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Technical appendix to 'Temporary stabilizations, sudden stops, and asset prices'

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Abstract
This is a technical appendix to our publication 'Temporary stabilizations, sudden stops, and asset prices' in the Review of Development Economics, Vol. 13 no. 2 (May 2009): 333-347.

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Technical Appendix to "Temporary Stabilizations, Sudden Stops, and Asset Prices"

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A Technical Appendix to "Temporary Stabilizations, Sudden Stops, and Asset prices"

A.1 Derivation of equation (5) .

The household's budget constraint in nominal terms can be written as

$$\dot{A}_t = -r\ E_t\ b_t - \dot{E}_t\ q_t\ b_t - \dot{q}_t\ E_t\ b_t + E_t\left(\ y + \tau_t - c_t\right),$$

where

$$A \equiv M - E\ q\ b$$

Alternatively,

$$\frac{\dot{A}_t}{E_t} = -r\ b_t - \varepsilon_t\ q_t\ b_t - \dot{q}_t\ b_t + \left(y + \tau_t - c_t\right), \tag{A.1}$$

Define $a \equiv \frac{A}{E} = m - q\ b$ as the real value of assets. Then

$$\dot{a}_t + \varepsilon_t\ a_t = \frac{\dot{A}_t}{E_t}.$$

Using the above (A.1) can be rewritten as

$$\dot{a}_t = -r\ b_t - \varepsilon_t\ q_t\ b_t - \dot{q}_t\ b_t - \varepsilon_t\left[m_t - q_t\ b_t\right] + \left(y + \tau_t - c_t\right),$$

Further, define $\rho \equiv \frac{(r+\dot{q})}{q}$ and $i \equiv \rho + \varepsilon$, and rewrite the above as

$$\dot{a}_t = \rho_t\ a_t +\ \ y + \tau_t - c_t - i_t\ m_t$$

which is (3) in the main text.

A.2 Proof of Proposition 2

The proof proceeds in five steps. First we derive some basic identities that are used later. Then, in step I, we derive the path of consumption described in Proposition 2.3 after assuming that the result stated in Proposition 2.1 holds. Step II shows that $\lambda_{\hat{T}} > \lambda^*$, i.e., the costate variable jumps on impact at $\hat{T}$. Next, in step III we derive a planar system in c_t and ρ_t that allows us to determine the path of ρ_t given the path of c_t and λ_t. Finally, in step IV it is shown that $q_{\hat{T}} < 1$, which is required to characterize the dynamic path of q_t as discussed in 3.2.3 and shown in Figure 4. In step V, we establish conditions that ensures that the cash-in-advance constraint binds in a borrowing-constrained equilibrium.

A.2.1 Basic identities

Allocations under perfect capital mobility To simplify algebra we assume that

$$u\left(c\right) = \ln c.$$

Using (6a):

$$\frac{c^{2^*}}{c^{1^*}} = \frac{1+r+\varepsilon^L}{1+r+\varepsilon^H} = \phi < 1. \tag{A.2}$$

From resource constraint (assume $k_0 = 0$)

$$c^{1^*}\left(1 - e^{-rT^*}\right) + c^{2^*}\ e^{-rT^*} = \bar{c} = r\ \bar{k} + y \tag{A.3}$$

Using (A.2) and (A.3):

$$c^{1^*} = \frac{\bar{c}}{1-(1-\phi)\ e^{-r\ T^*}} \Rightarrow c^{1^*} - \bar{c} = \frac{\bar{c}}{\frac{e^{rT^*}}{1-\phi} - 1} \tag{A.4}$$

