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Keywords
cattle markets, captive supplies, extended path algorithm

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I. Motivation and objectives

A number of empirical studies of fed cattle markets used regression analysis to document a negative correlation, in weekly or monthly time series data, between captive supply delivery volumes in a regional market and the market's spot price of cattle.[1] One of the most fundamental caveats of statistical analysis is that "correlation does not imply causation." Yet some market participants and commentators portray this negative correlation as evidence of the anti-competitive effects of captive supplies; in particular, as evidence that packers use captive supply procurement methods as a means of depressing spot market price.[2] A paper by Schroeter and Azzam (SA, 2004) raises the possibility that the negative correlation may simply be an essentially benign artifact of cattle delivery timing decisions made by market participants who behave competitively. The SA argument is suggestive but is based on an incomplete analysis of the market's underlying economic mechanisms and, so, is not entirely convincing. The purpose of this research is to undertake a more complete investigation of the effects of delivery timing decisions to determine whether they, in fact, might be responsible for the empirical regularity commonly found in data on cattle prices and captive supplies.

Although the SA analysis is incomplete, it is useful, nevertheless, to begin by reviewing it and identifying some of its shortcomings, since these shortcomings will

[1] One study that found this negative correlation is GIPSA's "Texas Panhandle" investigation of the cattle procurement activities of four large packing plants in the Texas Panhandle over a 66 week period in 1995 and 1996. Schroeter and Azzam (1999, 2003, 2004) provide extensive analysis of the GIPSA - Texas Panhandle data, including regressions that document the negative correlation between captive supplies and spot price. Earlier studies finding similar negative correlations include Elam (1992), Hayenga and O'Brien (1992), and Schroeder *et al.* (1993). Arguments by both plaintiffs and defense in the recent *Pickett et al. v. Tyson Fresh Meats, Inc. (IBP, Inc.)* case, (Civil No. 96-A-1103-N, U.S. District Court for the Middle District of Alabama, Northern Division), devoted significant attention to this negative correlation.

[2] It is tempting to try to resolve the causality question through the use of Granger causality tests. As Sims (1999) cautions, however, Granger causality "does not provide a statistical magic wand that allows us to discover true causal structures via data analysis, without substantive theory."

motivate the direction taken by the current research. The SA analysis addresses both marketing agreement and forward contract procurement methods. However, in this overview of the SA analysis, as well as in the alternative approach that will be offered to remedy several of the problems in SA, the focus will be on marketing agreements as the only captive supply procurement method. Narrowing the scope of the study in this way is an appropriate method of simplifying the analysis given the predominant role of marketing agreements among non-traditional procurement methods.[3]

The SA analysis of delivery timing incentives for marketing agreement cattle reflects features of typical marketing agreements and some important stylized facts of fed cattle markets:

1. Marketing agreements between packers and feeders typically cover full feedlot capacity. Thus, it is reasonable to think of feeders as falling into one of two categories, "marketing agreement feeders" or "spot market feeders," and it is reasonable to think of individual pens of cattle on feed as being earmarked for either spot market or marketing agreement sales.
2. Marketing agreements typically give the feeder discretion over the number of cattle to be delivered under the terms of the agreement, in any given week, but require that the feeder notify the packer of this number two weeks in advance of delivery.[4]

[3] For example, in the GIPSA - Texas Panhandle data analyzed in Schroeter and Azzam (1999), 73% of captive supply cattle were procured under marketing agreements. According to GIPSA's *Packers and Stockyards Statistical Report* (USDA, 2006), marketing agreement cattle made up 64% of the captive supply acquisitions of steers and heifers by the four largest beef packers in 2004.

[4] Appendix B of Schroeter and Azzam (1999) provides some evidence, from beef packing company documents and interviews with feedlot personnel, to substantiate the claims made in this and the previous paragraph.

3. Spot cattle are typically delivered to the plant approximately one week after sale.[5]

4. The formulas used for pricing marketing agreement cattle involve a base price, applicable to cattle of given quality characteristics, and a system of premia and discounts that are used to adjust the base price when the characteristics of delivered cattle deviate from those of the base carcass. Base prices can be set in a variety of ways but are typically tied to spot market prices paid the week prior to the week of delivery of the marketing agreement cattle.[6]

5. Delivery scheduling can, to some degree, take advantage of inter-temporal arbitrage opportunities. Expected price movements could make it profitable for cattle nominally ready for delivery one week to be delivered one or a few weeks early or late, even though they might suffer a small quality or weight discount as a result.

To briefly outline the SA argument, begin by considering the cattle on feed in the regional market's marketing agreement feedlots. Assume that each pen of marketing agreement cattle is characterized by a provisional delivery week; that is, a future week in which, assuming the physical development of the cattle proceeds as expected, it will be optimal to deliver if prices were to remain constant from week-to-week. Cattle that are "nominally ready" for delivery in one week would, however, be delivered either earlier or

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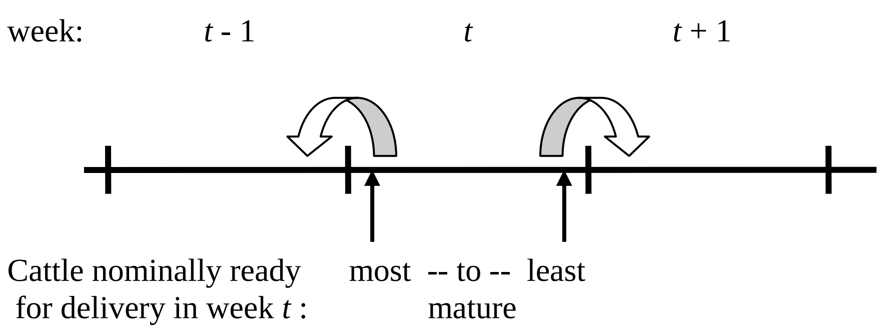
[5] For spot market pens of cattle in the GIPSA - Texas Panhandle data, the distribution of the lag, in days, between purchase and delivery has a mean of 6.98 and a standard deviation of 3.28.

[6] For example, the base price may be set at the level of a USDA Agricultural Marketing Service reported price for fed cattle in the relevant regional market the week prior to delivery. Or it may be set at the level of a quality-adjusted average price paid by the packer for spot market cattle slaughtered during the week of delivery of the marketing agreement cattle. But since the typical delivery lag for spot market cattle is approximately one week, this arrangement, too, ties the base price to the previous week's spot market price. Nine formulas accounted for virtually all of the marketing agreement cattle in the GIPSA - Texas Panhandle data set. Five of these formulas set the base price according to an AMS report. The remaining four tie the base price to the price of the packers' own cash market purchases.

later if expected price movements justified this kind of "off-schedule" delivery. The SA analysis incorporates these inter-temporal arbitrage opportunities in a limited way. Consider the time line depicted in the figure below. Imagine the collection of all of the pens of cattle on feed that are nominally ready for delivery in week *t* and think of this cohort being arrayed from left to right, from most mature pens to least mature pens, along the line segment corresponding to that week. SA assume that some of these cattle, those of intermediate maturity, *must* be delivered in their provisional delivery week. In other words, the analysis implicitly assumes that the costs of off-schedule delivery and the distribution of week-to-week price movements are such that it would never be optimal to deliver these cattle off-schedule. Some of the least mature of week *t*'s nominally-ready cohort, those farthest to the right along the line segment, would be candidates for a one week delay in delivery, however, if price were expected to increase sufficiently. Correspondingly, some of the most mature of the cattle nominally ready for delivery in week *t*, those on the far left of the segment, could be profitably rescheduled for delivery in week *t* - 1 given an anticipated price decrease of sufficient magnitude.



In week *t* - 2, marketing agreement feeders commit to their delivery numbers for week *t*. Again, the terms of conventional marketing agreement formulas tie the price paid for marketing agreement cattle delivered in week t to the spot market price the previous week. Thus if the expectation, based on week t - 2 information, is that price will increase from week t - 1 to t, $E_{t-2}[p_t - p_{t-1}] > 0$, feeders would decide, in week t - 2, to shift some of the least mature of the week-t-ready cohort to delivery in week t + 1, so that they would be paid the higher price, p_t. On the other hand, if the week t - 2 expectation is that price will fall between weeks t - 1 and t, $E_{t-2}[p_t - p_{t-1}] < 0$, some of the most mature of the week-t+1-ready cohort would be scheduled for delivery in week t instead, so that they could take advantage of the higher price, p_{t-1}, before the expected price decline. In either event, the number of cattle delivered off-schedule would increase with the absolute magnitude of the expected price change. Thus, considering the effects of scheduling decisions made in week t - 2 alone, the suggestion is that the number of marketing agreement cattle delivered in week t, would be negatively related to $E_{t-2}[p_t - p_{t-1}]$.

This kind of reasoning suggests the possibility of a link between delivery timing decisions and the week-to-week dynamics of price and captive supply delivery volumes in fed cattle markets. But, as noted earlier, there are several reasons why this simplistic analysis falls short of an explanation for the patterns of covariation that have been observed in actual data. First, the argument in SA is suggestive of a connection between captive delivery volumes and an *ex ante* expectation of a price *change*. It is not immediately clear whether this mechanism could generate the key empirical regularity that has been documented in several previous studies: a negative correlation between delivery volumes and the *level* of price.

