiber (i.e., the solid has a very large aspect ratio). For cellulose fiber slurries, the fibers have a density close to that of water and they form flocs at fiber mass fractions as low as 0.3% by weight, and continuous fiber networks at mass fractions greater than 1% [2]. When a gas is introduced into the fiber slurry, fiber network formation and flocculation can trap bubbles, preventing their rise to the surface. Individual bubbles must either bypass the flocs or coalesce with other bubbles to form a resultant bubble with a sufficient buoyant force to break through the fiber network [3]. If the fiber mass fraction is too

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high, preferential bubble rise paths may form in the fiber slurry where locally high fiber mass fraction regions (i.e., high floc concentration) prevent bubble ascension, and most bubbles are diverted to rise in locally low mass fraction regions. This process is typically referred to as channeling and is detrimental to any process where uniform transport characteristics are required for optimal performance.

Initial attempts to characterize gas flows in fiber slurry bubble columns were conducted by Walmsley [4]. Two cellulose fiber types were used in that study and both decreased the gas holdup when the mass fraction was greater than 0.6%. Walmsley concluded that the decrease in gas holdup implied an increase in bubble coalescence and/or channeling, which led to a reduction in the overall air-liquid interfacial area. Similar results were reported by Went et al. [5]. They further reported that when the fiber mass fraction was greater than ~1%, some of the fibers agglomerated into a high mass fraction region on the column bottom, lowering the fiber mass fraction in the upper column region. As a result, gas holdup decreased with increasing fiber mass fraction until $C \approx 1\%$, where a relatively constant gas holdup was reported for higher mass fractions.

Several investigators have studied gas flow regimes in GLF bubble columns. Homogeneous, transitional, and heterogeneous flow are typically observed in low fiber mass fractions systems, while pure heterogeneous flow is recorded in higher mass fraction systems [4, 6, 7]. Additional flow regime descriptors have been identified and depend on the specific system operating conditions [8-11].

This paper will report gas holdup results for three different GLF systems in a 10.2 cm semi-batch bubble column. Gas holdup is recorded over a range of fiber mass fractions ($0 \leq C \leq 1.6\%$) and superficial gas velocities ($U_g \leq 23$ cm/s).

EXPERIMENTAL PROCEDURES

Figure 1 shows a schematic representation of the semi-batch bubble column utilized in this study. The bubble column consists of two sections of a 10.2 cm inner diameter clear PVC cylinder with a total height of 233.7 cm. Air is injected into the system through a 1.3 cm thick acrylic aeration plate with 223 uniformly distributed 1 mm diameter holes, providing an open area of 2.22%. A gas plenum located below the aeration plate is filled with glass beads to promote uniform gas distribution into the bubble column. Two pressure transducers are located 91.4 cm apart along the column. Two mass flow meters are used to measure the gas flow rate in low and high ranges up to 100 L/min. The pressure transducers and mass flow meters are connected to a LabVIEW-based data acquisition system.

The GLF system is composed of air, water, and one of three cellulose fiber types; they include bleached softwood chemical pulp (softwood), bleached hardwood chemical pulp (hardwood), and bleached softwood chemithermomechanical pulp (BCTMP). Table 1 summarizes the relevant fiber characteristics. Softwood and BCTMP fiber utilize different processes to transform the tree to suitable papermaking fibers, where a significant difference is that BCTMP fibers contain lignin and softwood fibers do not [12].

Gas holdup (ε) is recorded in the GLF systems for various fiber mass fractions ($0 \leq C \leq 1.6\%$) over a range of superficial gas velocities ($U_g \leq 23$ cm/s), and is determined from the

