Structural wave propagation and sound radiation study through time and spatial processing

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Structural wave propagation and sound radiation study through time and spatial processing

Wei, Ruey-Chang, Ph.D.
Iowa State University, 1994
Structural wave propagation and sound radiation study through time and spatial processing

by

Ruey-Chang Wei

A Dissertation Submitted to the Graduate Faculty in Partial Fulfillment of the Requirements for the Degree of DOCTOR OF PHILOSOPHY

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For the Graduate College

Iowa State University
Ames, Iowa
1994

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DEDICATION

To my parents
and my beloved wife Fen-Hui.
TABLE OF CONTENTS

DEDICATION .......................................................... ii

CHAPTER 1. INTRODUCTION AND OBJECTIVES ........ 1
  Introduction ..................................................... 1
  Research Objectives .......................................... 6

CHAPTER 2. ANALYSIS TOOLS ............................... 9
  Time Domain Analysis ........................................ 9
    Signal processing and the synthetic force ............... 11
  K-space Analysis .............................................. 12
    K-space filtering technique ............................. 19
  Wave Speed Tracking ........................................ 21
    Phase speed and arrival time ............................ 22
    Reflection coefficient ................................... 23
    Simulation .................................................. 24

CHAPTER 3. SYSTEM STUDIED ................................. 36
  Introduction .................................................. 36
  Stress And Wave Propagation ............................... 38
  Energy Distribution ......................................... 56
CHAPTER 1. INTRODUCTION AND OBJECTIVES

Introduction

Vibrating structures and their radiated sound field have been of considerable interest in the past. Yet, how attachments to a structure or mechanical processing of the structure's material will change the sound radiation and wave propagation in the structures remains inadequately understood. Inhomogeneities generated by attachments such as ribs and stiffeners, or mechanical processing such as welding and bolting are very common in complex structures. These inhomogeneities seem insignificant compared to the size of the whole structure, but it is believed that these localized variations can be a major contributor to the farfield sound radiation. For example, in a submarine, Figure 1.1, the welds and ribs in the structure can cause significant sound radiation, therefore, this kind of study is very crucial for a structure that has sound radiation as a major concern.

It will be shown in this dissertation that inhomogeneities can affect the wave propagation in the structure, and therefore can significantly affect the structure's sound radiation. Because most of the attachments and mechanical processing to the structure can not be avoided, a better understanding of their role in wave propagation and sound radiation is very important. Consequently, experimental methods and analysis techniques to measure the sound radiation and wave propagation variations
Figure 1.1: Sound radiation caused by ribs and welds in a submarine

caused by a small discontinuity have been pursued in this research.

Many existing measurement and analysis techniques accurately record or present the properties that they were designed to characterize, but sometimes the information provided to the researcher does not reveal the total vibration and radiation picture. For example, the current techniques can not provide detailed information of sound radiation from different areas of a structure. For obvious academic and practical reasons, it would be advantageous to develop better experimental and analytical techniques, and to expand the understanding of the relationship between the inhomogeneities in the structure and the induced wave propagation and sound radiation changes.
One important aspect in sound radiation problems is locating sources of sound on a radiating structure, which not only concerns the sound power radiated by the structure but more importantly the location of the areas on the structure that radiate sound to the acoustic farfield. In the early stage of source locating study, far field pressure measurements were used for source identification [1] [2]. However, the results provided by this method is very difficult to interpret, because sound pressure is a scalar quantity and consequently it contains an additive value of source interactions. This shortcoming was overcome by an improved approach called the sound intensity method [3] [4]. The introduction of acoustic intensity measurements greatly enhanced the ability to analyze sound sources. However, the interpretation of the measured data and the distance away from the source that measurements were made were still important questions. Early work consisted of measuring the acoustic intensity at a few points relatively far from the source, then following the direction of the intensity vector arrow back to the source to imply the sound source location [4]. As the measurement process became more automated, the sound field was measured at more locations and closer to the source (this was called the conventional intensity measurement technique) [5] [6]. Source localization subsequently became more accurate. With this came the realization that both the active intensity and reactive intensity are needed for the source localization. Later, the development of Nearfield acoustical holography (NAH) provided a method for determining the acoustic field at the surface of a source [7]. Recently Loyau, Pascal, and Gaillard developed a method called broadband acoustic holography from intensity measurements (BAHIM) [8], which merges intensity and holography measurement techniques and provides an alternative method to performing NAH. With BAHIM, locations of
sources can be accurately determined, because its results are obtained at the source surface rather than further away from the source surface where conventional intensity measurements data are taken. This technique is especially suited for noise control. One disadvantage of the intensity/power analysis is that some areas of a structure will radiate sound and others will absorb sound. This is a very important phenomena to understand and quantify, but it does not directly indicate which areas of the structure radiate sound to the acoustic farfield.

In the study of a vibrating structure with inhomogeneities, one is not only interested in the location of the source, which could be the excitation point or the points where inhomogeneities occur, but also the causes of sound radiation from the inhomogeneities. The above techniques, NAH and BAHIM, cannot provide this understanding, because they cannot separate contributions from different points on the structure to the farfield. Therefore, this research proposes a k-space analysis for better insight into power distribution in sound radiation. The basic idea of k-space, or wavenumber space, analysis, is to transfer the real space data to the wavenumber domain, and process the data with filtering and other techniques, then transfer the data back to the real space for better interpretation. By doing so, not only can the location of the sources be identified and the sound power from different types of waves in the structure can be distinguished, but also different waves can be separated and examined individually.

The second goal of this research is to understand wave propagation in structures with inhomogeneities. Most acoustic work done in the past ignored the existence of evanescent waves generated by the inhomogeneities. The relationship between propagating waves and evanescent waves, and their roles in sound radiation were rarely
addressed. One of the reasons for this ignorance was the lack of proper analysis tools. Recently Fricke and Baggeroer presented a modal-slowness analysis technique for the study of plate vibration [9]. They used Radon transformation to transform the space-time data to slowness-time data, where slowness is defined as the inverse of the phase velocity of propagating wave. By this mapping technique, the flexural and longitudinal modes can be isolated from one another and may be analyzed individually. The drawback of this technique is that in a vibrating structure there are often many reflections and modes caused by boundaries or discontinuities. The complexity of wave propagation in a practical measurement greatly limits the resolution in slowness analysis, and therefore makes the separation of modes a very difficult task.

In this research, wave propagation is mainly studied by using the wave speed tracking technique, which determines the amplitudes of a wave propagating with a specified phase speed and frequency. This technique has been widely used in the study of seismology [10] [11], whose emphasis is to determine the group speed of waves propagating in the earth crust. The analysis used in such a study is equally valuable for applications of studying wave propagation in vibrating structures. By this technique, waves traveling at different phase speeds can be distinguished, and the speed change caused by any inhomogeneities in the structure can also be observed. Therefore one can study the relationship between a traveling wave and it's sound radiation contribution by coupling the wave speed tracking technique and k-space analysis. In addition, an integration method was developed, so that after the phase speed is determined, the reflection coefficients can be obtained, therefore the energy transfer in the structure can be studied.

Overall, by including all the mentioned techniques, this research focuses on the
study of the time history of the vibration and sound radiation. The backbone of this research is time domain processing rather than conventional frequency domain analysis. The idea of time domain processing is to measure the impulse response of a system, and the response due to any other force can then be calculated from the response function. Since the response function only needs to be measured once, this approach is very efficient and effective especially in the study of sound radiation.

**Research Objectives**

The first objective of this research is to develop an integrated acoustical and vibrational measurement and imaging system for source radiation analysis and vibration analysis. Under this main theme, the scope of this research includes:

1. development of reliable measurement techniques for the sound and vibration measurements,

2. development of effective and accurate analysis tools for interpretation of the results.

Hardware and a number of computer programs were developed for this purpose. Specifically, software associated with (1) data acquisition and (2) data processing was developed. The data acquisition includes monitoring and controlling the automatic data acquisition system. The data processing based on the improved techniques was also implemented. In addition, various supporting computer programs dealing with image enhancement (e.g., windowing and filtering) and graphics were also developed.

The second objective is to verify the validity of the proposed techniques by full-scale measurements. The experimental investigation has focused on a simple steel
Figure 1.2: Sound radiation caused by a rib welded to a simple steel beam

beam with a welded rib, Figure 1.2. The goal under this objective is to investigate the influence of a welded rib on the vibration and sound radiation of a steel beam, which involves:

1. development of theoretical analysis for wave motion in a structure with residual stress caused by the welding process,

2. full-scale measurements to verify the prediction from the proposed theory,

3. development of a means to overcome the influence of residual stress on wave propagation and sound radiation.

Under this objective, a plate equation with a static stress distribution was investigated to understand wave propagation in a simple beam with discontinuities. A series of simulations based on the modified plate equation was conducted to predict the response of the vibration and study the sound radiation resulting from different kinds of waves. Then a measurement procedure was designed and tested for its repeatability, and a set of measurements was performed in an anechoic chamber using the facilities implemented in the first stage of this research. Finally the experimental
results are analyzed and compared with theoretical calculations, and measures are taken to reduce the sound radiation caused by the discontinuities.

As a result of this research, a number of new concepts and new techniques significantly advancing acoustical and vibrational imaging were developed and experimentally verified. The significance of the results and suggestions for future perspectives of the research based on these merged techniques are discussed in the final chapter.
CHAPTER 2. ANALYSIS TOOLS

In this research, the data from the experiments will be analyzed with an array of tools. The tools are: (1) time domain analysis, (2) k-space analysis, and (3) wave speed tracking. These tools will provide important insights into the wave propagation and sound radiation being studied. The important features of the tools will be described in the following sections.

Time Domain Analysis

In this study, the oscillatory systems are assumed linear and time-shift invariant. Linearity means that the cause and effect are related linearly; e.g., if the load is doubled, the response will be doubled [12]. Consider a vibrating structure driven by the force \( f(t, x_0) \). The field quantity \( y(t, x, x_0) \) (acoustic pressure or particle velocity at position \( x \) in the fluid due to the force at \( x_0 \) on the structure) can be related to the drive force by the transfer function \( h(\tau, x, x_0) \), which is also referred to as the impulse response function. Note that \( h(\tau, x, x_0) \) only relates the drive force at \( x_0 \) to a particular field point \( x \); thus \( h(\tau, x, x_0) \) is needed for every spatial location of interest [13].

The linear system model allows the field quantity, \( y(t, x, x_0) \), to be expressed as a convolution integral in time of the drive force, \( f(t, x_0) \), and the impulse response
function, \( h(\tau, x, x_0) \):

\[
y(t, x, x_0) = \int_{-\infty}^{\infty} h(\tau, x, x_0) f(t - \tau, x_0) d\tau.
\] (2.1)

This can be calculated more conveniently in the frequency domain as a multiplication.

\[
y(\omega, x, x_0) = h(\omega, x, x_0) f(\omega, x_0).
\] (2.2)

The quantity \( h(\omega, x, x_0) \) is called the impulse response spectrum. Consider a measurement where the field quantity, pressure or velocity, \( y_0(t, x, x_0) \) is measured as a result of a driving force \( f_0(t, x_0) \), which can also be measured. Using the frequency spectra of these two quantities, the impulse response spectrum can be determined,

\[
h(\omega, x, x_0) = \frac{y(\omega, x, x_0)}{f(\omega, x_0)}.
\] (2.3)

This impulse response spectrum completely characterizes the response for any linear and time shift invariant force at position \( x_0 \) on the structure.

Once the impulse response spectrum is determined from measurements, the field quantity for any synthetic force can be calculated by using the frequency spectrum of the synthetic force and the measured impulse response spectrum in Equation (2.3),

\[
y_{\text{syn}}(\omega, x, x_0) = h(\omega, x, x_0) f_{\text{syn}}(\omega, x_0),
\] (2.4)

where \( y_{\text{syn}}(\omega, x, x_0) \) is the expected response if \( f_{\text{syn}}(\omega, x_0) \) is applied to the structure at point \( x_0 \). An inverse Fourier transform of Equation (2.4) gives the field quantity at a particular point as a function of time. By this procedure, the response of the structure's vibration and acoustic radiation for various driving forces can be studied.
It should be noted that the signal processing could be implemented in the time domain by convolution integrals, but all the processing in this research is performed in the frequency domain. The reason is that the convolution integral in the time domain becomes a multiplication in the frequency domain, which is much easier to implement. In addition, the FFT algorithm used for the Fourier transform is more efficiently performed on a computer than performing the convolution integral directly.

It can be concluded that analyzing wave propagation and sound radiation in the time domain provides a powerful means to identify and understand phenomena. For example, the speed at which waves propagate in a structure can be determined [14], and the sound radiation resulting from a particular incident wave on a discontinuity can be seen and quantified [13]. Also, the influence of different forcing functions on the response of a structure can be studied [13].

**Signal processing and the synthetic force**

Due to the nature of practical data acquisition, some measures need to be taken in order to prevent errors resulting from improper data processing. The measures that are taken are explained in detail by Mann et al. [13] and Kruger [15], so they will only be summarized here.

