Dual probe methods for determining soil thermal properties: numerical and laboratory study

James Robert Bilskie
Iowa State University
INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps. Each original is also photographed in one exposure and is included in reduced form at the back of the book.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.
Dual probe methods for determining soil thermal properties: Numerical and laboratory study

Bilskie, James Robert, Ph.D.
Iowa State University, 1994
Dual probe methods for determining soil thermal properties:
Numerical and laboratory study

by

James Robert Bilskie

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of the
Requirements for the Degree of
DOCTOR OF PHILOSOPHY

Department: Agronomy
Major: Soil Physics

Approved:

Signature was redacted for privacy.

In Charge of Major Work
Signature was redacted for privacy.
For the Major Department
Signature was redacted for privacy.
For the Graduate College

Iowa State University
Ames, Iowa
1994
DEDICATION

This dissertation is dedicated to my parents,

Joe and Ruth Bilskie.

These wonderful and wise people taught and demonstrated to me

the virtues of life which bring happiness and the fruit of labor.
<table>
<thead>
<tr>
<th>TABLE OF CONTENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>GENERAL INTRODUCTION</td>
</tr>
<tr>
<td>Explanation of Dissertation Format</td>
</tr>
<tr>
<td>VALIDATION OF A DUAL-PROBE METHOD FOR DETERMINING THERMAL PROPERTIES</td>
</tr>
<tr>
<td>ABSTRACT</td>
</tr>
<tr>
<td>INTRODUCTION</td>
</tr>
<tr>
<td>MATERIALS AND METHODS</td>
</tr>
<tr>
<td>RESULTS AND DISCUSSION</td>
</tr>
<tr>
<td>Sample thermal properties</td>
</tr>
<tr>
<td>Observed and calculated temperatures</td>
</tr>
<tr>
<td>Sand measurements</td>
</tr>
<tr>
<td>SUMMARY</td>
</tr>
<tr>
<td>REFERENCES</td>
</tr>
<tr>
<td>ANALYSIS OF A DUAL-PROBE METHOD FOR DETERMINING THERMAL PROPERTIES OF UNSATURATED SOIL MATERIALS</td>
</tr>
<tr>
<td>ABSTRACT</td>
</tr>
<tr>
<td>INTRODUCTION</td>
</tr>
<tr>
<td>MEASUREMENT METHODS</td>
</tr>
<tr>
<td>Calculation Methods</td>
</tr>
<tr>
<td>ERROR SOURCES</td>
</tr>
<tr>
<td>Model error</td>
</tr>
<tr>
<td>SP method</td>
</tr>
<tr>
<td>CF method</td>
</tr>
<tr>
<td>Measurement error</td>
</tr>
<tr>
<td>SP method</td>
</tr>
<tr>
<td>MEASUREMENT RESULTS</td>
</tr>
<tr>
<td>ERROR ANALYSIS OF MEASUREMENTS</td>
</tr>
<tr>
<td>Model error</td>
</tr>
<tr>
<td>Measurement error</td>
</tr>
<tr>
<td>SP method</td>
</tr>
<tr>
<td>CF method</td>
</tr>
</tbody>
</table>
GENERAL INTRODUCTION

The past thirty years have been an exciting time in soil science. The direction of research in the discipline is influenced by growing concern for the protection of soil, plant, wildlife and water resources. Great technological advances in measurement collection and information processing have guided our descriptions of complicated reactions and processes occurring in the soil both at the microscopic and ecosystem scale. Perhaps the most significant realization of this time is that soils must be regarded as a system with interacting and interdependent physical, chemical and biological components. This is exemplified in the study of water quality issues. The fate of a substance which has the potential to be transported from the near-soil-surface region to the water table is influenced by chemical and microbial transformations, movement between the liquid phase and solid phase through sorption reactions, by volatilization and by uptake by plants. These processes are in turn influenced by physical characteristics of the soil such as mineralogical and organic composition and temperature.

In light of the view of soil as an open system, it is imperative that we continually meliorate our ability to make prudent and accurate observations if our understanding of the subject is to be advanced. While many innovative measurement methods have been developed recently, deficiencies in this area persist as constraints on progress in our science. More complete observations must be incorporated into the models used to describe soil processes in an effort to refine and improve the models. Just as important as the development of new measurement techniques is the identification of precisely what information the measurements contain. This is accomplished through careful choice of theory during the design stage and complete error and sensitivity analyses. Additionally, the development of new measurement methods commonly provides ulterior benefits in the form of new discoveries and realizations.

With the preceding sentiment as motivation, the topic of this dissertation was chosen. The work involves the development of a measurement method for soil thermal properties. Thus an innovative method for measuring thermal properties is produced while providing a means to examine interaction between thermal and hydraulic aspects of the soil system. The method is based on a solution to the heat transfer equation and, therefore, is a description of energy transformation. Since soil thermal properties of a given soil are dependent on soil water properties, this method provides insight into the energy/mass balance of a soil.

Probe methods for measuring soil thermal properties are attractive because of the minimal disturbance to the soil under measurement and the simple instrumentation requirements. The single probe method (Jackson and Taylor, 1986) has become the standard technique for measuring thermal conductivity. Early probe method designs were modified to
include more than one probe and provided a method to measure thermal diffusivity (Larson, 1988) using point heat-source theory. Campbell et al. (1991) used theory for the instantaneous application of power to a line heat-source to measure volumetric heat capacity with good results.

The practicality of these probe methods for measuring soil thermal properties is recently realized because of advancements in data acquisition and processing equipment. The availability of data logging equipment provides a means to accurately and automatically collect outputs from a variety of transducer devices at selected time intervals and required resolution. The ability to accurately measure the temperature and the time associated with the temperature reduces the method error to a useful level. Ready access to powerful computers and numerical techniques (e.g., Press et al., 1986) allows efficient and accurate evaluation of data which was previously laborious and inefficient.

The dual-probe method evaluated in this work combines recently developed heat transfer theory with accurate measurement systems and powerful analytical methods to simultaneously determine soil thermal conductivity, volumetric heat capacity and thermal diffusivity. The device used to implement the theory consists of two small probes rigidly mounted at one end. One probe serves as a heat source and approximates a line heat-source. The second probe serves to sense the temperature change at a fixed distance when energy is introduced into the system via the heater probe.

Early studies of this method show good results but were performed only in limited applications. Bristow et al. (1994) described the basis for the method and present calculated thermal properties for a dry sand. Good results were obtained from their measurements but not consistently. Measurement parameters such as probe length, probe spacing, heating time and measurement time resolution were best first-approximations and can be optimized. Kluitenberg et al. (1994) provided an error analysis of the method which indicates that the measurement configuration can be optimized to give good results. The work of Bristow et al. (1994) provided optimism for the method and made results available for scrutiny. From the early work, it is apparent to this researcher that several aspects of this dual-probe method must be carefully addressed in order to develop it into a method useful for researchers in general. The careful evaluation of the method is the intent of this dissertation.

A logical first step in this evaluation is to subject the method to materials of which the thermal properties are known a priori. These measurements provide information about the accuracy and precision of the method and serve to identify limitations which can then be accepted or possibly refined.
The evaluation of any new method must include analyses of measurements collected over the entire range for which the method is intended to be used. This can be done for soils by making measurements for a range of soil types and water contents within those types. The application of first-order error analysis to these measurements will quantify the error contribution of each measured parameter.

Ewen and Thomas (1987) applied the coupled heat and mass transport theory of de Vries (1958) to probe devices to show that moisture migration can occur under the thermal gradients imposed by the heated probe in single-probe methods for determining thermal conductivity. To address the problem, constraints on applied power levels and the amount of time that power is applied were suggested since the moisture migration is dependent on the thermal gradients. The period of time that power is applied in the dual-probe method is much shorter, but the amount of power is significantly greater. Thus the consequence of convective heat transfer occurring in a measurement which assumes conduction heat transfer only must be determined.

The objectives of the study of this dissertation, therefore, were to (1) validate the method by applying it to materials of which the thermal properties are known or independently determined, (2) apply the method to soils having a range of conditions for which the method is intended to be used and (3) evaluate the effect of thermal gradients imposed by the instrument on moisture movement by applying coupled heat and mass transport theory.

Explanation of Dissertation Format

This study is presented in three sections. Each section was prepared according to general manuscript requirements for publication in a refereed scientific journal. The first paper, "Validation of a dual-probe method for determining thermal properties" will be submitted for publication in either Soil Science Society of America Journal or in Soil Science. The second and third papers, "Dual-probe method for determining thermal properties of unsaturated soils" and "Coupled heat and mass transport in dual-probe methods for determining soil thermal properties" will be submitted for publication in Soil Science Society of America Journal. A general summary follows the papers and contains a general summary of the work presented in the three papers. Literature cited in both the General Introduction and the General Summary is listed in Additional References.
VALIDATION OF A DUAL-PROBE METHOD FOR DETERMINING THERMAL PROPERTIES

A paper to be submitted to Soil Science Society of America Journal

J. R. Bilskie and R. Horton

ABSTRACT

Dual-probe methods which implement analytical solutions of the conduction heat transfer equation provide a means to simultaneously determine volumetric heat capacity, thermal conductivity and thermal diffusivity of soil. Thermal properties are determined from temperature monitored by a sensing probe over time at a known distance from a line heat-source. This study applies the dual-probe method to aqueous α-alumina colloidal suspensions and aqueous solutions of glycerol which have known or independently determined thermal properties to validate the method. Good agreement was found between the measured temperature distribution and the temperature described by the infinite line-source analytical solution using the known or independently determined thermal properties. The thermal properties were also estimated using nonlinear least-squares curve fitting. Relative error of the estimated heat capacity and thermal conductivity was less than 2 percent and 6 percent, respectively. Signal noise can be a problem in cases when the maximum temperature increase at the sensing probe is less than 0.5 °C. Good agreement was found when dual-probe temperature measurements performed on a saturated sand were compared with estimated temperature using curve fitted properties and using a mechanistic model.

INTRODUCTION

Soil is an agglomeration of constituents of various forms which are conjoined by mass and energy balances. An understanding of this dynamic system is dependent upon the ability to discern the properties used to describe the fundamental relationships which govern the balances. Though much progress has occurred in recent years, techniques which allow scientists and engineers to confidently quantify soil properties in a direct or indirect fashion are deficient for many of the soil properties. These deficiencies persist as constraints toward our understanding of soil systems. The determination of soil thermal properties is one such area that needs improved measurement techniques. A description of the thermal regime is necessary to evaluate
energy balances which influence the rate of physical, chemical and biological reactions. The development of measurement techniques which accurately characterize soil physical properties and subsequently enhance the ability to make objective observations must be continuously pursued to advance soil science.

One approach for quantifying thermal properties of porous media involves the use of probe devices. Probe methods for determining thermal conductivity in solid and porous materials were actually conceptualized in the late nineteenth century but only developed extensively for use in soils during the last forty years (Blackwell, 1956; de Vries and Peck, 1958). The principal advantages of probe methods are the insignificant disturbance imposed by the measuring device, the ability to take repeated measurements and the simplicity of the instrument. The single probe method for determining thermal conductivity (Jackson and Taylor, 1986) was the first application of probe methods to find widespread use. This method is based on the solution of the Fourier heat equation for a semi-infinite line heat-source in a homogeneous and isotropic medium. In practical application, a constant heat flux emanating from a small, cylindrical, heat-source probe produces a small temperature increase at the probe. The time response of the temperature change is a function of the thermal conductivity of the medium under measurement.

Recently, extensions of the early probe work have been developed for determining thermal properties other than thermal conductivity. Larson (1988) used a point heat-source in a probe and sensed the temperature response from thermistors in probes at up to ten locations surrounding the heat source to determine thermal diffusivity. Campbell et al. (1991) obtained good results for volumetric heat capacity measurements by applying instantaneous line source theory and measuring the temperature response at some known distance from the line heat-source. Most recently, Bristow et al. (1994) introduced another dual-probe method based on pulsed line heat-source theory which simultaneously determines thermal diffusivity and volumetric heat capacity. The ability to measure two soil thermal properties simultaneously is indeed valuable since the third property (thermal conductivity in this application) simply relates the other two, and the system is consequently completely described in the thermal domain.

The progress, especially recently, in the development of thermal property measurement methods provides optimism that soil thermal properties can be accurately and reliably determined. Kluitenbergen et al. (1993) and (1994) provide insightful evaluations and error analyses of the theoretical basis for the most recently suggested dual-probe measurement methods. Extensive error analyses in these works evaluate the underlying theory and the methods for calculating thermal properties from the measured temperature information.
Perhaps most important is the evaluation of the error stemming from the use of infinite line-source theory to describe heat flux from a probe of finite length and diameter. For probe dimensions and measurement arrangements similar to those used in this study, the amount of error in the calculation of thermal properties using the infinite line-source description is generally less than 1%. The results obtained to date substantiate the optimism for development of the dual-probe method.

The theoretical aspect of the method has been rigorously evaluated, but few measurements have been reported. A logical step in the evaluation of this method is to subject it to materials of known thermal properties. These measurements can support validity for the implementation of the theory. The theory is only useful if the inputs required for the theory are attainable using available technology both for probe construction and physical parameter measurements.

Data presented by Bristow et al. (1994) showed good agreement when the thermal properties determined using the dual-probe method were compared with independent determinations, but poor agreement was also common. Errors caused by contact resistance between the probe surface and the porous medium (Steinmanis, 1982) and moisture migration which can occur under relatively large thermal gradients (Philip and de Vries, 1957) were suggested by Bristow et al. (1994) as possible reasons for the poor agreement.

The problems of contact resistance and moisture migration can be alleviated in the application of this method to materials of known thermal properties if the material contains no pore space void of liquid. Using a saturated media reduces the number of factors which are sources of error. Once the method is validated by measuring a simple medium, the other problems can be addressed with a better fundamental knowledge of the method.