A second problem is that the thought model outlined above takes an essentially static approach to an inherently dynamic problem. For example, the number of cattle delivered in week t will also depend on decisions, made in week t - 3, that involve rescheduling delivery from week t to week t - 1 or vice versa; decisions that were ignored in the foregoing discussion. A complete model must recognize that, in reality, feeders face an entire interrelated series of decision problems.

A third problem is that the argument outlined above explicitly considers the conduct of marketing agreement feeders only. Spot market feeders, playing a similar game, will also make delivery timing decisions based on expectations of future price movements.[7] Needless to say, spot market feeder conduct plays a role in the determination of spot price. This observation highlights the most fundamental shortcoming of the simple thought-model proposed in SA: The argument takes price expectations as given and considers the implications for delivery scheduling, but it does not address the feedback from those delivery scheduling decisions into price determination. What is needed is a model that accounts for both "sides" of the market mechanism and in which the subjective price expectations that influence delivery timing are conditional forecasts based on the model itself. In other words, what is needed is a rational expectations equilibrium model of delivery timing decisions in a fed cattle market.

II. A rational expectations model of a regional market for fed cattle

The objectives of the current research are twofold: First, to assemble a model of a fed cattle market that addresses the delivery-timing incentive issues highlighted by SA, but in a way that avoids most of the shortcomings of that earlier analysis. And, second,

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[7] Spot market feeders' delivery timing "game" is played under somewhat different "rules," however. In particular, the decision to market this week will be based on observation of this week's price and an expectation, with only a one week forecast horizon, of next week's price.

to analyze the model to determine whether it is capable of explaining the observed negative correlations between spot market price and captive delivery volumes. For a simplified version of the model, one without inter-temporal arbitrage in delivery scheduling, it is possible to obtain some analytical results concerning circumstances under which the negative correlation would emerge. Incorporating inter-temporal arbitrage makes the model analytically intractable, however, so simulation methods must be used to investigate the effect of delivery timing incentives on the dynamics of price and captive supply volumes. The first step is to describe in detail how inter-temporal arbitrage opportunities in cattle delivery timing will be incorporated in the model.

II.1 Inter-temporal arbitrage opportunities in cattle delivery timing

As noted previously, the timing of marketing and delivery of fed cattle is dictated, to a large extent, by biological considerations that, at each point in time, compare the marginal benefit of further weight gain from a marketing delay against the additional cost of feed and the potential for a quality discount for overfat cattle. But sellers may have an incentive to deliver cattle slightly before or after optimal biological potential is reached if this provides an opportunity to take advantage of favorable price movements. The model incorporates these inter-temporal arbitrage opportunities in cattle delivery timing in essentially the same simple and relatively *ad hoc* way used in SA. To review and elaborate, suppose that a feeder owns a cohort of cattle that are "nominally ready" for delivery in week *t*. To say that these cattle are "nominally ready" for week *t* delivery means that it would be optimal, from the seller's perspective, to deliver them in week *t* if cattle prices were expected to remain constant from week to week. The model's assumption is that, given appropriate week-to-week changes in price, it would be profitable to deliver some of these cattle, the most mature among them, one week "early." Likewise, for appropriate week-to-week price movements, it would make sense to deliver some *other* cattle in the cohort, the least mature among them, one week "late." The proportions of the cohort delivered early or late would increase with the magnitude of the week-to-week price changes: With increases in the magnitude of an expected future price increase, for example, it becomes profitable to delay delivery of increasingly mature cattle. The model ignores the possibility of accelerating or delaying delivery of any cattle by two or more weeks, and the possibility that a given pen of cattle might be a candidate for late delivery, under some circumstances, and early delivery, under other circumstances.[8]

More formally, let N_{t-3} denote the market's number of (marketing agreement or spot market) cattle that are nominally ready for delivery in week t.[9] Let s denote the decision date for early delivery, in week t - 1, of some of these cattle. For example, in the case of marketing agreement cattle, $s = t - 3$, because of the stipulation in marketing agreements that requires feeders to fix delivery volume two weeks in advance. In the case of spot market cattle, $s = t - 2$, because spot market cattle delivered one week were

[8] These are relatively strong assumptions that implicitly impose restrictions on the distribution of week-to-week price changes that may occur, and on the arbitrage costs of "off-schedule" delivery. Without these assumptions, the optimal delivery scheduling rule would have to account for the fact that one benefit of delaying delivery by one week, for example, is the opportunity to acquire more information about the advisability of a two week delay. With the assumptions made here, this option value does not arise. So these assumptions greatly simplify the nature of the optimization problem while still allowing some scope for inter-temporal arbitrage in response to price movements.

[9] Henceforth, the convention on time subscripts dates a variable to the week in which its value is determined and becomes known. The size of the cohort of cattle nominally ready for delivery in any given week would be largely the result of placement decisions made many weeks earlier and, therefore, typically would be known, at least approximately, well in advance of the delivery week. In what follows, it is necessary that the sizes of the nominally ready cohorts be know at least three weeks prior to delivery, so the size of the week-t-ready cohort is denoted by N_{t-3}, although N_τ for any $\tau \le t-3$ would serve equally well.

typically sold the previous week. Recall that cattle, whether marketing agreement or spot market, delivered in week t are assumed to be paid the spot market price from the previous week: p_{t-1}. Using "$E_s[\cdot]$" to denote expectations conditioned on week s information, there would be an incentive for early delivery of some of the N_{t-3} week-t-ready cattle if $E_s[p_{t-2} - p_{t-1}]$ were positive. Assume that the proportion of the N_{t-3} week-t-ready cattle for which early delivery would be indicated can be represented by a function, $F_-(\cdot)$, of this expected price change: $F_-(E_s[p_{t-2} - p_{t-1}])$. $F_-(\cdot)$ takes the value zero for argument values less than or equal to zero (because cattle would not be delivered early if prices were expected to increase), and is continuous and non-decreasing for positive values of the argument (because a larger expected decrease in price would justify early delivery of a larger proportion of the cohort).

Now let r denote the decision date for late delivery, in week $t + 1$, of some of the week t ready cattle. For spot market cattle $r = t - 1$, whereas for marketing agreement cattle $r = t - 2$.[10] There would be an incentive for late delivery of some of the N_{t-3} week t ready cattle if $E_r[p_t - p_{t-1}]$ were positive. The proportion of week-t-ready cattle that would be delivered late is represented by $F_+(E_r[p_t - p_{t-1}])$ where $F_+(\cdot)$, like $F_-(\cdot)$, is equal to zero for non-positive values of the argument and continuous and non-decreasing otherwise.[11]

[10] The number of week-t-ready cattle that are delivered in week t would have to be determined in t - 1, in the case of spot market cattle, or in t - 2, in the case of marketing agreement cattle. Under the simple assumptions about inter-temporal arbitrage opportunities, any balance of the week-t-ready cattle that is not committed, at these dates, for week t delivery would, by default, be delivered in week t + 1.

[11] This simple characterization of inter-temporal arbitrage opportunities assumes that individual pens of cattle in the week-t-ready cohort might be candidates for early or late delivery, but not both. One way to incorporate this assumption, without restrictions on the expected week-to-week price changes, would be to require that $F_+(\cdot)$ and $F_-(\cdot)$, while non-decreasing, have asymptotes below 0.5.

II.1.1 Delivery timing for marketing agreement cattle

As noted above, the convention is to use a time subscript that identifies the week in which a variable's value is determined and becomes known. Thus, the number of the market's marketing agreement cattle nominally ready for delivery in week t is $NM_{t\text{-}3}$, the number of marketing agreement cattle delivered in week t is $QM_{t\text{-}2}$, and the spot market price in week t is p_t. The table below provides a reminder of these time indexing conventions.

Marketing agreement feeders:

week	t - 1	t	t + 1	t + 2	t + 3
Number of cattle nominally ready for delivery	$NM_{t\text{-}4}$	$NM_{t\text{-}3}$	$NM_{t\text{-}2}$	$NM_{t\text{-}1}$	NM_t
Number of cattle delivered	$QM_{t\text{-}3}$	$QM_{t\text{-}2}$	$QM_{t\text{-}1}$	QM_t	QM_{t+1}
Spot market price	$p_{t\text{-}1}$	p_t	p_{t+1}	p_{t+2}	p_{t+3}

Consider the determination of QM_t, the number of marketing agreement cattle delivered in week t + 2. The number of cattle nominally ready for delivery in this week is $NM_{t\text{-}1}$. In week t - 1, however, some of the week-t + 2-ready cattle might have been committed to early delivery in week t + 1; or some of the $NM_{t\text{-}2}$ week-t + 1-ready cattle could have been relegated to late delivery in week t + 2. The numbers of cattle involved in off-schedule delivery decisions made in week t - 1 are:

Number of week-t + 2-ready cattle delivered early (in week t + 1) =

$$F_{-}\left(E_{t-1}\left[p_t - p_{t+1}\right]\right) \cdot NM_{t-1} .$$

Number of week-t + 1-ready cattle delivered late (in week t + 2) =

$$F_{+}\left(E_{t-1}\left[p_{t+1} - p_t\right]\right) \cdot NM_{t-2} .$$

Note that one or the other of these expressions will be zero.[12] If the week t - 1 forecast is that spot market price will increase from week t to week t + 1 ($E_{t-1}\left[p_{t+1}-p_t\right]>0$), some of the week-t + 1-ready cattle will be delivered late but none of the week t + 2 ready cattle will be delivered early. If the week t - 1 forecast is that of a price decrease ($E_{t-1}\left[p_t-p_{t+1}\right]>0$), some week-t + 2-ready cattle will be delivered early, but no week-t + 1-ready cattle will be delivered late.

