INTRODUCTION

A formal yet widely applicable definition of inversion is difficult to find; this has led to Flax et al [1] to comment 'Inverse scattering means many things to many people'. However, the inverse problem, viewed with particular reference to defect characterisation, can be regarded as gaining information on the features of an unknown or concealed body which can be made to cause a disturbance in an interrogating field. Thus imaging can be classed a non parametric inversion and indeed imaging and inversion processes have been shown to be mathematically equivalent under certain conditions [2]. Solutions to an inverse problem can be divided into two groups: direct and indirect. The direct method involves mathematical operations (usually transforms) for which the experimental data are the input and the interpretation of those data is the output. Indirect inversion, on the other hand, means finding the best fit between experimental data and a previously assumed theoretical model, and is usually an iterative process.

This paper examines two inversion methods. The 1-D Born technique is a direct inversion algorithm utilising frequency information in the intermediate range (approx 0.5 to 2.5 ka, \( k = 2\pi /\text{wavelength}, \ a = \text{flaw radius} \)) which has been found to be robust in the presence of high noise levels.

An indirect inversion method has also been examined and initial results with bandlimited noisy synthetic data and experimental samples have been encouraging. The technique involves parameterising the features of the flaw about which information is required and then searching the parameter space for the best fit between flaw and ideal data.

INVERSION: PROBLEM AND SOLUTION

The inverse problem can be illustrated schematically as

\[
\text{observed disturbance} \xrightarrow{\text{infer}} \text{model of object}
\]
and

Forward Problem

model of object determine disturbance produced

More formally most inverse problems can be reduced to solving an integral equation of the type

\[ g(\tau) = \int_{0}^{\infty} K(\nu, \tau) p(\nu) \, d\nu \quad 0 \leq \tau < \infty \]

for \( p(\nu) \) from measurements of \( g(\tau) \), where \( K \) is the kernel relating the governing parameters \( \tau \) and \( \nu \); i.e. a Fredholm integral equation of the first kind (an equivalent matrix formulation can also be used).

Thus the objective of inverse methods can be viewed as the determination of one or more of the parameters in the governing equations or system of equations of some process. The problem of solving such equations is basically ill-conditioned and this causes difficulties in practical inversion techniques.

The potential method of solution to the inverse problem must accommodate four important considerations.

Existence

Before solution parameters are calculated it must be considered whether in fact a solution exists within the limitations of the inversion model assumptions. Although in some cases existence conditions can be established [4] it is also possible that the inversion algorithm will produce an 'answer' within a very limited model which is unrelated to the true solution. It may, therefore, be necessary to ensure that all possible solutions are included in the model.

Uniqueness

The solution of the inverse problem is not unique for cases involving experimental data which are incomplete or inexact (noisy) since incomplete data must result in an incomplete solution. Thus experimental data inversion cannot be regarded as a deterministic problem. However, methods do exist for establishing the significance of a particular solution [5,6] within a solution set. It may be the case that the common features (if any) in the range of possible solutions may be sufficient information. Alternatively, further assumptions about the physical situation or further experimental measurements could be made to narrow the class of possible solution. The considerations of existence, requiring a model with a large possible solution set, and uniqueness, requiring limitations on the solution set so that meaningful information can be obtained, must be balanced against each other.

Stability

A problem is defined as stable if the solution depends continuously on the data. If small perturbations in the data (such as noise) cause large changes in the solution, the problem is unstable or ill-conditioned. In fact, the majority of inverse problems are ill-conditioned [7]. The stability of a particular solution can be measured [8] and the problem is often dealt with by considering the smoothness of the model.
Solution Construction

This is the aspect of inversion which receives most attention and is obviously very dependent on the physics of the situation and the forward models available. What is required is an algorithm which finds a solution to a specified precision within a finite number of iterations. In order to solve an inverse problem the related forward problem must be fully understood and it is often the case that the forward problem must be solved in conjunction with the inverse problem [9] and these must be uncoupled in some way, say by a simplifying approximation of the forward problem.

