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Abstract

James Cassing and Jack Ochs' comment is, I believe, a very interesting extension of the analysis of my paper. Their two basic results are: (i) that congestion will occur in the search for jobs; and (ii) that given costly labor mobility, private decisions regarding voluntary quits will yield a socially optimal adjustment path if individuals have perfect foresight and if there is no congestion (externality) in the search process. Thus, they argue that even if factor prices are not rigid, the presence of congestion in the search process implies private decisions will not be socially optimal, and therefore that a subsidy will be needed to support the optimal plan.

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Comments

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International Trade, Factor-Market Distortions, and the Optimal Dynamic Subsidy: Reply

By Harvey E. Lapan[*]

James Cassing and Jack Ochs' comment is, I believe, a very interesting extension of the analysis of my paper. Their two basic results are: (i) that congestion will occur in the search for jobs; and (ii) that given costly labor mobility, private decisions regarding voluntary quits will yield a socially optimal adjustment path if individuals have perfect foresight and if there is no congestion (externality) in the search process. Thus, they argue that even if factor prices are not rigid, the presence of congestion in the search process implies private decisions will not be socially optimal, and therefore that a subsidy will be needed to support the optimal plan.

While I agree with the conclusions of the Cassing-Ochs paper, I disagree with the proof they present. In deriving the socially optimal plan, the authors state the objective is to choose $R^*(t)$ (the number of workers searching for jobs) "... to maximize national income ... over the planning horizon" (p. 952), yet the objective function chosen (equation (3)) reflects only wage income, and not national income. If maximization of national income is the objective, then I believe the objective function should be:

$$(1) \quad \max \sum_{t=1}^{N} [P_1' X_1(L_1(t)) + P_2' X_2(L_2(t))]$$

where the notation is the same as in their paper. Optimizing (1), subject to their equations (6)–(8) (and the definitional equations (4)–(5)), the optimality condition reduces to (15), as presented in their paper. However, if their objective function is used ((3) or (13)), the optimality condition will not, I believe, reduce to (15); the reason for this is that, in differentiating (13) with respect to $R_{N-\tau}$, Cassing-Ochs (implicitly) treat $W_1(N - \tau)$ and $W_2(N - \tau + 1)$ as constants.

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But, an increase in $R_{N-\tau}$ reduces $L_1(N - \tau)$, and increases $L_2(N - \tau + 1)$, which, from their (4) and (5), implies that $W_1(N - \tau)$ and $W_2(N - \tau + 1)$ change as $R_{N-\tau}$ changes. If the objective is maximization of *wage income* (as implied by their choice of objective function), then terms reflecting the changes in the wage rate due to changes in the control should appear in their objective function. Consequently, I believe that the criterion they present for an optimal path (15) is inconsistent with the objective function ((3) or (13)) that they use. On the other hand, if the objective is maximization of national income, as depicted by my (1), then I believe (15) reflects the appropriate optimality conditions. Nevertheless, I should stress that I do agree with their qualitative conclusion that congestion in the labor market will lead private decisions to be socially inefficient.

Furthermore, the model presented in my paper can readily be interpreted to consider the social optimality of private actions; the control model in no way assumes factor prices are rigid. The key assumption is

$$(2) \quad \dot{L}_c = \phi(u)L_m; \phi(0) = 0,$$
$$\phi' > 0, \phi'' \le 0$$

where $\dot{L}_c$ is the increase in employment in C (the sector in which labor's marginal value product is larger); L_m is the stock of potential workers in M ($L_c + L_m = L$, constant), and u is the unemployment rate in M.

While the discussion of the optimal subsidy in my paper presumes unemployment is involuntary, nothing precludes us from interpreting u as voluntary unemployment (this distinction is irrelevant for a centrally controlled solution). In terms of the Cassing-Ochs paper:

$$(3) \quad R(t) = uL_m$$

where R is voluntary unemployment. Thus,

(2) depicts the relationship between job hires in C and the number searching for employment there $(R(t))$. Moreover:

(4) $$(d\dot{L}_c/dR) = \phi'(u)$$

since $L_m(t)$ is given at t. Therefore, $\phi'(u)dt$ represents the probability that an individual searching for work for a time (dt) will find employment in C; it corresponds to $(1 - F(R))$ in the Cassing-Ochs paper. The assumption that $\phi'(0)$ is finite merely implies that, if there is only one searcher, it takes a nonzero amount of time for him (her) to find a job—a not unreasonable assumption.[1] From (4):

(5) $$(d^2\dot{L}_c/dR^2) = [\phi''(u)/L_m] \le 0$$

Since (5) reflects the change in the probability of finding a job as the number of people searching increases, $\phi'' \equiv 0$ corresponds to no congestion $(F'(R) = 0)$, whereas $\phi''(u) < 0$ corresponds to congestion in the labor market $(F'(R) > 0)$.

