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Lags and the Assignment Problem: A Note

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Abstract

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Disciplines

Economic Theory | Finance | Other Economics

LAGS AND THE ASSIGNMENT PROBLEM: A NOTE

by Harvey Lapan and Walter Enders*

Mundell [5] has demonstrated that monetary policy should be paired with external balance and fiscal policy with internal balance. This seminal article led to a voluminous literature[1] which attempted to rectify many of the problems inherent in Mundell's flow equilibrium model. Harry Johnson [2] has characterized this extension of Mundell's work as having "... lent itself to almost infinite mathematical product differentiation, with little significant improvement in quality of economic product ..." Although we do not fully agree with Harry Johnson—for there are many deficiencies in Mundell's model—we do believe that more can be said concerning the "Assignment Problem" in the context of Mundell's model. Specifically, we examine the effects of introducing discrete lags into the Mundell model. Part A of this note presents a generalized discrete time version of Mundell's model and shows that the "Principle of Effective Market Classification" cannot guarantee stability. Part B then examines how the presence of an outside lag affects the results of Part A.

A) The Basic Model[2]

The linearized version of the Mundell model can be represented by:[3]

$$Y_T = \alpha S_T + \beta r_T \quad (1)$$

$$B_T = \zeta Y_T + \lambda r_T \quad (2) \quad \text{or}$$

$$B_T = \alpha\zeta S_T + (\lambda + \zeta\beta) r_T \quad (2')$$

where: Y = income

r = interest rate

S = government surplus

B = balance of payments

and $\alpha < 0$; $\beta < 0$; $\zeta < 0$; $\lambda > 0$.

The problem posed is how to use monetary policy (represented by the interest rate) and fiscal policy (represented by the government surplus) to guarantee the attainment of full employment and balance of payments equilibrium. If policy makers knew the underlying structure of the economy they could directly set the values of the instruments at the levels which would produce the desired values of the targets. However, the underlying structure is assumed to be unknown so that policy makers must use a groping process to arrive at the desired values of the targets. In particular, two policy choices are available:[4]

(1) the government surplus can be altered in response to desired changes in income and the interest rate can be altered in response to desired changes in the balance of payments; or

2) the government surplus can be altered in response to desired changes in the balance of payments and the interest rate can be altered in response to desired changes in income.

Thus:[5]

Policy I $\quad \Delta S_T = a_1 [Y_T - Y^*]$

$$\Delta r_T = a_2 [B^* - B_T]$$

or Policy II $\quad \Delta r_T = b_1 [Y_T - Y^*]$

$$\Delta S_T = b_2 [B^* - B_T]$$

where: a_i and b_i are positive adjustment parameters

$$\Delta S_T = S_{T+1} - S_T$$

$$\Delta r_T = r_{T+1} - r_T$$

Y^* = desired level of income

B^* = desired balance of payments position

Mundell's graphical analysis implies specific values for the adjustment coefficients: i.e.,

$$a_1 = -\frac{1}{\alpha}; \quad a_2 = \frac{1}{\lambda + \zeta\beta}; \quad b_1 = \frac{-1}{\beta}; \quad b_2 = \frac{1}{\alpha\zeta}.$$

With these specific values, Mundell finds that Policy I guarantees stability while Policy II renders the system unstable.

This particular choice of the adjustment parameters assures that if only one instrument is changed then the desired magnitude of its corresponding target is achieved. These adjustment coefficients, then, imply that both the monetary and fiscal authorities know the underlying structure of the

economy and that the informational problem is one of policy coordination between the different branches of government. Since policy makers are faced with situations in which they know neither the behavior of other branches of government nor the underlying structure of the economic system, it is of interest to determine whether Policy I guarantees stability and Policy II instability, regardless of the choice of adjustment coefficients.

If Policy I is adopted, the characteristic equation of the system takes the form:

$$E^2 - d_0 E + d_1 = 0$$

where: $d_0 = 2 - a_2(\lambda + \zeta\beta) + a_1\alpha$

$$d_1 = 1 + a_1\alpha - a_2(\lambda + \zeta\beta) - a_1 a_2 \alpha\lambda$$

The necessary and sufficient conditions for stability are:[6]

$$1 - d_0 + d_1 = -a_1 a_2 \alpha\lambda > 0 \qquad (3)$$

$$1 + d_0 + d_1 = 4 + 2a_1\alpha - 2a_2(\lambda + \zeta\beta) - a_1 a_2 \alpha\lambda > 0 \qquad (4)$$

$$1 - d_1 = a_1 a_2 \alpha\lambda + a_2(\lambda + \zeta\beta) - a_1\alpha > 0 \qquad (5)$$

