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# Rejuveniles and growth

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Rejuveniles are "grown-ups who cultivate juvenile tastes in products and entertainment". In this note, we study a standard AK growth model of overlapping generations populated by rejuveniles. For our purposes, rejuveniles are old agents who derive utility from "keeping up" their consumption with that of the current young. We find that such cross-generational keeping up is capable of generating interesting equilibrium growth dynamics, including growth cycles. No such growth dynamics is possible either in the baseline model, one where no such generational consumption externality exists, or for almost any other form of keeping up. Steady-state growth in a world with rejuveniles may be higher than that obtained in the baseline model.

## **Keywords**

overlapping generations, growth cycles, keeping up preferences

## **Disciplines**

Economics

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## **Rejuveniles and Growth**

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# Rejuveniles and Growth

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## Abstract

*Rejuveniles are “grown-ups who cultivate juvenile tastes in products and entertainment”. In this note, we study a standard AK growth model of overlapping generations populated by rejuveniles. For our purposes, rejuveniles are old agents who derive utility from “keeping up” their consumption with that of the current young. We find that such cross-generational keeping up is capable of generating interesting equilibrium growth dynamics, including growth cycles. No such growth dynamics is possible either in the baseline model, one where no such generational consumption externality exists, or for almost any other form of keeping up. Steady-state growth in a world with rejuveniles may be higher than that obtained in the baseline model.*

*Key words:* Growth cycles, keeping up preferences, overlapping generations

*JEL Classifications:* E 13, E 32

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“Life can only be understood backwards, but it must be lived forward”

— Soren Kierkegaard

## 1 Introduction

Rejuvenile is a term coined by Christopher Noxon (2006) to describe “adults dedicated to indulging their inner child.” In her review in the WSJ (2006), Gurdon (2006) goes further to describe rejuveniles as “[This] curious modern hybrid, adult in physique yet deliberately madcap and childlike in tastes, habits,...”<sup>1</sup> In this note, we study a neoclassical world populated by overlapping generations of rejuveniles and seek to understand the impact of such preferences on economic growth. For our purposes, rejuveniles are old agents who derive utility from “keeping up” their consumption with that of the current young.<sup>2</sup>

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<sup>1</sup> Noxon (2003) provides more context. “Evidence of their presence is widespread. According to Nielsen Media research, more adults 18 to 49 watch the Cartoon Network than watch CNN. More than 35 million people have caught up with long-lost school pals on the Web site Classmates.com. (“There’s something about signing on to Classmates.com that makes you feel 16 again,” the “60 Minutes II” correspondent Vicki Mabrey reported.) Fuzzy pajamas with attached feet come in adult sizes at Target, along with Scoobie Doo underpants. The average age of video game players is now 29, up from 18 in 1990, according to the Entertainment Software Association. Hello Kitty’s cartoon face graces toasters. Sea Monkeys come in an executive set....And then there is Harry Potter, whose cross-generational popularity prompted the British publisher Bloomsbury to release an edition of the books with so-called grown-up covers.”

<sup>2</sup> Then there are people who match their consumption with that of the young by association. For example, there are 60 million grandparents in the United States – 72% of everyone over 50 in the US is a grandparent. Grandparents spend time and money with their grandchildren – over \$30 billion in annual spending. Research shows that going out to a restaurant and watching television together are the activities grandparents and grandkids do most.

Lately there has been some interest in dynamic macroeconomic models featuring “extended” preferences that deviate from the standard additive, time-separable, homothetic utility – the most notable being those that incorporate a minimum consumption requirement, habit persistence, or ‘keeping up with the Joneses’.<sup>3</sup> This note fits into this larger literature because it explores a consumption externality similar in spirit to the aforementioned, except for the fact that the externality studied here is *cross generational*. In fact, one can reinterpret the preferences studied in this paper as representing minimum consumption requirements imposed by the consumption patterns of generations other than one’s own, a keeping up with the senior and junior Joneses, if you will.<sup>4</sup>

More generally, we assume that an agent wishes to “keep up” her consumption with the rest of the population that is alive at the time, young and old alike. Within this “peer group”, we allow some members to have more influence than others. For example, it may be that when young, the agent wishes to keep her consumption more closely paced with members of her own cohort, rather than cohorts of the previous generation. When old, her consumption may be more heavily influenced by cohorts of the current, young generation (in which case, she would exhibit strong rejuvenile behavior).

In Section 2, we embed these extended preferences into a standard *AK* model of growth. In Section 3, somewhat counterintuitively, we find that the long run growth rate is higher in the presence of rejuveniles when compared to

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<sup>3</sup> Alvarez-Cuadado, Monteiro, and Turnovsky (2004) contain a nice discussion of the empirical underpinings of departures from time-separability.

<sup>4</sup> The classic reference on the empirical importance of minimum consumption requirements is Rosenweig and Wolpin (1993) who use Indian data to estimate a minimum consumption floor at 56% of the mean household food consumption.

the benchmark economy (one with no minimum consumption requirements of any kind). After all, for the old to keep up with the consumption of the young, the old would have had to save adequately in the past, and it is this forward-looking thriftiness that fosters growth. We also find that keeping up when young hurts growth.

Curiously enough, we also find that the model with juveniles has the potential to generate interesting dynamics in the growth rate, including *growth cycles*. The presence of the agent's own cohort in her peer group does not seem to matter much here – what does matter, and what sets our formulation apart from other papers looking at minimum consumption requirements, is the fact that we allow another generation to influence the agent's consumption decision – in this case, the old (when the agent is young) and youth (when the agent is old). Indeed, in the absence of juveniles, the economy at hand does not generate any growth fluctuations. The upshot is that juveniles raise the long run growth rate but their presence may also expose the economy to endogenous growth fluctuations.