First, since the measurement time must be finite, this aperture restriction leads to an error in the convolution process referred to as wraparound [7]. Fortunately, this wraparound error caused by convolution over a finite segment of data can be removed by a technique called zero-padding. Zero-padding extends the aperture of the original signal by adding zeros to the time history. In this study, zero-padding
was applied to the time signal to double its original length [13]. The impulse response and synthetic force spectra were first inverse Fourier transformed to time where zeros were added. The zero-padded signals were then Fourier transformed to the frequency domain. These spectra were then used in the processing outlined in the previous section.

Another problem in signal processing is that the bandlimited impulse response spectra have very sharp discontinuities, which create large errors in the inverse Fourier transform [16]. A Kaiser-Bessel bandpass filter was used to smooth out the spectrum at its ends.

In addition to being a filter, the Kaiser-Bessel filter is also used as a synthetic force in this research. This force is sometimes called a pseudo-impulse force, because a real impulse should have an uniform infinite frequency spectrum, but in this case, the Kaiser-Bessel filter is an impulse-like time signal with finite frequency content. The impulse-like synthetic force is preferred because one can observe the fast propagating incident and reflected waves in the structure without the interference of the on going driving force. The time history and frequency spectrum of a typical Kaiser-Bessel force is shown in Figure 2.1; the central frequency is 1000 Hz.

**K-space Analysis**

One goal of this research is to investigate sound radiation to the farfield by structural vibrations. This is important because not all the vibration in a structure will cause farfield sound radiation. To understand the sound radiation from a vibrating structure, consider an infinite, uniform plate. Assume this plate is in contact with a fluid that exists in the semi-infinite space $z > 0$, and a plane transverse wave of
Figure 2.1: Time history and frequency spectrum of a Kaiser-Bessel synthetic force
arbitrary frequency $\omega$ and wavenumber $\kappa$ is forced to travel in the plate (Figure 2.2.) The transverse plate displacement is

$$\eta(x, t) = e^{j(\omega t - \kappa x)}.$$  

(2.5)

In order to study the sound generated by this vibration, one can solve the wave equation and use the fluid momentum equation as a boundary condition. The acoustic pressure field can thus be found to be

$$p(x, y, z) = Ae^{-j\kappa x e - jk_z z},$$  

(2.6)
where $A$ is the amplitude of the sound pressure and

$$k_z = \begin{cases} \sqrt{k^2 - \kappa^2} & k^2 > \kappa^2 \\ -j\sqrt{k^2 - \kappa^2} & k^2 < \kappa^2, \end{cases}$$

where $k = \omega/c$ and $c$ is the sound speed in the fluid. From the resulting expression for the radiated sound pressure, it is found that the induced sound travels in the plane of the plate, x-direction, with the same wave number $\kappa$ as the transverse wave in the plate. This comes about in order to satisfy the boundary condition at the plate-fluid interface. On the other hand, in the z-direction, which is normal to the plate, two kinds of wave propagation are possible. First, when $k^2 > \kappa^2$, the wavenumber $k_z$ is a real number, hence the sound is propagating away from the plate with wavenumber $k_z$. Second, if $k_z$ is an imaginary number, $k^2 < \kappa^2$, the pressure field becomes

$$p(x,y,z) = A e^{-j\kappa z} e^{-\sqrt{\kappa^2 - k^2} z},$$

which has an exponentially decaying amplitude in the z-direction with increasing $z$. In the second case, one finds that only the surface waves exist, and the sound pressure will never propagate to the farfield. These are evanescent sound waves.

One can also reach the same results, by examining the expression for the radiated power per unit length of this plate [17], which is

$$P = \frac{1}{2} Re \left\{ \int_{-\infty}^{\infty} p(x) v_n^*(x) dx \right\}$$

$$= \frac{1}{4\pi} Re \left\{ \int_{-\infty}^{\infty} \frac{\pm \omega \rho_0 |V(k_x)|^2}{k^2 - k_x^2 - k^2 \kappa^2} dk_x \right\},$$

$$= \frac{1}{4\pi} Re \left\{ \frac{\pm \omega \rho_0 |V(\kappa)|^2}{(k^2 - \kappa^2)^{1/2}} \right\},$$

where $\rho_0$ is fluid density, $v_n^*(x)$ is complex conjugate of normal velocity of the plate, and $V(\kappa)$ is the wavenumber transform of $v_n(x)$. Without getting into the details.
of the mathematics, one can recognize that when $k^2 < \kappa^2$, the denominator of Equation(2.8)) will be pure imaginary. Therefore, the result of the integration will be an imaginary number, and the resulting power, which is the real part of the integral, will be zero. Thus only when the wavenumber of the vibration, $\kappa$, satisfies the condition $k^2 > \kappa^2$ does the vibration contribute to the sound power radiated by the plate. The sound power quantifies the sound that will reach the acoustic farfield.

From the above observations, one can extend the same analysis to a two-dimensional plate problem. The wavenumber of the vibration then becomes $\kappa = \sqrt{\kappa_x^2 + \kappa_y^2}$, where $\kappa_x$ and $\kappa_y$ are wavenumber components in the $x$ and $y$ direction respectively. In addition the radiation circle for a wave(or vibration) in a structure at certain frequencies can be defined. The radius of this circle is equal $k_0 = \omega/c_0$, where $c_0$ is the sound speed in the fluid. The relationship between $\kappa$ and the radiation circle is shown in Figure 2.3. And only the vibration with a wavenumber $\kappa$, which in inside of this circle, i.e. $\kappa < k_0$, can generate sound that radiates to the farfield.

Now consider the same plate used in Figure 2.2 and the ensuing discussion, but make it a finite plate in a baffle. The wavenumber spectrum, or k-space, of the vibration, $V(\kappa)$, is shown in Figure 2.4. Now there are many wavenumbers that exist (Fahy [17] has an excellent discussion of this case.) Only the components with wavenumbers less than $k_0$ will produce sound that radiates to the acoustic farfield. From this point of view, k-space becomes a powerful domain to study sound radiation from a vibrating structure. By observing the wavenumbers of the vibration, one can make a quick determination of the radiated sound power. Thus observing the change inside and outside of the radiation circle as conditions of the structure change, the change in sound radiation can be monitored.
Figure 2.3: Two-dimensional wave vector and radiation circle.
Figure 2.4: Identification of radiating wavenumber components
K-space filtering technique

The focus of this research is the sound radiated to the farfield, therefore only the wavenumbers of a structure's vibration inside of the radiation circle are of interest. In addition, the inhomogeneities in the structure are suspected because they can generate evanescent waves when incident by propagating waves. Because the wavenumber of an evanescent wave is zero, which is inside of the radiation circle, it’s contribution to sound radiation can be significant. In order to remove the nonradiating wavenumbers, and also observe the existence of the evanescent waves generated by the inhomogeneities, a k-space filter is considered. The k-space filter used in this study is an exponential filter, which mathematical expression is

$$k_{\text{filter}} = \left\{ \begin{array}{ll} 1 - \frac{\exp(-\beta)}{2} & kr < kc \\ \frac{\exp(\beta)}{2} & kr \geq kc, \end{array} \right.$$  (2.9)

where $\beta = \frac{1 - |kr|}{kc}$, $kr$ is the wavenumber of the propagating wave, $kc$ controls the width of the filter, and $s$ decides the slope of the filter. Example profiles of this filter with the same $kc$ and varied $s$ can be seen in Figure 2.5. The choice of parameters for the filter should be carefully chosen to avoid any distortion of the data, especially the amplitude, and it highly depends on the k-space spectrum of the structure being studied.

After the data is k-space filtered, the data is then inverse transformed back to the real space domain. From the filtered data, the areas on the structure that radiate sound to the farfield can be shown.
Figure 2.5: Profiles of k-space filter with $kc = 1$ at 2000 Hz
The speed of waves propagating in a structure can be measured as a means to separate wave types when they are superimposed. Specifically, since bending waves and longitudinal waves travel at different speeds, they can be distinguished. In this research, a filtering technique, developed by Mann and Williams [14], is used to calculate the amplitude of waves propagating through a region of a structure with a specified phase speed and frequency. It should be noted that this filtering technique for wave speed tracking should not be confused with the k-space filtering described in the previous section. The brief theory of the phase speed tracking technique is included here.

Consider a wave of frequency $\omega_1$ propagating at speed $c_1$. This wave can be assumed to have the form

$$u(t, x) = \cos[\omega_1(t - (x/c_1)) + \phi_1], \quad (2.10)$$

where $\phi_1$ is the phase of the wave. The filter used for speed tracking is

$$W(t, x, \omega_2, c_2) = \begin{cases} 
\cos[\omega_2(t - (x/c_2)) + \phi_2] & t_1 < t < t_2, \quad x_1 < x < x_2 \\
0 & \text{else},
\end{cases} \quad (2.11)$$

where $\omega_2$ is the frequency, $c_2$ is the phase speed and $\phi_2$ is the phase of the wave that filter is searching for. The filtering procedure involves multiplying the propagating wave (Equation(2.10)) by the filter (Equation(2.11)) and integrating over space and time.

$$A(\omega_2, c_2, t', x') = \int_{t_1}^{t_2} \int_{x_1}^{x_2} u(t, x)W(t, x, \omega_2, c_2)dx\,dt, \quad (2.12)$$
where $t' = (t_1 + t_2)/2$ and $x' = (x_1 + x_2)/2$. Because of the orthogonality of sine functions, this integral will give the largest value when $\omega_1 = \omega_2$, $c_1 = c_2$, and $\phi_1 = \phi_2$. Therefore, if there are several waves propagating at different phase speeds and frequencies, this filter can be used to distinguish them.

**Phase speed and arrival time**

In Equation (2.12), the wave speed tracking filter is centered at time $t'$ and location $x'$. Thus the result of the filtering can be viewed as the wave amplitude passing a region centered at the time $t'$ and point $x'$. If the dimensions ($X = x_2 - x_1$ and $T = t_2 - t_1$) of the filter are small, the results can be interpreted as the amplitude of a wave propagating through a point. This is important if one is going to use this filtering result to track the waves as they propagate and interact with boundaries or other discontinuities.

For this research, the filter is centered at one spatial point $x'$ while the time $t'$ and the phase speed $c_2$ are varied. This produces a calculation of the amplitude of waves propagating through the a small area as a function of time and phase speed. The resulting data is plotted as a so-called 'cp-t' (phase speed versus time) plot, in which grey shades are used to represent the amplitudes of the waves. One example will be shown later in this chapter. The peak in the cp-t plot gives time at which the wave arrives or leaves the point and the phase speed at which the wave is propagating. In addition to the phase speed and the arrival time, the change in the wave amplitudes (reflection coefficient) can also be computed from the same data, which will be discussed in the following section.
Reflection coefficient

The reflection coefficient is a very important indication of how discontinuities or inhomogeneities effect wave and energy propagation in the structure. The reflection coefficient requires knowing the amplitudes of the incident wave and wave reflected by a discontinuity. The wave speed tracking filter can be used to determine these because the incident and reflected waves will have approximately the same phase speed, and they travel in opposite directions. Thus if the incident wave has a positive wave speed, the reflected wave will appear at a negative phase speed. In terms of the data contained in the cpt file, this requires finding the peak at a positive phase speed and a corresponding peak at a negative phase speed. By determining the amplitudes of the waves at each phase speed the reflection coefficient is calculated from the ratios of the amplitudes.

A simple program was written to, first locate the highest amplitude (peak) in the cpt file, which is called \textit{maxval}(cpm, tm) with phase speed \textit{cpm} and arrival time \textit{tm}. Then the integration of amplitudes over a certain area around the peaks can be performed. The reason for the integration over some phase speed and time is that theoretically the peak of the dominant wave should be a ‘point’ in the cp-t plot. However the finite size of the phase speed tracking filter caused the the peak to be spread out; therefore, the area around the peak should be included in the calculation of the wave amplitude. The integration is performed over the data values in the cp-t plane as

\begin{equation}
intval = \int_{t_l}^{t_u} \int_{cp_l}^{cp_u} val(cp, t) \, dcpt,
\end{equation}

where \textit{intval} is the integrated value, \textit{val}(cp, t) is the amplitude at a certain point
(cp, t) in the cp-t plane, and the limits \( t_l, t_u, cp_l, \) and \( cp_u \) are decided as follows

\[
\begin{align*}
\text{\( t_l < tm \) and} & \quad \text{\( t_l \) is the smallest \( t \) that satisfies} \\
& \quad \text{\( \text{val}(cpm, t) \geq 0.5 \times \text{maxval}(cpm, tm) \)} \\
\text{\( t_u > tm \) and} & \quad \text{\( t_u \) is the largest \( t \) that satisfies} \\
& \quad \text{\( \text{val}(cpm, t) \geq 0.5 \times \text{maxval}(cpm, tm) \)} \\
\text{\( cp_l < cpm \) and} & \quad \text{\( cp_l \) is the smallest \( cp \) that satisfies} \\
& \quad \text{\( \text{val}(cp, tm) \geq 0.5 \times \text{maxval}(cpm, tm) \)} \\
\text{\( cp_u > cpm \) and} & \quad \text{\( cp_u \) is the largest \( cp \) that satisfies} \\
& \quad \text{\( \text{val}(cp, tm) \geq 0.5 \times \text{maxval}(cpm, tm) \).}
\end{align*}
\]

The reason that the number 0.5 was chosen is based on experience with the simulated data (results of one simulation will be shown in the next section.) After the integrations are performed for both the incident and reflected waves, the reflection coefficient can be calculated as

\[
\text{reflection coefficient} = \frac{\text{intval(incident)}}{\text{intval(reflected)}}.
\]

(2.14)

It should be noted that seldom is the area of integration for the incident wave the same as for the reflected wave, so the smaller area of two is chosen for the integration.

