Multiple cointegration and structural models: applications to exchange rate determination

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Multiple cointegration and structural models: Applications to exchange rate determination

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Multiple cointegration and structural models: applications to exchange rate determination

by

Selahattin Dibooglu

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For the Major Department

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For the Graduate College

Iowa State University
Ames, Iowa

1993
"Education is a bitter tree the fruit of which is sweet".

Aristotle

This work is dedicated to
the memory of my late brother,
teacher and educator,
Ibrahim Dibooglu
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CHAPTER I. INTRODUCTION

Structural models of the exchange rate have performed very poorly for the industrialized nations during the post-Bretton Woods period. Work pioneered by Meese and Rogoff (1983a, 1983b) demonstrates that the out-of-sample forecasts of structural models are no better than those of the random walk model. This result is readily explained if the fundamental determinants of exchange rates (e.g., money supplies, income levels, and interest rates) are themselves near-random walk processes. However, Baillie and Selover (1987), Baillie and McMahon (1989), and Kim and Enders (1991), among others, provide evidence that there are no cointegrating relationships between bilateral nominal exchange rates and the so-called fundamentals.

Potential estimation problems such as simultaneity, the reliance on limited information techniques, imposition of inappropriate constraints, misspecified dynamics, and small sample bias are often cited for this failure. Specification problems regarding nonlinearities, omitted variables, and improper expectation formation are mentioned as well (Meese 1990).

In spite of the difficulties, there is a renewed effort among economists to tackle these problems. New developments in econometric techniques, as exemplified by the recently growing time series literature, have facilitated these efforts considerably.

It has long been recognized in economics that non-instantaneous adjustment of
some dependent variables to their equilibrium value is not uncommon. A conventional method of modelling an adjustment process is using a Koyck transformation (e.g., Goldfeld 1976). With the development of cointegration and error-correction techniques, it is possible to combine a steady-state function with a complex set of dynamics. These techniques do not assume that the underlying relationship holds at each point in time, hence, raise the possibility that the underlying relationship may be stable even over periods of substantial institutional change and financial innovations. Finally, cointegration techniques do not rely on \textit{a priori} exogeneity assumptions. The nature of the adjustment process in the foreign exchange market makes cointegration and error correction techniques suitable for modelling the exchange rates.

One aim of this research is to estimate a structural exchange rate determination model. To this end we use the Johansen (1988) and Johansen and Juselius (1990) methodology to re-examine the existence of a cointegrating vector(s) between the exchange rate and the supposed fundamentals.\footnote{The Johansen procedure has some important advantages over the earlier Engle and Granger (1987) cointegration test in that it uses maximum likelihood methods, can detect the presence of multiple cointegrating vectors, and can be used to test hypotheses concerning restricted versions of the cointegrating vector(s).} The Johansen procedure has some

The second, and perhaps more ambitious, aim of the research is to propose a modelling strategy for models with cointegrated variables. In that regard we attempt to provide a reasonable interpretation for the existence of multiple cointegrating vectors.
We extend the suggestion of Bagliano, Favero, and Muscatelli (1991) and Smith and Hagan (1991) and interpret each cointegrating vector as either a behavioral or a reduced form equation stemming from a structural model. The issue is particularly germane since we find multiple cointegrating vectors between the exchange rate and the fundamentals in French/U.S. and Italian/U.S. data. We demonstrate that in the presence of multiple cointegrating vectors, the Johansen procedure allows us to impose "identifying" restrictions on each. The exactly identified equations can be properly considered to be equations resulting from a structural model. Given that these equations represent long-run static properties of the data, a more complex dynamic structure can be derived combining this prior analysis with the system. To that end, we propose using conventional innovation accounting (variance decomposition and impulse response functions) techniques based on the error-correction representation.

Chapter II summarizes the main points of the research in the area of structural exchange rate models. As a vast literature in this area has emerged, the Chapter is meant to be suggestive of the research in the area, not an extensive literature survey.

Chapter III summarizes cointegration techniques and suggests how these techniques can be used to model structural economic models. Chapter IV reviews the two-country Dependent Economy Model based on Dornbusch (1973, 1974). The model is particularly convenient in that it is familiar, has a simple formulation, and contains some straightforward identification restrictions. Moreover, it is general enough to be consistent with the Dornbusch (1976) "overshooting" model or with the Mussa (1976)
Monetary Model. We apply the procedure to estimate a structural model of the French Franc/U.S Dollar exchange rate in Chapter V. To preview our results, we find two distinct cointegrating vectors which we identify as (1) the money market equilibrium relationship and (2) the modified Purchasing Power Parity (PPP) relationship that is consistent with the Dependent Economy Model. We find that these relationships hold individually but the overall identification restrictions are rejected. We also estimate the error-correction representation (with the restricted and unrestricted equilibrium errors) in order to characterize the short-run properties of the system. Using conventional innovation accounting techniques, we show that a reasonable proportion of exchange rate variability is explained by the fundamentals. We repeat the analysis using Italian/U.S. data in Chapter VI. Conclusions, limitations of the methodology, and directions for further research are contained in Chapter VII.
Endnotes

1. In a recent paper, Adams and Chadha (1991) used the Johansen procedure and found strong evidence of cointegrating relationships between exchange rates and a set of monetary, fiscal, and current account measures.

2. In a sense this methodology bridges the conventional structural model framework with that of VAR approach of Sims (1980). The imposed structure on long run relations can be combined with VAR to derive short-run dynamics. At this point one may or may not elect to impose restrictions on short run dynamics. See the section on suggestions for further research.
Empirical Regularities of Exchange Rates

Since the collapse of the Bretton Woods system and the transition to the floating exchange rates in the early seventies, major trading country exchange rate behavior has been characterized by volatility which most economists had not anticipated. Many structural models based on economic theory were developed thereafter and it soon became clear that none of these models provided satisfactory explanation of exchange rate dynamics. Before discussing the nature of these models and their performance in some detail, it is instructive to note the empirical regularities that have characterized the domain of the research in this area in the past twenty years.¹

First, month-to-month or even daily bilateral exchange rate variability has been frequently large and almost totally unpredictable. It is not unusual for the spot exchange rate to change 2 percent on a daily basis, or 700 percent annually (Baillie and McMahon 1989). These changes can have dramatic resource allocation and wealth transfer effects if one considers the trading volume in the foreign exchange market.² As of mid-1989, for example, the average volume of trading was (net of double counting) $430 billion a day compared to the daily U.S. GNP of $22 billion and daily world trade in goods and services of only $11 billion (Froot and Thaler 1990).

Second, the variability of real exchange rates has been shown to be different in
alternative nominal exchange rate systems and can not be justified merely by different patterns of the underlying disturbances (Stockman 1983, Mussa 1986).

Third, the high variability of exchange rates is not matched with what are known as the economic fundamentals or forcing variables, with the exception of interest rate differentials. Also correlations between changes in exchange rates and changes in their respective fundamentals are not significant, and sometimes have the wrong sign (Meese 1990). One implication of the variability issue is that short term movements in the nominal rates are highly correlated with the real rates (since the nominal rates are much more volatile than relative national price levels), a fact damaging to the theory of purchasing power parity (henceforth PPP). Moreover, the deviations of the exchange rates from values implied by relative price levels are highly persistent (Huizinga 1987).

Another empirical regularity is noted when one plots the spot and the contemporaneous forward rate. For maturities up to a year, spot and forward rates tend to move in the same direction and approximately by the same amount. However if one shifts the forward rate one period ahead, the correlation disappears suggesting that the forward rate is a poor predictor of the future spot rate. Indeed many authors noted that the forward premium or discount on the dollar, for example, tended to systematically mispredict even the direction of the future movements of the dollar (Marrinan 1989).

Finally, there is no strong and systematic relationship between nominal or real exchange rates and the current account balances nor is there any significant relationship between the nominal or the real rates and differential rates of monetary expansion, with
the exception of some very highly inflationary economies (Mussa 1984).

Theories of Exchange Rate Determination

A challenging endeavor for economists has been building models that would provide plausible answers to various aspects of the above-mentioned phenomena. Pioneering works in open economy macroeconomics, such as the works of Mundell and Fleming (Mundell 1968, Fleming 1962), focused on the flow equilibrium brought about by a free float of the exchange rate. Accordingly, under a free float, the net excess demand for foreign exchange which equals the overall balance of payments (current plus the capital account) must be zero in equilibrium. This is combined with the traditional equilibrium condition for the goods market (the IS locus) and the money market (the LM locus) to solve for the endogenous variables of the model and to determine the comparative static effects of fiscal and monetary policy. In this model, given domestic and foreign price levels, a change in the exchange rate changes the terms of trade which leads to reallocation of demand, output, and employment.

Although the Mundell-Fleming model was a major breakthrough in incorporating asset markets and capital mobility into open economy macroeconomics, the focus on flow equilibrium has been regarded a fundamental flaw. The theoretical models developed thereafter tried to emphasize the distinction between stock and flow equilibria. For example, flow equilibrium as in a classical Marshallian demand and supply framework entails different motives for demand and supply sides and makes no
reference to the intertemporal aspect of equilibrium. In the equilibrium for a common stock, on the other hand, the motives for the buyers and sellers are the same. Since the exchange rate can be defined as the relative price of two national monies (the monetary approach) or as the relative price of domestic and foreign bonds (the portfolio approach), it is natural to apply the theory relevant to the determination of asset prices in modelling the exchange rate. Hence, exchange rates were basically determined by equilibrium conditions in the markets for the stocks of domestic and foreign assets such as money and government bonds. Models along these lines are called "asset market models" and classified as monetary or portfolio balance models depending upon the assumption regarding the substitutability of domestic and foreign assets and the wealth effects of a current account imbalance.

The flexible price monetary model

This version of the monetary model (Mussa 1976, Frenkel 1976, Bilson 1978a) assumes instantaneous price adjustment, hence PPP holds continuously. In addition, due to perfect capital mobility and substitutability of domestic and foreign bonds (asset holders are indifferent between domestic and foreign asset at the margin), uncovered interest parity must hold. The money market equilibrium in domestic and foreign markets completes the model.

More specifically, the flexible price monetary model assumes stable money demand functions for the home and foreign country:
(2.1) \[ m_t = p_t + \phi y_t - \lambda i_t \]

(2.2) \[ m_t^* = p_t^* + \phi^* y_t^* - \lambda^* i_t^* \]

where \( m \) is the money supply, \( y \) is real income, \( i \) is the nominal interest rate, \( p \) is the price level, \( t \) is time subscript; asterisks denote foreign counterparts and all variables except the interest rate are expressed in logarithms.

Prices are perfectly flexible and there are no restrictions on trade flows, so purchasing power parity holds instantaneously:

(2.3) \[ s_t = p_t - p_t^* \]

where \( s \) is the spot exchange rate, defined as the price of foreign currency in terms of domestic currency.

Combining Equations (2.1) through (2.3), and assuming that domestic and foreign money demand coefficients are equal \((\lambda = \lambda^*, \phi = \phi^*)\) we obtain

(2.4) \[ s_t = (m_t - m_t^*) - \phi(y_t - y_t^*) + \lambda(t - i_t^*) \]

which is the basic monetary model equation. This equation implies that a relative increase in the supply of domestic money causes a proportionate depreciation, whereas an increase in domestic real income causes an appreciation through raising the demand for domestic money and reducing the domestic price level.

Another assumption underlying the basic monetary model is perfect capital mobility and the substitutability of domestic and foreign assets. This assumption
suggests that uncovered interest parity will hold continuously:

\[(2.5) \quad i_t - i_t^* = E_t s_{t+1} - s_t\]

where \(E_t\) is the expectation operator conditional on available information at time \(t\).

Eliminating the interest rate differential from equation (2.4) using equation (2.5) we obtain

\[(2.6) \quad s_t = (m_t - m_t^*) - \phi(y_t - y_t^*) + \lambda(E_t s_{t+1} - s_t)\]

If expectations are assumed to be rational, then by iterating forward equation (2.6) can be expressed as

\[(2.7) \quad s_t = \left(\frac{1}{1+\lambda}\right) \sum_{j=1}^{\infty} \left(\frac{\lambda}{1+\lambda}\right)^j E_t[(m_{t+j} - m_{t+j}^*) + \phi(y_{t+j} - y_{t+j}^*)]\]

This expression implies that the current exchange rate is a function of the rational expectation of discounted future exogenous fundamentals, therefore the stochastic behavior of the exchange rate will depend on stochastic behavior of the fundamentals. Equation (2.7) also highlights some important aspects of exchange rate behavior: The exchange rate will be very sensitive to exogenous shocks in the fundamentals and revision of expectations will make the exchange rate very volatile.

The sticky price monetary model

More sophisticated versions of the monetary model especially those along the lines of Dornbusch (1976) do not assume price flexibility in the short run, so that PPP
is a long run phenomenon. Asset markets, on the other hand, clear continuously and uncovered interest parity holds.

Here, two versions of the model will be reviewed; the original model as developed in Dornbusch (1976), and the extension suggested by Frankel (1979), the real interest differential model (RID).

Dornbusch assumes that goods prices are sticky in the short run; they adjust gradually to a disequilibrium in the goods market. Asset markets clear continuously and are assumed to evolve in the same manner as in the simple monetary model except for the explicitly determined expectation formation. Asset market equilibrium conditions are given by

\begin{align}
2.8 & \quad m - p = -\lambda i + \phi y \\
2.9 & \quad i = i^* + \mu \\
2.10 & \quad \mu = \theta (\bar{s} - s)
\end{align}

where \( \mu \) is the expected depreciation rate, and a bar over the variable denotes long run value. Equations (2.8) and (2.9) are money market equilibrium and uncovered interest parity equations and are the continuous time versions of equations (2.1) and (2.5) respectively.

Equation (2.10) is the expectation formation equation. Accordingly, the exchange rate is expected to depreciate in proportion to the discrepancy between the long-run equilibrium exchange rate, \( \bar{s} \) and the current actual rate, \( s \). The regressive expectation
scheme in equation (2.10) can be shown to be consistent with rational expectations under certain circumstances.

Equations (2.8) through (2.10) can be combined to yield the asset market equilibrium:

\[(2.11) \quad m - p = -\lambda i^* - \lambda \theta (\bar{s} - s) + \phi \bar{y}\]

In the long run, however, the asset market equilibrium is

\[(2.12) \quad m - \bar{p} = -\lambda i^* + \phi \bar{y}\]

since the actual and expected depreciation rate is zero in the steady state. Subtracting (2.12) from (2.11) and rearranging yields

\[(2.13) \quad s = \bar{s} - (1/\lambda \theta)(p - \bar{p})\]

This equation reflects the simultaneous conditions of money market equilibrium and yield equalization, given expectation formation. A higher price level, for example, means a lower level of real balances, higher interest rates and given interest parity, an expectation of depreciation. This is possible only when the actual exchange rate is below the long run equilibrium value [which is equal to saying that the schedule represented by equation (2.13) is downward sloping in \((p, s)\) space].

The goods market can be characterized by exogenous output, imperfect substitutability of domestic goods and slowly adjusting prices; prices rise in proportion to excess demand:
where \( u \) is a constant representing exogenous demand shift factors such as exports or government expenditure, \( (s-p) \) is a measure of the relative price of domestic goods so that \( \delta(s-p) \) reflects the substitutability of domestic and foreign goods.

Solving equation (2.12) for the interest rate \( i \), substituting into (2.14), and expressing in terms of deviations from long run equilibrium yields

\[
(2.15) \quad \ddot{p} = \pi [\delta(s-s) + (\delta+\sigma/\lambda)(\bar{p}-p)]
\]

According to equation (2.15), in the steady state the increase in the price level creates an excess supply by raising the relative price of domestic goods and raises domestic interest rates through reduced real balances. The exchange rate must depreciate proportionally more in order to offset the deflationary effects of worsening terms of trade and higher interest rates. This means the \( \ddot{p} = 0 \) schedule will be positively sloped in \((p,s)\) space.

