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Joydeep Bhattacharya

*Iowa State University*, joydeep@iastate.edu

Robert R. Reed

*University of Kentucky*, RRREED@UKY.EDU

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## **Abstract**

In recent years, many countries have experienced a significant shift in demographic patterns towards the elderly. This phenomenon poses numerous challenges for the design of public pension programs and labor market policies. To better understand how public policy should be designed in response to an aging workforce, it is imperative to first make an assessment of how the lifecycle affects aggregate labor market activity, and in particular, unemployment. While much work has been done on exploring how the lifecycle influences individual labor market behavior, its impact on aggregate labor market outcomes is far less studied. This paper is an attempt at addressing this lacuna within the context of a lifecycle model with costly search and matching in the labor market. The lifecycle of workers in conjunction with frictions in the labor market produces an environment in which unemployment arises as a natural possibility and both young and old workers find themselves contemporaneously competing for the same jobs. The lifecycle is shown to have significant implications for aggregate labor market activity; it may even be responsible for an inefficient allocation of workers to jobs. Additionally, public policies designed to increase labor market participation among older workers may not necessarily enhance aggregate welfare.

## **Keywords**

search, labor market efficiency, unemployment, lifecycle

## **Disciplines**

Economics

# IOWA STATE UNIVERSITY

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# Aging, Unemployment, and Welfare in a Life-Cycle Model with Costly Labor Market Search\*

Joydeep Bhattacharya<sup>†</sup>

Robert R. Reed

Iowa State University

University of Kentucky

October, 2001

## Abstract

In recent years, many countries have experienced a significant shift in demographic patterns towards the elderly. This phenomenon poses numerous challenges for the design of public pension programs and labor market policies. To better understand how public policy should be designed in response to a aging workforce, it is imperative to first make an assessment of how the lifecycle affects aggregate labor market activity, and in particular, unemployment. While much work has been done on exploring how the lifecycle influences individual labor market behavior, its impact on aggregate labor market outcomes is far less studied. This paper is an attempt at addressing this lacuna within the context of a lifecycle model with costly search and matching in the labor market. The lifecycle of workers in conjunction with frictions in the labor market produces an environment in which unemployment arises as a natural possibility and both young and old workers find themselves contemporaneously competing for the same jobs. The lifecycle is shown to have significant implications for aggregate labor market activity; it may even be responsible for an inefficient allocation of workers to jobs. Additionally, public policies designed to increase labor market participation among older workers may not necessarily enhance aggregate welfare.

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**JEL Classification:** J 41, J 64, E 24

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<sup>†</sup>Corresponding author: Joydeep Bhattacharya, Department of Economics, Iowa State University, Ames IA 50011; Phone: (515) 294 5886; Fax: (515) 294 0221; E-mail: joydeep@iastate.edu

# 1 Introduction

Many developed countries around the world have been witnessing (and will continue to witness) a substantial “graying” of their populations. In the United States, for example, while only 1 out of 25 individuals was sixty-five years or older in 1900, this number rose to 1 out of 8 in 1994, and is expected to climb to 1 out of 5 by the year 2050.<sup>1</sup> The age composition of the work forces in many countries is also changing rapidly. Labor forces are expected to become significantly older. The European Union, for example, will likely see an increase in the share of workers aged 60 years and older from its 1995 value of 4% to a projected 15.9% in 2030. In many countries, labor force participation rates of older workers have been declining during the postwar period. In the US in 1950, 46% of men sixty-five years and older were active in the labor force, compared to only 16% in 1993. In France, since 1960, the participation rates among males 55 and older have fallen from 31.5% to 15%.<sup>2</sup> The combined effect of these and many other demographic changes is creating an unprecedented burden on the current younger generations of working individuals.<sup>3</sup> Not surprisingly, the world is starting to see a major reordering of political agendas and the triggering of generational tensions between those who pay and those who benefit. Many governments are responding to this crisis by introducing policies that increase labor market participation from older workers.<sup>4</sup> The question is, should they? What about the consequences of such policies for efficiency in labor markets? How should labor market policy be designed so that it can accommodate an aging labor force?

In order to understand some of the underlying inter-generational conflicts, it is imperative to make an assessment of how the lifecycle may affect aggregate labor market activity. A natural environment to study this is a model that permits explicit separation of the workforce into young and old workers.<sup>5</sup> Another prerequisite for studying endogenously-evolving patterns of labor mar-

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<sup>1</sup><http://www.census.gov/ipc/prod/ageame.pdf>

<sup>2</sup>Gruber and Wise (1999) attribute this drop in labor force participation rates mainly to the amazing generosity of early retirement schemes (60-70% of previous income) and long-term unemployment insurance programs. Similar programs with varying degrees of generosity are in place in Germany and Belgium.

<sup>3</sup>This problem is not unique to the United States. For example, the Commission on Global Aging in its 1999 report mentions that the payroll tax hikes needed to cover the rising costs of public pensions in Europe will add a cumulative \$5 per hour to real manufacturing costs by the mid-2020s.

<sup>4</sup>In the United States, Social Security regulations in recent years have been altered to encourage later labor force withdrawal and to increase penalties for early retirement; additionally, age discrimination laws have been extended to protect workers from mandatory retirement at any age. See Mulligan (2000).

<sup>5</sup>In related work, Shimer (2001) also studies the implications of population aging for the labor market. In contrast to our work, all workers in Shimer’s model are infinitely-lived even though in each period a new generation of workers is born. “Young” workers are individuals who were born in later periods than “old” workers and are therefore more likely to be unemployed since they have had less time to search for jobs in the labor market. See also Davidson,

ket activity is a setup in which unemployment can arise as an equilibrium outcome of the actions taken by independent actors. In this paper, we satisfy both these requirements by embedding a model of the labor market characterized by search and matching frictions, into an overlapping generations model with two-period lived agents. The former feature allows for the possibility of frictional unemployment to arise endogenously: workers search for (and may not find) employment opportunities in the labor market; similarly, firms may also face difficulties finding workers to fill their vacancies.<sup>6</sup> The lifecycle of workers explicit in the overlapping-generations setup in conjunction with the frictions in the labor market therefore produces a natural, rich environment in which both young and old workers may find themselves competing for the same jobs at the same time. It is this meeting of the young and the old in the labor market that produces interesting consequences for both. This paper is really a qualitative study of these consequences. More specifically, we are interested in exploring the isolated importance of the natural lifecycle in the determination of labor market participation, remuneration, and lifetime welfare of agents.

A few details of the model setup are in order here. At any date, there are some newly-born (young) agents and some old agents. All young agents are unemployed to begin with; they will incur some costs before they may search for employment opportunities. Firms post vacancies also at a cost and enter the labor market only if there are profits to be made from doing so. There is a standard economy-wide non-discriminating stochastic matching technology that connects vacancies to people. Once job matches are formed, production takes place. A match survives on to the next period with a high probability. This is the sense in which “jobs are durable”. If a worker gets separated from a match, she becomes a displaced worker, and may re-enter the labor market to seek employment the following period. At the start of any date, then, all the young workers, all the displaced workers (who are old), and all the old who did not get matched to jobs when young, enter the labor market seeking employment. The remaining agents in the economy are the old who hold jobs that have lasted from the previous period. All new matches produce the same amount of gross output.<sup>7</sup> Firms, however, incur training and re-tooling costs if they hire a young worker or an old unemployed worker but not if they continue an employment relationship from the previous

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Martin, and Matusz (1994).

<sup>6</sup>There are many competing ways (efficiency wages, agency problems, unions, etc.) to generate equilibrium unemployment. To the best of our knowledge, they would all require an explicit distinction (possibly in terms of health, productivity, desire for leisure etc.) between the young and the old. In contrast, our setup allows us to remain agnostic about those differences (except for the natural position of agents along the lifecycle) and yet produce equilibrium unemployment.

<sup>7</sup>This is the sense in which we abstract away from all differences between young and old workers and isolate only their lifecycle differences.

period.

We focus on those equilibria in which firms enter the labor market up to the point where all profits from entry are exhausted. We begin by providing a set of sufficient conditions under which a stationary equilibrium exists and is unique in this economy. We then proceed to characterize a steady-state by first examining the effects of increased severity of matching frictions on labor market outcomes. We find that while these affect young workers' wages, they have no effect on the wages of older workers. Even though matching frictions adversely affect the returns to an individual's labor market participation, they have ambiguous effects on the age-composition of the pool of unemployed workers. Increased productivity from job matching produces higher wages at each stage of the lifecycle, but also shifts the pool of unemployed to younger workers. Higher training costs lower young worker's wages, but have ambiguous effects on wages of older workers.

Unemployed old workers (be they displaced or never before employed) exert standard congestion externalities on young workers. Also recall that firms incur re-tooling costs if they hire an old unemployed worker but not if they continue an employment relationship from the previous period. It is precisely because of this reason and the fact that old (young) workers have one (two) period(s) of working life left, that firms care about whether they hire an old or a young worker. This opens up the possibility that the age composition of the labor force has critical consequences for job creation, and hence, welfare of all workers. In particular, if there is an abundance of old workers, it becomes harder for a firm to meet a young worker which renders the allocation of workers to jobs inefficient. It is in this context that government intervention to alter the age-composition of the labor force may be justified.

We go on to study a setting (“the discouraged worker case”) where all unemployed, old workers do not participate in the labor market. This stylized environment is meant to serve as a proxy for studying the effects of various public policies in the real world (such as social security and generous old-age unemployment programs) that directly or indirectly discourage older unemployed workers from participating in the labor market.<sup>8</sup> Numerical computations confirm the following flavor of results. Young workers more readily find jobs in this case than in the benchmark economy. This happens either because there are overall fewer workers searching for jobs or because there is more job creation (more firms looking for employees). For reasons described above, the lifecycle and hence

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<sup>8</sup>In France, for example, the “contrat de solidarité” recognized the “double need to encourage 55-59 year-old workers to stop work and to bring young workers into the labour market, as rising youth unemployment was a growing concern to society as a whole.” In many countries in Europe, a precondition to receiving unemployment benefits for people over the age of 55 is that they stop “seeking employment”. See the OECD (1995) study on “The Labor Market and Older Workers”.

the age composition of workers, becomes responsible for an inefficient allocation of workers to jobs in the labor market. We demonstrate via numerical examples that prohibiting older unemployed workers from entering the labor force may or may not remove this inefficiency. The answer depends on whether the resultant decongestion effect on young workers is stronger than the effect on overall job creation. It is our contention that the latter effect has remained largely ignored in popular discussions of optimal policy design in the wake of an aging labor force.

The rest of the paper is set out in the following manner. In Section 2, we lay out the environment in which workers and firms interact in the labor market. Section 3 focuses on our benchmark case where all workers actively participate in the labor market. Section 4 studies the “discouraged worker case” and discusses the labor market implications of employment policies intended to discourage labor market participation by older workers. Section 5 discusses how aging may lead to a possible inefficient allocation of workers to jobs. Some concluding remarks are presented in Section 6 below. The proofs of all the major results are contained in the appendices.

## 2 The Environment

We consider an economy consisting of an infinite sequence of two-period lived overlapping generations. Let  $t$  be the time index, with  $t = 0, 1, 2, \dots$ . The economy is populated by a continuum of two types of agents which we refer to as workers and firms. There is no growth in the population sizes of these agents.

### 2.1 Workers

The principal departure from the standard textbook model of search and matching in the labor market (as outlined in Pissarides, 2000 or Ljungquist and Sargent, 2001) is that workers in this model live only for two periods.<sup>9</sup> Consequently, in any period, there are two types of workers – young and old. Each generation of workers is of equal measure,  $\frac{1}{2}$ , so that the total population of workers is equal to 1 each period. At birth, all workers are unemployed. Furthermore, all workers are risk-neutral and have the same rate of time preference,  $\beta$ . Finally, individuals all have the same disutility of effort. There are no assets or saving instruments. As such, all income earned is consumed in that period itself.

