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Robust optimization vs. stochastic programming incorporating risk measures for unit commitment with uncertain variable renewable generation

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Keywords
Unit commitment, Renewable energy, Stochastic programming, Robust optimization, CVaR

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Robust Optimization vs. Stochastic Programming Incorporating Risk Measures for Unit Commitment with Uncertain Variable Renewable Generation

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Abstract Unit commitment seeks the most cost effective generator commitment schedule for an electric power system to meet net load, defined as the difference between the load and the output of renewable generation, while satisfying the operational constraints on transmission system and generation resources. Stochastic programming and robust optimization are the most widely studied approaches for unit commitment under net load uncertainty. We incorporate risk considerations in these approaches and investigate their comparative performance for a multi-bus power system in terms of economic efficiency as well as the risk associated with the commitment decisions. We explicitly account for risk, via conditional value at risk (CVaR) in the stochastic programming objective function, and by employing a CVaR-based uncertainty set in the robust optimization formulation. The numerical results indicate that the stochastic program with CVaR evaluated in a low-probability tail is able to achieve better cost-risk trade-offs than the robust formulation with less conservative preferences. The CVaR-based uncertainty set with the most conservative parameter settings outperforms an uncertainty set based only on ranges.

Keywords unit commitment · renewable energy · stochastic programming · robust optimization · CVaR

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1 Introduction

Unit commitment (UC), one of the most important tasks in electric power system operations, is an optimization problem to make the most cost-effective thermal generator commitment decisions for the system to meet forecast net load while satisfying operational constraints on the transmission system and generation resources [45]. As electricity generation from renewable resources increases, unit commitment faces challenges due to the high level of uncertainty in variable renewable resources such as wind power.

A common remedy to manage the variability and uncertainty in UC is to increase operating reserves [29]. Imposing reserve constraints on the UC problem, however, increases the total operating cost, and does not explicitly capture the uncertainty.

Two approaches for optimizing under uncertainty that have received substantial theoretical development – stochastic programming and robust optimization – have been applied in this context. Although several hybrid methods have also been devised for unit commitment, in this paper we focus on the capability of methods based purely on either probabilistic scenarios or uncertainty sets to control both the cost and the risk associated with day-ahead scheduling in the presence of uncertain variable renewable generation. We include a risk measure in the stochastic programming formulation and compare the results for two different formulations of the uncertainty set for robust optimization. Numerical results based on out-of-sample simulation suggest that the robust formulation with a "data-driven" uncertainty set provides an efficient cost/risk tradeoff if higher levels of risk are acceptable but the stochastic programming formulation minimizing expected cost in the very low probability upper tail dominates if risk is less tolerable.

Literature reviews of stochastic optimization based unit commitment have been recently done by Zheng et al. [50] and Tahanan et al. [38]. Stochastic unit commitment (SUC), which has been widely studied [8, 36, 39, 40], formulates the problem as a two-stage optimization problem using probabilistic scenarios. In the first stage, unit commitment decisions specify the binary status of generators to minimize start-up and shut-down costs as well as the expected cost of the second stage decisions. The second stage decisions on the dispatch of each generator committed in the first stage are then made for each scenario. To cope with the computational difficulties caused by a large number of scenarios, scenario reduction techniques are used frequently [15, 18]. Benders decomposition [41] and progressive hedging [11, 14] are two methods to efficiently solve the SUC with a two-stage structure.

Robust unit commitment (RUC) has also been studied extensively [7, 21, 22, 25, 26]. The unit commitment solutions of the RUC are immunized against all possible realizations of uncertainty in a given set. It provides a first stage commitment decision and a second stage dispatch decision while minimizing the dispatch cost under the worst-case realization [7]. Typically, the RUC is also presented in a two-stage formulation; however, there are also multi-stage formulations [26] and other variations [4] in the literature. Since RUC has a two-stage structure, it can be solved using Benders decomposition approaches [7].

Alternative stochastic optimization formulations and hybrid approaches have also been studied. The Interval UC [13,32,37,43], like RUC, is a scenario based approach that provides a solution by minimizing the cost of central net load forecast while keeping the lower and upper bounds feasible. It also guarantees the feasibility of transitions from lower to upper bound, and vice versa. Among the hybrid approaches, a unified stochastic and robust UC formulation has been extended in [48] to produce less conservative schedules than the RUC does.

Besides stochastic and uncertainty set-based robust methods, distributionally robust optimization (DRO) is a recently-proposed approach for UC with uncertain electricity supply. In this method, the uncertain parameter is represented as a random variable whose distribution is not known with certainty. A family of distributions may be specified by characteristics of its moments [16, 47] or a fixed discrete support with a bound on the distances among probability vectors [20, 49]. Two-stage formulations are proposed to optimize the expected cost with respect to the worst distribution in the specified family. Thus, DRO models inoculate solutions, on average, against missing or unreliable information on the range and likelihood of values for the uncertain parameters. In contrast, the focus of robust optimization or stochastic programming with a risk measure, respectively, is on optimizing against the worst cases of an uncertainty set or distribution that can be estimated reliably. Which approach is more appropriate depends on the decision makers' ability to use historical data to estimate descriptions of the uncertainty that will be valid for the study horizon, as well as on their level of risk aversion with respect to worst cases. There are ample and increasing amounts of data on wind energy. Electric power system operators are highly motivated to satisfy demand, even when extreme events occur. These observations motivate our focus on risk aversion, where risk can be reliably assessed as in SUC or RUC, rather than epistemic uncertainty about average performance as in DRO models for UC.

The concern of scenarios involving extremely rare events which lead to very costly solutions justifies using risk measures in stochastic UC models. In the literature, a common approach to considering risk is imposing chance constraints, which is equivalent to bounding the Value at Risk (VaR) of the loss. Chance-constrained UC models are used to find commitment schedules that are able to satisfy the power demand of the system with a user-defined reliability level [27, 28, 31, 33]. Wang et al. [42], however, proposed a UC model that includes features of both the two-stage stochastic program and the chance-constrained stochastic program. Conditional Value at Risk (CVaR) [34, 35] is a computationally superior measure for risk when compared to chance constraints. Chance-constrained models require extra binary variables, while CVaR can be formulated by more computationally tractable continuous variables. Huang et al. [19] present a two-stage stochastic UC model using CVaR in the constraints. Their model, however, requires a pre-defined maximum tolerable value for CVaR. Incorporating CVaR in the objective function overcomes this drawback. Bukhsh et al. [9] applied CVaR in this way to evaluate the risk associated with the mis-estimation of renewable energy. Asensio and Contreras [5] also optimize a weighted combination of expected cost and CVaR of the cost in order to underline the balance between risk and expected costs. We consider the combination of the expected cost and CVaR of the penalty cost in the second stage of the objective function in order to incorporate the risk associated with the possible shortages enforced in the system. In our RUC formulation, beside the uncertainty set based on ranges, we construct an uncertainty set based on historical data corresponding to CVaR.

There is an ongoing and lively debate in the UC literature to criticize or advocate different approaches of modeling the UC problem. Which approach provides the best schedules is an interesting question. Van Ackooij [3] compares four stochastic methods in unit commitment including probabilistically constrained programming, robust optimization and two-stage stochastic and robust programming focusing on computational aspects as well as flexibility and robustness. Pandzic et al. [32] compared their improved version of interval optimization for unit commitment with existing methods in terms of robustness and cost. Wu et al. [46] compared applications of scenario-based and interval optimization approaches to stochastic security-constrained unit commitment. They found that SUC produces less conservative schedules than the Interval UC but requires more computing resources. However, Cheung et al. [11] demonstrated that SUC instances of realistic scale could be solved within minutes using decomposition on an inexpensive computing cluster. We do not claim to resolve the debate but choose to focus on SUC and RUC in more detail and specifically in their worst case performance. As shortages in the unit commitment problem can result in costly consequences, we emphasize the use of risk measures to plan for the extremely rare events that lead to these consequences.