The determination of QM_t is completed with the outcome of decisions made at week t. Based on a week t forecast of the change in spot market price between weeks t + 1 and t + 2, marketing agreement feeders might decide to schedule some week-t + 3-ready cattle for early delivery (in week t + 2); or schedule some week-t + 2-ready cattle for late delivery in week t + 3:

Number of week-t + 3-ready cattle delivered early (in week t + 2) =

$$F_{-}\left(E_t\left[p_{t+1}-p_{t+2}\right]\right)\cdot NM_t .$$

Number of week-t + 2-ready cattle delivered late (in week t + 3) =

$$F_{+}\left(E_t\left[p_{t+2}-p_{t+1}\right]\right)\cdot NM_{t-1} .$$

Finally, QM_t, the number of marketing agreement cattle delivered in week t + 2, will be the number of week-t + 1-ready cattle that are delivered late, plus the number of week-t + 3-ready cattle that are delivered early, plus the number of week-t + 2-ready cattle that are delivered on schedule (neither early nor late):

$$\begin{aligned} QM_t = {} & F_{+}\left(E_{t-1}\left[p_{t+1}-p_t\right]\right)\cdot NM_{t-2}+F_{-}\left(E_t\left[p_{t+1}-p_{t+2}\right]\right)\cdot NM_t + \\ & \left(1-F_{-}\left(E_{t-1}\left[p_t-p_{t+1}\right]\right)-F_{+}\left(E_t\left[p_{t+2}-p_{t+1}\right]\right)\right)\cdot NM_{t-1}. \end{aligned} \tag{1}$$

[12] Of course, both will be zero if the week t - 1 forecast is that of no change in price between weeks t and t + 1.

II.1.2 Delivery timing for spot market cattle

The number of spot market cattle nominally ready for delivery in week t is denoted NS_{t-3}. Because spot market cattle delivered one week were typically sold the previous week, the number of cattle nominally ready for market in week t is therefore given by NS_{t-2}. The number of spot cattle delivered in week t (sold in week t - 1) is denoted QS_{t-1} and, as before, spot market price in week t is denoted p_t. The table below provides a reminder of these notational conventions for the case of spot market cattle.

Spot market feeders:

week	t - 1	t	t + 1	t + 2	t + 3
Number of cattle nominally ready for delivery	NS_{t-4}	NS_{t-3}	NS_{t-2}	NS_{t-1}	NS_t
Number of cattle nominally ready for market	NS_{t-3}	NS_{t-2}	NS_{t-1}	NS_t	NS_{t+1}
Number of cattle delivered	QS_{t-2}	QS_{t-1}	QS_t	QS_{t+1}	QS_{t+2}
Spot market price	p_{t-1}	p_t	p_{t+1}	p_{t+2}	p_{t+3}

Consider the determination of QS_t, the number of spot market cattle delivered in week t + 1. The number of cattle nominally ready for delivery in this week is NS_{t-2}. In week t - 1, the decision might have been made to market some of the NS_{t-2} cattle nominally ready for market in week t early, in week t - 1. Likewise, some of the NS_{t-3} cattle nominally ready for market in week t - 1 might have been withheld for late marketing in week t. The numbers of cattle involved in off-schedule delivery decisions made in week t - 1 are:

Number of cattle nominally ready for market in week t (delivery in week t + 1) that are marketed early, in week t -1 (delivered early, in week t) =

$$F_{-}\left(p_{t-1} - E_{t-1}[p_t]\right)\cdot NS_{t-2}\,.$$

Number of cattle nominally ready for market in week *t* - 1 (delivery in week *t*) that are marketed late, in week *t* (delivered late, in week *t* + 1) =

$$F_{+}\left(E_{t-1}[p_t] - p_{t-1}\right)\cdot NS_{t-3}\,.$$

Delivery timing decisions in week *t* could lead to the withholding, for one week, of some of the $NS_{t\text{-}2}$ cattle nominally ready for market in week *t*, or they could advance, by one week, the marketing of some of the $NS_{t\text{-}1}$ cattle that will be nominally ready for market in week *t* + 1:

Number of cattle nominally ready for market in week *t* + 1 (delivery in week *t* + 2) that are marketed early, in week *t* (delivered early, in week *t* + 1) =

$$F_{-}\left(p_t - E_t[p_{t+1}]\right)\cdot NS_{t-1}\,.$$

Number of cattle nominally ready for market in week *t* (delivery in week *t* + 1) that are marketed late, in week *t* + 1(delivered late, in week *t* + 2) =

$$F_{+}\left(E_t[p_{t+1}] - p_t\right)\cdot NS_{t-2}\,.$$

Finally, QS_t, the number of spot cattle delivered in week *t* + 1 will be the portion of $NS_{t\text{-}3}$ that was committed (in week *t* - 1) to late marketing in week *t*, plus the portion of $NS_{t\text{-}1}$ that was committed (in week *t*) to early marketing in week *t*, plus the portion of $NS_{t\text{-}2}$ that was marketed (in week *t*) and delivered (in week *t* + 1) on schedule:

$$\begin{aligned} QS_t &= F_{+}\left(E_{t-1}[p_t] - p_{t-1}\right)\cdot NS_{t-3} + F_{-}\left(p_t - E_t[p_{t+1}]\right)\cdot NS_{t-1} + \\ &\quad \left(1 - F_{-}\left(p_{t-1} - E_{t-1}[p_t]\right) - F_{+}\left(E_t[p_{t+1}] - p_t\right)\right)\cdot NS_{t-2}\,. \end{aligned} \qquad (2)$$

II.2 The behavior of beef packing plants

Consider the conduct of one of n representative beef packing plants serving the regional market. In week t, the plant manager knows the number of marketing agreement cattle scheduled for delivery to the plant in week $t + 1$, qm_{t-1}. The manager will enter week t's spot market with the objective of purchasing cattle, for delivery in week $t + 1$, up to the point at which marginal cattle cost plus marginal processing cost is equal to the expected marginal revenue product for sales of output in week $t + 1$. Each plant is assumed to be a price taker in both the regional cattle market and the national market in which output is sold.[13] So marginal cattle cost of spot market cattle purchased in week t is just p_t, and expected marginal revenue product is the expected value of the marginal product, or $(1+c)E_t\left[pw_{t+1}\right]\cdot y$, where pw_{t+1} is the boxed beef cutout value, representing the value of the carcass, in week $t + 1$, y is a carcass-to-live weight yield factor, and c is the value of non-carcass by-products expressed as a proportion of carcass value.

Let the packing plant's marginal processing cost, as a function of the number of cattle processed each week, q, be approximated locally by the following linear function: $MPC = a + Bq$, where a and B are parameters. The plant manager will then purchase qs_t spot market cattle in week t, where qs_t satisfies

$$p_t + a + B(qm_{t-1} + qs_t) = (1+c)E_t\left[pw_{t+1}\right]\cdot y\,.$$

Given the values of parameters, $E_t\left[pw_{t+1}\right]$, and qm_{t-1}, the pre-committed volume of week $t + 1$ captive deliveries, this equation implicitly defines the plant's demand for spot market cattle in week t, qs_t, as a function of the spot price p_t. Summing over the

[13] Again, the purpose of this inquiry is to investigate whether a dynamic model of price determination can explain the dynamics of price and captive supply volume without resort to assumptions of imperfectly competitive conduct.

market's *n* plants, dividing by *n*, and rearranging yields the aggregate demand, in indirect form, for spot market cattle in week *t*:

$$p_t = (1+c)E_t[pw_{t+1}]\cdot y - a - b(QM_{t-1} + QS_t), \tag{3}$$

where $b \equiv B/n$.

<u>II.3 Driving processes for the *NM*, *NS*, and *pw* variables</u>

The regional market's number of cattle nominally ready for delivery in a given week is determined, to a large extent by placement decisions made weeks earlier. The wholesale beef market, in which *pw* is determined, is essentially national in scope and insulated, to some extent, from the vagaries of the terms and volume of trade in a single regional fed cattle market. For these reasons, each of these variables; while it may influence the determination of the contemporaneous values of the model's other endogenous variables: QM_t, QS_t, and p_t; is assumed to be uninfluenced by them. Instead, each of NM_t, NS_t, and pw_t is assumed to be determined by its own past history and a contemporaneous random term and is represented by a flexible autoregressive stochastic specification:

$$NM_t = g_1(NM_{t-1}, NM_{t-2}, \ldots) + u_{1t} \tag{4}$$

$$NS_t = g_2(NS_{t-1}, NS_{t-2}, \ldots) + u_{2t} \tag{5}$$

$$pw_t = g_3(pw_{t-1}, pw_{t-2}, \ldots) + u_{3t} \tag{6}$$

where $u_{1t}, u_{2t},$ and u_{3t} represent zero-mean, serially uncorrelated, normally distributed stochastic error terms. The joint distribution of $u_{1t}, u_{2t},$ and u_{3t}, and the functional forms for $g_1(\cdot), g_2(\cdot)$ and $g_3(\cdot)$ remain to be specified.