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<sup>9</sup>We defend our assumption that workers live only for two periods by appealing to the absolute analytical intractability of a model with long but finitely lived agents, and to the fact that the primary purpose of our exercise is to provide insights of a qualitative nature. To that end, our basic point about the possible inefficiency of the allocation of workers to jobs can be made even in a two-period model.

## 2.2 Firms

Below, we will present a partial-equilibrium analysis of an industry in which firms produce a homogeneous consumption good each period using labor as the sole factor of production. Production is the result of a pairwise matching between one worker and a firm. Firms are infinitely-lived with a total population of measure  $\mathcal{F}$  in each period.<sup>10,11</sup> For ease of presentation, we assume that both workers and firms share the same discount factor  $\beta$ . All firms are homogeneous in that they all have the same technology. Firms maximize the present discounted stream of revenues net of all costs.

## 3 The Benchmark Model

Unlike in the standard neoclassical textbook model, trade in the labor market in this paper is not coordinated by a Walrasian auctioneer. Instead, we formally model a pairwise, random matching process between firms and workers and a bargaining process between them through which wages and payoffs are determined. The details are spelt out below.

### 3.1 Timeline

At the beginning of each period, the labor market opens. At that time, unemployed workers choose whether to search for vacancies or not. If they decide to search, they incur a search cost,  $s$ , which is expressed in terms of disutility of search. That is, we account for a worker's loss of leisure time that must be foregone when making the decision to search for a job. As described in Pissarides (2001),  $s$  represents the imputed value of leisure in terms of output (utility). On the other side of the market, firms make the decision whether to pay some upfront costs (described below) and enter the labor market to look for employees. Each firm may employ at most one worker. Let  $U_t$  ( $F_{v,t}$ ) denote the total mass of unemployed workers (unfilled vacancies) at the start of date  $t$ .

Unemployed workers and unfilled vacancies are brought together via a stochastic matching technology. Any unemployed (old or young) worker faces a probability  $\alpha_t$  of getting matched with a vacancy in period  $t$ . Within a period, firms and workers have at most one opportunity to meet and match. At the end of any period, a match formed at the start of that period dissolves (i.e., the

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<sup>10</sup>Assuming that firms are owned by coalitions of workers would require modeling the distribution of firms at each point in time. This would involve details that detract from the main focus of the paper – the interaction between the lifecycle and labor market activity.

<sup>11</sup>It is important to assume here that firms outlive any worker. This lets us sidestep issues that would arise when an young worker gets matched with an “old” (soon to die) firm.

employment relationship between a worker and a firm ends) with an exogenously given probability  $b$ .<sup>12</sup> Once formed, then, matches survive for a minimum length of one period and for a maximum length of two periods.

At the start of any date  $t$ , there are both young and old workers who may choose to participate in the labor market. All young workers are unemployed at this point. There is no on-the-job search; hence, old employed workers are not to be found searching in the labor market. The old workers fall under one of two employment categories, employed or unemployed.<sup>13</sup> At the start of date  $t$ , a worker may be unemployed and old either because she did not find a job when she was young [this happens with probability  $(1 - \alpha_{t-1})$ ], or she may have gotten separated from her previous job [this happens with probability  $\alpha_{t-1}b$ ]. With probability  $\alpha_{t-1}(1 - b)$ , she may remain attached to a match from the previous period.<sup>14</sup>

Firms incur costs of posting vacancies, given by  $a$ . Upon incurring this cost and searching for workers, all firms are equally likely to find a worker. The probability that a vacancy locates a worker is  $\theta_t$ . The probabilities of meeting a given type of worker (young or old), however, will depend on the participation of each type [ $u_{y,t}$  or  $u_{o,t}^u$ ] in the labor market. The probability of finding a young unemployed worker is  $\theta_t \tilde{u}_{y,t}$ , where  $\tilde{u}_{y,t} \equiv \frac{u_{y,t}}{U_t}$ . Similarly, the probability that a vacancy locates an unemployed old worker is  $\theta_t \tilde{u}_{o,t}$  where  $\tilde{u}_{o,t} \equiv \frac{u_{o,t}^u}{U_t}$  which is equal to  $\frac{\{(1-\alpha_{t-1})+\alpha_{t-1}b\}}{2}$ .

As in Oi (1962), we assume that there are costs which must be incurred at the beginning of the employment relationship. We refer to these as “hiring” costs and denote them as  $h$ .<sup>15</sup> Once matches are formed, production occurs. Let  $p$  denote the exogenously-determined market value of the firm’s output. Note that the net output from matching varies according to the duration of the match between a worker and a firm. Matches with new hires require the firm and the worker to incur the costs of training so that the net output from new matches is  $(p - h)$  while net output from a match with an old, retained worker is higher and is given by  $p$ . In this manner, one may view  $h$  as a cost incurred each time a firm makes a new hire or it may simply represent a productivity

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<sup>12</sup>The separation probability is exogenous implying that firms here cannot take rational/irrational decisions to “fire” workers at any point. This is a standard device (see Pissarides, 2000) used to capture certain aspects of real-world labor markets e.g., labor market turnover.

<sup>13</sup>For future reference, note that the terminology “benchmark model” refers to the setting where *all* unemployed workers, be they old or young or displaced old, search for employment. In Section 4 below, we will study an alternative setting where all the unemployed old (be they erstwhile displaced or not) are not allowed to participate in the labor markets.

<sup>14</sup>The limiting case  $b = 0$  corresponds to the situation where matches last for the entire two periods. In this case, jobs are “perfectly durable”.

<sup>15</sup>As described in Hutchens (1986), these may also be the result of Lazear (1981) contracts.

differential between new and old matches. In the latter sense, one may also view  $h$  as a parameter which reflects the importance of firm-specific human capital. As in Hutchens (1986), firms therefore have an incentive to hire those workers whose expected future tenure with them is longest.

Finally, the wage rate(s) for the different types of workers are then determined in accordance with the rules of Nash bargaining. At the end of the period, young employed workers learn their employment status for the following period (i.e., whether their current match survives to the next period or gets dissolved); at this time, old workers die.

To reiterate, the timing assumptions in our model are as follows. First, the labor market opens – unemployed workers and firms with vacancies look for trading opportunities. Matches are potentially formed. Second, the labor market closes. Then, production occurs, wages are paid and agents consume. Finally, young workers learn whether they will retain their jobs when they are old and the current old generation dies.

### 3.2 Workers' payoffs

We go on to compute the ex-ante expected payoffs in utility terms to the different types of workers. Recall that young workers begin a period unemployed, while old workers may begin the period either unemployed or employed. Let  $J_{y,t}$  be the expected lifetime utility of a newborn agent (young worker) at the start of date  $t$ , and  $w_{y,t}$  be the wage paid to a young worker in period  $t$ . Let  $J_{o,t}^e$  denote the expected lifetime utility of an old worker who begins period  $t$  employed at a firm. The wage paid to an old worker continuing employment is given by  $w_{o,t}^e$ .<sup>16</sup> Finally, let  $J_{o,t}^u$  denote the expected lifetime utility of an old worker who begins period  $t$  unemployed. The wage paid to an old unemployed worker is given by  $w_{o,t}^u$ . All old unemployed workers at  $t$ , irrespective of their past employment status, look identical and earn the same wage  $w_{o,t}^u$  if they find employment.

The following value functions govern the expected present discounted surplus of workers in each state:

$$J_{y,t} = \max \left\{ -s + \alpha_t [w_{y,t} + (1-b)\beta J_{o,t+1}^e + b\beta J_{o,t+1}^u] + (1-\alpha_t)\beta J_{o,t+1}^u, 0 \right\} \quad (1)$$

$$J_{o,t}^e = \max \{ w_{o,t}^e, 0 \} \quad (2)$$

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<sup>16</sup>Recall that workers' age is fully observable to everyone. That is, no worker can misstate his/her own age. In particular, no old worker can claim to be a young worker. As a result, all old workers, whether they were employed before and then got separated, or whether they did not find employment when young, all get paid the same wage  $w_{o,t}^u$ .

$$J_{o,t}^u = \max \{-s + \alpha_t w_{o,t}^u, 0\} \quad (3)$$

These expressions have intuitive interpretations. Consider for example, equation (1). Recall that if a worker does not seek employment, she does not incur the cost of search and simply gets 0 utility. If a young worker does seek employment, she must incur  $s$ . If she is matched (and this happens with probability  $\alpha_t$ ) she earns the wage  $w_{y,t}$  and may retain the job the following period. If she gets separated from her job (this happens with probability  $b$ ), then she will be in the state of being an unemployed old worker at the start of  $t + 1$ . Alternatively, she may retain the job, and find herself in the state of being an employed old worker in the following period. If she does not get matched (this happens with probability  $(1 - \alpha_t)$ ), she will find herself in the state of being unemployed at the start of  $t + 1$ .

We will focus on equilibria where all young workers find it in their best interest to search for jobs, i.e.,  $J_{y,t} > 0$  will hold. An equilibrium in which *every* type of unemployed worker will choose to search is characterized by  $J_{y,t} > 0$ ,  $J_{o,t}^e > 0$  and  $J_{o,t}^u > 0$ . For future reference, note that in a steady state in which all young workers search,

$$J_y = -s + \alpha [w_y + (1 - b)\beta w_o^e + b\beta(-s + \alpha w_o^u)] + (1 - \alpha)\beta(-s + \alpha w_o^u) \quad (4)$$

holds. Employed old workers always choose to carry on their employment and earn a wage. Unemployed old workers may or may not choose to work depending on the costs of search.

### 3.3 Firms' payoffs

As discussed above, firms with unfilled vacancies must search for unemployed workers to fill their vacancies. Firms begin each period in one of two possible states. They may have a vacancy, or they may be matched with an old worker from a previous employment relationship and will have a vacancy for sure the following period. Letting  $\Pi_{v,t}$  ( $\Pi_{f,t}$ ) be the expected lifetime profits of a firm that has an unfilled (filled) vacancy at the beginning of period  $t$ , the following Bellman equations describe the associated expected present discounted profits of a firm in each state:

$$\begin{aligned} \Pi_{v,t} = & -a + \theta_t \tilde{u}_{y,t} \{(p - h - w_{y,t}) + (1 - b)\beta \Pi_{f,t+1} + b\beta \Pi_{v,t+1}\} + \\ & \theta_t \tilde{u}_{o,t} \{(p - h - w_{o,t}^e) + \beta \Pi_{v,t+1}\} + (1 - \theta_t)\beta \Pi_{v,t+1} \end{aligned} \quad (5)$$

$$\Pi_{f,t} = (p - w_{o,t}^e) + \beta \Pi_{v,t+1} \quad (6)$$

When a firm posts a vacancy, it pays a vacancy-posting cost  $a$ . With probability  $\theta_t \tilde{u}_{y,t}$ , the firm gets matched with a young worker. In this setting, the firm earns net revenues of  $(p - h - w_{y,t})$  for the period. With probability  $b$ , the firm will lose the worker as a job separation occurs, and therefore will begin the following period with an unfilled vacancy. Otherwise, the firm will begin the following period with a filled vacancy and earn expected discounted profits of  $\Pi_{f,t+1}$ . With probability  $\theta_t \tilde{u}_{o,t}$ , the firm gets matched with an old unemployed worker instead. If that happens, the firm earns current net revenues of  $(p - h - w_{o,t}^u)$ , but will begin the next period with a vacancy since the old worker will die at the end of the period. If the firm finds no worker [and this happens with probability  $(1 - \theta_t)$ ] then it begins the next period with a vacancy and will earn expected present discounted profits of  $\Pi_{v,t+1}$ . Firms that begin the period with a filled vacancy do not pay the hiring costs and hence earn current net revenues of  $(p - w_{o,t}^e)$ ; however, they will need to look for a new worker in the following period. In passing, note that firms take  $\tilde{u}_{y,t}$  and  $\tilde{u}_{o,t}$  as given, when deciding whether to enter the labor market.<sup>17</sup>

For future reference, note that in a steady state [using (5) and (6)],

$$\Pi_v = \frac{-a + \theta \tilde{u}_y (p - h - w_y) + \theta \tilde{u}_y (1 - b) \beta (p - w_o^e) + \theta \tilde{u}_o (p - h - w_o^u)}{[1 - \theta \tilde{u}_y (1 - b) \beta^2 - \theta \tilde{u}_y (1 - b) b \beta^2 - \theta \tilde{u}_o \beta - (1 - \theta) \beta]}. \quad (7)$$

Some properties of  $\Pi_v$  are worth mentioning here. First, *ceteris paribus*, increased vacancy costs  $a$  and hiring costs  $h$  reduce  $\Pi_v$  for obvious reasons. It can also be checked that profits increase when it becomes easier for a firm to find a worker. Similarly, profits fall when workers' wages go up.