In Section 4, we go on to study several other ways of introducing minimum consumption requirements, such as: a) minimum consumption requirements that keep up with the level of development of the economy, b) keeping up with one's own past consumption (aka, habit persistence), and c) keeping up with one's parents consumption at parallel points in the lifecycle. Interestingly, *none* of these preferences can generate any form of endogenous growth fluctuations within our *AK* model framework.

Our work is in line with the recent paper by Alvarez-Cuadado, Monteiro, and Turnovsky (2004) who study alternative preference formulations (habit for-

mation, keeping up with the Joneses) within the context of a continuous time, infinitely-lived agent framework with a neoclassical production function. Our results are not strictly comparable given our overlapping generations structure; especially, their framework is not suited to study the cross generational keeping up preferences that is our focus. Additionally, while the generation of growth cycles is our focus, it is not theirs.<sup>5</sup>

Our note also relates to a part of a larger literature studying growth cycles (as opposed to periodicity in levels) in real neoclassical economies. To the best of our knowledge, all models to date that generate growth cycles are *technology* not preference driven (see, for example, Matsuyama (1999) or Walde (2005)). Our result on the existence of periodic growth equilibria is of some independent interest. There is a vast literature studying the possibility of periodic (even chaotic) equilibria in general equilibrium growth models, especially in overlapping generations models. Most of that literature is concerned with studying *nominal* cycles (i.e., fluctuations in price *levels*).<sup>6</sup> The rest of the literature has focussed on studying real cycles in the levels of the capital stock or output. As is well-known in that literature, complex dynamics (such as periodic equilibria) can emerge under assumptions such as limited market participation, imperfect competition, and multiple sectors. Additionally, as discussed in Azariadis (1993), a sufficiently strong income effect can cause savings to decline with an increase in the interest rate, creating a “backward-bending” savings function that can produce complex dynamics in overlapping

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<sup>5</sup> Alonso-Carrera, Caballé, and Raurich (2005) study the efficiency properties of the steady state in the standard infinitely-lived neoclassical model under alternative formulations of habit formation.

<sup>6</sup> See Grandmont (1985), Smith (1991), Bhattacharya and Russell (2003), or more recently, Bunzel (2006) and Koskela and Puhakka (2006).



generations models. In our model, periodic equilibria in the *real growth rate* emerge in a relatively standard economy; indeed, absent our assumptions on preferences (especially the presence of juveniles), our model economy would not produce endogenous fluctuations of any kind. Thus, ours is not *yet* another paper demonstrating the presence of complex dynamics in overlapping generation growth models. Our novelty lies in our ability to a) generate real growth cycles that are preference not technology driven, and b) to show that such growth cycles are not possible with almost any other kind of keeping up preferences.

## 2 The Model

We analyze a production economy inhabited by an infinite sequence of two-period lived overlapping generations, plus an initial old generation. At each date,  $t = 1, 2, 3, \dots$ , a new generation is born, consisting of a continuum of agents with mass 1. Each agent is endowed with one unit of time when young and is retired when old. Agents do not value leisure, so the allocation of work-time equals the time endowment of 1. In addition, each initial old agent is endowed with  $k_1 > 0$  units of capital.

There is a single final good produced using a production function  $F(K_t, L_t)$  where  $K_t$  denotes the capital input and  $L_t$  denotes the labor input at  $t$ . Let  $k_t \equiv K_t/L_t$  denote the capital-labor ratio (capital per young agent). Output per young agent at time  $t$  may be expressed as  $f(k_t)$  where  $f(k_t) \equiv F(K_t/L_t, 1)$  is the intensive production function. The final good can either be consumed in the period it is produced, or it can be stored to yield capital the following period. For reasons of analytical tractability, capital is assumed to depreciate

100% between periods.

We assume a standard  $Ak$  model with a Romer-style externality, where  $\alpha$  ( $1 - \alpha$ ) denotes capital's (labor's) share of output.<sup>7</sup> Firms in the economy are competitive and factors are paid their marginal product. Let  $w$  denote the wage and  $R$  denote the gross interest rate. For future reference,

$$w(k) = (1 - \alpha)Ak$$

and

$$R = \alpha A.$$

We now define cross-generational keeping-up preferences. We assume that agents have preferences represented by the atemporal utility function  $U(c_t, x_t)$  defined as

$$U(c_t, x_t) \equiv \frac{(c_t - \theta_t)^{1-\sigma}}{1-\sigma} + \frac{\beta(x_t - \delta_t)^{1-\sigma}}{1-\sigma}, \quad \sigma > 0, \theta_t \geq 0, \delta_t \geq 0, t \geq 1 \quad (1)$$

where  $c_t$  ( $x_t$ ) is the consumption of an agent of generation  $t$  when young (old) and  $\theta_t$  ( $\delta_t$ ) represents the minimum consumption requirement the agent faces when young (old). Peeking inside the consumption floors  $\theta$  and  $\delta$ , we posit that

$$\theta_t \equiv (\theta_y \bar{c}_t + \theta_o \bar{x}_{t-1}),$$

and

$$\delta_t \equiv (\delta_y \bar{c}_{t+1} + \delta_o \bar{x}_t).$$

Here, variables with “bars” represent average levels of consumption, taken as parametric by the agent. For example,  $\bar{c}_t$  represents the average level

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<sup>7</sup> In other words, we assume  $Y_t^i = A\bar{K}_t^\beta (K_t^i)^\alpha (L_t^i)^{1-\alpha}$  where  $i$  indexes a firm among a continuum of firms of unit measure, and  $\bar{K}$  denotes the average of all  $K^i$ 's. If one assumes that firms are all identical,  $L_t^i = 1$ , and  $\beta = 1 - \alpha$ , then it is easy to generate the expressions for  $w$  and  $R$  in the paper.

of consumption of the agent's generational cohorts when she is young. [Of course, in equilibrium,  $c_t = \bar{c}_t$  and  $x_t = \bar{x}_t$  will hold].  $\theta_y$  and  $\theta_o$  represent scalars capturing the strength of the influence of the current youth and current old's consumption, respectively, on the minimum consumption requirement of the agent when young. Similarly,  $\delta_y$  and  $\delta_o$  represent scalars capturing the strength of the influence of the current youth and current old's consumption, respectively, on the minimum consumption requirement of the agent when old. Within the context of these preferences, rejuvenile-like behavior is associated with  $\delta_y > 0$ .

