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D. M. Clatterbuck
Iowa State University

V. J. Garcia
Iowa State University

M. J. Johnson
Iowa State University

David C. Jiles
Iowa State University, dcjiles@iastate.edu

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Abstract

The Barkhausen model of Alessandro et al. [J. Appl. Phys. **68**, 2901 (1990)] has been extended to nonstationary domain walldynamics. The assumptions of the original model limit, its use to situations where the differential permeability, and time derivative of applied field are constant. The later model of Jiles et al. assumes that the Barkhausen activity in a given time interval is proportional to the rate of change of irreversible magnetization which can be calculated from hysteresis models. The extended model presented here incorporates ideas from both of these. It assumes that the pinning field and domain wall velocity behave according to the Alessandro model, but allows the rate of change of the magnetic flux to vary around a moving average which is determined by the shape of the hysteresis curve and the applied magnetic field wave form. As a result, the new model allows for changes in permeability with applied field and can also reproduce the frequency response of experimental Barkhausen signals.

Disciplines

Electromagnetics and Photonics | Engineering Physics

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D. M. Clatterbuck,[a)] V. J. Garcia, M. J. Johnson, and D. C. Jiles[b)]

Ames Laboratory and Center for Nondestructive Evaluation, Iowa State University, Ames, Iowa 50011

The Barkhausen model of Alessandro *et al.* [J. Appl. Phys. **68**, 2901 (1990)] has been extended to nonstationary domain wall dynamics. The assumptions of the original model limit, its use to situations where the differential permeability, and time derivative of applied field are constant. The later model of Jiles *et al.* assumes that the Barkhausen activity in a given time interval is proportional to the rate of change of irreversible magnetization which can be calculated from hysteresis models. The extended model presented here incorporates ideas from both of these. It assumes that the pinning field and domain wall velocity behave according to the Alessandro model, but allows the rate of change of the magnetic flux to vary around a moving average which is determined by the shape of the hysteresis curve and the applied magnetic field wave form. As a result, the new model allows for changes in permeability with applied field and can also reproduce the frequency response of experimental Barkhausen signals. © *2000 American Institute of Physics.* [S0021-8979(00)77908-4]

I. INTRODUCTION

Irreversible changes in magnetization occur when a domain wall moves suddenly from one local minimum in the free energy to another, causing discontinuous changes in the magnetization known as Barkhausen events. These discontinuous changes can be detected by selecting the components of the voltage induced in a coil encircling a sample over a given frequency range, usually in the range of tens to hundreds of kilohertz, while the sample is being magnetized. These components of voltage are linearly proportional to the discontinuous rate of change of flux in accordance with Faradays' law. As a result of the dependence of domain wall motion on the local conditions in the material through which the wall is moving, the Barkhausen effect has found use for probing both the microstructure and stress state of materials.

II. REVIEW OF EXISTING MODELS

A description of the domain wall dynamics giving rise to the Barkhausen effect has been proposed by Alessandro, Beatrice, Bertotti, and Montorsi[1] (ABBM). In this model the rate of change of magnetic flux $\dot{\phi}$ can be expressed as

$$\sigma G \dot{\phi} = H - H_s, \tag{1}$$

where σ is the electrical conductivity, G is a dimensionless coefficient (which reduces to $G = 0.1356$ if a wide slab is considered), and H is the local magnetic field. As the domain wall moves through a material it will experience an effective local pinning field H_c which is the result of interactions with discontinuities in the structure of the material. H_c is assumed to be a random function of the domain wall position. The interaction range of the domain wall with a given disconti-

nuity in the material can be described by a correlation distance[1] ξ, which represents the equivalent change in flux ϕ over the range of interaction

$$\frac{dH_c}{d\phi} + \frac{H_0 - \langle H_c \rangle}{\xi} = \frac{dW}{d\phi}, \tag{2}$$

where the white noise function $W(\phi)$ is characterized by $\langle dW \rangle = 0, \langle |dW|^2 \rangle = 2A d\phi$, with A being a coefficient which describes the variance of the fluctuations. Owing to the stationary assumptions of the model, both the power spectrum and amplitude probability distribution have analytical solutions in many regimes. Agreement of the ABBM model with experiments shows that the dynamics of a system with many interacting domain walls can be described by simply considering the statistical behavior of a single domain wall. However, the ABBM model is derived only for a limited case and is restricted to Barkhausen signals at constant applied field rates and in a small region of the hysteresis loop near the coercive point where the permeability is constant. It should also be noted that the model equations are restricted to soft magnetic materials where $\mu_r \gg 1$.