### 3.4 Matching

As discussed above, job creation in this model is the result of a two-sided matching process in which workers and firms engage in search and hiring activities. The details of this process are described below.

Unemployed workers and unfilled vacancies are brought together each period through a stochastic matching technology.<sup>18</sup> The matching technology describes the total number of matches,  $m_t = \mu M(U_t, F_{v,t})$ , that are formed at the beginning of each period, depending on the total masses of unemployed workers and unfilled vacancies. Since  $\alpha_t$  represents the probability that an unemployed worker will find any vacancy in period  $t$  and  $\theta_t$  is the probability that any unfilled vacancy

<sup>17</sup>The determination of  $\tilde{u}_y$  and  $\tilde{u}_o$  is relegated to Appendix A.

<sup>18</sup>Recall that  $U_t$  is the mass of unemployed workers and  $F_{v,t}$  is the total mass of unfilled vacancies at the beginning of period  $t$ .

will find an unemployed worker, it follows that the total number of workers who find employment ( $\alpha_t \cdot U_t$ ) must equal the total number of firms that filled their vacancies ( $\theta_t \cdot F_{v,t}$ ):

$$\alpha_t \cdot U_t = \theta_t \cdot F_{v,t} \tag{8}$$

It is important to note that  $\alpha_t$  and  $\theta_t$  are determined in equilibrium, and that both workers and firms take them as given when making their decisions. Noting that  $m_t = \theta_t \cdot F_{v,t}$ , we have

$$\alpha_t U_t = \theta_t F_{v,t} = m_t = \mu M(U_t, F_{v,t}) \tag{9}$$

the *matching condition*. As is fairly standard, we assume that the matching technology takes the Cobb-Douglas form:  $m_t = \mu(U_t)^\phi(F_{v,t})^{1-\phi}$  where  $\phi \in [0, 1)$ . An increase in either the number of unemployed workers or unfilled vacancies increases the number of matches each period, but at a decreasing rate. The extent of “lubrication” in the labor market is given by  $\mu > 0$ ; ceteris paribus, more matches occur when  $\mu$  is higher.

### 3.5 Bargaining and wage determination

The friction inbuilt into the job-firm matching process creates the possibility that a firm may remain unproductive or a worker may remain unemployed in any period. Firms and workers must therefore weigh the implications of finding themselves in these states (their outside options) when bargaining over their share of current and future surplus produced. In this connection, note that, matches, once formed, cannot be dissolved instantaneously and costlessly; they survive at least for a period.

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We now turn to the determination of the wage offer functions for both young and old workers. Matches between workers and unfilled vacancies leads to a surplus that is to be divided between the worker and the firm. Nash bargaining dictates that the total match surplus be shared by the firm and the worker; we denote the bargaining weight of the firm by  $\lambda$ , and that of the worker by  $1 - \lambda$ .<sup>20</sup> The net output from a match between a young worker (or old separated worker) and a firm is  $p - h$  and that between an old employed worker and a firm is  $p$ . The surplus generated from a match similarly depends on age and employment histories of workers involved in the match.

Old workers differ on the basis of past employment histories, which in turn, gets them different wages. Upon finding an old displaced worker, a firm earns current net revenues of  $(p - h - w_{o,t}^u)$  and

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<sup>19</sup>One thing to note here is that all wage contracts are of one-period length. That is, even when a match survives on to the second period, wages are determined by a fresh process of bargaining at the start of the second period.

<sup>20</sup>All workers have the same bargaining weight.

expected discounted profits, as of period  $t$ , of  $\beta\Pi_{v,t+1}$  (it will have a vacancy next period for sure). If the firm would not have found a worker in the labor market in period  $t$ , it would earn expected discounted profits of  $\beta\Pi_{v,t+1}$  as of period  $t$ . Thus, the gains from trade for a firm upon hiring an old worker in period  $t$  are given by:  $(p - h - w_{o,t}^u + \beta\Pi_{v,t+1}) - \beta\Pi_{v,t+1} = p - h - w_{o,t}^u$ .<sup>21</sup> The gains from trade for an old separated worker upon finding employment is given by  $w_{o,t}^u$ . There is no continuation payoff since the worker will die at the end of the period. The total net production generated is  $p - h$ . The worker's wage is therefore  $w_{o,t}^u = \lambda(p - h)$ . Using a very similar argument, it is clear that the wage paid to an old worker from a previous and unbroken employment relationship is  $w_{o,t}^e = \lambda p > w_{o,t}^u$ .

The gains from trade accruing to a firm from hiring a young worker will typically differ from that obtained by hiring an old worker. This is because the employment relationship between a young worker and a firm may be sustained for two periods as opposed to just one period with an old worker. The gains from trade for the firm are given by

$$p - h - w_{y,t} + b\beta\Pi_{v,t+1} + (1 - b)\beta\Pi_{f,t+1} - \beta\Pi_{v,t+1}$$

which upon simplification can be written as

$$p - h - w_{y,t} + (1 - b)\beta [\beta\Pi_{v,t+2} - \Pi_{v,t+1} + (p - w_{o,t+1}^e)].$$

The young worker's surplus from finding employment is given by

$$w_{y,t} + b\beta J_{o,t+1}^u + (1 - b)\beta J_{o,t+1}^e - \beta J_{o,t+1}^u$$

which in turn reduces to

$$w_{y,t} + (1 - b)\beta [w_{o,t+1}^e + s - \alpha_{t+1}w_{o,t+1}^u].$$

Then, Nash bargaining requires

$$\begin{aligned} & \lambda \{ w_{y,t} + (1 - b)\beta [w_{o,t+1}^e + s - \alpha_{t+1}w_{o,t+1}^u] \} \\ &= (1 - \lambda) \{ p - h - w_{y,t} + (1 - b)\beta [\beta\Pi_{v,t+2} - \Pi_{v,t+1} + (p - w_{o,t+1}^e)] \} \end{aligned}$$

Then, the young worker's wage function is given by

$$\begin{aligned} w_{y,t} &= (1 - \lambda) [(p - h) + (1 - b)\beta p] + (1 - \lambda) (1 - b)\beta (\beta\Pi_{v,t+2} - \Pi_{v,t+1}) - \\ & \quad \lambda(1 - b)\beta s + \lambda(1 - b)\beta \alpha_{t+1}w_{o,t+1}^u - (1 - b)\beta w_{o,t+1}^e \quad (10) \end{aligned}$$

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<sup>21</sup>Recall that firms are infinitely-lived.

The young workers' wage function has nice intuitive properties. *Ceteris paribus*, the easier it is for workers to find vacancies (higher  $\alpha$ ), the higher the wage paid to young workers. This is because a higher future probability of finding a job means a worker's option of waiting to find employment next period has higher *expected* utility. A higher search cost ( $s$  higher) is associated with lower wages because this lowers the value of waiting to finding employment next period. In addition, an increase in the expected profitability of a vacancy next period lowers  $w_{y,t}$  since this implies the firm has a higher value of outside opportunities besides the worker with whom the firm is currently matched. Finally, the effects of the job separation rate on the wage function for young workers are ambiguous. This is because gains from trade to both the worker and the firm are lower when jobs are less durable.

### 3.6 Stationary equilibria

Henceforth we focus exclusively on time-invariant equilibria. This will allow us to investigate the properties of long-run equilibria in the labor market. A steady state equilibrium is formally defined below.

**DEFINITION** *A steady-state equilibrium with free entry in the labor market consists of wage functions  $w_y^*$ ,  $w_o^e$ ,  $w_o^u$ , exogenously specified bargaining weights  $\lambda$  and  $(1 - \lambda)$  for the firm and the worker, and a quadruple  $(\alpha^*, \theta^*, U^*, F_v^*)$  satisfying the following conditions: (i) Nash bargaining; (ii) (Unrestricted Entry for firms):  $\Pi_v^* = 0$ ; and (iii) (Steady-State):  $\alpha^* U^* = \theta^* F_v^* = \mu M(U^*, F_v^*)$ .*

#### 3.6.1 The Steady-State Matching Condition

Recall that the matching condition for the economy is given by:

$$\alpha_t U_t = \theta_t F_{v,t} = m_t = \mu (U_t)^\phi (F_{v,t})^{1-\phi}$$

where  $\phi \in [0, 1]$ . Noting that  $U_t/F_{v,t} = \theta_t/\alpha_t$ , we may therefore write the *steady-state matching condition* as:

$$\hat{\theta} = \left[ \frac{\mu}{(\hat{\alpha})^\phi} \right]^{\frac{1}{1-\phi}} \quad (11)$$

Note that the steady-state matching condition implicitly defines a relationship between  $\alpha_t$  and  $\theta_t$  consistent with steady-state values for  $U_t$  and  $F_{v,t}$ . The *SS* locus is downward-sloping.

### 3.6.2 The Equilibrium Entry Condition

Firms choose to enter the labor market in order to search for an employee and they will continue to do so until all profit opportunities from new jobs are driven to zero. This “free-entry condition” dictates that the expected present value of future profits attributable to filling the marginal vacancy must equal the cost of vacancy-posting and hiring the next worker. Utilizing the wage functions described above in conjunction with  $\Pi_v^* = 0$  (from eq. 7), we can get a closed form expression for equilibrium entry by firms denoted by  $\hat{\alpha}$  :

$$\hat{\alpha} = \frac{(p-h) + \beta(1-b)s + (1-b)\beta p + \frac{\tilde{u}_o}{\tilde{u}_y}(p-h) - \left(\frac{a}{\hat{\theta}}\right) \frac{1}{\tilde{u}_y(1-\lambda)}}{(p-h)\lambda\beta(1-b)} \quad (12)$$

The intuition for why the  $EE$  locus is upward-sloping is as follows. *Ceteris paribus*, an increase in the probability with which a firm may find an unemployed worker (i.e., higher  $\theta$ ) encourages entry because it raises the expected profitability of a vacancy. Consequently, in order to keep firms indifferent between entering or not entering the market, the probability with which an unemployed worker finds employment opportunities must go up. This serves to raise (at least) young workers’ wages, thereby discouraging firm entry.

We summarize our discussion of the  $SS$  and  $EE$  loci in Lemma 1 below.