What sets our formulation apart from other papers looking at minimum consumption requirements is the fact that we allow a generation other than one's own to influence the agent's consumption decision – in this case, the old (when the agent is young) and youth (when the agent is old), and this pattern of influence may change over the agent's life. For example, consumption in an economy is dominantly “youth driven” if both  $\theta_y > \theta_o$  and  $\delta_y > \delta_o$ .

From our specification in (1), it is clear that we must assume

$$c_t \geq \theta_t \tag{2}$$

and

$$x_t \geq \delta_t. \tag{3}$$

holds in what follows.

Given the capital stock  $k_t$  and taking everyone else's consumption as given, the agent's choices for  $c_t$ ,  $x_t$ , and  $k_{t+1}$  conform to the following budget constraints:

$$c_t + k_{t+1} \leq (1 - \alpha)Ak_t \tag{4}$$

$$x_t \leq \alpha A k_{t+1}, \quad (5)$$

where we have incorporated in (4) and (5) the assumption that date  $t$  output  $y_t = A k_t$ . An agent's problem is to maximize (1) subject to (4) and (5). Furthermore, her optimal choices for  $c_t$ ,  $x_t$ , and  $k_{t+1}$  all have to be positive.

The first-order conditions for the agent's optimization problem are summarized by the following equation:

$$\frac{x_t - \delta_t}{c_t - \theta_t} = z, \quad (6)$$

where  $z \equiv (\beta \alpha A)^{1/\sigma}$ .

In the **benchmark** case, we set  $\delta_t = \theta_t = 0$ . Employing the two budget constraints (4) and (5) with equality, we can rewrite (6) as

$$\frac{\alpha A k_{t+1}}{(1 - \alpha) A k_t - k_{t+1}} = z.$$

Dividing the numerator and denominator of the left-hand side by  $k_t$ , we have:

$$\frac{\alpha A \gamma_t}{(1 - \alpha) A - \gamma_t} = z, \quad (7)$$

where  $\gamma_t \equiv y_{t+1}/y_t = k_{t+1}/k_t$  is the growth rate in output at date  $t$ . Solving for  $\gamma_t$ , we obtain our benchmark growth rate,  $\gamma_{bm}$ :

$$\gamma_{bm} = \frac{(1 - \alpha) A z}{\alpha A + z}. \quad (8)$$

Equation (6) for our specific formulation of minimum consumption requirements is

$$\frac{x_t - (\delta_y \bar{c}_{t+1} + \delta_o) \bar{x}_t}{c_t - (\theta_y \bar{c}_t + \theta_o \bar{x}_{t-1})} = z,$$

which after incorporating the definitions of the variables with bars, may be

rewritten as

$$\frac{(1 - \delta_o) x_t - \delta_y c_{t+1}}{(1 - \theta_y) c_t - \theta_o x_{t-1}} = z. \quad (9)$$

From the budget constraints (4) and (5), and (9), we write, for  $t \geq 2$ ,

$$\frac{(1 - \delta_o) \alpha A k_{t+1} - \delta_y ((1 - \alpha) A k_{t+1} - k_{t+2})}{(1 - \theta_y) ((1 - \alpha) A k_t - k_{t+1}) - \theta_o \alpha A k_t} = z.$$

A similar condition applies for date  $t = 1$ , as can be seen by incorporating the budget constraint for the initial old,  $x_0 = \alpha A k_1$ , in (9) for  $t = 1$ .

Dividing the numerator and denominator by  $k_t$ , we express this condition, for  $t \geq 1$ , in growth rates:

$$\frac{\gamma_t [(1 - \delta_o) \alpha A - \delta_y ((1 - \alpha) A - \gamma_{t+1})]}{(1 - \theta_y) ((1 - \alpha) A - \gamma_t) - \theta_o \alpha A} = z. \quad (10)$$

What is interesting about (10) is the presence of the future growth rate  $\gamma_{t+1}$ . The agent's belief about the consumption decisions of the future generation impact on consumption and saving decisions of the agent when young. It is by this mechanism that 'keeping up' has the potential to generate equilibrium cycles in *growth rates*. Note that if youth has no influence on the agent's consumption decision when old ( $\delta_y = 0$ ), i.e., no one is a rejuvenile, the future growth rate drops out of (10), and has no impact on the current growth rate  $\gamma_t$ .

Solving (10) for  $\gamma_{t+1}$  (assuming  $\delta_y > 0$ ) yields the difference equation, for  $t \geq 1$ ,

$$\gamma_{t+1} = \frac{z [(1 - \theta_y) (1 - \alpha) A - \theta_o \alpha A] + \gamma_t [\delta_y (1 - \alpha) A - (1 - \delta_o) \alpha A - z (1 - \theta_y)]}{\delta_y \gamma_t}. \quad (11)$$

A dynamic *growth equilibrium* for this economy is represented by a sequence

of positive growth rates  $\{\gamma_t\}_{t=1}^{\infty}$ ,  $\gamma_t \geq 0$ , that satisfies (11).

Not surprisingly, the system, with juveniles, displays indeterminacy, which can be indexed by the growth rate for date 1. Given an appropriate value for  $\gamma_1$ , (11) determines all rates of growth for  $t \geq 2$ .