Note the model's recursive structure. Given expectations, lagged values for all variables, and contemporaneous random error terms, equations (4), (5), (6) each, individually determine variables NM_t, NS_t, and pw_t, respectively. Given NM_t, equation (1) determines QM_t. And equations (2) and (3) jointly determine QS_t and p_t.

III. Calibrating the model

The objective of the current research is to investigate whether the model is capable of generating the pattern of covariation between price and captive supply volume that has been documented in previous studies using historical data, and to determine what role, if any, is played by the delivery timing issues raised by SA. Given this focus, it makes sense to calibrate the model to data reflective of a particular regional market and time frame associated with one such study. Schroeter and Azzam's 1999 GIPSA report which used data from the Texas Panhandle regional fed cattle market for a 66 week period from the week of Monday, February 6, 1995 through the week of Monday, May 6, 1996 is chosen as this benchmark study. The data used for calibration will include some summary statistics contained in the Schroeter and Azzam report, as well as some USDA Agricultural Marketing Service data for the region and time period. The other main source of model calibration information is Duewer and Nelson's (DN) 1991 study providing detailed cost and revenue estimates for beef slaughter and processing plants. DN's dollar value estimates are not directly applicable to the 1995-96 calibration period, however, because they are based on 1988 prices. The general procedure will be to use the DN figures to deduce estimates of normalized values, typically ratios of nominal values for costs or revenues. A scale degree of freedom will be eliminated, and levels of model parameters fixed, when equation (3), with the normalizations imposed, is required to hold at the means of p, pw, QM, and QS from the GIPSA - Texas Panhandle data.

III.1 Estimates of the parameters of the marginal processing cost and the value of marginal product functions: a, b, c, and y

The value of the carcass-to-live weight yield factor, y, is fixed at 0.63, a standard assumption given that dressing yields show little variation, either through time or across plants, about this 63% benchmark figure. Based on 1988 prices, DN estimated revenue per head for slaughter and process plants at $825.57.[14] Of this total, $717.22 represents boxed beef value. The difference reflects the value of non-carcass byproducts, and this difference, as a proportion of $717.22, gives the estimate of c: 0.1511.

Calibration of the parameters of the marginal processing cost function is complicated by the usual difficulty of obtaining reliable inferences about marginal cost from accounting data. The approach uses DN's estimates of packing plant costs and adopts average incremental costs as proxies for marginal cost. The model's local linear approximation to marginal processing cost is then calibrated to exhibit the same elasticity (at the means) with respect to output as evidenced by the DN data. DN estimate processing cost per head for slaughter and process plants of various capacities, operated either one or two shifts per day, for a selection of conventional shift lengths. For

[14] The focus is on slaughter and process plants because the GIPSA - Texas Panhandle plants produced boxed beef.

example, their Appendix Table 15 reports the following estimates for a 120-head-per-hour slaughter and process plant operated one shift per day:[15]

Hours of operation/shift/week	36 (4.5 8-hour days)	40 (5 8-hour days)	48 (6 8-hour days)
Plant slaughter/year (1000 head)	202.50	225.00	270.00
Fixed and variable cost/head ($/head)	81.06	78.38	76.88

Average incremental costs for the transition from 36 to 40, and from 40 to 48, hours of operation per week are $54.26/head and $69.38/head respectively. These are adopted as marginal cost estimates for this plant at the output levels corresponding to the midpoints between the output levels for 36 and 40 hours of operation (213,750 head/year) and between the output levels for 40 and 48 hours of operation (247,500 head/year). Thus, for a plant with these characteristics (120-head-per-hour, slaughter and process, 1 shift per day), within this normal range of operation, the estimates imply that increasing output by 15.8% (213,750 to 247,500 head/year) will increase marginal processing cost by 27.9% (54.26 to 69.38 $/head).

This exercise was replicated using DN's data for the 120-head-per-hour slaughter and process plant operated 2 shifts per day; and for four additional scenarios corresponding to plants with line speeds of 210-head-per-hour and 300-head-per-hour operated, in each case, either one or two shifts per day.[16] Among these six cases, the benchmark output expansion of 15.8% resulted in estimated marginal cost increases

[15] The shift lengths in the table below can be considered characteristic of "normal" operating procedures. The tables in DN also contain estimates for one shorter (32 hour per week) and one longer (50 hour per week) shift.
[16] These estimates appear in DN's Appendix Tables 16, 19, 20, 23, and 24.

ranging from a low of the 27.9% calculated above to a high of 30.1% with an average value of 29.3%.

Again, the calibration of the local linear approximation for the marginal processing cost function assumes that it exhibits the same average elasticity at the means that is implied by the DN data. Thus a 15.8% output increase through a normal range of operation will be assumed to result in a 29.3% increase in marginal cost:

$$a + B\,(1.158)\left(\overline{qm} + \overline{qs}\right) = 1.293\left[a + B\left(\overline{qm} + \overline{qs}\right)\right],$$

where $\overline{qm} + \overline{qs}$ is a representative weekly slaughter volume (of marketing agreement and spot market cattle) for a typical plant. Given *n* plants and recalling that $b \equiv B/n$, the equation becomes

$$a + b\,(1.158)\left(\overline{QM} + \overline{QS}\right) = 1.293\left[a + b\left(\overline{QM} + \overline{QS}\right)\right],$$

where $\overline{QM} + \overline{QS}$ is a representative weekly slaughter volume for the market as a whole. Solving yields a proportionality relationship between parameters *a* and *b*:[17]

$$b\left(\overline{QM} + \overline{QS}\right) = -2.17\,a\,. \qquad (7)$$

For the *p* and *pw* series, data roughly corresponding to the GIPSA - Texas Panhandle study's region and time frame were obtained from USDA Agricultural Marketing Service sources. Data for *pw* were derived from estimated composites of boxed beef cutout values, FOB Omaha, for the weeks of 2/6/95 through 5/6/96. For each reporting day, daily box beef cutout values were averaged for four carcass weight/grade categories: light choice, heavy choice, light select, and heavy select. Each weekly value

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[17] The estimates imply that marginal processing cost is increasing and elastic with respect to output at the mean. Imposing these features on the local linear approximation requires $b > 0$ and $a < 0$.

was calculated as the simple average of the daily averages for reporting days within the week. The 66-week average of the resulting *pw* series is $100.07/cwt carcass.

Data for the *p* series were derived from weighted average prices of steers in the 1100-1250 lb. live weight category in lots grading 35-65% select or choice sold in feedlots in the Texas Panhandle and Western Oklahoma region. The value of *p* for a given week was taken to be the head-weighted average of daily average prices for reporting days within the week. The 66-week average of these prices is $64.88/cwt. live.

In the GIPSA - Texas Panhandle data, spot market and marketing agreements were the predominant, but not the only, fed cattle procurement methods. Spot market cattle and marketing agreement cattle accounted for 71.3% and 21.0% of slaughter, respectively, with the balance of 7.7% made up of forward contract and packer-fed cattle. In the calibration of the model, in which only spot and marketing agreement procurement methods play a role, the representative total weekly slaughter is normalized to the value 100, and then the spot market and marketing agreement components are fixed at values that reflect the same proportional relationship between the two components as in the GIPSA - Texas Panhandle data: $\overline{QS} = 77.25,\ \overline{QM} = 22.75$.

Making use of the estimated values for *c* and *y*, and replacing variables with GIPSA - Texas Panhandle sample means, equation (3) becomes:

$$\overline{p} = 1.1511 \cdot \overline{pw} \cdot 0.63 - a - b\left(\overline{QM} + \overline{QS}\right).$$

Using $\overline{p} = 64.88$, $\overline{pw} = 100.07$, and $\overline{QM} + \overline{QS} = 100$, the above equation can be solved simultaneously with equation (7) to yield: $a = -6.5706$ and $b = 0.1426$.

III.2 Specifying forms for the off-schedule delivery functions

$F_-(\delta)$ ($F_+(\delta)$) is the proportion of the cohort of cattle on feed that would be nominally ready for delivery in a given week but would be delivered one week early (late) given an expected price decrease (increase) of δ from one week to the next. $F_-(\delta)$ ($F_+(\delta)$) will equal zero for negative values of δ because cattle would not be delivered early (late) if price were expected to increase (decrease). Although this approach to modeling the delivery timing decision in relatively *ad hoc*, it is clear that $F_-(\cdot)$ and $F_+(\cdot)$ implicitly reflect the distribution, among pens of cattle, of off-schedule delivery costs within any cohort of cattle nominally ready for delivery in a particular week. Thus it is natural to expect that $F_-(0)=F_+(0)=0$ and that both $F_-(\cdot)$ and $F_+(\cdot)$ would be initially increasing in their arguments: A larger expected price decrease (increase) would increase the proportion of a week's cohort of nominally ready cattle for which early (late) delivery would be profitable.