**Lemma 1** *Given a  $\tilde{u}_o$  and  $\tilde{u}_y$  consistent with a steady state equilibrium with unrestricted entry, the combination of  $\alpha$  and  $\theta$  that make a firm indifferent between entering and not entering the labor market in search of an employee is given by the pair*

$$\hat{\alpha} = \frac{(p-h) + \beta(1-b)s + (1-b)\beta p + \frac{\tilde{u}_o}{\tilde{u}_y}(p-h) - \left(\frac{a}{\hat{\theta}}\right) \frac{1}{\tilde{u}_y(1-\lambda)}}{(p-h)\lambda\beta(1-b)}, \quad (13)$$

and

$$\hat{\theta} = \left[ \frac{\mu}{(\hat{\alpha})^\phi} \right]^{\frac{1}{1-\phi}} \quad (14)$$

Figure 1 provides a possible configuration of the  $EE$  and  $SS$  loci with the equilibrium marked as the pair  $(\hat{\alpha}, \hat{\theta})$ . Table 1 summarizes the comparative static properties of the  $EE$  and the  $SS$  loci. The parameters that shift the  $EE$  locus alone are  $p, h, s, \beta, b, \lambda, a, \tilde{u}_o, \tilde{u}_y$ .

Parameter	<i>EE</i> locus	<i>SS</i> locus
$a$	↙	no effect
$p$	ambiguous	no effect
$h$	↗	no effect
$\lambda$	↗	no effect
$s$	↗	no effect
$\mu$	no effect	↗
$b$	ambiguous	no effect
$\tilde{u}_o$	↗	no effect
$\tilde{u}_y$	ambiguous	no effect
$\beta$	↙	no effect
$\phi$	no effect	↗

**Table 1: Comparative Statics**

### 3.6.3 Existence and uniqueness

As is clear from (13) and (14), the *EE* and *SS* loci are non-linear. Hence, the issue of existence of an intersection to the two loci is not trivial. The next lemma proposes a set of conditions that are sufficient for the existence and uniqueness of a unique solution to (13) and (14).

**Proposition 1** *Define*

$$X \equiv \frac{\frac{(p-h)}{(1-b)} \left[ 1 + \frac{\tilde{u}_o}{\tilde{u}_y} \right] + \beta(s+p)}{(p-h)\lambda\beta},$$

and

$$n \equiv \left( \frac{a}{(p-h)\lambda\beta} \right) \frac{1}{\tilde{u}_y(1-\lambda)(1-b)}.$$

A sufficient condition for (13) and (14) to have a unique solution  $\hat{\alpha} \in (0, 1)$  and  $\hat{\theta} \in (0, 1)$  is given by

$$n + \mu^{\frac{1}{\phi}} < X < 1 + n.$$

When  $\hat{\alpha} \in (0, 1)$  and  $\hat{\theta} \in (0, 1)$  hold,  $\tilde{u}_y \in (0, 1)$  and  $\tilde{u}_o \in (0, 1)$  hold too.

Recall that  $\hat{\alpha}$  and  $\hat{\theta}$  were computed holding  $\tilde{u}_y$  and  $\tilde{u}_o$  fixed. That is, firms take  $\tilde{u}_y$  and  $\tilde{u}_o$  as given and compute  $\hat{\alpha}$ , the probability with which an unemployed worker will get matched in the

steady state. In general equilibrium,  $\tilde{u}_y$  and  $\tilde{u}_o$  are endogenous [see eqs. (29)-(30) in the appendix]. Proposition 2 below computes the general equilibrium steady state incorporating the values of  $\tilde{u}_y$  and  $\tilde{u}_o$  from eqs. (29)-(30) into (13) and (14).

**Proposition 2** *A general equilibrium steady state in the baseline case is jointly characterized by*

$$\alpha^* = \frac{\frac{2a}{\theta^*(1-\lambda)} - \beta(1-b)(s+p) - 2(p-h)}{\frac{a(1-b)}{\theta^*(1-\lambda)} - (1-b)(\lambda\beta+1)(p-h)}, \quad (15)$$

and

$$\theta^* = \left[ \frac{\mu}{(\alpha^*)^\phi} \right]^{\frac{1}{1-\phi}}. \quad (16)$$

Sufficient conditions for the existence of a valid solution to (15) and (16) are spelt out in the next lemma:

**Lemma 2** *A set of sufficient conditions for a valid solution [ $\alpha^* \in (0,1)$  and  $\theta^* \in (0,1)$ ] to (15) and (16) are  $\mu < 1$  and*

$$\frac{1}{(1-\lambda)} \left[ \frac{a(1+b)}{\beta(1-b)(s+p) + (p-h)(1+b)(1+\lambda\beta)} \right] > \mu^{\frac{1}{1-\phi}}.$$

Given the non-linearity in (15) and (16), the model does not permit an exact closed-form solution to (15) and (16) in general. Some progress can be made towards an exact closed form solution in the case when  $\phi = 0.5$ .

**Lemma 3** *Suppose  $\phi = 0.5$ . Then, (15) and (16) reduce to the quadratic*

$$\alpha^2 - \alpha \left[ \frac{2}{1-b} + \frac{(\lambda\beta+1)(p-h)\mu^2(1-\lambda)}{a} \right] + \frac{\mu^2(1-\lambda)}{a} \left[ (s+p)\beta + \frac{2(p-h)}{1-b} \right] = 0.$$

*The only valid solution to this quadratic is given by*

$$\alpha_1^* = \frac{\left[ \frac{2}{1-b} + \frac{(\lambda\beta+1)(p-h)\mu^2(1-\lambda)}{a} \right] - \sqrt{\left[ \frac{2}{1-b} + \frac{(\lambda\beta+1)(p-h)\mu^2(1-\lambda)}{a} \right]^2 - \frac{4\mu^2(1-\lambda) \left[ (s+p)\beta + \frac{2(p-h)}{1-b} \right]}{a}}}{2}.$$

*Then  $\theta^*$  may be computed from  $\theta^* = \frac{\mu^2}{\alpha_1^*}$ .*

That is, even though mathematically, there are two solutions to (15) and (16) that are possible, there is a unique economically-meaningful solution.

### 3.7 Age-earnings profiles

Workers in our model have different bargaining positions based purely on their position along the life-cycle. Lemma 4 computes the wages that different workers get as a result.

**Lemma 4** *In a steady-state equilibrium under free entry,*

$$w_y^* = \lambda(p - h) [1 + (1 - \lambda)\beta(1 - b)\alpha^*] - (1 - \lambda)\beta(1 - b)s \quad (17)$$

$$= \lambda(p - h) + (1 - \lambda)\beta(1 - b)J_o^{u*} \quad (18)$$

$$w_o^{u*} = \lambda(p - h)$$

$$w_o^{e*} = \lambda p.$$

Lemma 4 provides several important insights into the age-earnings profiles in our model. The first is that young workers' wages are always at least as high as wages of newly-hired old workers since  $w_y^* - w_o^{u*} = (1 - \lambda)\beta(1 - b)J_o^{u*} \geq 0$ . That is, old displaced workers suffer earnings losses.<sup>22</sup> In our model, these can be attributed to differences in bargaining power at each stage of the lifecycle. Young workers who make job contacts have the outside option of job search in the following period, an option unavailable to old workers. Thus, younger workers may earn beyond their share of their current marginal product, but older workers cannot.

This leads to an ambiguity in the slope of an individual's age-earnings profile. Although young workers' wages are higher than the wages of newly-hired old workers, it is possible that they are higher than wages of old, retained workers. The model can also generate a more conventional upward sloping age-earnings profile. The intuition is as follows. While the ability to search for employment later introduces the possibility that a young worker will obtain more than her current share of output from matching, it is also clear that future job search is costly to such a worker. Thus, higher search costs tend to reduce her outside option in bargaining over wages. In sum, higher search costs tend to reduce young workers' wages, but generally tend to not have effects on older workers' wages since older workers do not have the ability to search for jobs in the future.

### 3.8 Numerical Experiments in the Benchmark Model

It is instructive to classify the main parameters in the benchmark model into four sets, those affecting (i) the durability of jobs ( $b$ , the rate of job destruction), (ii) search/matching frictions ( $s$ ,

<sup>22</sup>Earnings losses by displaced workers are discussed in Jacobson et. al. (1993), and more recently, Rodriquez and Zavodny (2000).

$\mu$ ,  $a$ , and  $\phi$ ), (iii) matching productivity ( $p$  and  $h$ ), and (iv) bargaining conditions (the firms' share of the surplus,  $\lambda$ ). We begin by producing a baseline example.

**Example 1** *Let the parameters of the economy be as follows:  $\beta = 0.9$ ,  $s = 0.05$ ,  $a = 0.2$ ,  $\mu = 0.4$ ,  $\phi = 0.5$ ,  $b = 0.3$ ,  $p = 1$ ,  $h = 0.3$ , and  $\lambda = 0.5$ . Then, it can be easily checked that (15) and (16) have an unique economically-meaningful solution given by  $(\alpha^*, \theta^*) = (0.413, 0.387)$ . At this equilibrium,  $w_y^* = 0.380$ ,  $w_o^{u*} = 0.350$ ,  $w_o^{e*} = 0.5$ ,  $J_y = 0.298$ ,  $J_o^u = 0.0947$ ,  $\tilde{u}_y^* = 0.585$ ,  $\tilde{u}_o^* = 0.415$ ,  $F_v^* = .913$ ,  $F_f^* = 0.145$ ,  $F^* = 1.06$ .*

Starting from the above benchmark set of parameters, we vary each parameter in isolation so as to gain some insight into the effect of each factor on aggregate labor market outcomes. It is important to distinguish between the *direct* effects of each parameter from their *indirect* effects. *Direct* effects capture the impact of each parameter taking the steady-state probability of finding a vacancy ( $\alpha^*$ ) as given; *indirect* effects include the impact indirectly through its influence on  $\alpha^*$ . In many cases (not all), these two effects act in the same qualitative direction. In each numerical example that we consider, the direct effects are dominant.

### 3.8.1 Job durability

We begin by first describing how an increase in the rate of job destruction (lower durability of employment) affects labor market activity. For given wages, a higher rate of job separation ( $b$  higher) lowers the expected profitability of a firm (hence discouraging firm entry) because it must incur the costs of finding workers again in the following period. Since there are fewer firms searching for the given population of workers, the probability of any worker finding a job is lower ( $\alpha^*$  lower) and the probability of firms finding unemployed workers is higher ( $\theta^*$  higher).

What is the impact of a higher rate on job separation on young workers' wages? First, a higher rate of job destruction affects the likelihood that workers will be unemployed again in the future. They are more likely to have to search for a job again when old. In addition, since the higher rate of job destruction lowers the probability of finding a job, the expected returns from searching for a job in the following period are also lower. This lowers a young workers' outside option when bargaining with the firm and therefore young workers' wages. Consequently, the higher rate of job destruction lowers the expected lifetime utility of young workers. However, it has no effect on the lifetime utility of employed older workers. The indirect negative effect of the job destruction rate on

the probability of matching in the labor market lowers the expected lifetime utility of unemployed old workers.

Finally, the higher rate of job destruction shifts the age-composition of the unemployed towards old workers. This is primarily because finding a job in the current period is less likely to translate into a young worker remaining employed when old.

### 3.8.2 Search and matching frictions

We next turn to a discussion of the effects of changing various parameters relating to matching frictions in our economy.