The method of solution is dependent on the initial formulation of the problem (for example, matrix, integral or differential) and include trial and error techniques, approximations, iterative procedures, transforms, and exact analytical solutions.

BORN INVERSION

The basic technique for obtaining suitable ultrasonic signals for 1-D Born Inversion is well documented elsewhere [10,11]. Therefore, only the major differences between our experimental/development protocol and other published works will be outlined. The transducers used were a specially constructed lead metaniobate probe and a commercial contact probe. The samples were diffusion bonded Titanium alloy or maraging steel blocks containing spheroidal or ellipsoidal voids.

The Born Inversion gives good sizing results for strong scatterers such as voids in metals although it is derived for weak scatterers. The inversion was therefore examined using exact scattering data from spheroidal voids and experimental data. The technique operates on the real part of the back-scattered frequency spectrum which has been time-shifted such that the flaw centre corresponds to the time origin. A comparison of the real part of the time-shifted spectra for experimental data [11], exact theoretical data [12] and Born theoretical data [13], which have each gone through the same signal processing procedure shows good correspondence.

Analysis of the back-scattered time domain signal from a weak scatterer shows reflections from the front and back of the inclusion as well as other contributions, not fully resolved due to the signal being obtained from a bandlimited frequency spectrum. A similar plot for a void also shows a secondary signal due to creeping wave circumnavigating the flaw surface.

It is expected, therefore, that the Born Inversion will work reasonably well for voids in elastic media which support a creeping wave of detectable magnitude. However, it should be noted that for a weak scatterer the signal path difference between front and back face is proportional to 4a, (a = flaw radius), whereas for a void where the creeping waves are tangentially launched and re-radiated the path difference is \((2+\pi)a\). This is a 28% change in pathlength if the longitudinal host velocity and creeping wave velocity are approximately equal. In fact, if strong and weak scattering ideal data are treated in an equivalent manner in the inversion algorithm the strong scatterer does give larger radius predictions although not as significantly different as expected from path and velocity differences (see Table 1).

However, for small voids, low signal to noise ratios or voids with rough uneven surfaces it is unlikely that creeping waves will be detected. The effect on the inversion of degrading the creeping wave contribution,
however, is not serious. For example, reducing the secondary impulse amplitude by over 80% causes a change of less than 8% in the Born radius prediction.

The second major problem was that of accurately obtaining the time shift required to locate the flaw centroid and various methods were investigated to this end.

(i) Area function: This method examines the cross-sectional flaw area normal to the direction of wave propagation which is a maximum at the flaw centroid [14].

(ii) Maximum flatness: Using ideal Born data it is found that if the Born predicted radius is calculated for a large number of time shifts, a pattern is produced where the correct time shift is located where the function is maximally flat [15].

(iii) Minimisation of imaginary part (MOIP): It can be shown that for an ideal Born scatterer only the real part of the back-scattered spectrum exists if the flaw centroid corresponds to the time origin. Owing to noise, the imaginary part of the experimental data spectrum remains finite; however, the integral of this (imaginary part of spectrum against frequency) tends to a minimum at the correct time shift.

(iv) Low-frequency examination: Low-frequency information in the range $ka < 0.5$ can be used to predict the required time shift, however, it was found that experimental data was inaccurate in this region.

It was found that for good data, i.e. frequency spectra covering adequate bandwidth (approximately 0.5 to 2.5 $ka$) and containing no anomalous signals, the three methods of time-shifting predicted time shifts usually within one or at most two resolution points of the measurement system. The correspondence or otherwise was then used to classify the data as suitable or unsuitable for further processing.

On inversion of experimental data according to the Born equation, a further good/bad classification was carried out. Because the experimental system is bandlimited the characteristic function is a smoothed step function and the radius location therefore has to be estimated using:

(a) area under function/peak of function
(b) distance corresponding to the point that is 50% of the peak value.

Radius predictions by these two methods which did not agree within 10% were rejected, being of inadequate bandwidth or too noisy. Table 1 shows the time shifts and corresponding radius predictions for some data from spherical voids.