The contributions of this paper are summarized as follows. First, we present a stochastic programming formulation of UC in which we consider the combination of the expected cost and the CVaR of the penalty cost in the second stage in order to incorporate the risk associated with the possible shortages. We refer to this problem as SUC-CVaR. Second, for the robust unit commitment, referred to as RUC, we investigate two formulations of the uncertainty set over which the net load may vary. One formulation of the uncertainty set is defined by a lower bound and an upper bound with a budget of uncertainty [7]. The second set is constructed as a combination of historical scenarios using a data driven approach [6] that is related to CVaR. The performance of these methods is assessed in out-of-sample simulation. Because the results of all approaches depend strongly on the risk parameter used, we also provide insights on how the choice of the parameter value affects the tradeoff between cost and risk. We focus numerical comparisons of results on the worst case performance. As a contribution on the solution method, a branch and cut algorithm is adopted to improve the computational efficiency of Benders decomposition for RUC.

This paper is organized as follows. In Section 2, we introduce the mathematical model along with the definitions of sets, parameters and variables. We explain scenarios and uncertainty sets in Section 3. Numerical experiments and simulation are presented in Section 4 in which we make some comparisons among the different formulations and uncertainty sets. Finally, we conclude this paper in Section 5.

2 Mathematical Models

We consider the unit commitment problem for a multi-bus power system based on the formulation presented in [10]. Two different approaches are applied to model the uncertainty of net load, which equals load less the available variable renewable generation. The first approach is two-stage stochastic programming considering risk. In this approach, the unit commitment decisions are made in the first stage before the uncertain parameter values are realized, and the economic dispatch amounts are then determined in the second stage for each scenario. In other words, in the first stage, we decide which units must be on in each period of time, and in the second stage, the dispatch decisions on power flows are made based on the net load values. The objective function is to minimize first stage costs plus the CVaR of the second stage costs. The second approach is robust optimization. We applied the structure introduced in [7] for the robust optimization model. The objective function has two parts, reflecting the two-stage nature of the decision. The first part is the commitment cost and the second part is the worst case second-stage dispatch cost.

2.1 Sets, Parameters, and Decision Variables

Sets:
$\mathcal{B}$: Set of buses
$\mathcal{L} \subset \mathcal{B} \times \mathcal{B}$: Set of transmission lines
$\mathcal{L}_O(b)$: Set of lines from bus b
$\mathcal{L}_I(b)$: Set of lines to bus b
$\mathcal{G}$: Set of thermal generators
$\mathcal{G}(b) \subset \mathcal{G}$: Set of generators at bus $b \in \mathcal{B}$
K: Set of indices of the time periods.
$\mathcal{I}_j$: Set of time intervals of stairwise startup function of thermal unit j
$\mathcal{S}$: Set of scenarios
$\mathcal{U}$: Generic uncertainty set of the net loads
$\mathcal{U}_1$: Periodwise box uncertainty set of the net loads
$\mathcal{U}_2$: Data-driven uncertainty set of the net loads
Parameters:
$d_{bks}(d_{bk}(u))$: Net load at bus $b \in \mathcal{B}$ in period $k \in K$ for scenario $s \in \mathcal{S}$ (uncertainty set member $u \in \mathcal{U}$) (MW)
π_s: Probability of scenario s

RE_ℓ: Reactance of line $\ell \in \mathcal{L}$ (ohm)
TL_ℓ: Thermal limit (capacity bound) for line $\ell \in \mathcal{L}$ (MW)
$\underline{P}_j$: Minimum power output of unit $j \in \mathcal{G}$ (MW)
$\overline{P}_j$: Maximum power output of unit $j \in \mathcal{G}$ (MW)
RD_j: Ramp-down limit of unit j (MW/h)
RU_j: Ramp-up limit of unit j (MW/h)
SD_j: Shut-down ramp limit of unit j (MW/h)
SU_j: Start-up ramp limit of unit j (MW/h)
DT_j: Minimum down-time of unit j (h)
UT_j: Minimum up-time of unit j (h)
SDC_j : Shut-down cost coefficient of unit j (\$)
$v_j(0)$: Unit j's on/off status at time 0 (initial condition) (0/1)
$\hat{v}_j(0)$: Unit j's down-time/up-time status at time 0 (0/1)
$p_j(0)$: Power output of unit j in period 0 (initial condition) (MW)
$a^i_j, i = 1, \ldots, n$: The slope of the ith segment of piecewise linear total production cost function of unit j
$b^i_j, i = 1, \ldots, n$: The intercept of the ith segment of piecewise linear total production cost function of unit j
$h^i_j, i = 1, \ldots, n-1$: Breakpoint between the ith and $(i+1)$st segment of piecewise linear total production cost function of unit j
Λ_1, Λ_2: Penalty weights for power balance slack variables (\$/MWh)
$\varrho_j(i)$: Start-up cost of unit j if started up in time interval i
a_j: No-load cost of unit j (\$)
B_ℓ: Inverse of (non-zero) reactance on line $\ell \in \mathcal{L}$ (mho)
ITO_j: Number of time periods unit j must be online initially;
$ITO_j = \min(|K|, \max(0, \texttt{round}((UT_j - \hat{v}_j(0))/\tau)))$ (number of time periods)
ITF_j: Number of time periods unit j must be offline initially;
$ITF_j = \min(|K|, \max(0, \texttt{round}((DT_j + \hat{v}_j(0))/\tau)))$ (number of time periods)
γ: Tail probability parameter of CVaR term
Δ: Budget of uncertainty in uncertainty set $\mathcal{U}_1$
α: Tail probability parameter of uncertainty set $\mathcal{U}_2$

First Stage Decision Variables:
v_{jk}: Binary variable: equals 1 if unit j is online in period k and 0 otherwise
c^u_{jk}: Start-up cost incurred by unit j in period k (\$)
c^d_{jk}: Shut-down cost incurred by unit j in period k (\$)

Second Stage Decision Variables:
$p_{jks}(p_{jk}(u))$: Power output of unit j in period k for scenario $s \in \mathcal{S}$ (uncertainty set member $u \in \mathcal{U}$) (MW)
$\overline{p}_{jks}(\overline{p}_{jk}(u))$: Maximum available power output of unit j in period k for scenario s (uncertainty set member $u \in \mathcal{U}$) (MW)
$\theta_{bks}(\theta_{bk}(u))$: Phase angle for bus b during time period k for scenario s (uncertainty set member $u \in \mathcal{U}$) (radians)
$w_{\ell ks}(w_{\ell k}(u))$: Line power for line $\ell \in \mathcal{L}$ in time period k for scenario $s \in \mathcal{S}$ (uncertainty set member $u \in \mathcal{U}$) (MW)
$c^P_{jks}(c^P_{jk}(u))$: Total production cost of unit j in period k for scenario s (uncer-

tainty set member $u \in \mathcal{U}$) (\$)
η: Auxiliary decision variable, equal to Value at Risk, to linearize CVaR
ξ_s: Dispatch (production plus penalty) cost for scenario s (\$)
r_s: Deviation of dispatch cost in scenario s above the Value at Risk
$\alpha^+_{bks}, \alpha^-_{bks}(\alpha^+_{bk}(u), \alpha^-_{bk}(u))$: Power balance slack variables at bus b in period k for scenario s (uncertainty set member $u \in \mathcal{U}$) (MW)