This study was undertaken to perform a fundamental evaluation of the dual-probe method described by Bristow et al. (1994). The dual-probe method was used to measure the temperature distribution in a saturated suspension and a solution for which the thermal properties are known or independently determined. Thus concerns about contact resistance and moisture migration are alleviated and the validity of the infinite line-source solution and the measurement method can then be evaluated. The values of the thermal properties calculated from these measurements are interpreted considering potential sources of error. The method was also used to determine thermal properties of a saturated sand. The results of the sand measurements were compared to values derived from the thermal property estimation method of de Vries (1966).
MATERIALS AND METHODS

The Fourier heat equation can be solved to describe the temperature distribution when energy is introduced into a system via an infinite line source and heat transfer is by conduction. Equation [1] is the solution of the heat equation for the case of power applied in the pulsed mode (De Vries, 1952):

\[
T(r,t) = \begin{cases} 
T_1(r,t); & 0 < t \leq t_0 \\
T_2(r,t); & t > t_0 
\end{cases}
\]

[1]

where

\[
T_1(r,t) = -\frac{q}{4\pi\lambda} \left[ \text{Ei} \left( \frac{-r^2 \rho c}{4\lambda t} \right) - \text{Ei} \left( \frac{-r^2 \rho c}{4\lambda t_0} \right) \right]
\]

[2]

\[
T_2(r,t) = \frac{q}{4\pi\lambda} \left[ \text{Ei} \left( \frac{-r^2 \rho c}{4\lambda(t - t_0)} \right) - \text{Ei} \left( \frac{-r^2 \rho c}{4\lambda t} \right) \right]
\]

[3]

and \( T \) is the temperature (C) at time \( t \) (s) and distance \( r \) (m) from the line source, \( t_0 \) is the length of time that power is applied to the line-source (s), \( q \) is the quantity of heat liberated per unit length per unit time (J m\(^{-1}\) s\(^{-1}\)), \( \lambda \) is the soil thermal conductivity (J m\(^{-1}\) s\(^{-1}\) °C\(^{-1}\)), \( \rho c \) is the soil volumetric heat capacity (J m\(^{-3}\) °C\(^{-1}\)), and \( \text{Ei} \) is the exponential integral. Recall that thermal conductivity and volumetric heat capacity are related by the quotient \( \lambda/\rho c \) which defines the thermal diffusivity, \( \kappa \) (m\(^2\) s\(^{-1}\)).

A dual-probe device as shown in Fig. 1 was constructed to implement the theory described by Eq. [1]. The shaft portion of stainless steel hypodermic needles (diameter = 0.813 mm) was cut to length 30 mm plus the thickness of the base disc. Enameled Evanohm wire (Wilbur B. Driver Co., Newark, NJ) of diameter 0.075 mm and resistance 1141 ohm/m was pulled into the exposed 30 mm of the heater probe needle. Copper-constantan thermocouples were placed 15 mm from the probe ends in both the heater probe and the sensor probe. Both probes were then filled with high thermal conductivity, low electrical conductivity epoxy to hold the heater wire and thermocouples in place and to provide electrical insulation. Spacing between the heater and sensor probe was fixed by the spacing between holes drilled
Figure 1. Mechanical drawing of dual probe apparatus
into the base disc. The probe needles were fixed in the base disc by friction from the close fit of the needles in the drilled holes and the epoxy which filled the inside of the upper cylinder. Extension cables for the heating wires and the thermocouples were routed through the upper cylinder. The diameter of the probe device body was chosen to match the inner diameter of the probe holder. The probe holder was then attached to the sample cylinder with machine screws thus providing rigid placement of the probes in the sample material.

Figure 2 shows the measurement arrangement. The CR21X datalogger (Campbell Scientific Inc., Logan, UT) was used to switch the current from a constant current power supply, monitor the temperature in the heater and sensor probes and monitor the voltage drop across the precision series resistor. Knowing the voltage drop across the series resistor, the resistance value of the series resistor and the resistance of the heating wire allows calculation of the value $q$ in Eqs. [2] and [3].

Suspensions of $\alpha$-alumina ($Al_2O_3$) in water were prepared in varied proportions to obtain test materials with a range of volumetric heat capacities. The volumetric heat capacity of these materials can be calculated from known constituent fractions using Eq. [4].

$$\rho c_{susp} = f_{Al_2O_3}\rho c_{Al_2O_3} + f_{H_2O}\rho c_{H_2O}$$  \[4\]

In Eq. [4], $f$ is the volume fraction of the $\alpha$-alumina or water. The thermal conductivity was measured using the method of Jackson and Taylor (1986). The colloidal suspensions were made by slowly adding $\alpha$-alumina in powder form to water while maintaining the pH at 4.0 using 0.10 N HNO₃ and continuously agitating using a stirring plate. Electrostatic stabilization by pH control of the $\alpha$-alumina in an aqueous media ensures that the mixture remains a homogeneous suspension (Schilling and Aksay, 1991). Particles of $\alpha$-alumina of submicron size and with a narrow particle-size distribution (Sumitomo Chemical, America, NY, NY) were used for the suspension. Suspensions with $\alpha$-alumina volume fractions of 0.24, 0.40, 0.50 and 0.54 were prepared. The high viscosity of $\alpha$-alumina mixtures with volume fractions greater than 0.54 made it prohibitively difficult to stir.

Solutions of glycerol and water with volume fractions for glycerol ranging from 0.0 to 1.0 were also used to produce liquid materials with a range of thermal properties. To prevent convective flow which can be induced by thermal gradients generated by the presence of the heater probe, a small and equal amount of agar was added to increase the viscosity of the solution. The final product was a gel-like substance. It was assumed that the agar does not alter the thermal behavior of the solutions. A form of Eq. [4] with glycerol as the added
Figure 2. Dual probe method measurement arrangement and instrumentation schematic
constituent instead of \( \alpha \)-alumina was used to calculate the volumetric heat capacity of the glycerol solutions. We note here that this method does not strictly hold for solutions but only for mixtures.

Measurements were also performed on a saturated sand sample. The fine sand was packed to a bulk density of 1.72 g cm\(^{-3}\), saturated, and the dual-probe measurements performed. The thermal conductivity and volumetric heat capacity values estimated using the dual-probe method are compared to values calculated using the method of de Vries (1966).

The intended use of the dual-probe method is to estimate thermal properties from measured temperature distribution data. The \( \alpha \)-alumina, glycerol and sand data were analyzed using a nonlinear least-squares curve-fitting technique (CF) based on the work of Press et al. (1986). This curve-fitting method uses the Levenberg-Marquardt method of minimization for nonlinear problems. The first partial-derivatives of Eq. [11] with respect to each of the fitted parameters--volumetric heat capacity and thermal conductivity--are needed.

RESULTS AND DISCUSSION
Sample thermal properties

The thermal properties of the test materials are presented in Table 1. The volumetric heat capacity values were calculated using Eq. [4]. Thermal conductivities are averages of four determinations using the single probe method of Jackson and Taylor (1986). The coefficients of variation of the thermal conductivity measurements are 2 % or less. Published thermal conductivity values for glycerol and water are 0.296 (\( J \, m^{-1} \, ^{\circ}C^{-1} \, s^{-1} \)) and 0.58 (\( J \, m^{-1} \, ^{\circ}C^{-1} \, s^{-1} \)), respectively (Weast, 1986). The range of component fractions was chosen to cover as much of the range of soil thermal properties as possible. Volumetric heat capacity values for soil typically range from 1.2 to 3.0 (MJ m\(^{-3}\) °C\(^{-1}\)) and thermal conductivities from 0.2 to 3.0 (J m\(^{-1}\) °C\(^{-1}\) s\(^{-1}\)). The volumetric heat capacity of the test materials was generally higher than typical soil values, and the thermal conductivity values were a good match.

The measurement of thermal conductivity by the single probe method (Jackson and Taylor, 1986) was performed using both the heater probe of the dual-probe device (length = 30 mm and diameter = 0.83 mm) and a separate conductivity probe of length 22.5 cm and diameter 1.0 mm. It was found that the thermal conductivities obtained using the shorter heater probe of the dual-probe device were consistently higher than those using the 22.5 cm probe. This difference was generally 6% to 7% and is consistent with the findings of Bristow et al. (1994) for measurements on dry sand and our findings during measurements on a variety of materials.
Table 1. Thermal properties of α-alumina and glycerol compounds

<table>
<thead>
<tr>
<th>Volumetric fraction of Al₂O₃ or Glycerol in water</th>
<th>Volumetric heat capacity (MJ m⁻³ °C⁻¹)</th>
<th>Thermal conductivity (J m⁻¹ s⁻¹ °C⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.24 Al₂O₃</td>
<td>3.91</td>
<td>1.32 (0.0281)</td>
</tr>
<tr>
<td>0.40 Al₂O₃</td>
<td>3.79</td>
<td>2.37 (0.0361)</td>
</tr>
<tr>
<td>0.50 Al₂O₃</td>
<td>3.63</td>
<td>3.05 (0.0301)</td>
</tr>
<tr>
<td>0.54 Al₂O₃</td>
<td>3.58</td>
<td>3.32 (0.0315)</td>
</tr>
<tr>
<td>1.00 Al₂O₃</td>
<td>3.08</td>
<td>na</td>
</tr>
<tr>
<td>0.00 glycerol (water)</td>
<td>4.17</td>
<td>0.60 (0.0072)</td>
</tr>
<tr>
<td>0.20 glycerol</td>
<td>3.95</td>
<td>0.52 (0.0079)</td>
</tr>
<tr>
<td>0.40 glycerol</td>
<td>3.72</td>
<td>0.44 (0.0045)</td>
</tr>
<tr>
<td>0.60 glycerol</td>
<td>3.50</td>
<td>0.40 (0.0034)</td>
</tr>
<tr>
<td>0.80 glycerol</td>
<td>3.27</td>
<td>0.34 (0.0043)</td>
</tr>
<tr>
<td>1.00 glycerol</td>
<td>3.05</td>
<td>0.30 (0.0043)</td>
</tr>
</tbody>
</table>

†Calculated by summation of constituent parts using published values (Eq. [4]).
‡Determined using single probe method (Jackson and Taylor, 1986).
§Published values of volumetric heat capacity from Robie et al., (1978) and Weast (1986) for 25 °C.
Values in parenthesis are standard deviation for 4 measurements.
Review of literature on the single probe method for thermal conductivity reveals that consideration of probe length and diameter dimensions is important in avoiding error due to axial heat flow. Blackwell (1956) used a numerical example to demonstrate an analytical analysis of the axial heat flow error in the use of probe methods for determining thermal conductivity. Error due to axial heat flow would result in a higher measured thermal conductivity value. The numerical example indicated that a length-to-diameter ratio of greater than 25 results in an axial flow error of less than 1%. The heater probe of the dual-probe device used in this study has a length-to-diameter ratio of approximately 35, and, therefore, axial flow error should not be significant. Thus the discrepancy between results which use probes with different length-to-diameter ratios cannot be attributed to axial flow error.

The disagreement between thermal conductivity measurements was pursued. Careful measurements performed during this study indicate that heat can escape the medium under measurement through the connecting wires for the thermocouple and heating element. To examine this heat loss, cable temperature was monitored on the cables connecting the dual-probe device to the datalogger and power supply. Several thermocouples were attached to the cables over a range of distances from the probe, and the cables with attached thermocouples were then thermally insulated. An increase in cable temperature was observed while power was applied to the heater probe followed by a decrease shortly after power was removed. The temperature increase was greater at locations nearer the probe indicating that this temperature increase was not from power dissipation in the cable due to current flow to the heating element. The fraction of the total power dissipated by the heater probe which can be conducted through the connecting cables will be greater as the length to diameter ratio of the probe decreases. Consideration must be given to probe design to account for heat loss via the electrical cables if the dual-probe device is to be used for single probe thermal conductivity measurements.

**Observed and calculated temperatures**

The results of the measurements on the \( \alpha \)-alumina suspension are presented in Fig. 3. For all cases of the \( \alpha \)-alumina measurements the temperature distribution by the analytical solution is well represented by the data in both shape and amplitude. Relative to water, \( \alpha \)-alumina has a higher thermal conductivity and a lower volumetric heat capacity. This results in increased thermal diffusivities at higher volume fractions of \( \alpha \)-alumina with more rapid temperature changes and narrower profiles. This behavior would be more apparent in Fig. 3 had all the measurements been performed at the same heat input level. The relative amount of noise on the data is dependent on the maximum temperature increase. A significant amount of
Figure 3. Temperature response measured 5.0 mm from line heat-source (X) and calculated using analytical solution (—) for a range of α-alumina/water volume fractions.
noise is apparent in the 0.54 volume fraction sample because of the small temperature increase. The choice of applied power level will affect the signal-to-noise ratio and will have an influence on the calculation of thermal properties. This measurement characteristic deserves attention when the measurements are made and can be especially important for the method suggested by Bristow et al. (1994), since only the information contained in the maximum temperature datum point is used for the calculation.

The results of the measurements on the aqueous glycerol materials are presented in Fig. 4. As in the α-alumina measurements, the amount of power applied to the heater probe was not the same for all cases so a direct comparison of temperature increase and shape of curve is not possible. If an equal amount of power were used for all the materials, the material with the highest thermal diffusivity (the 0.0 fraction glycerol) would have the highest maximum temperature value and the narrowest width of temperature response. The time when the maximum temperature occurs is much later in the glycerol solutions compared to the α-alumina suspensions because glycerol has a much lower thermal conductivity value. Additionally the glycerol temperature response curve is broader with smaller slope values on both sides of the peak.

The agreement between the measured data and the expected values for both the α-alumina and glycerol is good. However there are small discrepancies, and these are difficult to evaluate without a benchmark for comparison. Fig. 5 is presented to allow more complete evaluation of Figs. 3 and 4. In Fig. 5, Eq. [1] was used to generate temperature distributions using two thermal diffusivity values. To demonstrate the effect which deviations of thermal conductivity and volumetric heat capacity would have on the distributions, Eq. [1] was used to generate additional temperature distributions with the thermal property values five percent below the base value and five percent above. The difference in the temperature distribution caused by the five percent changes in thermal conductivity at the two diffusivity values is shown in the upper two graphs while the lower two graphs show the effect for volumetric heat capacity.