MONTE CARLO/HEDGEHOG PROTOCOL

In this section the Monte Carlo/Hedgehog search for ultrasonic defect sizing is described and the reasons for choosing this indirect trial and error type routine detailed. The method was first proposed by Valyus [3] and is commonly used in seismology.

The information required from the inversion must first be parameterised. For ultrasonic defect sizing these parameters could be flaw radius, density and flaw longitudinal and shear wave velocity, thus characterising
### Table 1 - Timeshifts and corresponding radius predictions for synthetic and experimental data

<table>
<thead>
<tr>
<th>Input data</th>
<th>Timeshift range (μs)</th>
<th>Radius area/peak (μm)</th>
<th>Radius 50% peak (μm)</th>
<th>Nominal radius (μm)</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideal Born data</td>
<td>1.85</td>
<td>197</td>
<td>188</td>
<td>200</td>
<td>No noise</td>
</tr>
<tr>
<td>Ideal Born data</td>
<td>1.85 - 199 - 201</td>
<td>192</td>
<td>200</td>
<td>10 dB S/N</td>
<td></td>
</tr>
<tr>
<td>Ideal Born data</td>
<td>1.78 - 120 - 129</td>
<td>142 - 158</td>
<td>200</td>
<td>10 dB S/N</td>
<td></td>
</tr>
<tr>
<td>Ideal Void data</td>
<td>0.8</td>
<td>212</td>
<td>203</td>
<td>200</td>
<td>No noise</td>
</tr>
<tr>
<td>Ideal Void data</td>
<td>0.8 - 209 - 210</td>
<td>198 - 203</td>
<td>200</td>
<td>10 dB S/N</td>
<td></td>
</tr>
<tr>
<td>Ideal Void data</td>
<td>0.74 - 109 - 140</td>
<td>138 - 148</td>
<td>200</td>
<td>10 dB S/N</td>
<td></td>
</tr>
<tr>
<td>Expt data Void in Ti alloy</td>
<td>-0.08 - 0.1</td>
<td>195 - 202</td>
<td>193 - 198</td>
<td>200</td>
<td>Accept result</td>
</tr>
<tr>
<td>Expt data Void in Ti alloy</td>
<td>-0.13 - 0.14</td>
<td>238 - 240</td>
<td>229 - 237</td>
<td>300</td>
<td>Accept result</td>
</tr>
<tr>
<td>Expt data Void in Ti alloy</td>
<td>-0.09 - 0.18</td>
<td>113 - 188</td>
<td>148 - 182</td>
<td>200</td>
<td>Reject result</td>
</tr>
</tbody>
</table>

The flaw composition and size. A four dimensional discretised parameter space is therefore set up which should contain all the potential solutions. The space can be examined using any conventional searching technique, in practice the Monte Carlo is used and a random point chosen. At each point, the theoretical scattering function for the given parameter is calculated. The synthetically generated data is then compared to experimental data from a flaw of unknown characteristics. The degree of correspondence between the two data sets is calculated by standard techniques (least squares best fit) and recorded. In the Monte Carlo another random point is chosen and the process repeated.

The Hedgehog, however, utilises the information obtained from the correspondence calculations; if the point on the parameter space is determined to be a 'good fit' to the experimental data then the nearest neighbours in that space are also examined. If any of these are found to be 'good fits' then their nearest neighbours are examined. This process continues until all the points adjacent to a point within the 'good fit' region have been examined. The algorithm then reverts to the Monte Carlo and another random point is chosen for comparison with experimental data. The process continues until all regions of the parameter space have been examined.

The advantages of the Hedgehog routine are:
1) It can locate and describe the minimum regions (i.e. best fit regions) of a multivariate function.
2) Unstable solutions appear as isolated points. The point distribution can be regarded as a measure of solution stability.
3) Any priori information available about the defect to be characterised can be used to restrict the parameter space. Further, the parameter space can, if necessary, be extended virtually indefinitely (the limitation being the computing power available) to encompass all possible solutions.
4) The algorithm can be used in conjunction with any forward model which can be adequately parameterised or indeed a combination of models (say one for cracks, one for volumetric defects). The Hedgehog is only limited by the forward models available.
5) Experimental deviations from the ideal can be incorporated in the forward model.
6) When zones of interest/minimum regions are located a finer mesh spacing (i.e. finer spacing between points in the discretised parameter space) can be generated for more detailed examination.