One simple functional form for $F_-(\cdot)$ and $F_+(\cdot)$ is

$$F_-(\delta) \text{ or } F_+(\delta) = \begin{cases} 0 & \text{for } \delta \le 0 \\ \dfrac{\delta}{\Delta p_{\max}} \cdot \overline{f} & \text{for } 0 < \delta \le \Delta p_{\max} \\ \overline{f} & \text{for } \delta > \Delta p_{\max} \end{cases} \tag{8}$$

Defined in this way, $F_-(\cdot)$ or $F_+(\cdot)$ would increase linearly from 0 to a maximum of $\overline{f}$ as the expected price change increases from 0 to $\Delta p_{\max}$.[18] One critical assumption of the

[18] $F_+(\cdot)$ and $F_-(\cdot)$ need not be symmetrically defined. Even if both functions exhibited the same general form, they could incorporate different values of $\Delta p_{\max}$ and $\overline{f}$.

model is that, within any cohort of cattle on feed, there are some pens that must be delivered in the "scheduled" week and *distinct* sets of pens that are potential candidates for delivery one week early or late. One way to insure the above specification's consistency with this assumption is to require $\overline{f} < 0.5$. So for example, with $\Delta p_{\max} = 1.00$ and $\overline{f} = 0.10$, the implication is that up to 10% of a week's nominally ready cohort delivered either early or late, and this maximum volume of off-schedule delivery would occur with an expected price change of $1.00/cwt. or more.

III.3 Calibrating the driving processes for the predetermined variables

Obtaining data to inform the calibration of the *NS* and *NM* processes (equations (4) and (5)) is a challenge because the concept of a "nominally ready" cohort of cattle, while intuitively appealing, exists only in the abstract. For example, feeders do not routinely note and record the total number of cattle in the pens that they expect will be ready for market in, say, 3 or 4 weeks. Data on weekly cattle marketings could not be expected to mirror the *NS* and *NM* processes because marketing numbers confound week-to-week movements in numbers of nominally ready cattle with the effects of any off-schedule delivery decisions. Because the number of cattle nominally ready for delivery in any one week is dictated, to a large extent, by placement decisions made four to six months earlier, data on placements might offer some insights into the time series behavior of *NS* and *NM*. There are two problems with the use of placement data, however. First of all, weekly (as opposed to monthly) data on placements are not readily available. Secondly, data on placements show a much stronger monthly seasonal component than do data on marketings, suggesting that much of the month-to-month variation in placement numbers is offset by seasonal differences in length of time on feed. Thus one would expect that month-to-month fluctuations in placements would exaggerate the variation in numbers of nominally ready cattle.

Given these calibration difficulties in the cases of the *NS* and *NM* processes, the unconditional means of these series are simply fixed at the GIPSA - Texas Panhandle means of spot and marketing agreement weekly deliveries (normalized to 77.25 and 22.75, respectively) and then low-order stationary autoregressive specifications for equations (4) and (5) are used while allowing for contemporaneous correlation between the error terms u_{1t} and u_{2t}.

The stochastic process for the output price series, the model's equation (6), was calibrated by estimating autoregressive specifications for the demeaned *pw* series by ordinary least squares. A third order autoregressive process appeared to provide an adequate specification: When a fourth lag was added to the model, the adjusted R^2 decreased. Estimating the third order process using 66 weeks of data yielded the following coefficient estimates and standard errors (in parentheses):

$$\left(pw_t - \overline{pw}\right) = \underset{(0.129)}{1.040}\left(pw_{t-1} - \overline{pw}\right) - \underset{(0.184)}{0.307}\left(pw_{t-2} - \overline{pw}\right)$$

$$+ \underset{(0.125)}{0.154}\left(pw_{t-3} - \overline{pw}\right) + u_{3t}$$

Standard error of regression = 1.691; adjusted $R^2 = 0.835$

The point estimates of the parameters satisfied the stationarity conditions for the autoregressive process.[19] The result of a Breusch-Godfrey test for serial correlation (up to order 6) in the u_{3t} process was that the hypothesis of no autocorrelation could not be rejected at conventional significance levels.[20] Consequently the specification for the stochastic process of beef packers' output price (equation (6)) is

$$\begin{aligned}\left(pw_t - \overline{pw}\right) = 1.040\left(pw_{t-1} - \overline{pw}\right) - 0.307\left(pw_{t-2} - \overline{pw}\right)& \\ + 0.154\left(pw_{t-3} - \overline{pw}\right) + u_{3t}&\end{aligned} \tag{6}$$

where $\overline{pw} = 64.88$ and u_{3t} $i.i.d.\ N\left(0, (1.691)^2\right)$.

IV. Solving the model by the extended path algorithm

The model consisting of equations (1) through (6) is a dynamic, nonlinear, rational expectations model.[21] Equations (1) and (2) amount to "no-arbitrage" conditions, for marketing agreement and spot market feeders respectively, insuring that feeders have no opportunity to increase expected net returns by rescheduling deliveries. Equation (3) is the demand for spot market cattle derived from profit maximization by beef packing plants acting as price takers in both the fed cattle and boxed beef markets. Equations (4),

[19] Enders (2003) gives the stationarity conditions for a third-order autoregressive process represented as $y_t - \sum_{i=1}^{3} a_i y_{t-1} = 0$. These include 1 - a_1 - a_2 - a_3 > 0 and three other inequalities, all of which are satisfied at the values of the point estimates of the parameters.

[20] See Greene (2000). The test is performed by regressing the residuals on the models explanatory variables and six (in this case) lags of the residuals. On the null hypothesis of no autocorrelation in u_{3t}, the R^2 from this supplementary regression times the number of observations (66) is distributed as a χ_6^2. The calculated value of the statistic was 5.39, well below the critical value for even a 10% significance level test (10.64).

[21] Although the functional forms for $F_-(\cdot)$ and $F_+(\cdot)$ are somewhat arbitrary, any reasonable specifications would have to contain essential non-linearity insofar as these functions' values will be zero for non-positive arguments yet increasing over at least some range of positive values for the arguments.

(5), and (6) characterize the stochastic processes governing the evolution of the model's pre-determined variables: the sizes of each week's nominally ready cohorts of marketing agreement and spot market cattle, and the price in the national market for the boxed beef output.

The model is a *rational expectations* model in that it involves conditional forecasts, based on the model itself, of future values of endogenous variables. In particular, solving the model for variables dated t requires conditional forecasts of prices in periods $t + 1$ and $t + 2$, and obtaining these forecasts requires, in turn, solution of the model for these dates. Fair and Taylor (1983) developed an iterative numerical procedure, the "extended path algorithm," for solving models with this feature.

IV.1 A simple illustration

The logic of the extended path algorithm can be illustrated using the following simple example. Consider the model with endogenous variables x_t and y_t:

$$y_t = f\left(x_t, x_{t-1}, E_t\left[x_{t+1}\right]\right)$$

$$x_t = g\left(y_t\right) + u_t$$

$$u_t \quad i.i.d. \;\; N\left(0, \sigma^2\right)$$

for t = 1, 2, . . ., where $x_0 = \bar{x}_0$ is given. Solution of the model for period t requires the expectation, conditioned on current information, of x_{t+1}. The basic idea of the algorithm is to start with initial (essentially arbitrary) guesses for expectations of a finite sequence of future xs permitting recursive solution of the model forward over an "extended path." These solutions are used to stochastically simulate the expectations leading to refined estimates which are then used to update the initial guesses. With the updated expectations, the model can be recursively solved over the extended path once again. This process can be iterated in an obvious way until the provisional guesses for future expected values sufficiently closely match the conditional expectations calculated from the model. In the literature, these iterations over the vector of expectations are called "Type II" iterations.[22]

In more detail, numerical solution of the model for date $t = 1$ would start with initial guesses for

$$E_1[x_2], E_1[x_3], E_1[x_4], \ldots, E_1[x_{k+1}],$$

where k denotes the length of the extended path. For $h = 1, 2, \ldots, H$, draw

$$u_1^h, u_2^h, \ldots, u_k^h \quad i.i.d. \;\; N(0, \sigma^2)$$

and solve the model recursively for

$$(y_1^h, x_1^h), (y_2^h, x_2^h), \ldots, (y_k^h, x_k^h)$$

using $x_0 = \bar{x}_0$ and the initial guesses for expectations. Then, for $s = 2, 3, \ldots, k$, calculate the sample counterpart to $E_1[x_s]$ as

$$\frac{1}{H}\sum_{h=1}^{H} x_s^h .$$

These refined estimates of the expectations of future xs are then compared to the initial guesses. If the two sets of estimates are not yet sufficiently close, as judged by an appropriate convergence criterion, the updated estimates replace the initial guesses and the entire process is repeated. When convergence in the vector of expectations is achieved, a final value of u_1 is drawn from $N(0, \sigma^2)$ and the current estimate of $E_1[x_2]$ is

[22] "Type I" iteration is the term reserved for any iterations that may be required in a numerical solution algorithm needed to solve the model's nonlinear equations for a given period and a given vector of expectations.

used to solve the model for $t = 1$ yielding values for y_1 and x_1. Accuracy can be improved by using a more severe convergence criterion and by increasing H, the number of stochastic simulation draws. The results should also be checked for sensitivity to the choice of k, the length of the extended path. The value of k should be increased until further incremental increases have negligible impacts on the solution. With the model now solved for $t = 1$, the process can be continued to generate entire solution time series.