The order of magnitude difference in the thermal diffusivity value has a large effect on the shape of the temperature distributions. The narrower profile at higher diffusivity values was also evident in the comparison of the α-alumina and glycerol results. Changes in each of the thermal property values have a distinctly different effect. A change in thermal conductivity of plus or minus five percent has little effect on the temperature in the vicinity of the temperature peak. Conductivity change is expressed most strongly on the leading and trailing portions of the curve, and the same percent change in conductivity has a larger effect at the
Figure 4. Temperature response measured 5.0 mm from line heat-source (×) and calculated using analytical solution (—) for a range of glycerol/water volume fractions.
Figure 5. Temperature distribution at 5 mm from a line heat-source using Eq. [1] with a $t_0$ value of 10 s and for two thermal diffusivity values. Top graphs show effect of changing the thermal conductivity (COND) by plus and minus 5 percent. Bottom graphs show effect of changing the volumetric heat capacity (VHC) by plus and minus 5 percent.
Diffusivity = $1 \times 10^{-6}$ (m$^2$ s$^{-1}$)  

Diffusivity = $1 \times 10^{-7}$ (m$^2$ s$^{-1}$)
lower diffusivity value. Changes in volumetric heat capacity influence the entire curve but are most pronounced at the temperature peak. A reduction of the volumetric heat capacity of five percent results in a greater change in the temperature at the peak than an increase of five percent. A review of Figs. 3 and 4 using the information provided by Fig. 5 suggests that the agreement between actual and measured thermal properties is well within 5%.

Table 2 presents the results of thermal properties calculation using non-linear curve-fitting for the α-alumina, glycerol and sand data previously discussed. Relative error is defined by

\[
\% e_r = \frac{X_{CF} - X_{STD}}{X_{STD}} \times 100
\]

with \(X\) the thermal property being estimated. There was no bias in the relative error values. The agreement between standard and fitted values of heat capacity is very good as reflected in the small relative error values. The agreement for thermal conductivity is also good, but the relative error values are higher. The higher error values for the conductivity compared to the heat capacity are expected since there is more error involved in determining the conductivity through single probe method measurement than in calculating the heat capacity from accurately measured laboratory weights. It is also noted that the curve fitting method gives equal weighting to all data points. The previous discussion of Fig. 5 demonstrated that the temperature distribution holds varying degrees of sensitivity depending of the position on the distribution curve.

In order to determine the precision of the dual-probe method for determining thermal properties, a set of 75 measurements, each spaced one hour in time, were performed on a glycerol sample. The mean and standard deviation for heat capacity were 3.05 and 0.03 MJ m\(^{-3}\) °C\(^{-1}\). The mean and standard deviation for thermal conductivity were 0.300 and 0.005 W m\(^{-1}\) °C\(^{-1}\). The method was accurate and precise in the determination of glycerol thermal properties.

**Sand measurements**

Measurements were taken on a fine sand with the same measurement arrangement used on the α-alumina and glycerol materials. The thermal properties of the saturated sand were estimated using the method of de Vries (1966) and calculated using curve fitting. For the de
<table>
<thead>
<tr>
<th>Test material</th>
<th>Volumetric heat capacity (MJ m(^{-3}) °C(^{-1}))</th>
<th>Thermal conductivity (J m(^{-1}) °C(^{-1}) s(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CF  (\pm) % T</td>
<td>CF  (\pm) % T</td>
</tr>
<tr>
<td>0.24 Al(_2)O(_3)</td>
<td>3.94 (.093)</td>
<td>0.8</td>
</tr>
<tr>
<td>0.40 Al(_2)O(_3)</td>
<td>3.72 (.077)</td>
<td>1.7</td>
</tr>
<tr>
<td>0.50 Al(_2)O(_3)</td>
<td>3.62 (.082)</td>
<td>0.2</td>
</tr>
<tr>
<td>0.54 Al(_2)O(_3)</td>
<td>3.52 (.079)</td>
<td>1.8</td>
</tr>
<tr>
<td>0.00 glycerol</td>
<td>4.16 (.045)</td>
<td>0.2</td>
</tr>
<tr>
<td>0.20 glycerol</td>
<td>3.93 (.045)</td>
<td>0.5</td>
</tr>
<tr>
<td>0.40 glycerol</td>
<td>3.75 (.069)</td>
<td>0.8</td>
</tr>
<tr>
<td>0.60 glycerol</td>
<td>3.51 (.023)</td>
<td>0.2</td>
</tr>
<tr>
<td>0.80 glycerol</td>
<td>3.25 (.030)</td>
<td>0.6</td>
</tr>
<tr>
<td>1.00 glycerol</td>
<td>3.01 (.034)</td>
<td>1.2</td>
</tr>
<tr>
<td>saturated sand</td>
<td>2.67</td>
<td>2.8</td>
</tr>
</tbody>
</table>

\(\dagger\) Relative error. Values in parentheses are standard deviations for 4 measurements.

Table 2. Estimated values of volumetric heat capacity and thermal conductivity using non-linear least-squares curve fitting (CF) and error values relative to known or independently measured thermal property values.
Vries estimation, a volumetric heat capacity for quartz of 1.91 MJ m\(^{-3}\) °C\(^{-1}\) (Robie et al., 1978) was assumed. The estimation yielded a saturated sand thermal conductivity of 3.035 (J m\(^{-1}\) °C\(^{-1}\) s\(^{-1}\)) and a volumetric heat capacity of 2.746 (MJ m\(^{-3}\) °C\(^{-1}\)). The results from curve fitting are presented in Table 2. In the case of the sand, the value according to de Vries (1966) was used for the STD value. The agreement between measured, fitted and estimated using the de Vries method is very good. The results are presented graphically in Fig. 6. The time at which the maximum temperature occurs in the saturated sand is even earlier than for the 0.54 fraction α-alumina because the lower heat capacity of the saturated sand results in a higher thermal diffusivity value. Thus the heat travels more rapidly from the heater probe to the sensing probe 5 mm away than for the other materials tested. The agreement between the measured and the estimated temperature distribution is generally very good with only a slight discrepancy near the peak.

**SUMMARY**

Dual-probe methods for simultaneously determining volumetric heat capacity, thermal conductivity and thermal diffusivity have recently emerged as potentially useful measurement techniques because of new theoretical developments and the availability of instrumentation with the required accuracy and versatility. Previous studies have rigorously evaluated the theoretical basis for the method and presented measurements for air-dry soils. This study subjects the method to measurements on materials with known or independently determined thermal properties. Sample materials included an aqueous colloidal suspension of α-alumina, an aqueous solution of glycerol and a saturated quartz fine sand.

Thermal conductivity was determined independently to provide a basis for comparison of the dual-probe results. The method of Jackson and Taylor (1986) was applied to measurements on the α-alumina and glycerol samples using both the heater probe of the dual-probe device and using a probe with a greater length-to-diameter ratio. A larger proportion of the applied heat was found to escape during the small probe measurements which resulted in erroneously large conductivity values.

Temperature was sensed by a thermocouple installed in a probe a known distance from the heater probe. There was good agreement between the measured temperature and the temperature described by the analytical solution of the heat equation upon which the method is based. Signal noise associated with measured temperature data became a problem when temperature increase at the sensing probe was less than 0.5 °C. Thermal properties were estimated using nonlinear least-squares curve fitting of the measured temperature data. The
Figure 6. Temperature response of a saturated sand measured 5.0 mm from line heat-source (X), estimated using the method of de Vries (1966) (-) and estimated by nonlinear curve-fitting (- -).
curve-fitted thermal property values were expressed as relative error with respect to the known thermal values, and the mean relative error for heat capacity and conductivity were 0.8 % and 2.6 %, respectively.

A comparison between temperature measurements on a saturated sand and temperature as estimated using both curve fitting and the method of de Vries (1966) gave very good agreement.

REFERENCES


ANALYSIS OF A DUAL-PROBE METHOD FOR DETERMINING THERMAL PROPERTIES OF UNSATURATED SOIL MATERIALS

A paper to be submitted to Soil Science Society of America Journal

J. R. Bilskie and R. Horton

ABSTRACT

Dual-probe methods can be used to simultaneously determine soil volumetric heat capacity, thermal conductivity and thermal diffusivity. The measurement method is nondestructive and simple in instrumentation requirements. Preliminary studies showed good results when the method was applied to a dry sand, but poor results have been reported from some measurements. This study was conducted to apply a dual-probe method to two soil materials over a range of water contents. Thermal properties are calculated from the measured temperature distribution data using explicit expressions for the thermal properties from the analytical solution of the conduction heat equation and using nonlinear least-squares curve-fitting. Error analyses were performed to determine the contribution to total error from each of the properties measured by the instrumentation system. The error in probe spacing was the major error contribution for all thermal properties using the curve-fitting method and for heat capacity using the explicit method. The explicit method for calculating thermal conductivity was most sensitive to error in the time resolution of the measured temperatures. For all analyses, error was greater when thermal diffusivity was higher. This increase is not linear and is only significant at the highest diffusivity values typically found in sandy soils. The results indicate that dual-probe methods can provide accurate determinations of soil thermal properties.

INTRODUCTION

Dual-probe methods for determining soil thermal properties have recently been developed and evaluated by a variety of investigators and have potential as a means to completely describe the soil thermal regime from nondestructive measurements. The simplicity of the measurement is a significant advantage and the small spatial scale of the determination can allow monitoring of thermal properties in close proximity to the soil surface of other
objects. Results reported to date indicate that the basis for these methods is theoretically sound and accurate results can be obtained.

The dual-probe methods invoke conduction heat transfer theory to describe the temperature distribution over time at a known distance from a constant heat flux line-source. Campbell et al. (1991) used the analytical solution of the conduction heat equation for an instantaneous heat pulse from an infinite line source to measure the volumetric heat capacity of a Royal sandy loam (coarse-loamy, mixed, mesic Xerollic Camborthid). They found excellent agreement between measured and calculated values over a range of water contents and bulk densities. Kluitenberg et al. (1993) performed a rigorous error analysis of the Campbell et al. (1991) method and generally found the calculated volumetric heat capacity to differ by less than one percent from theoretical determinations using both the finite probe length and finite probe radius descriptions in a pulsed mode. This is especially significant when the simple data requirements of the Campbell et al. (1991) method are considered.

Bristow et al. (1994) used the pulsed mode solution of the conduction heat transfer equation to derive a method which simultaneously determines thermal diffusivity and volumetric heat capacity. Since the third thermal property, thermal conductivity, is simply the product of heat capacity and diffusivity, a complete thermal description is obtained. While the Bristow et al. (1994) work is principally a description of the implementation of pulsed mode conduction heat transfer theory in dual-probe devices, some results are presented for measurements on two dry sand soils and a dry clay soil. A comparison of the results obtained from applying the dual-probe method to results using independent methods shows mixed degrees of agreement. Suggested reasons for the lack of consistent agreement include error in the independent methods for determining the thermal properties, error in the dual-probe method resulting from contact resistance between the probe and the soil and error due to instrumentation equipment. It is also noted that error due to moisture migration under the thermal gradients generated by the heat probe might occur but was unlikely under the dry conditions of the soils tested. Bilskie et al. (1994) applied the method to colloidal suspensions and solutions of which the thermal properties were known or independently determined. In their study, the use of liquid materials alleviated the problem of contact resistance and the viscosity of the materials was sufficient to prevent movement by convection under the thermal gradients. Good results were reported for all measurements.

The dual-probe method is only useful if it is capable of determining soil thermal properties with an amount of error and over the range of conditions commonly found for soils. These conditions include various soil types and a range of water contents. Confidence in the
measurement results can only be obtained if the method is robust for thermal property
determination over the entire range. Additionally, the degree of confidence must be quantified
by identifying the accuracy and precision that can be expected from the method. This requires
careful evaluation of the errors associated both with making the measurements and with the
assumptions of the theory which are the basis for the method. Dual-probe estimates of soil heat
capacity and thermal conductivity have been obtained for dry soils (Bristow et al., 1994) and
saturated soil (Bilskie et al., 1994). To date the dual-probe methods have not been tested on
moist, unsaturated soils.

The objectives of this study are (1) to collect dual-probe measurements on different
soils over a wide range of water contents, (2) to calculate and compare the thermal properties
from these measurements using both the method proposed by Bristow et al. (1994) and a
nonlinear least squares curve-fitting, and (3) to quantify the sources of error which are prevalent
in the measurements. By addressing these topics, this study will provide an evaluation of the
dual-probe method for determining thermal properties for a range of water content conditions
and for different soil types.

MEASUREMENT METHODS

A dual-probe device as shown in Fig. 1 was constructed. The shaft portion of stainless
steel hypodermic needles (diameter = 0.813 mm) was cut to a length of 30 mm plus the
thickness of the base disc. Enameled Evanohm wire (Wilbur B. Driver Co., Newark, NJ) of
diameter 0.075 mm and resistance 1141 ohm/m was pulled into the heater probe needle.
Copper-constantan thermocouples were placed 15 mm from the probe end opposite the disc
base in both the heater probe and the sensor probe. Both probes were then filled with high-
thermal-conductivity, low-electrical-conductivity epoxy to hold the heater wire and
thermocouples in place and provide electrical insulation. Spacing between the heater and sensor
probe was fixed by the spacing between holes drilled into the base disc. Spacing of 5 mm was
used for this study. The probe needles were fixed in the base disc by friction from the close fit
of the needles in the drilled holes and the epoxy which filled the inside of the upper cylinder.
Extension cables for the heating wire and the thermocouples were routed through the upper
cylinder. The diameter of the probe device body was chosen to match the inner diameter of the
probe holder. The probe holder was then attached to the sample cylinder with machine screws
thus providing rigid placement of the probes in the sample material.

The CR21X datalogger (Campbell Scientific Inc., Logan, UT) was used to switch the
current from a constant current power supply, monitor the temperature in the heater and sensor
Figure 1. Mechanical drawing of dual probe apparatus
probes and monitor the voltage drop across a precision resistor in series with the power supply and the heating element as shown in Fig. 2. Knowing the voltage drop across the series resistor, the resistance value of the series resistor and the resistance of the heating wire allows calculation of the amount of heat added to the system.