### 3 Characterizing Equilibria

#### 3.1 Existence

Our minimum consumption requirements (2) and (3) place restrictions on the magnitude of the primitives of the model. These are summarized by

**Assumption 1**  $(1 - \alpha)(1 - \theta_y) > \alpha\theta_o$ .

**Assumption 2**  $(1 - \delta_o)(1 - \theta_y) > \delta_y\theta_o$ .

Note both assumptions place the restriction that the influence on the consumption decision of an agent's own peer group cannot be too great, i.e.,  $\theta_y, \delta_o < 1$ .

The constraint (2) requires

$$\begin{aligned} (1 - \theta_y)c_t \geq \theta_o \bar{x}_{t-1} &\Leftrightarrow (1 - \theta_y)((1 - \alpha)Ak_t - k_{t+1}) \geq \theta_o \alpha Ak_t \\ &\Leftrightarrow [(1 - \theta_y)(1 - \alpha) - \alpha\theta_o]Ak_t \geq (1 - \theta_y)k_{t+1} \end{aligned}$$

and since  $k_{t+1} > 0$ , we require Assumption 1. This assumption also restricts the growth rate, in the first instance, to be no greater than  $\bar{\gamma}$ , where  $\bar{\gamma} \equiv (1 - \alpha)A - \frac{\alpha A \theta_o}{1 - \theta_y}$ .

Examining (10), we see the model also places the restriction that the growth rate must be greater than  $\underline{\gamma}$ , where  $\underline{\gamma} \equiv \max \{0, (1 - \alpha) A - \alpha A (1 - \delta_o) / \delta_y\}$ , for otherwise, the numerator in (10) is negative while the denominator is positive. (This condition also follows from (3)). Assumption 2 ensures  $\underline{\gamma} < \bar{\gamma}$ . Hence, a valid equilibrium growth sequence  $\{\gamma_t\}_{t=1}^{\infty}$  defined by (11) additionally requires  $\underline{\gamma} < \gamma_t < \bar{\gamma}$  for  $t \geq 1$ .

Let  $h(\gamma_t)$  denote the function described by the right-hand side of (11). Given Assumptions 1 and 2,  $h(\cdot)$  has the following properties:<sup>8</sup>

**P-1**  $h'(\gamma_t) < 0$  and  $h''(\gamma_t) > 0$  over the interval  $(\underline{\gamma}, \bar{\gamma})$ .

**P-2**  $h(\underline{\gamma}) > \underline{\gamma}$  and  $h(\bar{\gamma}) < \bar{\gamma}$ .

Together, P-1 and P-2 ensure a *unique steady-state* growth rate (denoted  $\gamma^*$ ) exists for all dates  $t \geq 1$ . It is easy to verify using (11) that at a steady state  $\gamma^*$ ,

$$\gamma^* = \frac{a}{\gamma^*} + b \quad (12)$$

holds where

$$\begin{aligned} a &\equiv \frac{z [(1 - \theta_y) (1 - \alpha) A - \theta_o \alpha A]}{\delta_y} \\ b &\equiv \frac{[\delta_y (1 - \alpha) A - (1 - \delta_o) \alpha A - z (1 - \theta_y)]}{\delta_y}. \end{aligned} \quad (13)$$

Solving (12) yields  $\gamma^* = \frac{1}{2}b \pm \frac{1}{2}\sqrt{4a + b^2}$  where the root,  $\frac{1}{2}b - \frac{1}{2}\sqrt{4a + b^2}$ , is negative and hence economically invalid.

Properties P-1 and P-2 are not sufficient, however, to ensure  $h(\cdot)$  maps into  $[\underline{\gamma}, \bar{\gamma}]$ , whereby a sequence  $\{\gamma_t\}_{t=1}^{\infty}$  obtains for *any* initial  $\gamma_1 \in [\underline{\gamma}, \bar{\gamma}]$ . This requires

<sup>8</sup> Condition P-1 can be verified directly by inspection of the right-hand side of (11). Condition P-2 is verified in the Appendix.

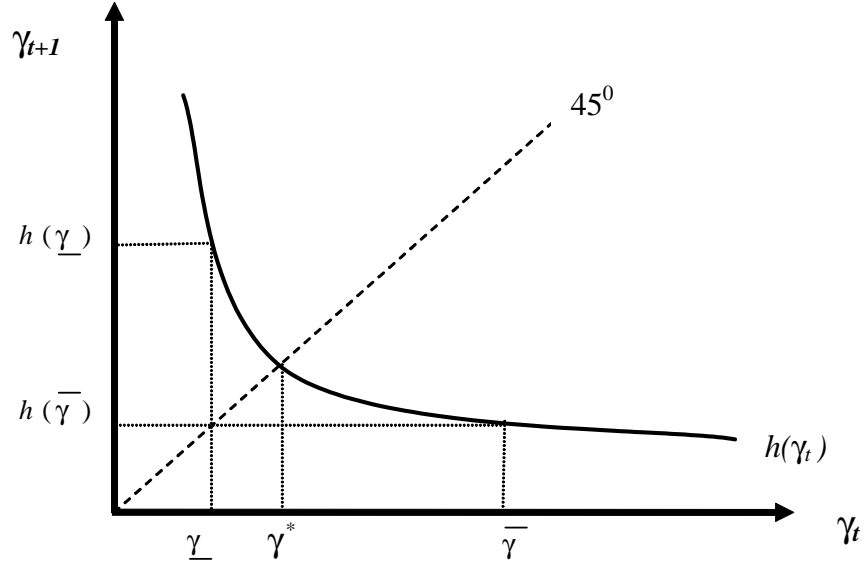


Fig. 1. The mapping  $h(\cdot)$

**Assumption 3**  $(\delta_y(1 - \alpha) - \alpha(1 - \delta_o))A \geq (1 - \theta_y)z$ .

Note that Assumption 3 implies  $\underline{\gamma} = (1 - \alpha)A - \alpha A(1 - \delta_o)/\delta_y$ . These assumptions also yield the properties

**P-3**  $h'(\gamma^*) \geq -1$

**P-4**  $\underline{\gamma} \leq h(\bar{\gamma})$  and  $h(\underline{\gamma}) \leq \bar{\gamma}$ ,

which ensures  $h(\cdot)$  maps into  $[\underline{\gamma}, \bar{\gamma}]$ .