IV.2 Adapting the basic method to the current model

The model of equations (1) through (6) involves complications that require extensions of the basic method outlined above. The extension to six endogenous variables and six equations is straightforward. There are three jointly normally distributed stochastic error terms, perhaps exhibiting some contemporaneous correlations among them. Standard random number generator routines are available to easily handle this kind of problem. Unlike the simple illustrative case, the model of equations (1) through (6) involve expectations with both one- and two-period forecast horizons but this, too, requires only a trivial modification of the basic procedure because provisional estimates of expectations with forecast horizons up to the length of the extended path are available at each iteration of the process.

A more substantive complication in the model of equations (1) through (6) is the presence of two "view-points;" that is, two dates, t and t - 1, upon which expectations are conditioned. This complication is handled in the following way. The solution of the model for the initial period, $t = 1$, begins with a vector of provisional guesses for "current-view-point" expectations:

$$E_1[p_2], E_1[p_3], E_1[p_4], \ldots, E_1[p_{k+2}],$$

and a vector of provisional guesses for "lagged-view-point" expectations; that is, expectations conditioned on date $t = 0$ information:

$$E_0[p_1], E_0[p_2], E_0[p_3], \ldots, E_0[p_{k+1}].$$

The extended path algorithm is used to obtain a solution for period $t = 1$ by iterating over the current-view-point expectations until convergence, while holding the lagged-view-point expectations fixed throughout. Advancing to period $t = 2$, the vector of current-view-point expectations is reset to initial arbitrary guesses and the vector of lagged-view-point expectations is set to the vector of convergent values of the current-view-point expectations from the solution for $t = 1$. Thus, the updating of the estimates of lagged-view-point expectations occurs as the algorithm proceeds from one solution date to the next. To insure convergence in both lagged- and current-view-point expectations, the first several periods of the generated time series are discarded, and the results are checked for sensitivity to the length of this initial "burn-in" period.

IV.3 Outline of the extended path algorithm

The extended path algorithm for the dynamic rational expectations model of delivery timing was implemented by a FORTRAN program using subroutines from the NAG Library for random number generation and for the key computational step in solution of the model for a given date: finding the root of a nonlinear equation. The following is an outline of the extended path algorithm.[23] Throughout, let t denote the date for which the model is currently being solved.

[23] A copy of the program code is available from the author upon request.

Set values of lagged endogenous variables needed for solution of the model for the first solution date (t = 1): NS_0, NS_{-1}, NS_{-2}, NM_0, NM_{-1}, NM_{-2}, pw_0, pw_{-1}, pw_{-2}, QM_0, and p_0.

Specify initial "estimates" (guesses) for lagged view-point expectations (expectations conditioned on information available one period prior to the first solution date): $E_0\lfloor p_j \rfloor$ for $j = 1, 2, \ldots, k+4$, where k is the length of the extended path.

Set t = 1.

(*) Specify initial "estimates" (guesses) for current view-point expectations (expectations conditioned on information available on the current solution date): $E_t\lfloor p_{t+j} \rfloor$ for $j = 1, 2, \ldots, k+4$.

(**) For $h = 1, 2, \ldots, H$, draw $\left(u_{1t}^h, u_{2t}^h, u_{3t}^h\right), \left(u_{1t+1}^h, u_{2t+1}^h, u_{3t+1}^h\right), \ldots, \left(u_{1t+k+2}^h, u_{2t+k+2}^h, u_{3t+k+2}^h\right) i.i.d.\ N(0, \Omega)$, where Ω is the covariance matrix of the stochastic error terms, and solve the model recursively for periods t, t + 1, . . ., t + k + 2; yielding solutions for price: p_s^{*h} for s = t, t + 1, . . ., t + k + 2.

Calculate $\frac{1}{H}\sum_{h=1}^{H} p_s^{*h}$ for s = t + 1, t + 2, . . ., t + k + 2, and compare to $E_t\lfloor p_{t+j} \rfloor$ for j = 1, 2, . . ., k + 2. If the old estimates and the new estimates of current view-point expectations are not sufficiently close, update the old guesses and go to (**).

If the old guesses and the new estimates are sufficiently close, draw $\left(u_{1t}, u_{2t}, u_{3t}\right) \sim N(0, \Omega)$ and solve the model for period t.

Reset lagged view-point expectations to convergent values of current view-point expectations: $E_t\lfloor p_{t+j} \rfloor$ for $j = 1, 2, \ldots, k+4$. Reset lagged endogenous variables to reflect solutions for the current period.

If t is less than the number of periods that are to be solved, set t = t + 1 and go to (*).

V. Replicating the Schroeter/Azzam regressions with simulated data

As mentioned in section I, one of the studies that found a negative correlation between captive supply delivery volume and spot market price is GIPSA's "Texas Panhandle" investigation (Schroeter and Azzam, 1999). In that study, data on cattle procurement by four large packing plants in the Texas Panhandle in 1995 and 1996 were used as a basis for investigating the empirical relationship between spot price and captive deliveries in the region's fed cattle market.[24] The report includes results of time series regressions estimated using 66 weekly observations. In the notation of the GIPSA report (p. 40, equation (3)), the regression models were of the following form:

$$\begin{aligned} average\ price\ in\ week\ t = \gamma_0 + \gamma_1 AVGVAL_t + \gamma_2 Q_t + \\ \gamma_3 (non-cash\ deliveries\ in\ week\ t) + \\ \gamma_4 WEEK_t + \gamma_5 WEEK2_t + \varepsilon_t \end{aligned} \tag{9}$$

The dependent variable represents the region's average spot market cattle price in week *t*, and corresponds to p_t of the simulation model. $AVGVAL_t$ is a measure of the price of the packers' output in week *t*, the counterpart of the simulation model's pw_t. Q_t is the number of steers and heifers purchased on the spot market by the GIPSA study's four Texas plants in week *t*, the counterpart of the simulation model's QS_t. "Non-cash (captive) deliveries in week t" were measured in two ways: by the total number of head of fed cattle procured by non-cash methods and delivered to the four Texas plants in week *t*, and by this number expressed as a proportion of the plants' total slaughter volume in week *t*.[25] The GIPSA study's total head measure of captive deliveries corresponds to the simulation model's QM_{t-2}. Finally, $WEEK_t$ is a simple time trend and $WEEK2_t$ is the square of $WEEK_t$, so that these two terms contribute a quadratic time trend to the regression. With these adjustments in notation, and concentrating on the version of the

[24] The analysis implicitly assumed that the purchases of the four surveyed plants represent essentially all of the trading volume in the relevant regional market.

[25] In the simulation model, marketing agreements represent the only non-cash procurement method. Non-cash purchases by the four plants in the GIPSA investigation included marketing agreement cattle, forward contract cattle, and packer-fed cattle, although marketing agreements accounted for 73% of captive supplies.

model with the "total head" measure of captive deliveries, the Schroeter/Azzam regression model becomes:

$$p_t = \gamma_0 + \gamma_1 pw_t + \gamma_2 QS_t + \gamma_3 QM_{t-2} + \gamma_4 WEEK_t + \gamma_5 WEEK2_t + \varepsilon_t \tag{10}$$

The regressions were estimated using each of the alternative measures of captive deliveries combined with each of four different, but very similar, measures of the dependent variable. The estimation was conducted using the Yule-Walker procedure to correct for serial correlation in ε_t.[26] Again, concentrating on the results obtained using the "total head" measure of captive deliveries (Table VII.2.2 in Schroeter and Azzam, 1999), one robust outcome, as previously mentioned, is a significantly negative estimated coefficient on the captive supply variable ($\hat{\gamma}_3 < 0$). Another consistent finding across regressions is a significantly positive estimated coefficient on spot market quantity ($\hat{\gamma}_2 > 0$).[27]

To explore the simulation model's implications for the time series properties of the endogenous variables, start with a simple version of the model for which some analytical results can be easily obtained. Assume no inter-temporal arbitrage in delivery scheduling, corresponding to $F_+(\cdot)$ and $F_-(\cdot)$ functions that are identically zero. In this case, equations (1) and (2) reduce to

$$QM_t = NM_{t-1} \tag{1'}$$

$$QS_t = NS_{t-2} \tag{2'}$$

With these simplifications, equation (3) becomes

[26] Estimation was also conducted using a two-stage-least-squares procedure to account for the endogeneity of Q_t, producing results that were very similar qualitatively.
[27] This sign pattern also characterized the results of two-stage-least-squares estimation although, in that case, the spot market quantity coefficient estimates were insignificantly positive.