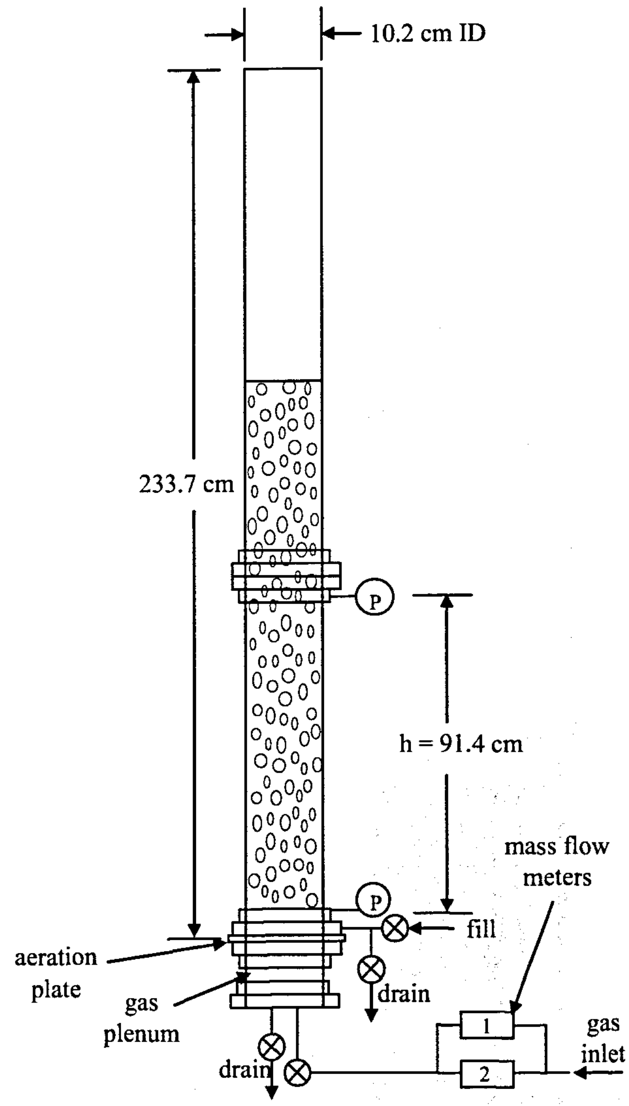

Figure 1. Semi-batch bubble column experimental facility.

Table 1. Properties of the cellulose fibers used in this study.

Property	Hardwood	Softwood	BCTMP
Wood Species	Eucalyptus	65-75% Northern Black Spruce, 20-25% Jackpine, 5-10% Balsam Fir	Northern Pine
Particle Average Fiber Length (mm)	0.69	1.2	0.8
Length-Weighted Average Fiber Length (mm)	0.78	2.31	1.91
Coarseness (mg/100m)	6.9	13.1	29.5
Number of fibers per gram (millions)	21.4	6.37	4.25

column pressure drop. In a semi-batch system, the frictional pressure drop is negligible, so the total pressure drop corresponds to the hydrostatic head; in this case,

$$\varepsilon = 1 - \frac{\Delta P}{\Delta P_o} \quad (1)$$

where ΔP is the pressure drop between the two pressure transducers with $U_g > 0$, and ΔP_o is the corresponding pressure drop with $U_g = 0$. For a gas-liquid system, ΔP_o equals the liquid hydrostatic head; for the GLF systems, ΔP_o corresponds to the fiber slurry hydrostatic head.

Experiments are performed at specified fiber mass fractions (C), where the actual fiber mass added to the system is determined from

$$M_f = CM_t \tag{2}$$

The total mass of the fiber-water mixture M_t is determined from $M_t = \rho_{eff}V$, where ρ_{eff} is the effective slurry density determined from

$$\frac{1}{\rho_{eff}} = \frac{C}{\rho_f} + \frac{1-C}{\rho_w} \tag{3}$$

The moisture-free cellulose fiber density is $\rho_f = 1500$ kg/m^3 and V is the volume of the fiber-water mixture.

All fibers are disintegrated from dry lap fiber sheets. The fiber sheets are originally torn into small pieces and then a specified mass of oven-dry fiber is weighed; they are then soaked in tap water for 24 hours before the pieces of fiber sheet are disintegrated in a British Disintegrator. The concentrated fiber suspension is then transferred to the bubble column and additional tap water is added to raise the suspension to a bubble column height of 1.22 m (12 column diameters). The column is then operated for approximately 20 minutes at a high gas flow rate to ensure the slurry is well mixed. The gas flow rate is then adjusted to the lowest value of interest to begin data acquisition and then incremented sequentially every five minutes to collect additional data points.

RESULTS AND DISCUSSION

Air-Water

Base-line gas holdup measurements were first completed in an air-water system. As shown in Fig. 2 for C = 0%, gas holdup is linearly dependent on superficial gas velocity at low gas flow rates; this is an indication of homogeneous flow [13-15]. Visual observations indicate bubbles at low superficial gas velocities are relative small, uniformly dispersed, and rise in an almost vertical fashion, which are other characteristics of homogeneous flow conditions.

As the superficial gas velocity is increased, gas holdup deviates from a linear variation, but continues to increase, marking the onset of the transitional flow regime. Visually, bubbles begin to move laterally resulting in bubble-bubble interactions and the development of a serpentine flow pattern.