The asset market equilibrium condition (equation 2.13) and the goods market steady state equilibrium condition (equation 2.15 with \( \ddot{p} = 0 \)) will give a unique solution for \((p,s)\). Short run dynamics of the system follow directly from these equations.

Of the short run dynamics, the "overshooting" result is the most celebrated one. It simply states that the exchange rate jumps above or "overshoots" its long run equilibrium value in response to an increase in nominal money stock. Mathematically, by totally differentiating equation (2.11), noting that \( d\delta = dm, dp=0, \) and that \( y \) and \( i^* \)
are constant, we obtain

\[
\frac{ds}{dm} = 1 + \frac{1}{\theta \lambda}
\]

which is greater than one. The intuition behind this result is as follows: Following an unanticipated permanent monetary disturbance, all prices have to rise proportionally, but since prices do not adjust immediately, excess money balances drive domestic interest rates down. For this to be compatible with the equilibrium in the foreign exchange market, traders must expect appreciation of the domestic currency to compensate for lower interest rates. This requires the exchange rate initially to jump above or "overshoot" its long run equilibrium. When prices adjust to restore the purchasing power parity, the exchange rate moves (falls) to its long run equilibrium level. The model can also be extended to accommodate imperfect capital mobility in which case "undershooting" takes place.

In the Dornbusch model money is not neutral in the short run. This is due to price stickiness, which is aimed at explaining short term deviations from purchasing power parity. In the long run, however, the model has typical characteristics of the monetary model. The overshooting scenario has some implications which are compatible with empirical regularities: That exchange rate fluctuates more than national price levels and because of price stickiness, changes in nominal exchange rates drive the real exchange rates (i.e., they are correlated). Although these implications seem to have claim to realism, Diba (1986) noted the following: If the exchange rates respond quickly
to new information while commodity prices adjust with a lag, we should expect changes in the spot rate to be useful predictors of future changes in nominal prices and there is no evidence supportive of this fact (cited in Marrinan 1989).

Another version of the sticky price monetary model is due to Frankel (1979). Frankel’s version is different in that it tries to capture the effects of secular inflation. In inflationary environments, uncovered interest parity should be modified so that real rates of return are equalized. More specifically, Frankel maintains that prices are sticky in the short run; PPP holds as a long run relation:

\[ (2.17) \quad \delta - \bar{p} - \bar{p}^* \]

Replacing equation (2.4) with its equivalent long run version and noting that in the long run, the expected depreciation rate is just equal to expected inflation differential we obtain

\[ (2.18) \quad \delta = (\bar{m} - \bar{m}^*) - \phi(\bar{y} - \bar{y}^*) + (\Delta p - \Delta \bar{p}) \]

where \( \Delta p \) is the expected domestic inflation rate. Assume that income growth is exogenous (or simply equal to zero), and expectations are rational, then the expected inflation rate is simply the expected future monetary growth rate. Frankel specifies the random walk as a benchmark specification for the monetary growth rate. In this case, the expected future monetary growth rate, hence the expected inflation rate is equal to the current monetary growth rate, \( \pi - \pi^* \). Equation (2.18) becomes
In the short run, the exchange rate expectations are given by the following relation which is a modified version of equation (2.10):

\[(2.20) \quad \Delta s^e = \theta(\bar{s} - s) + (\pi - \pi^*)\]

This equation states that in the long run when the exchange rate lies on its equilibrium path, it is expected to increase at the rate \((\pi - \pi^*)\). Combining equation (2.20) with the uncovered interest parity relation \(\Delta s^e = (i - i^*)\), we obtain

\[(2.21) \quad (s - \bar{s}) = (1/\theta)[(i - \pi) - (i^* - \pi^*)]\]

which states that the gap between the current exchange rate and its long run equilibrium value is proportional to the real interest differential.

If we combine equation (2.21) with equation (2.18) we get a general monetary equation of exchange rate determination:

\[(2.22) \quad s = (m - m^*) - \phi(y - y^*) + \lambda(\pi - \pi^*) - (1/\theta)[(i - \pi) - (i^* - \pi^*)]\]

or

\[(2.23) \quad s = (m - m^*) - \phi(y - y^*) - (1/\theta)(i - i^*) + \left(\frac{\lambda}{\theta}\right)(\pi - \pi^*)\]

Note that when the level of the money supply rather than the monetary growth rate follows a random walk, the expected long-run inflation differential \((\pi - \pi^*)\) is zero. In this case equation (2.22) reduces to

\[(2.24) \quad s = (m - m^*) - \phi(y - y^*) - (1/\theta)(i - i^*)\]
which is the Dornbusch version of the semi-reduced form. Thus, equation (2.24) as well as equation (2.4) are nested models which can be econometrically tested by estimating equation (2.23).

**The portfolio balance model**

The substitutability of domestic and foreign non-money assets is an important assumption in the monetary approach. It allows for the aggregation of domestic and foreign bonds into a single market (the bond market) which can be excluded from analysis by application of Walras’ Law. As for the portfolio balance approach, domestic and foreign bonds are imperfect substitutes which makes their supply relevant. This implies that the uncovered interest parity will not hold and there will be a risk premium. Also, the portfolio balance model is a stock-flow model in which current account imbalances have a feedback effect on wealth hence on long run equilibrium. Among the influential contributions to the portfolio balance model, one can mention Kouri (1976), Allen and Kenen (1980), Dornbusch and Fisher (1980), Isard (1983), and Branson (1984).

In the portfolio balance model, the exchange rate is determined (at least in the short run) by supply and demand in financial asset markets. The determination of the exchange rate in the short run can be explained with the following simple model. Suppose the private sector wealth is composed of three assets: Money (M), domestic bonds (B), and foreign bonds denominated in foreign currency (F). We can treat B as
government debt held by the private sector, and \( F \) as the level of net claims on foreigners, hence current account surplus/deficit gives the rate of change in \( F \). Assuming that domestic residents hold domestic money only, and that the country is small so that the foreign interest rate \( i^* \) is given, equilibria in the asset markets are achieved by equating the supplies of assets to their desired levels:

\[
\begin{align*}
(2.25) & \quad M - M(i,i^*)W & M_r < 0, & M_{r^+} < 0 \\
(2.26) & \quad B - B(i,i^*)W & B_r > 0, & B_{r^+} < 0 \\
(2.27) & \quad SF - F(i,i^*)W & F_r < 0, & F_{r^+} > 0 \\
(2.28) & \quad W = M + B + SF
\end{align*}
\]

where \( S \) is the level of the exchange rate. We follow the convention that a \( Y_r \) denotes the partial derivative of the function \( Y \) with respect to the argument \( r \). The function \( M(i,i^*) \) has the interpretation as the desired level of wealth to be held in money. Similarly \( B(i,i^*) \), and \( F(i,i^*) \) are the desired proportions of wealth to be held in domestic and foreign bonds respectively. In this case the demand functions are homogeneous in wealth and are expressed in nominal terms.

Since we are concerned with determination of the exchange rate in the short run, we can treat the supply of assets as fixed. Also, we are abstracting from exchange rate expectations so that the expected rate of depreciation is zero. In this simple framework, the exchange rate \( S \), and the domestic interest rate \( r \), adjust to equate the supply and demand for financial assets. For example, an increase in \( M \) (monetary policy) raises
financial wealth, so as agents try to rebalance their portfolios by buying domestic bonds, the domestic interest rate falls. At the same time when they buy foreign bonds F, the demand for foreign currency goes up depreciating the domestic currency (S rises). The effects of fiscal expansion can be analyzed similarly. An increase in B has an ambiguous effect on the exchange rate because the substitution and wealth effects operate in opposite directions: When B increases, the level of wealth rises so the demand for foreign assets increases which depreciates the exchange rate. On the other hand, with the increase in B, bond prices go down while domestic interest rates rise. The increase in domestic interest rates tend to reduce the demand for foreign bonds. The magnitude of this substitution effect depends on the degree of substitutability of domestic and foreign bonds in the portfolios. If domestic and foreign bonds are not close substitutes, the wealth effect dominates the substitution effect and the exchange rate depreciates.

In the long run, the adjustment process is rather complex since one must take into account the effect of exchange rate on the current account balance which represents the change in net domestic holdings of foreign assets (i.e., the supply of foreign assets). Thus, an exchange rate change has demand as well as supply effects in the long run, and a disturbance leads to a chain of stock-flow adjustments.

In this scheme, the exchange rate affects the current account balance. The current account balance represents the change in the supply of foreign assets and as such, a change in the exchange rate affects the level of wealth through the current account
which also affects asset demands, which, in turn, affects the exchange rate. Thus, the portfolio balance model is a dynamic model in which the exchange rate is determined by a dynamic adjustment process which involves the asset markets, the current account, price levels, and the rate of asset accumulation.

An increase in the money supply brought about by an open market purchase of domestic bonds, for example, has effects beyond an immediate depreciation of the domestic currency. If the economy was initially in equilibrium (zero trade balance and zero net holdings of foreign assets) the increase in the exchange rate will induce a positive trade balance through improvements in competitiveness, assuming that the Marshall-Lerner (stability) condition holds. This means a current account surplus and accumulation of foreign assets. An increase in foreign asset holdings induces an appreciation of the domestic currency and this begins to worsen the trade balance. Meanwhile, the increase in the money supply pushes domestic prices up, which adds to the worsening trade balance.

A zero trade balance will not restore equilibrium, since an acquired positive level of net foreign asset holdings will bring a stream of interest payments, hence for a zero current account balance, the trade balance must go into deficit. This induces a further appreciation of the exchange rate to its long run equilibrium level. In this scenario, the exchange rate jumps in response to the increase in the money supply in order to clear the asset markets, then adjusts slowly as a result of the induced current account imbalances. Note that this scenario is an alternative derivation of the "overshooting"
which does not involve price stickiness. In the rational expectations version of the model (e.g., Branson 1984), the same results apply with respect to the effects of various shocks but the exchange rate reacts immediately to the expected current account imbalances.

**The equilibrium approach**

The asset market approach is not based on individual optimizing behavior, though subsequent work has explored the consistency of its hypothesized behavior with the individual optimization paradigm. More recently, models that attempted to use individual utility maximization in exchange rate determination have appeared. This approach, known as the equilibrium approach, incorporates real factors such as productivity, government spending, investment, current account and budget deficit into the models using intertemporal optimization framework so that the optimal paths are simultaneously determined. The idea of the equilibrium approach to exchange rates can be explained as follows: Changes in future economic conditions or current changes in the supply of or demand for goods have an impact on time paths of real variables such as consumption, investment and the current account. These real disturbances will also alter equilibrium relative prices including the exchange rates (Stockman 1987). Changes in real rates are partly accomplished through changes in nominal rates, hence repeated disturbances to supplies or demands create a correlation between changes in nominal and real rates. According to this approach, the idea that changes in real rates
are due to real shocks with a large permanent component can explain the fact that changes in real exchange rates tend to persist for very long periods of time or to be nearly permanent.

In more sophisticated models of this nature, it is not possible to find closed form solutions to the exchange rate, although the first order conditions for utility maximization place restrictions on comovements of the model’s endogenous variables, a phenomenon analogous to asset pricing literature (Meese 1990).

Empirical Evidence and Structural Exchange Rate Models

A substantial body of literature has emerged that attempted to test implications of exchange rate models either by testing reduced/semi-reduced forms, or testing the out-of-sample forecasting performance of the models. In the forecasting performance tests, forecasts of the models are compared to a standard such as the random walk process. In this section, empirical evidence for the structural models will be reviewed.

Tests of the reduced forms

Earlier tests of the reduced form of the monetary approach (flexible price or real interest differential versions) employed data from the 1920s or from the 1970s up to 1978. These studies used econometric versions of equations (2.4) or (2.24) and the evidence was largely supportive. Frenkel (1976) tested a version of equation (2.4) for the German hyperinflation period 1920 through 1923 and reported results in favor of the
basic monetary model. Bilson (1978b) tested equation (2.6) with the expected depreciation rate, $E_t (s_{t+1} - s_t)$ replaced by the forward premium and without identical domestic and foreign money demand elasticities. The study used data for the Deutsche Mark- British Pound exchange rate over the period January 1972 through April 1976 and reported partially successful results. Although the coefficients were of expected signs and the model explained 92% of the variation in the exchange rate, it did not fare well against a random walk model in terms of goodness of fit. Finally Frankel (1979) tested equation (2.24) replacing the expected inflation rate with the long term interest differential on the grounds that the domestic and foreign long term real interest rates are equalized. The study used data for the Deutsche Mark/U.S. Dollar-exchange rate over the period July 1974 through February 1978 and reported significant coefficients of expected signs. This implied that the Dornbusch model and the simple monetary model were rejected in favor of the real interest differential model.

Although these selected studies found evidence supportive of the monetary model, tests that extended the sample period to include post-1978 data shed serious doubts on the ability of the monetary approach to explain the exchange rate. Indeed, most of the estimates had the wrong sign, the equations lacked explanatory power, and autocorrelation was a serious problem. Particularly troubling are the studies of the Deutsche Mark/U.S. Dollar, which found that an increase in the money supply leads to an appreciation of the domestic currency. Frankel (1982) refers to this phenomenon as the "mystery of multiplying marks".
Various explanations have been proposed for the failure of the monetary models. Dornbusch (1980) incorporates the current account impact into the exchange rate equation within rational expectations and portfolio balance frameworks with a partial success. Haynes and Stone (1981) suggests that imposing the constraint that foreign and domestic money demand elasticities be equal is inappropriate. The study asserts that linear constraints not only cause bias in parameter estimates, but the bias that results from subtracting variables leads to sign reversals if the variables are positively correlated. Having proven that, the authors report expected signs for unconstrained coefficient estimates in equation (24). Another suggestion has been made by Frankel (1982) which introduced wealth into money demand equations. Other studies focused on the relative instability of money demand equations which is attributed to shifts in the velocity of money. Frankel (1984), for example, introduced a relative velocity shift term into the money demand equation which led to some success in the results.

The possible endogeneity of some right hand side variables in equation (2.24) was another concern in empirical literature. Driskill and Sheffrin (1981) contends that because the reduced form of the basic monetary model (equation 2.6) has the expected depreciation term on the right hand side, the estimates are not consistent as the expected depreciation rate is correlated with the error term. After deriving "observable reduced form equations" within the context of rational expectations, the study estimates these equations simultaneously subject to implied overidentifying within- and cross-equation parameter constraints. The use of a rational expectations solution led to other attempts to
Hoffman and Schlaginhauf (1983) estimated the forward looking solution to the flexible price monetary model (equation 2.7) directly. They assumed that a typical variable in the fundamentals is generated by an ARIMA(1,1,0) process. The study then tested the validity of a rational expectations model along with the coefficient restrictions implied by the flexible price monetary model. While the rational expectations restrictions were satisfied, the monetary model restrictions were rejected for the Deutsche Mark among a sample of the French Franc and British Pound against the U.S. Dollar.

Woo (1985) considered the rational expectations solution to the monetary model with partial adjustment in the demand for real balances. This study modelled the exogenous fundamentals by a quadratic deterministic trend and used the residuals to avoid using non-stationary fundamentals. The results yielded plausible parameter estimates.

Other applications of the rational expectations framework to the monetary model include the variance bounds/excess volatility tests or testing for "bubbles" which are typical of the asset market literature. Variance bounds methods were originally used by Shiller (1979) in the context of the volatility of long-term bond returns. The idea within the context of the simple monetary model can be explained as follows: Consider the exchange rate, given by the simple monetary model forward looking solution, \( s_n \), in equation (2.7). The ex-post rational exchange rate, \( s^* \), can be obtained by replacing the
expected values of future exogenous fundamentals with their actual realized values.

