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Coordinating forward and reverse flows of products for a manufacturer-retailer supply chain model

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Keywords
Remanufacturing, Coordination, Pricing

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Coordinating forward and reverse flows of products for a manufacturer-retailer supply chain model

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In this paper, we formulate and analyze a manufacturer-retailer supply chain model with product remanufacturing. This model takes into account both forward flow of the new product as well as reverse flow of the used product. Specifically, we assume that the manufacturer can control the wholesale price of the new product as well the transfer price of the used product from the retailer while the retailer can control the retail price of the new product and the collecting price of the used product. Under this assumption, we compare and contrast the coordinated scenario vs. the uncoordinated scenario. Managerial insights and a numerical example are illustrated.

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1. Introduction

In this paper, we study a single manufacturer- single retailer supply chain model with forward flow of the new product and reverse flow of the used product for remanufacturing. We will assume that the manufacturer can control the wholesale price of new product as well the transfer price of used product from the retailer; while the retailer can control the retail price of new product and the collecting price of used product. Under this assumption, this paper integrates a linear approximation of the product return rate function into the pricing decision of the supply chain members.

Many extant remanufacturing literatures assume that the used products are returned at a predetermined rate, for example, in Toktay et al. (2000), a used camera is returned with probability *p*, independently of all other cameras. In these models, product returns are assumed to be an exogenous process and the remanufacturer passively accepts product returns.

On the other hand, a few literatures address the problem of how to actively control the product returns using financial incentives. "In a market-driven system, end-users are motivated to return end-of-life products by financial incentives, such as deposit systems, credit toward a new unit, or cash paid for a specified level of quantity." (Guide and Van Wassenhove, 2001). In Savaskan et al. (2004), to achieve the return rate τ, the collecting party must pay a total collecting cost $C_L \tau^2 + A\tau\, D(p)$, where $C_L \tau^2$ is a concave investment cost, A is variable collecting cost per unit and $D(p)$ is the demand; the collector then can optimize over τ to maximize the profit.

In our paper, we model the market-driven behavior of used product returns in the framework of pricing decision of the manufacturer-retailer supply chain. The key contribution of our model is the integration of a linear approximation of the product return rate function and the pricing decision of the retailer and the manufacturer under uncoordinated and coordinated scenarios. In our model, the optimal prices and the optimal return rates are simultaneously determined.

The rest of the paper is organized as follows. In section 2, we formulate the supply chain model with a linear approximation of the return rate function. Then in Section 3 we solve the supply chain model and provide model analysis. Section 4 provides a numerical example. Finally, conclusion and future work are discussed in Section 5. Various interesting findings are presented throughout the paper.

2. The supply chain model

2.1 Model environment

We consider a manufacturer-retailer model with product remanufacturing (see figure 1). The manufacturer can manufacture a new product from raw materials with unit cost of *cm*, or remanufacture a returned product into a new one with unit cost of *cr*. We assume there is no difference between the manufactured and remanufactured products (this is true for some products, e.g. photocopiers, see Kerr, 2000), and $\Delta = cm - cr > 0$. We also assume that the supply chain is manufacturer-driven, with manufacturer as the Stackelberg leader (see Ertek and Griffin, 2002), i.e., the manufacturer can control the unit wholesale price of new products (*w*) and the unit transfer price of returned products from the retailer (*b*); while the retailer can control the unit retail price of new products (*p*) and the unit collecting price of returned products from end-customers (*c*).