Allocations under borrowing constraints First observe that $c = c^{1^*} > \bar{c}$ for $t \in \left[0, \hat{T}\right]$. At $t = 0$, as households decide to consume $c^{1^*} \geq \bar{c}$, there is a discrete portfolio readjustment given by

$$\triangle b = \triangle m = \triangle c = c^{1^*} - \bar{c}$$

Hence,

$$b_0 = \bar{b} + c^{1^*} - \bar{c}; \tag{A.5}$$

As $m = c^{1^*}$ for $t \in [0, \hat{T})$, h remains at its value h_0 for $t \in [0, \hat{T})$ as follows from (11). Hence, the economy's net stock of foreign assets at $\hat{T}$, $k_{\hat{T}} = h_0 - b_{\hat{T}}$. Integrating the resource constraint (14) over $t \in [0, \hat{T})$, and noting that $\bar{k} = k_0$ (economy's net stock of foreign assets can not change discretely), we get

$$k_{\hat{T}}\ e^{-r\hat{T}} - \bar{k} = \frac{y - c^{1^*}}{r}\left(1 - e^{-r\hat{T}}\right).$$

Substituting $\bar{c} = y + r\bar{k}$ and $k_{\hat{T}} = h_0 - b_{\hat{T}}$ in the above yields

$$\left(b_0 - b_{\hat{T}}\right) e^{-r\hat{T}} = \frac{\left(\bar{c} - c^{1^*}\right)}{r}\left(1 - e^{-r\hat{T}}\right),$$

which with (A.4) leads to

$$b_{\hat{T}} = \bar{b} + \frac{\bar{c}}{r}\frac{\left(e^{r\hat{T}} - 1 + r\right)}{\frac{e^{rT^*}}{1-\phi} - 1}$$

Next using the above equation with the definition of $\hat{c}$ and again using the fact that $y - r\bar{b} = (1-r)\,\bar{c}$

$$\begin{aligned}\hat{c} &= \frac{y - r\ b_{\hat{T}}}{1-r} \\ &= \bar{c}\left[\frac{\frac{e^{rT^*}}{1-\phi}(1-r) - e^{r\hat{T}}}{\frac{e^{rT^*}}{1-\phi}\left(1-(1-\phi)\,e^{-rT^*}\right)(1-r)}\right]\end{aligned}$$

which using (A.4) yields

$$\frac{\hat{c}}{c^{1^*}} = 1 - \frac{e^{-r\left(T^*-\hat{T}\right)}(1-\phi)}{(1-r)} \tag{A.6}$$

A.2.2 Step I: Consumption dynamics

With $c_{\hat{T}} = c^{1^*}$ equation (21) directly yields

$$c_t = \hat{c} + \left(c^{1^*} - \hat{c}\right)\ e^{-(1-r)\left(t-\hat{T}\right)}, \quad t \in [\hat{T}, T^*). \tag{A.7}$$

For for $t \geq T^*$ the economy is in steady state:

$$\begin{aligned}c_t = c_{T^*} &\equiv r\ k_{T^*} + y = r\ \left(h_{T^*_-} - b_{\hat{T}}\right) + y \\ &= r\ c_{T^*_-} + (1-r)\ \ \hat{c}\end{aligned}$$

where we have used the fact that $k_{T^*} = k_{T^*_-}$. Additionally, $h_{T^*_-} = m_{T^*_-} = c_{T^*_-}$. Further, from (A.7)

$$c_{T^*_-} = \hat{c} + \left(c^{1^*} - \hat{c}\right)\ e^{-(1-r)\left(T^*-\hat{T}\right)}$$

which combined with previous equation yields

$$c_{T^*} = \hat{c} + r\ \left(c^{1^*} - \hat{c}\right)\ e^{-(1-r)\left(T^*-\hat{T}\right)} \text{ for all } t \geq T^* \tag{A.8}$$

Equation (A.7) and (A.8) together are presented as (27) in Proposition 2.

A.2.3 Step II: The path of λ_t

Using (22a), (22b), and (A.7) gets

$$\lambda_{\hat{T}} = \lambda_{T^*}\ e^{-\left(1+r+\varepsilon^L\right)\left(T^*-\hat{T}\right)} + \int_{\hat{T}}^{T^*} \frac{e^{-\left(1+r+\varepsilon^L\right)\ \left(t-\hat{T}\right)}}{\left(c^{1^*} - \hat{c}\right)\ e^{-(1-r)\left(t-\hat{T}\right)} + \hat{c}}dt \tag{A.9}$$