Further increases in superficial gas velocity result in a continued increase in gas holdup, bubble coalescence and breakup, and heterogeneous flow conditions. Some investigators indicate that a local maximum in gas holdup may be observed in the transitional flow regime [7, 15], but recent results indicate this is valid when the aeration plate open area is less than 1% [16]. Since the aeration plate open area in this study is 2.22%, a local gas holdup maximum is not observed.

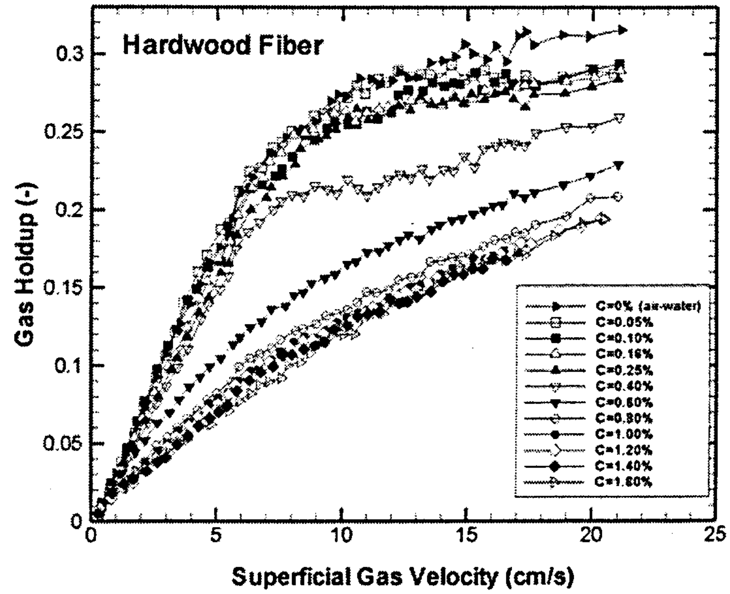

Figure 2. Gas holdup as a function of superficial gas velocity for hardwood fiber slurries.

Cellulose Fiber

Hardwood Fiber

When hardwood fiber is added to the bubble column (Fig. 2), the bubble column hydrodynamics are visually similar to those of the air-water system when the hardwood fiber mass fraction is low ($C \leq 0.4\%$). Homogeneous flow is observed when the superficial gas velocity is $U_g \lesssim 5$ cm/s. Transitional and heterogeneous flow are also observed at higher superficial gas velocities, but the gas holdup in the heterogeneous regime is slightly lower than that recorded for the air-water system when $C \leq 0.25\%$. When C = 0.4%, a more significant reduction in gas holdup is recorded in the heterogeneous regime. When $C \geq 0.6\%$, homogeneous flow is not observed, even at very low superficial gas velocities and the flow regime is defined as pure heterogeneous over the entire range of superficial gas velocities [17]. This trend has been observed in other GLF systems [4, 5, 7, 9].

The decrease in gas holdup with increasing fiber mass fraction has been attributed to the presence of larger bubbles and gas channeling in fiber suspensions. It has been shown that with the addition of as little as 0.1% fiber by mass, the bubble shape is flatter than those observed in pure water systems. The flatter bubbles tend to interact with other bubbles near the injection region, leading to bubble coalescence. The resulting larger bubbles rise faster in the fiber suspension and reduce gas holdup [9]. Additionally, fiber flocculation traps small bubbles until they coalesce with other bubbles to form a large enough bubble to break through the fiber floc, this also leads to faster bubble rise velocities and smaller bubble residence times, reducing gas holdup [7].

When $C \gtrsim 0.8\%$, further increases in hardwood fiber mass fraction have a small, if any, effect on gas holdup; this trend is the result of fiber settling and gas channeling, particularly at the lower superficial gas velocities. Hence, the nominal value of fiber mass fraction may be increasing, but local fiber mass