2.2 Constraints

As in [10], the constraints include UC constraints and non-UC constraints. The UC constraints; i.e., those with only UC variables, include minimum up-time and down-time constraints as well as start-up constraints:

- **Minimum up-time constraints:**

$$\sum_{k=1}^{ITO_j} [1 - v_{jk}] = 0, \quad \forall j \in \mathcal{G} \tag{1}$$

$$\sum_{n=k}^{k+UT_j-1} v_{jn} \geq UT_j [v_{jk} - v_{j,k-1}],$$
$$\forall j \in \mathcal{G}, \forall k = ITO_j + 1, \ldots, |K| - UT_j + 1 \tag{2}$$

$$\sum_{n=k}^{|K|} \{v_{jn} - [v_{jk} - v_{j,k-1}]\} \geq 0,$$
$$\forall j \in \mathcal{G}, \forall k = |K| - UT_j + 2, \ldots, |K| \tag{3}$$

- **Minimum down-time constraints:**

$$\sum_{k=1}^{IFT_j} v_{jk} = 0, \qquad \forall j \in \mathcal{G} \tag{4}$$

$$\sum_{n=k}^{k+DT_j-1} [1 - v_{jn}] \geq DT_j [v_{j,k-1} - v_{jk}],$$
$$\forall j \in \mathcal{G}, \forall k = ITF_j + 1, \ldots, |K| - DT_j + 1 \tag{5}$$

$$\sum_{n=k}^{|K|} \{1 - v_{jn} - [v_{jk} - v_{j,k-1}]\} \geq 0,$$
$$\forall j \in \mathcal{G}, \forall k = |K| - DT_j + 2, \ldots, |K| \tag{6}$$

- **Start-up costs:**

$$c^u_{jk} \geq \varrho_j(i) \left(v_{jk} - \sum_{m=1}^{min(k-1,i)} v_{j,k-m} \right), \forall j \in \mathcal{G}, k \in K, i \in \mathcal{I}_j \tag{7}$$

– **Shut-down costs:**

$$c^d_{jk} \geq SDC_j(v_{j,k-1} - v_{jk}), \quad \forall j \in \mathcal{G}, k \in K \tag{8}$$

– **Variable bounds:**

$$v_{jk} \in \{0,1\}, \quad \forall j \in \mathcal{G}, \forall k \in K \tag{9}$$

Nominal versions of the non-UC constraints are formulated as follows:

– **Line power:**

$$w_{\ell k} = B_\ell \left(\theta_{BF_\ell,k} - \theta_{BT_\ell,k}\right), \quad \forall \ell \in \mathcal{L}, \ k \in K \tag{10}$$

$$\theta_{1,k} = 0, \quad \forall k \in K \tag{11}$$

– **Power balance:**

$$\sum_{j \in \mathcal{G}(b)} p_{kj} + \sum_{\ell \in \mathcal{L}_I(b)} w_{\ell k} - \sum_{\ell \in \mathcal{L}_O(b)} w_{\ell k} + \alpha^+_{bk} - \alpha^-_{bk} = d_{bk}, \quad \forall b \in \mathcal{B}, k \in K \tag{12}$$

– **Generation limits:**

$$\underline{P}_j v_{jk} \leq p_{jk} \leq \bar{p}_{jk}, \forall j \in \mathcal{G}, k \in K \tag{13}$$

$$0 \leq \bar{p}_{jk} \leq \overline{P}_j v_{jk}, \quad \forall j \in \mathcal{G}, k \in K \tag{14}$$

– **Ramp-up, start-up and shut-down ramp rate:**

$$\bar{p}_{jk} \leq p_{j,k-1} + RU_j v_{j,k-1} + SU_j[v_{jk} - v_{j,k-1}] + \overline{P}_j[1 - v_{jk}], \quad \forall j \in \mathcal{G}, k \in K \tag{15}$$

$$\bar{p}_{jk} \leq p_{j,k+1} + RD_j v_{j,k+1} + SD_j[v_{jk} - v_{j,k+1}] + \overline{P}_j v_{j,k+1}, \quad \forall j \in \mathcal{G}, k = 1, \ldots, |K| - 1 \tag{16}$$

– **Ramp-down limits on the power output:**

$$p_{j,k-1} - p_{jk} \leq RD_j v_{jk} + SD_j[v_{j,k-1} - v_{jk}] + \overline{P}_j[1 - v_{j,k-1}], \quad \forall j \in \mathcal{G}, k \in K \tag{17}$$

– **Total production cost:** For $j \in \mathcal{G}, k \in K$,

$$c^P_{jk} = \begin{cases} a^1_j p_{jk} + b^1_j, & 0 \leq p_{jk} \leq h^1_j, \\ a^2_j p_{jk} + b^2_j, & h^1_j \leq p_{jk} \leq h^2_j, \\ \vdots \\ a^n_j p_{jk} + b^n_j, & p_{jk} \geq h^{n-1}_j \end{cases} \tag{18}$$

– **Variable bounds:**

$$0 \leq p_{jk} \leq \overline{P}_j, \qquad \forall j \in \mathcal{G}, k \in K \tag{19}$$

$$0 \leq \bar{p}_{jk} \leq \overline{P}_j, \qquad \forall j \in \mathcal{G}, k \in K \tag{20}$$

$$\alpha_k^+, \alpha_k^- \geq 0, \qquad \forall k \in K \tag{21}$$

2.3 Stochastic Programming Unit Commitment Model Including CVaR (SUC-CVaR)

In stochastic programming, we use CVaR as a tractable measure to model the risk associated with the imbalances of net load. In contrast to chance-constrained models requiring additional binary variables, CVaR involves only linear constraints and continuous variables, making it computationally attractive. We include the CVaR of dispatch costs in the objective function to manage the risk associated with production cost, shortage and excess of generation relative to net load. Considering the probability density function of cost and γ as a parameter indicating the right tail probability of that function, CVaR$_\gamma$ is defined as the expected value in the worst $100\gamma\%$ of the cost distribution. A stochastic programming model including CVaR in the objective function after liniearization [35] can be formulated as follows:

$$\begin{aligned}
\min \quad & \sum_{k \in K} \sum_{j \in \mathcal{G}} \{c_{jk}^u + c_{jk}^d + a_j v_{jk}\} + \left\{\eta + \frac{1}{\gamma} \sum_{s \in S} \pi_s r_s \right\} & \\
s.t. \quad & \xi_s = \sum_{k \in K} \sum_{j \in \mathcal{G}} \pi_s c_{jks}^P + \Lambda_1 \sum_{b \in \mathcal{B}} \sum_{k \in K} \alpha_{bks}^+ + \Lambda_2 \sum_{b \in \mathcal{B}} \sum_{k \in K} \alpha_{bks}^-, & \forall s \in S, \\
& r_s \geq 0, & \forall s \in S, \\
& r_s \geq \xi_s - \eta, & \forall s \in S, \\
& \text{Constraints (1)-(9)}, & \\
& \text{Constraints (10)-(21)}, & \forall s \in S.
\end{aligned}$$

Note that the SUC model uses second stage decision variables defined with s subscripts and enforces these constraints for each $s \in S$. The RUC model uses second stage decision variables defined as functions of u and enforces these constraints for all $u \in \mathcal{U}$. To avoid duplicating the constraints, we refer to nominal version of Constraints (10)-(21) in both formulations.