**Vacancy posting costs** Higher vacancy costs have effects similar to higher rates of job destruction. Higher vacancy costs reduce firms' expected profits from posting vacancies, implying that fewer firms will enter the market. Consequently, workers will face more difficulties finding jobs ( $\alpha^*$  lower) and firms will be more likely to fill their vacancies ( $\theta^*$  higher). Otherwise, all the effects of higher vacancy costs are qualitatively the same as the higher rate of job destruction with the exception of the age-composition of the unemployed. In contrast to the effects of higher job destruction, economies with higher vacancy costs will have a higher proportion of unemployed young workers. This happens because vacancy costs affect the overall "demand" for labor whereas the job destruction rate affects only the probability of retaining a job once employed.

**Matching frictions** An overall reduction in the economy's matching frictions (higher  $\mu$ ) has the same effect as a reduction in the costs of posting vacancies except for the age-composition of the unemployed. A reduction in matching frictions implies that all workers are more likely to find jobs. Hence, the mass of unemployed old workers is lower which shifts the unemployment pool towards a higher proportion of young workers.

**Elasticity of matching technology** We can also discuss the effects of an increase in the elasticity of the matching technology with respect to the mass of unemployed workers (a higher value of  $\phi$ ). To understand the effects of  $\phi$ , it is important to first consider a limiting case where  $\phi = 1$ . In this case, the total number of matches is fixed and independent of the number of vacancies. The congestion problem arising from an increase in the number of vacancies is most severe. It follows that an increase in  $\phi$  is associated with a lower number of matches (for a given number of vacancies created); consequently, firms face more difficulties finding workers to fill their vacancies.

As a result, fewer firms enter the market and so  $\alpha^*$  will be lower and  $\theta^*$  will be higher. The effects of  $\phi$  on the remaining endogenous variables follow from its effects on  $\alpha^*$  and  $\theta^*$ .

**Worker search costs** As discussed above, the costs of search only affects the wages of young workers since old workers do not have the outside option of searching for jobs in a later period. Since the *direct* effect of higher search costs is a reduction in young workers' wages, this raises the expected profitability for firms from posting vacancies. Consequently, more firms choose to enter the market thereby raising the probability that workers will find jobs and lowering the likelihood that vacancies get filled. Since workers are more likely to find jobs and some workers will retain them ( $b < 1$ ), economies with higher search costs will have relatively younger pools of unemployed workers.

### 3.8.3 Match output

We now turn to studying the effects of the productivity of matches. Matches may be more productive either from an increase in the value of the firm's output  $p$ , or from a reduction in hiring costs,  $h$ . An increase in the productivity of matching through either possibility is associated with higher probabilities of matching for workers and a lower probability of filling a vacancy since more firms enter the market. Although an increase in  $p$  is associated with higher wages for all workers, the change in training costs only affects wages in matches for new hires,  $w_y^*$  and  $w_o^{u*}$ . This is because training costs are sunk from the perspective of matches between firms and retained employees. Either increase in the productivity of matches leads to an increase in  $\tilde{u}_y^*$ .

### 3.8.4 Bargaining power

We conclude with a discussion of the effects of an increase in workers' bargaining power,  $1 - \lambda$ . In this case, the *direct* and *indirect* effects of an increase in  $1 - \lambda$  counteract. On the one hand, an increase in bargaining power gives workers a bigger share of the surplus when matched; on the other hand it lowers workers' wages because less firms decide to post vacancies and workers are less likely to find matches ( $\alpha^*$  lower). Thus, an increase in workers' bargaining power has ambiguous effects on their expected lifetime utility. Unemployed workers (both young and old) obtain lower expected utility when  $1 - \lambda$  is higher because it is more difficult for them to find jobs while older workers who have retained their jobs from the previous period are better off.

## 4 The “Discouraged Worker” Case

We go on to study a setting (“the discouraged worker case”) where all unemployed, old workers do not participate in the labor market. This stylized environment is meant to serve as a proxy for studying the effects of various public policies in the real world (such as social security and generous old-age unemployment programs) that directly or indirectly discourage older unemployed workers from participating in the labor market. In France, for example, the “contrat de solidarité” recognized the “double need to encourage 55-59 year-old workers to stop work and to bring young workers into the labour market, as rising youth unemployment was a growing concern to society as a whole.” In many countries in Europe, a precondition to receiving unemployment benefits for people over the age of 55 is that they stop “seeking employment”. In this section, we focus on the labor market consequences of such policies.

We now reformulate our benchmark model to analyze some of the aggregate labor market outcomes associated with differing degrees of labor market participation according to age. In contrast to Section 3, we assume that old workers who are unemployed do not search for jobs. For ease of comparison, we focus solely on stationary equilibria. The details are sketched below.

Using the same notation as in Section 3, the returns to search for workers in each state are given by

$$J_y = \max \{-s + \alpha [w_y + (1 - b)\beta J_o^e + b\beta J_o^u] + (1 - \alpha)\beta J_o^u, 0\} \quad (19)$$

$$J_o^e = \max \{w_o^e, 0\} \quad (20)$$

and,

$$J_o^u = 0. \quad (21)$$

It is easy to see that (4) is replaced by

$$J_y = -s + \alpha [w_y + (1 - b)\beta w_o^e] \quad (22)$$

Firms face the same problem as before except that there are no old unemployed workers they can match with. Accordingly,

$$\Pi_v = \frac{-a + \theta \tilde{u}_y (p - h - w_y) + \theta \tilde{u}_y (1 - b)\beta (p - w_o^e)}{[1 - \theta \tilde{u}_y (1 - b)\beta^2 - \theta \tilde{u}_y (1 - b)b\beta^2 - (1 - \theta)\beta]}$$

continues to hold. For future reference, note that since  $\tilde{u}_y = 1$ ,

$$\Pi_v = \frac{-a + \theta(p - h - w_y) + \theta(1 - b)\beta(p - w_o^e)}{[1 - \theta(1 - b)\beta^2 - \theta(1 - b)b\beta^2 - (1 - \theta)\beta]}. \quad (23)$$

An old employed worker will earn  $w_o^e = \lambda p$  just as before. A young worker's surplus from finding employment is now given by

$$w_y + (1 - b)\beta J_o^e = w_y + (1 - b)\beta w_o^e = w_y + (1 - b)\beta \lambda p$$

while a firm's surplus from a match with a worker is (as before) given by  $p - h - w_y + b\beta\Pi_v + (1 - b)\beta\Pi_f - \beta\Pi_v$ . Nash bargaining implies

$$w_y = \lambda(p - h) - \lambda\beta(1 - b)(1 - \beta)\Pi_v + (1 - \lambda)(1 - b)\beta\lambda p - (1 - \lambda)(1 - b)\beta\lambda p \quad (24)$$

$$= \lambda(p - h) - \lambda\beta(1 - b)(1 - \beta)\Pi_v \quad (25)$$

Under free entry,  $\Pi_v = 0$  holds and hence (24) implies

$$w_y = \lambda(p - h). \quad (26)$$

That is, a young worker's wage is simply equal to her share of the one-period match surplus. The fact that old workers do not search for jobs effectively eliminates the outside options available to young workers, unlike the situation in (10).

Under free entry,  $\Pi_v = 0$ , and hence [from (23)]

$$a = \theta\tilde{u}_y(p - h - w_y) + \theta\tilde{u}_y(1 - b)\beta(p - w_o^e)$$

holds. Using (26) and  $w_o^e = \lambda p$ , it follows that

$$\frac{a}{\theta\tilde{u}_y} = (1 - \lambda)(p - h) + (1 - \lambda)(1 - b)\beta p.$$

Analogous to (13) and (14), we now have

$$\hat{\theta} = \frac{a}{\tilde{u}_y(1 - \lambda)[p - h + (1 - b)\beta p]} \quad (27)$$

and

$$\hat{\alpha} = \left( \frac{\mu}{\theta^{1-\phi}} \right)^{\frac{1}{\phi}}. \quad (28)$$

It follows from (27) that  $\alpha$  and  $\theta$  are independent of each other [unlike (13) and (14)]. This is to be expected since workers (by assumption) do not have any outside options to use when bargaining over their wages – so,  $\alpha$  does not enter the wage equation and therefore firms' incentives to enter the market.

## 4.1 Comparison with the Benchmark Model

The table below documents values of various endogenous variables of interest computed for both the baseline and the discouraged worker case, using the same set of parameters as in Example 1 from Section 3.

Variable	benchmark	discouraged
$\alpha$	0.413	0.532
$\theta$	0.387	0.301
$\tilde{u}_y$	0.585	1
$\tilde{u}_o$	0.415	0
$w_y$	0.380	0.350
$w_o^u$	0.350	NA
$w_o^e$	0.500	0.500
$J_y$	0.298	0.304
$J_o^u$	0.0947	0
$J_o^e$	0.500	0.500
$F$	1.06	1.07
$F_f$	0.145	0.186
$F_v$	0.913	0.885

**Table 2: Comparison with the Benchmark Model**

It is clear that the lack of labor market participation by the old has a significant impact on aggregate labor market outcomes. First, young workers more readily find jobs in the discouraged worker case than in the benchmark economy. This happens because there may be fewer workers searching for jobs, and in some cases, there is more firm entry. More firms may choose to post vacancies in the discouraged worker economy because all matches have higher durability. We find that this is the case when the separation probability is low, hiring costs are high, and the workers' cost of search is low. If there are significant costs in starting new hires, the profitability from matching with new hires is low. In these cases, firms earn higher expected profits when old workers do not search for jobs.

## 5 Aging, Unemployment, and Welfare

We conclude by asking the following question: within the context of our model, would it benefit society in the aggregate if all the old unemployed workers stayed out of the labor market? On the face of it, the answer seems obvious. An additional old worker that searches and finds a job adds to society's total output. At the same time, such a worker increases aggregate search costs (because she herself searches). Simple logic would then suggest that if the worker's addition to society's output (net of search cost) is positive, it is a good idea for the old to seek employment.

At the societal level, however, there are additional costs that accrue when the old compete with the young for jobs. By increasing congestion in the labor market, old workers reduce the possibility that a young worker will find a job. Why is this welfare-reducing? Recall that a young worker who finds a job will hold the same job with high probability for two periods while an old worker will hold the job for at most a period. This means that firms will incur less vacancy costs if they find younger workers. These costs can be especially important since all workers produce the same gross output in a match. However, increased durability of jobs can also be a mixed blessing. There will be fewer vacancies left to fill for the next generation; furthermore, tighter labor market conditions may reduce future firm entry. On net, it is therefore not transparent if policies that keep the old displaced and/or unemployed workers out of the labor market are necessarily welfare-improving.

Because we are concerned with the equilibrium effects of labor market participation or non-participation by old and unemployed workers, we chose as our welfare criterion, a population-based average of expected lifetime utility for each group of workers. In terms of our earlier notation, this measure of social welfare is given by:

$$W \equiv \frac{1}{2}J_y + \frac{1}{2}\alpha(1-b)J_o^e + \frac{1}{2}[(1-\alpha) + \alpha b]J_o^u.$$

The results of our numerical exercises are summarized in Table 3 below. Our benchmark set of parameters is the same as in Example 1 from Section 4. Again, we change one variable at a time, keeping all others fixed.