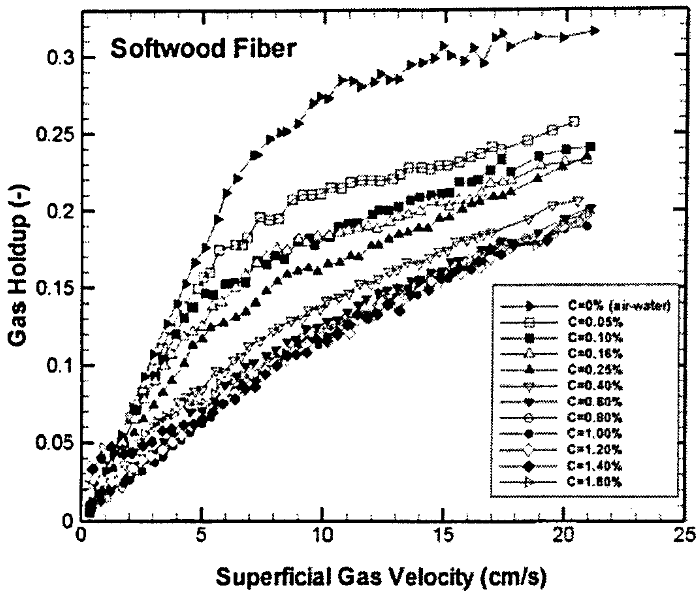

Figure 3. Gas holdup as a function of superficial gas velocity for softwood fiber slurries.

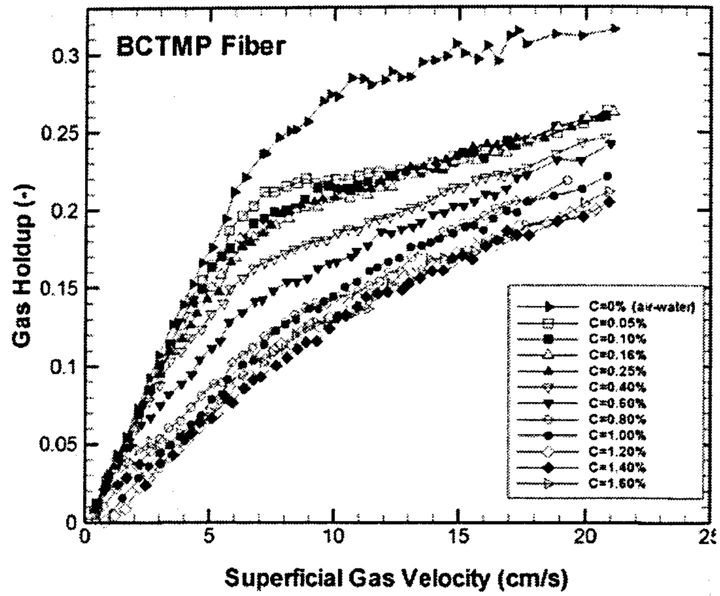

Figure 4. Gas holdup as a function of superficial gas velocity for BCTMP fiber slurries.

fractions may be much higher in the lower column region and nearly constant in the upper column region, resulting in a small, if any, change in gas holdup.

Softwood Fiber

When softwood fiber is added to the bubble column, similar gas holdup trends result (Fig. 3). However, the gas holdup in the heterogeneous regime is substantially reduced from that of the air-water system, even when the softwood fiber mass fraction is as low as C = 0.05%. Softwood fiber is generally longer and has a much higher coarseness than hardwood fiber [12]; these fiber characteristics encourage fiber flocculation [18], resulting in bubble coalescence and a reduction in gas holdup. Softwood fiber suspensions result in pure heterogeneous flow when C ≥ 0.4%. When C ≥ 0.6%, there is a negligible difference in the gas holdup when $U_g > 5$ cm/s.

At very low superficial gas velocities ($U_g < 3$ cm/s), the gas holdup is higher when C ≥ 1.2% because of gas entrainment. The fiber system is originally mixed by using a high gas flow rate to create a turbulent environment and then the gas flow rate is reduced to its lowest value. When the gas flow rate is reduced, the fiber suspension immediately flocculates to form a continuous fiber network creating a pseudoplastic material [19, 20]. This prevents some of the gas in the fiber suspension from exiting the system. Thus, the gas holdup for the high fiber mass fraction systems at the low superficial gas velocities have a considerable amount of entrained gas. When the gas flow rate is high enough to create a sufficient amount of turbulence, the slurry becomes fluidized and the entrained gas exits the system; this is shown by the C = 1.2-1.6% results following the other data when U_g is high enough. This trend is not observed in the hardwood fiber suspensions because of differences in the fiber characteristics; softwood is longer and has a higher coarseness than hardwood, encouraging fiber flocculation [18]. The softwood flocs retain small bubbles when the turbulence intensity is low, such as when $U_g < 3$ cm/s.