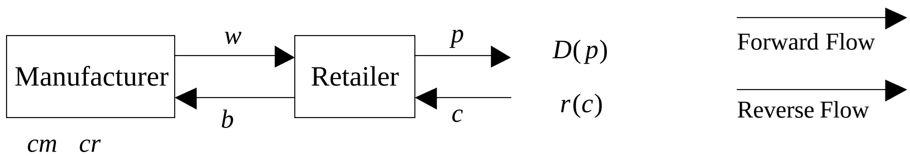

Figure 1. The manufacturer-retailer supply chain with remanufacturing

While there are many forms of demand function (see e.g. Gallego and Van Ryzin, 1994), the demand of new products in our paper is assumed to be a linear function of retail price *p*, i.e., $D(p) = \alpha - \beta \times p$. The reverse flow is characterized by the return rate of used products, $r(c)$, which we assume is a linear function of the collecting price *c*. Specifically, $r(c)$ has the form: $r(c) = \min\{kc, 1\}$ in which k is the marginal return rate, i.e., one unit increase in c will result in k unit increase of the product return rate. k reflects the end-customers' tendency toward product returning. $r(c)$ is bounded above by 1, when all the used products are returned to the supply chain. 1/k is the upper bound of retailer's collecting price, because any collecting price larger than 1/*k* would not improve the return rate any more. The linear function of $r(c)$ in our model is a first-degree approximation of many actual return rate functions (See Figure 2).

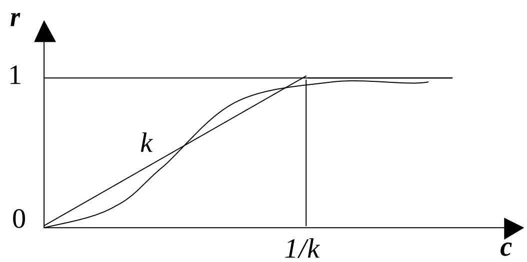

Figure 2. Return rate function r(c)

We also have the following assumptions in our model: (1) all returned products are equal in quality and can be used for remanufacturing; (2) both the manufacturer and the retailer have access to the whole supply chain structure and cost information; (3) the model is considered in an equilibrium setting, i.e., transitional stages to the equilibrium of the manufactured and remanufactured products are not considered in this paper.

2.2 Model formulation

We first consider the profit of each member and the whole supply chain. The retailer's profit, given wholesale price w and transfer price b, is

$$\begin{aligned}\Pi^{R} &= (\alpha - \beta \times p) \times (p - w) + (\alpha - \beta \times p) \times r(c) \times (b - c) \\ &= (\alpha - \beta \times p)[p - w + (b - c) \times r(c)]\end{aligned} \tag{1}$$

The retailer's profit contains two parts: one is the profit from selling a new product; the other is the profit from collecting a used product and transferring it to the manufacturer. Accordingly, the manufacturer's profit is

$$\begin{aligned}\Pi^{M} &= (\alpha - \beta \times p) \times [w - cm \times (1 - r(c)) - cr \times r(c)] - (\alpha - \beta \times p) \times r(c) \times b \\ &= (\alpha - \beta \times p)[w - cm + (\Delta - b) \times r(c)]\end{aligned} \tag{2}$$

Due to remanufacturing, the average unit production cost of the manufacturer is $cm \times (1 - r(c)) + cr \times r(c)$, i.e., $r(c)$ of total new products will be from remanufactured, and the remaining $1 - r(c)$ will be manufactured from raw materials. The second term in Π^{M} reflects the cost of transferring the used products from the retailer to the manufacturer. Finally, the total supply chain profit is

$$\Pi^{T} = \Pi^{R} + \Pi^{M} = (\alpha - \beta \times p) \times [p - cm + (\Delta - c) \times r(c)] \tag{3}$$

We consider two scenarios for this supply chain model:

Scenario 1: uncoordinated supply chain

In the uncoordinated supply chain scenario, the manufacturer and the retailer optimize respectively their own profits by manipulating the variables under their own control. As stated above, we assume that the manufacturer has sufficient power and acts as a Stackelberg leader. Specifically, the retailer will maximize Π^{R} over p, c for a given pair of w and b. Because of the information sharing assumption, the manufacturer will know exactly the retailer's optimal retailer price $p^{*}(w,b)$ and optimal collecting price $c^{*}(w,b)$ (reactive functions). By taking into account these reactive functions, the manufacturer will maximize Π^{M} over w and b.

Scenario 2: coordinated supply chain

In the coordinated supply chain scenario, the manufacturer and the retailer jointly determine the optimal prices (p^{*} and c^{*}) to maximize overall supply chain profit Π^{T}.