Variable	Welfare (Benchmark)	Welfare (Discouraged Worker Case)
$b = 0.25$	0.266	0.267
$b = 0.30$	0.249	0.245
$b = 0.35$	0.232	0.223
$s = 0.0$	0.303	0.270
$s = 0.05$	0.249	0.245
$s = 0.10$	0.195	0.220
$a = 0.191$	0.262	0.258
$a = 0.20$	0.249	0.245
$a = 0.250$	0.191	0.191
$\mu = 0.35$	0.181	0.182
$\mu = 0.40$	0.249	0.245
$\mu = 0.42$	0.278	0.273
$\phi = 0.45$	0.251	0.263
$\phi = 0.50$	0.249	0.245
$\lambda = 0.48$	0.248	0.245
$\lambda = 0.50$	0.249	0.245
$\lambda = 0.55$	0.247	0.242
$h = 0.25$	0.274	0.262
$h = 0.30$	0.249	0.245
$h = 0.35$	0.224	0.228
$p = 0.95$	0.213	0.214
$p = 1.00$	0.249	0.245
$p = 1.05$	0.279	0.271

**Table 3: Welfare Comparison**

The numerical results in Table 3 provide insight into the possible welfare effects of policies that discourage labor market participation by older workers. We find that net welfare effects involve the following trade-off: labor market participation by the old increases the amount of employment, but the allocation of jobs shifts towards workers with lower expected job tenures. The net welfare effects involve one or more of the following aspects of the labor market: (i) the durability of jobs, (ii) search/matching frictions, (iii) match productivity, and (iv) bargaining conditions.

Given our numerical results from Table 3, we can attempt to understand if policies that discourage labor market participation have welfare-improving effects. Although we illustrate that the lifecycle may lead to an inefficient allocation of workers to jobs, our numerical results suggest that policies which explicitly discourage labor market participation by older workers may be unwarranted. In many cases, we find that there is little or no gain from such policies. Notable exceptions are economies with higher search costs and costs of new hires (alternatively, more firm-specific human capital accumulation).

## 6 Concluding Remarks

Many countries have recently started to confront the consequences of an aging population. This poses numerous problems for public pension programs, many of which significantly impact the labor market because they encourage retirement. In response to the increasing financial burdens of ever-expanding pension programs, governments have begun seeking ways to promote more labor market participation by older workers. This raises some important questions about the impact and desirability of these policies on aggregate labor market outcomes. As a first step towards addressing these issues, this paper studies the implications of aging for labor market behavior by constructing a lifecycle model with costly search and matching in the labor market. In addition to the congestion problems associated with increased labor market participation by older workers, we illustrate that aging may lead to an inefficient allocation of workers to jobs since young workers have longer expected job tenures than older workers and because firms economize on hiring costs when they retain a worker for two periods. Despite this possibility for labor market inefficiency, it is not clear that policies aimed at encouraging retirement are welfare-improving.

It is important to note here that satisfaction of the standard “Hosios (1990) condition” does not guarantee efficiency in our model only because there are workers of different types (old and employed, old and never before employed, and old and displaced) and they all have the same

bargaining weight. We conjecture that allowing workers of different types to have different bargaining weights may eliminate the inefficiency. Indirectly, our present analysis suggests that, in some cases, efficiency may be somewhat restored within our setup by prohibiting older unemployed workers from participating in the labor market thereby allowing us to be perfectly agnostic as to the weights of different worker types in wage bargaining.

In passing, also note that the current structure does not directly yield any insights into the desirability (or lack thereof) of age-discrimination laws. Age discrimination, according to Cain (1986), is defined as lesser opportunities for older workers that do not reflect lower productivity. In the model, firms cannot restrict or direct their search only to young workers. Additionally, the matching technology is non-discriminating as far as age is concerned. The model does however produce equilibria where aggregate *worker* welfare may be higher when the older unemployed workers do not search. In this sense, if all workers collectively agree on some form of government intervention that effectively mimics employer age-discrimination, their welfare as a *group* could go up under certain circumstances.

Our current endeavour is incomplete. In order to draw further insights into these issues, we intend to extend the current model to explicitly introduce various aspects of age-specific public policies into our framework. In Bhattacharya and Reed (2001), we introduce endogenously funded old-age public pension programs and a variety of other policies which relate to aging and labor market behavior. Furthermore, an additional aspect of aging is that older workers may have a higher value of leisure time or a higher disutility cost of working than younger workers. In related research, Bhattacharya, Mulligan, and Reed (2001) study this issue and ask if this provides a rationale for public pension programs to induce retirement. Although it is possible that public retirement programs can improve the allocation of workers to jobs in the labor market, they find that countries pay their elderly substantially more than labor market search theory implies their jobs are worth.

# Appendix

## A Steady state measures for workers and firms in the baseline model

Below, we compute the steady-state measures of active firms and workers that are interacting in the labor market. To that end, it is instructive to collect all the notation in one place. Denote by

- $u_{o,t}^u \equiv$  mass of unemployed old workers at date  $t$  who did not find jobs at date  $t - 1$
- $u_{o,t}^s \equiv$  mass of unemployed old workers at date  $t$  who got separated from their jobs at date  $t - 1$
- $e_{o,t} \equiv$  mass of agents who found employment at date  $t - 1$  and are employed at the start of date  $t$
- $u_{y,t} \equiv$  mass of young newborn (unemployed) agents at date  $t$  ( $u_{y,t} = \frac{1}{2}$  by assumption)
- $U_t \equiv$  mass of unemployed workers at the start of date  $t$
- $\tilde{u}_{y,t} \equiv \frac{u_{y,t}}{U_t}$  is the probability that a given unemployed worker is young
- $\tilde{u}_{o,t} \equiv \frac{u_{o,t}^u + u_{o,t}^s}{U_t}$  is the probability that a given unemployed worker is old

Also recall that

- $\alpha_t \equiv$  probability that any unemployed worker finds a vacancy
- $\theta_t \equiv$  probability that any vacancy (firm) finds an unemployed worker
- $b_t \equiv$  probability that any employed worker gets separated from his job

Then, it follows that

$$U_t \equiv u_{o,t}^u + u_{o,t}^s + u_{y,t},$$

$$e_{o,t} \equiv \alpha(1-b)u_{y,t},$$

$$u_{o,t}^u + u_{o,t}^s + e_{o,t} = \frac{1}{2},$$

and the flow into unemployment must satisfy

$$U_{t+1} = (1-\alpha)u_{y,t} + \alpha b u_{y,t} + u_{y,t+1}$$

In a steady state then, it follows that

$$u_o^s = \frac{\alpha b}{2}$$

$$u_o^u = \frac{(1-\alpha)}{2}$$

$$U = \frac{(1-\alpha)}{2} + \frac{\alpha b}{2} + \frac{1}{2} = \frac{2 - \alpha(1-b)}{2}$$

$$\tilde{u}_y = \frac{1}{2 - \alpha(1-b)} \tag{29}$$

$$\tilde{u}_o = \frac{1 - \alpha(1-b)}{2 - \alpha(1-b)} \tag{30}$$

Firms take  $\tilde{u}_y$  and  $\tilde{u}_o$  as given when making their entry decisions.

We now proceed to compute the masses of firms with and without a vacancy. Let  $F_t \equiv$  total mass of firms in existence at the start of date  $t$ ,  $F_{v,t} \equiv$  total mass of firms with a vacancy at start of date  $t$ , and  $F_{f,t} \equiv$  total mass of firms with a filled vacancy at start of date  $t$ .

If a firm has a vacancy at  $t$ , it can find itself in one of three possible situations at  $t+1$ . First, a) it does not find a worker at  $t$  and hence will have a vacancy at  $t+1$ , or b) it finds a young worker this period; this worker gets separated with probability  $b_t$  in which case the firm will have a vacancy next period, or the worker does not get separated [with probability  $(1-b_t)$ ] in which case the firm will not have a vacancy at  $t+1$ , and c) it finds an old worker this period in which case the firm will definitely have a vacancy at  $t+1$ . Then, it follows that the flow into  $F_{v,t+1}$  is given by

$$F_{v,t+1} = F_{f,t} + (1-\theta)F_{v,t} + \theta\tilde{u}_{y,t}bF_{v,t} + \theta\tilde{u}_{o,t}F_{v,t}$$

while the flow into  $F_{f,t+1}$  is given by

$$F_{f,t+1} = \theta\tilde{u}_{y,t}(1-b)F_{v,t}.$$

Accounting restrictions require that

$$F_{v,t} + F_{f,t} = F_t$$

hold. It can be easily shown that the steady state masses of firms are as follows:

$$F = \frac{\alpha}{2} \left[ \frac{2 - \alpha(1 - b)}{\theta} + (1 - b) \right]$$

$$F_v = \frac{\alpha}{2\theta} [2 - \alpha(1 - b)]$$

$$F_f = \frac{\alpha(1 - b)}{2}$$

where  $F_f = e$  has to hold.

## B Steady state measures in the discouraged worker case

Since only the young are assumed to actively search, the mass of unemployed agents is  $\frac{1}{2}$ . The measure of old, employed agents is  $\frac{1}{2}\alpha(1 - b)$ . For firms, as before, the flow of vacancies into period  $t + 1$  is given by:

$$F_{v,t+1} = F_{f,t} + (1 - \theta)F_{v,t} + \theta bF_{v,t}$$

The flow of filled vacancies into period  $t + 1$  is given by:

$$F_{f,t+1} = \theta(1 - b)F_{v,t}$$

Accounting restrictions require

$$F_{v,t} + F_{f,t} = F_t$$

Imposing steady state on the above three equations provides a system of 3 equations in 3 unknowns:

$F_v$ ,  $F_f$ , and  $F$  from where it follows that

$$F_f = \frac{1}{2}\alpha(1 - b) \quad F_v^* = \frac{1}{2}\left(\frac{\alpha}{\theta}\right) \quad F = \frac{1}{2}\alpha\left\{(1 - b) + \frac{1}{\theta}\right\} \quad (31)$$

## C Proof of Proposition 1

Recall that

$$\hat{\alpha} = \frac{(p - h) + \beta(1 - b)s + (1 - b)\beta p + \frac{\tilde{u}_o}{\tilde{u}_y}(p - h) - \left(\frac{\alpha}{\theta}\right) \frac{1}{\tilde{u}_y(1 - \lambda)}}{(p - h) \lambda \beta (1 - b)}$$

which upon rearrangement reduces to

$$\hat{\alpha} = \frac{\frac{(p-h)}{(1-b)} \left[ 1 + \frac{\tilde{u}_o}{\tilde{u}_y} \right] + \beta(s + p) - \left(\frac{\alpha}{\theta}\right) \frac{1}{\tilde{u}_y(1-\lambda)(1-b)}}{(p-h) \lambda \beta}$$

We can solve for  $\hat{\alpha}$  and  $\hat{\theta}$  as follows (recall  $\tilde{u}_y$  and  $\tilde{u}_o$  are taken as given) from the following equation:

$$\left(\frac{\mu}{\theta^{1-\phi}}\right)^{\frac{1}{\phi}} = \frac{\frac{(p-h)}{(1-b)} \left[1 + \frac{\tilde{u}_o}{\tilde{u}_y}\right] + \beta(s+p) - \left(\frac{a}{\theta}\right) \frac{1}{\tilde{u}_y(1-\lambda)(1-b)}}{(p-h)\lambda\beta}$$

Using the definitions of  $X$  and  $n$ , this reduces to

$$\left(\frac{\mu}{\theta}\right)^{\frac{1}{\phi}} \theta + \frac{n}{\theta} = X \quad (32)$$

Define

$$H(\theta) \equiv \left(\frac{\mu}{\theta}\right)^{\frac{1}{\phi}} \theta + \frac{n}{\theta}$$

This is the left hand side of (32). The following results are easy to show:

1.

$$\lim_{\theta \rightarrow 0^+} \left(\frac{\mu}{\theta}\right)^{\frac{1}{\phi}} \theta = \infty$$

This means that near  $\theta = 0$ , the function  $H(\cdot)$  starts off near  $\infty$ . Hence,  $\theta = 0$  is ruled out as a solution. This proves that if there is any solution, it must have  $\theta > 0$ .