BCTMP Fiber

When BCTMP fiber is used to form the fiber suspension, similar trends are observed (Fig. 4). The most significant difference with BCTMP fiber is that a small amount of foam formation is observed when this fiber type is used. Although surface tension, electrical conductivity, total dissolved solids content, and pH are measured using filtrate samples from all experiments, no significant consistent difference is recorded for all fiber types. However, the BCTMP fiber used in this study was produced using sodium sulfite. Although the resulting fiber was washed and neutralized after beaching, it may still contain a small amount of lignosulfonates. Since lignosulfonate is water-soluble and a soap, it is believed that it is responsible for the foam that is produced with the BCTMP fiber.

Comparison Between Cellulose Fiber Types

The characteristics of the three cellulose fiber types used in this research are very different (see Table 1), resulting in considerable differences in experimental behavior and results. While the softwood fiber has a tendency to stick together in the upper column region, the BCTMP fiber system has a tendency to produce significant amounts of foam in the upper column region. The hardwood fiber did not show either of these two behaviors. As shown in Fig. 5, the effect of fiber type on gas holdup at a low mass fraction is significant. The overall gas holdup for the hardwood system is very high through the transition and heterogeneous flow regimes. The gas holdup in the BCTMP system is also slightly higher than that of the softwood system through these flow regimes, and it is also observed that the overall gas holdup for the softwood fiber system remains the lowest over the entire range of fiber mass fractions. As the fiber mass fraction in the system is increased, the differences in overall gas holdup between fiber types decreases significantly. When C = 0.80%, the overall gas holdup for the softwood system remains below that of the

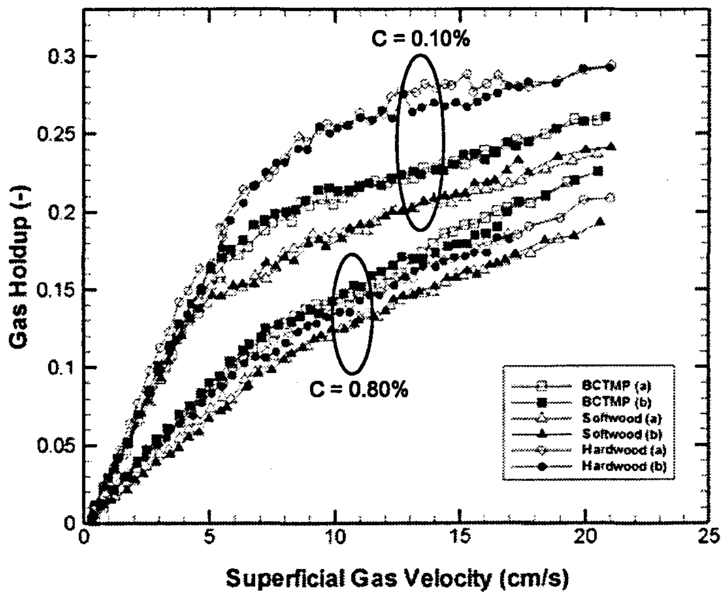

Figure 5. Gas holdup comparisons for different cellulose fiber types and C = 0.1 and 0.8%.

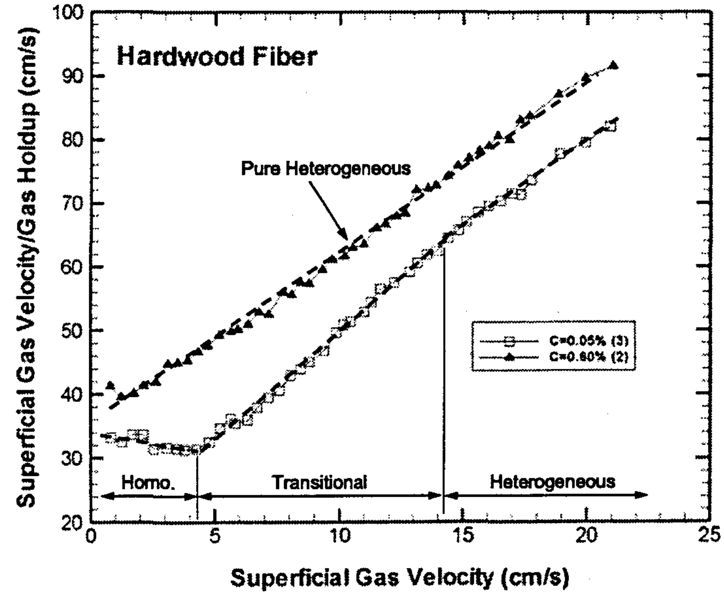

Figure 6. Sample drift flux plot for Hardwood fiber.

BCTMP or hardwood system, but all of the results are significantly closer to each other than at C = 0.10%.