2.4 Robust Unit Commitment Model (RUC)

The RUC formulation incorporates uncertainty only in terms of ranges of the uncertain parameters. The following formulation is based on the model presented in [7].

$$\begin{aligned}
\min \quad & \sum_{k\in K}\sum_{j\in\mathcal{G}}\{c^u_{jk}+c^d_{jk}+a_j v_{jk}\}+\max_{u\in\mathcal{U}}\Bigg\{\sum_{k\in K}\sum_{j\in\mathcal{G}}c^P_{jk}(u)+\\
& \Lambda_1\sum_{b\in\mathcal{B}}\sum_{k\in K}\alpha^+_{bk}(u)+\Lambda_2\sum_{b\in\mathcal{B}}\sum_{k\in K}\alpha^-_{bk}(u)\Bigg\}\\
s.t. \quad & \text{Constraints (1)-(9)},\\
& \text{Constraints (10)-(21)}, \quad \forall u\in\mathcal{U}.
\end{aligned}$$

where $\mathcal{U}$ is the uncertainty set of the net loads. The only uncertain parameters in this formulation are the net load values, d_{bk}, on the right-hand-sides of the power balance constraints for each bus b at each time period k. For this study, we consider two distinct definitions for the uncertainty set. We will explain how we construct them from scenarios in Section 3.

The above formulation can be recast in the following equivalent form.

$$\begin{aligned}
&\min_{y_1} \sum_{k\in K}\sum_{j\in\mathcal{G}}\{c^u_{jk}+c^d_{jk}+a_j v_{jk}\}+\max_{u\in\mathcal{U}}\min_{y_2\in\Omega(y_1,u)}\Bigg\{\sum_{k\in K}\sum_{j\in\mathcal{G}}c^P_{jk}(u)+\\
&\quad \Lambda_1\sum_{b\in\mathcal{B}}\sum_{k\in K}\alpha^+_{bk}(u)+\Lambda_2\sum_{b\in\mathcal{B}}\sum_{k\in K}\alpha^-_{bk}(u)\Bigg\}\\
&s.t.\ \text{UC Constraints: (1)-(9)}\\
&\quad \text{where}\\
&\quad y_1=\text{UC (first stage) variables}\\
&\qquad =\{v_{jk},c^u_{jk},c^d_{jk}\forall j\in\mathcal{G},k\in K\}\\
&\quad y_2(u)=\text{Non-UC (second stage) variables}\\
&\qquad =\Big\{p_{jk}(u),\bar{p}_{jk}(u),\theta_{jk}(u),w_{jk}(u),c^P_{jk}(u),\alpha_{bk}(u),\alpha^+_{bk}(u),\alpha^-_{bk}(u),\\
&\qquad\quad \forall b\in\mathcal{B},j\in\mathcal{G},k\in K\}\\
&\quad \Omega(y_1,u)=\{y_2|\text{Non-UC Constraints: (10)-(21), given } y_1,u\}.
\end{aligned} \tag{22}$$

3 Scenarios vs. Uncertainty Sets

In the stochastic programming approach, the uncertain parameter vector is captured by a number of discrete probabilistic scenarios, whereas in the robust optimization approach, the range of its values is defined by a continuous set. To make a reasonable comparison, it is necessary to define uncertainty sets consistent with the scenarios used for the stochastic programming model. For RUC, constructing a proper uncertainty set plays an important role in determining the conservativeness of the model [17]. The uncertainty set is often defined by a lower bound and an upper bound on the uncertain parameter based on the mean value and volatility of the distribution. Jiang et al. [22] introduced a two-dimensional uncertainty set to describe the uncertain problem parameters. Lorca and Sun [25] proposed dynamic uncertainty sets.

Dai et al. [12] applied a multi-band uncertainty set that helps to avoid overly conservative solutions.

We consider two approaches to develop the uncertainty sets. The first approach is to assume the net load for each time period at each node falls between a lower and an upper bound, which can be set by certain percentiles of the random load and wind power output based on historical data [48] or by a fixed percentage of the nominal load [7]. Note that a scenario specifies the net load for each hour and each bus in the scheduling horizon. A parameter called the budget of uncertainty is defined to control the deviation of all loads from their nominal values. According to [7] the uncertainty set can be described as follows:

$$\mathcal{U}_1 := \left\{ d_{bk} : \sum_{b \in N_d} \frac{|d_{bk} - \bar{d}_{bk}|}{\hat{d}_{bk}} \leq \Delta_k, d_{bk} \in [\bar{d}_{bk} - \hat{d}_{bk}, \bar{d}_{bk} + \hat{d}_{bk}], \forall b \in N_d, \forall k \in K \right\}$$

where N_d is the set of buses that have uncertain load, $\bar{d}_{bk}$ is the nominal value of load and $\hat{d}_{bk}$ is the maximum possible deviation of load of bus b at time k from the nominal value. The parameter Δ_k is the budget of uncertainty, taking values between 0 and $|N_d|$. When $\Delta_k = 0$, the robust formulation corresponds to the deterministic case. As Δ_k increases, the uncertainty set expands, which results in more conservative UC solutions. The maximum amount of deviation that can be considered for the net load in each period is $|N_d|$.

Our second approach is to construct uncertainty sets using historical realizations of the random variables by applying a connection between convex sets and a specific class of risk measures [6]. To elaborate this connection, here we focus on avoiding shortage of electricity generation to meet the net load; i.e., we wish to guarantee, in some sense, that

$$\sum_{j \in \mathcal{G}(b)} p_{kj} + \sum_{\ell \in \mathcal{L}_I(b)} w_{\ell k} - \sum_{\ell \in \mathcal{L}_O(b)} w_{\ell k} \geq d_{bk}, \quad \forall b \in \mathcal{B}, k \in K. \tag{23}$$

Bertsimas and Brown point out that, for random $\tilde{d}_{bk}$, a CVaR constraint formulated as

$$CVaR_\alpha \left(\sum_{j \in \mathcal{G}(b)} p_{kj} + \sum_{\ell \in \mathcal{L}_I(b)} w_{\ell k} - \sum_{\ell \in \mathcal{L}_O(b)} w_{\ell k} - \tilde{d}_{bk} \right) \leq 0, \quad \forall b \in \mathcal{B}, k \in K. \tag{24}$$

says, roughly, that expected generation in the 100 % of worst cases is no less than the net load. Further, their Theorem 3.1 states that the feasible set of decisions corresponding to this constraint is identical to the set of decisions that satisfies

$$\sum_{j \in \mathcal{G}(b)} p_{kj} + \sum_{\ell \in \mathcal{L}_I(b)} w_{\ell k} - \sum_{\ell \in \mathcal{L}_O(b)} w_{\ell k} \geq d_{bk}(u), \quad \forall b \in \mathcal{B}, k \in K. \tag{25}$$

for every $u \in \mathcal{U}_2$ as defined below. Here, the tail probability $\alpha = r/N$ where r and N are described in the following.

Given N data points; i.e., $s_1, s_2, \ldots, s_N$, the uncertainty set corresponding to CVaR_α is

$$\mathcal{U}_2 := \text{conv}\left(\left\{\frac{1}{\alpha}\sum_{i\in I}\pi_i s_i + (1 - \frac{1}{\alpha}\sum_{i\in I}\pi_i)s_j : \right.\right.$$
$$\left.\left. I \subseteq \{1, \ldots, N\}, j \in \{1, \ldots, N\} \setminus I, \sum_{i\in I}\pi_i \leq \alpha\right\}\right),$$

where conv($\cdot$) denotes the convex hull.