The disadvantages are:

1) The algorithm can be computationally demanding depending on the complexity of the forward model. However, a database of signals can be built up which would reduce computing time considerably.
2) The problem of existence can only be dealt with by expanding the parameter space.
3) Additional processing is required to distinguish between different minimum regions within a parameter space.

Results

Experiments were first performed with noisy bandlimited synthetic data to examine the performance of the algorithm under controlled conditions. This confirmed that the least square calculation was unaffected by white, gaussian, uncorrelated noise even at 3 dB signal-to-noise and that a bandwidth of 0.5 to 1.8 ka still give good sizing results. However, the algorithm was degraded by loss of low-frequency data such that a bandwidth of 1 to 2.5 ka proved inadequate.

Initial results with (fairly noisy) experimental data are encouraging and are shown in Table 2. A data comparison is shown in Fig. 1 and an example of the graphical output of the Hedgehog program is shown in Fig. 2.

CONCLUSIONS

The Born algorithm works for types of data for which it is not theoretically derived. The bandwidth requirements of the Born are on the limits of commercially available probes. However a protocol for matching flaw size to probe bandwidth has been developed and has been used to measure the quality of data input to the inversion.

An indirect inversion method (Hedgehog) has also been developed and initial experimental results with fairly poor quality data have been encouraging. Evaluation of the Hedgehog with synthetic data has shown it to be robust in the presence of uncorrelated Gaussian noise and limited bandwidths. Indirect methods can utilise all the available information about an unknown scatterer and do not involve approximations or simplifications of the forward model and the consequent inaccuracies.
Table 2 - Hedgehog inversion results on synthetic data and real data from diffusion bonded samples

<table>
<thead>
<tr>
<th>Sample data</th>
<th>Nominal radius (µm)</th>
<th>Best prediction (µm)</th>
<th>Range of first 4 predictions (µm)</th>
<th>Bandwidth (ka)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ti spherical void (Synthetic data)</td>
<td>250</td>
<td>260</td>
<td>260-290</td>
<td>0.1-2.5 (10 dB S/N)</td>
</tr>
<tr>
<td>Ti-6Al-4V spherical void</td>
<td>250</td>
<td>260</td>
<td>260-290</td>
<td>0.5-2.5 (10 dB S/N)</td>
</tr>
<tr>
<td>Face A</td>
<td>200</td>
<td>200</td>
<td>180-210</td>
<td>0.4-2.02</td>
</tr>
<tr>
<td>C</td>
<td>200</td>
<td>200</td>
<td>190-240</td>
<td>0.4-2.08</td>
</tr>
<tr>
<td>D</td>
<td>200</td>
<td>200</td>
<td>180-210</td>
<td>0.4-2.05</td>
</tr>
<tr>
<td>Maraging Steel spherical void</td>
<td>300</td>
<td>280</td>
<td>270-300</td>
<td>0.4-1.65</td>
</tr>
<tr>
<td>Face D</td>
<td>300</td>
<td>300</td>
<td>290-320</td>
<td>0.45-1.65</td>
</tr>
<tr>
<td>A</td>
<td>200</td>
<td>250</td>
<td>230-260</td>
<td>0.3-1.11</td>
</tr>
<tr>
<td>C</td>
<td>400</td>
<td>440</td>
<td>430-460</td>
<td>0.6-2.5</td>
</tr>
</tbody>
</table>

Fig. 1 Comparison of the backscattered magnitude spectra for ideal and experimental data from a 200 µm radius void in Ti-alloy
Fig. 2  Output from Hedgehog program showing the degree of correspondence between ideal and experimental data for various flaw radii and compositions (long. vel.) (only good correspondence points shown)

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REFERENCES

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