Equation 3 is always satisfied as long as a_1 and a_2 are greater than zero; Equations 4 and 5 are satisfied by the shaded areas in Figures 1 and 2 respectively. Moreover, Equation 4 can be seen to be the only tight constraint since it never intersects Equation 5.[7]

It should be noted that Mundell's choice of the adjustment coefficients (point M in Figures 1 and 2) always produces stability. However, Policy I does not always guarantee that the system will converge
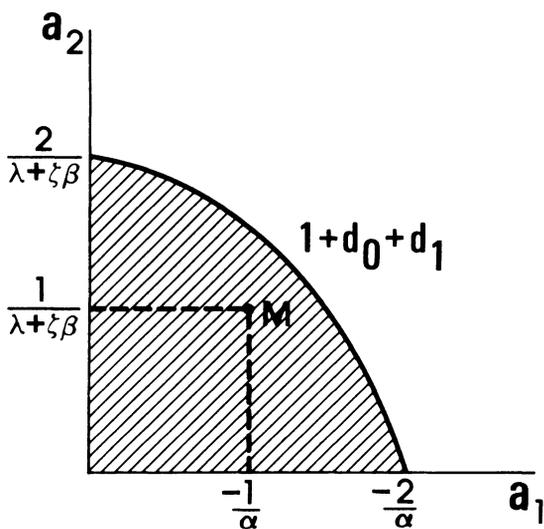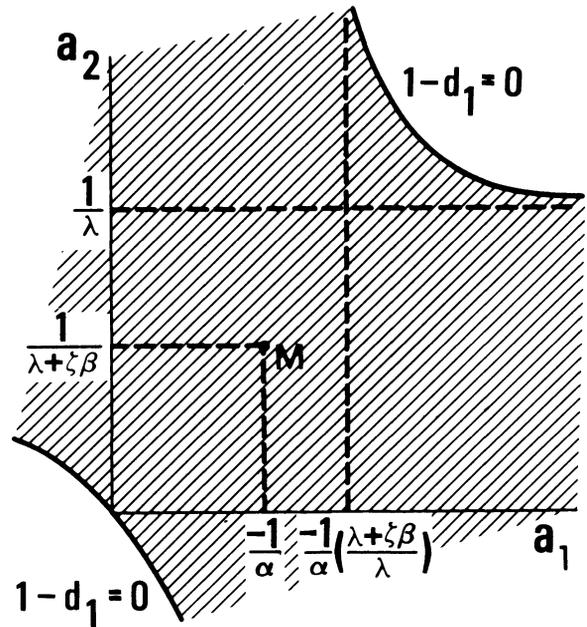

FIGURE 2.

FIGURE 1.

when the possibility of overshooting is recognized. Thus, the "Principle of Effective Market Classification" does not always hold. Further, since increases in λ, $\zeta\beta$, and $-\alpha$ cause the intercepts in Figure 1 to fall twice as fast as the Mundell point, it can be concluded that the problems associated with overshooting increase as the targets become more sensitive to the instruments. Thus, acquisition of information concerning the underlying structure of the system becomes more important as targets become more sensitive to instruments.

Given the set of adjustment coefficients which insure stability, it is interesting to note that the Mundell point yields the fastest speed of adjustment.[8] On minimizing the absolute value of the largest characteristic root, we find $a_1 = -1/\alpha$; $a_2 = 1/(\lambda + \zeta\beta)$ and the characteristic roots are $\pm(\zeta\beta/(\lambda + \zeta\beta))^{1/2}$. Notice that the speed of adjustment is independent of α, decreases as $\zeta\beta$ increases and increases as λ increases.[9]

With regard to Policy II, the system is unstable with any positive choice of adjustment coefficients. Under Policy II, the characteristic equation becomes:

$$E^2 - d_2 E + d_3$$

where $d_2 = 2 + b_2\beta - \alpha\zeta b_1$

$$d_3 = 1 + b_1 b_2 \alpha\lambda + b_2\beta - \alpha\zeta b_1$$

The necessary and sufficient conditions for stability are violated since the sum of the coefficients is negative, i.e., $1 - d_2 + d_3 = b_1 b_2 \alpha\lambda < 0$.