Assuming the probability distribution of sample points s_i as $\pi_i = 1/N$, each of which represents one observation of demand $\{d_{bk}\}$, and considering $\alpha = r/N$ for some $r \in \mathbb{Z}^+$, this has the interpretation of the convex hull of all r-point averages the demand observations. Let

$$\mathcal{F} := \left\{\frac{1}{\alpha}\sum_{i\in I}\pi_i s_i + (1 - \frac{1}{\alpha}\sum_{i\in I}\pi_i)s_j : \right.$$
$$\left. I \subseteq \{1, \ldots, N\}, j \in \{1, \ldots, N\} \setminus I, \sum_{i\in I}\pi_i \leq \alpha\right\}.$$

be the set of all possible points for a certain α value. Because $\mathcal{F}$ has finitely many elements, we can write it as $\mathcal{F} = \{f_1, \ldots, f_m\}$ for some finite m. To define the uncertainty set, we introduce a decision variable μ_i corresponding to each $f_i, i = 1, \ldots, m$. Since each scenario has dimension $|\mathcal{B}| \times |K|$, the elements of uncertainty set are represented as f_{ibk}. The convex hull term in the definition of $\mathcal{U}_2$ can be formulated as the following:

$$\mathcal{U}_2 := \Big\{(\mu, d) \in \mathbb{R}^m \times \mathbb{R}^{|\mathcal{B}|\times|K|} : \sum_{i=1}^{m}\mu_i f_{ibk} = d_{bk}, \forall b, k,$$
$$\sum_{i=1}^{m}\mu_i = 1, \mu_i \geq 0, i = 1, \ldots, m\Big\}.$$

Here, the parameter α takes values between 0 and 1. As α decreases, the uncertainty set enlarges, so that the resulting robust solutions are more conservative and the system is protected against a higher degree of uncertainty.

4 Numerical Experiments

To test the approaches, we adopted the modified 24-bus IEEE RTS-96 system [44] with 32 generators and 38 transmission lines using data from [2]. The data set includes hourly demand data for one year. Three wind farms are added to the grid as in [32] with wind scenarios extracted from the NREL wind data sets [1]. The penalty cost coefficients were arbitrarily set with values of $\Lambda_1 =$

40 \$/MWh for deficits and $\Lambda_2 = 1$ \$/MWh for excess energy. These relatively modest values allow shortages to occur in an exaggerated way which allows comparison of risks. They are revisited in a sensitivity study at the end of this section.

The wind data set contains 365 scenarios, assumed as equally likely. We used the fast forward selection algorithm [18] to reduce the number of scenarios to 20, which is twice as many as used in [32]. The reduced set of scenarios included one assigned a probability of 0.58, three with probabilities of 0.09, 0.11 and 0.13, respectively, and the remainder with probabilities below 0.05. They are categorized as high-, medium- and low-probability, respectively, and the corresponding net load scenarios, aggregated over the buses, are plotted as 24-hour time series in Fig. 1. The problems are solved using these 20 scenarios, and out-of-sample testing of the solutions is done by Monte Carlo simulation with the other 345 scenarios. A similar out-of-sample testing scheme was employed in [24] to compare the results of robust optimization using a convex hull uncertainty set with stochastic programming using a CVaR objective. The plot also shows the bus-aggregated worst cases identified in each of the RUC optimizations, as described below. To construct the uncertainty set from the historical data, we considered a fixed percentage of the nominal value as the maximum deviation of net load. Bertsimas et al. set the net load deviation to be 10% of the nominal value for a system with shallow penetration of wind power [7]. Wu et al. set the uncertainty to be 15% and 25% of the total installed wind power generation capacity based on observational data [46]. According to this, we set the maximum deviation ($\hat{d}_{bk}$) of net load of bus b at time k as 25% of the nominal value. Also, in this numerical study, $\Delta_k = \Delta$ for all values of k.

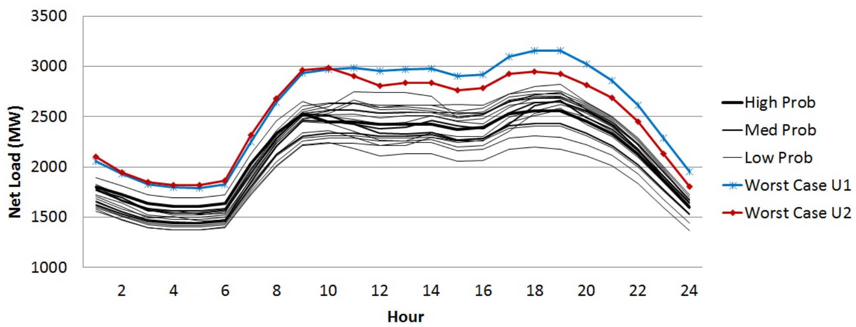

Fig. 1 Probabilistic scenarios and worst cases in each uncertainty set ($\Delta = |N_d|$ is used to find the worst case for $\mathcal{U}_1$ and α=0.05 is used to find the worst case for $\mathcal{U}_2$)

4.1 Improved solution algorithm

A Benders decomposition based algorithm proposed in [7] was applied to solve the RUC. However, this method, called "cutting plane" in the sequel, follows an iterative approach in which each iteration involves solving a mixed-integer program (MIP) and afterwards, having solved some subproblems, an inequality is added to the formulation. Therefore, the formulation becomes progressively larger and, hence, more time-consuming to solve.

We adopted a branch and cut modification [30] for this Benders decomposition which makes it orders of magnitude faster. Rather than solving a MIP in each iteration, we can solve the initial master problem in a single branch-and-bound tree and, in each node with an integer feasible solution, solve the underlying subproblems and add inequalities dynamically to the initial formulation. By employing this approach we must explore only one branch-and-bound tree. Hence, the method is likely to be more efficient. The steps of the decomposition are summarized in Appendix A. For more details on this approach, we refer to [23].

We implemented these algorithms in Python 2.7 and employed CPLEX Python API 12.5 as the integer programming solver. Table 1 summarizes computational results of the Benders decomposition based algorithm proposed in [7] and the adopted branch-and-cut approach for this Benders decomposition. We computed the gaps for the cutting plane approach by

$$\text{Gap\%} = \frac{U^{BD} - L^{BD}}{U^{BD}} \times 100.$$

Results show that the cutting plane method was not able to solve any of the instances to optimality within the 3 hour time limit whereas the branch-and-cut method solved all the instances well within the given time limit. The proposed method found the optimal solution for all but one of six instances in less than 45 minutes. We also observed that decrements of the gap in the cutting plane approach slow down as the algorithm progresses. This implies that one should expect a much longer time to close the gap with the cutting plane approach.

4.2 Results

The results of solving the problem using different approaches were evaluated according to the costs of optimally dispatching the committed units in the out-of-sample simulation. Total costs include dispatch and unit commitment cost:

$$\text{Total Cost} = \text{Dispatch Cost} + \text{Unit Commitment Cost},$$

where

$$\text{Dispatch Cost} = \text{Production Cost} + \text{Penalty Cost}.$$

Table 1 Comparison of the proposed branch-and-cut method with the existing cutting plane approach of [7] on instances with different settings. The columns show CPU time in seconds, number of iterations and % gap for the cutting plane approach, and CPU time in seconds and % gap for the branch-and-cut. A "-" means the instance solved to optimality.