Given that the future fundamentals are not contained in the information set at time t, the ex-post rational rate, $s^*_t$, will differ from $s_t$ by a rational forecast error, $v_t$, which is uncorrelated with $s_t$ (i.e., $v_t = s^*_t - s_t$). Therefore we can write

$$\text{(2.29)} \quad \text{var}(s^*_t) = \text{var}(s_t) + \text{var}(v_t), \quad \text{var}(s^*_t) \geq \text{var}(s_t)$$

where the inequality follows directly from the assumptions made above.

Huang (1981) used an excess volatility test similar in nature along with some tests of equality restrictions of the cross covariance functions. The study covered data from March 1973 through March 1979 for the U.S Dollar/Deutsche Mark, U.S. Dollar/British Pound, and British Pound/Deutsche Mark exchange rates. In all tests, the data was found to be incompatible with the monetary approach and/or the efficient market hypothesis.

The variance bounds test is also an indirect test of the existence of speculative bubbles. This is because the variance bounds hypothesis is valid if there is no speculative bubble, therefore the failure of the hypothesis can be taken to indicate the presence of a rational bubble\textsuperscript{11}. Meese (1986) tested for speculative bubbles directly using British Pound/U.S. Dollar, Japanese Yen/U.S Dollar, and Deutsche Mark/U.S. Dollar monthly data over the period 1973-82. The author rejects the joint null hypothesis of no bubbles and stable autoregressive processes for relative money supplies and real incomes for the German and British data. The Japanese data provide no
conclusive evidence. The study further tests whether the exchange rate is cointegrated with money supplies and/or incomes using the Granger and Engle method based on Granger and Engle (1984). The results indicate no cointegration suggesting that the cointegration test is not conclusive with respect to speculative bubbles.

Earlier empirical evidence regarding the sticky price monetary model has been supportive. Wallace (1979) studied the free float of the Canadian Dollar in 1950s and found evidence in favor of the Dornbusch model. Also Driskill (1981) tested the Dornbusch model for the Swiss Franc U.S. dollar exchange rate using quarterly data over the 1973-79 period. The study uses reduced form exchange rate and price level equations consistent with the Dornbusch model and a more general stock-flow model with imperfect capital mobility. The advantage of the generalized model is that it makes no a priori assumption about overshooting/undershooting leaving the issue as an empirical question. While some of the constraints implied by the Dornbusch model were violated, the study found that the elasticity of the exchange rate with respect to an unanticipated monetary disturbance exceeded unity (roughly 2) indicating overshooting. Also, the adjustment to a new long run equilibrium was found to take longer than 2 years and the process exhibited non-monotonic patterns. Other studies did not confirm the Dornbusch model. Backus (1984) used quarterly data for the Canadian Dollar over the period 1973-80 and found statistically insignificant coefficient estimates. Papell (1988) used a systems approach to capture the model dynamics. He allowed income and interest rates to be endogenous and estimated them with the model which he expressed
as a vector autoregressive moving average reduced form with nonlinear parameter constraints and rational expectations. With the system approach, the author argues, one can model domestic income and interest rates endogenously in addition to prices and exchange rate, and avoid the use of proxies for expected inflation. The study used quarterly data for the effective exchange rates of Germany, Japan, United Kingdom, and the United States over the period 1973.Q1 through 1984.Q4. In addition to reporting plausible and statistically significant structural coefficients, the study could not reject cross equation restrictions for Japan, United Kingdom, and the United States. Also parameter estimates indicated exchange rate overshooting for Germany, and undershooting for Japan. The results for the United States and the United Kingdom were inconclusive.

Recently, Lee and Enders (1991) decomposed nominal and real exchange rate movements into components induced by nominal and real factors using the technique developed by Blanchard and Quah (1989). The study used monthly data over the period 1973-89, for the Deutsche Mark, Canadian dollar, and Japanese Yen against U.S. Dollar. Overall, nominal shocks account for no more than 5% of real exchange rate movements, and have only a minor role in explaining nominal exchange rate movements. Moreover, the authors find little evidence supportive of exchange rate overshooting.

As we mentioned before, autocorrelation has been a major problem in empirical testing of the exchange rate reduced forms. The near unit autoregressive coefficient of
the residual series suggests that the unexplained factors have a near permanent effect on the level of the exchange rate, or to put it another way, the exchange rate is not cointegrated with the fundamentals.

Several studies examined the cointegration between the exchange rate and the fundamentals. Baillie and Selover (1987) tested whether equation (2.24) exists as a cointegrating relationship using the procedure developed by Engle and Granger (1987). They used monthly data for the British Pound, Deutsche Mark, Japanese Yen, Canadian Dollar, and French Franc against the U.S. Dollar over the period March 1973 through December 1983. The results were disappointing in that the monetary model reduced form did not even represent a long run equilibrium relationship. The authors further tested the validity of PPP as a long run relationship. With the exception of the French Franc/U.S. Dollar exchange rate, the PPP was rejected as a long run equilibrium relationship. McNown and Wallace (1989) tested the validity of equation (2.4) as well as equation (2.24) as cointegrating relationships for the bilateral exchange rates of the United Kingdom, Japan, Germany, France, Canada, and the United States with three different base countries, and over several time periods ending June 1986. The results were no different: The monetary model reduced forms did not exist as long run equilibrium relationships.

Baillie and Pecchenino (1992) used the flexible price monetary model forward looking solution (equation 2.7) and derived a theoretical cointegrating relationship between the nominal exchange rate, money supply differential, and the output
differential. They tested this implication along with the existence of money market equilibrium relationships (equations 2.2 and 2.3) for the United Kingdom and the United States using the procedure developed by Johansen (1988). They used monthly data from March 1973 through May 1990, and concluded that the implication of the monetary model was rejected. The authors also performed various unit root tests on the real exchange rate with no conclusive evidence. However, they found a unique cointegrating vector among money market variables for both the United States and the United Kingdom. This leaves the persistent deviations from PPP as the only source of rejection for the equilibrium monetary model.

Tests of the portfolio balance model have proven difficult because it is difficult to obtain data on holdings of financial assets broken down by the currency of denomination. Branson, Halttunen, and Mason (1977) used a logarithmic reduced form which is linear in $m_t, m_t^*, f_t, f_t^*$, where $f_t = \log F_t$, assuming that in the short run, the price level and the output level can be taken as fixed. The study measures $f_t$ and $f_t^*$ by the cumulated current accounts of the domestic and foreign country respectively. The study reports coefficients of expected signs but they are mostly non-significant. The study further hypothesizes the possible endogeneity of the money supply, and tries to capture that by a reaction function for the money supply. Estimating the system with two-stage least squares, the authors produce more significant parameter estimates with greater explanatory power.

Frankel (1983) estimated single equation portfolio balance model equations for
the U.S. Dollar/Deutsche Mark exchange rate using monthly data over the period January 1974 through October 1978 and assuming various asset preference combinations (U.S. bonds are held by U.S. residents only etc). The results were very poor in that all coefficients on stock variables appeared to be of incorrect sign and were significantly so.

Subsequent tests of the portfolio balance model involved using a reduced form equation which synthesized the monetary and portfolio balance approaches. The equation incorporates the risk premium into equilibrium exchange rate reduced form of the real interest differential model given by equation (2.23). Accordingly, investors diversify the risk from exchange rate variability according to expected relative rate of return (risk premium):

\[(2.30) \quad \frac{B}{SF} = g(i-i^*-\Delta s^*)\]

where \(B\) and \(F\) are supplies of domestic and foreign bonds and \(g\) is a positive valued function of the risk premium. Assuming a log linear form for equation (2.29) and combining with equation (2.23) one can show that\(^4\) the exchange rate can be written as

\[(2.31) \quad s = \beta_0 + \beta_1(m-m^*) + \beta_2(y-y^*) + \beta_3(\pi-\pi^*) + \beta_4(i-i^*) + \beta_5(b-f)\]
where $\beta_1, \ldots, \beta_5$ are coefficients which are functions of structural model parameters, and $f$ and $b$ are the logarithms of $F$, and $B$ respectively. Note that equation (2.30) adds another variable, the relative bond supply as explanatory variable to the real interest differential reduced form which is given by equation (2.23). This form can be used (and has been used) to discriminate between various versions of the monetary model and the portfolio balance model, since each of the models places sign and exclusion restrictions on the $\beta_i$'s.

Different versions of equation (2.30) have been estimated by, among others, Hooper and Morton (1982), and Frankel (1983, 1984). Hooper and Morton (1982) extended the Dornbusch- Frankel specifications by allowing large and sustained changes in real exchange rates. The change in the real exchange rate was modelled to be primarily related to the current account. The current account provides information about long run equilibrium exchange rate, hence current account movements produce changes in real exchange rate through expectations. The current account also affects exchange rate through risk premium. Here, risk premium is modelled as a linear function of cumulated current account and foreign exchange market intervention flows. The study employed monthly and quarterly data for the U.S. dollar effective exchange rate over the period 1973 through 1978. The results confirmed the theory; all coefficient estimates were significant and of expected sign. In all equations, the risk premium coefficients had incorrect signs and were insignificant suggesting that portfolio preferences are not important, at least in the short run. However, the presence of significant current account
effects through expectations imply that the portfolio preferences are important in the long run.

Frankel (1983) estimated equation (2.30) for the Deutsche Mark/U.S. Dollar exchange rate over the period January 1974 through October 1978 and found that the coefficients of the monetary model variables were of correct sign. However, when corrected for autocorrelation, most of the coefficients were no longer significant.

Frankel (1984) estimated a different version of equation (2.30) without the relative bond supply, but modelling instead the risk premium as a solution to the asset market equilibrium conditions. Using monthly data from January 1974 through mid-1981 with currency specific sample periods, and using a sample of currencies consisting of Deutsche Mark, British Pound, Japanese Yen, and Canadian Dollar all against the U.S. Dollar, Frankel reported significant risk premium terms, but at the expense of insignificant monetary model terms.

**Forecasting performance of the structural models**

Another avenue of empirical literature concerns testing the forecasting performance of exchange rate models. It is by now an "empirical regularity" that these models have extremely poor fit even within sample (i.e., using actual realized values of the explanatory variables) and yield forecasts no better than a naive random walk model. This has been dramatized initially by two papers by Meese and Rogoff (Meese and Rogoff 1983, 1984) who found that during the 1973-81 period, no structural or time
series technique could outperform the random walk model at any forecast horizon shorter than twelve months.

The original Meese and Rogoff study (Meese and Rogoff 1983) employed data from March 1973 through June 1981 for the British Pound, Japanese Yen, Deutsche Mark all against the U.S. Dollar, and trade-weighted dollar exchange rates using the flexible price (equation 2.4), real interest differential (equation 2.24), and the monetary/portfolio synthesis (equation 2.30) reduced forms. Comparing the performance of these equations to the performance of the random walk model, the study concluded that none of these models outperformed the random walk in 1, 3, 6, and 12 month forecasting horizons. The study used alternatively the forecasts of forward exchange rate, a unit autoregressive model of the spot rate, and a vector autoregression model, but with no success. The study used "rolling regression" method and the comparison was based on the mean error, mean absolute error, and root mean square error. Moreover, the authors tried various alternative methods (estimating the models in first differences, not constraining home and foreign elasticities to be the same, using different measures of the money supply etc) but without any success; the random walk still dominated.

Meese and Rogoff (1984) constrained the model coefficients to have values based on "theoretical and empirical literature on money demand and PPP". Using vector autoregression, the authors show that the instruments used in estimating structural models may not be exogenous which motivates using the constrained coefficient method. The study concluded that although in forecast horizons up to a year the random
walk outperformed the constrained coefficient reduced forms, the structural models did better in terms of root mean squared error in forecasts beyond a year.

Following the Meese and Rogoff studies, an ample literature emerged on the forecasting performance of the exchange rate models. Wolf (1987), and Alexander and Thomas (1987) relaxed the assumption that the structure of the exchange rate models is stable over time. Both used Kalman filter and the Meese and Rogoff sample currencies (Alexander and Thomas extended the sample period up to 1985) with only minor success. Wolf found that of the sample currencies considered, exchange rate models performed better than a random walk only in Deutsche Mark/U.S. Dollar exchange rate. At forecasting horizons less than a year, Alexander and Thomas reported uniformly poor performance for the structural models.

In addition to confirming the Meese and Rogoff studies, Ahking and Miller (1987) investigated the time series properties of nominal money stocks, real incomes, long and short term interest rates, cumulative trade balances, and the price levels all of which are believed to be important fundamental determinants of exchange rates. They found that with few exceptions all of these variables were not random walks, and suggested that using observed values instead of equilibrium values is inappropriate and might be causing the misspecification of the empirical models.

Somanath (1987) introduced the lagged dependent variable to the explanatory variables and showed that this improves the forecasting ability of the models suggesting non-instantaneous adjustment of the actual exchange rate to its equilibrium value
suggested by structural model reduced forms. Indeed, Boothe and Glassman (1987), in this context, proposed using "error correction models" in modelling the exchange rate.

**Summary and Conclusions**

Three main views regarding exchange rate determination have evolved since the early 1970s; the monetary approach (flexible price, sticky price, and real interest differential versions); the portfolio balance approach, and the equilibrium approach. We have reviewed the theoretical and empirical literature on these approaches with a special emphasis on the asset market approach. Although these models have a considerable theoretical appeal, most did not fare well against empirical evidence. This failure can be traced to specification as well as estimation problems. For example most models assume PPP (at least in the long run) which hinges upon very restrictive assumptions (e.g., no change in equilibrium relative prices). The sizable "noise" component of the exchange rate and the non-instantaneous adjustment process makes modelling the exchange rate very difficult using conventional econometric techniques.

In the following chapters, we will attempt to apply a less restrictive econometric technique which does not assume that the underlying relationship holds at each point in time or impose a specific adjustment process. Furthermore, given the performance of PPP in the post-Bretton Woods period, we allow for real shocks that alter equilibrium relative prices to cause deviations from PPP.
Endnotes

1. Empirical regularities of exchange rates are surveyed, among other places, in Mussa (1984) and MacDonald (1988). For a comprehensive recent survey of exchange rate economics, see MacDonald and Taylor (1992).

2. Since the volatility of exchange rates is not matched by the price levels, a change in the nominal exchange rate means change in the real exchange rate.

3. Instead of looking at correlations between levels, it is more accurate to look at the correlations between first differences because of stationarity considerations.

4. The flexible price monetary model is also referred to as "the simple monetary model" or "the basic monetary model".


6. Stockman (1987) defines the exchange rate as the relative price of foreign goods in terms of domestic goods which is known as "terms of trade".

7. The choice of random walk is no coincidence since it represents "no forecast" at all.

8. I.e., placing coefficients on the right hand side variables, adding an intercept and an error term.

9. The autoregressive residuals with near unit coefficients is interesting to note, since they "roughly" imply lack of cointegration. See the subsequent discussion in this section.

11. "Bubble" refers to fluctuation in asset prices driven by speculation due to self-fulfilling expectations which is not warranted by market fundamentals. Rational expectations and rationality itself does not necessarily imply that the asset price will be equal to the value implied by the fundamentals. I.e., bubbles are related to multiple rational expectations solutions. For a general discussion, see Blanchard and Watson (1982). In the context of exchange rates, MacDonald and Taylor (1989) discusses speculative bubbles.

12. If the exchange rate and the fundamentals are cointegrated, then the equilibrium error is stationary, hence bubbles can not exist. But lack of cointegration may or may not be consistent with bubbles.