$$p_t = (1+c)E_t[pw_{t+1}] \cdot y - a - b(NM_{t-2} + NS_{t-2}) \tag{3'}$$

Regarding the model's driving processes, assume that the deviations of NM_t and NS_t from their unconditional means are first-order autoregressive processes:

$$nm_t = m_1 nm_{t-1} + u_{1t} \tag{4'}$$

$$ns_t = s_1 ns_{t-1} + u_{2t} \tag{5'}$$

where nm_t and ns_t represent NM_t and NS_t in deviation form. The innovations in these processes, u_{1t} and u_{2t}, are assumed to be independent of the innovation, u_{3t}, in the pw_t process given by equation (6). Finally, assume that the vector (u_{1t}, u_{2t}) is *i.i.d.* normal with

$$E[u_{1t}^2] = \sigma_1^2, \quad E[u_{2t}^2] = \sigma_2^2, \quad \text{and} \quad E[u_{1t}u_{2t}] = \rho\sigma_1\sigma_2 \quad \text{for all } t.$$

Return to equation (10), substitute for p_t, QS_t, and QM_t using equations (1'), (2'), and (3'), transfer the term in pw_t to the left-hand-side, drop the quadratic time trend for simplicity, and subtract unconditional means from both sides of the equation. The result is a regression equation with dependent variable $y_t \equiv v_t - b(nm_{t-2} + ns_{t-2})$, where v_t incorporates current and lagged values of pw_t and is independent of nm_s and ns_s for all *t* and *s*; and vector of regressors $x_t \equiv (ns_{t-2}, nm_{t-3})$. The ordinary least squares estimates of this regression, based on a sample of size *T*, are given by

$$\begin{pmatrix} \hat{\gamma}_2 \\ \hat{\gamma}_3 \end{pmatrix} = \left(\sum_{t=1}^{T} x_t' x_t \right)^{-1} \left(\sum_{t=1}^{T} x_t' y_t \right),$$

with

$$p\lim\begin{bmatrix}\hat{\gamma}_2\\ \hat{\gamma}_3\end{bmatrix}=\left(p\lim\frac{1}{T}\sum_{t=1}^{T}x_t'x_t\right)^{-1}\left(p\lim\frac{1}{T}\sum_{t=1}^{T}x_t'y_t\right)$$

$$= -b\begin{bmatrix}\dfrac{\sigma_2^2}{1-s_1^2} & \dfrac{s_1\rho\sigma_1\sigma_2}{1-m_1s_1}\\ \dfrac{s_1\rho\sigma_1\sigma_2}{1-m_1s_1} & \dfrac{\sigma_1^2}{1-m_1^2}\end{bmatrix}^{-1}\begin{bmatrix}\dfrac{\rho\sigma_1\sigma_2}{1-m_1s_1}+\dfrac{\sigma_2^2}{1-s_1^2}\\ \dfrac{m_1\sigma_1^2}{1-m_1^2}+\dfrac{s_1\rho\sigma_1\sigma_2}{1-m_1s_1}\end{bmatrix}. \tag{11}$$

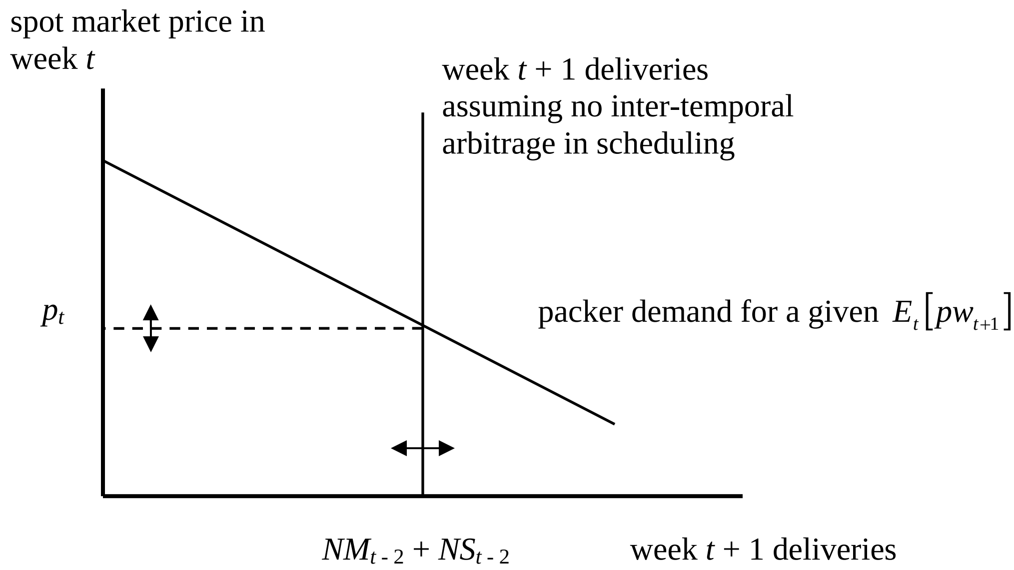
For purposes of illustration, momentarily restrict attention to a case involving one further simplification: $s_1 = 0$, so that the NS_t process is serially uncorrelated. Noting that *b* is a positive parameter, equation (10) yields for this special case:

$$sign\left(p\lim\begin{bmatrix}\hat{\gamma}_2\\ \hat{\gamma}_3\end{bmatrix}\right)=sign\begin{bmatrix}-\left(\dfrac{\rho\sigma_1}{\sigma_2}+1\right)\\ -m_1\end{bmatrix}. \tag{12}$$

As noted earlier, the Schroeter/Azzam regressions from the GIPSA Texas Panhandle study consistently yielded $\hat{\gamma}_2 > 0$ and $\hat{\gamma}_3 < 0$. For the special case underlying equation (12), this sign pattern would be expected to emerge if the nm_t process exhibits positive first-order autocorrelation ($m_1 > 0$) and if $\rho\sigma_1/\sigma_2 < -1$. This last condition could be satisfied if the innovation in the nm_t process (u_{1t}) has a greater standard deviation than the innovation in the ns_t process (u_{2t}); that is, $\sigma_1 > \sigma_2$; and if these two innovations are negatively correlated contemporaneously: $\rho < 0$.[28]

[28] The plausibility of these restrictions on the parameters of the NM_t and NS_t processes is somewhat difficult to assess because these variables are unobservable. In the calibration of the model, based on data from the GIPSA Texas Panhandle study, the mean of QM_t is roughly 30% of the mean of QS_t. Presumably, a similar proportional relationship would hold between the means of NM_t and NS_t. The requirement that $\sigma_1 > \sigma_2$ therefore implies much greater relative week-to-week variability in NM_t than in NS_t and, presumably, in QM_t than in QS_t. C. Robert Taylor (2006), the plaintiff's chief economist in the recent *Pickett* case (See footnote 1.), reports that Tyson's weekly captive supply slaughter exhibited a coefficient of variation that was 2.8 times higher than the coefficient of variation for their weekly cash

The intuition of this result can be seen graphically.



For a given $E_t[pw_{t+1}]$, equation (3') gives week t price, p_t, as a decreasing function of total deliveries in week t + 1. In the present case of no inter-temporal arbitrage in delivery timing, week t + 1 deliveries, $QM_{t-1} + QS_t$, are given by $NM_{t-2} + NS_{t-2}$. Thus, for a given $E_t[pw_{t+1}]$, p_t and $NM_{t-2} + NS_{t-2}$ will be negatively correlated. Given $\sigma_1 > \sigma_2$ and $\rho < 0$, however, "large" realizations of $NM_{t-2} + NS_{t-2}$ will tend to result from "large" realizations of NM_{t-2} and "small" realizations of NS_{t-2}. Thus p_t will be negatively correlated with $NM_{t-2} = QM_{t-1}$ and positively correlated with $NS_{t-2} = QS_t$. Furthermore, given positive serial correlation in the NM_t process ($m_1 > 0$), the negative correlation between p_t and $NM_{t-2} = QM_{t-1}$ could induce a negative correlation between

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cattle slaughter. There is some intuition to support $\rho < 0$. NM_t and NS_t are determined, in large part, by feeder cattle placements made five months (a typical length of time on feed) prior to week t. With a fixed supply of feeder cattle to be allocated among feedlots, one would expect that larger than normal allocations to spot market feedlots would tend to go along with smaller than average allocations to marketing agreement feedlots in any given week.

p_t and $NM_{t-3} = QM_{t-2}$. In terms of the coefficients of equation (10), the result is consistent with the $\hat{\gamma}_2 > 0, \hat{\gamma}_3 < 0$ pattern found in the Schroeter/Azzam regressions.[29]

VI. Simulations

The simulation-based inquiry into the effects of inter-temporal arbitrage in delivery scheduling will employ the following strategy. Start with parametrizations for which the model with no inter-temporal arbitrage implies $p\lim\hat{\gamma}_2 > 0$, a consistent finding in the Schroeter/Azzam estimations of equation (9), and $p\lim\hat{\gamma}_3 = 0$, implying no (partial) correlation between captive deliveries and contemporaneous spot market prices. Then incorporate inter-temporal arbitrage into the model using the specifications of the $F_-(\cdot)$ and $F_+(\cdot)$ functions given in equtation (8) with $\Delta p_{\max} = 1.00$ and $\bar{f} = 0.10$. Again, this implies that a maximum of 10% of a weeks nominally ready cohort of cattle could be rescheduled for delivery one week early or late, and that this maximum volume of off-schedule delivery would occur with an expected price change of $1.00/cwt. or more. Then use the extended path algorithm to carry out simulations of the model with inter-temporal arbitrage, generating several ensembles of the model's endogenous variables for each parametrization. The length of the time series in the Schroeter/Azzam regressions, 66 weeks, is chosen as the length of the simulated ensembles. For each ensemble, estimate equation (10) to reveal the sampling distributions of $\hat{\gamma}_2$ and $\hat{\gamma}_3$.