		Cutting plane			Branch-and-cut	
		Time(s)	Iter.	G(%)	Time(s)	G(%)
	$\Delta = 0.1\|N_d\|$	10800	20	5.99	1115.51	-
	$\Delta = 0.5\|N_d\|$	10800	19	7.57	1628.50	-
RUC-$\mathcal{U}_1$	$\Delta = 1.0\|N_d\|$	10800	20	6.59	2609.47	-
	$\alpha = 0.1$	10800	19	6.18	4758.98	-
	$\alpha = 0.5$	10800	2	14.11	1530.52	-
RUC-$\mathcal{U}_2$	$\alpha = 0.8$	10800	6	12.26	1993.57	-

The production cost is computed from Equation (7) as:

$$\text{Production Cost} = \sum_{j\in\mathcal{G}}\sum_{k\in K} c^P_{jk}.$$

We also compute the penalties of deficit and excess of demand requirements as

$$\text{Penalty Cost} = \Lambda_1 \sum_{k\in K}\sum_{b\in\mathcal{B}} \alpha^+_{bk} + \Lambda_2 \sum_{k\in K}\sum_{b\in\mathcal{B}} \alpha^-_{bk}.$$

The unit commitment cost includes the start-up, shut-down cost and no-load costs:

$$\text{Unit Commitment Cost} = \sum_{k\in K}\sum_{j\in\mathcal{G}} \{c^u_{jk} + c^d_{jk} + a_j v_{jk}\}.$$

Fig. 2 shows a comparison of empirical distributions of the total cost in the out-of-sample simulation. The plots suggest that RUC models result in slightly higher total cost compared to SUC-CVaR.

Details of the costs including their means and confidence interval widths are summarized in Table 2. We comment on the results of this table by focusing on settings for each approach with almost equal expected total cost; i.e., SUC-CVaR with $\gamma = 0.02$, RUC-$\mathcal{U}_1$ with $\Delta = 0.5|N_d|$ and RUC-$\mathcal{U}_2$ with $\alpha = 0.5$. The unit commitment cost of RUC-$\mathcal{U}_2$ is significantly lower than in SUC-CVaR and RUC-$\mathcal{U}_1$ while it has higher penalty cost.

To compare uncertainty sets $\mathcal{U}_1$ and $\mathcal{U}_2$, one can see that although the unit commitment cost of RUC-$\mathcal{U}_2$ with $\alpha = 0.5$ is lower than the unit commitment cost of RUC-$\mathcal{U}_1$ with $\Delta = 0.1|N_d|$, RUC-$\mathcal{U}_2$ results in higher violation and penalty cost, and at the same time slightly more production cost. This conclusion is reinforced by Table 3, which contains mean and confidence intervals of the violation of constraints (12). SUC-CVaR has almost same expected production cost as RUC-$\mathcal{U}_1$, but less than that of RUC-$\mathcal{U}_2$. Overall, the cost comparison indicates that the RUC-$\mathcal{U}_1$ model is more conservative than either SUC-CVaR or RUC-$\mathcal{U}_2$.

Table 2 Mean and 95%-confidence intervals (CI) of dispatch, production, penalty, unit commitment and total costs. Unit commitment cost is identical for all scenarios; therefore, no CIs for it are shown. Boldface indicates cases selected for analysis in the text.

			Costs				
			Dispatch	Production	Penalty	Unit Commitment	Total
SUC-CVaR	$\boldsymbol{\gamma = 0.002}$	Mean	680994	680092	902	192136	**873130**
		CI	(668893, 693096)	(668145, 692039)	(469, 1336)	-	(861028, 885231)
	$\gamma = 0.02$	Mean	669477	668194	1283	178920	848397
		CI	(656905, 682049)	(655846, 680543)	(718, 1848)	-	(835825, 860969)
	$\gamma = 0.2$	Mean	666501	664561	1940	174212	840714
		CI	(653653, 679350)	(652057, 677065)	(1130,2749)	-	(827865, 853562)
RUC-$\mathcal{U}_1$	$\varDelta = \lvert N_d \rvert$	Mean	703311	703297	14	256919	960230
		CI	(692446, 714175)	(692431, 714164)	(7, 20)	-	(949366, 971095)
	$\varDelta = 0.5\lvert N_d \rvert$	Mean	701530	701518	12	248316	949847
		CI	(690438, 712623)	(690424, 712613)	(7, 18)	-	(938754, 960940)
	$\boldsymbol{\varDelta = 0.1\lvert N_d \rvert}$	Mean	675299	674975	324	199300	**874599**
		CI	(662846, 687753)	(662590, 687360)	(176, 473)	-	(862146, 887053)
RUC-$\mathcal{U}_2$	$\alpha = 0.1$	Mean	684348	684247	101	197596	881943
		CI	(672194, 696501)	(672112, 696382)	(22, 179)	-	(869790, 894097)
	$\boldsymbol{\alpha = 0.5}$	Mean	720399	718346	2053	156836	**877235**
		CI	(706916, 733882)	(705231, 731462)	(1301, 2805)	-	(863752, 890718)
	$\alpha = 0.8$	Mean	725826	714114	11712	133357	859182
		CI	(710541, 741110)	(700712, 727516)	(8809, 14615)	-	(843898, 874467)

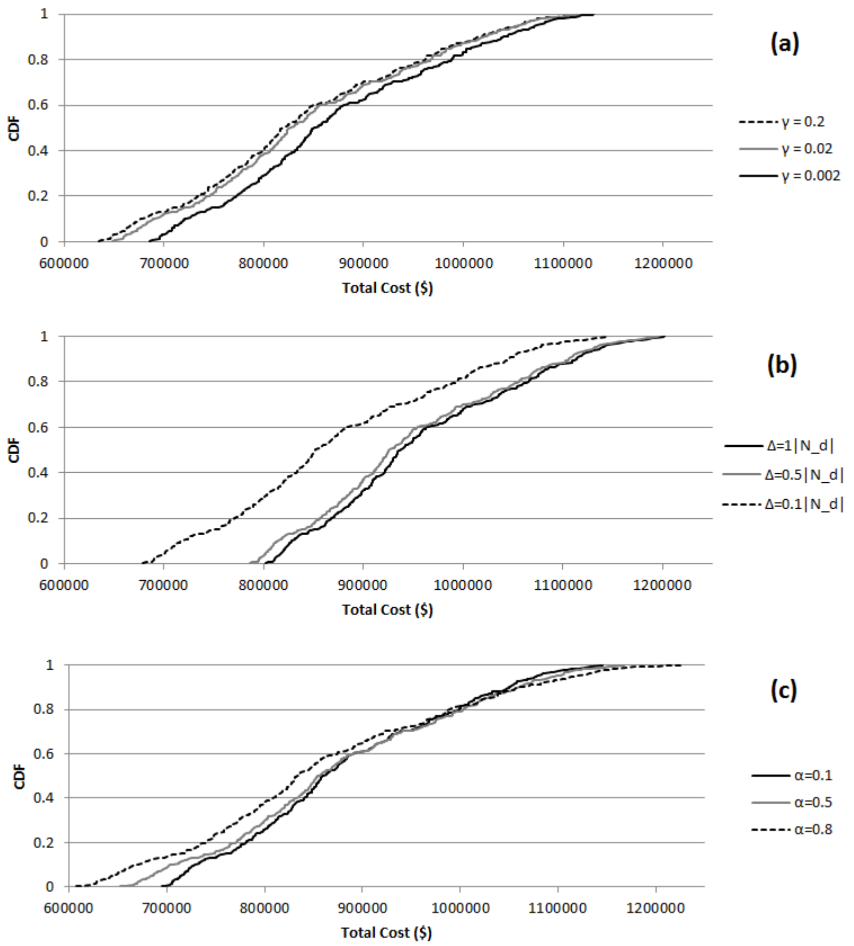

Fig. 2 Distributions of total cost from (a) SUC-CVaR, (b) RUC-$\mathcal{U}_1$, (c) RUC-$\mathcal{U}_2$ models

The level of conservatism can be adjusted in all three methods by adjusting the extent of the uncertainty sets considered in the RUC formulations or the tail probability in the SUC-CVaR formulation. Fig. 3 presents a Pareto chart to illustrate the tradeoff between cost and reliability. It indicates that, if a decision-maker emphasizes cost by setting Δ small, or α or γ large, then SUC-CVaR achieves the most efficient combinations of expected cost and expected shortage. When these parameters are adjusted for higher conservatism, the box-uncertainty RUC-$\mathcal{U}_1$ dominates SUC-CVaR . However, by setting the most stringent risk-minimizing parameter settings in the data-driven RUC-$\mathcal{U}_2$, a better cost-reliability tradeoff is achieved. We note that shortages occur even when a very low-probability tail is used in the CVaR portion of the SUC