3. Model solution and analysis

In this section, we first explain briefly the solution procedures for the two scenarios in subsection 3.1 and 3.2. Then we summarize the solution and provide analysis in subsection 3.3.

3.1 Solution procedure for uncoordinated scenario

In the uncoordinated scenario, we first assume that the optimal $c^{*} \le 1/k$. By setting

$$\frac{\partial}{\partial c}\Pi^{R} = 0 \quad \text{and} \quad \frac{\partial}{\partial p}\Pi^{R} = 0$$

we can get $c^{*} = b/2$ and $p^{*} = \alpha/(2\beta) - kb^{2}/8 + w/2$ (Π^{R} is jointly concave in p and c). By replacing c and p in Π^{M} with c^{*} and p^{*}, we can get $\Pi^{M} = (-b(b + cr)k + cm(b\,k - 2) + 2w)(4\alpha + b^{2}k\beta - 4w\beta)/16$, which is a polynomial of w and b. By setting

$$\frac{\partial}{\partial w}\Pi^{M} = 0 \quad \text{and} \quad \frac{\partial}{\partial b}\Pi^{M} = 0$$

and solving the equations simultaneously, we get three solutions ($w_{1} = \alpha/(2\beta) + cm/2 + k\Delta^{2}/8$, $b_{1} = \Delta$), (w_{2}, b_{2})

and (w_3, b_3). However $\Pi^M(w_2, b_2) = 0$ and $\Pi^M(w_3, b_3) = 0$; and (w_1, b_1) is the only solution that $\Pi^M > 0$. By checking the Hessian matrix of Π^M with respect to w and b at (w_1, b_1), we find that (w_1, b_1) would be a global optimal solution if $\alpha - (cm - k\Delta^2/4)\beta > 0$. This condition is always satisfied because when $b = b_1 = \Delta$ and $c^* = b/2 = \Delta/2$, the total profit of the supply chain is $\Pi^T = (\alpha - \beta \times p) \times [p - cm + (\Delta - c) \times r(c)] = (\alpha - \beta \times p)(p - cm + k\Delta^2/4)$. For $\Pi^T > 0$, the conditions that $0 \le cm - k\Delta^2/4 < p$ and $\alpha - \beta \times p > 0$ are true for all feasible p's, which leads to $\alpha - (cm - k\Delta^2/4)\beta > 0$. Because $\Pi^T \ge 0$ is the least requirement for the supply chain, the condition $\alpha - (cm - k\Delta^2/4)\beta > 0$ is always true. Thus $(w^* = \alpha/(2\beta) + cm/2 + k\Delta^2/8, b^* = \Delta)$ is the optimal solution when $c^* < 1/k$ or $\Delta/2 \le 1/k$. We can also find p^*, Π^{M*}, Π^{R*} and Π^{T*} by replacing b and w with b^* and w^* in appropriate formulas. When $\Delta/2 > 1/k$, the optimal solution is obtained at $c^* = 1/k$. Using similar procedures as above, we can get corresponding p^*, w^*, b^*, Π^{M*}, Π^{R*} and Π^{T*} for the case of $\Delta/2 > 1/k$. The ultimate solution is a combination of the two solutions of $\Delta/2 \le 1/k$ and $\Delta/2 > 1/k$ (i.e., a minimum of the two solutions, except retailer price p^* which is a maximum of the two, see Table 1).

3.2 Solution procedure for coordinated scenario

In the coordinated scenario, we first assume that the optimal $c^* \le 1/k$. By maximizing Π^T over p and c, we can get $c^* = \Delta/2$ and $p^* = \alpha/(2\beta) + cm/2 - k\Delta^2/8$ (Π^T is jointly concave in p and c). We then can get Π^{T*} by replacing p and c with p^* and c^*. When $c^* > 1/k$, the optimal solution is obtained at $c^* = 1/k$. Using similar procedures as above, we can get corresponding p^* and Π^{T*}. The ultimate solution is a combination of the two solutions of $\Delta/2 \le 1/k$ and $\Delta/2 > 1/k$ (i.e., a minimum of the two solutions, except retailer price p^* which is a maximum of the two, see Table 1).