**Simulation**

As described in previous sections, from the vzt file, the phase speed of propagating waves can be computed at a specified frequency using the wave speed tracking technique. The tracking filter is centered at one space location, and moved through the time domain searching for the maximum amplitude and the corresponding phase
speed. The process can be carried out at several positions along the beam. By doing this with the same wave, the phase speed of the wave as it propagates down the beam can be obtained.

To demonstrate this process, simulated data is used. The simulated plain beam has the same properties as the beams used in the experiments, Figure 2.6. The beam is a half inch thick HY-80 steel beam with dimensions of 243.84 cm x 5.08 cm. The density of the beam is $7.6 \times 10^3 \text{ kg/m}^3$, and the Young's modulus is $2.0 \times 10^{11} \text{ N/m}^2$. The dispersion equation of the simulated beam is $k = 0.232\sqrt{\omega}$, where $k$ and $\omega$ (radians per second) is the wavenumber and frequency of the propagating waves in the beam respectively. The simulated beam is excited over the frequency band of 500-1000 Hz. In addition to the propagating waves, the evanescent waves were added at the excitation point. The amplitude of the evanescent wave is one-fifth the amplitude of the propagating waves; a choice based on the experimental experience. The reflection coefficients are 0.9 and 0.3 at the right and left ends of the simulated beam respectively. The space and time plot of this wave propagation can be seen in Figure 2.7, called a vz-t plot. It can be seen in the vz-t plot, waves start to propa-
gate from the excitation point in two directions and reflect off the boundaries with decreased amplitudes. Therefore the wave paths become more complicated, because of the increasing number of the reflections, and identification becomes more difficult as time proceeds. It should be noted that due to the higher reflection coefficient on the right end, the reflected waves from the right end are stronger and more evident than the left reflected waves in the plot. The following calculations will focus on the right end boundary.

First, by using the phase speed filtering technique, one can construct cp-t plots for the incident and reflected waves at 750 Hz and at position $x = 1.0m$, Figures 2.8 and 2.9. The darkest points (peaks) in the plots are the highest amplitudes of the waves, and the positive and negative phase speeds indicate the incident and reflected waves respectively. The negative phase speed of the reflected wave indicates that it is traveling in the opposite direction of the incident wave. In this particular case, the wave speed of the maximum amplitude is 293.94 m/sec, arriving at a time of 0.00387 sec. This is the incident wave. Another peak occurs in Figure 2.9 at -293.94 m/sec at an arrival time of 0.00746 sec. This is the reflected wave. Both speeds are in agreement with what was input to the simulations.

In the same way, the phase speed distribution of the dominant wave at 750 Hz is plotted in Figures 2.10 and 2.11. The positive phase speeds represent right-propagating waves, Figure 2.10, and the negative phase speeds represent left-propagating waves, Figure 2.11. The size of the phase speed tracking filter used is 20 points in space, so the filter was centered from the 11th to 54th measurement points. In addition, because of the filter size, the calculation of the phase speed up to the 17th point included the excitation point, which is located between the 6th and 7th point.
Figure 2.7: A vz-t plot of the simulated beam at frequency: 500-1000 Hz
Figure 2.8: A cp-t plot of a simulated incident wave at 750 Hz in a steel beam at point $x = 1.0m$
Figure 2.9: A cp-t plot of a simulated reflected wave at 750 Hz in a steel beam at point $x = 1.0m$
A noticeable high wave speed was picked up at the 11th point, and decayed as the filter moved away from the excitation point. When the calculation did not include the excitation point, the speed of propagating waves is stable, around 293 m/s. The high wave speed near the excitation point could come from the evanescent or the propagating waves.

The oscillation of the phase speed in Figures 2.10 and 2.11 is due to the resolution of the speed tracking filter. There are 100 steps in the phase speed searching, and in this case the range of phase speed searching is from 100 to 700 m/s. Therefore, the resolution in phase speed is 6 m/s, the amount of oscillation in Figures 2.10 and 2.11.

To verify the source of the high phase speed at the excitation point, the same simulated data, but without the evanescent waves at the excitation point, was processed. The results for the simulated data without the evanescent waves are also shown in Figures 2.10 and 2.11, along with previous results. Both results are very similar, therefore the high phase speed in the right-propagating wave is not from the evanescent waves. The calculation of the phase speed of the right-propagating wave did increase near the excitation point. On the other hand, in the left-propagating wave which is the reflection from the right end, the increase around the drive location is not seen.

In order to understand the phase speed increase at the drive location for the right-propagating wave, a beam with a slower flexural phase speed was used for simulations. The results of the phase speed of right-propagating and left-propagating waves are shown in Figures 2.12 and 2.13. Again, the existence of the evanescent wave did not change the phase speed. The increase of phase speed in the right-propagating waves
is not as significant as in Figure 2.10. It is difficult to conclude the cause of the speed increase in the right-propagating waves near the excitation point. However, for most of the beam's length, the phase speed tracking filter provides an accurate estimation of phase speed of the propagating waves.

The calculation of the reflection coefficient is based on the data in Figures 2.8 and 2.9. The total integration time and speed are 0.00267 sec. and 381.82 m/sec. The reflection coefficient from the computation is 0.855. For the same beam at 1000 Hz, the reflection coefficient is 0.871. The results of the reflection coefficient is very close to the value used in the simulation. From the simulation results, the accuracy of filtering technique is verified, and the accurate estimates of the reflection coefficient can be obtained.
Figure 2.10: Phase speed of right-propagating wave of the simulated beam at $f = 750 \text{ Hz}$
Figure 2.11: Phase speed of left-propagating wave of the simulated beam at \( f = 750 \) Hz
Figure 2.12: Phase speed of right-propagating wave of the simulated beam with slower flexural phase speed at $f = 750$ Hz
Figure 2.13: Phase speed of left-propagating wave of the simulated beam with slower flexural phase speed at $f = 750$ Hz
CHAPTER 3. SYSTEM STUDIED

Discontinuities on a structure are known to contribute significantly to wave propagation in the structure and to sound radiation. The discontinuities can couple different wave types. In the worst case scenario, energy from waves that do not radiate sound is coupled into waves that do radiate sound. In general, discontinuities in a structure consist of changes in geometry, attachments of internal structures, and attachments of supporting structures for example. With all discontinuities there are welds, bolts, and deformations caused by machining, all which represent inhomogeneities in the structure. Note the distinction is made between discontinuities (attachments) and inhomogeneities.

Introduction

Consider the structural inhomogeneities shown in Figure 3.1. All of them are frequently encountered on real structures, but there is little understanding of how they influence wave propagation in the structure and the resulting sound radiation. Often they are ignored because they are small compared to a wavelength of a propagating wave in the structure, and they appear to be small compared to the discontinuity. However, there is evidence that the inhomogeneities do influence wave propagation in structures.
Figure 3.1: Examples of structural inhomogeneities and discontinuities
Associated with each inhomogeneity are four variations from homogeneous conditions: (1) material variations, (2) geometric variations, (3) prestress, and (4) residual stress. Of these, the influence of local prestress and residual stress is least considered in the literature. Prestress is induced by contact forces due to tightening a bolt or a clamp, for example, and residual stress is caused by cold working a material or the material cooling after welding. Throughout this dissertation the phrases prestress and residual stress will be used interchangeably because in both cases, the static stress is superimposed upon the dynamic stress caused by wave propagation.

**Stress And Wave Propagation**

In order to get an estimate of the influence of prestress, a plate equation for flexural motion with moderately large amplitudes will be used [18][19]. A plate equation with prestress only in one direction is:

\[
\nabla^2(D\nabla^2 w) + \rho \frac{\partial^2 w}{\partial t^2} - \frac{\partial}{\partial x} \left( N_x \frac{\partial w}{\partial x} \right) = 0,
\]

(3.1)

where \( w \) is the plate's displacement, \( D \) is the material constant, and \( N_x \) is the prestress in the \( x \) direction integrated through the plate thickness.

The simplest case to consider is a uniform prestress over the entire plate caused by a tensile or compressive force at the ends of the plate. In this case, if waves are only propagating in the \( x \) direction, then the plate equation becomes

\[- D \frac{\partial^4 w}{\partial x^4} + \rho \frac{\partial^2 w}{\partial t^2} - N_x \frac{\partial^2 w}{\partial x^2} = 0.
\]

(3.2)

Now take both spatial and temporal Fourier transforms of this equation, and solve the resulting characteristic equation, \( Dk_x^4 - \omega^2 \rho + N_x k_x^2 = 0 \), for the propagating
waves. The wavenumber of the propagating wave is

\[ k_x = \pm \sqrt{-\beta + \sqrt{\beta^2 + k_0^4}}, \]  

(3.3)

where \( \beta = \frac{N_x}{2D} \) and \( k_0^4 = \frac{\rho \omega^2}{D} \) is the "original" wavenumber. The positive sign is for right-propagating waves and the negative sign is for left-propagating waves. Near a typical weld the prestress ranges between 100 MPa and 400 MPa [20][21]. Considering a 4 cm thick steel plate, \( D = 1.17 \times 10^6 \text{ N-m}, \rho = 8.0 \times 10^3 \text{ kg/m}^3 \). Because \( N_x \) is the prestress in the \( x \) direction integrated through the plate thickness, for this case of an uniform prestress, \( N_x \) is equal to the prestress times the thickness of the plate. Therefore, \( N_x \) ranges from 4 MPa-m to 16 MPa-m. First consider a wave at 100 Hz without prestress. The wavenumber of the propagating wave will be \( k_0 = 7.21 \text{ m}^{-1} \). The wavenumber with prestress is:

\[
\begin{align*}
    k_x &= 7.09 \text{ m}^{-1} \quad \text{for } N_x = 4 \text{ MPa-m} \\
    k_x &= 6.75 \text{ m}^{-1} \quad \text{for } N_x = 16 \text{ MPa-m}.
\end{align*}
\]

This indicates a change from the unstressed state of 2% and 6%. In terms of sound radiation, this change in wavenumber can move non-radiating waves into the radiation circle, causing significant radiation.

Now consider the case of a spatially varying prestress, again assuming there is prestress only in the \( x \) direction. The same steel plate as above is used for demonstration, and the prestress is assumed as a normal distribution function of space,

\[ N_x(x) = A e^{-\left(\frac{x^2}{2\tau^2}\right)}, \]  

(3.4)

where \( A \) is the maximum value of \( N_x(x) \). For the calculations to follow it will be assumed that \( A = 16 \text{ MPa-m} \). The variable \( \tau \) defines the width and slope of the normal
function. Examples of $N_x(x)$ plotted for various $\tau$ is shown in Figure 3.2. Using the original form of the plate equation, Equation (3.1), and the spatial distribution for $N_x(x)$, Equation (3.4), the plate equation can be written as

$$D \nabla^4 w + \rho \frac{\partial^2 w}{\partial t^2} - N_x(x) \frac{\partial^2 w}{\partial x^2} - \frac{\partial N_x(x)}{\partial x} \frac{\partial w}{\partial x} = 0. \quad (3.5)$$

Now consider the case of a plane wave, propagating through a plate described by Equation (3.5). The equation for the plane wave is

$$w(x) = e^{j(k_x x - \omega t)}, \quad (3.6)$$

where $k_x$ will be a function that depends on $x$. By substituting the equation for the plane wave into Equation (3.5), the following dispersion equation is obtained

$$Dk_x^4 + N_x(x)k_x^2 - jx\tau N_x(x)k_x - \omega^2 \rho = 0. \quad (3.7)$$

There are four solutions for $k_x$ that will satisfy Equation (3.7). Thus there are four waves that one can expect to find in the plate. These roots, $k_x$, of Equation (3.7) can be solved for numerically if $N_x(x)$ is known.