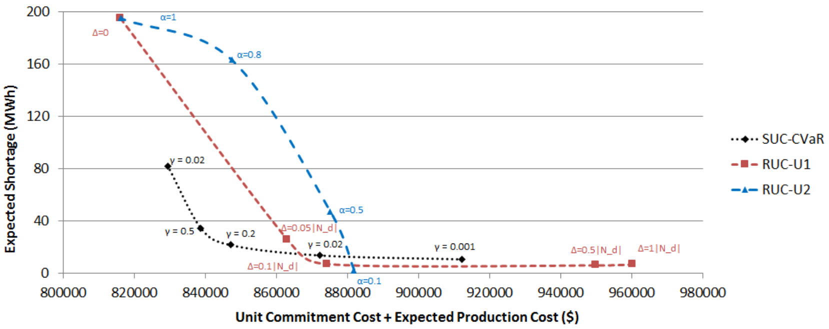

Fig. 3 Pareto chart of expected shortage vs. unit commitment cost plus expected production cost.

objective. This unavoidable occurrence was also noted in [51] due to the approximation introduced by scenario reduction. Because the confidence intervals on the expected cost and shortage values overlap, these findings should be verified in more extensive numerical tests using the particular system parameters and penalty values chosen by the system operator.

In another experiment, we assess the sensitivity of the models using intermediate values of the risk parameters with respect to penalty coefficients (i.e., Λ_1 and Λ_2). Figure 4 shows that as the penalty coefficients rise, the RUC models respond with higher total cost whereas SUC-CVaR does not react as much.

Because the RUC models are designed to protect against the worst case, it is instructive to compare their worst case performances. As a byproduct of the unit commitment optimization, we obtain the net load sequence that embodies the worst case along with the UC schedule optimized for that case. These worst case bus-aggregated load sequences are shown in Fig. 1; note that the aggregation causes them to appear to fall outside the envelope of scenarios used to construct the uncertainty sets. We evaluated the commitment decisions of each method and each risk parameter setting by dispatching them against the worst-case net loads identified in the largest uncertainty sets according to both formulations. Fig. 5 shows a Pareto chart of the penalty cost vs. the unit commitment plus production cost for the worst case of $\mathcal{U}_1(\Delta = |N_d|)$. This is the sequence of net loads for which RUC-$\mathcal{U}_1$ optimizes with $\Delta = |N_d|$. The same chart for the worst case of $\mathcal{U}_2$ with $\alpha = 0.05$ appears similar. The most efficient trade-offs between cost and mismatch penalty are achieved by the SUC-CVaR model if cost is emphasized and by the RUC-$\mathcal{U}_2$ model if reliability is prioritized by the choice of its risk parameter value.

Table 3 Mean, 95% Lower Confidence Limit (LCL) and 95% Upper Confidence Limit (UCL) of Violations (MWh)

			Violations		
			Total	Shortage	Excess
SUC-CVaR	$\gamma = 0.002$	Mean	13.67	12.90	0.77
		LCL	7.58	6.83	0.08
		UCL	19.77	18.97	1.47
	$\gamma = 0.02$	Mean	21.62	21.60	0.02
		LCL	12.36	12.34	-0.02
		UCL	30.87	30.85	0.05
	$\gamma = 0.2$	Mean	34.47	34.45	0.02
		LCL	20.59	20.57	-0.02
		UCL	48.35	48.33	0.05
RUC-$\mathcal{U}_1$	$\Delta = \vert N_d \vert$	Mean	6.84	0.00	6.84
		LCL	3.62	-	3.62
		UCL	10.06	-	10.06
	$\Delta = 0.5\vert N_d \vert$	Mean	6.12	0.00	6.12
		LCL	3.62	-	3.62
		UCL	8.91	-	8.91
	$\Delta = 0.1\vert N_d \vert$	Mean	7.16	7.16	0.00
		LCL	3.98	3.98	-
		UCL	10.33	10.33	-
RUC-$\mathcal{U}_2$	$\alpha = 0.1$	Mean	2.54	1.97	0.56
		LCL	1.06	0.55	0.13
		UCL	4.01	3.39	1.00
	$\alpha = 0.5$	Mean	47.15	42.38	4.77
		LCL	31.89	27.11	2.69
		UCL	62.41	57.65	6.84
	$\alpha = 0.8$	Mean	163.21	163.21	0.00
		LCL	123.37	123.37	-
		UCL	203.06	203.06	-

5 Conclusions

Robust optimization and stochastic programming have been extensively discussed and studied as alternatives to optimize unit commitment schedules under uncertainty. A popular impression has arisen that the robust approach, with its focus on the worst case, is better able to control risk while stochastic programming emphasizes expected values. However, the stochastic programming formulation can easily accommodate a risk measure. Moreover, the results of both methods depend strongly on the model for the uncertain parameters – either the uncertainty set or the probabilistic scenarios employed in the optimization. To compare both approaches on the same information basis, we constructed uncertainty sets by two methods based on the same reduced set of scenarios used in the stochastic programming formulation. The schedules found by each approach for various risk parameter values were evaluated in an out-of-sample simulation.

The numerical results indicate that the cost-risk trade-off achieved by any approach is strongly influenced by the value of its respective risk parameter. By incorporating risk in the stochastic programming formulation in terms of

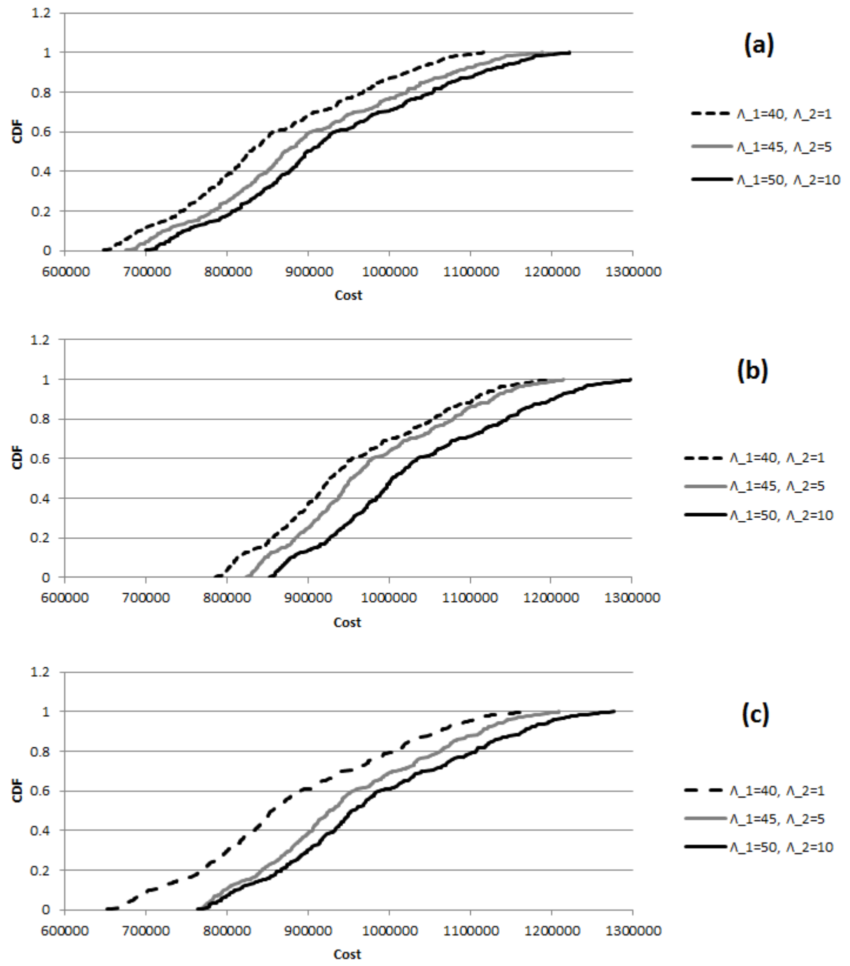

Fig. 4 Comparison of different penalties for (a) SUC-CVaR ($\gamma = 0.02$), (b) RUC-$\mathcal{U}_1$ ($\Delta = 0.5|N_d|$), (c) RUC-$\mathcal{U}_2$ ($\alpha = 0.5$)