14. For proof, see Frankel (1983), pp 101-103.
CHAPTER III. MULTIPLE COINTEGRATION AND STRUCTURAL MODELS

Cointegration

Before attempting to discuss the idea of cointegration in some detail, it is useful to define a related concept, namely integration. A series is said to be integrated of order d, denoted I(d), if it requires to be differenced d times to achieve a stationary, invertible, non-deterministic autoregressive moving average (ARMA) representation. Typically, the order of integration is determined through a testing procedure, which is known as "testing for unit roots". The most common methods in the literature are those based on the works of Dickey and Fuller (Dickey and Fuller 1979, 1981). The augmented Dickey-Fuller method involves running the following regression to determine the presence of a unit root in the series $y_t$:

$$
\Delta y_t = \alpha + \pi y_{t-1} + \sum_{j=1}^{k} \gamma_j \Delta y_{t-j} + u_t
$$

where $\Delta$ is the first difference operator, $\alpha$, $\pi$, and $\gamma_j$ ($j=1,\ldots,k$) are regression coefficients, and $u_t$ is a white noise error term. The maximum lag, $k$, is to be chosen such that the error term $u_t$ is approximately white noise. Here the null hypothesis of a unit root is given by $\pi = 0$, while the alternative hypothesis that the series is stationary implies that $\pi < 0$. Under the null hypothesis, the distribution of the involved t-statistic is non-standard and the empirical distribution is tabulated in Fuller (1976).

Following Engle and Granger (1987), the idea of cointegration can be presented
formally as follows: Suppose the variables in the n-dimensional vector $Y_t$ are $I(d)$. The series contained in $Y_t$ are said to be cointegrated, if there exist a non-null vector $\beta$ such that $z_t = \beta'Y_t$ is $I(d-b)$, $b > 0$, where $\beta$ is called the cointegrating vector. For example, if the vector $Y_t$ contains two variables, $y_t$ and $w_t$, both of which are $I(1)$ and cointegrated, then they have an error correction representation:

$$\Delta y_t = A(L)\Delta y_{t-1} + B(L)\Delta w_{t-1} + \alpha_1 z_{t-1} + \epsilon_{1t}$$

$$\Delta w_t = C(L)\Delta y_{t-1} + D(L)\Delta w_{t-1} + \alpha_2 z_{t-1} + \epsilon_{2t}$$

where $(1 - \theta)$ is the cointegrating vector, $z_t = y_t - \theta w_t$, $A(L)$, $B(L)$, $C(L)$, $D(L)$ are finite order lag polynomials, and $\epsilon_{it}$ ($i = 1, 2$) are disturbance terms. It is common to interpret the cointegrating relation $y_t = \theta w_t$ as a long run equilibrium relation, while the error correction term $z_t$ can be interpreted as a temporary divergence from the long run equilibrium. The terms $\alpha_1$ and $\alpha_2$ are adjustment coefficients and are measures of the speed by which the system adjusts in response to last period’s deviation from the equilibrium. The importance of the error correction terms can be attributed to the Granger Representation Theorem which states that the error correction terms (the stationary linear combinations) Granger causes changes in at least some of the variables in the model (Engle and Granger 1987, 255-259).

In practice, cointegration is also determined through a testing procedure and there are numerous methods in the literature. Tests based on a static regression and those based on the cointegrating rank are the most commonly used procedures.
Static regression tests do not attempt to estimate the number of cointegrating vectors; they try to distinguish between no cointegration versus at least one cointegrating vector. Engle and Granger's two step procedure (Engle and Granger 1987) is along this line and it can be implemented as follows: The variables in the n-dimensional vector $Y_t$ are first tested for the order of integration. If all are of the same order, say $I(1)$, then they are partitioned as $(y_{1t}, y_{2t})$ where $y_{1t}$ is a scalar variable and $y_{2t}$ is an $(n-1 \times 1)$ vector. The choice of $y_{1t}$ is arbitrary as long as its coefficient in the cointegrating vector is non-zero. The hypothesis of cointegration can be tested by considering the following regression:

$$(3.4) \quad y_{1t} = \theta' y_{2t} + u_t$$

If the variables in $Y_t$ are cointegrated then there exists at least one $(n-1 \times 1)$ vector $\theta$ such that $u_t = y_{1t} - \theta' y_{2t}$ is $I(0)$. Since $u_t$ is not known, its estimate is constructed by running ordinary least squares (OLS) to equation (3.4). Now, testing for cointegration amounts to testing whether the estimated $u_t$ series is $I(0)$ through unit root tests. An augmented Dickey-Fuller test or Phillips-Perron test (Phillips and Perron 1988) can be applied as in the univariate case. The null hypothesis is no cointegration against the alternative of cointegration. However, the critical values of these tests can not be used due to the multivariate nature/ dimension of the vector $y_{2t}$. Critical values for augmented Dickey-Fuller t-test and Phillips-Perron $Z(\pi)$ and $Z(t_n)$ tests are tabulated in Phillips and Ouliaris (1990) and Engle and Yoo (1987) for the dimension of $y_{2t}$ ranging from 1 to 5.
The Engle and Granger procedure has been criticized on the basis of considerable small sample bias in the parameter estimates (Banerjee et al. 1986). Also, this test procedure does not have well defined limiting distributions, and testing for cointegration is not a straightforward procedure (Hall 1989). For example, testing for unit roots in the individual variables is a prerequisite for the cointegration test, yet the critical values are not adjusted accordingly and there is no theory that would allow for this adjustment (Campbell and Perron 1991). One serious limitation of the Engle and Granger procedure is that it does not address the issue of multiple cointegrating vectors. Procedures based on the cointegrating rank are meant to address the issue of multiple cointegrating vectors and one commonly used method is Johansen’s maximum likelihood procedure (Johansen 1988).

Johansen considers a general autoregressive representation of the n-dimensional vector $Y_t$:

$$Y_t = \pi_1 Y_{t-1} + \cdots + \pi_k Y_{t-k} + \mu + \varepsilon_t \quad t=1,\ldots,T. \tag{3.5}$$

where $\varepsilon_t$ is a normally independently identically distributed n-dimensional vector with zero mean and covariance matrix $\Omega$. Considering the fact that our variables of interest are of integrated nature, and generally are I(1), it is customary to write equation (3.5) in first difference form:

$$\Delta Y_t = \sum_{j=1}^{k-1} \Gamma_j \Delta Y_{t-j} + \Pi Y_{t-k} + \mu + \varepsilon_t \tag{3.6}$$
The issues concerning cointegration can be analyzed by comparing the stochastic properties of both sides of equation (3.6). Assume for a moment that the variables in $Y_t$ are $I(1)$. Then, the left hand side of (3.6) is stationary. The right hand side will be stationary provided that the terms of $\Pi Y_{t-k}$ are stationary. Three interesting cases can be identified using an argument related to the rank of the $\Pi$ matrix:

(i) rank($\Pi$) = $n$ (full rank), in which case all elements of $Y_t$ are stationary;
(ii) rank($\Pi$) = 0 (null matrix), there is no linear combination of $Y_t$ which is stationary;
(iii) $0 < \text{rank}(\Pi) = r < n$, there exist $(n \times r)$ matrices $\alpha$ and $\beta$ such that

$$\Pi = \alpha \beta'$$

In case (iii) for $\Pi Y_{t-k}$ to be stationary, $\beta'Y_{t-k}$ must be stationary. Hence the columns of $\beta$ give the $r$ linearly independent cointegrating vectors. Note that the partition $\Pi = \alpha \beta'$ is not unique as for any nonsingular $G$ matrix, $\Pi = (\alpha G)(G^{-1}\beta')$ is also a valid partition. This is because the data can only give information about the space spanned by $\alpha$ and $\beta$. The suggested solution is that one normalizes each column of $\beta$ with respect to one element which is known a priori to be non-zero.

The estimation of $\alpha$ and $\beta$ proceeds as follows: $\Delta Y_t$ is regressed on the lagged differences of $\Delta Y_t$ and a set of residuals, $R_{ct}$, is obtained; also $Y_{t-k}$ is regressed on the
same lagged differences and another set of residuals, $R_{sh}$, is obtained. The likelihood function is proportional to

\[
L(\alpha, \beta, \Omega) = |\Omega|^{-\frac{1}{2}} \exp\left[-\frac{1}{2} \sum_{t=1}^{T} (R_{ot} + \alpha \beta') R_{ot}' \Omega^{-1} (R_{ot} + \alpha \beta')\right]
\]

If $\beta$ were fixed, the likelihood function could be maximized with respect to $\alpha$ and $\Omega$ by regressing $R_{ot}$ on $-\beta' R_{sh}$ to give

\[
\hat{\alpha}(\beta) = -S_{ok} \beta (\beta' S_{kk} \beta)^{-1}, \quad \hat{\Omega}(\beta) = S_{oo} - S_{ok} \beta (\beta' S_{kk} \beta)^{-1} \beta' S_{ko}
\]

where

\[
S_{ij} = (1/T) \sum_{t=1}^{T} R_{it} R_{jt}', \quad i, j = o, k.
\]

Therefore, maximizing the likelihood function is equivalent to minimizing

\[
|S_{oo} - S_{ok} \beta (\beta' S_{kk} \beta)^{-1} \beta' S_{ko}|
\]

and it can be shown that (3.12) will be minimized when

\[
|\beta' S_{kk} \beta - \beta' S_{ko} S_{oo}^{-1} S_{ok} \beta| / |\beta' S_{kk} \beta|
\]

attains a minimum with respect to $\beta$. The maximum likelihood estimator of $\beta$ is found by solving

\[
|\lambda S_{kk} - S_{ko} S_{oo}^{-1} S_{ok}| = 0
\]
so that \( n \) estimated eigenvalues \((\hat{\lambda}_1 > \hat{\lambda}_2 > \ldots > \hat{\lambda}_n)\), and the corresponding \( n \) eigenvectors \((\hat{\phi}_1, \ldots, \hat{\phi}_n)\) are formed. The matrix of eigenvectors, \( V \), is normalized such that

\[
(3.15) \quad V^\prime S_{kk} V = I
\]

and the \( r \) cointegrating vectors are given by the \( r \) statistically significant eigenvectors, i.e.,

\[
(3.16) \quad \hat{\beta} = (\hat{\phi}_1, \ldots, \hat{\phi}_r)
\]

and \( \alpha \) is constructed following equation (3.8).

In order to find the statistically significant cointegration vectors Johansen suggests the following likelihood ratio statistics (also known as trace and maximum eigenvalue statistics):

\[
(3.17) \quad \lambda_{tr} = -T \sum_{i=q+1}^{n} \ln(1-\hat{\lambda}_i)
\]

\[
(3.18) \quad \lambda_{max} = -T \ln(1-\hat{\lambda}_{q+1})
\]

The trace statistic which is given by equation (3.17) tests the null hypothesis \( r \leq q \) against the general alternative \( r = n \), while the maximum eigenvalue statistic defined in equation (3.18) tests the null \( r = q \) against the alternative hypothesis that \( r = q + 1 \). The distributions of these statistics are not standard, since under the null hypothesis, the estimated eigenvalues correspond to \( n-r \) nonstationary common trends rather than stationary linear combinations of the data. The empirical distributions are multivariate versions of the Dickey-Fuller distribution and are derived in terms of a multivariate
Brownian motion. The critical values are tabulated in Johansen and Juselius (1990), for values of n-r between 1 and 5.

After finding the significant cointegrating vectors, one can test hypotheses of the form \( \beta = H \varphi \) which amounts to testing linear restrictions on all vectors. These tests can be carried out using the following likelihood ratio test

\[
(3.19) \quad \lambda_{LR} = -T \sum_{i=1}^{r} \ln\left(\frac{1 + \hat{\lambda}_i}{1 - \hat{\lambda}_i}\right)
\]

where \( r \) is the number of significant cointegrating vectors found by applying the tests defined in equations (3.17) and (3.18), and \( \hat{\lambda}_i \) and \( \tilde{\lambda}_i \) are the estimated characteristic roots from the unrestricted and restricted models respectively. The asymptotic distribution of this statistic is shown in Johansen (1988) to be \( \chi^2 \) with \( rs \) degrees of freedom, where \( s \) is the number of restrictions imposed on the cointegration vectors.

**Multiple Cointegration and Structural Models: A Methodology**

Although the Johansen procedure provides a framework within which one can estimate the significant cointegrating vectors and test restrictions upon them, it is difficult to interpret more than one cointegrating relationship in many applications. One practical solution in dealing with multiple cointegrating vectors is to consider economic theory and pick up the vector that has a "reasonable interpretation". However this is too ad hoc and the existence of multiple cointegrating relationships consists of an important piece of information one can not ignore. An important question is whether an
underlying structure can be identified in the case of multiple cointegrating vectors.

Recently, Bagliano, Fevero and Muscatelli (1991) have suggested a procedure to test a structural model with cointegrated variables, which they applied to Italian money demand. The procedure involves applying the Johansen procedure to all the simultaneous equation model variables as an initial step to find whether the number of significant cointegration relationships exactly match the number of behavioral equations in the model. Having confirmed that, one can apply the Engle and Granger OLS procedure to estimate the cointegration vectors implied by the structural model and construct the error correction terms. These terms are then imposed as restrictions on the unrestricted model. The validity of these restrictions can be tested by applying the likelihood ratio test.

Along these lines, Smith and Hagan (1991) also suggested interpreting each cointegrating relationship as a reduced form or behavioral relationship stemming from a structural model. These attempts capture very important themes, but there are many unanswered questions. First, applying OLS to estimate individual structural relationships as cointegrating vectors is not a reliable procedure since it assumes a unique cointegrating vector among the reduced set of variables. The OLS will not give consistent parameter estimates when this assumption is invalid. It is desirable to test this assumption by applying the Johansen procedure to the reduced set of variables. Second, cointegration framework does not allow for the identification of the short run dynamics. In many applications information about the adjustment process is crucial in
understanding the interrelationships in the system. Finally, a unified approach to identification, testing and the derivation of short run dynamics is required. In this section we suggest a unified approach to identifying structural relationships in systems with cointegrated variables, testing the validity of these relationships, and deriving the adjustment processes.

We follow Bagliano, Fevero and Muscatelli (1991) and Smith and Hagan (1991) in interpreting each cointegrating relationship as a reduced form or behavioral relationship stemming from a structural model. In order to identify the structure, we propose beginning with a flexible model that can accommodate a broad class of short run dynamic behaviors. If the model is true in the sense that it approximates the behavior that generated the data, then each behavioral relationship of the model must hold as a cointegrating relationship. Imposing the structure of the model by estimating its behavioral relationships as cointegrating relationships is the second step. Since cointegrating relationships are long run relationships by definition, they can not distinguish between models that have the same long run relationships but different adjustment processes. We propose using conventional innovation accounting and impulse response functions to derive short run dynamics of the system.

**Econometric issues**

According to Engle and Granger (1987), a single cointegrating vector has a straightforward interpretation as a long-run "equilibrium" relationship. Nevertheless, the
existing literature has paid scant attention to the existence and interpretation of multiple cointegrating vectors. As we illustrate later, each of the \( r \) equations represented by \( \beta'Y_t \) can be interpreted as an independent behavioral or reduced-form long-run relationship.

The econometric problem is to properly identify each of the cointegrating vectors. The resolution of this identification problem is trivial when each cointegrating relationship has a direct interpretation as a behavioral relationship. For example, as we will demonstrate later in Chapter VI, two cointegrating relations in the money market can be interpreted as money supply and demand relations if proper theoretical criteria are met. One can also impose restrictions on the cointegrating vectors to see if such interpretation is possible. The problem is that one cannot always impose and test arbitrary sign and exclusion restrictions on individual cointegrating vectors.

In order to identify behavioral equations from a structural model we undertake the following steps:

1. Pretest all variables to be included in the \( Y_t \) matrix for stationarity. It is generally inappropriate to mix variables which are integrated of different orders. Having selected the appropriate variables, use the Johansen procedure to obtain the number of cointegrating vectors. This is the tentative number of behavioral or reduced form relations in the model.

2. Economic theory may suggest the existence of certain structure on the variables. The existence of hypothesized zero restrictions can be tested by the trace and maximum eigenvalue statistics. In order to identify each behavioral relation, one can
impose the zero identifying restrictions by running the Johansen procedure with the appropriate variables excluded. If the remaining variables are then found to be cointegrated, the exclusion restriction is deemed to be appropriate. If no cointegrating vector is found, the restriction suggested by the model must be rejected.