Typically, the roots are complex numbers, which can be represented as

$$k_x = k_x^r + jk_x^i, \quad (3.8)$$

where $k_x^r$ and $k_x^i$ are real numbers, and represents the real part and imaginary part of $k_x$ respectively. By substituting this root back into the expression for the plane wave, one can get the solution of $w(x)$ as

$$w(x) = e^{j(k_x^r x - \omega t)}e^{-k_x^i x}. \quad (3.9)$$

From the plane wave solution, Equation (3.9), it is shown that the real part of $k_x$ describes the propagation of the wave, and the imaginary part of $k_x$ describes the
Figure 3.2: Examples of $N_x(x)$ with different $\tau$
spatial decay of the wave's amplitude as it propagates. It should be noted that
the sign of the $k_x$ gives the direction of propagation; a positive $k_x$ will be a wave
propagating in the positive x-direction, a right-propagating wave, and a negative
$k_x$ will be a wave propagating in the negative x-direction, a left-propagating wave.
Therefore, the types of waves from the roots of Equation(3.7) can be catalogued as,

1. a complex root: a right-propagating wave with a decaying amplitude,

2. a complex root: a left-propagating wave with a decaying amplitude,

3. a pure imaginary root: an evanescent wave with zero wavenumber.

As an example, consider a steel beam. The prestress using Equation(3.4) is
assumed to have $\tau = 1.0 \times 10^4$ and $A = 16$ MPa-m. At 1000 Hz one can find two
types of roots. Two roots are complex numbers which indicate propagating waves
with wavenumbers and amplitudes that vary in the region of the prestress. One root
has a positive real part, a right going wave, and the other has negative real part
is a left going wave. The other type of root is pure imaginary, which represents
evanescent waves. The comparison of the real part of propagating wavenumber with
no prestress and with prestress at different $\tau$ is shown in Figures 3.3 and 3.4 for 1000
Hz and 200 Hz respectively. The width of the prestress, controlled by $\tau$, affects the
range of influence of the prestress on the real part of the wavenumber.

The wavenumber decreases because of the prestress, but not significantly for
high frequency waves (1000 Hz in this case). For the 200 Hz wave, the real part
of the wavenumber not only decreases near the center of the prestress, it increases
significantly right before the peak of the prestress, especially when $\tau = 1.0 \times 10^5$. 
Figure 3.3: Comparison of real part of wavenumber at $f = 1000$ Hz, $A = 16$ MPa-m with different $\tau$
Figure 3.4: Comparison of real part of wavenumber at $f = 200$ Hz, $A = 16$ MPa-m with different $\tau$
The increase is around 15%. This shows that the low frequency propagating wave is more sensitive than high frequency waves to the slope of the prestress distribution.

If the amplitude of the prestress, $A$ in Equation (3.4), is increased from 16 MPa-m to 160 MPa-m, the change in the real part of the wavenumber at the peak of the prestress is more than 6% at 1000 Hz, Figure 3.5. The decreased wave number means an increase in phase speed. If a lower frequency is examined, $f = 200$ Hz, the influence of prestress on the real part of the wavenumber is quite different from the results at 1000 Hz. Not only is the change more significant, up to 60% at $A = 160$ MPa-m, but it increases and also decreases in different regions of the prestress, Figure 3.6. In other words, the phase speed of a propagating wave at low frequency fluctuates greatly with the presence of a local distribution in the prestress. Therefore measuring a change in phase speed could be a good indication of the existence of the prestress.

As for the imaginary part of the complex roots, as shown in Equation (3.9), it causes a decay in the amplitude of the propagating wave. But from the results shown in Figure 3.7, the change of amplitude from prestress is very small, less than 1%, unless with a very high amplitude prestress distribution. However at a lower frequency, the amplitude of the propagating wave is more sensitive to the prestress. In Figure 3.8, the amplitude decrease is around 2.5% at $A = 16$ MPa-m and $f = 200$ Hz.

The evanescent waves, caused by the pure imaginary roots of Equation (3.5), are non-propagating waves that have an exponentially decaying amplitude. The wavenumbers (imaginary part) of the evanescent waves are plotted in Figure 3.9 at $f = 1000$ Hz and $\tau = 1.0 \times 10^4$ for different prestress amplitudes, and in Figure 3.10 at
Figure 3.5: Comparison of real part of wavenumber at $f = 1000$ Hz, $\tau = 1.0 \times 10^4$ with different prestress amplitude
Figure 3.6: Comparison of real part of wavenumber at $f = 200$ Hz, $\tau = 1.0 \times 10^4$ with different prestress amplitude
Figure 3.7: Comparison of amplitudes of propagating waves at $f = 1000$ Hz, $\tau = 1.0 \times 10^4$ with different prestress amplitude
Figure 3.8: Comparison of amplitudes of propagating waves at $\Lambda = 16$ MPa-m, $\tau = 1.0 \times 10^4$ at different frequencies
f = 1000 Hz and A = 16 MPa-m for various \( \tau \). It is observed that the wavenumber of the evanescent wave changes with the existence of prestress \( N_0(x) \), but the change is not symmetric with respect to the center of the prestress.

The amplitude of the evanescent wave is shown in Figures 3.11 and 3.12. Even though the wavenumber changes significantly as shown in Figures 3.9 and 3.10, the amplitude of the evanescent wave is very similar with variations in the prestress amplitude and \( \tau \). The reason is that the decay occurs in a very small region of space, so the difference among the amplitude of the evanescent wave is almost not recognizable. The nonsymmetry in wavenumber is also shown in the amplitude, especially in Figure 3.11 for \( A = 160 \) MPa-m. It can be concluded that the amplitude of evanescent wave is not affected very much by the prestress, even though the wavenumber did change. If various frequencies were investigated, it can be shown that the decay of the amplitude of the evanescent wave is sensitive to the frequency. The lower the frequency the faster the spatial decay of the evanescent wave, Figure 3.13.

To summarize, the prestress alters wave propagation in the following ways:

1. It changes wavenumbers as the waves propagate and changes phase speed.

2. It changes the amplitude of the wave.

3. It has more influence on propagating waves at lower frequency.

4. It has little influence on the amplitude of evanescent waves.

Thus, because of the ability to change the phase speed, prestress has potential to move energy from subsonic (nonradiating) waves to supersonic (radiating) waves causing increased sound radiation. In this study, experiments will be performed to observe this influence.
Figure 3.9: Comparison of wavenumber of evanescent waves at $f = 1000$ Hz, $\tau = 1.0 \times 10^4$ with different prestress amplitude
Figure 3.10: Comparison of wavenumber of evanescent waves at $f = 1000$ Hz, $A = 16$ MPa-m with different $\tau$.
Figure 3.11: Comparison of amplitude of evanescent waves at $f = 1000$ Hz, $\tau = 1.0 \times 10^4$ with different prestress amplitude
Figure 3.12: Comparison of amplitude of evanescent waves at $f = 1000$ Hz, $A = 16$ MPa-m with different $\tau$
Figure 3.13: Comparison of amplitude of evanescent waves at $A = 16 \text{ MPa-m}$ and $\tau = 1.0 \times 10^4$ with various frequencies.
Energy Distribution

In Chapter 2, the radiation circle was discussed. It was shown that only the waves with wavenumbers inside of the radiation circle can radiate sound to the acoustic farfield. From previous sections in this chapter, it was shown that the prestress in the structure can change the wavenumber. In this section a one-dimensional system is used to study the contribution to the farfield from different waves with the existence of prestress. The analysis is based on the k-space spectra of waves and their relationship with the radiation circle.

A steel beam with a prestress distribution at its middle section is used for simulations. The spatial distribution of the prestress will create inhomogeneities at the middle section of the beam. When a wave is incident on this section of the beam, not only are waves transmitted and reflected, there is also an evanescent wave which decays exponentially away from the discontinuity that will be created, Figure 3.14.

For the propagating waves (incident, reflected, and transmitted) in the finite beam, the waves can be viewed as a cosine wave times a rectangular window in space, see Figure 3.15. The reason for the multiplication of the window is because

![Figure 3.14: Waves propagation in a steel beam with prestress](image-url)
the limitation of measurement and the region of wave propagation. For example, the incident wave ends when the wave reaches the discontinuity, this sets up the right end of the window, and because the measurement is over a finite distance, the left end of the window is present.

![Figure 3.15: Propagating wave and rectangular window](image)

Thus the measured wave, $w_m$, can be expressed mathematically as

$$w_m(x) = w(x) \cdot h(x), \quad (3.10)$$

where

$$w(x) = Ap \cos(2\pi k x)$$

$$h(x) = \begin{cases} 1, & |x| < x_0 \\ 1/2, & |x| = x_0 \\ 0, & |x| > x_0, \end{cases}$$

and $Ap$ is the amplitude of the propagating wave. By the convolution theorem the multiplication in the space domain can be expressed as a convolution in the k-space domain [16], which is

$$W_m(k) = W(k) * H(k), \quad (3.11)$$

where $W(k)$ and $H(k)$ are the Fourier transformations of $w(x)$ and $h(x)$ respectively, and $*$ is the convolution operator. By performing the spatial Fourier transform on
\( w(x) \) and \( h(x) \), one can determine that

\[ W(k) = \delta(k - k_x), \tag{3.12} \]

and

\[ H(k) = 2\pi_0 \frac{\sin(2\pi_0 k)}{2\pi_0 k}. \tag{3.13} \]

Therefore, the convolution in k-space is

\[ W(k) * H(k) = 2\pi_0 \frac{\sin(2\pi_0 (k - k_x))}{2\pi_0 (k - k_x)}. \tag{3.14} \]

It can be concluded that the k-space spectrum of a propagating wave is a sinc function with its center at \( k = k_x \).

The evanescent wave with an exponentially decaying amplitude can be seen in Figure 3.16. There is no need for windowing because the amplitude of the evanescent wave dies out within the range of the measurement. Mathematically the expression in the real space for the evanescent wave and the k-space transformation of the evanescent wave are:

\[ y(x) = A_e \exp(-\alpha |x|), \tag{3.15} \]

\[ W(x) = 2\pi_0 \frac{\sin(2\pi_0 (k - k_x))}{2\pi_0 (k - k_x)}. \]

Figure 3.16: Evanescent wave with amplitude \( A_e \).
and

\[ Y(k) = \frac{2\alpha}{\alpha^2 + (2\pi k)^2}, \quad (3.16) \]

where \( A_e \) is the amplitude of the evanescent wave, and \( \alpha \) is the decaying rate of evanescent wave; a larger \( \alpha \) will cause the evanescent wave to decay at a faster rate in space. The \( \alpha \) used in the following simulations is based on the simulation results in a previous section, especially from Figure 3.13.

With the expressions for the k-space spectra for both propagating waves and the evanescent wave, one can calculate the contribution of each wave to the farfield sound radiation. The sum of the spectra can be broken down into the following

\[ W(k) + Y(k) = V_i^+(k) + V_t^-(k) + V_t^+(k) + V_e(k), \quad (3.17) \]

where \( V(k) \) stands for the spectrum of different waves, the subscript of \( V(k) \), \( i, r, t, \) and \( e \) represent the incident, reflected, transmitted, and evanescent wave respectively. The sign in the superscript of \( V(k) \) indicates the direction of propagation, for example, the incident wave is a right-going wave with a + sign, so is transmitted wave. On the other hand the reflected wave is a left-going wave with a – sign. The evanescent wave has no superscript because it is centered at the discontinuity and is not a wave that propagates.

The following power calculation will be divided into three groups, right-going waves at positive wavenumbers, which are incident and transmitted waves, the left-going wave at negative wavenumbers, which is the reflected wave, and the evanescent wave centered at zero wavenumber.

For example, consider a beam of 2.44m with an assumed reflection coefficient at the prestress of 0.2 and the evanescent wave that has an assumed decay of \( \alpha = 2.0 \).
From the experimental data, the amplitude of the evanescent wave is usually one-fifth of the amplitude of the incident wave, therefore in this simulation it is assumed that $A_e = 0.2 \times A_p$. The resulting k-space spectra at 750 Hz is plotted in Figure 3.17. The spectrum peak at positive wavenumbers is the summation of the incident and transmitted waves, the peak at negative wavenumbers is the reflected wave, and the evanescent wave has a peak at $k = 0$. Two vertical lines in the plot indicate the location of the radiation circle, in this particular case, $k_0 = 13.86 m^{-1}$. By this plot, one can integrate the spectra inside of the radiation circle to compute sound power from the different waves. The percentage of the total power radiated by the different waves is summarized in Table 3.1 for different reflection coefficients and for different values of $\alpha$. The material properties of the beam and the frequency are kept constant. The k-space spectra of these cases are shown in Figures 3.17-3.20.

Table 3.1: The percentage of total radiated power from different waves at 750 Hz

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>Reflection Coefficient</th>
<th>Incident + Transmitted</th>
<th>Reflected Wave</th>
<th>Evanescent Wave</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.2</td>
<td>76.01 %</td>
<td>8.45 %</td>
<td>15.54 %</td>
</tr>
<tr>
<td>1.0</td>
<td>0.8</td>
<td>50.68 %</td>
<td>33.78 %</td>
<td>15.54 %</td>
</tr>
<tr>
<td>5.0</td>
<td>0.2</td>
<td>77.36 %</td>
<td>8.48 %</td>
<td>15.16 %</td>
</tr>
<tr>
<td>5.0</td>
<td>0.8</td>
<td>50.91 %</td>
<td>33.93 %</td>
<td>15.16 %</td>
</tr>
</tbody>
</table>

From Table 3.1, it can be concluded that the evanescent wave is an important contributor to the sound radiation, because its major spectrum peak is always inside the radiation circle. It should be noted that the peak of the evanescent wave is almost invisible in Figures 3.19 and 3.20 when $\alpha$ is increased to 5.0. The decrease of the evanescent wave's peak is because at the higher $\alpha$ value the energy of the evanescent wave is spread over a wider range of wavenumbers. The spreading also
causes a decrease in the percentage of sound radiated by the evanescent wave, but not very significantly (from 15.54 % to 15.16 %.) Moreover the change in the reflection coefficient does not affect the sound radiation from the evanescent wave, it only transfers the radiated power from the transmitted wave to the reflected wave or vice versa.