CVaR with a sufficiently low tail probability, the stochastic programming formulation can achieve the most efficient combinations of cost and risk when a decision maker emphasizes cost. However, when a higher level of conservatism is preferred, robust optimization models can achieve the most efficient combinations of cost and risk. Between the two uncertainty set formulations for robust optimization, the data-driven method that incorporates probabilities of scenarios as well as their ranges of values achieves better cost-risk trade-offs than the one based on ranges alone when the risk parameter is set to its most stringent value. The two CVaR-based approaches, SUC-CVaR and RUC with

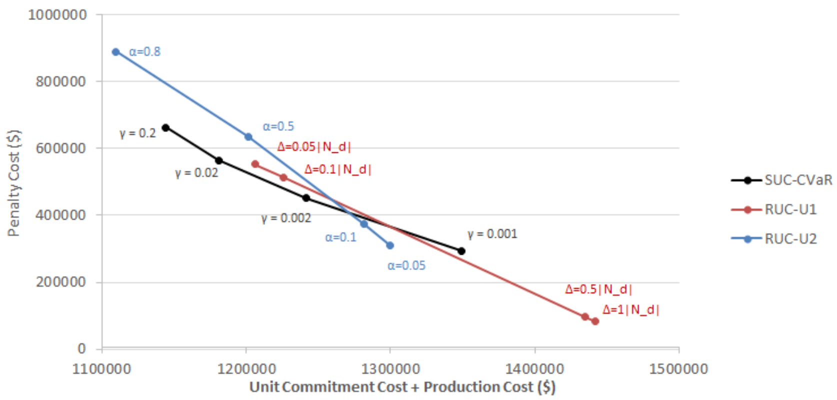

Fig. 5 Pareto chart of penalty cost vs. unit commitment cost from dispatching unit commitment schedules found by different formulations and settings in the worst case of $\mathcal{U}_1(\Delta = |N_d|)$)

CVaR-based uncertainty set, provide schedules that perform best should the worst case occur.

Appendix A Solution Algorithm

We write y_1 for the UC variables that are independent of uncertainty; i.e., the unit commitment, start-up and shut-down cost variables. Also, we write y_2 for the dispatch variables (recall that the dispatch variables may depend on the values of uncertain parameters, e.g. power output, phase angle, production cost). The uncertain parameters (in our problem, the hourly net load) can vary on a set denoted as $\mathcal{U}$. Suppose that the RUC is summarized as

$$\min_{y_1, y_2} \quad \left(c^T y_1 + \max_{u \in \mathcal{U}} \; b^T y_2(u) \right) \tag{26a}$$

$$s.t. \quad \mathbf{F} y_1 \leq \mathbf{f}, \tag{26b}$$

$$\mathbf{H} y_2(u) \leq \mathbf{h}, \qquad u \in \mathcal{U} \tag{26c}$$

$$\mathbf{A} y_1 + \mathbf{B} y_2(u) \leq \mathbf{g}, \qquad u \in \mathcal{U} \tag{26d}$$

$$\mathbf{E} y_2(u) = u, \qquad u \in \mathcal{U} \tag{26e}$$

$$y_1 \in \mathbb{R}^{n_1} \times \{0,1\}^{p_1}, \tag{26f}$$

$$y_2(u) \in \mathbb{R}^{n_2}, \qquad u \in \mathcal{U}. \tag{26g}$$

In Problem (26), the oubjective function (26a) minimizes a combination of unit commitment costs, such as start-up and shut-down costs, and the worst case of the dispatch costs, such as production and shortage costs. Constraint (26b) only defines feasibility of the unit commitment variables. Constraint (26c) involves both the unit commitment decisions and dispatch variables, such as ramp-up and ramp-down constraints. In constraint (26d), we only constraint dispatch variables, such as power balance equations. Note that dispatch variables depend on the uncertain parameter u. Finally, we have restrictions on decision variables: unit commitment variables are mixed-integer where n_1 is the number of continuous variables and p_1 is the number of binary decisions, and dispatch variables are n_2 continuous variables.

We can associate these constraints to those of our formulation in Section 2: Constraint (26b) contains constraints (1)-(8), Constraint (26c) contains (10), (11), (19), Constraint (26d) contains (14)-(18), Constraint (26e) contains (12), (13), Constraint (26f) contains (9), and Constraint (26g) contains (20)-(22).

In this formulation, the second term of the objective function represents the worst case of the dispatch cost. By including this second term, we ensure that the unit commitment problem remains feasible, thus robust, under any realization of uncertainty.

Note that the dispatch constraints depend on both the unit commitment variable y_1 and the uncertain parameter u. Hence, we write $\Omega(y_1, u)$ as a feasible set defined by the dispatch constraints. We let

$$\Omega(y_1, u) = \{y_2 : \text{(26c)}, \text{(26d)}, \text{(26e) and (26g) are satisfied for fixed } y_1 \text{ and } u\}.$$

Problem (26) can be equivalently reformulated as

$$\begin{aligned} &\min_{y_1} c^T y_1 + \max_{u \in \mathcal{U}} \min_{y_2 \in \Omega(y_1,u)} b^T y_2 \\ &s.t. \text{ Constraints (26b), (26f).} \end{aligned} \tag{27}$$

One may observe that $\min_{y_2 \in \Omega(y_1,u)} b^T y_2$ is actually the dispatch problem for a fixed unit commitment decision y_1 and uncertain parameter u. Now, by maximizing the optimal cost of the dispatch problem over all possible $u \in \mathcal{U}$, the worst case dispatch decision is obtained.

To solve Problem (27), we reformulate it as follows:

$$\begin{aligned} &\min_{y_1, \gamma} c^T y_1 + \gamma \\ &s.t. \text{ (26b), (26f)} \\ &\qquad \gamma \geq S(y_1, u), \forall u \in \mathcal{U}, \end{aligned} \tag{28}$$

where

$$S(y_1, u) = \min_{y_2 \in \Omega(y_1,u)} b^T y_2. \tag{29}$$

We write $R(y_1)$ as the worst case of the dispatch problem:

$$R(y_1) = \max_{u \in \mathcal{U}} S(y_1, u). \tag{30}$$

Note that in our problem formulation, since $R(y_1)$ represents the worst case dispatch cost, we write $\gamma \geq 0$ without loss of optimality. This problem can be then reformulated as

$$\begin{aligned} \min_{y_1,\gamma} \; & c^T y_1 + \gamma \\ s.t. \; & (26b), (26f) \\ & \gamma \geq R(y_1), \\ & \gamma \geq 0. \end{aligned} \tag{31}$$

Problem (31) is solved using a Benders decomposition approach applying a banch and cut approach. For more details on how to solve subproblem (30) and master problem (31), we refer to [23].

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