Models which describe the Barkhausen effect over the entire hysteresis loop have been proposed by Sablik and Augustymak [2] and Jiles, Sipahi, and Williams[3] (JSW) and subsequently developed further.[4,5] This latter model assumes that the rate of irreversible change in magnetization is proportional to the level of Barkhausen activity in a given time interval. While not explicitly stated, this implies that the Barkhausen signal is the result of a linear combination of many individual events occurring at different locations within in the material. In order to account for the random nature of the Barkhausen effect, it was suggested that the number of Barkhausen events in a given time period $N(t_n)$ is related to the to the number of events in the previous time period $N(t_{n-1})$ by some random increment. This increment is small enough to ensure that the number of events remain correlated with those in the previous time period. Since the number of possible locations for a Barkhausen event to occur

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[a)]Present address: Lawrence Berkeley National Lab and Dept. of Materials Science and Mineral Engineering, University of California, Berkeley, CA 94720.
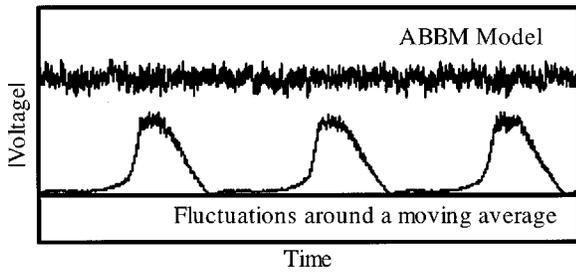[b)]Electronic mail: gauss@ameslab.gov

FIG. 1. The ABBM model gives fluctuations around a constant average, while the proposed model describes fluctuations around a moving average.

FIG. 3. The effect of σ on the envelope with A chosen to compensate for the effect of variance. As σ is increased the result is an increased broadening and lag in the envelope.

in the material is large, but the probability that an event occurs at a specific point in the material is low, Poisson statistics describe the variation of the number of events from one time period to the next. This leads to a model equation of the form

$$\dot{M}_{JS}(t_n) = (M_{disc})\chi'_{irr}\dot{H}[N'(t_{n-1}) + \delta\sqrt{N'(t_{n-1})}], \quad (3)$$

where M_{JS} represents the Barkhausen activity in terms of the "jump sum" as discussed by Swartzendruber *et al.*,[6] $\langle M_{disc}\rangle$ is the average jump size, χ'_{irr} is the irreversible differential susceptibility, H is the rate of change of magnetic field with time, and δ is a random number with a mean value of zero and a standard deviation of unity. The advantage of this model is that it is able to describe the Barkhausen effect at all points on the hysteresis curve and under the action of a variable rate of change of applied field and thus does not require the assumption of a stationary process.

Consider the ABBM model in the particular case where $\xi \to \infty$. This corresponds to an unlimited range of correlation between the domain wall and a given pinning site in the material, or alternatively to a sufficiently slow domain wall velocity. Equation (2) then reduces to $dH_c = dW$, which can also be expressed as $dH_c = \delta\sqrt{2A\,d\phi}$, where δ is again a normal distribution with a mean of zero and a variance of unity. Following the earlier work,[1] this leads to

$$\sigma G \frac{d\dot{\phi}}{dt} = \dot{H}_a - \dot{H}_\infty - \delta\sqrt{2A\,d\phi}, \quad (4)$$