The above discussion is summarized pictorially in Figure 1 below. The upshot is that under Assumptions 1-3, the law of motion for the equilibrium growth rate is downward sloping and that there is a unique stationary growth rate.



### 3.2 Comparative Statics

In this subsection, we address two related issues: i) how does ‘keeping up’ affect the steady-state growth rate, and ii) how steady-state growth in this model compares with the benchmark growth rate  $\gamma_{bm}$ , that is, growth in a model where  $\theta_t = \delta_t = 0$ .

**Proposition 1** *‘Keeping up’ when young lowers the steady-state growth rate; ‘keeping up’ when old raises the growth rate.*

A formal proof of Proposition 1, provided in the Appendix, involves describing how the curve in Figure 1 will shift with a change in one of the four keeping up parameters,  $\theta_i, \delta_i, i = y, o$ . The intuition behind this proposition, however, is simple. A greater value for either keeping up parameter ( $\theta_y$  or  $\theta_o$ ) requires the agent, when young, to devote more of her wage income to current consumption and less to saving. This in turn leads to a lower growth rate. On the other hand, the greater the keeping up parameters  $\delta_y, \delta_o$ , the more the agent saves in order to meet the greater consumption commitment when old, which increases the growth rate. Counter to what might be expected, rejuvenile behavior actually contributes to greater economic growth; after all, for the old to keep up with the consumption of the young, the old would have had to save adequately in the past and it is this forward-looking thriftiness that fosters growth.

Given the ambiguous way in which keeping up, in general, affects growth, it would seem that the steady-state growth rate in this model may be either greater or less than the benchmark growth  $\gamma_{bm}$ . However, with Assumption 3, we have:

**Proposition 2** *If  $\theta_y < \delta_o$ , the steady-state growth rate in this model is greater than in the benchmark case,  $(1 - \alpha)Az / (\alpha A + z)$ .*

The condition  $\theta_y < \delta_o$ , along with Assumption 3, implies  $\gamma_{bm} < h(\bar{\gamma})$ . Since  $h(\gamma_t) > h(\bar{\gamma})$  for all  $\gamma_t \in [\underline{\gamma}, \bar{\gamma}]$ , the proposition is true.

Having explored the properties of the steady state growth rate, we now move on to study the dynamics of the growth rate. We are particularly interested in the possibility that the growth rate may exhibit endogenous cyclical fluctuations. From P-1, we know that in the presence of juveniles ( $\delta_y > 0$ ), the law of motion  $h(\cdot)$  is negatively sloped everywhere suggesting the possibility of such fluctuations near the steady state. We address this potential next.

### 3.3 Cycles

Heuristically, we can describe the possibility of growth cycles as follows. Suppose the young at date  $t$  believe that when they are old, the young at  $t + 1$  will have fairly low levels of consumption (i.e., they save a lot and the growth rate  $\gamma_{t+1}$  is high). This will imply ‘keeping up’ with the young at  $t + 1$  will not require a large amount of savings on the part of the young at date  $t$  – hence they consume more when young and the growth rate  $\gamma_t$  is low. But what makes the young choose to save more at  $t + 1$ ? If the young at date  $t + 2$  face a similar prospective future as the young at  $t$ , they will choose a high level of consumption, thereby making ‘keeping up’ more difficult for the old at  $t + 2$  (the young at  $t + 1$ ), which is countered by greater saving by the young at date  $t + 1$ .

It is convenient, for our purposes here, to rewrite the law-of-motion (11). Let

$\gamma^*$  denote the steady-state of (11), and let  $\Gamma \equiv h'(\gamma^*) < 0$  (see P-2) . We can then write (see the Appendix)

$$\gamma_{t+1} = (1 + \Gamma) \gamma^* - \frac{(\gamma^*)^2 \Gamma}{\gamma_t}. \quad (14)$$

Equation (14) can be used to characterize *stable* 2-period flip cycles.

Let  $\gamma_o$  ( $\gamma_e$ ) denote the growth rate at odd (even) dates, respectively. In a 2-period cycle, we then have

$$\begin{aligned} \gamma_e \gamma_o &= (1 + \Gamma) \gamma^* \gamma_o - (\gamma^*)^2 \Gamma \\ \gamma_e \gamma_o &= (1 + \Gamma) \gamma^* \gamma_e - (\gamma^*)^2 \Gamma \end{aligned} \quad (15)$$

A trivial solution to (15), of course, is  $\gamma_e = \gamma_o = \gamma^*$ , the 1-cycle or the steady state. However, we seek solutions to (15) with  $\gamma_o \neq \gamma_e$ . From (15), it is clear no such solutions exist unless  $\Gamma = -1$ , in which case the system (15) is underdetermined and a continuum of solutions exist. For a given value for  $\gamma_o$ ,  $\gamma_e = (\gamma^*)^2 / \gamma_o$  holds, as can be seen from (15) evaluated at  $\Gamma = -1$ . We provide two examples below.