The air-dry test soil materials, a fine, quartz sand (Granusil, Unimin Corporation, Portage, WI) and a fine-loamy, mixed, mesic Aquic Hapludoll were ground, passed through a 2-mm sieve and uniformly packed into sample containers. Bulk density for the sand and loam were 1.70 and 1.23 g cm⁻³, respectively. The sample containers were 7-cm long sections of PVC pipe with inside diameter of 7.8 cm. A piece of #4 Whatman filter paper (Whatman Laboratory Division, Springfield Mill, Kent, England) was attached to one end of the sample container to serve as a bottom and as a path for water to be removed during desorption.

The packed soil samples were slowly saturated with tap water from the bottom, and the dual-probe device was installed in the sample. Measurements were taken first for the determination of thermal conductivity using the single probe method (Jackson and Taylor, 1986) and then for the determination of thermal properties using dual-probe methods. The saturated sample was then placed in a desorption chamber. This apparatus consists of a cylindrical, plexiglass container with small holes drilled in the base plate. The holes were covered with 0.45 micron thick nylon disks (MSI, Westboro, MA) upon which the soil sample was placed. The nylon material is air-impermeable for the range of desorption tensions used and serves as the boundary between the pressurized desorption cell and the atmosphere. Each soil sample was successively desorbed to a maximum tension of 300 cm for the sand and 1000 cm for the loam. Upon equilibration at each applied tension, data sets were collected for the thermal properties and water content was determined by measuring the mass of water removed during desorption. Power was applied to the heater probe for 10 s, and the amount of power was chosen to give a temperature increase at the sensor probe of approximately 1 °C. The desorption chamber method of water desorption and measurement was used for tension values up to 500 cm, and a pressure plate apparatus was used for water desorption at higher tensions. Five single probe measurement sets at 10-min intervals and ten dual-probe sets at 30-min intervals were taken at each water content. A measurement set consists of a reading of all measured parameters (temperatures and voltage) every one second and for five minutes. The collection of measurements performed sequentially over a water content range provided water retention and thermal property information simultaneously.
Figure 2. Dual probe method measurement arrangement and instrumentation schematic
Calculation Methods

Two methods are presented here for calculating the thermal properties from the measured data. The measured data consist of the temperature over time as measured by the thermocouple in the probe located a known distance from the line heat-source. The method given by Bristow et al. (1994) is based on the analytical solution of the conduction heat equation for pulsed heat input from an infinite line source and is given by

\[ T(r,t) = \begin{cases} 
T_1(r,t); & 0 < t \leq t_0 \\
T_2(r,t); & t > t_0 
\end{cases} \]  

where

\[ T_1(r,t) = -\frac{q}{4\pi \kappa \rho c} \text{Ei} \left( \frac{-r^2}{4\kappa t} \right) \]  

\[ T_2(r,t) = \frac{q}{4\pi \kappa \rho c} \left[ \text{Ei} \left( \frac{-r^2}{4\kappa (t - t_0)} \right) - \text{Ei} \left( \frac{-r^2}{4\kappa t} \right) \right] \]

and \( T(r,t) \) is the temperature change over time, \( t \) (s), and at distance, \( r \) (m), from the heat source, \( q \) the heat input to the line source (J m\(^{-1}\) s\(^{-1}\)), \( \kappa \) is the thermal diffusivity (m\(^2\) s\(^{-1}\)), \( t_0 \) is the length of time that power is applied to the line-source (also referred to as the pulsewidth) and \( \text{Ei} \) is the exponential integral.

If Eq. [3] is differentiated and set equal to zero, both \( \kappa \) and \( \rho c \), the volumetric heat capacity (J m\(^{-3}\) °C\(^{-1}\)) can be explicitly expressed as

\[ \kappa = \frac{r^2}{4} \left[ \frac{1}{t_m - t_0} - \frac{1}{t_m} \right] \left[ \ln \left( \frac{t_m}{t_m - t_0} \right) \right] \]

and
with \( t_m \) the time when the maximum temperature increase, \( T_m \), occurs. The product of Eqs. [4] and [5] yield thermal conductivity, \( \lambda \) (J m\(^{-1}\) °C\(^{-1}\) s\(^{-1}\)). This method for calculating the thermal properties is hereafter referred to as the single-point (SP) method. The SP method has the advantages of simplicity and small data storage requirements. However, poor signal-to-noise ratio or poor resolution of the measured data will yield inaccurate results.

Nonlinear least-squares curve-fitting (CF) was applied to Eq. [1] using the Levenberg-Marquardt method given by Press et al. (1986). The function to be fitted by the CF method is

\[ T = T(t; \rho c, \lambda) \]

and the objective function, \( O(\rho c, \lambda) \), is

\[ O(\rho c, \lambda) = \sum_{i=1}^{n} \left( \frac{T_i - T(t_i; \rho c, \lambda)}{\sigma_i} \right)^2 \]

with the subscript \( i \) the \( i \)th temperature datum point of \( n \) data points, and \( \sigma_i \) is the standard deviation of the single data point. The first partial derivatives of the temperature, \( T \), with respect to the thermal property being estimated are required for this method. The properties volumetric heat capacity and thermal conductivity are used in this work.

**ERROR SOURCES**

The thermal property numerical value obtained by applying a calculation method to the data from dual-probe device measurements is the sum of the actual thermal property value and some amount of error. For the dual-probe method as considered in this study, the total amount of error can be attributed to one of three error sources. The three sources are model error, measurement error and application error. Model error comes from using infinite line heat-source theory to describe the probe device which has finite length and cylindrical dimensions. Measurement error stems from the inability to exactly measure the physical parameters needed to calculate the thermal properties. These parameters include temperatures, voltages and elapsed
times and are measured with electronic instrumentation. Kluitenberg et al. (1994) developed theory for placing bounds on both the model error and measurement error for the SP method. This theory will comprise a portion of the evaluation in this study. Application error is comprised of error due to the nonideal behavior of both the probe device and the sample under measurement. The probe device demonstrates nonideal behavior because the probes have thermal properties which are different from the material being measured and can result in a distortion of the heat flux in the vicinity of the probes. For this study, the probes are assumed to have infinite thermal conductivity and zero heat capacity; this is the most limiting case. Non ideal behavior of the sample material is the result of deviation from the assumptions of homogeneity and isotropic thermal properties, imperfect contact between the probe and sample and the movement of moisture due to the thermal gradients imposed by heater probe. Distortion of heat flux can also result from the fact that the probe is fixed to a holder at one end and free at the other end. The application error can be minimized through design and careful measurement techniques but is not easily quantified. The following section describes how the magnitude of the model and measurement errors can be bounded when using both the SP and the CF methods.

**Model error**

**SP method**

The use of Eq. [1] to describe a cylindrical probe device with finite length is an approximation which allows explicit expression of the thermal properties as stated in Eqs. [4] and [5]. The use of this approximation must be justified by examining the resultant error for a range of conditions typical for application of the method. Kluitenberg et al. (1994) present the analytical solution of the transient heat equation for a cylindrical probe device of finite length. This solution is more complex than Eq. [1] and is not easily expressed in terms of the thermal properties. A rigorous comparison of the finite cylindrical solution and Eq. [1] show that the magnitude of model error is dependent on the sample thermal properties, the physical arrangement of the probe (probe length and spacing) and amount of time that power is applied to the probe.

A bounding value for model error can be determined in the following manner. A temperature distribution is generated using the finite, cylindrical solution with chosen thermal property values. The $t_m$ and $T_m$ values needed for Eqs. [4] and [5] are taken from this distribution and the properties thermal diffusivity and volumetric heat capacity are calculated using Eqs. [4] and [5]. Designating the results of these calculations as hat values, the bounds on relative error, $e_r$, are then given as


\[ e_r(\kappa) = \frac{\hat{\kappa} - \kappa}{\kappa} \] \hspace{1cm} [8]

and

\[ e_r(\rho c) = \frac{\hat{\rho}c - \rho c}{\rho c} \] \hspace{1cm} [9]

with \( \kappa \) and \( \rho c \) the values used in the finite cylindrical solution. Since thermal conductivity is the product of thermal diffusivity and volumetric heat capacity, the relative error of conductivity is the sum of the diffusivity and heat capacity relative errors expressed as

\[ e_r(\lambda) = e_r(\kappa) + e_r(\rho c) \] \hspace{1cm} [10]

The evaluation of the model error over a range of soil conditions will enable informed interpretation of the results and identification of the limitations.

**CF method**

A comparison of the infinite line-source solution and the finite cylindrical solution for soil thermal properties and probe arrangements similar to those used in this study shows nearly identical temperature values from time equals zero to the peak of the temperature distribution. Beyond the peak, the two solutions diverge. The divergence is principally due to the finite length of the probe while the finite radius has little influence for typical probe dimensions. For this study, only the increasing temperature portion of the temperature distribution curve is used for curve-fitting. By imposing this restriction, the model error bound for the CF method is the same as for the SP method.

**Measurement error**

**SP method**

The measurement of several physical parameters is necessary before Eqs. [4] and [5] can be used to calculate thermal properties. Errors associated with the parameter measurements will translate into errors in the determination of thermal properties. Identifying the reason for the error and the magnitude will allow optimization of the method.
Using first-order error analysis, Kluitenberg et al. (1994) derived expressions for the relative error of the calculated thermal properties with respect to the measured parameters. The relative error expressions for volumetric heat capacity and thermal conductivity are presented here in a general form.

\[
\frac{\Delta \rho c}{\rho c} = a_{q,\rho c} \frac{\Delta q}{q} + a_{Tm,\rho c} \frac{\Delta T_m}{T_m} + a_{r,\rho c} \frac{\Delta r}{r} + a_{t_m,\rho c} \frac{\Delta t_m}{t_m} + a_{t_0,\rho c} \frac{\Delta t_0}{t_0} \tag{11}
\]

\[
\frac{\Delta \lambda}{\lambda} = a_{q,\lambda} \frac{\Delta q}{q} + a_{Tm,\lambda} \frac{\Delta T_m}{T_m} + a_{t_m,\lambda} \frac{\Delta t_m}{t_m} + a_{t_0,\lambda} \frac{\Delta t_0}{t_0} \tag{12}
\]

The individual relative error coefficients in Eqs. (11) and (12), \(a_{\text{parameter.property}}\), are summarized in Table 1. Constant coefficients in Table 1 mean that the magnitude of the associated error term can only be improved by decreasing the uncertainty of its measurement or increasing the magnitude of the parameter. Increasing the parameter magnitude beyond the values used in this study will generally have adverse effects. In all cases when the coefficient is variable, it is dependent on \(t_m, t_0, r, \) and \(k\). As an example for applying the coefficients, consider the coefficient value of -2 for the spacing term, \(r\), in the calculation of heat capacity. If the measured spacing value were 0.0055 m, and the actual spacing were 0.0050, \(\Delta r/r\) would have a value of 0.10. The coefficient -2 then means that the heat capacity would be underestimated by a factor of 0.20 or 20%.

Values for the individual relative error terms \(\Delta q/q, \Delta T_m/T_m, \Delta r/r, \Delta t_m/t_m\) and \(\Delta t_0/t_0\) are derived from measurements and are subject to measurement error. First-order error analysis can also be applied to the measured parameters to define error bounds for the measurement error.

The value for \(q\) is dependent on the current flow through the heating element in the heater probe and on the resistance of the heating element. The current flow is determined by
Table 1. Summary of coefficients of thermal property relative error description

<table>
<thead>
<tr>
<th>thermal property</th>
<th>parameter</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>heat capacity</td>
<td>q</td>
<td>+1</td>
<td>-2</td>
<td>-1</td>
<td>variable</td>
</tr>
<tr>
<td>conductivity</td>
<td>r</td>
<td></td>
<td></td>
<td></td>
<td>variable</td>
</tr>
<tr>
<td></td>
<td>T_m</td>
<td></td>
<td></td>
<td></td>
<td>variable</td>
</tr>
<tr>
<td></td>
<td>t_m</td>
<td></td>
<td></td>
<td></td>
<td>variable</td>
</tr>
<tr>
<td></td>
<td>t_0</td>
<td></td>
<td></td>
<td></td>
<td>variable</td>
</tr>
</tbody>
</table>
measuring the voltage drop across a series resistor of known value (Fig. 2). A value for \( q \) can be derived from

\[
q = I^2 R_p
\]  

[13]

with \( I \) the current through the heating element (amp) and \( R_p \) the resistance of the heating element (ohm/m). The current, \( I \), can be determined from

\[
I = \frac{E}{R_s}
\]  

[14]

with \( E \) the voltage drop across the series resistor (volts) and \( R_s \) the value of the series resistor (ohm). Applying first-order error analysis to Eq. (13) yields

\[
\frac{\Delta q}{q} = \frac{2\Delta E}{E} - \frac{2\Delta R_s}{R_s} + \frac{\Delta R_p}{R_p}
\]  

[15]

The values for \( \Delta E \), \( \Delta R_s \) and \( \Delta R_p \) depend on how accurately \( E, R_s \) and \( R_p \) can be measured. The value for \( \Delta E \) is the accuracy of the datalogger and is derived from the datalogger performance specifications. A precision resistor was used for the series resistor, but the specified tolerance value was not used for \( \Delta R_s \). The value for \( \Delta R_s \) was more accurately determined by applying a known current through the resistor and measuring the voltage drop. The \( \Delta R_s \) value is then dependent on the instruments used to measure the current and voltage drop. Instruments are readily available which are capable of these measurements with a degree of accuracy significantly better than 1 %.

The value for \( \Delta T_m \) is the error in determining \( T(r,t_m) \) for Eq. [1]. Since \( T(r,0) = 0 \), this value is a change in temperature instead of an actual temperature magnitude. Therefore only imprecision but not inaccuracy of the temperature measurement contributes to this error. For the datalogger used in this study, the \( \Delta T_m \) term depends on the thermocouple voltage measurement resolution, the linearity of the thermocouple temperature/voltage relationship and the error from using a polynomial expression to convert the measured thermocouple voltage to a temperature. Both errors are very small and a maximum value of 0.02 °C for \( \Delta T_m \) is derived for the measurement arrangement used here.
The probes of the probe device are short and rigid but some distortion of spacing is possible when the probes are inserted into a material to be measured. The r coefficient value of -2 listed in Table 1 for heat capacity indicates that the calculation is quite sensitive to the spacing parameter. To determine bounds for this error, the probe device was repeatedly inserted into a clear, plexiglas container with soil material packed in the same manner as the samples used in the other measurements. The probe device was inserted near the wall of the container so the probes could be viewed from outside the container, and the spacing was measured. The maximum spacing change value observed, 0.00015 m, was used to bound this error for this set of measurements.