Private individuals, in deciding whether to quit work and search, compare the opportunity cost of search to the expected benefits. Letting $V(t)$ represent expected (private) net benefits of search:[2]

(6) $$V(t) = [\phi'(u) \int_t^T (W_c(\theta)$$
$$- W_m(\theta))e^{-r(\theta - t)}d\theta - W_m(t)]dt$$

In (6) $W_m(t)dt$ is the opportunity cost of searching for a time interval dt, $\phi'(u)dt$ is the probability of finding a higher paying job, and the integral represents the net dis-

counted value of the higher wage rate.[3] Of course, T reflects the end of the horizon for the prospective searcher. If $V(t)$ is positive at $u = 0$, some search is worthwhile; otherwise, none will be undertaken.

The socially optimal plan is given in my earlier paper; from the maximum principle (my (11)):

(7) $$q\phi'(u) - PF'_m(N_m) \le 0;$$
$$u[q\phi'(u) - PF'_m] = 0; \qquad N_m \equiv L_m(1 - u)$$

where q—the costate variable—is the (current) social value of an increase in $L_c(t)$. The differential equation for q is (my (16)):

(8) $$\dot{q} = (r + \phi)q - (F'_c - P(1 - u)F'_m) =$$
$$(r + \phi - u\phi')q - (F'_c - PF'_m)$$

Consider the term $(\phi - u\phi')$; along an optimal path, $u^*(t)$ is given. Define

(9) $$\epsilon(t) \equiv \phi(u^*) - u^*\phi'(u^*)$$

Given that $\phi'' \le 0$, $\phi(0) = 0$, and $\phi'(0)$ is finite, then $\epsilon(t) \ge 0$ everywhere. Moreover, $\epsilon(t) \equiv 0$ if either (i) $\phi'' \equiv 0$, or (ii) $u^*(t) \equiv 0$ for all t. Thus:

(10) $$\epsilon(t) > 0$$
if, and only if, $\phi'' < 0$ and $u^*(t) > 0$

Integrating (8), using (9) and the transversality condition $q(T) = 0$ yields

(11) $$q(t) = \int_t^T [e^{-r(\theta - t)}e^{-\int_t^\theta \epsilon d\lambda}$$
$$\cdot (F'_c(\theta) - PF'_m(\theta))]d\theta$$

In (11), r is constant, but ϵ is understood to depend on time (if $\phi'' < 0$ and $u^*(t) > 0$). Given $q(t)$, $u^*(t)$, the optimal unemployment rate at t, is determined from (7).

In order to compare the socially optimal plan to atomistic decisions, we must specify how W_c and W_m are determined. If these parameters do not reflect the current marginal value product of employed workers,

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[1] The assumption that labor mobility is costless means, in our context, that $\phi'(u)$ is infinite.

[2] Formally, the decision is not only whether to search for a job, but when. Define $\tilde{V}(t) = V(t)e^{-rt}$, so that $\tilde{V}(t)$ is the discounted value of search at t. If $0 \le \tilde{V}(t) < \tilde{V}(t + dt)$, then search at t is not desirable, even though it will eventually become so; i.e., $u(t) = 0$, but $u(\tau) > 0$, some $\tau > t$. However, for $\phi'' \le 0$, $r \ge 0$, and $\tilde{V}(t + dt) \ge 0$, it is readily shown that $u(t) = 0$ implies $\tilde{V}(t) \ge \tilde{V}(t + dt)$. Consequently, not all search will be postponed: $u(\tau) > 0 \rightarrow u(t) > 0$ for all $t < \tau$. Similarly, if employment never falls to zero in M (as is guaranteed by the Inada derivative conditions), then a competitive solution with perfect foresight implies $\tilde{V}(t) \le 0$. Therefore, for a competitive solution, $u(t) > 0$ implies $\tilde{V}(t) = 0$; and $u(t) = 0$ implies $\tilde{V}(\tau) \le 0$ for all $\tau > t$. Throughout, we assume individuals are risk neutral.