where H_a is the applied field and H_m is a correction to the magnetic field due to magnetostatic or demagnetizing effects. Over short time periods, this equation will be domi-
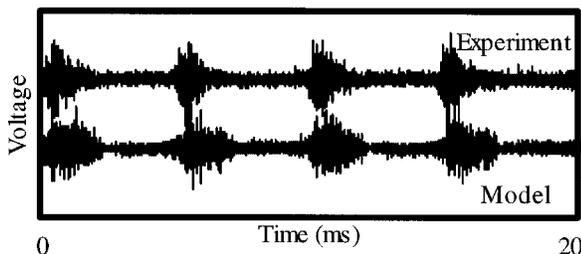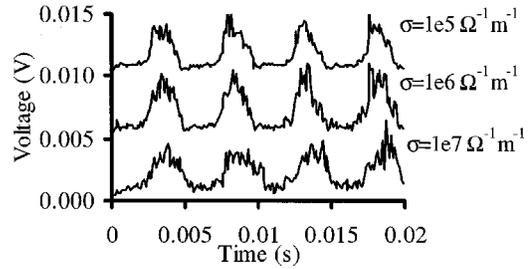
nated by the third term on the right, and thus if we consider the change in ϕ during a short time increment Δt, we obtain a simplified ABBM model of the form

$$\dot{\phi}(t_n) = \dot{\phi}(t_{n-1}) + \delta\frac{\sqrt{2A\Delta t}}{\sigma G}\sqrt{\dot{\phi}(t_{n-1})}. \quad (5)$$

The JSW model described by Eq. (3) can be rewritten as

$$\dot{M}_{JS}(t_n) = \dot{M}_{JS}(t_{n-1}) + \delta\sqrt{(M_{disc})\chi'_{in}\dot{H}}\sqrt{\dot{M}_{JS}(t_{n-1})}, \quad (6)$$

which shows that the ABBM model can be reduced to a mathematical structure similar to the JSW model under these conditions.

III. PROPOSED MODEL

In order to arrive at a model that can describe the dynamics of the Barkhausen events which occur as a sample is magnetized around an entire hysteresis loop, the ABBM equation can be written as

$$\frac{d\dot{\phi}}{dt} + \frac{\dot{\phi} - \langle\dot{\phi}\rangle}{\tau} = -\frac{1}{\sigma G}\frac{dH_c}{dt}. \quad (7)$$

We can see that the original ABBM model describes the variation of $\dot{\phi}$ from a constant average value $\langle\dot{\phi}\rangle$. This is shown graphically in Fig. 1 for the case of continual domain wall motion. In the proposed model $\dot{\phi}$ does not have a single average value for all times, but instead it has an ensemble average value which is a function of time, $\langle\dot{\phi}\rangle(t)$. This is equivalent to suggesting that if one applies a cyclic time varying magnetic field, the material will sweep out a hysteresis curve, and that the fine details of this curve will be different during each cycle due to the statistical nature of the Barkhausen effect. Over many cycles the hysteresis curve at

FIG. 2. Comparison of experimental Barkhausen signal and results of model in the time domain.
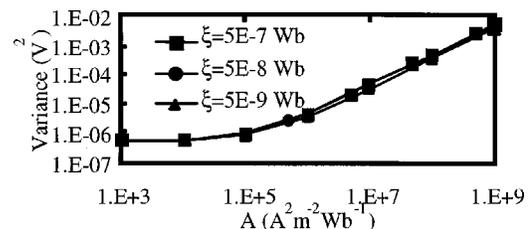

FIG. 4. Effect of A on variance at various values of ξ. For large values of A the variance increases linearly. There was no effect on the variance due to ξ, except at small values where there is slight increase in variance with ξ.
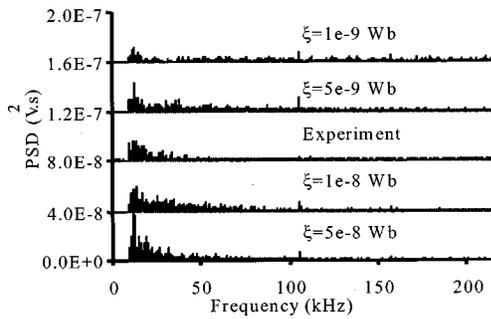
FIG. 5. Effect of ξ on the PSD at constant variance. As ξ is decreased, the PSD is flattened.
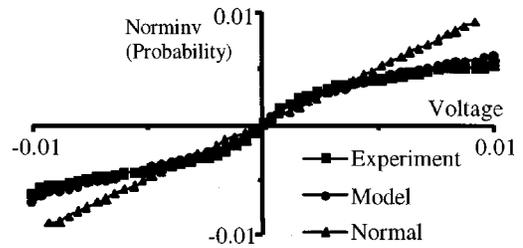