3.3 Solution analysis

The solution of the supply chain model is listed in the Table 1.

Table 1. Solution of the supply chain model

	Scenario 1 (Uncoordinated)	Scenario 2 (Coordinated)
w^*	$\min\left\{\frac{\alpha}{2\beta} + \frac{cm}{2} + \frac{k\Delta^2}{8}, \frac{\alpha}{2\beta} + \frac{cr}{2} + \frac{3}{2k} + \varepsilon\right\}^*$	n/a
b^*	$\min\left\{\Delta, \frac{2}{k} + \varepsilon\right\}^*$	n/a
p^*	$\max\left\{\frac{3\alpha}{4\beta} + \frac{cm}{4} - \frac{k\Delta^2}{16}, \frac{3\alpha}{4\beta} + \frac{1}{4k} + \frac{cr}{4}\right\}$	$\max\left\{\frac{\alpha}{2\beta} + \frac{cm}{2} - \frac{k\Delta^2}{8}, \frac{\alpha}{2\beta} + \frac{1}{2k} + \frac{cr}{2}\right\}$
c^*	$\min\left\{\frac{\Delta}{2}, \frac{1}{k}\right\}$	$\min\left\{\frac{\Delta}{2}, \frac{1}{k}\right\}$
Π^{M*}	$\min\left\{\frac{(\alpha - (cm - k\Delta^2/4)\beta)^2}{8\beta}, \frac{(\alpha - (cr + 1/k)\beta)^2}{8\beta}\right\}$	n/a
Π^{R*}	$\min\left\{\frac{(\alpha - (cm - k\Delta^2/4)\beta)^2}{16\beta}, \frac{(\alpha - (cr + 1/k)\beta)^2}{16\beta}\right\}$	n/a
Π^{T*}	$\min\left\{\frac{3(\alpha - (cm - k\Delta^2/4)\beta)^2}{16\beta}, \frac{3(\alpha - (cr + 1/k)\beta)^2}{16\beta}\right\}$	$\min\left\{\frac{(\alpha - (cm - k\Delta^2/4)\beta)^2}{4\beta}, \frac{(\alpha - (cr + 1/k)\beta)^2}{4\beta}\right\}$

$^{*}\varepsilon \geq 0$ and the same ε for w^{*} and b^{*}. In calculating $\min\{\cdot,\cdot\}$, set $\varepsilon = 0$.

From Table 1, we can notice that depending on $\Delta/2 > 1/k$ or $\Delta/2 < 1/k$, the optimal solution is drawn from one set of two solutions. An interesting observation is that when $\Delta/2 > 1/k$, the optimal whole price is $w^{*} = \alpha/(2\beta) + cr/2 + 3/(2k) + \varepsilon$ and optimal transfer price is $b^{*} = 2/k + \varepsilon$ ($\varepsilon \geq 0$, and ε is same for w^{*} and b^{*}). This is because when $\Delta/2 > 1/k$, the retailer will set the collecting price to $c^{*} = 1/k$ and thus the product return rate $r(c^{*}) = 1$, i.e., every new product sold will be returned eventually. Therefore, only the difference of w and b affects the profits of the manufacturer and the retailer in our model.

As to the profit, we notice that in the uncoordinated scenario Π^{R*}, Π^{M*} and Π^{T*} have linear relationships ($\Pi^{M*} = 2\Pi^{R*} = 2\Pi^{T*}/3$). Also we can derive that the upper bound of Π^{R*} is $(\alpha - cr\times\beta)^2/(16\beta)$ when $k = \infty$; while the lower bound of Π^{R*} is $(\alpha - cm\times\beta)^2/(16\beta)$ when $k = 0$ (Similar for Π^{M*} and Π^{T*}).

Finally, we can find the relationship between the uncoordinated scenario and the coordinated scenario. The collecting price c^{*} is the same in both scenarios; while the retailer price has the relationship $2p_u^{*} = p_c^{*} + \alpha/\beta$ (p_u^{*} is the optimal retailer price in uncoordinated case, p_c^{*} is the optimal retailer price in coordinated case). Also the total profit of the uncoordinated scenario is only 75% of that of the coordinated scenario.