One of the research goals is to locate the evanescent wave caused by the inhomogeneities in the structure. This involves separating the propagating waves and the evanescent wave. As mentioned previously the k-space filtering technique can help to achieve this goal under certain circumstance. But Figure 3.19 shows that the sidelobes of the propagating waves' spectra inside the radiation circle are difficult to separate from the evanescent wave. One can choose to investigate a lower frequency propagating wave, so that the peaks of the propagating waves will be outside of the radiation circle, but the sidelobes can still not be avoided, especially when the amplitude of the evanescent wave is significantly smaller than the propagating waves. The other approach is to choose a material or geometry of the structure with a lower flexural wave speed, so at the same frequency the main peaks and sidelobes of the propagating waves in k-space can be moved away from the radiation circle. The identification of the evanescent wave will then be possible.

As an example, consider the same beam as before but with a slower flexural wave speed, \( \alpha = 5.0 \), a reflection coefficient of 0.2, and a frequency of 750 Hz. The k-space spectrum of this new case is shown in Figure 3.21. It is clear that the peaks and sidelobes of the propagating waves are far from the radiation circle, compared with Figure 3.19. Not only is it easier now to locate the evanescent wave, the contribution of sound radiation from the evanescent wave also jumps from 15.16 % to 35.69 %.
The results for various reflection coefficients and values for $\alpha$ with a lower flexural wave speed is shown in Table 3.2.

Table 3.2: The percentage of total radiated power of a slower flexural phase speed beam at 750 Hz

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>Reflection Coefficient</th>
<th>Incident + Transmitted</th>
<th>Reflected Wave</th>
<th>Evanescent Wave</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.2</td>
<td>57.26 %</td>
<td>6.36 %</td>
<td>36.38 %</td>
</tr>
<tr>
<td>1.0</td>
<td>0.8</td>
<td>38.17 %</td>
<td>25.45 %</td>
<td>36.38 %</td>
</tr>
<tr>
<td>5.0</td>
<td>0.2</td>
<td>57.88 %</td>
<td>6.43 %</td>
<td>35.69 %</td>
</tr>
<tr>
<td>5.0</td>
<td>0.8</td>
<td>38.59 %</td>
<td>25.72 %</td>
<td>35.69 %</td>
</tr>
</tbody>
</table>
Figure 3.17: K-space spectra of a finite steel beam with a wave incidents on a discontinuity; $\alpha = 1.0$, reflection coefficient = 0.2, $f = 750$ Hz
Figure 3.18: K-space spectra of a finite steel beam with a wave incidents on a discontinuity; $\alpha = 1.0$, reflection coefficient = 0.8, $f = 750$ Hz
Figure 3.19: K-space spectra of a finite steel beam with a wave incidents on a discontinuity; $\alpha = 5.0$, reflection coefficient $= 0.2$, $f = 750$ Hz
Figure 3.20: K-space spectra of a finite steel beam with a wave incidents on a discontinuity; $\alpha = 5.0$, reflection coefficient = 0.8, $f = 750$ Hz
Figure 3.21: K-space spectra of a finite steel beam with a slower flexural phase speed incidents on a discontinuity; $\alpha = 5.0$, reflection coefficient = 0.2, $f = 750$ Hz
CHAPTER 4. CHIRP SIGNAL

A linear chirp is a waveform whose instantaneous frequency increases linearly with time between two specified frequencies. It is used in sonar and radar applications, where it has well known properties for target detection and for estimating target range and velocity [22]. Even birds, whales, bats and other animals are known to use chirplike acoustic signal for ranging and communication. Its capability to concentrate power is desirable to maximize the signal-to-noise ratio [23]. In addition, the low peak-to-rms amplitude ratio (crest factor) of a chirp also makes it a good candidate for broadband system excitation [24]. In the past, random noise was most commonly used to excite a system over a frequency band, not many researchers in sound and structural vibration have applied the chirp signal. This study intends to show the characteristics of chirp and use it as an excitation source. The comparison of results from a chirp and random noise will also be included and discussed in this chapter.

Formulation

For a linear chirp waveform, its instantaneous frequency varies linearly with time, so that in the expression of a chirp, the frequency rate $\mu$ is a constant, and phase $\theta(t)$ is a quadratic function of time. Therefore the waveform can be written as [22]
where $A$ is a constant waveform maximum amplitude

$$\mu = (fu - f_l)/T$$

$f_u, f_l$ are upper and lower band limits

$T$ is period of chirp

$\alpha$ is a constant phase factor.

From a previous study [23], the Fourier transform of this finite-length waveform will lead to power leakage in the frequency domain if there is a discontinuity at the “ends” of the finite-length section. In order to reduce the leakage, the waveform amplitude must be zero at the “ends” of the waveform. By forcing this to be so, both the zero order and first order derivatives of the waveform are guaranteed to be continuous, and leakage can be eliminated. This condition can be satisfied by setting $\alpha = 0$, and $\theta(T)$ is a multiple of $2\pi$, say $\theta(T) = 2m\pi$, where $m$ is an integer. Now from Equation (4.1) and applying the imposed condition, the following relationship can be found,

$$2\pi[f_lT + (fu - f_l)T/(2T)] = 2m\pi$$

$$(fu + f_l)T/2 = m$$

or

$$Tf_c = m,$$

(4.2)

where $f_c$ is the central frequency of the band. Therefore, the product of the time period and the central frequency of the waveform must be an integer in order to
reduce leakage in the frequency spectrum.

An example of a chirp signal using Equation (4.1) with the condition in Equation (4.2), with frequency = 0-400 Hz and period $T = 0.1$ sec., is shown in Figure 4.1.

**Fourier Transform Of Chirp Signal**

In order to understand the frequency content of the chirp, the Fourier transform of this signal is needed. The expression for the chirp is the same as in the previous section, but in order to simplify the derivation, amplitude $A$ and phase $\alpha$ are assumed.
to be 1 and 0 respectively, so the form of chirp now is,

\[ x(t) = \sin[2\pi(f_t + \mu t^2/2)] = \sin(2\pi q), \quad (4.3) \]

where \( q = (f_t + \mu t^2/2) \). The Fourier transform of chirp with a duration of \( T \) is

\[
X(f) = \int_{-\infty}^{\infty} \sin(2\pi q)e^{-j2\pi ft}dt
= \int_{0}^{T} \sin(2\pi q)e^{-j2\pi ft}dt
= \frac{1}{2} \int_{0}^{T} \left[ \sin2\pi(q + ft) + \sin2\pi(q - ft) \right]
- j[\cos2\pi(q - ft) - \cos2\pi(q + ft)]dt, \quad (4.4)
\]

where \( j = \sqrt{-1} \).

In order to further simplify the notation, the following substitutions are chosen,

\[ 2\pi(q + ft) = Lt^2 + 2Mt, \quad (4.5) \]
\[ 2\pi(q - ft) = Lt^2 + 2Nt, \quad (4.6) \]

where

\[ L = \pi(f_u - f_l)/T, \]
\[ M = \pi(f_l + f), \]
\[ N = \pi(f_l - f). \]

Using this notation Equation(4.4) is rewritten as,
\[ X(f) = \frac{1}{2} \int_0^T \left[ \sin(Lt^2 + 2Mt) + \sin(Lt^2 + 2Nt) \right] \]
\[-j \left[ \cos(Lt^2 + 2Mt) - \cos(Lt^2 + 2Mt) \right] dt. \quad (4.7)\]

Now the integral can be divided into 4 parts, and using an integral table [25], the first term of Equation (4.7) can be solved as

\[ \int_0^T \sin(Lt^2 + 2Mt) dt \]
\[ = \sqrt{\frac{\pi}{2L}} \left[ \cos \left( \frac{M^2}{L} \right) S \left( \frac{LT + M}{\sqrt{L}} \right) - \sin \left( \frac{M^2}{L} \right) C \left( \frac{LT + M}{\sqrt{L}} \right) \right. \]
\[- \cos \left( \frac{M^2}{L} \right) S \left( \frac{M}{\sqrt{L}} \right) + \sin \left( \frac{M^2}{L} \right) C \left( \frac{M}{\sqrt{L}} \right) \right], \quad (4.8)\]

where S and C are Fresnel integrals, and defined as

\[ S(x) = \frac{2}{\sqrt{2\pi}} \int_0^x \sin(t^2) dt \]
\[ = S_1(\sqrt{\frac{2}{\pi}} x) \]
\[ = \int_0^{\sqrt{\frac{2}{\pi}} x} \sin \left( \frac{\pi t^2}{2} \right) dt \quad (4.9)\]

\[ C(x) = \frac{2}{\sqrt{2\pi}} \int_0^x \cos(t^2) dt \]
\[ = C_1(\sqrt{\frac{2}{\pi}} x) \]
\[ = \int_0^{\sqrt{\frac{2}{\pi}} x} \cos \left( \frac{\pi t^2}{2} \right) dt. \quad (4.10)\]

For the ease of programming, other forms of Fresnel integrals, namely \( S_1 \) and \( C_1 \), are used. Finally the results of integral of first term is

\[ \int_0^T \sin(Lt^2 + 2Mt) dt \]
The rest of the integral can be done similarly. The final result for the Fourier transform of the chirp is

\[
X(f) = \frac{1}{2} \sqrt{\frac{2}{\pi L}} \left[ \{\cos\left(\frac{M^2}{L}\right)S_1(\sqrt{\frac{\pi}{L}} \frac{LT+M}{\sqrt{L}}) - \sin\left(\frac{M^2}{L}\right)C_1(\sqrt{\frac{\pi}{L}} \frac{LT+M}{\sqrt{L}})ight.ight.
\]
\[\left. - \cos\left(\frac{M^2}{L}\right)S_1(\sqrt{\frac{\pi}{L}} \frac{M}{\sqrt{L}}) + \sin\left(\frac{M^2}{L}\right)C_1(\sqrt{\frac{\pi}{L}} \frac{M}{\sqrt{L}})\right]\]
\[+ \{\cos\left(\frac{N^2}{L}\right)S_1(\sqrt{\frac{\pi}{L}} \frac{LT+N}{\sqrt{L}}) - \sin\left(\frac{N^2}{L}\right)C_1(\sqrt{\frac{\pi}{L}} \frac{LT+N}{\sqrt{L}})ight.
\]
\[\left. - \cos\left(\frac{N^2}{L}\right)S_1(\sqrt{\frac{\pi}{L}} \frac{N}{\sqrt{L}}) + \sin\left(\frac{N^2}{L}\right)C_1(\sqrt{\frac{\pi}{L}} \frac{N}{\sqrt{L}})\}ight] - j\left[ \{\cos\left(\frac{M^2}{L}\right)C_1(\sqrt{\frac{\pi}{L}} \frac{LT+M}{\sqrt{L}}) + \sin\left(\frac{M^2}{L}\right)S_1(\sqrt{\frac{\pi}{L}} \frac{LT+M}{\sqrt{L}})ight.ight.
\]
\[\left. - \cos\left(\frac{M^2}{L}\right)C_1(\sqrt{\frac{\pi}{L}} \frac{M}{\sqrt{L}}) - \sin\left(\frac{M^2}{L}\right)S_1(\sqrt{\frac{\pi}{L}} \frac{M}{\sqrt{L}})\}ight]
\[+ \{\cos\left(\frac{N^2}{L}\right)C_1(\sqrt{\frac{\pi}{L}} \frac{LT+N}{\sqrt{L}}) + \sin\left(\frac{N^2}{L}\right)S_1(\sqrt{\frac{\pi}{L}} \frac{LT+N}{\sqrt{L}})ight.
\]
\[\left. - \cos\left(\frac{N^2}{L}\right)C_1(\sqrt{\frac{\pi}{L}} \frac{N}{\sqrt{L}}) - \sin\left(\frac{N^2}{L}\right)S_1(\sqrt{\frac{\pi}{L}} \frac{N}{\sqrt{L}})\}\right].
\] (4.12)

In Figure 4.2, an example of the Fourier transformed of the chirp signal, with the frequency ranging from 0 Hz to 400 Hz, and period \(T = 0.1\) sec., is shown. It is noticed that except near the limits of the frequency band, the chirp signal provides a very uniform power spectrum. The oscillation of the spectrum in the center of the frequency range is within 2 dB, which is very good compared with random noise over the same frequency range. This again shows the potential of a chirp for being a broadband excitation source.
Comparison With Random Noise

In order to show the capability of a chirp signal, the improved chirp signal was
coded into a data acquisition program, which can generate a stable chirp signal that
then was amplified and fed to a shaker. Measured data from sensors were taken by
the same program right after the structure was excited by the shaker.