(3) It is possible to test the overall structure of the model by forming the error correction terms (residuals from the cointegrating vectors) which correspond to the structural relationships and imposing them on the VAR as suggested by Bagliano, Favero, and Muscatelli (1991). This is a test of fixing the cointegrating vectors. However, we use structural equilibrium errors found in step two instead of using those found by static OLS regression. This test is conditional on all parameter values, hence is too strong for testing the zero identifying restrictions. Additional evidence can be provided by dynamic analysis of the system (innovation accounting based on the error-correction representation).

(4) It is also possible to reestimate equation (3.6) using the same structural error correction terms. This is the error-correction representation of the system. Finally, innovation accounting (variance decomposition, and impulse response functions) can be used to obtain information concerning the dynamics of the restricted system.

(5) As a final step, one can form error correction terms from the unrestricted cointegrating vectors and do variance decomposition and impulse response with the unrestricted system. Comparing the results in step 4 and 5 provides another method of testing the plausibility of the underlying theory.
Summary and Conclusions

In this chapter, cointegration techniques are summarized with a special reference to the Johansen procedure based on Johansen (1988). In addition, a modelling strategy for models with cointegrated variables is proposed. The idea is to interpret each cointegrating vector as a structural or reduced form equation stemming from a theoretical model. To identify the structure, one can impose the zero exclusion restrictions to see if hypothesized relations exist. If the structure is identified, the resulting cointegrating vectors are long run relations by definition, and have interpretations as steady state functions. Errors from these functions can be combined to the error correction representation and the short run dynamics can be derived using innovation accounting techniques.
Endnotes

1. As most macroeconomic time series are either I(1) or I(0), we do not discuss multiple unit roots. If the series is suspected to be integrated of order higher than one, the unit root test can be applied successively to the differenced series.

2. If the series is suspected to have a deterministic trend, one can include the time trend as a regressor. The most commonly used trend model is the linear one.

3. When \( \pi > 0 \), the series is non-stationary as well, but in this case conventional econometric techniques are still applicable.

4. These methods are surveyed in detail in Campbell and Perron (1991), and Muscatelli and Hurn (1991).

5. When there are multiple cointegrating vectors, the Engle-Granger procedure yields inconsistent parameter estimates.

6. Another test of cointegration based on the cointegrating rank has been proposed by Stock and Watson (1988).

7. It is also inappropriate to arbitrarily mix trending variables with integrated variables. If the system contains integrated variables as well as trending variables, an additional requirement for cointegration is that those linear combinations which cancel the unit roots also cancel the deterministic trends. For details see Campbell and Perron (1991).

8. At this stage, it is possible to obtain the VAR in the form of equation (3.6) using the appropriate number of error-correction terms. The unrestricted cointegrating vectors obtained in step (1) yield the long-run relationships embedded in the system. Standard innovation accounting techniques (Variance Decompositions and Impulse Response Functions) can be used to analyze the dynamics of the unrestricted system. For details on innovation accounting in a VAR framework with cointegrated variables see Lutkepohl and Reimers (1992).
CHAPTER IV. THE DEPENDENT ECONOMY MODEL

The Model

In this section we present a simple model that is compatible with most theoretical monetary models of exchange rate determination. In addition, the model allows real shocks to cause permanent deviations from PPP. It is based on Dornbusch (1973, 1974) and is called the "dependent economy model" to emphasize the small country assumptions. The country is assumed to be a price taker in the world market for both importables and exportables. This assumption implies that terms of trade is exogenously given, which allows for the aggregation of importables and exportables into a composite commodity called traded goods. The model is particularly convenient in that it is familiar, has a simple formulation, and contains straightforward identification restrictions.

Consider a simple two-country version of the Dependent Economy Model. Equilibrium in the money markets requires:

\[ m_t = k + p_t + \eta y_t - \lambda r_t ; \quad \eta, \lambda > 0 \]

\[ m^*_t = k^* + p^*_t + \eta^* y^*_t - \lambda^* r^*_t ; \quad \eta^*, \lambda^* > 0 \]

where \( m \) is the log of the domestic money supply; \( p \) is log of the domestic price level; \( y \) is log of domestic output (=income); \( r \) is domestic interest rate; \( t \) is time subscript; (*) denotes the foreign counterpart of domestic variables; and \( k, \theta, \) and \( \lambda \) are constants.
Following conventional practices, we assume $\eta = \eta^*$ and $\lambda = \lambda^*$ so that we can form:

$$m_t - m_t^* = (k - k^*) + (p_t - p_t^*) + \eta(y_t - y_t^*) - \lambda(r_t - r_t^*)$$  

(4.3)

Recent work on the demand for money suggests that equations (4.1) and (4.2) [hence equation (4.3)] hold only as long-run equilibrium relationships. Thus, we depart somewhat from the original Dornbusch formulation in that our equation (4.3) is not intended to represent equilibrium at a point in time; rather, it represents the long-run/cointegrating relationship in the money market.

**Purchasing power parity**

The relationship between the exchange rate and national price levels is an important ingredient of most theoretical exchange rate models. Monetary models of the exchange rate, such as Mussa (1976), usually link national price levels (i.e., $p$ and $p^*$) by assuming the existence of PPP. However, this theory has been subject to extensive empirical investigation since its original formulation by Cassel (1922) and the results have not been favorable enough to assume it without question. Baillie and Selover (1987), Baillie and Pecchenino (1992), among others, provide evidence that the exchange rate and national price levels are not even cointegrated. The questionable performance of PPP particularly during post-Bretton Woods period suggests the use of an alternative version of PPP.

It is well-known that PPP theory hinges upon restrictive theoretical assumptions.
As has been emphasized in Chapter II, differential speeds of adjustment in asset and goods markets lead to transitory departures from PPP. Permanent departures from PPP arise as a consequence of real shocks that alter equilibrium relative prices. In our model we will assume that productivity differentials across tradables and nontradables alter equilibrium relative prices which in turn affect the nominal exchange rate. This can be illustrated more formally as follows:

Let the domestic price level be a weighted average of the prices of traded goods and non-traded goods:

\[
(4.4) \quad p_t = \theta p^T_t + (1 - \theta) p^N_t
\]

where:

\[
\rho_t = p^N_t - p^T_t
\]

and, \(p^T\) is the log of the price of traded goods; \(p^N\) is log of the price of non-traded goods; \(\rho\) is log of the relative price of non-traded good; and \(\theta\) is a share parameter. A similar relationship exists for the prices in the foreign country.

Assuming commodity arbitrage in the traded good only:

\[
(4.5) \quad p^T_t - p^{*T}_t + s_t
\]

where \(s_t\) is the log of the domestic currency price of foreign currency.

Combining equations (4.4) and (4.5) and assuming that the share parameters for the two countries are equal (i.e., \(\theta = \theta^*\)), yields:

\[
(4.6) \quad s_t - p_t - p^{*}_t = (1 - \theta)(\rho_t - \rho^*_t)
\]
Equation (4.6) is the modified Purchasing Power Parity (PPP) relationship; as long as there are no relative price changes, relative PPP will hold. However, relative price shocks (i.e., changes in $p-p^*$) can cause deviations from PPP.

An important feature of the Dependent Economy Model is that the relative price of non-traded goods is determined by productivity differentials across sectors. To best understand the relationship between productivity and relative prices, consider Figure 4.1.

![Diagram](image)

Figure 4.1. Productivity, relative prices and the exchange rate
The production possibility curve between tradables and non-tradables is given by AB. Let the relative price of the non-traded good be given by the slope of line CD; hence, $p_t = OC/OD$. Note that line EF is parallel to CD. Assuming full employment, at this particular value of $p_t$, production takes place at point H. As shown in the Figure, consumption takes place at point K (on the indifference curve labeled I). The market for the non-traded good clears since domestic production equals domestic demand. The excess demand for the tradable good can be satisfied by importing $EC = KH$ units of tradables.

The depiction of a nation with a trade deficit is necessarily a short-run phenomenon; given that a nation cannot forever borrow from abroad, long-run equilibrium requires that net imports equal zero. The dynamic adjustment of the system is such that the wealth reduction associated with the trade deficit leads to a reduction in expenditure levels. Let the income expansion curve be given by line OK. Hence, any decline in expenditures must be accompanied by a relative price change so as to preserve equilibrium in the market for non-tradables. Thus, long-run equilibrium must occur on the production possibilities curve somewhere along line segment JH. The point is that there is a unique long-run relative price of non-tradables for any given set of preferences and technology. As can be inferred from Figure 4.1, given preferences, an increase in the relative productivity of tradables will be associated with a relatively high price of the non-traded good. Given equation (4.6), a higher price of the non-traded good implies appreciation of domestic currency.
Closing the model entails (i) linking domestic and foreign interest rates (usually through uncovered interest parity), (ii) the appropriate specification of the goods market clearing relationships (e.g., sticky prices versus full market clearing), and (iii) linking spot and forward exchange rates. We elect not to make any specific assumptions regarding these issues; our intent is to construct a model which is consistent with this framework. The error-correction representation can suggest the proper specifications regarding the dynamics of the model.

Summary and Conclusions

In this chapter, we have presented an illustrative monetary model of exchange rate determination. The questionable performance of PPP during the post-Bretton Woods period suggests an alternative specification of PPP. With small country assumptions, we model productivity differentials between tradables and non-tradables as the driving force that alter equilibrium relative prices and hence the exchange rate. Equilibrium in the money market completes the model.
1. There is no theoretical reason to constrain the domestic and foreign behavioral parameters of the money demand functions to be equal. However, to conserve degrees of freedom, our estimation strategy requires this assumption. Moreover, equation (4.3) is a weaker restriction than (4.1) and (4.2) alone. For example, if there is "missing" money for both countries individually, equation (4.3) may still be quite stable.

2. For example, Boughton (1991) examines the long-run properties of the demand for money in five large industrial countries (United States, Japan, Germany, France, and the United Kingdom) under the hypothesis that the long-run functions are stable but that the dynamic adjustment processes are complex. To capture this property of the money demand functions the author suggests using cointegration techniques. The results broadly support this hypothesis. See, also Baba, Hendry and Starr (1992) and works cited therein for a review of recent empirical studies of the demand for money.

3. A detailed treatment and survey can be found in Officer (1976). More recent evidence is provided by MacDonald (1988), pp 214-218.

4. Note that a permanent change in preferences can lead to a permanent change in the relative price of the non-tradable. Unfortunately, we have no clear means of measuring preferences, hence, we abstract from this issue in our empirical estimations.
Identification and Estimation of the Model

In this section, we apply the methodology we suggested in Chapter III to the French Franc/U.S. Dollar exchange rate. The model we seek to identify is the Dependent Economy model presented in Chapter IV.

Making specific reference to the Dependent Economy Model, we let the 7 x 1 column vector $Y_t$ consist of the variables $(m_t-m^*_t)$, $(p_t-p^*_t)$, $(y_t-y^*_t)$, $(r_t-r^*_t)$, $s_t$, a measure of relative productivity (denoted by $pr_t-pr^*_t$), and a constant. If equations (4.3) and (4.6) hold as long-run relationships, we should find at least two independent cointegrating vectors for $Y_t$.

Given that there are two such cointegrating vectors, it should be possible to impose a set of zero restrictions such that the coefficient on $pr_t-pr^*_t$ is zero in one of the vectors and that the coefficients on $m_t-m^*_t$, $y_t-y^*_t$, and $r_t-r^*_t$ are zero in the other vector. These restrictions are imposed by running the cointegration test without the corresponding variables to see if the hypothesized long run relations exist. If these criteria are met, it is possible to test the imposed identifying restrictions as well as other specific restrictions pertaining to the coefficients (e.g., the normalized price coefficient is unity in the PPP relation).
Data

Our data set consists of variables consistent with the Dependent Economy Model. These variables are also typically included in the simple monetary model of exchange rate determination such as that of Dornbusch (1976), Mussa (1976) or Frenkel (1976). Specifically, we include relative money supplies (M1), the short-term interest rate differential (as measured by call money rate), relative output levels (as measured by real GNP), the relative price level (as measured by the GNP deflators), and nominal exchange period average rate. In addition, we include a measure of the relative productivity between tradables and non-tradables. We follow the usual practice measuring relative productivity in the tradable sector as the index of industrial employment to industrial output. This ratio for France divided by the ratio for the U.S. is our measure of international relative productivity. The data is quarterly from 1971.Q3, (the end of the Bretton Woods period) to 1990.Q4 for France. We report only bilateral results concerning the U.S. and France. All data are taken from the ROM-disk edition of International Financial Statistics.

The unrestricted cointegrating relationships

Before implementing the identification procedure we outlined above, we investigate the stochastic properties of the individual series (we are particularly interested in the order of integration of the various series). Table 5.1 reports the results of Augmented Dickey-Fuller (1979, 1981) unit root tests using 4 and 8 quarter lags
Table 5.1. Augmented Dickey-Fuller statistics for the exchange rate and the "fundamentals"

<table>
<thead>
<tr>
<th>Variable</th>
<th>Lags</th>
<th>ADF-t statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>s</td>
<td>4</td>
<td>-1.76</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>-1.65</td>
</tr>
<tr>
<td>p-p*</td>
<td>4</td>
<td>-1.43</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>-1.89</td>
</tr>
<tr>
<td>m-m*</td>
<td>4</td>
<td>-1.81</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>-2.11</td>
</tr>
<tr>
<td>y-y*</td>
<td>4</td>
<td>-1.72</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>-1.16</td>
</tr>
<tr>
<td>r-r*</td>
<td>4</td>
<td>-2.86</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>-2.64</td>
</tr>
<tr>
<td>pr-pr*</td>
<td>4</td>
<td>-2.38</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>-2.25</td>
</tr>
</tbody>
</table>

(the 5% critical value is -2.90). It is clear from the table that a unit root cannot be rejected for all series, with a possible exception of the short term interest rate differential.

Next we use the Johansen procedure to determine the number of cointegrating relationships for the Franc/Dollar nominal exchange rate, relative money supply, relative price level, short-term interest rate differential, relative output level, and the relative productivity level. Since the data are quarterly, and we have a potential degrees of
freedom problem with 8 lags, we use a lag length of 4. Likelihood ratio test statistics and their critical values regarding the number of long run equilibrium relationships in the system are presented in Table 5.2.