FIG. 6. Amplitude probability distribution for experimental Barkhausen signal and model. Both the proposed model and experiment results show similar systematic deviations from a normal distribution.

any given time in the cycle will generally lie close to the average value for that time as in the bottom half of Fig. 1. We can then rewrite the model in terms of deviations from this ensemble average. Since Barkhausen jumps actually correspond to irreversible changes in magnetization $\dagger_{\mathrm{irr}}$ rather than changes in magnetic induction, we rewrite the model in terms of SI_{irr} rather than ϕ, where S is the cross sectional area, where $\tau = \sigma GS\chi_{\mathrm{irr}}$. The pinning field H_c is given by

$$\frac{d\dot{I}_{\mathrm{irr}}}{dt} + \frac{\dot{I}_{\mathrm{irr}} - \langle \dot{I}_{\mathrm{irr}} \rangle}{\tau} = -\frac{1}{\sigma GS}\frac{dH_c}{dt}, \tag{8}$$

where $\tau = \mu_a \sigma GS\chi'_{\mathrm{in}}$. The pinning field H_c is given by

$$\frac{dH_c}{dI_{\mathrm{irr}}} + \frac{S(H_c - \langle H_c \rangle)}{\xi} = \frac{dW}{dI_{\mathrm{irr}}}, \tag{9}$$

and

$$\langle dW \rangle = 0, \quad \langle |dW|^2 \rangle = 2ASdI_{\mathrm{irr}}. \tag{10}$$

Due to the nonlinear, multivalued properties of the differential susceptibility, both $\langle i_{\mathrm{irr}} \rangle$ and τ will now be functions of the position on the hysteresis curve and can be related to time t through the applied field wave form $H_a(t)$ by

$$\langle \dot{I}_{\mathrm{arr}} \rangle = \mu_0 \chi'_{\mathrm{irr}}[H_a(t)]\dot{H}_a(t). \tag{11}$$

The extended Barkhausen model based on these equations is able to generate signals which closely resemble the experimental data around a hysteresis loop, as shown in Fig. 2.

The first step in modeling the experimental Barkhausen signals from nickel was to select a value of σ. The effect of σ on the Barkhausen voltage "envelope" is shown in Fig. 3. Simulations show that $\sigma = 1 \times 10^5 \, \Omega^{-1} \, \mathrm{m}^{-1}$ yields a Barkhausen envelope which matches well with the experimental data.

The effect of the parameters A and ξ on the variance of the Barkhausen signal is shown in Fig. 4. Simulations were also performed with systematically varying values of ξ and A selected so that the variance was the same as the variance of the experimental data. The power spectral density (PSD) for

this series of simulations for various values of ξ are shown in Fig. 5. Agreement with the experimental PSD was found for $\xi = 1 \times 10^{-8} \, \mathrm{W\,b}$. The time domain of this simulation was compared to the experimental results of Fig. 2, and the amplitude probability distribution for both are shown in Fig. 6. By linearizing the probability with respect to a normal distribution, we find that both the experimental data and the simulation show systematic deviations from a normal distribution with the same variance.

IV. CONCLUSIONS

The ABBM model was only derived for cases of constant permeability and constant rate of change of applied field at locations near the coercive point. However, the model is able to describe the amplitude distribution and power spectrum of the Barkhausen signal under these restricted conditions. The model has been combined with the JSW model, which is applicable at all points on the hysteresis curve. A more general model therefore results which can describe the fluctuations in Barkhausen emissions around a moving average. Comparisons with experimental data obtained on a nickel sample show that the model is able to accurately reproduce the time domain, amplitude probability distribution, and power spectrum of the Barkhausen signal.

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