A “free-free” steel beam was investigated, whose natural frequencies and mode
shapes were measured and calculated using two excitation sources: a chirp signal and
random noise. Results are compared in the following sections.

**Measurements on a steel beam**

A steel beam, with dimensions: 2.5x2.8x233.5 cm, was suspended by two strings near its ends, the boundary condition was considered “free-free.” A shaker was attached 10 cm from one end, while a laser vibrometer was used to measure the normal velocity of the beam. The non-contact sensor used in this case can reduce the effects from the added mass of traditional contact sensors such as an accelerometer. The experimental setup is shown in Figure 4.3.

The purpose of conducting initial measurements was to investigate the validity of chirp excitations. Therefore, measurement were taken using both a chirp and a random noise excitation. In this case, the central frequency of the chirp was 200 Hz with a bandwidth of 200 Hz, and a period of 1 second. The frequency range of random noise was 0 to 400 Hz. Resonances of the beam in the range of 100 to 300 Hz were theoretically calculated for a “free-free” steel beam [26] to compare with the two measurements. The resonances in this particular frequency range were analytically calculated to be $f_3 = 127.6$ Hz for the third mode and $f_4 = 210.1$ Hz for the fourth mode. The frequency response functions of the beam shown in Figures 4.4 and 4.5 for the chirp and random excitations, show resonances at $f_3 = 133$ Hz and $f_4 = 210$ Hz, which were very close to the theoretical predictions. The slight difference between theoretical and experimental results can be attributed to rough estimation of beam’s material quality (eg. density, Young’s modulus, and etc.). It should be noted that the frequency range of the chirp is 100 to 300 Hz, therefore in Figure 4.4 no peak is found outside the frequency range. On the other hand, the frequency range of the
Figure 4.3: Experiment setup for the beam velocity measurement
random noise is 0 to 400 Hz, so more modes were excited.

After the resonances of interest \((f_3\) and \(f_4\)) were found, a set of 13 measurement points was performed for both excitations. The distance between two neighboring measurement points was 10 cm, so the whole measurement only covered half of the total length of the beam. Again, the theoretical mode shapes were calculated and compared with two measurement results. It should be noted that the values in Figures 4.6 and 4.7 were all normalized for ease of comparison. For both frequencies, very good agreement can be found among the three results. The chirp results are even closer to the theoretical values than random noise particularly near the peaks of the mode shapes.

From the results of initial experiments, Figures 4.4- 4.7, the chirp signal excitation had shown the same capability of multi-frequency signal as random noise. In some cases, the results using the chirp signal were even better than using the random noise. In addition, because random noise is a steady state excitation, the input power is divided by the frequency components in the bandwidth. On the other hand, for the chirp signal, the high energy levels are always concentrated at one frequency at every instant. The high signal-to-noise ratio makes the chirp signal a more efficient broad band signal than the random noise. Moreover, since the chirp waveform is a transient signal, it not only provides information in the frequency domain but also in the time domain. Because of these advantages, the chirp signal was chosen to be the excitation signal for the experiments in this research.
Figure 4.4: Frequency response function of a beam excited by chirp signal
Figure 4.5: Frequency response function of a beam excited by random noise
Figure 4.6: Mode shape comparison of free-free beam at $f_3 = 133$ Hz
Figure 4.7: Mode shape comparison of free-free beam at $f_4 = 219$ Hz
CHAPTER 5. EXPERIMENTAL PROCEDURES AND DATA PROCESSING

A series of experiments was made on simple models. Simple models were chosen so that the influence on sound radiation due to the discontinuities and inhomogeneities could be isolated. Complicated structures at this stage of the research will cloud the issues. The experiments are explained in the following sections. The analysis techniques described in previous chapters will be applied to the measured data; therefore, an overview of the data processing will also be given in this chapter.

Experimental Setup

All the measurements that are presented in this dissertation were made in an anechoic chamber. The chamber has approximate interior dimensions of 3.81 m x 3.81 m x 2.79 m, with a 175 Hz cutoff frequency for the fiberglass wedges installed over the walls, ceiling, and floor. A computer controlled two-dimensional scanner was installed in the chamber to move the sensor for the measurements. The x and y axes of the scanner were driven by two independent stepping motors.

The structure used in the experimental investigation is a half inch thick HY-80 steel beam with the dimensions of 243.84 cm x 5.08 cm. The beam was hung from the ceiling of the chamber with two baffles on the sides. The baffles were made of
3/4-inch-thick high-density particle-board. The experimental setup can be seen in Figure 5.1. The bottom end of the steel beam was immersed in a bucket of silicon sand, which can significantly damp waves incident on that end in order to reduce many of the reflections from the bottom end. Reducing reflections at the bottom end can help avoid too many waves traveling in the beam at one time, which can make the study of wave propagation easier. The beam was driven by a B & K 4809 Vibration Exciter. The connection between the exciter and the beam was a 0.48 cm x 5.08 cm brass stinger with a force transducer attached between the beam and the stinger. The location of the excitation is 50 cm from the bottom of the beam.

Two types of data were taken during the measurement. First, the input force to the beam was measured using a PCB 208A02 Force Transducer. Second, the nearfield sound pressure generated by the vibration of the beam was measured using a B & K 1/4-inch pressure microphone, which is kept 2 cm from the surface of the beam. In this research instead of a direct measurement of normal velocity of the beam, the nearfield pressure was measured because it closely matched the normal surface velocity of the beam. The microphone also can avoid some shortcomings in the velocity measurement, for example, if the accelerometer or other contact sensors were used for velocity measurements, not only would it be very tedious to move the sensor from one measurement point to another, but the mass of the sensor will alter the actual vibration. Even if a remote sensor like a laser vibrometer is used, in order to get enough reflections for a better signal-to-noise ratio, a highly retroreflective tape has to be attached to the measured surface, a very time-consuming process. Some initial measurements in this research have shown that the results from the microphone and a laser vibrometer are in very good agreement. Therefore, for the convenience
Figure 5.1: Experiment setup of the beam measurement
of the measurement, all the experimental results shown in this dissertation are from data obtained with a microphone and force transducer.

**Experimental Procedures**

A block diagram of the experimental instrumentation is given in Figure 5.2. The Concurrent 7000 workstation controls the movement of the microphone, generates the signal for the shaker, samples the signals from the microphone and the force transducer, calculates the complex spectra between signals from the microphone and the force transducer, and stores the resulting spectra. The whole experimental procedure is automated. It takes about one hour to measure 64 points over the beam.

At each measurement point, the data acquisition program on the workstation, will first generate a chirp signal, the frequency range of the chirp in this study is 500 to 1500 Hz. The chirp signal is then amplified and fed to the shaker, which starts the vibration of the beam. When the vibration starts, the force transducer on the beam and the microphone on the scanner both pick up the response. Before the signals reach the workstation, they are first amplified: the microphone signal is amplified 50 dB to 70 dB, and the force transducer signal is amplified 10 dB to 20 dB. The signals are amplified so that they are well within the dynamic range of the A/D converter of the computer. The signals are then passed through a low-pass anti-aliasing filter. The amplified and filtered signals are digitally sampled by the workstation, which uses an array processor to calculate the 4096 point FFT's and the 2048 point auto-and cross-spectra. Twenty frequency averaging are taken at every measurement point to reduce the random noise in the signals.

After the data acquisition is completed at one measurement point, the worksta-
Figure 5.2: Block diagram of the experimental instrumentation
tion sends the commands to the motor controller which causes the scanner to move the probe to the next measurement location. It should be noted that the data acquisition does not resume until the movement of the microphone is completed and fully stopped. The distance between adjacent measurement points in all experiments is 2.5 cm. The first point of the measurement is 13.75 cm below the excitation point. The discontinuity of interest is between measurement points 36 and 37.

In order to guarantee the repeatability and quality of the measurements, a 'standard' procedure of setting the beam and baffle was devised. Therefore not only are the experimental results repeatable, also it is felt that the experiments are free of errors that could result from different setups of the experiments.

**Mechanical Process**

In order to understand the influence of discontinuities and inhomogeneities on the wave propagation in structures; two mechanical processes, a weld and an attachment, will be applied to the tested beam to simulate a realistic structure. By these processes, the structural variations causing the change in wave propagation can be separated and studied. The structural variations are: material property changes, geometry changes, and residual stress changes. In addition to these two processes, heat treatment will also be applied to a welded structure in order to remove the residual stress, so the effects of other variations can be isolated. The details of these mechanical processes will be discussed in the following sections.
Weld

Welding was introduced to this research to simulate one of the types of discontinuities found in real structures. In this study, welding was used to attach a T-bar to the steel beam. The welding was done by the Machine Shop of the Engineering Research Institute (ERI) at Iowa State University. The welding technique applied by ERI is referred as MIG (inert-gas-shielded, metal-arc welding.)

Two particular features of MIG make it a popular choice in industry. First, the filler wire is fed continuously into the weld by a wire-feed mechanism through a torch or gun. This mechanism not only provides a faster welding speed, but because of the no-stop process the weld spatter is also minimized. The result is a smooth and good-appearing weld surface, and a substantial cost saving in metal finishing. Second, the inert gas will shield the filler wire during the welding procedure, therefore the weld area is surrounded by the inert gas blanket to protect it from atmospheric contamination. The introduction of the inert-gas-shield provides a better quality weld. Because MIG is so widely used in the industry, it was chosen in this research.

Attachment

Structures consisting of plates stiffened by a system of orthogonal ribs have found wide application for aircraft, ships, bridges, buildings, as well as in many other branches of contemporary structural engineering. These stiffening elements, represent a relatively small part of the total weight of the structures, and substantially influence the structures strength, stiffness and stability. As important as the rib’s role is in structural strength, the sound radiation induced by this small part of the structure should not be ignored either, especially in the structures where sound is a major
In this study, in addition to the measurements on the plain steel beam, a T-bar made of the same HY-80 steel was welded to the beam. Measurements were made with the plain steel beam, and the same measurement was performed with the T-bar welded to the beam. Then the results of the beam with and without the T-bar were compared to see the influence of the T-bar on wave propagation and sound radiation. The geometry of the T-bar is shown in Figure 5.3. It should be noted that the T-bar itself is constructed by welding together two small pieces.

![Figure 5.3: Geometry of the T-bar](image)

**Heat treatment**

Welding was used to attach the T-bar to the steel beam. Usually the welds are performed at very high temperatures, above the melting point of the structure's material. The cooling around this local area causes residual stresses as the material shrinks. This residual stress may lead to cracking or fracture. In this study, the effect of the residual stress on the wave propagation needs to be removed in order to
separate this effect from other factors, i.e. the added mass and change in geometry caused by the attachment. Heat treating the beam will redistribute the residual stress in the beam so that it is small and approximately uniform. This is desired in an attempt to remove the influence of the residual stress. Therefore heat treatment was applied to the beam, and the results from the beam before and after the heat treatment were compared to see the contribution of the residual stress.

There are many types of equipment that can perform heat treatment, most commonly used are furnaces and salt baths. But due to the size of the beams, 243.84 cm long, a flexible ceramic heating pad was chosen to perform the heat treatment. The temperature and power of the heating pad is controlled by a Cooperheat Twin Heat Module (THM). The power of the heat treatment is provided by a welding machine. The temperature on the surface of the heated beam is measured by a thermocouple and fed back to the THM. When the required temperature is reached, the temperature is maintained for a period of time, called soaking, to insure relief of the residual stress in the thickest part of the structure. After the soaking, the temperature is gradually reduced at a rate that will insure an approximately uniform temperature throughout the material.

In this research, two heating pads were placed near the base of the T-bar, and two high temperature insulation blankets were wrapped around the beam and the heating pads. This setup was placed on a steel welding table. The blankets provide a very uniform heating and cooling process. For the HY-80 steel beam used in this research, the temperature suggested by the manufacturer for the stress relieving is 1100 F. With the power percentage timer on THM set to 6, it took around one hour to bring the temperature on the beam from room temperature to 1100 F. It then
took another one hour for the soaking. Finally the temperature was brought back to the room temperature by turning off the power of the welding machine. Usually it takes more than 12 hours to completely cool off the beam.

Data Processing

There are many ways to interpret and analyze the measured data. In order to avoid the confusion with different kinds of data files, this section is devoted to the explanation of postprocessing of the experimental data. A general procedure for data processing is shown in the block diagram of Figure 5.4. In addition to how the data is created, the relationship between the data and the data presentation (graphs) will also be explained.