The value for λ_{T^*} is obtained by using (6a), (A.8) and the fact that $\rho = r$ for $t \geq T^*$. Thus

$$\lambda_{T^*} = \frac{1}{1 + r + \varepsilon^H} \frac{1}{\hat{c} + r \ \left(c^{1*} - \hat{c}\right) e^{-(1-r)\left(T^* - \hat{T}\right)}},$$

which using (A.6) can be rewritten as

$$\lambda_{T^*} = \frac{1}{\left(1 + r + \varepsilon^L\right) \ c^{1*}} \left[\phi \frac{1}{1 - \frac{e^{-r \ \left(T^* - \hat{T}\right)}(1-\phi)}{(1-r)} + r \ \left(\frac{e^{-r \ \left(T^* - \hat{T}\right)}(1-\phi)}{(1-r)} \right) e^{-(1-r)\left(T^* - \hat{T}\right)}} \right] \tag{A.10}$$

Observe that $\lambda^* = \frac{1}{(1+r+\varepsilon^L) \ c^{1*}}$. Then, using (A.6) in (A.9) and combining with (A.10) yields

$$\lambda_{\hat{T}} = \lambda^* \left[\begin{array}{c} \phi \frac{e^{-\left(1+r+\varepsilon^L\right)\left(T^* - \hat{T}\right)}}{1 - \frac{e^{-r \ \left(T^* - \hat{T}\right)}(1-\phi)}{(1-r \ )} \left[1 - r \ e^{-(1-r)\left(T^* - \hat{T}\right)}\right]} \\ + \int_{\hat{T}}^{T^*} \frac{\left(1+r+\varepsilon^L\right) \ e^{-\left(1+r+\varepsilon^L\right) \ \left(t - \hat{T}\right)}}{1 - \frac{e^{-r \ \left(T^* - \hat{T}\right)}(1-\phi)}{(1-r \ )} \left[1 - \ e^{-(1-r)\left(t - \hat{T}\right)}\right]} dt \end{array} \right] \tag{A.11}$$

Clearly, for $\varepsilon^L = \varepsilon^H, \ \phi = 1$. Then $\lambda_{\hat{T}} = \lambda^*$. Furthermore,$\lambda_{\hat{T}} = \lambda^*$ if $\left(T^* - \hat{T}\right) \to 0$ or $\left(T^* - \hat{T}\right) \to \infty$. In all these cases there are no intertemporal price distortions and the economy remains at steady state. Thus for a finite T^* and $\hat{T} \neq T^*$, in general $\lambda_{\hat{T}} \neq \lambda^*$. Hence, on impact λ may jump up or down; in particular, $\lambda_{\hat{T}} > \lambda^*$ iff

$$\phi \frac{e^{-\left(1+r+\varepsilon^L\right) \ T^*}}{1 - \frac{1-\phi}{1-r} \left(e^{-r \ T^*} - r \ e^{-T^*}\right)} + \int_0^{T^*} \frac{\left(1 + r + \varepsilon^L\right) \ e^{-\left(1+r+\varepsilon^L\right) \ t}}{1 - \frac{1-\phi}{1-r} \left(e^{-r \ T^*} - e^{-r \ T^*} e^{-(1-r)\left(t - \hat{T}\right)}\right)} dt > 1 \tag{A.12}$$

where $T^* \equiv T^* - \hat{T}$. From (A.12) it can be verified that the LHS depends on T^* in a non-monotonic way. For $r = 0.04$, it can be verified numerically that (A.12) holds for all $0 < \varepsilon^L < \varepsilon^H < 30$ (i.e., a devaluation rate of 3000%) for any value of T^*.

A.2.4 Step III: The planar system in c_t and ρ_t

Differentiating (6a) with respect to time yields

$$u''\left(c_t\right) \dot{c}_t = \dot{\lambda}_t \ \left(1 + r + \varepsilon_t\right) + \lambda_t \ \dot{\rho}_t, \tag{A.13}$$

where $I_t = 1 + r + \varepsilon^L, t \in [0, T^*)$, and $I_t = 1 + r + \varepsilon^H, t > T^*$. Substituting (6a), (6b), and (A.7), into (A.13) yields

$$\dot{\rho}_t = \left(\rho_t - r + \gamma c_t \frac{u''(c_t)}{u'(c_t)} \left(1 - \frac{\hat{c}}{c_t} \right) \right) (1 + r + \varepsilon_t) . \tag{A.14}$$

The above equation along with (21) constitutes a dynamic planar system; Figure 3 presents the phase diagram for c_t and ρ_t for $t \in [\hat{T}, T^*)$.