A value for \( \Delta t_m \) will indicate how accurately the time when the peak temperature occurs can be determined. Since temperatures are measured at discrete time intervals, this accuracy will depend on the time interval programmed into the datalogger. The time interval used for the measurements in this study was 1.0 s which results in a \( \Delta t_m \) value of 0.5 s.

A value for \( \Delta t_o \) is dependent on the structure of the datalogger program and accuracy of the clock which controls the microprocessor in the datalogger. The programming decision to determine whether power is to be switched is made early in the program and is not affected by other parts of the program. The inaccuracy of the internal clock can be assumed zero which leads to \( \Delta t_o \) being insignificant and is given a value of zero.

**CF method**

The measurement error component for the CF method cannot be described using first-order error analysis since there are no explicit expressions to analyze. The terms \( T_m \) and \( t_m \) are not present in the CF method leaving \( q \), \( r \) and \( t_0 \) as measurement error sources. The effect of measurement error in the CF method was established by determining the accuracy for measuring the parameters \( q \), \( r \) and \( t_0 \) and then calculating the thermal properties using the curve-fitting method with the parameters set at the limits of accuracy for measuring the parameters. The accuracy for measuring \( q \), \( r \) and \( t_0 \) was determined in the same manner as for the SP method.

**MEASUREMENT RESULTS**

Dual-probe measurements were performed on the sand and loam samples over a range of water contents beginning at saturation. Water retention curves are presented in Fig. 3. The results of a subset of these measurements were chosen to represent the entire water content range and are presented in Figs. 4 and 5. Temperature values measured at 5.0 mm from the
Figure 3. Soil water retention curves
Figure 4. Measured temperature (X) and calculated temperature using the curve fitting method (O) and the single point method (Δ) for sand soil over a range of water tensions.
Figure 5. Measured temperature (X) and calculated temperature using the curve fitting method (O) and the single point method (Δ) for the loam over a range of water tensions.
heater probe are shown with estimated temperature distributions using Eq. [1] and thermal properties calculated with the SP and CF methods. The amount of power applied to the heater probe during the measurements was chosen to give a maximum temperature increase at the sensing probe of approximately 1 °C. The power was applied for 10 s beginning at time equal zero in all measurements.

In viewing Figs. 4 and 5, there is little apparent difference in the measured temperature data for a given soil over the range of water content conditions. If the same power had been applied for all measurements on a given sample, the magnitude of the maximum temperature value would have differed over the entire water content range by approximately 0.17 °C and 0.29 °C for the sand and loam soil, respectively. This suggests that in field applications of the method, the power level would not require adjustment over the range of expected water contents thus simplifying instrumentation system requirements.

The agreement between the data and the temperature curves using values estimated with the SP and CF methods is very good for all measurements on the loam. The measured and estimated temperatures for the sand soil contain notable discrepancies. In the sand soil there are discrepancies between measured and estimated values with the greatest discrepancy for the thermal properties estimated using the SP method. The source of this discrepancy is the topic of the error analysis portion of this study and will be discussed later.

Values of volumetric heat capacity and thermal conductivity calculated by applying the SP and CF methods to all measured data are presented in Table 2. The higher water-holding capacity of the loam results in larger volumetric heat capacities for the range of tensions used in this study. In the narrow range where there are water contents common to both materials, the volumetric heat capacity of the sand is slightly greater than that of the loam. Bower and Hanks (1962) found the heat capacity of feldspar to be about ten percent higher than quartz using radiation calorimetry, but few accurate heat capacity measurements exist for minerals in the conglomerate form of soil where bulk density must be accounted for. The thermal conductivity of the sand is always greater than that of the loam. Where the data in Table 2 allow comparison of thermal conductivity at similar volumetric water contents, the conductivity of the sand is greater by a factor of three. Both the lower air-filled porosity and the higher quartz fraction of the sand contribute to the higher conductivity, but the lower air-filled porosity is the predominant factor.

The heat capacity and thermal conductivity information of Table 2 is presented graphically in Fig. 6 along with thermal properties as estimated using the method of de Vries (1966). Heat capacity is linearly related to water content, and this is well reflected in both the
Table 2. Results of dual probe measurements on sand and loam soil materials over range of water contents

<table>
<thead>
<tr>
<th>Soil</th>
<th>tension content (cm)</th>
<th>water volumetric heat capacity SP (m$^3$ m$^{-3}$)</th>
<th>volumetric heat capacity CF ($10^6$ J m$^{-3}$ $^\circ$C$^{-1}$)</th>
<th>thermal conductivity SP (J m$^{-1}$ $^\circ$C$^{-1}$ s$^{-1}$)</th>
<th>thermal conductivity CF (J m$^{-1}$ $^\circ$C$^{-1}$ s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>sand</td>
<td>0</td>
<td>0.351</td>
<td>2.66 (0.056)</td>
<td>2.68 (0.017)</td>
<td>2.87 (0.198)</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>0.330</td>
<td>2.49 (0.052)</td>
<td>2.59 (0.013)</td>
<td>3.58 (0.208)</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.309</td>
<td>2.43 (0.049)</td>
<td>2.46 (0.009)</td>
<td>3.21 (0.212)</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>0.196</td>
<td>1.97 (0.043)</td>
<td>2.05 (0.018)</td>
<td>2.50 (0.232)</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>0.048</td>
<td>1.43 (0.039)</td>
<td>1.44 (0.009)</td>
<td>2.05 (0.240)</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>0.036</td>
<td>1.49 (0.041)</td>
<td>1.47 (0.011)</td>
<td>1.61 (0.204)</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>0.031</td>
<td>1.51 (0.027)</td>
<td>1.45 (0.021)</td>
<td>1.63 (0.118)</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>0.027</td>
<td>1.38 (0.026)</td>
<td>1.33 (0.010)</td>
<td>1.49 (0.111)</td>
</tr>
<tr>
<td></td>
<td>300</td>
<td>0.022</td>
<td>1.37 (0.020)</td>
<td>1.36 (0.007)</td>
<td>1.48 (0.103)</td>
</tr>
<tr>
<td>loam</td>
<td>0</td>
<td>0.533</td>
<td>2.93 (0.002)</td>
<td>2.94 (0.015)</td>
<td>1.50 (0.076)</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.512</td>
<td>2.85 (0.009)</td>
<td>2.86 (0.051)</td>
<td>1.42 (0.069)</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>0.491</td>
<td>2.76 (0.005)</td>
<td>2.76 (0.009)</td>
<td>1.27 (0.002)</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.436</td>
<td>2.54 (0.011)</td>
<td>2.55 (0.008)</td>
<td>1.20 (0.060)</td>
</tr>
<tr>
<td></td>
<td>75</td>
<td>0.396</td>
<td>2.42 (0.011)</td>
<td>2.43 (0.063)</td>
<td>1.17 (0.006)</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>0.322</td>
<td>2.24 (0.017)</td>
<td>2.25 (0.007)</td>
<td>1.09 (0.086)</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.310</td>
<td>2.26 (0.016)</td>
<td>2.22 (0.021)</td>
<td>1.04 (0.133)</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.295</td>
<td>2.17 (0.015)</td>
<td>2.16 (0.096)</td>
<td>1.05 (0.115)</td>
</tr>
</tbody>
</table>

Values in parentheses are standard deviation for 10 measurements.
Figure 6. Calculated thermal properties with water content
sand and loam graphs with slightly poorer results at the lowest water contents. There is good agreement between both the SP and CF methods. The thermal conductivity values calculated using the SP method agree in trend with the values calculated with the CF and de Vries method, but show some deviation from a smooth relationship to water content. This deviation in the case of conductivity suggests that conductivity is more sensitive to the measured data than heat capacity when the CF and SP calculation methods are applied.

**ERROR ANALYSIS OF MEASUREMENTS**

**Model error**

The relative model error values for heat capacity and thermal conductivity for the range of thermal diffusivity values found in the measured samples are presented in Fig. 7. The use of the infinite line source solution to describe the finite cylindrical source used in this study will result in a very small error in the calculation of volumetric heat capacity. The effect of model error on the calculation of conductivity results in errors values greater than 1% for the sand since the diffusivity for this relatively high-bulk-density fine sand was always greater than \(1.0 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}\). A \(t_0\) value of 10 s was used for the measurements of this study, but the conductivity model error is also presented here for \(t_0\) values of 6 s and 8 s to demonstrate that the error can be reduced by decreasing \(t_0\). The range of diffusivity values for the loam was between \(4 \times 10^{-7}\) and \(5 \times 10^{-7} \text{ m}^2 \text{ s}^{-1}\) which results in a conductivity relative error less than 1%.

A small reduction in the amount of model error can be achieved by increasing probe length, but this can result in increased error in the spacing error for heat capacity since longer probes will be more likely to bend when inserted into soil.

**Measurement error**

**SP method**

Table 3 summarizes the error terms of Eqs. [11] and [12]. It is noted here that the values in this table are error bounds and thus reflect the maximum possible value for the error. The error term for probe spacing is not included in Table 3 because it is the same for all measurements. A value for \(\Delta r\) of 0.00015 m was determined from the measurements described in the error sources section. This value and the probe spacing of 0.00497 m for the probe device used in this study results in a \(\Delta r/r\) value of 0.0302. Since the coefficient for the \(\Delta r/r\) term is \(-2\), the upper error bound for this term is 6.04%. This value demonstrates the sensitivity of the method to probe spacing. Unlike some of the other error sources, spacing error cannot be reduced by controlling other measurement parameters. An increase in probe spacing will
Figure 7. Thermal conductivity and volumetric heat capacity components of model error for diffusivity values of measured samples
Table 3. Results of error analysis for SP thermal property calculation method

<table>
<thead>
<tr>
<th>Tension (cm)</th>
<th>Tm (°C)</th>
<th>tm (s)</th>
<th>$\frac{\Delta T_m}{T_m}$ (%)</th>
<th>$\frac{\Delta t_m}{t_m}$ (%)</th>
<th>$\frac{\Delta t_m}{t_m}$ (%)</th>
<th>$\Delta q/q$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0.94</td>
<td>13</td>
<td>2.128</td>
<td>0.463</td>
<td>1.781</td>
<td>-2.599</td>
</tr>
<tr>
<td>7</td>
<td>0.72</td>
<td>13</td>
<td>2.778</td>
<td>0.890</td>
<td>3.423</td>
<td>-3.316</td>
</tr>
<tr>
<td>15</td>
<td>0.93</td>
<td>13</td>
<td>2.151</td>
<td>0.698</td>
<td>2.685</td>
<td>-3.735</td>
</tr>
<tr>
<td>25</td>
<td>0.98</td>
<td>13</td>
<td>2.041</td>
<td>0.599</td>
<td>2.304</td>
<td>-2.918</td>
</tr>
<tr>
<td>40</td>
<td>0.97</td>
<td>13</td>
<td>2.062</td>
<td>0.884</td>
<td>3.400</td>
<td>-3.329</td>
</tr>
<tr>
<td>60</td>
<td>1.03</td>
<td>13</td>
<td>1.942</td>
<td>0.467</td>
<td>1.796</td>
<td>-2.591</td>
</tr>
<tr>
<td>90</td>
<td>1.01</td>
<td>14</td>
<td>1.980</td>
<td>0.463</td>
<td>1.654</td>
<td>-2.591</td>
</tr>
<tr>
<td>150</td>
<td>1.18</td>
<td>14</td>
<td>1.695</td>
<td>0.465</td>
<td>1.661</td>
<td>-2.591</td>
</tr>
<tr>
<td>300</td>
<td>1.11</td>
<td>14</td>
<td>1.802</td>
<td>0.465</td>
<td>1.661</td>
<td>-2.591</td>
</tr>
<tr>
<td>Loam</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0.94</td>
<td>18</td>
<td>2.128</td>
<td>0.090</td>
<td>0.250</td>
<td>-1.618</td>
</tr>
<tr>
<td>10</td>
<td>1.03</td>
<td>19</td>
<td>1.942</td>
<td>0.100</td>
<td>0.263</td>
<td>-1.494</td>
</tr>
<tr>
<td>25</td>
<td>0.85</td>
<td>19</td>
<td>2.353</td>
<td>-0.002</td>
<td>-0.007</td>
<td>-1.698</td>
</tr>
<tr>
<td>50</td>
<td>0.94</td>
<td>19</td>
<td>2.128</td>
<td>0.039</td>
<td>0.103</td>
<td>-1.619</td>
</tr>
<tr>
<td>75</td>
<td>1.21</td>
<td>19</td>
<td>1.653</td>
<td>0.073</td>
<td>0.192</td>
<td>-1.551</td>
</tr>
<tr>
<td>200</td>
<td>0.96</td>
<td>19</td>
<td>2.083</td>
<td>0.082</td>
<td>0.216</td>
<td>-1.532</td>
</tr>
<tr>
<td>500</td>
<td>1.02</td>
<td>20</td>
<td>1.961</td>
<td>0.103</td>
<td>0.258</td>
<td>-1.407</td>
</tr>
<tr>
<td>1000</td>
<td>1.06</td>
<td>19</td>
<td>1.887</td>
<td>0.074</td>
<td>0.195</td>
<td>-1.548</td>
</tr>
</tbody>
</table>
reduce the error contribution of this term, but increasing probe spacing will increase model error (Kluitenberg, 1994).