Using the \( \lambda \) test, we can reject the null hypotheses of \( r = 0 \) and \( r \leq 1 \) (against alternatives that \( r \geq 1 \) and \( r \geq 2 \), respectively) at conventional significance levels. The calculated value of \( \lambda \) for the null of \( r = 2 \) is barely rejected at the 5% significance level. Given that there are 2 or more cointegrating vectors, the \( \lambda_{\text{max}} \) test for a null of \( r = 2 \), against the specific alternative \( r = 3 \), cannot be rejected at the 95% level. Hence, there is strong evidence for two cointegrating relationships in the system. Allowing for a constant as the seventh element in the cointegrating vectors, the 7 x 2 cointegrating matrix \( \beta \) and the adjustment matrix \( \alpha \) are given by:

\[
(5.1) \quad \beta' = \begin{bmatrix} 2.71 & -2.90 & -2.91 & 10.96 & -1.72 & 11.72 & -2.3 \\ 1.21 & 0.22 & -11.86 & -21.21 & 0.60 & 24.24 & 0.14 \end{bmatrix}
\]

\[
(5.2) \quad \alpha = \begin{bmatrix} -0.0087 & 0.0178 & 0.0155 & -0.0061 & 0.0266 & -0.0108 \\ 0.0008 & 0.0002 & -0.0048 & 0.0022 & 0.0012 & -0.0263 \end{bmatrix}
\]

Normalizing each of the two cointegrating vectors with respect to the exchange rate, the two long-run equilibrium relationships are:

\[
(5.3) \quad s_t = 1.0713(m_t - m_t^*) + 1.0743(p_t - p_t^*) - 4.0485(y_t - y_t^*) + 0.6354(r_t - r_t^*) - 4.330(pr_t - pr_t^*) + 0.8502
\]

\[
(5.4) \quad s_t = -0.1860(m_t - m_t^*) + 9.8116(p_t - p_t^*) + 17.552(y_t - y_t^*) - 0.5006(r_t - r_t^*) - 20.057(pr_t - pr_t^*) - 0.1160
\]
Table 5.2. Cointegration tests for the French Franc and the "fundamentals"

<table>
<thead>
<tr>
<th>Null</th>
<th>$\lambda_t$</th>
<th>$\lambda_u(.95)$</th>
<th>$\lambda_{\max}$</th>
<th>$\lambda_{\max}(.95)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r=0$</td>
<td>135.9</td>
<td>102.1</td>
<td>53.8</td>
<td>40.3</td>
</tr>
<tr>
<td>$r=1$</td>
<td>82.1</td>
<td>76.1</td>
<td>28.5</td>
<td>34.4</td>
</tr>
<tr>
<td>$r=2$</td>
<td>53.7</td>
<td>53.1</td>
<td>22.6</td>
<td>28.1</td>
</tr>
<tr>
<td>$r=3$</td>
<td>31.1</td>
<td>35.0</td>
<td>13.0</td>
<td>22.0</td>
</tr>
<tr>
<td>$r=4$</td>
<td>18.2</td>
<td>20.0</td>
<td>11.8</td>
<td>15.7</td>
</tr>
<tr>
<td>$r=5$</td>
<td>6.4</td>
<td>9.2</td>
<td>6.4</td>
<td>9.2</td>
</tr>
</tbody>
</table>

As reported, there are no obvious explanations for the two equilibrium equations. Admittedly, the first equation (the most "significant" of the two cointegrating vectors) has many of the properties of a reduced form structural model of the exchange rate. The franc price of the dollar moves proportionately to the relative supply of francs, and relative increases in real French GNP and/or productivity act to appreciate the franc. The positive relationship between the interest rate differential and the franc price of the dollar is consistent with uncovered interest parity. Uncovered interest rate parity implies that the French interest rate will exceed the U.S. rate only when individuals anticipate a depreciation of the Franc. If such expectations are discounted into the current exchange rate, the current franc price of the dollar will be positively related to the interest rate differential. The second relationship is not directly interpretable. Note, however, that the exchange rate, relative prices and productivities do have signs consistent with the modified PPP relationship.
The structural equilibrium relationships

Our suggested solution to the problem of interpretation is to impose structure on the cointegrating relationships. Two of the key long-run equilibrium relationships in the Dependent Economy Model are the money market equilibrium relationship and the modified PPP relationship. The issue is whether these cointegrating vectors are consistent with the estimated structural relationships. In essence, we seek to impose the identification restrictions on the estimated $\beta$ matrix. Note that we can exactly identify the two relationships if we can impose the restrictions that $\beta_{11} = \beta_{16} = 0$ and $\beta_{22} = \beta_{24} = \beta_{26} = 0$, and still find "reasonable" equilibrium relationships. Normalization of the resulting two equations with respect to the relative money supplies and exchange rate respectively, yields the estimated structural relationships. These restrictions can be tested using a likelihood ratio test. We can also test restrictions concerning structural model parameters as long as these restrictions are homogenous. Cointegration tests carried out without the corresponding subset of variables are presented in Tables 5.3 and 5.4.

As shown in Table 5.3, the estimated values of $\lambda_{\omega}$ and $\lambda_{\text{max}}$ strongly indicate a single cointegrating vector between relative money supplies, price levels, income levels, and interest rates. Moreover Table 5.4 indicates that we cannot reject a null of a single cointegrating vector between the exchange rate, relative price levels, and relative productivity at conventional significance levels.
Table 5.3. Cointegration tests for money market variables

<table>
<thead>
<tr>
<th>Null</th>
<th>$\lambda_r$</th>
<th>$\lambda_r(.95)$</th>
<th>$\lambda_{\text{max}}$</th>
<th>$\lambda_{\text{max}}(.95)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r=0$</td>
<td>60.6</td>
<td>53.1</td>
<td>35.0</td>
<td>28.1</td>
</tr>
<tr>
<td>$r=1$</td>
<td>25.6</td>
<td>35.0</td>
<td>15.3</td>
<td>22.0</td>
</tr>
<tr>
<td>$r=2$</td>
<td>10.3</td>
<td>20.0</td>
<td>6.9</td>
<td>15.7</td>
</tr>
<tr>
<td>$r=3$</td>
<td>3.5</td>
<td>9.2</td>
<td>3.5</td>
<td>9.2</td>
</tr>
</tbody>
</table>

Table 5.4. Cointegration tests of the modified purchasing power parity

<table>
<thead>
<tr>
<th>Null</th>
<th>$\lambda_r$</th>
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Appropriately normalizing these two cointegrating vectors, we find the following long-run relationships:

\[(5.5) \quad m_t - m_t^* = 1.396(p_t^* - p_t^*) + 4.2239(y_t^* - y_t^*) + 0.326(r_t^* - r_t^*) - 0.029\]

\[(5.6) \quad s_t = 2.72(p_t^* - p_t) - 4.53(pr_t^* - pr_t) - 2.27\]
The money market "equilibrium" relationship is reasonably well-behaved, although the implied income elasticity of demand is rather large while the estimated price elasticity of 1.396 is "too high". Although the point estimate of the interest rate elasticity is of the wrong sign, the $\chi^2_{df=1}$ test statistic for the null that the elasticity is zero is 0.916, indicating that the hypothesis cannot be rejected. Further, we test whether money and prices have equal coefficients of opposite sign. A $\chi^2_{df=1}$ test rejects this restriction at 1.3% significance level. Since the interest term is not significant, we estimate the money market equilibrium relationship without interest rate differential:

$$m_t - m_t^* = 1.406(p_t - p_t^*) + 3.891(y_t - y_t^*) - 0.0041$$

This relation looks reasonable and will serve as our final money market equilibrium relationship.

The modified-PPP relationship also has the "correct" signs. Relative increases in the French price level are associated with increases in the price of the dollar, whereas relative increases in French productivity are associated with decreases in the price of the dollar. Again, the coefficient on the price level is too large; the $\chi^2_{df=1}$ test statistic that this coefficient is unity can be rejected at less than 1% significance level. Finally, a joint test that the price term is unity while the productivity term is zero (strict PPP holds) yields a $\chi^2_{df=2}$ of 20.38, suggesting that the restriction is clearly rejected.

Other than the magnitude of the coefficient on the price terms, we have identified the two key long-run relationships which are consistent with a structural
exchange rate model. Moreover, our findings are compatible with those of MacDonald (1992) and Patel (1991). Both of these papers discuss reasons why measurement errors in price level series can account for such results which appear to conflict with long-run money neutrality.

**Test of fixing the cointegrating vectors**

In this section we simultaneously test all restrictions summarized by fixing the parameters of the two cointegrating vectors corresponding to the identified structural relationships. In a sense, this is an indirect test of the validity of the zero exclusion restrictions. Consider the restricted \( \beta' \) matrix:

\[
\begin{bmatrix}
0 & -1 & 1.41 & 3.89 & 0 & 0 & -0.0041 \\
-1 & 0 & 2.72 & 0 & 0 & -4.53 & -2.27
\end{bmatrix}
\]

which corresponds to equations (5.6) and (5.7). A likelihood ratio test, which is distributed as \( \chi^2_{6-10} \), yields the value of 35.69 indicating that the restrictions are clearly rejected at conventional significance levels. However, this test is overly strong in that it is conditional on all parameter values; rejection can be due to the fixing of any one of the parameters. Hence, the overall support for the model is mixed. Additional evidence concerning the model can be provided by an analysis of the dynamic system.

**The Analysis of the Dynamic System**

Since the identified model relationships are long run relationships in nature, they can be compatible with many short-run adjustment processes. In this section we use
conventional innovation accounting and impulse response functions to investigate the short run dynamics of the system. We undertake this task using two alternative equilibrium specifications. First, we use equilibrium errors obtained from the identified structural relationships which are given by equations (5.6) and (5.7). In the second specification we take a statistical approach. Without assuming any structure, we use equilibrium errors obtained from equations (5.3) and (5.4) and repeat the same exercise. Comparing the results provides another method of assessing the validity of the identified structure.

**Variance decomposition and impulse response functions of the "restricted system"**

In this step, we re-estimate the system imposing the restricted cointegrating vectors on the system. Manipulating equation (3.6) yields the error-correction representation:

\[
\Delta Y_t = A(L)\Delta Y_{t-1} + \alpha \beta' Y_{t-1} + \mu + \epsilon_t
\]

where: \( A(L) \) is an \( n \times n \) matrix with elements which are \( k \)-order polynomials in the lag operator \( L \). Here the \( \beta' Y_{t-1} \) are the residuals formed using equations (5.6) and (5.7). This equation is estimated with \( Y_t \) containing the variables: relative money supplies, productivity, income, prices, and the exchange rate. We now can proceed with the usual innovation accounting analysis typical of VAR analysis.
**Variance decomposition** The top portion of Table 5.5 reports the Variance Decomposition Analysis for four different forecasting horizons using a Choleski Decomposition.\(^7\)

The order of the variables is that implied by the Table: money → productivity → income → prices → exchange rate. Note that money and productivity "explain" the preponderance of their forecast error variance. After 4 quarters, income and prices explain approximately 50% of their forecast error variance. Possibly the most important result is that the exchange rate is affected by other variables in the system. The exchange rate shares a long-run equilibrium with other variables and these other variables explain approximately 35% of exchange rate forecast error variance. This is supporting evidence that we have identified an exchange rate determination model. Although the exchange rate follows near random walk behavior we have found other variables which share common trends with the exchange rate. Notice also that the exchange rate accounts for relatively little of the forecast error variance of these other variables.

The lower portion of the table shows the contemporaneous correlation matrix of the innovations. The correlation coefficients between income and productivity innovations (0.51), and productivity and price innovations (0.37), are high. The middle portion of the Table shows the Variance Decomposition at a 12 quarter forecasting
Table 5.5. Variance Decomposition and the correlation matrix of the innovations

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Reverse Order

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horizon using the reverse ordering exchange rate → prices → income → productivity → money. As expected from the correlation coefficients, the importance of productivity for income and for prices is diminished. Without further restrictions, it is not possible to easily disentangle productivity and income shocks, and productivity and price shocks. For our purposes, the key feature is that the money supply continues to explain a sizable proportion of exchange rate movements (20.1 %) even with the reverse ordering. Moreover, exchange rate movements explain relatively little (less than 6%) of the forecast error variance of the other variables.

Impulse response functions  The dynamic relationships among the variables can be best understood by examining the impulse response functions (we present the impulse responses for the first ordering only). Figures 5.1a and b give the responses to one standard deviation shock in the money supply (note that the system is expressed in first differences). In response to a typical money supply shock, both the price level and the price of the foreign currency increase. As predicted by the Dependent Economy Model, domestic money expansion is associated with domestic inflation and a depreciation of the domestic currency. After the third quarter the price level, but not the exchange rate, begins to fall back towards its original level.\(^8\) Notice also that there is no evidence of overshooting as would be predicted by the Dornbusch model. The money supply innovation appears to have "Keynesian" effects in that it is associated with what appear to be permanent effects on output and productivity (Figure 5.1a). A typical money
Figure 5.1. Responses to a money supply shock: a) productivity and output responses
Figure 5.1. (continued) b) price and exchange rate responses
supply shock is associated with increases in both income and productivity; since the changes are never negative, the increases appear to be permanent. Our explanation of the findings is that the ordering of the variables is crucial in such interpretations; reversing the ordering (so that productivity and income come before money) yields results that income (or productivity) shocks increase the money supply.

The "own" effects of productivity innovations (see Figure 5.2a) and the effects on income quickly die out after the first quarter. Although the contemporaneous effect on prices is positive (Figure 5.2b), there is a general decline in prices after the productivity shock. Surprisingly, the productivity shock has little effect on the exchange rate. The implication is that the productivity shocks are responsible for deviations from PPP; however, it is prices which respond to productivity shocks.

Responses to a price shock are given in Figures 5.3a and b. The price shock has no contemporaneous effect on either productivity or output. However in second and third quarters productivity rises then falls back. Similarly output rises in the second quarter then falls. These can be explained as supply side effects.

Finally, exchange rate shocks (see Figures 5.4a and b) have small price level effects and the own effects disappear gradually. On the other hand, an exchange rate shock has Keynesian effects in that a currency depreciation is associated with increases in domestic output and productivity.
Figure 5.2. Responses to a productivity shock: a) productivity and output responses
Figure 5.2. (continued) b) price and exchange rate responses
Figure 5.3. Responses to a price shock: a) productivity and output responses
Figure 5.3. (continued)   b) price and exchange rate responses
Figure 5.4. Responses to an exchange rate shock: a) productivity and output responses
Figure 5.4. (continued)  b) price and exchange rate responses
Variance decomposition and impulse response functions with unrestricted equilibrium errors

In this section we do variance decomposition and impulse response analysis using errors from the unrestricted cointegrating vectors. Since no structure is imposed on the cointegrating vectors, the dynamic analysis with unrestricted equilibrium errors can be insightful as to how much we gain by imposing a structure consistent with a theoretical model. Also, the unrestricted system should reveal the importance of the interest rates in the short run dynamic behavior of the variables. The method we employ is the same as the one in the previous section with the following order of variables in the Choleski decomposition: money → productivity → interest rates → income → prices → exchange rate.

Variance Decomposition  Table 5.6 gives variance decomposition at four different forecasting horizons, and the correlation matrix of the innovations. The top portion of the table reports results with an order implied by the table. Comparing variance decomposition of the restricted system and the unrestricted system, one can notice some differences. Money explains a larger portion of its forecast error variance in the unrestricted system. Productivity innovations, on the other hand, explain a smaller proportion of their own forecast error variance. In addition to explaining output and price variability, productivity innovations explain the variability of interest rates.

Probably the most important result of Table 5.6 is that exchange rate accounts
Table 5.6. Variance decomposition with "unrestricted" equilibrium errors

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<th>y-y*</th>
<th>p-p*</th>
<th>s</th>
</tr>
</thead>
<tbody>
<tr>
<td>m-m*</td>
<td>1.00</td>
<td>-0.08</td>
<td>-0.06</td>
<td>0.01</td>
<td>0.18</td>
<td>0.17</td>
</tr>
<tr>
<td>pr-pr*</td>
<td>1.00</td>
<td>0.42</td>
<td>0.60</td>
<td>0.34</td>
<td>-0.10</td>
<td></td>
</tr>
<tr>
<td>r-r*</td>
<td>1.00</td>
<td>0.42</td>
<td>0.27</td>
<td>-0.23</td>
<td></td>
<td></td>
</tr>
<tr>
<td>y-y*</td>
<td>1.00</td>
<td>0.19</td>
<td>-0.13</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>p-p*</td>
<td></td>
<td>-0.18</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

for a larger proportion of its forecast error variance. In other words, other variables in
the system explain a larger proportion of the forecast error variance in the restricted
(structural) model. After 12 quarters, the variables in the system explain only 23 % of
the forecast error variance of the exchange rate in the unrestricted model as compared to
35 % in the restricted model. This piece of evidence is supportive of the Dependent
Economy Model.

As in the restricted model, doing variance decomposition with the order of the
variables reversed does not alter the main results, although it does alter results concerning the order of highly correlated variables. The correlation matrix of the innovations is similar to that of the restricted system except interest rate innovations are highly correlated with both productivity and output innovations. Since interest rates are absent in the restricted system and are correlated with productivity and output, it is difficult to know whether adding the interest rates to the system improves its explanatory power.