As described earlier, the auto- and cross- spectra of signals from the microphone and the force transducer are stored by the data acquisition workstation, the data file is called an spc file. After a measurement is finished, the spc file is transferred to a Digital DECstation 5000 for data processing. The Concurrent 7000 workstation is used for data acquisition and storage. The data postprocessing, calculations, and plotting of the results are done on the faster DECstation. The first file created on the DECstation from the spc file is a vzw file, which contains the impulse frequency response function of the measured data as a function of space (measurement location). Then the vzw file is zero-padded in space and time to twice its original size to remove the wraparound error, k-space filtering can also follow if some wavenumbers are intended to be removed. If the k-space filtering is required, the vzw file will be first transferred to k-space, then transferred back to the vzw format after filtering.

In order to calculate the response of the measured system, the impulse response
function is multiplied by a synthetic force at every measurement point. The resulting data is then inverse Fourier transformed to the time domain, and a vzt file is obtained. A vzt file contains the time history of the response caused by the synthetic force. So if a vz-t plot is created from a vzt file, one can observe the paths of wave propagation in the beam as time proceeds. From the vzt file, the radiated power, phase speed, and reflection coefficient can be computed at every measurement point. The file resulting from the phase speed calculation is called a cpt file, which can generate a cp-t plot (shown in a previous section.) All the files and plots mentioned here will be used in the next chapter for the presentations of the results, and all the techniques for creating these files were described in previous chapters.
Figure 5.4: Block diagram of the data processing
CHAPTER 6. RESULTS ANALYSIS

This chapter presents the experimental results from the measurements of the HY-80 steel beams. Three cases were investigated:

1. a plain steel beam,

2. a steel beam with a welded T-bar,

3. a steel beam with a welded T-bar after heat treatment.

It should be noted that in each case three samples are measured, so that a confidence interval can be established for the measurements and data processing results. Because the changes in wave propagation and sound radiation due to the inhomogeneities are usually small, the establishment of the confidence interval can help to determine if the changes are from the inhomogeneities or a variance in measurement results. In most of the results, the confidence interval will first be introduced, then the averaged values of the three samples will be compared for all the cases. Each beam for every case was measured and the data processed in the same manner. The results from these cases are analyzed in the following sections. The sound radiation and phase speed change over the inhomogeneities are the focus of the analysis.
Sound Radiation

The study of sound radiation in this research, not only has the goal of estimating the contribution to sound radiation from the inhomogeneities in the structure, but also to locate the inhomogeneities from this information. In the sense of the radiated power, the inhomogeneities can be considered as other sound sources in addition to the excitation point on the structure.

Time response

One direct way to view all the excitation on the structure is to observe the time history of the structure's vibration. As described in Chapter 3, the measured data can be multiplied with a synthetic force and a vzt file will be obtained. The synthetic force used in this chapter is a Kaiser-Bessel filter with energy focused in the frequency range from 500 Hz to 1000 Hz, Figure 6.1.

The vzt plots of the resulting vzt files for three cases are shown in Figures 6.2-6.4. Because of the similarity among the vzt plots for the three samples, only the vzt plot of one sample is presented here. From the result for the plain beam, Figure 6.2, a very clear propagating path is shown which is dominated by an initial wave reflecting at the two ends of the beam. The location of the excitation is also clear, because the highest amplitude (darkest color) is above the shaker, and the waves originate from this point. The response patterns are similar in results for the beam with the welded T-bar, before and after heat treatment, Figures 6.3 and 6.4. The vibration pattern does get more complicated after the wave is first incident on the T-bar, at the location $x = 88.75$ cm. Additional reflections are created after this and the constructive and destructive interference of the waves becomes evident as the waves progress, and the
Figure 6.1: Time history and frequency spectrum of a Kaiser-Bessel synthetic force used in the experimental data, \( f = 500-1000 \text{ Hz} \)
Figure 6.2: A vz-t plot of a 1/2-inch HY-80 steel beam at frequency: 500-1000 Hz
Figure 6.3: A vz-t plot of a 1/2-inch HY-80 steel beam with welded T-bar at frequency: 500-1000 Hz
Figure 6.4: A vz-t plot of a heat treated 1/2-inch HY-80 steel beam with T-bar at frequency: 500-1000 Hz
propagating paths of the individual waves soon become very difficult to recognize.

From these results, it is hard to determine the contribution to the sound radiation from the T-bar even with the changed vibration pattern. And from this data the effect on the response of the beam caused by heat treating the welded T-bar cannot be seen. Therefore, from a vz-t plot, one can observe the wave propagation through the beam, but no conclusive information about sound radiation from the T-bar is provided.

**K-space filtering**

As mentioned in previous chapters, k-space filtering can be used to remove propagating wave components that do not radiate sound to the farfield, in hopes that the evanescent wave can be revealed. Before the k-space filter is applied, it is important to understand the contents of the k-space spectra. The real space data, a vzw file, can be transformed into the wavenumber domain with a spatial Fourier transform. The result is a vkw file. A vkw file is the frequency spectra, $\omega$, of the vibration as a function of the wavenumber, $k$.

The vkw plots of the plain beam and the beam with a T-bar (without and with heat treating) are shown in Figures 6.5-6.7. The two darkest vertical stripes in the plots are the dominant propagating waves. From the simulations in Chapter 3, it was shown that the spectra of the evanescent waves can be concealed with the sidelobes of the spectra of the propagating waves. In order to 'uncover' the spectra of evanescent waves, the high wavenumber components (darkest stripes) need to be filtered out, so that the spectra of the evanescent wave, centered at zero wavenumber, can be observed. The k-space filter used here, described in Chapter 2, has the filter parameters of $s = 0.1$ and $kc = 0.6$. The values of the filter were chosen so that most
Figure 6.5: A $vk-w$ plot of a 1/2-inch HY-80 steel beam at frequency: 500-1000 Hz
Figure 6.6: A $v_k-w$ plot of a 1/2-inch HY-80 steel beam with welded T-bar at frequency: 500-1000 Hz
Figure 6.7: A $v_k$-$w$ plot of a heat treated 1/2-inch HY-80 steel beam with T-bar at frequency: 500-1000 Hz
wavenumbers outside of radiation circle can be removed. The same filter was applied to three cases, and then inverse transformed back to real space and time, a vzt file.

A filtered vzt file is a file with fewer wavenumber components, so that the evanescent waves can be shown. The results of the filtered data are shown in Figures 6.8-6.10 for the three cases. For the plain beam, Figure 6.8, the filtered vz-t plot loses some resolution due to the loss of higher wavenumbers through the k-space filtering. The excitation point (darkest point) can be seen in Figure 6.8 to radiate the most sound to the farfield. The large sound radiation at the right edge of the figure is due to truncation in the measurement. There is also some sound radiation from the flexural wave as it propagates.

As for the beams with the welded T-bar, both the excitation point and the location of T-bar show very significant sound radiation in Figures 6.9 and 6.10. At this point it is not conclusive that the evanescent waves around the T-bar alone are responsible for the farfield sound radiation. If the radiated sound at the T-bar is due to the evanescent waves, then the heat treatment should redistribute the stress and change the evanescent wave. The experimental results for the heat treated beam with T-bar did not show much change from the results before heat treating. Therefore there are several possibilities:

1. The heat treatment was not performed properly, and as a result the influence of the residual stress was not removed or reduced.

2. The radiation from the T-bar is not caused by the evanescent wave.

3. The radiation from the T-bar is not a result of the residual stress.

The heat treatment performed in this research followed the guidelines from the
Figure 6.8: A filtered v-t plot of a 1/2-inch HY-80 steel beam at frequency: 500-1000 Hz
Figure 6.9: A filtered vzt plot of a 1/2-inch HY-80 steel beam with welded T-bar at frequency: 500-1000 Hz
Figure 6.10: A filtered vz-t plot of a heat treated 1/2-inch HY-80 steel beam with T-bar at frequency: 500-1000 Hz
manufacturer and other heat treatment handbooks. If the radiation is not caused by the evanescent wave due to the residual stress, then the removal of the residual stress of course will not change the results. This aspect will be discussed with simulations in the next section. Finally, it is also possible that the radiation from the T-bar is generated by factors other than the residual stress, for example, the mass of the T-bar, and the geometry change caused by the T-bar. These other factors are beyond the scope of this study.

As a conclusion, the k-space filter helps to locate the sources of farfield sound radiation on a structure, but does not give enough information to determine what is really causing the farfield radiated sound. At this point the heat treatment has not conclusively separated the residual stress as a factor in the sound radiation from the attachment.

**Radiated sound power**

It is important to understand quantitatively the various contributions to sound radiation from the T-bar. The sound power can be calculated from a vzt file by integrating the filtered squared sound pressure over time. It should be noted that the sound pressure was measured in the acoustic nearfield, but not all the sound in the nearfield will be propagated into the farfield. If the original vzt file (without filtering) is integrated, the result is the sound distribution in the nearfield, which is not equivalent to the sound power radiated to the farfield.

Figures 6.11- 6.13 show the integrated sound fields for the three cases. Each figure contains results for the three samples, so that a confidence interval can be established. The confidence interval for each case is around 2 dB. The pressure
field has a clear drop at the T-bar, Figures 6.11 and 6.12, which is greater than the confidence interval, however there is no clear indication of the sources of farfield sound radiation. The comparison of three cases, based on the averages of the results for the three samples, is shown in Figure 6.14. It is clear that the differences among the results for the three cases are within the confidence interval except directly above the T-bar, in other words, the acoustic near field is not affected significantly by adding the T-bar or applying the heat treatment.

As for the farfield radiated sound, the integrated power for each case is shown in Figures 6.15-6.17. The major peaks in the plain beam results are the shaker and the end reflections. The additional peak in the results with the T-bar is at the location of the T-bar. The deviations in the results from the three samples range from 2 dB to 10 dB in different cases, but it is obvious that the T-bar has an important contribution to the sound radiation, Figures 6.16 and 6.17. From the comparison among the cases, Figure 6.18, the beams with the T-bar have significant sound radiation from the T-bar, which does not exist in the case of plain beams. It is interesting to note that the contribution to the farfield sound from the T-bar is large even though it is quite small compared to the size of the whole beam. However, the heat treating did not affect the sound radiation from beams with a T-bar to a level that is outside of the confidence interval. Therefore, the influence of heat treating on sound radiation cannot be evaluated at this stage.

**Simulation**

The wave propagation in the HY-80 steel beam can be simulated, but instead of a T-bar attachment, a boundary with the reflection coefficient of 0.2 is placed where
Figure 6.11: Integrated sound field of a plain beam at $f = 500-1000$ Hz
Figure 6.12: Integrated sound field of a plain beam with T-bar before heat treatment at $f = 500-1000$ Hz
Figure 6.13: Integrated sound field of a plain beam with T-bar after heat treatment at $f = 500-1000$ Hz
Figure 6.14: Averaged sound field of three cases at $f = 500-1000$ Hz
Figure 6.15: Radiated sound field of a plain beam at $f = 500-1000$ Hz
Figure 6.16: Radiated sound field of a plain beam with T-bar before heat treatment at $f = 500-1000$ Hz
Figure 6.17: Radiated sound field of a plain beam with T-bar after heat treatment at $f = 500$-1000 Hz
Figure 6.18: Averaged radiated sound field of three cases at \( f = 500-1000 \) Hz
the T-bar would be. The value of 0.2 was chosen for the reflection coefficient to simulate the reflection coefficient of the T-bar, which will be discussed later in this chapter. The original vz-t plot of this simulation is shown in Figure 6.19, it should be noted that only one reflection is simulated for the bottom end and new boundary of the beam. Because of the small reflection, the new boundary is almost invisible. The same k-space filter used in the last section is applied to this data, and the filtered vz-t plot is shown in Figure 6.20. Again, filtering allows the location of the new boundary to be recognized, but there is no evanescent wave created in this simulation. The farfield sound radiation caused by the low wavenumbers are most likely the sidelobes of the propagating wave inside of the radiation circle. The sidelobes are partially caused by the reflection at the T-bar location.

The radiated sound power obtained by integration of the filtered vz-t file over time is shown in Figure 6.21. The radiated sound from the simulation is similar to the results for beams with the T-bar. Instead of the T-bar, the sound radiation of the simulation is generated by the new boundary in addition to the excitation and end reflection. It can be concluded that the reflections at the boundary alone can contribute significantly to the farfield sound radiation.

From this simulation, it is proved that the sound radiation from the T-bar observed in the last section could also come from the low wavenumber propagating waves. Moreover the interferences among reflections created by the T-bar can generate a similar sound radiation. Therefore, it is not necessary that the evanescent wave be the sole contributor of the radiated sound. There is still not enough evidence to pin point the role of the evanescent wave in farfield sound radiation.
Figure 6.19: A vz-t plot of a simulated 1/2-inch HY-80 steel beam at frequency 500-1000 Hz.
Figure 6.20: A filtered vzt plot of a simulated 1/2-inch HY-80 steel beam frequency: 500-1000 Hz
Figure 6.21: Radiated sound field of the simulated beam at $f = 500$-1000 Hz
Beam with slower flexural phase speed

As described in Chapter 3, a beam with a lower flexural phase speed can make the k-space spectra separation of the propagating and evanescent waves easier, because the spectral peak of the propagating waves are further away from the radiation circle. A steel beam with a welded T-bar with the same geometry as the HY-80 steel beam was also investigated, however the thickness of the new beam is 1/4-inch, and the material is different from HY-80. The difference in thickness and material quality gives the new beam a slower flexural phase speed.