Figure 5 also shows that replicate runs for the given experimental conditions are repeatable for this natural fiber system. Small variations exist among individual data points, but general trends are reproducible.

Regime Transitions

The Zuber-Findlay drift flux model [21] was used to determine the superficial gas velocity at which flow regime transition occurs. The drift flux model has been successfully applied to GLS systems [22] and to GLF systems [7, 10]. The drift flux for a semi-batch bubble column is defined as:

$$\frac{U_g}{\varepsilon} = C_o U_g + U_\infty \tag{4}$$

where C_o is a distribution parameter that gauges the velocity and gas holdup profile uniformity, and U_∞ usually represents the drift flux velocity. The drift flux model is applicable to heterogeneous flow conditions and has been used to identify the flow regime by plotting U_g/ε as a function of U_g [15]. Using this method, two hardwood fiber mass fractions (C = 0.05% and 0.8%), which represent two distinct flow characteristics, are selected to show how the analysis is completed (Fig. 6). When homogeneous, transitional, and heterogeneous flow are observed, the onset of transition is identified by the intersection of the first two linear regions on the drift flux plot, as shown in Fig. 6. The intersection of the second and third linear region identifies fully heterogeneous flow. A similar analysis was completed for all other conditions. For C = 0.8%, the drift flux model fits the data well, indicating that only heterogeneous flow exists for all superficial gas velocities. This confirms that with increasing fiber mass fraction, the flow regime will undergo the change from three possible flow regimes (homogeneous, transitional, and heterogeneous) to pure heterogeneous flow.

Figure 7 shows the superficial gas velocity at which the transitional regime begins as a function of fiber mass fraction for the three different cellulose fiber types. Multiple experiments were completed at each fiber mass fraction and the transitional superficial gas velocity was determined for each experiment. Although there is considerable scatter in the cellulose fiber transitional superficial gas velocity, the superficial gas velocity at which transitional flow is first observed generally decreases with increasing cellulose fiber mass fraction. This result was also observed for Rayon fiber systems [7]. For a given fiber mass fraction, the transitional superficial gas velocity is also generally highest (lowest) for BCTMP (hardwood) fibers in most cases.

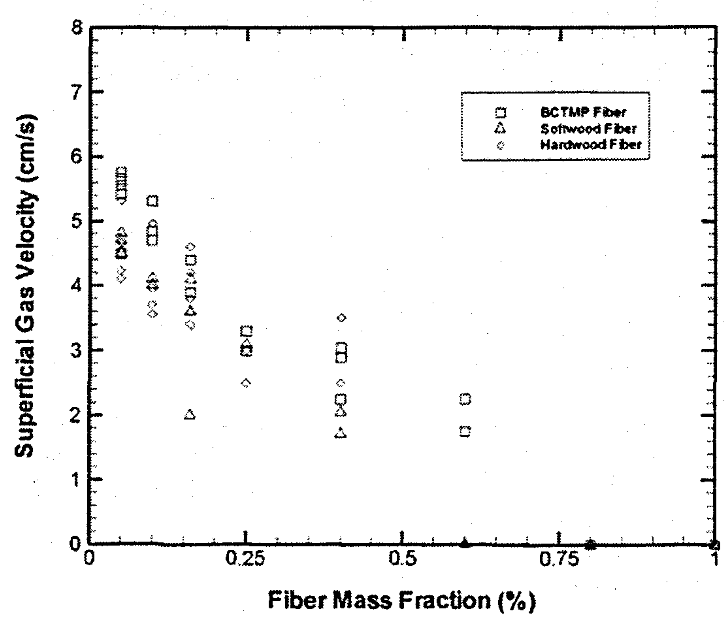

Figure 7. The superficial gas velocity at which the transitional flow regime is first observed.

CONCLUSIONS

Gas holdup results were presented for a 10.2 cm semi-batch bubble column filled with cellulose fiber slurries for a range of fiber mass fraction ($0 \leq C \leq 1.6\%$) and superficial gas velocities ($U_g \leq 23$ cm/s). Gas holdup decreased with increasing fiber mass fraction until $C \sim 0.8\%$, where only small, if any, additional changes were recorded. Homogeneous, transitional, and heterogeneous flow conditions were observed in low fiber mass fraction systems whereas only pure heterogeneous flow was observed at higher mass fractions. Hardwood fibers generally produced the highest gas holdup and softwood fibers produced the lowest. The Zuber-Findlay drift flux model was used to determine the superficial gas velocity at which transitional flow was first observed; this value decreased with increasing fiber mass fraction.

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