The relative error for the maximum temperature increase term, $\Delta T_m/T_m$, has a constant coefficient of -1 for both heat capacity and conductivity and is therefore dependent only on the accuracy of determining $T_m (\Delta T_m)$ and the $T_m$ value. Since $T_m$ is approximately 1 °C and the derived value for $\Delta T_m$ is 0.02 °C, the value for this relative error term is about 2 %. The coefficient for this term is -1 for both heat capacity and conductivity meaning that if measured temperature is greater than actual temperature this term will contribute to an underestimation of the thermal properties by 2 % at maximum. Since the slope of the temperature distribution curve is small in the vicinity of the peak, a positive value of $\Delta T_m$ is more likely to occur than a negative one, and understimation will generally result.

The coefficients $a_{t_m, \rho C}$ and $a_{t_m, \lambda}$ are dependent on the probe spacing, pulsewidth, diffusivity and $t_m$. The probe spacing and pulsewidth are fixed for all measurements presented here leaving diffusivity and $t_m$ which are related to each other (see Eq. [41]). Both coefficients have larger absolute values for the sand than the loam, because the coefficient value is proportional to the diffusivity, and the sand has higher diffusivity values. The values presented in Table 3 were calculated using the measured $t_m$ values which have a resolution of ± 0.5 s. The absolute value of $a_{t_m, \lambda}$ increases as the value of $t_m$ decreases. An error of -0.5 s in determining $t_m$ can increase the value of $a_{t_m, \lambda}$ by 25 % at high diffusivity values. Both coefficients have greater values when diffusivity is higher. This is apparent in Table 3 when the values for sand and loam are compared. The value of the coefficients can not be readily reduced by modifying the measurement arrangement and so any reduction in this contribution to error must come from a reduction in the time interval between temperature measurements. Changing the time interval from 1 s to 0.25 s will reduce the error by a factor of 4. The lower limit for time interval will be restricted by the number of measurements and calculations which the datalogger is programmed to performed. A more complex program will force a larger time interval. The calculations using the SP method were repeated by determining $t_m$ with a resolution of 0.25 s instead of 1.0 s. All values of calculated conductivity and heat capacity were within 3 % of the CF values. Additionally, the deviations in Fig. 4 for the 0 cm and 90 cm data were alleviated.

The relative error associated with the value of applied power, $\Delta q/q$, cannot be reduced significantly by adjusting the measurement arrangement. The value of $\Delta q$ can be minimized by
carefully measuring the resistance values of the series resistor and the heating element and by choosing the voltage range of the datalogger which will give the greatest accuracy.

**CF method**

The error in calculated thermal property values using the CF method which will result from measurement error in \( q \) is the same as the value of \( \Delta q \) since \( T \) is linearly dependent on \( q \) in Eq. [1]. This was confirmed by using adjusted values for \( q \) in the CF calculation.

The sensitivity of the CF method to error in probe spacing was evaluated by curve-fitting the measured data with \( r \) in Eq. [1] set to \( r-\Delta r \) and with \( r \) set to \( r+\Delta r \). Values used were \( \Delta r = 0.00015 \) m and \( r = 0.00497 \) m. Error in probe spacing did not affect the calculation of thermal conductivity. The magnitude of the error in the calculated heat capacity was slightly less than the amount found in the SP method with a maximum value of 5.8 %. The error was greater when the same amount of probe spacing change was in the direction of the probes moved apart than when moved together. It was found that the amount of error increased with diffusivity in the case of the increased probe spacing and decreased with diffusivity for decreased probe spacing.

**CONCLUSIONS**

This analysis of dual-probe methods for determining soil thermal properties on soils with different texture and over a range of water contents provides insight into the accuracy of the methods and identifies the aspects of the methods which can be modified to optimize results. Soil thermal properties were determined for a sand and a loam using measurements of temperature over time at a known distance from a heater probe. The calculation methods included nonlinear least-squares curve-fitting (CF) and explicit expressions derived from the analytical solution of the conduction heat equation for a pulsed infinite line source (SP). The thermal property values determined by the two methods were used in the analytical solution to allow comparison of the measured data and estimated values using the determined properties. The agreement was very good for the CF method for both soil materials. The SP method is especially sensitive to the time resolution of the measurements and yielded more error for the sand than the loam because of the narrower temperature distribution of the sand measurements.

The error component of the calculated thermal properties can result from model error, measurement error and the nonideal behavior of the probe device and porous media. The relative error which stems from model error in the estimation of thermal conductivity was a maximum of 1.5 % for the measurements of this study while the relative error of heat capacity
was less than 0.2%. The conductivity error can be reduced by using shorter periods of applied power to the heater probe. The model error for both properties increases with thermal diffusivity.

First-order error analysis was applied to the SP method to evaluate the contribution of measurement error to the estimation of heat capacity and conductivity. The major contribution to error in heat capacity estimation was uncertainty in spacing between the heater probe and the sensing probe. The error associated with determining the time of maximum temperature increase is the major contributor to conductivity error. At the highest diffusivity values, the absolute value of the time of maximum temperature term exceeded 10%. This term can be reduced to much smaller values by reducing the time interval of measurement collection.

REFERENCES


COUPLED HEAT AND MASS TRANSPORT IN DUAL PROBE METHODS FOR DETERMINING SOIL THERMAL PROPERTIES

A paper to be submitted to Soil Science Society of America Journal

J. R. Bilskie and R. Horton

ABSTRACT

Probe methods are an attractive means for measuring soil thermal properties in laboratory and field situations because of the nondestructive nature of the method and the ability to perform repeated measurements over time in changing soil conditions. Recently, dual probe methods have received much attention because volumetric heat capacity, thermal conductivity and thermal diffusivity can all be obtained from a single measurement set. When power is applied to the heater probe, large thermal gradients can develop in the vicinity of the heater probe. These gradients are conducive for moisture movement, and this convective heat transfer can result in errors since the method is based on conduction heat transfer. The consequences of convective heat transfer were evaluated by using a finite element numerical model to describe the transient coupled heat and mass transport. The model was used to simulate measurements on a sand, a loam and a silt loam over a range of water contents. Use of this model indicates that moisture migration occurs only in the region near the heater probe but not beyond 1.5 mm from the probe. The simulations indicate that the convective heat transfer component is not a factor when temperature at the heater probe does not exceed 80 °C.

INTRODUCTION

Heat probe methods are an attractive means for measuring soil thermal properties in laboratory and field situations. The methods can provide repeated measurements over time in changing soil conditions and with little disturbance of the material being measured. The single probe method for measuring thermal conductivity as described by Jackson and Taylor (1986) has been thoroughly evaluated and is now considered the standard for deriving conductivity. Recently, multiple probe configurations have been devised which can be used to estimate volumetric heat capacity and diffusivity. In the dual-probe method, power is introduced to the system of interest as a constant heat flux by converting electrical energy to thermal energy via a
heating element installed in a small probe. The temperature is monitored at a known distance from the heat source by a thermocouple installed in a parallel probe. Campbell et al. (1991) used theory for an instantaneous application of power to an infinite line source to determine volumetric heat capacity knowing the maximum temperature increase at a known distance from the heating probe. A rigorous theoretical evaluation of this relatively simple method was made by Kluitenberg et al. (1993) by comparing it to results obtained using finite and cylindrical heat source probes operated in a pulsed mode. The Campbell et al. (1991) method was found suitable for determining volumetric heat capacity for a range of conditions and was robust for sensing changes in volumetric heat capacity.

Theory for pulsed application of power to an infinite line source was applied by Bristow et al. (1994) to derive a dual-probe method for simultaneously measuring volumetric heat capacity and thermal diffusivity. The use of pulsed power application theory versus instantaneous application theory more realistically describes the measurement method. The ability to completely characterize the thermal properties of soils is a significant advancement in soil measurement techniques but has not been extensively tested in a variety of soil moisture and texture conditions as has the single probe method.

Though the single and the multiple probe methods are based on similar conduction heat transfer theory, the measurement procedure differs in the amount of power which is applied via the heating element. In order to obtain good resolution of a temperature measurement at some distance from the heating element, typically 5 mm to 8 mm, a relatively large heat flux at the line source is necessary. The single probe method requires temperature increases at the heat source which are typically less than 5 °C. However, in order to get a 0.5 °C increase at a distance 6 mm from the heat source in a dry sandy soil, the temperature increase at the heat source can exceed 60 °C with thermal gradients in excess of 100 °C cm⁻¹. These large thermal gradients can be conducive for water movement. Since the method is based on theory which considers heat transfer by conduction only, convective heat transfer can introduce error into the method.

Heat transfer from a line source surrounded by soil involves several mechanisms. When power is applied to the line source, thermal gradients are generated. Heat transfer by conduction will occur in proportion to the soil thermal conductivity. If thermal gradients are sufficient, moisture will move in the direction away from the line source as liquid and/or vapor. The reduced amount of water near the probe will result in lower thermal conductivity which will increase the thermal gradients since the heat flux is constant. If conditions are conducive for vapor transport, the thermal gradients might actually be decreased because of the more efficient transfer of energy by the vapor than by the liquid or solid constituents. Besides the thermal
gradient, the amount of vapor transport will be dependent on the degree of saturation and the pore size distribution. When vapor condenses away from the vaporization point, the energy transformation is exothermic and the temperature will increase. The movement of moisture also changes the heat capacity which will affect thermal gradients. Furthermore, the thermally induced mass transport can produce water pressure potential gradients in the direction toward the line source, and these gradients can reduce water flux away from the line source or even reverse the flux direction under some conditions.

Considering the thermal gradients which can be generated during the use of dual probe methods, there is little doubt that mass transport can occur concurrently with heat transport. The validity of applying conduction heat transfer theory to the dual probe method is contingent on identifying the conditions which are suitable for convection heat transfer. Theory describing moisture movement under thermal gradients was presented by Philip and de Vries (1957). Subsequent work by de Vries (1958) contains refinements of this theoretical description as well as simulations of heat and water transport for a variety of soils.

The objective of this study is to evaluate the potential error which can result from moisture migration under the large thermal gradients that are generated during the use of a dual probe method in determining soil thermal properties. This evaluation is performed by applying coupled heat and mass transport theory incorporated into a transient finite element model.

**DESCRIPTION OF DUAL PROBE METHOD**

Dual probe methods for determining soil thermal properties are based on analytical solutions of the transient heat equation for an infinite line source. Heat transfer is assumed to be by conduction only. For the case of a constant heat flux input for a period of time \( t_o \), the solution describing the temperature distribution \( T(r,t) \) (°C) is (de Vries, 1952; Kluitenberg et al., 1993)

\[
T(r,t) = \begin{cases} 
T_1(r,t); & 0 < t \leq t_o \\
T_2(r,t); & t > t_o 
\end{cases}
\]  

[1]

where

\[
T_i(r,t) = -\frac{q\rho c}{4\pi\lambda} Ei\left(\frac{-r^2\rho c}{4\lambda t}\right)
\]  

[2]
\[ T_2(r,t) = \frac{q \rho c}{4\pi\lambda} \left[ E_i \left( \frac{-r^2 \rho c}{4\lambda(t-t_0)} \right) - E_i \left( \frac{-r^2 \rho c}{4\lambda t} \right) \right] \]  \[ [3] \]

and \( T \) is the temperature at a distance from the line source, \( r_s \) (m), and at time, \( t \) (s), \( t_0 \) is the length of time that power is applied to the line-source (s), \( q \) is the line-source strength per unit length per unit time (J m\(^{-1}\) s\(^{-1}\)), \( \lambda \) is the soil thermal conductivity (J m\(^{-1}\) s\(^{-1}\) °C\(^{-1}\)), \( \rho c \) is the soil volumetric heat capacity (J m\(^{-3}\) °C\(^{-1}\)), and \( E_i \) is the exponential integral. The thermal conductivity and volumetric heat capacity are related by the quotient \( \lambda/\rho c \) which defines the thermal diffusivity, \( \kappa \) (m\(^2\) s\(^{-1}\)).

Typical devices for implementing these solutions consist of two parallel probes of length 3 cm and diameter 1 mm. A heating element is installed in one probe and provides a means for introducing energy into the system being measured. The second probe, which is at a rigidly fixed distance 5 to 8 mm from the heating probe, contains a temperature sensing element such as a thermocouple or thermistor. The temperature at this sensing probe is the temperature distribution \( T(r,t) \).

Instrumentation requirements for the method are simple and include a current source for the heating element and a datalogger. The datalogger (1) controls the time of power application to the heating element, (2) monitors the applied power level and (3) monitors the temperature at the sensing probe. Thermal diffusivity, conductivity and volumetric heat capacity can be obtained from Eq. [1] by solving explicitly for these properties (Bristow et al., 1993 and Campbell et al., 1991) or by using nonlinear curve fitting methods.

MODEL DESCRIPTION

Benjamin et al. (1990) developed a two-dimensional model around the one-dimensional pressure-potential-based coupled heat and water transport model of Milly and Eagleson (1980). (1990). The medium is assumed to be nondeforming and isotropic both hydraulically and thermally. Spatial heterogeneity is allowed by using a potential-based system. The Galerkin finite element method as described by Pinder and Gray (1977) is used for the numerical implementation. An overview of the underlying theory of the model is presented here. The reader is referred to the work of Benjamin et al. (1990) and Milly and Eagleson (1980) for numerical implementation and model details. The movement of heat and moisture during application of dual probe methods is evaluated using a modified version of a coupled heat and mass transport model developed by Benjamin et al. The modifications only affect the format of the output data and the underlying model is not changed.
Heat component

The production, storage and transport of thermal energy in a porous medium can be described in one-dimensional form by (Milly, 1982)

\[
\begin{align*}
\left[ C + L \theta \frac{\partial \rho_v}{\partial T} - \left( \rho_1 W + \rho_v L \right) \frac{\partial \theta}{\partial T} \right] \frac{\partial T}{\partial t} \\
+ \left[ L \theta \frac{\partial \rho_v}{\partial \psi} - \left( \rho_1 W + \rho_v L \right) \frac{\partial \psi}{\partial T} \right] \frac{\partial \psi}{\partial t}
\end{align*}
\]

\[= \nabla \cdot \left[ \lambda T + \rho_1 \left( LD_{\psi v} + \gamma TD_{T \psi} \right) \psi \right] - c_1 q_m \cdot \nabla T \tag{4} \]

where \(C\) is the combined volumetric heat capacity of the solids, the liquid water and the water vapor (J m\(^{-3}\) °C), \(L\) is the latent heat of vaporization of water (J kg\(^{-1}\)), \(\theta\) is the volumetric air content (m\(^3\) m\(^{-3}\)), \(\rho_1\) and \(\rho_v\) are the density of liquid and vapor (kg m\(^{-3}\)), respectively, \(W\) is the heat of wetting (J kg\(^{-1}\)), \(\psi\) is the volumetric water content (m\(^3\) m\(^{-3}\)) and \(\psi\) is the matric potential (m). On the right-hand side of Eq. [4], \(D_{\psi v}\) is the transport coefficient for vapor movement under matric potential gradients (m s\(^{-1}\)), \(D_{T \psi}\) is the transport coefficient for movement of adsorbed water under thermal gradients (m\(^2\) s\(^{-1}\)), \(c_l\) is the volumetric heat capacity of the liquid water (J m\(^{-3}\) °C) and \(q_m\) is the water flux (m s\(^{-1}\)).