4. Numerical example

In this section, we use a numerical example to illustrate the supply chain model. As we stated before, the marginal return rate k reflects the end-customers' tendency towards product returning and plays an important role in our model, we will therefore particularly focus on how k affects the performances of the supply chain in this numerical example.

The parameters used in the numerical example are $\alpha = 1000$, $\beta = 20$, cm=10, and cr=8; we also vary the parameter k from 0 to 2 to demonstrate its effect. The performance of the supply chain is sketched in Figure 3. The graphs in the upper row are of the uncoordinated scenario; the graphs in the lower row are of the coordinated scenario.

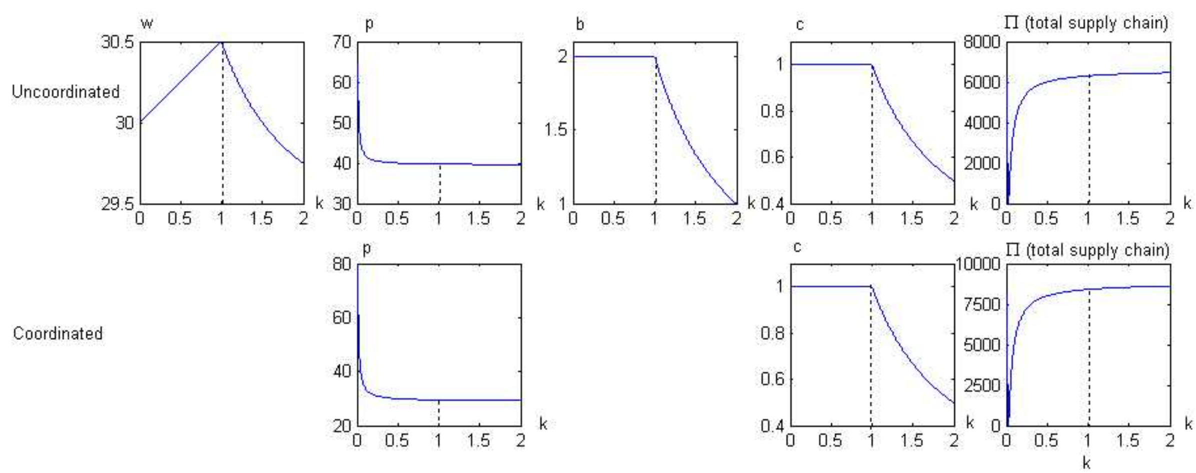

Figure 3. The supply chain performance (numerical example)

We can notice that there are two phases in each graph: $k \leq 2/\Delta = 1$ and $k \geq 2/\Delta = 1$. For example, the wholesale price w first increases and then decreases; while the retailer price keeps decreasing all the way, but at different rate in the two phases. The manufacturer retains no direct savings from remanufacturing ($\Delta = cm - cr = 2$) in the first phase, while starts to retain part of such direct savings in the second phase (b^* is starting to decrease). We can also see that the retailer price is lower and total supply chain profit is higher in the coordinated case than that in the uncoordinated case.

5. Conclusions and future work

In this paper, we analyzed a manufacturer-retailer supply chain model with product remanufacturing. A linear approximation of the product return rate function, based on the market-driven behavior of used product returns, was modeled and integrated into the pricing decision of the retailer and the manufacturer. Two scenarios, uncoordinated and coordinated supply chain, were solved and compared.

Future research can extend our model in several ways. At the present time, we are investigating the cause and degree of "inefficiency" (profit loss due to the decentralized decision process of the uncoordinated scenario vs. the coordinated scenario) in the forward and reverse flow separately. We are also studying the case when the return rate function *r(c)* has a general form ($0 \le r(c) \le 1$, $r'(c)$ exists and $r'(c) > 0$) and its impact on the supply chain model, including the feasibility of various coordination mechanisms, such as fixed franchise fee, quantity discount, etc. Further extension can also consider using piecewise linear approximation of *r(c)* in our model.

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