**Example 1** *Let  $\alpha = 1/3$ ,  $\beta = 1$ ,  $\sigma = 2/3$ ,  $A = 3$ ,  $\theta_y = 2/3$ ,  $\theta_o = 0$ ,  $\delta_y = 1/3$ ,  $\delta_o = 2/3$ . In this case, the steady-state growth rate  $\gamma^* = \sqrt{2}$ . An example of an equilibrium with 2-cycles is summarized by the pair  $(\gamma_o, \gamma_e) = (5/3, 1.20)$ .*

**Example 2 (Youth Driven Consumption)** *Retain the same parameter values as above except for  $\delta_y$  and  $\delta_o$ . Let  $\delta_y = 1/2$  and  $\delta_o = 1/3$ . The steady-state growth rate  $\gamma^* = 2/\sqrt{3}$ . An example of a 2-cycle equilibrium pair is  $(\gamma_o, \gamma_e) = (4/3, 1)$ .*

Examining (11) and (14) reveals that  $\Gamma = -1$  when  $b$ , in (13), equals zero. This

in turn implies stable 2-period flip cycles are possible whenever the parameters  $\delta_y$ ,  $\delta_o$ , and  $\theta_y$  satisfy

$$\delta_y (1 - \alpha) A - (1 - \delta_o) \alpha A = z (1 - \theta_y). \quad (16)$$

The parameter values of the two examples above were selected to conform to this condition.

By a similar construction, using (14), it is easy to show that cycles of higher periodicity are not possible. These results are summarized in the following proposition.

**Proposition 3** *Stable cycles exist only in the case where  $\Gamma = -1$  (see (16)) and in that instance, only cycles of periodicity two are possible.*

If  $\Gamma > -1$ , all equilibrium sequences with  $\gamma_1 \neq \gamma^*$ , converge, in an alternating, periodic fashion, to the steady-state  $\gamma^*$ .

#### 4 Alternative Forms of ‘Keeping Up’ Preferences

Part of our interest in studying these preferences was to identify whether endogenous fluctuations in growth rates could arise purely from preferences. Recall that a necessary condition for any kind of volatility in growth rates is that the law of motion for growth not be positively sloped everywhere. One thing we know for sure is that the benchmark model (with  $\theta = \delta = 0$ ) will not deliver growth cycles. We also now know that in the presence of juveniles, flip growth cycles are possible. Phrased differently, then, our issue becomes: can alternative reasonable specifications of keeping up preferences produce a (somewhere) negatively sloped law of motion?

We identify three popular alternatives found in the literature: i) keeping up with a consumption minimum defined as a function of current output (used most recently in Alvarez and Diaz, 2005), ii) keeping up with a standard of living established by one's parents (also known as generational keeping up, used in de la Croix and Michel, 2002), and iii) keeping up with one's past consumption (also known, simply as habit formation, used for example in Alessei and Lusardi, 1997, or Bunzel, 2006). We discuss briefly how each of these alternatives would work in our current framework.

#### 4.1 Consumption Minimum as a Function of Current Output

One popular form of 'keeping up', intended to mimic a 'keeping up with the Joneses' argument assumes that agents desire to keep up with a consumption minimum defined as a function of current output. Often this consumption minimum is interpreted as a time-evolving poverty line. In this formulation,  $\theta_t = \theta y_t$  and  $\delta_t = \delta y_{t+1}$ , with  $1 - \alpha > \theta$  and  $\alpha > \delta$ . Constraints (2) and (3) are replaced with

$$\begin{aligned} c_t &\geq \theta y_t \\ x_t &\geq \delta y_{t+1} \end{aligned}$$

The marginal conditions for the agent's problem, with the changes indicated above, can be summarized by (6). The counterpart to (10) is

$$\frac{\alpha A k_{t+1} - \delta A k_{t+1}}{((1 - \alpha) A k_t - k_{t+1}) - \theta A k_t} = z,$$

which reduces to

$$\frac{(\alpha - \delta) A \gamma_t}{(1 - \alpha) A - \gamma_t - \theta A} = z. \tag{17}$$

Solving for the growth rate, one gets

$$\gamma_t = \frac{Az(1 - \alpha - \theta)}{(\alpha - \delta)A + z},$$

for all  $t \geq 1$  and hence it is clear that no fluctuations in growth rates are possible here. Here too the stationary growth rate may be greater or less than baseline  $\gamma_{bm}$ ; the difference,  $\gamma_t - \gamma_{bm}$  is

$$-\frac{zA[(\alpha A + z)\theta - \delta(1 - \alpha)A]}{(\alpha A + z)((\alpha - \delta)A + z)},$$

which is greater than or less than 0 depending on whether  $\delta(1 - \alpha)A \gtrless (\alpha A + z)\theta$ .

#### 4.2 *Generational Keeping Up*

The overlapping generations framework allows us to consider the possibility that parents may, in part, shape the consumption decisions of their offspring. de la Croix and Michel (2002) consider such ‘keeping up’ effects in a neoclassical growth model. These sorts of preferences can generate cycles in *levels*. We briefly describe here how they would work in a model with *Ak* technology and *growth rates*.<sup>9</sup>

We assume the agent’s utility depends on how much her own consumption differs from the consumption of her parents (denoted  $c_{t-1}$  and  $x_{t-1}$  above and taken as given by the agent) at *parallel* points in their life. The parameters

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<sup>9</sup> Our discussion here is somewhat broader as we allow the parent’s consumption when young and old to affect the child’s utility both when young *and* old, while de la Croix and Michel (2002) assume the parent’s consumption directly only affects the child’s utility when young.