Each of the bracketed terms on the left-hand side of Eq. [4] contain a storage term and a production term and the right-hand side accounts for the gradient of both sensible and latent heat flux. Thus Eq. [4] has the form of the conservation of heat equation.

The water vapor density is described using an expression from Marshall and Holmes (1979). Both \(\psi\) and \(T\) are variables in the expression which leads to the partial derivative of \(\rho_v\) with respect to both \(\psi\) and \(T\). The temperature dependence of volumetric water content is described by (Milly and Eagleson, 1980)
\[
\frac{\partial \theta}{\partial T} = -\frac{\partial \theta}{\partial \psi} c_\psi \psi \exp[-c_\psi (T - T_0)]
\]  

[5]

where \(c_\psi\) is a constant temperature correction factor (°C\(^{-1}\)), and \(T_0\) is the initial temperature.

For the thermal conductivity value, the method of an apparent thermal conductivity (de Vries, 1966) is used and is given here as

\[
\lambda = 1.25 \left[ \frac{\sum_{i=s,c,o} K_i x_i \lambda_i}{\sum_{i=s,c,o} K_i x_i} \right]
\]

[6]

with

\[
K_i = \frac{1}{3} \sum_{j=1}^{3} \left( 1 + \left[ \left( \frac{\lambda_i}{\lambda_a} \right) - 1 \right] g_j \right)^{-1}
\]

[7]

and \(\lambda_i\) are the thermal conductivity for the soil constituents sand, s, silt and clay, c, and organic matter, o. The terms \(g_i\) are dimensionless shape factors and \(\lambda_a\) is the thermal conductivity of air (J m\(^{-1}\) s\(^{-1}\) °C\(^{-1}\)).

For the specific heat the constituent summation method of de Vries (1966) is used and presented here as

\[
C = x_s c_s + x_c c_c + x_o c_o + c_i \theta
\]

[8]

where \(x_s\), \(x_c\), and \(x_o\) are the volume fractions of sand, silt and clay combined and organic matter, respectively (m\(^3\) m\(^{-3}\)) and \(c_s\), \(c_c\) and \(c_o\) are the respective volumetric heat capacities (J m\(^3\) °C\(^{-1}\)).

Moisture component

Conservation of water mass gives

\[
\frac{\partial}{\partial t} (\rho_l \theta + \rho_v \theta_v) = -\nabla \cdot \mathbf{q}_m
\]

[9]

thus accounting for both liquid and vapor transport. The moisture flux, \(\mathbf{q}_m\), is defined as
where $D_{rv}$ is the diffusion coefficient for vapor flux under a matric potential gradient, $K$ is the hydraulic conductivity and $k$ is a unit vector with direction opposite the force of gravity. Both the heat and the moisture component are strictly correct only for isotropic medium since the conductivity and diffusion term are isotropic.

The hydraulic properties are specified using (van Genuchten, 1980)

\begin{equation}
\theta = \theta_r + \frac{\theta_s - \theta_r}{\left[1 + \left(\frac{\alpha |\psi|}{\theta_r}\right)^n\right]^m} \tag{11}
\end{equation}

\begin{equation}
K(\psi) = \frac{K_s \left\{1 - \left(\frac{\alpha |\psi|}{\theta_r}\right)^n \left[1 + \left(\frac{\alpha |\psi|}{\theta_r}\right)^n\right]^{-m}\right\}^2}{\left[1 + \left(\frac{\alpha |\psi|}{\theta_r}\right)^n\right]^{2m}} \tag{12}
\end{equation}

where $\theta_r$ is the residual water content (m$^3$ m$^{-3}$), $\theta_s$ is the saturated water content (m$^3$ m$^{-3}$), $\alpha$ (m$^{-1}$), $m$ and $n$ (both dimensionless) are empirical constants used in the function describing the relationship between matric potential and volumetric water content, $|\psi|$ is the absolute value of the matric potential (m) and $K_s$ is the saturated hydraulic conductivity (m s$^{-1}$)

SIMULATIONS

The coupled heat and mass model was used to simulate transient thermal and hydraulic behavior in the vicinity of the heater probe for the three hypothetical soils described in Table 1. A range of initial water potentials and applied heater power levels were used for the simulations. The principle of symmetry was used in choosing the finite element grid presented in Fig. 1 which also identifies the conditions of the grid boundaries. Grid node spacing ranged from 0.1 mm near the heat source node to 1.0 mm near the outer boundaries. The overall grid is a 2.5 cm square. In addition to the parameters listed in Table 1, the bulk density, initial temperature and initial water potential are the only required inputs. A 0.5 s time step was chosen which the program automatically reduced if convergence did not occur within a
Table 1. Hypothetical soil characteristics

<table>
<thead>
<tr>
<th>soil</th>
<th>percent sand</th>
<th>percent clay</th>
<th>saturated water content ( (m^3 \ m^{-3}) )</th>
<th>saturated hydraulic conductivity ( (cm \ h^{-1}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>sand</td>
<td>92.0</td>
<td>5.0</td>
<td>0.423</td>
<td>3.3</td>
</tr>
<tr>
<td>loam</td>
<td>37.5</td>
<td>20.0</td>
<td>0.540</td>
<td>1.4</td>
</tr>
<tr>
<td>silt loam</td>
<td>23.0</td>
<td>22.5</td>
<td>0.558</td>
<td>1.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>soil</th>
<th>( \alpha ) ( (cm^{-1}) )</th>
<th>( n )</th>
<th>residual water content ( (m^3 \ m^{-3}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>sand</td>
<td>0.020</td>
<td>3.00</td>
<td>0.033</td>
</tr>
<tr>
<td>loam</td>
<td>0.023</td>
<td>1.46</td>
<td>0.051</td>
</tr>
<tr>
<td>silt loam</td>
<td>0.036</td>
<td>1.65</td>
<td>0.182</td>
</tr>
</tbody>
</table>
Figure 1. General configuration of simulation grid with boundary conditions. The dashed borders on the right side and bottom indicate that the grid extends in these directions but element size remains the same.
prescribed number of iterations. The automatic time step reduction was only necessary under
drier soil conditions when large temperature gradients resulted from large heat inputs. The time
for simulating power applied to the heating probe \( t_0 \) in Eq. (1) was 10 s. Figures 2 and 3
present test soil hydraulic and thermal characteristics over the range of water contents.

RESULTS AND DISCUSSION

Figure 4 was generated using Eq. (1) with \( t_0 = 10 \) s and a thermal diffusivity value of
10^{-7} \text{ m}^2 \text{ s}^{-1} to show the family of temperature distributions for a range of distances from the line
heat-source. The temperature distribution profile for the distance of 5.0 mm from the line
source is similar to the distribution seen at the sensing probe when the dual probe method is
used. It is interesting to note the very large rate of temperature change at locations near the line
source both when power is first applied and when it is removed. During the time that power is
applied, the temperature gradient between distances 0.1 mm and 0.2 mm quickly exceeds 100 °C
\text{ cm}^{-1}; however, immediately upon removal of power from the line heat-source the gradient
rapidly drops to less than 1 °C \text{ cm}^{-1} and converges to 0 within 30 s.

The large temperature gradients in the vicinity of the line heat-source are conducive to
water migration in both the liquid and the vapor form. Under the conditions encountered with
the use of a dual probe device, i.e. large thermal gradients for short periods of time, water
movement near the line source under the thermal gradients will predominately be in vapor
phase. In addition to the vapor pressure gradient which results from the thermal gradients, the
amount of vapor transport is dependent on the availability of pore space through which the
vapor can move. The transport coefficient associated with vapor movement under thermal
gradient, \( D_{vt} \), is dependent on degree of saturation and has been shown to be minimum under
both very dry conditions because there is little water to vaporize and under very wet conditions
because there is no pore space for vapor movement. The water content at which a maximum
value occurs is dependent on the soil texture and structure.

Figures 5, 6 and 7 present the results of simulations performed on three different soil
types for a range of initial matric potential values. A summary of the simulation conditions and
results is presented in Table 2. The amount of power applied at the line heat-source for the
simulations was chosen to give a temperature increase of 1.0 °C at a distance 5.0 mm from the
line source. These values represent typical conditions present during actual soil measurements
and provide a standard for comparison among the range of conditions. It has been found
during laboratory measurements that a minimum increase in temperature at the sensing probe of
0.5 °C is needed to provide sufficient temperature measurement resolution. This minimum
Figure 2. Hydraulic properties of soils used in simulations
Figure 3. Thermal properties of soils used in simulations
Figure 4. Temperature response for range of distances (mm) from line source for $t_0 = 10\ s$
Figure 5. Water content change over time in sand soil at the line heat-source (0.0 mm) and a range of distances away from the line heat-source for initial tension value of 100 cm.
Figure 6. Water content change over time in loam soil at the line heat-source (0.0 mm) and a range of distances away from the line heat-source for initial tension values of 100 cm, 200 cm and 500 cm
Figure 7. Water content change over time in silt loam soil at the line heat-source (0.0 mm) and a range of distances away from the line heat-source for initial tension values of 100 cm and 200 cm.
### Table 2. Initial water conditions, relative applied power level and temperature response for simulations

| Soil Texture | $|\psi_0|$ (cm) | $\theta(\psi_0)$ (m$^3$ m$^{-3}$) | $*q_r$ | $T_{m(0)}$ (°C) | $t_m$ (°C) |
|--------------|-------------|-------------------------------|--------|-----------------|----------|
| Sand         | 10          | 0.421                         | 0.939  | 57.6            | 22.0     |
|              | 25          | 0.393                         | 0.900  | 57.4            | 21.5     |
|              | 50          | 0.278                         | 0.747  | 55.5            | 20.5     |
|              | 100         | 0.123                         | 0.538  | 53.1            | 19.5     |
| Loam         | 10          | 0.523                         | 0.996  | 67.2            | 26.0     |
|              | 25          | 0.487                         | 0.942  | 66.9            | 26.5     |
|              | 50          | 0.432                         | 0.868  | 66.6            | 26.5     |
|              | 100         | 0.359                         | 0.741  | 63.3            | 25.5     |
|              | 200         | 0.287                         | 0.676  | 63.8            | 24.5     |
|              | 500         | 0.209                         | 0.573  | 63.8            | 23.5     |
| Silt Loam    | 10          | 0.534                         | 1.000  | 69.0            | 27.0     |
|              | 25          | 0.479                         | 0.925  | 68.8            | 27.0     |
|              | 50          | 0.410                         | 0.828  | 67.8            | 26.5     |
|              | 100         | 0.340                         | 0.734  | 67.0            | 26.0     |
|              | 200         | 0.286                         | 0.680  | 67.5            | 25.0     |

$\psi_0 =$ initial soil matric potential.

$*q_r =$ relative line heat source applied power level. Silt loam at 10 cm = 1.0.

$T_{m(0)} =$ maximum temperature at line heat source. Initial temperature = 20 °C.

$t_m =$ time of maximum temperature increase at $r = 5.0$ mm.
resolution is needed to accurately calculate the thermal properties. Simulations were performed on all three soils for initial matric potential values of -10 cm, -25 cm, -50 cm, -100 cm, -200 cm and -500 cm. The results are not presented for the cases when water content at the line heat-source changed by less than 0.01 m$^3$ m$^{-3}$. In these cases with insignificant mass transport, the temperature change throughout the soil system is adequately described using conduction heat transfer theory. Initial soil temperature is 20 °C in all cases.

At tensions of 50 cm and less, over 65 percent of the pore space in the sand soil is occupied with water and heat flux away from the line heat-source is predominately by conduction heat transfer. The large quartz content of the sand in combination with the large water content under these conditions results in relatively large thermal conductivity. When the initial tension is 100 cm the water content is 12.3 percent and the movement of vapor is plausible. Figure 5 shows a small increase in water content at the distances 0.14 mm, 0.2 mm and 0.4 mm during the first 2 s while water is moving in a wave-like fashion in the direction away from the line source. It is noted here that this is a radial system and the soil volumes encompassed by $0.0 \leq r \leq 0.14$ and $0.14 \leq r \leq 0.2$ are approximately equal and the volume defined by $0.2 \text{ mm} < r < 0.4 \text{ mm}$ is approximately five times greater than that defined by $0.14 \text{ mm} < r < 0.2 \text{ mm}$.

An evaluation of the water movement is aided by considering Eq. [10]. The gravity term (third term on the right-hand side) can be ignored for the short time periods considered here. A slightly different form of Eq. [10] and neglecting gravity flow is presented in Eq. [13].

$$q_m = K \nabla \psi + D_{\psi v} \nabla \psi + D_{\psi v} \nabla T$$\hspace{1cm}[13]$$

The isothermal vapor diffusivity and the thermal diffusivity terms are further expanded as

$$D_{\psi v} = \frac{D_a}{\rho_l} \tau \theta_a \frac{\partial \rho_v}{\partial \psi}$$ \hspace{1cm}[14]$$

with $D_a$ the molecular diffusion coefficient of water vapor in air (m$^2$ s$^{-1}$) and $\tau$ the tortuosity of the air-filled pore space and
\[
D_{TV} = \frac{D_n}{\rho_1} f \xi \frac{\partial \rho_v}{\partial T}
\]

with \(f\) a factor derived by Philip and de Vries (1957) accounting for vapor flow in the presence of liquid islands in the pore space and \(\xi\) is a correction also derived by Philip and de Vries (1957) to account for thermal gradients being larger across individual pores than on the macroscopic scale.