Applying the same synthetic force as in the previous section, the response of the new beam is shown in Figure 6.22. It is shown in this vz-t plot, that the reflections from the T-bar are stronger in the new beam than in the 1/2-inch H-Y steel beam. The location of the T-bar can be determined by the drastic change in sound pressure over the T-bar. The interference among waves in the new beam make the paths of propagation impossible to recognize after the T-bar. From the results after k-space filtering, Figure 6.23, the location of the T-bar is clearly distinguishable with a high amplitude. The radiated sound field of this beam is shown in Figure 6.24. The contribution to the acoustic farfield is again significant from the T-bar.

The beam with a slower phase speed did make the localization of the sources easier, but the existence and influence of the evanescent wave is still not conclusive. Even though the spectra of the propagating waves are further away from the radiation circle, the sidelobes still can spill into the radiation circle and contribute to the sound radiation.
Figure 6.22: A $v_z$-$t$ plot of a 1/4-inch steel beam of slower flexural phase speed at frequency: 500-1000 Hz
Figure 6.23: A filtered vz-t plot of a 1/4-inch steel beam of slower flexural phase speed at frequency: 500-1000 Hz
Figure 6.24: Radiated sound field of a beam with slower flexural phase speed at $f = 500-1000$ Hz
Wave Phase Speed

In Chapter 2, the wave speed tracking technique was successfully performed on simulated data. The wave phase speed distribution of the simulated beam was obtained after the calculations. In this section, the experimental data is processed by the same tracking filter to study the influence of the T-bar on wave propagation in the beam.

Wave phase speed change

Now the phase speed tracking technique is applied to the experimental data of the 1/2-inch HY-80 steel beam. Because three samples were measured in each case, the average of the three samples and the confidence interval were computed for the phase speed calculation. In the resulting plots, the solid lines represent the averaged values, and the dotted lines represent the confidence interval. Moreover, shading is included in some plots to indicate the confidence interval more clearly.

In Figure 6.25, the phase speeds of the right-propagating waves are shown for three cases. The high phase speeds near the shaker that were observed in the previous simulation in Chapter 2, Figure 2.10, are also seen in the three cases. Again, this may not be the change in phase speed of the propagating waves, it could be an effect of the tracking filter itself. In order to see the change around the T-bar more clearly, only the 21st to 54th points are plotted in Figures 6.26 and 6.27, in which only two cases are compared. In Figures 6.26 and 6.27, the confidence intervals of two cases are overlapped except near the T-bar. The separation of the confidence intervals near the T-bar indicates that there is a decrease in phase speed caused by the T-bar, and the change is not from experimental or signal processing errors. The decrease in
the phase speed is around 4%, from around 290 m/s for the plain beams to about 275 m/s, for the beams with a T-bar before and after heat treating. There is no significant effect from heat treating. The comparison of the averaged phase speed of the three samples of the three cases is shown in Figure 6.28.

As for the left-propagating wave, the decrease in phase speed is more significant. It is around 7% near the T-bar. The heat treatment did not affect the phase speed of the left-propagating wave either. The confidence intervals of the data before and after heat treating coincide. The results of three samples with the confidence intervals and the averaged phase speed of the three cases are shown in Figures 6.29 and 6.30. The rise in phase speed at the excitation point caused by the evanescent waves is not seen, and the phase speed for the plain beam is around -290 m/s, proving that the existence of the T-bar causes the phase speed to change. This change is not from the detection of evanescent waves at the T-bar, because evanescent waves, having much higher phase speeds, are expected to increase the measured phase speed. So from the discussions in previous chapters, the change in phase speed of the propagating waves will also affect the sound radiation of the beam.

**Reflection coefficient**

In Chapter 2, the calculation of the reflection coefficient was performed successfully on the cpt files of the simulation, here the same technique is applied to the experimental data. The focus of this section is to compute the reflection coefficient of the T-bar, so the calculations of the cpt files are made at point 36. The T-bar is between the 36th and 37th measurement points. As recalled in the vz-t plots of the beams with the T-bar, the left-propagating waves are dominated by the reflections at
Figure 6.25: Phase speed of right-propagating wave of 1/2-inch HY-80 steel beam at 750 Hz
Figure 6.26: Phase speed of right-propagating wave of 1/2-inch HY-80 steel beam at 750 Hz (details)
Figure 6.27: Phase speed of right-propagating wave of 1/2-inch HY-80 steel beam at 750 Hz (details)
Figure 6.28: Averaged phase speed of right-propagating wave of 1/2-inch HY-80 steel beam at 750 Hz
Figure 6.29: Phase speed of left-propagating wave of 1/2-inch HY-80 steel beam at 750 Hz
Figure 6.30: Averaged phase speed of left-propagating wave of 1/2-inch HY-80 steel beam at 750 Hz
the top end of the beam. In order to calculate only the reflections off the T-bar and avoid the reflections from the top end, the calculation was restricted to a narrower time range; 1 msec to 5 msec. The arrival time of the reflections from the top end is around 8 msec.

The reflection coefficients of the T-bar for the three samples before and after heat treatment are shown in Table 6.1. The reflection coefficients do not vary much among cases or samples, the reflection coefficient of the T-bar at 750 Hz is around 0.16. The effect of heat treating in this case is again insignificant. It should be noted that if the reflections off the T-bar are stronger, then the peak of the reflection in the cp-t plot will be easier to separate from the peak of the reflection off the top end. Then the calculation of the reflection coefficient of the T-bar are expected to be more accurate.

<table>
<thead>
<tr>
<th>Sample</th>
<th>T-bar Beam</th>
<th>Heated-T-bar Beam</th>
</tr>
</thead>
<tbody>
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<td>Sample 1</td>
<td>0.162</td>
<td>0.158</td>
</tr>
<tr>
<td>Sample 2</td>
<td>0.154</td>
<td>0.158</td>
</tr>
<tr>
<td>Sample 3</td>
<td>0.171</td>
<td>0.174</td>
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</table>
CHAPTER 7. CONCLUSIONS AND RECOMMENDATIONS

Summary And Conclusions

This study has been concerned with new experimental and analytical techniques for understanding the relationship between wave propagation and sound radiation in a structure with inhomogeneities. Several new or merged techniques were developed and tested.

The analysis tools used in this study include: time domain analysis, k-space filtering, and wave speed tracking. The time domain analysis assumes the studied system is linear and time shift invariant, and the impulse spectrum characterizes the response of the system. In this research, the impulse response frequency spectrum is measured experimentally. The response of the structure due to different synthetic forces can be examined based on the measurement of the impulse response. Zero-padding and filtering were used to avoid errors in the signal processing. The Kaiser-Bessel bandpass filter, used for both filtering and the synthetic force, provides an impulse-like time signal with finite frequency content.

K-space filtering was shown to be a good tool in the study of sound radiation. The concept of the radiation circle was built, and later in the Chapter 3 was expanded to investigate the contributions to the radiated sound field from different waves in the structure. K-space filtering involves removing energy in spatial wavenumbers.
that are outside the radiation circle, leaving only the energy that radiates to the acoustic farfield. The application of the k-space filter can be used to locate the area of a structure that radiates sound to the farfield, but the separation of wavenumbers from different waves is not possible with such a simple filter.

The formulation of the wave speed tracking technique was shown, and successfully tested on synthetic data. With the wave speed tracking technique, the phase speed and arrival time of propagating waves can be computed, and the reflection coefficient at a boundary can be calculated. The phase speed change over the distance traveled by a wave is detected by this technique as an indication of inhomogeneities in the measured structure.

The excitation signal used in the measurement is a linear chirp, which is an impulse-like signal. The Fourier transform of the chirp signal was derived, and the resulting frequency spectrum is very uniform making the chirp a good candidate for the broad frequency band excitation. In addition, the high signal-to-noise ratio makes excitation of every mode in a structure possible. A steel beam was excited by both the chirp and random noise and measurements were taken. Both signals excited the same mode shapes for the beam which are in good agreement with theoretical predictions. Moreover, the transient property of the chirp provides information in the time domain in addition to the frequency information. With all the advantages, the chirp was chosen as the excitation signal in the experimental portion of this research.

The system used to test the proposed techniques is a simple steel beam. In order to create the inhomogeneities, a T-bar was welded to the beam. In this research, residual stress created by the welding process is of interest. Therefore the residual stress was built into the plate equation, and the modified wavenumbers were solved
for. The simulated data shows that residual stress has significant effects on the wavenumbers and amplitudes of waves in the structure.

A very consistent experimental setup and a repeatable experimental procedure were designed and tested to minimize the errors caused by the setup or mishandling of the equipment. In addition, three samples were measured for every case of the research, so the confidence interval of the final results could be established. Knowing the confidence interval allowed differences among the experimental cases to be separated from the random errors in the experiments and data processing.

As a result of k-space filtering, the area of the beam with a T-bar welded to it was shown to radiate significant sound to the acoustic farfield, even though the size of the T-bar is small. The radiated sound was from the waves with wavenumbers inside of the radiation circle, which include evanescent and propagating waves. The simulations show the important role of evanescent waves in sound radiation; however, it is difficult to separate the wavenumbers of evanescent waves from the wavenumbers of propagating waves. Therefore, in the experimental data, the contribution from the evanescent wave to the farfield radiated sound cannot be evaluated. Moreover, the heat treating should have redistributed the residual stress, and changed the evanescent waves at the T-bar. However, the sound power distribution did not significantly change after heat treating, so the role of evanescent waves in sound radiation is not clearly identified in the experimental results. Even though experimentally, the influence of residual stress was not proven, the addition of residual stress to the plate equation, and its effect are still important to be understood for research involving structures with residual stress.

Another possible cause of sound radiation at the T-bar is reflections at the T-bar.
Simulation of a beam with a new boundary in place of the T-bar has shown the same sound power distribution as the one for the beam with a T-bar. The reflections at the T-bar could be the reason for the increase in sound radiation around the T-bar.

The T-bar also caused the phase speed of propagating waves to decrease near the T-bar. The decrease in phase speed at the T-bar is 4% to 7%. The calculations of phase speeds for propagating waves were generally successful, except for the presence of the evanescent wave. Both in the experiments and the simulations, a great increase in phase speed was observed when the phase speed calculation includes the point of excitation, where large amplitude evanescent waves exist. However, the decrease in phase speed over the T-bar should not be the result of evanescent waves, because the speed of evanescent waves is much faster than the propagating waves and should cause an increase in the wave speed estimation.

As a conclusion, the proposed techniques have proved to be very useful in the study of wave propagation and sound radiation for structures. The time history of the vibration, k-space spectra, farfield radiated sound, phase speed distribution, and reflection coefficient are determined by merging several analysis techniques. The T-bar attachment to the simple beam plays a significant role both in the farfield sound radiation and wave propagation. Some unanswered questions about the T-bar are worthy of future study.

**Recommendations For Future Work**

The beam with a welded T-bar is used in this dissertation mainly to demonstrate the capabilities of the proposed analysis techniques. However, through the study, the T-bar has shown important influences on the beam's vibration and sound radiation.
Since this type of stiffened structure is very common in actual machines and building construction, it is important to fully investigate this problem.

At this stage, the major uncertainty in the study of the beam with the T-bar is the role of the evanescent wave in the sound radiation. One difficulty of the present technique is to calculate, in the k-space spectra, the separate contributions due to evanescent and propagating waves. In future research, a more sophisticated k-space filter, such as a recursive filter, should be employed, so that the spectra of different waves can be totally separated, and the contribution of the evanescent wave to the radiated sound field can be clearly evaluated.

Attaching the T-bar to a structure causes several variations to a structure, each of which could have an effect on the problem of study. These variations include:

1. material variation
2. geometry variation
3. residual stress variation

In this research, only residual stress variation was explored, but the change in vibration and sound radiation of the beam could come from a combination of these variations. It is important to understand the influence of each factor. Therefore, more experiments should be designed and performed to separate these factors and evaluate their contribution to the change in the structure's vibration and sound radiation. The residual stress was included in the plate equation in this research, but more realistic models should be established to include all the factors, so that the response of a structure with an attachment can be better understood. Only when knowledge of
the influence of all factors is obtained, can measures be applied to reduce or minimize their effects on the structure’s vibration and sound radiation.

Heat treatment was applied in this research to reduce the effect of residual stress, but no significant change was observed. More study is needed to understand the effect of heat treatment, but more importantly the residual stress in the structure should be measured. If the distribution of the stress can be obtained quantitatively, then the effect of heat treating on redistributing the stress can be monitored, and one can have a better understanding of the role of the residual stress in this kind of study.

Further investigation of phase speed tracking is needed to explain the high phase speed at the excitation point. Finally, in addition to the one-dimensional problem studied in this research, the measurement and the analysis techniques should be expanded to two-dimensional structures like plates and shells.
BIBLIOGRAPHY


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