2.

$$H'(\theta) = \frac{1}{\phi} \left(\frac{\mu}{\theta}\right)^{\frac{1}{\phi}-1} \left(\frac{-1}{\theta^2}\right) \theta + \left(\frac{\mu}{\theta}\right)^{\frac{1}{\phi}} \left(\frac{-n}{\theta^2}\right) < 0$$

3. Recall that the right hand side of (32) is a constant. Then, a sufficient condition for a solution to (32) be a positive fraction is that

$$H(\theta = 1) < X$$

This would imply that there is an intersection between  $H(\cdot)$  and  $X$  to the left of  $\theta = 1$ . Recall  $H(1) = \mu^{\frac{1}{\phi}} + n$ . Then, we have

$$\mu^{\frac{1}{\phi}} + \left(\frac{a}{(p-h)\lambda\beta}\right) \frac{1}{\tilde{u}_y(1-\lambda)(1-b)} < \frac{\frac{(p-h)}{(1-b)} \left[1 + \frac{\tilde{u}_o}{\tilde{u}_y}\right] + \beta(s+p)}{(p-h)\lambda\beta}$$

or,

$$\begin{aligned} \mu^{\frac{1}{\phi}} &< \frac{\frac{(p-h)}{(1-b)} \left[1 + \frac{\tilde{u}_o}{\tilde{u}_y}\right] + \beta(s+p)}{(p-h)\lambda\beta} - \left(\frac{a}{(p-h)\lambda\beta}\right) \frac{1}{\tilde{u}_y(1-\lambda)(1-b)} \\ &= \frac{\frac{(p-h)}{(1-b)} \left[1 + \frac{\tilde{u}_o}{\tilde{u}_y}\right] + \beta(s+p) - \frac{a}{\tilde{u}_y(1-\lambda)(1-b)}}{(p-h)\lambda\beta} \\ &= \frac{1}{(p-h)\lambda\beta} \left[ \frac{(p-h)}{(1-b)} + \frac{1}{\tilde{u}_y(1-b)} \left\{ (p-h)\tilde{u}_o - \frac{1}{(1-\lambda)} \right\} + \beta(s+p) \right] \end{aligned}$$

Therefore, a sufficient condition for  $\hat{\theta} < 1$  is that

$$\mu^{\frac{1}{\phi}} < \frac{1}{(p-h)\lambda\beta} \left[ \frac{(p-h)}{(1-b)} + \frac{1}{\tilde{u}_y(1-b)} \left\{ (p-h)\tilde{u}_o - \frac{1}{(1-\lambda)} \right\} + \beta(s+p) \right] \quad (33)$$

hold or  $\mu^{\frac{1}{\phi}} < X - n$ . So far, we have established a condition under which  $\hat{\theta} \in (0, 1)$ . We still need to show that the corresponding  $\hat{\alpha}$  lies in  $(0, 1)$ .

4. One thing to note is that  $X > 1$ . To see this, write  $X$  as

$$\frac{\frac{(p-h)}{(1-b)} \left[ 1 + \frac{\tilde{u}_o}{\tilde{u}_y} \right] + \beta(s+p)}{(p-h)\lambda\beta} = \frac{\left[ 1 + \frac{\tilde{u}_o}{\tilde{u}_y} \right]}{\lambda\beta(1-b)} + \underbrace{\frac{\beta(s+p)}{(p-h)\lambda\beta}}_{>0}$$

Since  $\left[ 1 + \frac{\tilde{u}_o}{\tilde{u}_y} \right] > 1$  and  $\lambda\beta(1-b) < 1$ , it follows that  $X > 1$ .

5. Recall from (32) that

$$\hat{\alpha} = X - \frac{n}{\hat{\theta}}$$

Notice that  $\frac{d\hat{\alpha}}{d\hat{\theta}} > 0$  and so the maximum value that  $\hat{\alpha}$  can take corresponds to the maximum value that  $\hat{\theta}$  can take (which is 1). Then, a sufficient condition for  $\hat{\alpha} < 1$  is that  $X < 1 + n$ .

6. Combining all these, we can say that there exists a unique equilibrium  $\hat{\alpha}$  and  $\hat{\theta}$  both positive fractions if the following sufficient condition on parameters is satisfied:

$$n + \mu^{\frac{1}{\phi}} < X < 1 + n$$

In passing, notice that under this sufficient condition,  $\hat{\alpha}$  and  $\hat{\theta}$  are both positive fractions, and that fact alone immediately implies that

$$\tilde{u}_y = \frac{1}{2 - \hat{\alpha}(1-b)} \in (0, 1)$$

and

$$\tilde{u}_o = \frac{1 - \hat{\alpha}(1-b)}{2 - \hat{\alpha}(1-b)} \in (0, 1).$$

## D Proof of Lemma 4

In a steady state, the gains from hiring a young worker for the firm are given by  $p - h - w_y + b\beta\Pi_v + (1-b)\beta\Pi_f - \beta\Pi_v$ . Using  $\Pi_f = (p - w_o^e) + \beta\Pi_v$ , it follows that these gains may be written as  $p - h - w_y - \beta(1-b)(1-\beta)\Pi_v + (1-b)\beta(p - w_o^e)$ . The young worker's surplus from finding

employment is given by  $w_y + b\beta J_o^u + (1-b)\beta J_o^e - \beta J_o^u$  which upon simplification yields  $w_y + \beta(1-b)s - \beta(1-b)\alpha w_o^u + (1-b)\beta w_o^e$ . Under non-symmetric bargaining,

$$(1-\lambda)[w_y + \beta(1-b)s - \beta(1-b)\alpha w_o^u + (1-b)\beta w_o^e] = \lambda[p - h - w_y - \beta(1-b)(1-\beta)\Pi_v + (1-b)\beta(p - w_o^e)]$$

must hold. This readily simplifies to

$$w_y = \lambda(p - h) + \lambda\beta(1-b)[(p - w_o^e) - (1-\beta)\Pi_v] - (1-\lambda)\beta(1-b)[s - \alpha w_o^u + w_o^e]$$

which upon further simplification yields

$$w_y = (p - h)[\lambda + (1-\lambda)\lambda\beta(1-b)\alpha] - \beta(1-b)(1-\beta)\Pi_v - (1-\lambda)\beta(1-b)s.$$

When  $\Pi_v^* = 0$ ,

$$w_y = \lambda(p - h)[1 + (1-\lambda)\beta(1-b)\alpha] - (1-\lambda)\beta(1-b)s$$

holds. ■

## E Proof of results in Table 1

Note that (13) can be written as

$$\hat{\alpha} = X - \frac{n}{\hat{\theta}}.$$

An obvious necessary condition for  $\hat{\alpha} > 0$  is

$$\frac{\frac{(p-h)}{(1-b)} \left[ 1 + \frac{\tilde{u}_o}{\tilde{u}_y} \right] + \beta(s+p)}{(p-h)\lambda\beta} > \frac{n}{\hat{\theta}}$$

which translates to a lower-bound restriction on  $\theta$ :

$$\hat{\theta} > \frac{n(p-h)\lambda\beta}{\frac{(p-h)}{(1-b)} \left[ 1 + \frac{\tilde{u}_o}{\tilde{u}_y} \right] + \beta(s+p)}.$$

For  $\hat{\alpha} > 0$ , it is however sufficient that either

$$\frac{1}{(1-b)\lambda\beta} \left( 1 + \frac{\tilde{u}_o}{\tilde{u}_y} \right) > \frac{n}{\hat{\theta}} \quad (34)$$

or

$$\frac{\beta(s+p)}{(p-h)\lambda\beta} > \frac{n}{\hat{\theta}} \quad (35)$$

hold. Henceforth, we will maintain the sufficient conditions spelt out in (34) and (35). Now, from (13):

$$\hat{\alpha} = \frac{(p-h) + \beta(1-b)s + (1-b)\beta p + \frac{\tilde{u}_o}{\tilde{u}_y}(p-h) - \left(\frac{a}{\hat{\theta}}\right) \frac{1}{\tilde{u}_y(1-\lambda)}}{(p-h)\lambda\beta(1-b)}$$

Then, the following are immediate:

$$\begin{aligned}\frac{\partial \hat{\alpha}}{\partial a} &= - \left( \frac{1}{\hat{\theta}} \right) \frac{1}{\tilde{u}_y (1 - \lambda) (p - h) \lambda \beta (1 - b)} < 0 \\ \frac{\partial \hat{\alpha}}{\partial s} &= \frac{\beta (1 - b)}{(p - h) \lambda \beta (1 - b)} > 0 \\ \frac{\partial \hat{\alpha}}{\partial \tilde{u}_o} &= \frac{\frac{1}{\tilde{u}_y} (p - h)}{(p - h) \lambda \beta (1 - b)} > 0 \\ \frac{\partial \hat{\theta}}{\partial \mu} &= \frac{1}{1 - \phi} \left( \frac{\mu}{\hat{\alpha}} \right)^{\frac{\phi}{1 - \phi}} \left( \frac{1}{\hat{\alpha}} \right) > 0 \\ \frac{\partial \hat{\theta}}{\partial \phi} &= \hat{\theta} \ln \left( \frac{\mu}{\hat{\alpha}} \right) \frac{1}{(1 - \phi)^2} > 0\end{aligned}$$

The following are also easy to verify:

$$\begin{aligned}\frac{\partial \hat{\alpha}}{\partial \beta} &= \frac{1}{\beta} \left[ \frac{n}{\hat{\theta}} - \frac{\left( 1 + \frac{\tilde{u}_o}{\tilde{u}_y} \right)}{(1 - b) \lambda \beta} \right] < 0 \text{ by (34).} \\ \frac{\partial \hat{\alpha}}{\partial h} &= \frac{1}{p - h} \left[ \frac{(s + p)}{(p - h) \lambda} - \frac{n}{\hat{\theta}} \right] > 0 \text{ by (35).} \\ \frac{\partial \hat{\alpha}}{\partial \lambda} &= \frac{X}{\lambda} + \frac{n}{\hat{\theta}} \frac{1 - 2\lambda}{\lambda (1 - \lambda)} > 0 \text{ if } \lambda < 0.5 \\ \frac{\partial \hat{\alpha}}{\partial b} &= \frac{1}{(1 - b)^2} \left[ \frac{\left( 1 + \frac{\tilde{u}_o}{\tilde{u}_y} \right)}{\lambda \beta} - \frac{n}{\hat{\theta}} \right] \text{ is of ambiguous sign.} \\ \frac{\partial \hat{\alpha}}{\partial p} &= \frac{1}{p - h} \left[ \frac{n}{\hat{\theta}} - \frac{(s + h)}{(p - h) \lambda} \right] \text{ is of ambiguous sign.} \\ \frac{\partial \hat{\alpha}}{\partial \tilde{u}_y} &= \frac{1}{\tilde{u}_y} \left[ \frac{n}{\hat{\theta}} - \frac{\tilde{u}_o}{\tilde{u}_y} \frac{1}{(1 - b) \lambda \beta} \right] \text{ is of ambiguous sign.}\end{aligned}$$

## F Proof of Lemma 1

In a steady state with free entry,  $\Pi_v = 0$  holds. This implies

$$a = \theta \tilde{u}_y (p - h - w_y) + \theta \tilde{u}_y (1 - b) \beta (p - w_o^e) + \theta \tilde{u}_o (p - h - w_o^u) \quad (36)$$