The total water flux occurs in response to both thermal gradients as vapor flux and in response to the matric potential gradients in both the liquid and vapor states. The thermal gradients are a result of the heat flux into the system at the line source. Matric potential gradients develop because vapor moves by thermal gradients and condenses to liquid in cooler soil. The direction of the thermal and matric potential gradients is opposite for \(t \leq t_0\). The simulated thermal gradients reach a maximum value within the first two to three seconds at locations near the line heat source (see Fig. 4). The thermal vapor diffusivity term continues to increase as water moves out of the pores nearer the line source and the value of \(f\) increases. As the water is moving away from the line source, matric potential gradients in the direction toward the line source are generated and the isothermal vapor diffusivity term increases though the hydraulic conductivity decreases.

When power is removed, water immediately moves in the direction back toward the line source principally in response to matric potential gradients since the thermal gradients rapidly dissipate. For the distances 0.2 mm and 0.4 mm from the line source, even though the water has decreased over the time that power is applied to the line source, it continues to decrease immediately after power is removed to supply water to the still drier regions closer to the line source. The thermal gradients rapidly dissipate when no heat flux input is present at the line source and water movement is now due to matric potential gradients only. For \(t \leq t_0\) the direction of the thermal and matric potential gradients are opposing. For \(t > t_0\) the gradients for water flux are essentially unidirectional and the movement toward the line source is more rapid than when movement is in the direction away from it. The system reaches near re-equilibration after only 10 s.

In order to evaluate the behavior of the sand at different initial water contents, a simulation was performed at an initial tension of 150 cm using the same applied power level as used at 100 cm. A tension of 150 cm corresponds to a water content of 0.075 m^3 m^-3. Figure 8 shows a greater amount of water movement during and after application of power when
Figure 8. Water content change over time in sand soil at the line heat-source (0.0 mm) and a range of distances away from the line heat-source for initial tension value of 150 cm. Applied power level is same as in Fig. 5.
compared to an initial tension of 100 cm. The lower water content resulted in larger temperature gradients because of the reduced thermal conductivity and resulted in more void pore space for vapor movement. The maximum temperature at the line source was 6 °C greater at the higher tension. The total water flux both during and after power application and the total time to reach re-equilibration was greater for the drier sand. In both cases there was nearly complete water content re-equilibration after only 10 s.

The porosity and pore size distribution of the loam soil result in more water retained for a given matric potential than the sand. As in the case for the sand, there is no significant water movement at initial matric potentials above -100 cm. The higher water content for the loam means a higher heat capacity value, and the amount of power required to cause a 1.0 °C temperature increase at 5.0 mm from the line source and the temperature increase at the line source is greater.

The total amount of water that moves in the loam soil increases as initial matric potential decreases (Fig. 6) even though the applied power level decreases and the thermal gradients are similar. Referring to Eq. [15], the value of \( f \) increases as matric potential decreases, but the value of the partial derivative of vapor density with temperature decreases by a greater amount. Thus the thermal vapor diffusivity term cannot be responsible for the increase in water flux. Both the matric potential gradient and the isothermal vapor diffusivity terms are greater at lower matric potentials meaning that the reduction in hydraulic conductivity has a significant effect on movement for \( t < t_0 \). It takes longer for the soil water in the loam to re-equilibrate when the initial water content is lower because there is more total water movement.

The coincidence of the water content for the tension of 200 cm in the loam and the silt loam provides an opportunity to evaluate the effect of hydraulic properties on the movement of water in the dual probe method. The applied power level is the same for both the loam and the silt loam at 200 cm, the thermal gradients are only slightly higher for the silt loam and the thermal properties are similar. There is substantially greater water movement in the silt loam, and an examination of the parameters describing the two soils shows the difference lies in the hydraulic characteristics. These characteristics are quantified by the parameters \( \alpha, n, \) and \( \theta_r \). These parameter values are higher for the silt loam. Simulations not presented here show an inverse relationship between the value of the residual water content and the amount of water which moves. However, as the values of both \( \alpha \) and \( n \) increase, the result is more water movement. These hydraulic parameters define both the water retention and the hydraulic conductivity properties. This behavior demonstrates the sensitivity of thermal and isothermal mass transport to hydraulic properties.
The *q_r value presented in Table 2 is the relative amount of power which must be applied at the line heat-source to cause a 1.0 °C increase in the temperature at the sensing probe 5.0 mm from the line source. The silt loam soil at a tension of 10 cm required the greatest amount of power to get a 1.0 °C increase at 5.0 mm from the line source and is given a *q_r value of 1.0. The high values of *q_r for the silt loam along with the corresponding highest T_m values and the lowest t_m values are a result of the relatively low thermal diffusivity of the silt loam. The lower quartz composition of this soil also contributes to this effect.

The model results describing water movement under thermal and matric potential gradient indicate that there is significant water movement under relatively dry conditions, but this movement is limited to locations very near the line heat-source. Specifically of interest in this study is the effect which water movement has on the temperature response at the position of the sensing probe. It is this temperature which is used to calculate the thermal properties. If the movement of water in the region between the line heat-source and the sensing probe causes heat flux behavior to deviate significantly from that of a simple conduction heat transfer system, the use of conduction heat transfer to determine thermal properties is not valid.

In order to evaluate the effect of water movement on the temperature distribution, the model was modified by setting the \( VT \) term in Eq. [10] equal to zero. This removes all water movement under thermal gradient and forces the model to describe the problem in terms of heat transfer by conduction only. This modified version of the model was validated by comparing the generated temperature distribution to the analytical solution for the heat equation (Eq. [1]). As in the evaluation of water movement, the level of applied power in these comparison simulations is that amount which results in a 1.0 °C increase at a distance 5.0 mm from the line heat-source. The results, Figs. 9, 10 and 11, are presented as the difference in the temperature as determined using the complete model and the temperature using conduction heat transfer \( (VT = 0) \) only.

There is essentially no water movement in the sand when the initial matric potential is greater than -100 cm, and, under these conditions, there is no difference in the temperature distribution as determined using the coupled heat and mass model and the conduction heat transfer model. At an initial matric potential of -100 cm in the sand soil, Fig. 5 indicates that water immediately moves away from the line heat-source. Fig. 9 shows that this reduces the thermal conductivity at the line source, the heat moves less efficiently away from the line source, and the temperature increases more rapidly than under conduction only heat transfer. It should be noted that after 10 s of power application to the line heat-source the temperature at the line source is only 0.5 °C higher than in the case of purely conduction heat transfer. At very early
Figure 9. Difference in temperature distribution in sand soil using complete couple heat and mass model ($T_{\text{comp}}$) and conduction heat transfer only model ($T_{\text{cond}}$) at 100 cm initial tension.
Figure 10. Difference in temperature distribution in loam soil using complete couple heat and mass model ($T_{\text{comp}}$) and conduction heat transfer only model ($T_{\text{cond}}$) at 100 cm, 200 cm and 500 cm initial tension.
Figure 11. Difference in temperature distribution in silt loam soil using complete couple heat and mass model (Tcomp) and conduction heat transfer only model (Tcond) at 100 cm, 200 cm initial tension
times during power application, the reduction in heat flux in the direction away from the line source when thermal gradient induced water flux occurs results in the temperature for conduction only temperature distribution being slightly greater. Then as water continues to be forced away from the line source, the temperature distribution under the coupled heat and mass conditions is greater though only by a small amount.

A comparison of the temperature distribution of the loam (Fig. 10) and the sand soils at an initial tension of 100 cm shows similar differences between the two models. However, the applied power to the loam soil to achieve a 1.0 °C temperature increase at 5.0 mm from the line source is 1.44 times that used for the sand. Though approximately the same amount of water moves away from the line source in both soils, the higher actual water content of the loam at a tension of 100 cm keeps the thermal conductivity higher. In addition to the higher thermal conductivity value of the loam, the greater heat flux is enhanced by a larger thermal gradient in the vicinity of the line source.

Under drier conditions of the loam the differences between the model results increase both at the line source and at distances away from the line source even though the amount of power applied at the line heat-source is less with drier soils. For the loam soil at an initial tension of 500 cm there is a delay before any difference between the models occurs at the distances 0.2 mm and 0.4 mm. This delay corresponds with the time that the water content at these positions begins to decrease. Both models show the same temperature distributions at these positions until the water content drops below the initial water content. The initial water content is the water content seen by the conduction-only model for all times.

At a tension of 100 cm the water content of both the loam and the silt loam are similar. Though the amount of power applied to the silt loam is about 5 percent less than that applied to the loam, Fig. 11 shows that the difference in model results is significant. Since the bulk densities of the two soils are also similar, the difference demonstrates the contribution of the quartz mineral component to a higher thermal conductivity.

The preceding discussions describe the water and temperature behavior in the vicinity of a line heat-source. The temperature results clearly show that the difference in system behavior when described using coupled heat and mass transport as compared to purely conduction heat transfer rapidly abates at distances farther from the line source. The use of these models over a range of soil types and conditions under the restriction that line heat-source temperature does not exceed 80 °C indicates that (1) there is no water migration under thermal gradient at locations greater than 1.50 mm from the line source, and (2) water migration does not affect the temperature distribution at locations greater than 2.0 mm from the line source. Therefore, the
use of these models indicates that conduction heat transfer is appropriate for describing the temperature distribution when using dual probe devices under the conditions presented in this work.

REFERENCES


GENERAL SUMMARY

A dual probe method for simultaneously determining soil volumetric heat capacity, thermal conductivity and, consequently, thermal diffusivity was developed and evaluated in this work. The theoretical basis for the method is the analytical solution to the conduction heat equation for an infinite line source in the pulsed mode. This theory is implemented in a measurement device consisting of two parallel probes rigidly mounted to a base. One probe contains a heating element and serves as a heater probe to simulate a line source. The other probe contains a thermocouple for measuring the temperature response over time at the probe spacing distance. The small size of the probes results in insignificant disturbance to the soil system being measured. There are no special instrumentation requirements which makes the method simple and inexpensive.

Several concerns surfaced during preliminary studies including (1) the validity of using cylindrical probes of finite length to simulate an infinite line heat-source, (2) convective heat transfer stemming from thermal gradient imposed moisture migration and (3) contact resistance between the probe surface and the media being measured. These concerns were addressed in this work by subjecting the method to materials of known or independently determined thermal properties which contained no void pore space. The materials used were an aqueous colloidal suspension of aluminum oxide and an aqueous solution of glycerol. Comparisons between measured temperature and temperature described by the analytical solution using the known thermal properties indicate that the assumptions associated by implementing the theory with the dual probe device are valid. Application of the method to a saturated sand also provided good results when estimated thermal properties from the dual probe method were compared to the properties estimated by the de Vries (1966) method.

Measurements on a sand and a loam were taken over a range of water contents. A rigorous error analysis which accounted for both model error and measurement error was applied to the measurements. Thermal properties were calculated from the measurements using the analytical solution explicitly expressed in terms of volumetric heat capacity and thermal diffusivity and using nonlinear least-squares curve fitting. For the explicit calculation method, both model and measurement error increased with thermal diffusivity. Volumetric heat capacity was found to be most sensitive to uncertainties in probe spacing while thermal conductivity was most sensitive to the resolution in determining the time when the temperature at the sensing probe reaches the maximum value. When curve fitting was used, the probe spacing error contributed most to total error. By carefully choosing the probe physical arrangement and measurement parameters, the errors associated with the model and measurement parameters can
be reduced to below a few percent even on soils with high diffusivity values. Volumetric heat capacity and thermal conductivity calculated using both methods were compared to estimated values using the method of de Vries (1966). Comparison was very good throughout most of the water content range with some deviation for the driest conditions.

The dual probe methods are based on an analytical solution of the heat equation which assumes heat transfer is by conduction only. When power is applied to the heater probe, large thermal gradients can result and these gradients are conducive for moisture migration which is convective heat transfer. This convection is a source of error in the method.

The consequences of convective heat transfer were evaluated by applying coupled heat and mass transport theory to the problem. A transient finite element numerical model was used to evaluate the problem for a range of applied power levels, soil mineral compositions and initial water contents. Thermal gradients in the immediate vicinity of the heater probe were greatest for a dry sand. Moisture migration occurred only in the region near the heater probe but never beyond 1.5 mm from the probe. A comparison of the temperature over time at the location of the sensing probe was performed using the numerical model and the conduction analytical solution. The comparison indicates that the convective heat transfer component is not a factor when temperature at the heater probe does not exceed 80 °C.

The work presented in this dissertation indicates that dual probe methods which implement analytical solutions of the conduction heat equation are useful for determining volumetric heat capacity, thermal conductivity and, consequently, thermal diffusivity. Since heat capacity is linearly related to water content, the information from the dual probe measurements can also be used to determine water content. Use of the probe device causes minimal disturbance to the system of interest because of the small probe size. The method is inexpensive to use due to the simplicity of the probe design and the use of commonly available instrumentation devices.
ADDITIONAL REFERENCES


ACKNOWLEDGMENTS

This dissertation is the result of a team effort, and I wish to acknowledge the other members of the team. I have received support in ways which cannot be described here but are familiar to those lucky enough to have a family like mine. My wife, Jan, and three daughters, Jenny, Emily and Karli, have been a constant source of love and support throughout this challenging endeavor. We have all grown so much during this time.

My friend and major advisor, Dr. Bob Horton, made my scholastic pursuit at Iowa State University possible through his generous support, insightful evaluations of my work and constant motivation. Our friendship has continuously grown throughout my graduate studies.

I wish to thank Dr. Peter Wierenga for the inspiration, motivation and thoughtful advice. He sparked my early interests in soil physics, and I am fortunate to have Peter as a friend and colleague.

Dr. Keith Bristow and Dr. Gerard Kluitenberg were always so free with assistance to me as I needed it, and I am grateful. Their unselfish contributions ring throughout this dissertation.