A.2.5 Step IV: Proof of $q_{\hat{T}} < 1$

Noting that $q_{\hat{T}} > 1$ is impossible, we only need to rule out $q_{\hat{T}} = 1$. Suppose $q_{\hat{T}} = 1$. Then (25) implies that

$$\lambda_{\hat{T}} = e^{-r\ \left(T^* - \hat{T}\right)}\ \lambda_{T^*} + r \int_{\hat{T}}^{T^*} e^{-r\ \left(u - \hat{T}\right)}\ \lambda_u\ du \tag{A.15}$$

where λ_u using (22a) is given by

$$\lambda_u = \lambda^* \left[\begin{array}{c} \phi \frac{e^{-\left(1+r+\varepsilon^L\right)\ (T^* - u)}}{1 - \frac{1-\phi}{1-r}\left(e^{-r\ (T^*-u)} - r\ e^{-(T^*-u)}\right)} \phi \\ + \int_u^{T^*} \frac{\left(1+r+\varepsilon^L\right)\ e^{-\left(1+r+\varepsilon^L\right)\ (t-u)}}{1 - \frac{1-\phi}{1-r}\left(e^{-r\ (T^*-u)} - e^{-r\ (T^*-u)}\ e^{-(1-r)(t-u)}\right)} dt \end{array} \right] \tag{A.16}$$

But the solution obtained from (A.15) and (A.16) can be consistent with the expression for $\lambda_{\hat{T}}$ in (A.9) only if $T^* - \hat{T} \to 0$ or $T^* - \hat{T} \to \infty$. In both cases from $\lambda_{\hat{T}} = \lambda^*$ and both (A.9) and (A.15) are trivially satisfied. Otherwise, (A.15) does not hold. Hence, for all finite and non-zero $T^* - \hat{T}$, we conclude that $q_{\hat{T}} < 1$. This conclusion is numerically verified.

A.2.6 Step V: Conditions for binding cash-in-advance constraint

For (4) to bind under perfect capital mobility we need $1 + r + \varepsilon^L > 1$ for all t, which easily holds since $\varepsilon^L > 0$ by assumption. Under borrowing-constrained equilibrium described by Proposition 2 it is sufficient to check whether (4) holds at $\hat{T}$. Since $c_{\hat{T}} = c^{1^*}$, checking if $1 + i_{\hat{T}} > 1$ requires that

$$u'\left(c^{1^*}\right) = \lambda_{\hat{T}}\ \left(1 + i_{\hat{T}}\right) > \lambda_{\hat{T}}$$

Note that $u'\left(c^{1^*}\right) = \lambda^*\ \left(1 + r + \varepsilon^L\right)$. Then using (A.11) the above condition becomes

$$\phi \frac{e^{-\left(1+r+\varepsilon^L\right)\ T^*}}{1 - \frac{1-\phi}{1-r}\left(e^{-r\ T^*} - r\ e^{-T^*}\right)} + \int_0^{T^*} \frac{\left(1 + r + \varepsilon^L\right)\ e^{-\left(1+r+\varepsilon^L\right)\ t}}{1 - \frac{1-\phi}{1-r}\left(e^{-r\ T^*} - e^{-r\ T^*} e^{-(1-r)\left(t - \hat{T}\right)}\right)} dt < 1 + r + \varepsilon^L$$

Fix ϕ. Then, for any $\phi < 1$ there exists $\bar{\varepsilon}$ such that for all $\varepsilon^L > \bar{\varepsilon}$, the above condition holds. For the parameter values we consider in our numerical simulations $\varepsilon^L > \bar{\varepsilon} = 0$ ensures that the above condition holds. For other parameter values $\bar{\varepsilon}$ can be appropriately chosen.