[3] The integral should run from $(t + dt)$ to T, but for small dt, the difference is of the second order of smallness $((dt)^2)$. Note that our formulation of the problem permits discounting, whereas Cassing-Ochs consider only the case where the private and social discount rates are zero.

it is clear private actions will not be optimal. Thus, assume

$$(12) \quad W_c(t) = F_c'(L_c(t)); \quad W_m(t) = PF_m'(N_m(t))$$

where C is the numeraire. Using (12) and (11), (7) becomes:

$$(7') \quad \phi'(u) \int_t^T [e^{-r(\theta-t)} e^{-\int_t^\theta \epsilon d\lambda} \cdot (W_c(\theta) - W_m(\theta)) d\theta] - W_m(t) \leq 0$$

Comparing (7') to (6), the private decision rule, we see that the expressions differ only in the term involving ϵ; if $\epsilon(t) \equiv 0$, the two expressions coincide, assuming individuals have perfect foresight. If there is no congestion ($\phi'' \equiv 0 \equiv \epsilon(t)$), then private decisions are socially optimal. Moreover, in my earlier paper I showed that for $\phi'' \equiv 0$, $N_m(t)$ increases over time (for $u(t) > 0$). This implies that all separations occur initially. As time passes, some individuals find jobs in C, whereas others return to their "original" jobs in M.[4] The pool of searching workers declines over time (this is also true for $\phi'' < 0$).

If $\phi'' < 0$, but $u^*(t) \equiv 0$ (i.e., if wage differentials are small, relative to r, or the length of the plan is short), then $\epsilon(t) \equiv 0$ and private decisions will again be socially optimal. However, if $\phi'' < 0$, and $u^*(t) > 0$ for some t, then (6) and (7') no longer coincide. The congestion—or externality—causes private decisions to be socially suboptimal. Comparing (6) and (7') we see that, starting from the same initial allocation of labor, the *initial* unemployment rate under private actions $u^P(t)$ will exceed $u^*(t)$, as stated by Cassing-Ochs. Clearly, the *initial* private unemployment rate is higher because private decision makers ignore the congestion caused by additional entries into the pool of searchers; the optimal plan

properly recognizes these congestion costs.

However, this does not imply that private decisions lead to unemployment rates that are *everywhere* higher than for the optimal path. The higher initial $u^P(t)$ implies that at any future time, more workers will be employed in C under the private solution than under the socially optimal plan ($L_c^P(t) > L_c^*(t)$). Moreover, since the decision rule for terminating search—or labor transfers—is the same for private decisions and socially optimal ones; and since the terminal period of full employment increases as L_c increases, it immediately follows that full employment is restored sooner under private actions than under the socially optimal plan. Consequently, with congestion, the initial private unemployment rate is higher than is socially optimal, but ultimately it falls below the unemployment rate along the optimum path.[5] Full employment is achieved sooner under private actions, more labor is transferred to the higher wage sector, and national income—during the latter stages of the plan—is larger under private actions. Of course, the discounted value of national income over the whole plan is less under private actions.

The principal point raised by Cassing-Ochs, I believe, is that even if factor prices are not rigid, private actions will not be socially optimal if congestion occurs in the search process. This point—with which I completely agree—follows directly from the externality generated by too many people searching for jobs at any one time. Private decisions will be socially optimal if: (i) no congestion occurs; (ii) individuals have perfect foresight; (iii) the private and social planning horizons and discount rates are equal; and (iv) wages adjust instantaneously. Should any of these conditions fail to hold, then some intervention is required to support the socially optimal plan.

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[4] This assumes no search is necessary in M—workers, having been employed there, know where to look for work. For symmetry, one should assume it is necessary to search for jobs in M as well as C; this, in turn would discourage initial quits. However, neither my original paper—nor the Cassing-Ochs specification—incorporates this assumption.

[5] Of course, if an optimal plan were instituted at any future moment, given the labor allocation provided by private decisions, then the socially optimal unemployment rate would be lower. However, it makes more sense to contrast the time path generated by private decisions to that generated by a plan that is optimal over the whole planning period.

REFERENCES

J. Cassing and J. Ochs, "International Trade, Factor-Market Distortions, and the Optimal Dynamic Subsidy: Comment," *Amer. Econ. Rev.*, Dec. 1978, *68*, 950–55.

H. E. Lapan, "International Trade, Factor Market Distortions, and the Optimal Dynamic Subsidy," *Amer. Econ. Rev.*, June 1976, *66*, 335–46.