Since

$$w_o^u = \lambda (p - h)$$

$$w_o^e = \lambda p$$

and

$$w_y = \lambda(p-h)[1 + (1-\lambda)\beta(1-b)\alpha] - (1-\lambda)\beta(1-b)s$$

holds, (36) implies

$$a = \theta\tilde{u}_y(p-h-w_y) + \theta\tilde{u}_y(1-b)\beta(p-\lambda p) + \theta\tilde{u}_o(p-h - [\lambda(p-h)])$$

which after simplification yields,

$$\frac{a}{\theta} \frac{1}{\tilde{u}_y} = (p-h-w_y) + (1-\lambda)(1-b)\beta p + \frac{\tilde{u}_o}{\tilde{u}_y} (p-h)(1-\lambda) \quad (37)$$

Using the expression for  $w_y$  reveals

$$\begin{aligned} p-h-w_y &= p-h-\lambda(p-h)[1 + (1-\lambda)\beta(1-b)\alpha] + \beta(1-b)(1-\lambda)s \\ &= (p-h)[(1-\lambda)\{1 - \lambda\beta(1-b)\alpha\}] + \beta(1-b)(1-\lambda)s \end{aligned}$$

Then, it follows from (37) that

$$\begin{aligned} \frac{a}{\theta} \frac{1}{\tilde{u}_y} &= (p-h)[(1-\lambda)\{1 - \lambda\beta(1-b)\alpha\}] + \beta(1-b)(1-\lambda)s + (1-\lambda)(1-b)\beta p + \frac{\tilde{u}_o}{\tilde{u}_y} (p-h)(1-\lambda) \\ &= (1-\lambda) \left[ (p-h)\{1 - \lambda\beta(1-b)\alpha\} + \beta(1-b)s + (1-b)\beta p + \frac{\tilde{u}_o}{\tilde{u}_y} (p-h) \right] \end{aligned}$$

and so,

$$\left(\frac{a}{\theta}\right) \frac{1}{\tilde{u}_y(1-\lambda)} = (p-h)\{1 - \lambda\beta(1-b)\alpha\} + \beta(1-b)s + (1-b)\beta p + \frac{\tilde{u}_o}{\tilde{u}_y} (p-h) \quad (38)$$

holds. Rearranging (38) yields (13).

## G Proof of Proposition 2

Recall that  $\tilde{u}_y = \frac{1}{2-\alpha(1-b)}$ , and  $\tilde{u}_o = \frac{1-\alpha(1-b)}{2-\alpha(1-b)}$ . Using these in (38), we get

$$\begin{aligned} \left(\frac{a}{\theta}\right) \frac{1}{\tilde{u}_y(1-\lambda)} &= \frac{a\{2-\alpha(1-b)\}}{\theta(1-\lambda)} = [(p-h)\{1 - \lambda\beta(1-b)\alpha\} + \beta(1-b)s + (1-b)\beta p + [1-\alpha(1-b)](p-h)](p-h) \\ &= (p-h)\{2-\alpha(1-b)(\lambda\beta+1)\} + \beta(1-b)(s+p). \end{aligned}$$

It follows that

$$\frac{a\{2-\alpha(1-b)\}}{\theta(1-\lambda)} - (p-h)\{2-\alpha(1-b)(\lambda\beta+1)\} = \beta(1-b)(s+p)$$

Collecting terms involving  $\alpha$ , we get

$$\frac{2a}{\theta(1-\lambda)} - \frac{a(1-b)}{\theta(1-\lambda)}\alpha + (1-b)(\lambda\beta+1)(p-h)\alpha = \beta(1-b)(s+p) + 2(p-h)$$

or,

$$\left[ \frac{a(1-b)}{(1-\lambda)} \right] \left( \frac{\alpha}{\theta} \right) - [(1-b)(\lambda\beta+1)(p-h)]\alpha = \frac{2a}{\theta(1-\lambda)} - \beta(1-b)(s+p) - 2(p-h)$$

from where it follows that

$$\alpha = \frac{\frac{2a}{\theta(1-\lambda)} - \beta(1-b)(s+p) - 2(p-h)}{\frac{a(1-b)}{\theta(1-\lambda)} - (1-b)(\lambda\beta+1)(p-h)}.$$

■

## H Proof of Lemma 2

For convenience, rewrite (16) and (15) as

$$\theta = \frac{a}{(1-\lambda)} \frac{2 - \alpha(1-b)}{\beta(1-b)(s+p) + (p-h)[2 - \alpha(1-b)(1 + \lambda\beta)]} \quad (39)$$

and

$$\theta = \left[ \frac{\mu}{\alpha^\phi} \right]^{\frac{1}{1-\phi}}. \quad (40)$$

It is easy to check that (from (40))

$$\frac{d\theta}{d\alpha} = -\frac{\phi}{1-\phi} \frac{\theta}{\alpha} < 0.$$

Also,

$$\lim_{\alpha \rightarrow 0} \left[ \frac{\mu}{\alpha^\phi} \right]^{\frac{1}{1-\phi}} = \infty \quad \lim_{\alpha \rightarrow \infty} \left[ \frac{\mu}{\alpha^\phi} \right]^{\frac{1}{1-\phi}} = 0$$

So, the  $\theta$  - locus starts off near  $\infty$  and is downward-sloping and eventually goes to zero. The first condition we need for an intersection between (39) and (40) in the (0,1) range is that

$$\lim_{\alpha \rightarrow 1} \left[ \frac{\mu}{\alpha^\phi} \right]^{\frac{1}{1-\phi}} < 1 \Leftrightarrow \mu < 1.$$

Then, if we additionally satisfied the restriction that

$$\lim_{\alpha \rightarrow 1} \frac{a}{(1-\lambda)} \frac{2 - \alpha(1-b)}{\beta(1-b)(s+p) + (p-h)[2 - \alpha(1-b)(1 + \lambda\beta)]} > \lim_{\alpha \rightarrow 1} \left[ \frac{\mu}{\alpha^\phi} \right]^{\frac{1}{1-\phi}}$$

we would have ensured that there was at least one intersection between (39) and (40) in the (0,1) range. ■

## I Proof of Lemma 3

When  $\phi = 0.5$ , then (16) implies  $\theta^* = \frac{\mu^2}{\alpha^*}$ . Then, (15) implies

$$\alpha = \frac{\frac{2a\alpha}{(1-\lambda)\mu^2} - \beta(1-b)(s+p) - 2(p-h)}{\frac{a(1-b)\alpha}{(1-\lambda)\mu^2} - (1-b)(\lambda\beta+1)(p-h)}. \quad (41)$$

It is easily checked that (41) reduces to

$$\alpha^2 \frac{a(1-b)}{(1-\lambda)\mu^2} - \alpha(1-b)(\lambda\beta+1)(p-h) = \frac{2a\alpha}{(1-\lambda)\mu^2} - \beta(1-b)(s+p) - 2(p-h)$$

which simplifies to

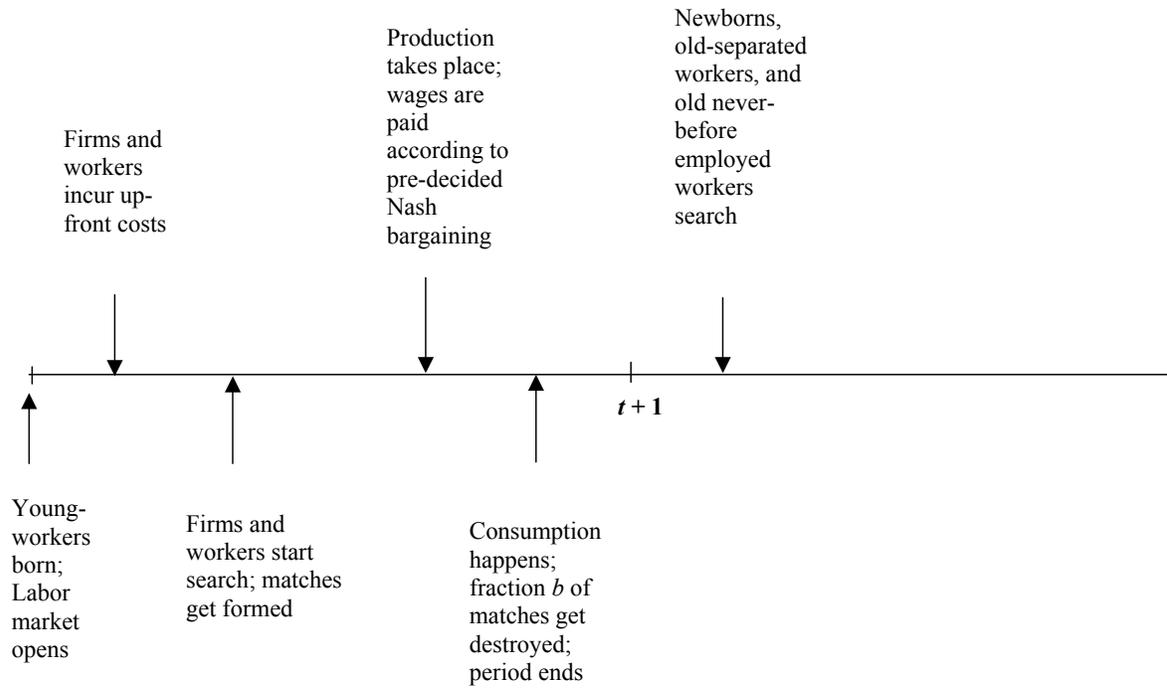
$$\alpha^2 \frac{a(1-b)}{(1-\lambda)\mu^2} - \alpha \left[ \frac{2a\alpha}{(1-\lambda)\mu^2} + (1-b)(\lambda\beta+1)(p-h) \right] + \beta(1-b)(s+p) + 2(p-h) = 0.$$

Further simplification yields the desired quadratic. ■

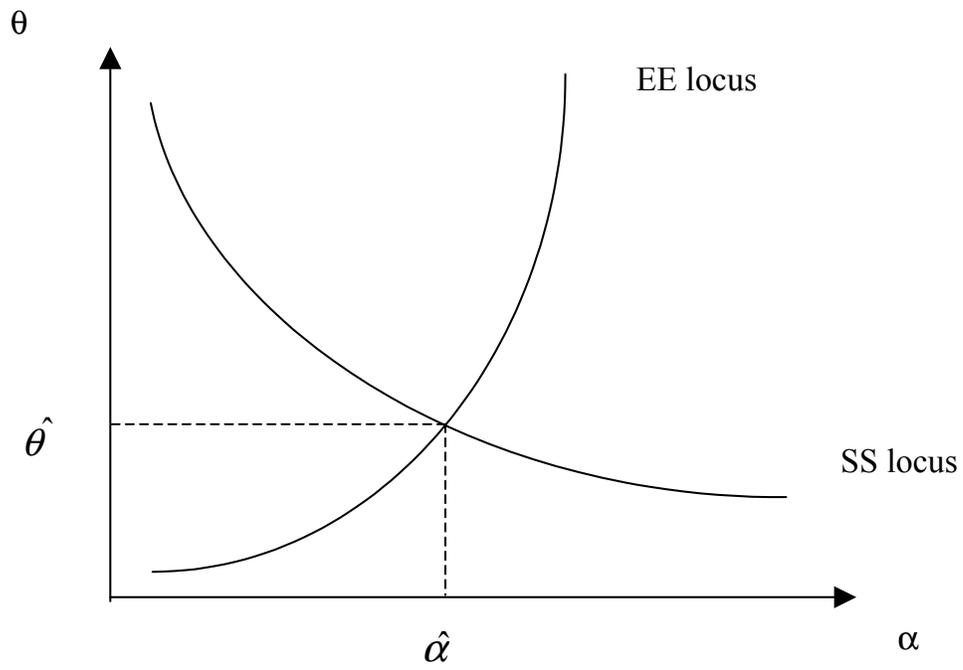
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**Figure 1:** Timeline of events



**Figure 2:** EE and SS loci