**Impulse response functions** The impulse response functions for the unrestricted system are given in Figures 5.5a through 5.9b. Money supply shock appears to have no productivity effect in the unrestricted model. However output, price and exchange rate effects are the same as in the structural model (Figures 5.5a and b). Figure 5.5c gives the interest rate response to a money supply shock. The money supply seems to have no permanent effect on the interest rates. The initial response is negative as expected; subsequent changes are both positive and negative with no overall long-run effect.

Responses to a productivity shock are similar to the structural model as well (Figures 5.6a and b). The interest rate shock appears to have a small effect on output; after an initial positive change in output, the change is negative. The interest rate effect on productivity is overall negative since negative changes dominate the initial positive change (Figure 5.7a).
Figure 5.5. Responses to a money supply shock in the unrestricted system: a) productivity and output responses
Figure 5.5. (continued)  b) price and exchange rate responses
Figure 5.5. (continued)  c) interest rate response
Figure 5.6. Responses to a productivity shock in the unrestricted system: a) productivity and output responses
Figure 5.6. (continued)  b) price and exchange rate responses
Figure 5.7. Responses to an interest rate shock in the unrestricted system: 
a) productivity and output responses

The exchange rate and price responses to an interest rate shock are quite interesting (Figure 5.7b). The price and exchange rate respond by moving in opposite directions, with an initial exchange rate appreciation. Since changes are both positive and negative, the interest rate has a transitory effect on prices and exchange rate.

The output and productivity responses to a price shock appear to be different in
Figure 5.7. (continued)  b) price and exchange rate responses
the unrestricted model (Figure 5.8a). In the unrestricted model productivity changes are primarily negative whereas they are positive in the structural model. The price and exchange rate responses appear to be similar, however (Figure 5.8b).

Finally, the output and productivity responses to an exchange rate shock have similar "Keynesian" effects except the productivity effect is magnified in the unrestricted model (Figure 5.9a). Exchange rate and price responses to an exchange rate shock are almost identical in both models (Figure 5.9b)

**Summary and Conclusions**

This chapter applies the methodology suggested in Chapter-III to identify a structure compatible with the Dependent Economy Model presented in Chapter IV. The cointegration tests indicate the presence of two distinct cointegrating vectors which do not have straightforward interpretations. Imposing zero identifying restrictions, we find that the two restricted cointegrating vectors are consistent with a) a money market equilibrium relationship, b) modified PPP relationship of the Dependent Economy Model. Using equilibrium errors from the restricted and unrestricted versions of the cointegrating vectors, we proceed with a dynamic analysis based on the error correction representation. Impulse response and variance decomposition results are somewhat supportive of the model.
Figure 5.8. Responses to a price shock in the unrestricted system: a) productivity and output responses
Figure 5.8. (continued) b) price and exchange rate responses
Figure 5.9. Responses to an exchange rate shock in the unrestricted system: a) productivity and output responses
Figure 5.9. (continued) b) price and exchange rate responses
Endnotes

1. In a sense, such structural identification of the cointegrating vectors is straightforward as compared with classical identification. Here, it is not necessary to make assumptions as to which variables are endogenous (since the validity of the Johansen procedure does not require such assumptions). Moreover, exact identification is facilitated since each equation in the system can be estimated directly.

2. It is common practice to proxy relative productivity in the traded good sector to that in the non-tradable sector by manufacturing productivity alone. Thus for the French/U.S. case, $p_r/p_r^*$, is defined to be: French manufacturing employment/French index of industrial production all divided by the corresponding ratio for the U.S.

3. It would be interesting to use a "system approach" in which shocks from other countries affect the macroeconomic variables in a given country. Since we have to rely on asymptotic theory, degrees of freedom considerations preclude us from pursuing this subject.

4. Diagnostic tests indicate that the residuals from the 4 lag model approximate white noise.

5. However, there are questions as to what the elasticities "should be" in the studies of the demand for money. Boughton (1991) found, among other things, that the traditionally accepted restrictions about long-run homogeneity with respect to the price level and unitary or less than unitary real income elasticity are questionable. Moreover, this conclusion is robust with respect to a variety of estimation strategies.

6. Again, cointegration tests indicate the presence of a unique cointegrating vector. The tests are not presented due to space considerations.

7. We proceed by excluding the interest rate from all equations since it is absent from both cointegrating vectors. Hence, it does not affect the long-run equilibrium relationships among the variables. Of course, the interest rate may affect the dynamic relationships without affecting the long-run relationships among the variables. See below for details of the effects of including the interest rate differential in the error-correction model.
8. Notice that there is no requirement that the variables return back to their original levels. For example, a money supply innovation is a temporary change in the first-difference of the relative money supply. Clearly, a temporary change in the first-difference of the money supply can have a permanent affect on its level and the level of all of the other variables in the system.
CHAPTER VI. AN APPLICATION TO THE ITALIAN LIRA/U.S. DOLLAR EXCHANGE RATE

Identification and Estimation

In this section we first investigate the existence of cointegrating relationships consistent with the Dependent Economy Model. As in Chapter V, we will try to extract structural information from the VAR before attempting to estimate the structural equilibrium relationships directly. If the model is correct we should find equilibrium relationships consistent with money market equilibrium and modified purchasing power parity. Again, these relationships can be tested together to assess the validity of the structural model. Additional evidence on short run dynamics can shed further light on the model. Before implementing these steps, we investigate the order of integration for individual series.

Augmented Dickey-Fuller unit root tests regarding the degree of integration are given in Table 6.1 (the 5 % critical value is -2.90). The table indicates that we fail to reject the null of a unit root in all series (with the possible exception of the short-run interest rate differential). For our purposes we use the long term interest differential, i-i*, instead of the short term interest differential. All other variables to be included in the VAR are the same as in the French case. The quarterly data runs from 1971.Q3 through 1990.Q3 and are, again, taken from International Financial Statistics.

Next, we test for the number of cointegrating relationships between the exchange
Table 6.1. Augmented Dickey-Fuller statistics for the exchange rate and the "fundamentals": The Italian case

<table>
<thead>
<tr>
<th>Variable</th>
<th>Lags</th>
<th>ADF-t statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>s</td>
<td>4</td>
<td>-1.76</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>-1.69</td>
</tr>
<tr>
<td>p-p*</td>
<td>4</td>
<td>-2.17</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>-2.83</td>
</tr>
<tr>
<td>m-m*</td>
<td>4</td>
<td>-2.48</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>-2.11</td>
</tr>
<tr>
<td>y-y*</td>
<td>4</td>
<td>-2.18</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>-2.76</td>
</tr>
<tr>
<td>r-r*</td>
<td>4</td>
<td>-2.35</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>-3.12</td>
</tr>
<tr>
<td>i-i*</td>
<td>4</td>
<td>-2.44</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>-1.96</td>
</tr>
<tr>
<td>pr-pr*</td>
<td>4</td>
<td>-0.96</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>-0.82</td>
</tr>
</tbody>
</table>

rate and the fundamentals. Table 6.2 reports the calculated $\lambda_u$ and $\lambda_{\text{max}}$ statistics used to determine the number of cointegrating vectors among the Lira/Dollar exchange rate, relative money supplies, relative price levels, relative output levels, long-term interest rate differential, and the relative productivity levels. Again, 4 lags appear sufficient for the error process to be approximately white noise.

Using the $\lambda_u$ test, we cannot reject the null hypotheses that $r\leq5$ and $r\leq4$ at the 95
Table 6.2. Cointegration tests for the Italian Lira and the fundamentals

<table>
<thead>
<tr>
<th>Null</th>
<th>$\lambda_r$</th>
<th>$\lambda_r(95)$</th>
<th>$\lambda_{max}$</th>
<th>$\lambda_{max}(95)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>r=0</td>
<td>194.4</td>
<td>102.1</td>
<td>69.6</td>
<td>40.3</td>
</tr>
<tr>
<td>r=1</td>
<td>124.9</td>
<td>76.1</td>
<td>50.7</td>
<td>34.4</td>
</tr>
<tr>
<td>r=2</td>
<td>74.2</td>
<td>53.1</td>
<td>32.5</td>
<td>28.1</td>
</tr>
<tr>
<td>r=3</td>
<td>41.7</td>
<td>34.9</td>
<td>23.0</td>
<td>22.0</td>
</tr>
<tr>
<td>r=4</td>
<td>18.7</td>
<td>20.0</td>
<td>13.5</td>
<td>15.7</td>
</tr>
<tr>
<td>r=5</td>
<td>5.1</td>
<td>9.2</td>
<td>5.1</td>
<td>9.2</td>
</tr>
</tbody>
</table>

% significance level. Hence, using this test, we conclude that there cannot be more than 4 cointegrating vectors. Using the $\lambda_{max}$ test we can reject a null of $r=3$ against the alternative of $r=4$ at the 95% but not the 97.5% level of significance. For expositional purposes, we present results assuming only three cointegrating vectors.

The $7 \times 3$ cointegrating matrix $\beta$, and the adjustment matrix $\alpha$ are given by:

\[
\beta' = \begin{bmatrix}
-0.15 & -2.17 & 2.76 \\
3.02 & 3.15 & -12.34 \\
0.26 & -3.04 & 1.46 \\
\end{bmatrix} \quad \begin{bmatrix}
19.24 & -5.25 & -3.19 \\
-2.61 & -0.71 & 19.38 \\
16.72 & 2.54 & -0.41 \\
\end{bmatrix}
\]

\[
\alpha = \begin{bmatrix}
0.0141 & -0.1130 & -0.0066 \\
0.0285 & 0.0077 & -0.0004 \\
0.0441 & 0.0157 & 0.0094 \\
\end{bmatrix} \quad \begin{bmatrix}
-0.0140 & -0.0057 & -0.0227 \\
-0.0001 & 0.0277 & -0.0336 \\
-0.0132 & -0.0068 & -0.0059 \\
\end{bmatrix}
\]
Normalizing each of the three cointegrating vectors with respect to the exchange rate, the three long-run equilibrium relationships are:

\[(6.3)\quad s_t = -14.29(m_t - m_t^*) + 2.76(p_t - p_t^*) + 19.24(y_t - y_t^*) - 5.25(i_t - i_t^*) - 21.06(p_{rt} - p_{rt}^*) - 118.37\]

\[(6.4)\quad s_t = -1.04(m_t - m_t^*) + 4.09(p_t - p_t^*) + 0.86(y_t - y_t^*) + 0.23(i_t - i_t^*) - 6.42(p_{rt} - p_{rt}^*) + 4.26\]

\[(6.5)\quad s_t = 11.33(m_t - m_t^*) - 5.44(p_t - p_t^*) - 62.44(y_t - y_t^*) - 9.49(i_t - i_t^*) + 1.53(p_{rt} - p_{rt}^*) - 67.49\]

Unlike the French case, the unrestricted cointegrating vectors do not have recognizable properties. Next, we carry out cointegration tests on money market and modified purchasing power parity variables. Table 6.3 presents the \(\lambda_u\) and \(\lambda_{\text{max}}\) statistics used to test for the number of cointegrating vectors among the four money market variables. Both \(\lambda_{\text{max}}\) and \(\lambda_u\) tests suggest that there are exactly two independent cointegrating vectors. Normalizing each with respect to the money supply yields:

\[(6.6)\quad m_t - m_t^* = 9.40 + 0.97(p_t - p_t^*) - 5.02(i_t - i_t^*) + 10.57(y_t - y_t^*)\]

\[(6.7)\quad m_t - m_t^* = 5.49 + 0.63(p_t - p_t^*) + 0.88(i_t - i_t^*) + 4.61(y_t - y_t^*)\]
Table 6.3. Cointegration tests for money market variables: The Italian case

<table>
<thead>
<tr>
<th>Null</th>
<th>$\lambda_{max}$</th>
<th>$\lambda_{(95)}$</th>
<th>$\lambda_{max}$</th>
<th>$\lambda_{(95)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r=0$</td>
<td>90.7</td>
<td>53.1</td>
<td>46.8</td>
<td>28.1</td>
</tr>
<tr>
<td>$r=1$</td>
<td>43.9</td>
<td>34.9</td>
<td>26.4</td>
<td>22.0</td>
</tr>
<tr>
<td>$r=2$</td>
<td>17.5</td>
<td>20.0</td>
<td>13.3</td>
<td>15.8</td>
</tr>
<tr>
<td>$r=3$</td>
<td>4.3</td>
<td>9.2</td>
<td>4.3</td>
<td>9.2</td>
</tr>
</tbody>
</table>

Equation (6.6) has all of the properties of a money demand function: money demand is proportional to the relative price level (at usual significance levels), negatively related to the interest rate differential, and positively related to relative income levels. However, the magnitudes of the implied semi-interest rate and income elasticities of demand for money are too high. Equation (6.7) can be interpreted as a money supply function; the nominal money supply is allowed to accommodate increases in the price level (note that the coefficient is less than unity) and real income levels. Moreover, increases in the interest rate differential are associated with a larger money supply.

We also performed tests to determine whether the modified-PPP relationship was a cointegrating vector (test statistics are reported in Table 6.4). At the 95% significance level, there is a single cointegrating vector for the Lira/Dollar exchange rate, relative price levels, and relative productivity levels. Normalizing with respect to the exchange rate:
(6.8) \[ s_t = 2.092.04(p_t - p_t^*) - 3.89(pr_t - pr_t^*) \]

Notice that the modified-PPP relationship for the Lira/Dollar case is similar to that for the Franc/Dollar case. The signs are all correct. However, in this case we can also impose the proportionality restriction between prices and nominal exchange rates (we cannot reject the null of proportionality at the 13% level of significance). The evidence so far is consistent with the Dependent Economy Model.

Table 6.4. Cointegration tests of the modified purchasing power parity: The Italian case

<table>
<thead>
<tr>
<th>Null</th>
<th>( \lambda_{tr} )</th>
<th>( \lambda_{tr}(95) )</th>
<th>( \lambda_{max} )</th>
<th>( \lambda_{max}(95) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r=0 )</td>
<td>41.9</td>
<td>34.9</td>
<td>22.2</td>
<td>22.0</td>
</tr>
<tr>
<td>( r=1 )</td>
<td>19.8</td>
<td>20.0</td>
<td>14.4</td>
<td>15.8</td>
</tr>
<tr>
<td>( r=2 )</td>
<td>5.3</td>
<td>9.2</td>
<td>5.3</td>
<td>9.2</td>
</tr>
</tbody>
</table>

Test of fixing the cointegrating vectors

In the previous section, we have seen that imposing the zero exclusion restrictions allows for the identification of three structural equations consistent with the Dependent Economy Model. The restricted cointegrating vectors corresponding to the
variables $s$, $(m-m^*)$, $(p-r^*)$, $(y-r^*)$, $(i-i^*)$, $(pr-pr^*)$ and an intercept are

$$\begin{align*}
\beta' &= (0 \quad -1 \quad 0.97 \quad 10.57 \quad -5.02 \quad 0 \quad 9.4) \\
\beta' &= (6.9) \\
\beta' &= (-1 \quad 0 \quad 2.04 \quad 0 \quad 0 \quad -3.89 \quad 2.09)
\end{align*}$$

A $\chi^2_{dfr-12}$ test that the true structure is given by equation (6.9) yields the value 46.53 suggesting that the restriction is rejected. As in the French/U.S. model, the parametric structure of the model is rejected.

**The Short Run Dynamics of the System**

In order to investigate the short run dynamics of the system we do variance decomposition and impulse response analysis. As in the French/U.S. model, the task is undertaken in two steps; equilibrium errors from the structural model are used and then the results are compared with unconstrained model dynamics.