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B) Outside Lags

The above treatment of the assignment problem assumes that instruments affect targets instantaneously. In order to see how lags may alter the stability properties of the system, we consider a simple lag structure in which the interest rate effect on income is lagged one period. This lag structure seems reasonable for illustrative purposes since, *a priori*, one can expect that the interest rate effect on income takes longer to operate than any other outside lag. Using this lag, we can represent the model as follows:[10]

$$Y_T = \alpha S_T + \beta r_{T-1} \qquad (6)$$

$$B_T = \zeta Y_T + \lambda r_T \qquad (7)$$

$$\Delta S_T = a_1 [Y_T - Y^*] \qquad (8)$$

$$\Delta r_T = a_2 [B^* - B_T] \qquad (9)$$

Under this policy, the characteristic equation takes the form:

$$E^3 + d_4 E^2 + d_5 E + d_6$$

where
$$d_4 = -[2 + a_1\alpha - a_2\lambda]$$

$$d_5 = (1 - a_2\lambda)(1 + a_1\alpha) + a_2\zeta\beta$$

$$d_6 = -a_2\zeta\beta$$

The necessary and sufficient conditions for stability are:[11]

$$1 + d_4 + d_5 + d_6 = -a_1 a_2 \alpha\lambda > 0 \qquad (10)$$

$$3 - d_4 - d_5 + 3d_6 = 4 + a_1 a_2 \alpha\lambda - 4a_2\zeta\beta > 0 \qquad (11)$$

$$1 - d_4 + d_5 - d_6 = 4 + 2a_1\alpha - 2a_2(\lambda - \zeta\beta)$$
$$- a_1 a_2 \alpha\lambda > 0 \qquad (12)$$

$$1 - d_6^2 + d_4 d_6 - d_5 = -a_2^2 \zeta\beta(\lambda + \zeta\beta)$$
$$+ a_2(\lambda + \zeta\beta)(1 + a_1\alpha) - a_1\alpha > 0 \qquad (13)$$

Condition 10 holds for all a_1, a_2 greater than zero, while condition 11 can be shown to be redundant given conditions 12 and 13.[12] Thus, 12 and 13 alone define the region of stability, and the shaded areas of Figures 3 and 4 depict this region, for the cases in which $\lambda \leq \zeta\beta$ and $\lambda > \zeta\beta$, respectively.[13]

As in the unlagged case, the possibility of overshooting can produce instability. In comparing the effect of the lag on the region of stability it is apparent that Equation 2 is tight vis-a-vis Equation 12 since subtraction of 2 from 12 yields $4a_2\zeta\beta$, which is always positive. Moreover, from Figures 1, 3, and 4 it is apparent that the lagged case will be stable for some values of (a_1, a_2) for which the

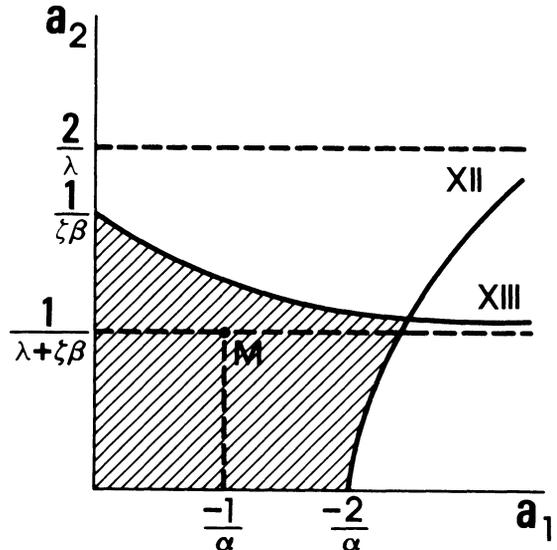FIGURE 3. ($\lambda \leq \zeta\beta$)
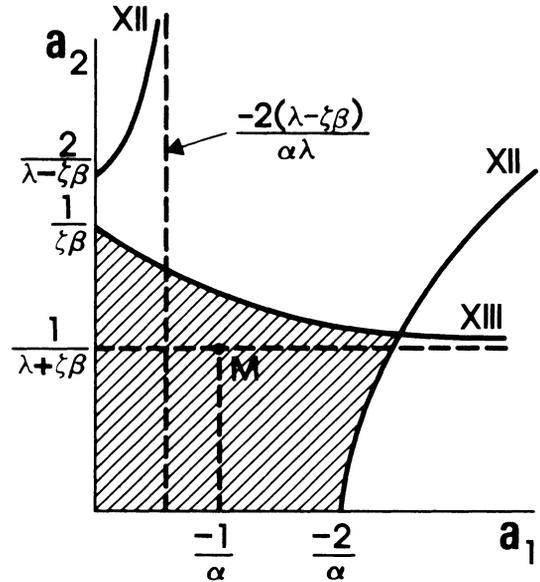
FIGURE 4. ($\lambda > \zeta\beta$)

unlagged case is unstable (e.g., $a_1 > -2/\alpha$). If Equation 2 never intersects Equation 13 in the positive quadrant, then the lag unambiguously increases the range of stability. Solving these equations simultaneously, it can be shown that if $\lambda > 2[\sqrt{13} - 3]\zeta\beta \simeq 1.2\zeta\beta$, then the lag unambiguously increases the region of stability. Thus, the lag tends to be stabilizing as capital becomes more mobile since a greater degree of capital mobility will reflect itself in increasing λ relative to $\zeta\beta$.[14]