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[29] The simple case in which $s_1 = 0$, is meant to be illustrative. Using equation (11), the same sign pattern can be shown to occur with $s_1 = 0.5, m_1 = 0.5, \sigma_1 = 4.0, \sigma_2 = 2.0,$ and $\rho = -0.7$, and with $s_1 = 0.5, m_1 = 0.5, \sigma_1 = 4.0, \sigma_2 = 3.0,$ and $\rho = -0.9$, for example.

The parameter scenarios will assume that nm_t and ns_t are first-order autoregressive processes with innovations that are uncorrelated with the innovation in the pw_t process. The first scenario with further assume $s_1 = 0$. From equation (12), $s_1 = 0$ combined with $m_1 = 0$, $\sigma_1 / \sigma_2 = 2.0$, and $\rho = -0.7$ implies $p\lim \hat{\gamma}_2 > 0$ and $p\lim \hat{\gamma}_3 = 0$.

The second and third parameter scenarios will maintain the assumption of first-order autoregressive processes for nm_t and ns_t, but will allow $s_1 \neq 0$. From equation (11), it can be shown that $p\lim \hat{\gamma}_3 = 0$ requires

$$\rho^2 = \frac{2m_1^2 s_1 - m_1 - m_1^3 s_1^2}{m_1^2 s_1 - m_1^2 s_1^3 - s_1 + s_1^3}. \tag{13}$$

So, for example, with $m_1 = 0.4$ and $s_1 = 0.6$, equation (13) implies $\rho = \pm 0.84632727$. Taking the negative value for ρ and combining these parameters with $\sigma_1 / \sigma_2 = 2.0$, from equation (11) the implication is $p\lim \hat{\gamma}_2 > 0$ and $p\lim \hat{\gamma}_3 = 0$ as desired. Likewise, with $m_1 = 0.1$, $s_1 = 0.5$, equation (13) implies $\rho = \pm 0.49304933$. The negative value for ρ with $\sigma_1 / \sigma_2 = 6.0$ again gives $p\lim \hat{\gamma}_2 > 0$ and $p\lim \hat{\gamma}_3 = 0$.

For each of the three parameter scenarios, the extended path algorithm was used to generate ten ensembles of the model's endogenous series, each 66 weeks long.[30] For each of the three parameter scenarios, equation (10) was estimated by ordinary least-

[30] The tuning parameters of the extended path algorithm were set as follows. The length of the extended path and the length of the burn-in period were set at 20 weeks. The number of stochastic simulation draws for each Type II iteration was 2000. The criterion for declaring convergence of Type II iterations was that the maximum (over the extended path) absolute difference between old and updated expectation estimates be no bigger than 0.05; that is, 5 cents/cwt. Convergence typically required between 8 and 12 Type II iterations for each solution period. The simulation of each ensemble required approximately 20 minutes of CPU time on a UNIX workstation with a 500 MHz processor.

squares using each of the ten simulated ensembles of data for that scenario.[31] The table below provides some summary statistics on the sampling distribution of *t*-values for $\hat{\gamma}_2$ and $\hat{\gamma}_3$. A negative sign for the *t*-statistic indicates a negative parameter estimate.

Parameter Scenario		*t*-statistics for test of H_0: parameter = 0	
		γ_2	γ_3
$\sigma_1 = 6.0, \sigma_2 = 3.0$	mean	-6.74	-1.69
$\rho = -0.7$	max	-4.44	-0.15
$m_1 = 0.0, s_1 = 0.0$	min	-8.36	-3.95
$\sigma_1 = 6.0, \sigma_2 = 3.0$	mean	-6.43	-3.58
$\rho = -0.84632727$	max	-4.68	-2.73
$m_1 = 0.4, s_1 = 0.6$	min	-9.01	-5.02
$\sigma_1 = 6.0, \sigma_2 = 1.0$	mean	-5.68	-2.22
$\rho = -0.49304933$	max	-3.75	-0.24
$m_1 = 0.1, s_1 = 0.5$	min	-8.19	-3.75

The introduction of inter-temporal arbitrage has dramatic effects on the implications of the model for the time series properties of spot market price and quantity, and captive supply delivery volume. In each of the three scenarios, parameters were fixed at values for which the model with no inter-temporal arbitrage would imply $p\lim\hat{\gamma}_2 > 0$ and $p\lim\hat{\gamma}_3 = 0$. Yet when inter-temporal arbitrage is introduced, the model's simulated data generate uniformly negative estimates of both γ_2 and γ_3. The estimates of γ_2 were statistically significant at the 1% level in every case. The estimates of γ_3, the coefficient of captive delivery volumes, were statistically significant at the 1% level for 53% (16/30) of the ensembles, and at the 10% level in 73% (22/30) of the

[31] In Schroeter and Azzam (1999), estimation was conducted using the Yule-Walker procedure to correct for serially correlated error terms. In the regressions using simulated data, Durbin-Watson statistics suggested that no serial correlation correction was necessary. The results in the table are from estimation of equation (10) with the quadratic time trend omitted. Estimates of the coefficients of the trend terms were seldom significant. Including these terms had little effect on estimates of γ_2 and γ_3.

ensembles. These findings do illustrate the potential of inter-temporal arbitrage in delivery scheduling to generate a negative correlation between price and captive supply deliveries and, to that extent, they provide some support for the suggestion that this empirical regularity may simply be an artifact of delivery timing decisions made by price-taking agents. These findings do not, however, fully rationalize regression results like those obtained in Schroeter and Azzam, 1999: The negative estimates of γ_2, emerging from the model's simulated data, are in stark contrast to the Schroeter/Azzam regression results obtained using actual market data.

VII. Summary and Conclusions

The purpose of this investigation was to attempt to provide an explanation for an empirical regularity often found in fed cattle market data: Price in a regional market tends to be negatively correlated with the market's contemporaneous volume of captive supply deliveries. Some have suggested that this correlation is evidence of a causal relationship *from* captive supplies *to* spot price, whereby packers can strategically engineer lower spot market prices by engaging in more captive supply procurement activity. A recent paper by Schroeter and Azzam (SA, 2004) suggests that the negative correlation might, instead, be an artifact of cattle delivery timing decisions made by price-taking market participants. The purpose of this investigation, more specifically, was to formalize the SA argument in a dynamic rational expectations model of cattle delivery timing, and use the model as a basis for testing the SA conjecture.

In the model, cattle feeders preparing cattle for sale, either on the spot market or under the terms of marketing agreements, schedule delivery timing to take advantage of expected price changes. Competitive packing plants conduct spot market procurement operations with a profit maximization objective. Random elements are introduced through stochastic specifications describing the evolution of the model's pre-determined driving processes: wholesale beef price and the sizes of cohorts of cattle "nominally ready" for delivery each week. Actual market data, some of it obtained as part of the GIPSA - Texas Panhandle study (Schroeter and Azzam, 1999), were used to calibrate the model.

The simple version of the model, without active delivery timing decisions, can be solved analytically to show that, for certain parametrizations, results consistent with previous regression findings do obtain. In particular, given certain parameter scenarios, the implications of the simple model are qualitatively compatible with the regression results reported in the GIPSA - Texas Panhandle study (Schroeter and Azzam, 1999). Unfortunately, it is difficult to judge the realism of the parameter scenarios giving rise to these results because the modeling construct of the "nominally-ready-for-delivery" cohort, while intuitively plausible, is unobservable, leaving little concrete basis for calibrating the parameters of these processes.

When active delivery scheduling decisions are incorporated into the model, it becomes analytically intractable, requiring numerical solution using the extended path algorithm. In the analysis of this version of the model, set parameters at values which would imply no correlation between spot price and captive deliveries if there were no active delivery scheduling. Starting from these benchmarks, a simulation/regression approach is used to discover the effects of inter-temporal arbitrage. The results provide only partial support for the SA conjecture: There is a strong tendency for inter-temporal arbitrage in delivery scheduling to induce negative correlations between spot market price and captive supply delivery volumes. The results do not fully rationalize market-data regression findings like those in Schroeter and Azzam (1999), however. Although experimentation has been limited to only a few parameter scenarios, other key aspects of the Schroeter/Azzam regression findings have not, as yet, been replicated with data from the simulation model.

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