$\theta$  and  $\delta$  determine how much weight the agent places on ‘keeping up’ in each stage in life. We replace (2) and (3) with

$$\begin{aligned} c_t &\geq \theta c_{t-1} \\ x_t &\geq \delta x_{t-1} \end{aligned}$$

Like our framework with juveniles, generational keeping up introduces a time dynamic in the equation for the equilibrium growth rate. The first order conditions for the agents’ problem for dates  $t \geq 1$  can be summarized by (6), with  $\theta_t = \theta c_{t-1}$  and  $\delta_t = \delta x_{t-1}$ . The counterpart to (10) for dates  $t \geq 2$  is

$$\frac{\alpha A k_{t+1} - \delta \alpha A k_t}{(1 - \alpha) A k_t - k_{t+1} - \theta [(1 - \alpha) A k_{t-1} - k_t]} = z,$$

which readily reduces to

$$\frac{\alpha A \gamma_t \gamma_{t-1} - \delta \alpha A \gamma_{t-1}}{(1 - \alpha) A \gamma_{t-1} - \gamma_t \gamma_{t-1} - \theta [(1 - \alpha) A - \gamma_{t-1}]} = z. \quad (18)$$

This yields the law-of-motion for the growth rate  $\gamma_t$  for dates  $t \geq 2$  as follows:

$$\gamma_t = \gamma_{bm} + \frac{\delta \alpha A + \theta z}{\alpha A + z} - \frac{\theta (1 - \alpha) A z}{(\alpha A + z) \gamma_{t-1}} \equiv H(\gamma_{t-1}) \quad (19)$$

A quick examination of equation (19) reveals that  $H'(\gamma_{t-1}) \geq 0$  implying that generational keep up preferences cannot generate endogenous fluctuations in the growth rate.

Unlike in the juvenile formulation above, however, the initial growth rate  $\gamma_1$  is not indeterminate. For date  $t = 1$ , the marginal conditions can be summarized by

$$\frac{\alpha A k_2 - \delta \alpha A k_1}{(1 - \alpha) A k_1 - k_2 - \theta c_0} = z,$$

where  $k_1$  and  $c_0$  are given. Dividing the numerator and denominator by  $k_1$

yields

$$\gamma_1 = \frac{\delta\alpha A + (1 - \alpha)Az - z\theta c_0/k_1}{\alpha A + z}$$

or

$$\gamma_1 = \gamma_{bm} + \frac{\delta\alpha A - z\theta c_0/k_1}{\alpha A + z}$$

Although  $c_0$  and  $k_1$  are given at date 1, they presumably are selected together and satisfy a budget constraint similar to (4) and (5). Let  $c_0 = (1 - \alpha)Ak_0 - k_1$ , where  $k_0$  is given. Substituting in expression above, and dividing the numerator and denominator by  $k_1$ , we have

$$\gamma_1 = \gamma_{bm} + \frac{\delta\alpha A + z\theta}{\alpha A + z} - \frac{\theta(1 - \alpha)Az}{(\alpha A + z)\gamma_0}.$$

which is of the same form as (19).

### 4.3 Habit Formation

In this formulation, we set  $\theta_t = 0$  in (2) and replace  $\delta_t$  in (3) with

$$x_t \geq \delta c_t.$$

Unlike the form of ‘keeping up’ presented in the previous sections and the other two alternatives listed above, the agent, when young, will *internalize* this minimum requirement when selecting  $c_t$ . The counterpart to (6) in this case is:

$$\frac{(x_t - \delta c_t)}{c_t} = \tilde{z}, \tag{20}$$

where  $\tilde{z} \equiv (\beta(\alpha A + \delta))^{1/\sigma}$ . Incorporating the budget constraints (4) and (5)



into this expression, we derive the counterpoint to (10) as

$$\frac{(\alpha A k_{t+1} - \delta((1 - \alpha) A k_t - k_{t+1}))}{(1 - \alpha) A k_t - k_{t+1}} = \tilde{z},$$

which reduces to

$$\frac{(\alpha A \gamma_t - \delta((1 - \alpha) A - \gamma_t))}{(1 - \alpha) A - \gamma_t} = \tilde{z}. \quad (21)$$

The growth rate in this economy is constant for all  $t \geq 1$  (hence no possibility of endogenous fluctuations) and satisfies:

$$\gamma_t = \frac{\tilde{z}(1 - \alpha) A}{\alpha A + \tilde{z}}.$$

Comparing this solution with the baseline growth rate  $\gamma_{bm}$  is easy since it is in a form similar to the baseline with  $\tilde{z}$  replacing  $z$ . Since the baseline is increasing in  $z$ , and since  $\tilde{z} > z$ , it follows that the growth rate here is greater than baseline. This of course makes perfect sense and is consistent with the general finding in our ‘keeping up’ formulation: the habit formation parameter motivates the agent to find ways to increase her consumption when old - the way to do this is increase saving when young, which leads to a higher growth rate.

## 5 Conclusion

Real growth cycles (cycles in the growth rate of real per capita income) are observed in almost every country around the world. Economists have sought to generate these cycles within the neoclassical paradigm. Toward that end, they have had to rely on changing the formulation of technology away from the usual neoclassical textbook specifications. In this note, we ask the question: Can a simple change in preferences deliver growth cycles? The only preference

structure that has the potential to generate such cycles is one where agents face minimum consumption requirements imposed by the consumption patterns of generations other than one's own. We define juveniles as old agents who derive utility from "keeping up" their consumption with some measure of the consumption of the current young. We show that juveniles raise the long run growth rate but their presence may also expose the economy to endogenous growth fluctuations.

# Appendix

**1. Proof of P-2** Suppose  $\underline{\gamma} = (1 - \alpha) A - \alpha A (1 - \delta_o) / \delta_y$ . The difference  $h(\underline{\gamma}) - \underline{\gamma}$  can be written as:

$$h(\underline{\gamma}) - \underline{\gamma} = \frac{\alpha z [(1 - \theta_y) - (1 - \theta_y) \delta_o - \theta_o \delta_y]}{\delta_y [(1 - \alpha) \delta_y + \alpha \delta_o - \alpha]}.$$

The numerator of this difference is positive, by Assumption 2. The assumption that  $(1 - \alpha) A - \alpha A (1 - \delta_o) / \delta_y > 0$  ensures that the denominator is also positive; hence,  $h(\underline{\gamma}) > \underline{\gamma}$ .