Note that the effect of the lag of r_T on income

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increases the range of a_1 for which the system converges; thus, if there is uncertainty about the values of λ, $\zeta\beta$, and α, but it is believed that monetary policy affects income with a lag, then relatively greater weight should be attached to the speed of adjustment of the government surplus.

In summary, we have seen that the introduction of lags into the Mundell model does not alter the policy prescription of assigning r to the balance of payments and S to Y. However, the presence of outside lags can narrow the range of stability, and means that some care must be exercised in choosing the speed of adjustment of the respective targets—too high a speed of adjustment can render the system unstable, whereas slow speeds of adjustment retard the convergence of the system.

Notes

1. For example see: Levin [3], Mathieson [4], Tsiang [6].
2. Interested readers are invited to write to the authors for a graphical presentation of the results in this section.
3. Mundell assumes that the exchange rate is fixed so that it can be subsumed in the functional forms of Equations 1 and 2. Thus, it is assumed that the exchange rate is not a policy tool although relaxing this assumption would not rule out the possibility of overshooting discussed below.
4. It is not necessary to consider intermediate cases wherein each target is adjusted in response to two instruments since intermediate cases can be considered to be simultaneous usage of both Policy I and Policy II with a weight attached to each. If any pure policy proves inferior to the other, the weight attached to that policy should be zero.
5. This policy assignment slightly differs from Mundell's in that here we assume the monetary and fiscal authorities operate simultaneously whereas Mundell assumes that the policy makers alternate.
6. See Gondolfo [1, p. 107] for the derivation of these conditions.
7. Set Equations 4 and 5 equal to zero and solve for a_2, i.e.,

$$a_2 = \frac{4\lambda \pm \sqrt{-16\lambda\zeta\beta}}{2\lambda(\lambda + \zeta\beta)}.$$

Thus, the solution for the common intersection of 4 and 5 is imaginary.
8. Min. $d_0 + (d_0 - 4d_1)^{1/2}$

S.T. $d_0 > 0$

$a_1 > 0$

$a_2 > 0$

Max. $d_0 - (d_0 - 4d_1)^{1/2}$

S.T. $d_0 \le 0$

$a_1 > 0$

$a_2 > 0$

In each case the Kuhn-Tucker conditions are satisfied at a unique point with $d_0 = 0$ and $-\alpha a_1 = a_2(\lambda + \zeta\beta) = 1$.
9. See Gondolfo [1, p. 16] for an explanation of characteristic roots and the relationship between the value of the roots and convergence. We note here, that the greater is the largest characteristic root (in absolute value) the slower is the speed of adjustment.
10. We only consider Policy I in the lagged case since Policy II is unstable even in the presence of lags.
11. See Gondolfo [1, p. 107] for derivation of these stability conditions.
12. Assuming Equation 11 holds with equality, we have for 12 and 13:

$$-a_2^2\lambda(\lambda + \zeta\beta) + 4a_2(\lambda + \zeta\beta) - 4 \ge 0 \qquad (12')$$

$$-[-a_2^2\lambda(\lambda + \zeta\beta) + 4a_2(\lambda + \zeta\beta) - 4][1 - a_2\zeta\beta] \ge 0. \qquad (13')$$

Since, when 11 holds with equality, $1 - a_2\zeta\beta \ge 0$, it follows that (12') and (13') must be of opposite sign. Further, note that when Equations (11) and (12) intersect, (13) passes through the same point. Graphing (11), using this information, it is readily seen that whenever Equation (11) is violated, either (12) or (13) must also be violated.
13. The vertical, intercepts of XII and XIII in Figure 4 are not important, since both lie above the region of stability for the unlagged case. If $\lambda < 3\zeta\beta$, XII and XIII do not intersect for $a_1 < -2/\alpha$; if $\lambda > 3\zeta\beta$, then for small values of a_1, XII becomes the tight constraint.
14. λ will increase as the degree of capital mobility increases since any given change in the interest rate will produce greater improvements in the balance of payments. Since S reflects the marginal propensity to impact and β reflects the impact of monetary policy on income levels, this product reflects the potency of monetary policy on the balance of trade.

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