**Variance decomposition and impulse response analysis using structural equilibrium errors**

The short run dynamics of the system are derived in the same manner as in the French model; here we use equilibrium errors derived from the structural cointegrating vectors given by equation (6.9). The order of variables in Choleski orthogonalization is productivity, output, interest rates, money stocks, prices and exchange rates.
Variance decomposition  Table 6.5 gives variance decomposition of the restricted system for four different forecasting horizons, and the correlation matrix of the innovations. The upper part of the table pertains to the regular order implied by the table. The table indicates that productivity explains a sizable proportion of its own forecast error variance as well as the variance of output, prices, and somewhat the exchange rate. Money and interest rate innovations explain about half of their error variance after 12 quarter lags. The forecast error variance of prices is explained by innovations in other variables, notably productivity and exchange rate innovations. Exchange rate innovations, on the other hand, explain a greater proportion of the forecasting error variance in all other variables with the exception of money supply. The most important point is that exchange rate error variance is explained by other variables in the system; roughly 37% of the variance is explained, 17% by productivity innovations alone. This is supportive evidence that the exchange rate shares common trends with the fundamentals as specified by the Dependent Economy Model3.

A caution about the aforementioned results is in order. The correlation matrix of the innovations given at the bottom of Table 6.5 reveals that the correlation between productivity and output (0.60) and productivity and prices (0.46) is rather high. This implies that the order in the orthogonalization of the innovations will matter. This can be seen from the reverse order variance decomposition. The output innovations replace productivity in explaining a large proportion of the variation in the forecast error of other variables in the system.
Table 6.5. Variance decomposition with restricted equilibrium errors: The Italian case

<table>
<thead>
<tr>
<th>Lags</th>
<th>Innovation in</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>k</td>
<td>pr-pr*</td>
</tr>
<tr>
<td>pr-pr*</td>
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<td></td>
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<td>12</td>
<td>62.9</td>
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<td>y-y*</td>
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<td></td>
<td>12</td>
<td>36.9</td>
</tr>
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<td>i-i*</td>
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<td>0.5</td>
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<tr>
<td></td>
<td>4</td>
<td>5.6</td>
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<td></td>
<td>8</td>
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<td>17.0</td>
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<td>m-m*</td>
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<td>3.4</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>12.2</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>12.6</td>
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<td>12</td>
<td>28.6</td>
</tr>
<tr>
<td>s</td>
<td>1</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>9.4</td>
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<td></td>
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<td>14.3</td>
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<td></td>
<td>12</td>
<td>17.0</td>
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</table>
Table 6.5. (continued)

<table>
<thead>
<tr>
<th>k</th>
<th>pr-pr*</th>
<th>y-y*</th>
<th>i-i*</th>
<th>m-m*</th>
<th>p-p*</th>
<th>s</th>
</tr>
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<tbody>
<tr>
<td>pr-pr*</td>
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<td>31.1</td>
<td>20.4</td>
<td>1.8</td>
<td>6.3</td>
<td>12.8</td>
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<tr>
<td>y-y*</td>
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<td>8.7</td>
<td>29.6</td>
<td>2.0</td>
<td>5.6</td>
<td>7.4</td>
</tr>
<tr>
<td>i-i*</td>
<td>12</td>
<td>6.2</td>
<td>10.4</td>
<td>46.8</td>
<td>6.6</td>
<td>1.1</td>
</tr>
<tr>
<td>m-m*</td>
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<td>4.6</td>
<td>4.6</td>
<td>6.1</td>
<td>64.3</td>
<td>13.6</td>
</tr>
<tr>
<td>p-p*</td>
<td>12</td>
<td>9.4</td>
<td>11.7</td>
<td>1.6</td>
<td>4.2</td>
<td>31.2</td>
</tr>
<tr>
<td>s</td>
<td>12</td>
<td>11.5</td>
<td>9.6</td>
<td>2.1</td>
<td>5.4</td>
<td>4.2</td>
</tr>
</tbody>
</table>

Correlation Matrix

<table>
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<tr>
<th>pr-pr*</th>
<th>y-y*</th>
<th>i-i*</th>
<th>m-m*</th>
<th>p-p*</th>
<th>s</th>
</tr>
</thead>
<tbody>
<tr>
<td>pr-pr*</td>
<td>1.00</td>
<td>0.60</td>
<td>0.07</td>
<td>0.18</td>
<td>0.46</td>
</tr>
<tr>
<td>y-y*</td>
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<td>0.17</td>
<td>0.29</td>
<td>0.20</td>
<td>0.08</td>
</tr>
<tr>
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Impulse response functions

The impulse response functions using structural equilibrium errors for the Italian/U.S. case are presented in Figures 6.1a through 6.6c. In response to a productivity shock, output and productivity increase for one quarter and they fall back (Figure 6.1a). The contemporaneous effect of a productivity innovation on prices and the exchange rate is positive as well (Figure 6.1b). Since changes in prices and exchange rate become negative after the second quarter, these variables seem to return to their original level.
Figure 6.1. Responses to a productivity shock: a) productivity and output responses
Figure 6.1. (continued)  b) price and exchange rate responses
The productivity and own effects of an output shock are positive (Figure 6.2a) and seem to be permanent. The money stock and interest rate respond (Figure 6.2b) by fluctuating more persistently (the interest rate response being mostly negative). The overall effect of an output shock on money stock and interest rate seem to be transitory. The initial price and exchange rate responses to an output shock (Figure 6.2c) are negative. In response to an output shock, the exchange rate appreciates, then depreciates. Eventually both prices and the exchange rate return to their original level.

The productivity and output responses to an interest rate shock (Figure 6.3a) are mostly negative. On the other hand, the initial interest rate and money stock responses (Figure 6.3b) are positive. Note that the short run adjustment process is not constrained to follow any particular path compatible with a theoretical model. The dynamics of the money market and possibly the ordering of the variables is responsible from such unanticipated results. Since changes in interest rates and money stocks are positive and negative, both return to their original level. Price and exchange rate responses to an interest rate shock are negative as well (Figure 6.3c). An increase in the interest rate in favor of the home country leads to the appreciation of the domestic currency. This effect seems to be permanent.

The responses to a money stock shock are given by Figures 6.4a, b, and c. Productivity and output respond by fluctuating in opposite directions persistently with no permanent effect. More interestingly, the money stock seems to have little interest rate effect. The own effects of a money stock shock die down alternating between
Figure 6.2. Responses to an output shock: a) productivity and output responses
Figure 6.2. (continued) b) money stock and interest rate responses
Figure 6.2. (continued)  
c) price and exchange rate responses
Figure 6.3. Responses to an interest rate shock: a) productivity and output responses
Figure 6.3. (continued) b) money stock and interest rate responses
Figure 6.3. (continued)  c) price and exchange rate responses
Figure 6.4. Responses to a money stock shock: a) productivity and output responses
Figure 6.4. (continued)  

b) interest rate and money stock responses
Figure 6.4. (continued)  c) price and exchange rate responses
positive and negative values. Initial price and exchange rate responses are positive. Since the responses alternate between positive and negative values, the money stock seems to have a temporary effect on prices and the exchange rate. Note that, when expressed in levels the exchange rate effect in the Italian model is more compatible with the overshooting hypothesis.

The price shock appears to have an initial positive output effect and a small productivity effect (Figure 6.5a). The own effects of a price shock die quickly after an initial increase. Surprisingly, exchange rate effects of a price shock are negative possibly through the changes in other variables in the system (Figure 6.5b).

Finally, responses to an exchange rate shock are given in Figures 6.6a, b, and c. Productivity and output responses are mostly positive implying "Keynesian" effects (Figure 6.6a). The money stock alternates between positive and negative values whereas the interest rate response is mostly negative (Figure 6.6b). The price effect is positive; prices go up in response to an exchange rate shock for about three quarters, then they fall back. Own exchange rate effects die down quickly.

As compared to the French/U.S model the adjustment process seems to be slightly different. For example the own effect of an exchange rate shock appears to die down gradually in the French/U.S model whereas it disappears relatively quickly in the Italian/U.S. model. In other words, the exchange rate adjusts to its equilibrium value slowly in the French/U.S. model.
Figure 6.5. Responses to a price shock: a) productivity and output responses
Figure 6.5. (continued) b) price and exchange rate responses
Figure 6.6. Responses to an exchange rate shock: a) productivity and output responses
Figure 6.6. (continued) b) interest rate and money stock responses
Variance decomposition and impulse response functions with unrestricted equilibrium errors

As in the French/U.S model we take a statistical approach and compare the results with the structural approach. Using unrestricted equilibrium errors, variance decomposition results are given in Table 6.6.
Table 6.6. Variance decomposition with unrestricted equilibrium errors: The Italian case

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Some results appear to be similar to the structural case; productivity (and output in the reverse order) still explain a large proportion of the variance in other variables. A careful examination of the table reveals some differences. First, interest rates explain a larger proportion of nominal variables (money stocks, prices and the exchange rate) in the unrestricted system. Second, the exchange rate does not explain a large proportion of the variability in other variables in the unrestricted system. Finally, and most importantly, other variables explain a greater proportion of exchange rate variability in the unrestricted system. Unlike the French/U.S case, the variance decomposition results
in the unrestricted system fare better.

The above conclusion does not extend to the impulse response functions. A sample of impulse response functions of the unrestricted system is given in Figures 6.7a through 6.11b. Although some impulse response functions are similar to the restricted case, most of them are too magnified suggesting stability problems. For example, the output and productivity responses to an exchange rate shock are explosive (Figure 6.11a). Similarly price and exchange rate responses to a productivity shock (Figure 6.7b) seem to be magnified as well.

**Summary and Conclusions**

In this chapter we applied the identification framework developed in previous chapters to the bilateral Italian/U.S. data. The cointegration test indicated three independent vectors which we identified as the money supply, money demand, and modified purchasing power parity relationships. The dynamic impulse response and variance decomposition based on the error correction representation revealed mixed evidence regarding the Dependent Economy Model. With errors from the unrestricted versions of the cointegrating vectors, about 47% of exchange rate variability is explained as compared to 37% when structural equilibrium errors are used. However this conclusion does not extend to the impulse response functions. Dynamic impulse response functions of the unrestricted model have explosive patterns as compared to structural model impulse response functions which have properties consistent with the Dependent Economy Model.
Figure 6.7. Responses to a productivity shock in the unrestricted system:
a) productivity and output responses
Figure 6.7. (continued)  b) price and exchange rate responses
Figure 6.8. Responses to an interest rate shock in the unrestricted system: a) productivity and output responses
Figure 6.8. (continued)  b) price and exchange rate responses
Figure 6.9. Responses to a money stock shock in the unrestricted system: a) productivity and output responses
Figure 6.9. (continued)  b) price and exchange rate responses
Figure 6.9. (continued) c) interest rate and money stock responses
Figure 6.10. Responses to a price shock in the unrestricted system: 
a) productivity and output responses
Figure 6.10. (continued)  b) price and exchange rate responses
Figure 6.11. Responses to an exchange rate shock in the unrestricted system: a) productivity and output responses
Figure 6.11. (continued)  b) price and exchange rate responses
Endnotes

1. Long term rates can be justified by a term structure argument.

2. The coefficients of these variables will appear in the same order in the cointegrating matrix, $\beta$, and the adjustment matrix, $\alpha$.

3. One should remember that one of the research agendas in the literature is whether the exchange rate is a random walk. Considering the fact that the exchange rate has a large noise component, the Dependent Economy Model fares well as a theoretical model.
Summary and Concluding Discussion

One of the major aims of this study is to provide a modelling strategy for models with cointegrated variables. A reasonable interpretation for the existence of multiple cointegrating vectors among a given set of variables is to think of each of the individual cointegrating vectors as a behavioral or reduced form equation resulting from a structural model. Given this natural interpretation, structural identification can be quite straightforward since it is not necessary to specify the set of endogenous versus exogenous variables. Exact identification of the behavioral equations can be facilitated by the imposition of zero restrictions on the cointegrating vector(s). Moreover, it is possible to test these restrictions. Given that these structural relations hold as steady state functions, it is possible to incorporate the deviations from these equilibria into the dynamic analysis which is represented by the error-correction form. Traditional impulse response and variance decomposition techniques can be used to derive short-run properties of the system. Restricted and unrestricted versions of the cointegrating vectors/ equilibrium errors can be used in the dynamic analysis to evaluate the plausibility of the underlying theory.

We illustrated the procedure using the set of variables suggested by the Dependent Economy Model of exchange rate determination. Bilateral data for France
and the U.S. revealed two independent cointegrating relationships among the exchange rate, relative money supplies, relative price levels, relative income levels, interest rate differential, and relative productivity levels. We were able to impose zero restrictions on the two cointegrating relationships; the restricted vectors can be reasonably interpreted as a money market equilibrium equation and the modified-PPP equation. Using Italian and U.S. data, we found three cointegrating relationships which we sensibly interpret as money demand function, money supply function, and the modified-PPP relationship. We have found that the hypothesized structural relationships of the Dependent Economy Model exist as long-run equilibrium relationships, though a test of fixing the cointegrating along the lines of the estimated structure was not supportive of the model.

Interpreting the restricted cointegration vectors as long-run behavioral relationships has important implications for the error-correction representation of the model. The resulting impulse response functions have the property that all variables necessarily return to levels consistent with the long-run behavior posited by a structural model. Using the French/U.S. data, the restricted dynamics appears to be consistent with a "Keynesian" model. In response to a "typical" domestic money shock, the relative price level, output level, productivity level, and price of foreign currency all tend to increase. Possibly the most important result of the dynamic analysis is that the exchange rate is affected by other variables in the system. The exchange rate shares a long-run equilibrium with other variables and these other variables explain approximately 35% of exchange rate forecast error variance. Also the exchange rate accounts for relatively
little of the forecast error variance of these other variables in the French/ U.S. model.

The dynamic analysis of the unrestricted system in the French/U.S case reinforces the above conclusion. Without imposing any structure on the long run relationships, other variables in the system account for only 23 % of the exchange rate forecast error variance. This evidence is in favor of the Dependent Economy Model.

The dynamic analysis of the Italian/U.S. model is not as conclusive. Although other variables explain a larger proportion of exchange rate variability in the unrestricted system (about 47 % as compared to 37 % in the restricted system), some of the impulse response functions exhibit explosive patterns suggesting stability problems.

Although the exchange rate follows near random walk behavior we have found other variables which share common trends with the exchange rate. Overall, given the simple nature of the model, the evidence is not discouraging.

Suggestions for Further Research

One problem with standard impulse response analysis is that there is no unique way to obtain orthogonalized innovations. Our dynamic response functions are conditional on the ordering in a Choleski decomposition. However, the willingness to interpret the cointegrating vectors as behavioral equations can be helpful in the estimation of a more appropriate set of response functions. Consider the French/U.S. case, in which there are two behavioral equations among a total of six variables. In the context of the structural model, at least two of the six variables must be endogenous
and *as many as* four can be exogenous. (Unfortunately, the terminology is ambiguous since the term "exogenous" in a structural model does not exactly correspond to the term when used in an econometric context; here, we refer to the structural context). In the Dependent Economy Model, the exchange rate and the relative price level are typically taken to be endogenous. In principle, it is possible to incorporate the decomposition technique developed by Blanchard and Quah (1989) into the dynamic analysis. The dynamic impulses of the exchange rate and relative price level innovations can be constrained to have no long-run effects on the levels of the exogenous variables. In this case long run behavior as well as short-run dynamics are constrained according to some theoretical considerations. Another avenue is to pursue the structural VAR approach (Bernanke 1986) in orthogonalizing the innovations. In this case the structural context can be helpful in modelling the innovations. We leave these thoughts as suggestions for further research.

One of the implicit assumptions we have made is that the innovations in the French/U.S. model are orthogonal to that of the Italian/U.S. model. An interesting question related to specification is whether third country shocks (or shocks from a group of countries) affect the exchange rate; another prospect for future research.

Finally one can use the methodology we have suggested to test alternative models of exchange rate determination. The hypothesized long run relations of different models and their short run dynamics can be investigated using our "structural cointegration" framework.
REFERENCES


Hooper, P., and Morton, J. (1982). Fluctuations in the dollar: A model of nominal and
real exchange rate determination. *Journal of International Money and Finance, 1*, 39-56.


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