If  $(1 - \alpha) A - \alpha A (1 - \delta_o) / \delta_y < 0$ ,  $\underline{\gamma} = 0$ . Since  $\lim_{\gamma \rightarrow 0} h(\gamma) = \infty$ , the result  $h(\underline{\gamma}) > \underline{\gamma}$  holds.

At  $\gamma = \bar{\gamma}$ ,

$$\bar{\gamma} - h(\bar{\gamma}) = \frac{\alpha A [(1 - \theta_y) (1 - \delta_o) - \delta_y \theta_o]}{\delta_y (1 - \theta_y)},$$

which is positive, by Assumption 2.

**2. Proof of P-3** We have:

$$h'(\gamma_t) = \frac{-[A(1 - \theta_y)(1 - \alpha) - \alpha A \theta_o] z}{\delta_y \gamma_t^2},$$

which can be written as

$$h'(\gamma_t) = \frac{-a}{\gamma_t^2}$$

where  $a$  is defined in (13). When Assumption 3 holds with equality,  $b$ , in (13), equals zero and  $\gamma^* = \sqrt{a}$ , so  $h'(\gamma^*) = -1$ . When Assumption 3 holds with strict inequality,  $b > 0$  and  $\gamma^* > \sqrt{a}$ , hence  $h'(\gamma^*) > -1$ .

**3. Proof of P-4** First,  $\underline{\gamma} = [\delta_y(1-\alpha) - \alpha(1-\delta_o)]A/\delta_y > 0$ , by Assumption 3.

The difference  $h(\bar{\gamma}) - \underline{\gamma} = 0$ . The difference  $\bar{\gamma} - h(\underline{\gamma})$  equals

$$\bar{\gamma} - h(\underline{\gamma}) = \frac{\alpha[(1-\theta_y)(1-\delta_o) - \delta_y\theta_o][A(\delta_y(1-\alpha) - \alpha(1-\delta_o)) - (1-\theta_y)z]}{\delta_y(1-\theta_y)[\delta_y(1-\alpha) - \alpha(1-\delta_o)]}.$$

Assumption 3 ensures  $\delta_y(1-\alpha) > \alpha(1-\delta_o)$ . Assumption 2 and Assumption 3 ensure the numerator above is positive.

**4. Proof Proposition 1** We analyze below the impact on  $h(\gamma_t)$  (11) of a marginal change in each of the keeping up parameters. These indicate how the curve in Figure 1 will shift for a change in each of these variables; the result of the proposition then follows.

- a.  $\frac{\partial \gamma_{t+1}}{\partial \theta_y} = \frac{-((1-\alpha)A - \gamma_t)z}{\delta_y \gamma_t} < 0$ .
- b.  $\frac{\partial \gamma_{t+1}}{\partial \theta_o} = \frac{-\alpha Az}{\delta_y \gamma_t} < 0$ .
- c.  $\frac{\partial \gamma_{t+1}}{\partial \delta_y} = \frac{\alpha A}{\delta_y} > 0$ .
- d.  $\frac{z(1-\theta_y)\gamma_t + A[-(1-\alpha)(1-\theta_y)z + \alpha(1-\delta_o)\gamma_t + \alpha\theta_o z]}{\delta_y \gamma_t}$ .

In last case, the numerator is increasing in  $\gamma_t$ . Evaluated at  $\gamma_t = \underline{\gamma}$ , the numerator equals

$$\alpha A(1-\delta_o)[A(\delta_y(1-\alpha) - \alpha(1-\delta_o)) - (1-\theta_y)z] + \delta_y\theta_o\alpha Az,$$

which is positive by Assumption 3.

### 5. Proof Proposition 2 Form the difference

$$h(\bar{\gamma}) - \gamma_{bm} h(\bar{\gamma}) - \gamma_{bm} = \frac{\alpha A [A(\delta_y(1-\alpha) - \alpha(1-\delta_o)) - (1-\delta_o)z]}{\delta_y(\alpha A + z)}.$$

From Assumption 3,  $A(\delta_y(1-\alpha) - \alpha(1-\delta_o)) > (1-\theta_y)z$ , which is greater than  $(1-\delta_o)z$  if  $\theta_y < \delta_o$ .

**6. Derivation of (14)** By definition,  $\Gamma \equiv \frac{-z[(1-\theta_y)(1-\alpha)A - \theta_o\alpha A]}{\delta_y(\gamma^*)^2}$ . We can then write

(11) as

$$\gamma_{t+1} = \frac{-\Gamma(\gamma^*)^2}{\gamma_t} + \frac{[\delta_y(1-\alpha)A - (1-\delta_o)\alpha A - z(1-\theta_y)]}{\delta_y}.$$

Evaluating this expression at the steady-state ( $\gamma_t = \gamma_{t+1} = \gamma^*$ ), we have:

$$\gamma^* = -\Gamma\gamma^* + \frac{[\delta_y(1-\alpha)A - (1-\delta_o)\alpha A - z(1-\theta_y)]}{\delta_y}, \text{ or}$$

$$\frac{[\delta_y(1-\alpha)A - (1-\delta_o)\alpha A - z(1-\theta_y)]}{\delta_y} = (1 + \Gamma)\gamma^*.$$

We can then write (11) as

$$\gamma_{t+1} = (1 + \Gamma)\gamma^* - \frac{\Gamma(\gamma^*)^2}{\gamma_t}.$$

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