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SOCIAL SECURITY AND INTERGENERATIONAL REDISTRIBUTION*

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Many countries around the world have large public pension programs with significant cross-cohort redistribution. This paper provides a rationale for such programs in a lifecycle framework with search and matching frictions in the labor market. In the model, public pension programs alter the age composition of the labor force by inducing the jobless elderly to retire. This improves the allocation of workers to jobs, raises firm entry and may also improve welfare. By requiring a long history of labor market attachment as a precondition to receiving benefits, these programs raise the future value of current employment for the young. This redistributes bargaining strength and income from the young to the old.

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1 Introduction

At least a hundred and fifty countries around the world have public pension programs. As documented in Gruber and Wise (1999) and Mulligan and Sala-i-Martin (2004), these programs share several common striking features. First, over 70% of countries pay pension benefits in a way as to discourage work by their elderly citizens. This is starkly evident from the fact that retirement, nearly everywhere, is a necessary condition for receiving full public pension benefits. In addition, governments use a variety of "stick and carrot" measures to dissuade the elderly from seeking work: high implicit taxes and earnings penalties on income earned beyond a certain age act as sticks while generous benefits act as carrots. Second, an important prerequisite for receiving public pension benefits in almost every case is a documented long history of labor market participation. Finally, public pension programs generally have pay-as-you-go features implying substantial intergenerational redistribution. In this paper, we provide a rationale for pension programs with these features. We also offer a novel channel by which cross-cohort redistribution may take place.

Explanations for the existence of social security are classified by Mulligan and Sala-i-Martin (2004) as either political theories or efficiency theories. The former view social security as redistribution, "the outcome of a political struggle." The elderly are the winners of a political contest in which the reward is a pension (see Boldrin and Rusticini, 2001). Efficiency theories, on the other hand, identify market inefficiencies (e.g., imperfect financial markets) and explain how a pension program might be created to alleviate them. We follow the latter route and isolate a novel market inefficiency arising out of search frictions in the labor market. We go on to demonstrate that public pensions can increase aggregate welfare by partly removing such distortions.

Consider a labor market characterized by search and matching frictions and in which individuals at different positions along the lifecycle compete with each other for the same jobs. In such a setting, assuming a fixed stock of available job vacancies, increased labor market participation by the old would restrict the employment prospects of the young. Public pension programs, by encouraging retirement of the elderly, may therefore improve the ability of young workers to find jobs. Moreover, such programs can also provide incentives for firms to create more vacancies. As discussed in Oi (1962), firms incur fixed costs while establishing employment relationships with workers. If the age composition of the labor force is heavily skewed towards the elderly, then firms may not find it profitable to create vacancies.[1] Why? While post-match bargaining with the worker ensures a correct division of the post-match surplus created by the job, it does not compensate the firm for its pre-match sunk cost. The firm can better spread this expense if it gets matched with a young worker (one with a long expected tenure) than an old worker (one with a short tenure). In short, firms would earn higher profits by hiring young workers rather than the old. In such an environment, it is possible that the allocation of workers to jobs would be improved by removing some old jobless workers from the labor market. We argue that pension programs act in this way so as to foster job creation and raise welfare.[2,3]

These channels establish that social security programs can help alleviate distortions in the labor market along the extensive margin. We also illustrate how these programs may be responsible for reducing inefficiencies along the intensive margin. To see this, notice that young jobseekers with more time remaining in the job market are in a superior bargaining position compared to their older jobless counterparts. As such, firms would have to part with a large fraction of the post-match surplus in order to attract these people to work. On its own, this effect would hamper firm entry and potentially reduce aggregate welfare. We identify an entirely novel route by which pension programs may undo this bargaining advantage of the young and foster cross-cohort redistribution. By

[1] This assumes that age discrimination laws limit firms' abilities to sort across workers.

[2] In related work, Shimer (2001) also studies the implications of population aging for the labor market. He shows that young workers work for less due to their lack of labor market experience while we show this effect can be entirely induced by public pension programs. In contrast to our work, all workers in his model are infinitely-lived and in each period, a new generation of workers is born. Our methodology is most closely related to Pissarides (1992) who utilizes a two-period overlapping generations model with labor market frictions to study the implications of the loss of productivity that may accompany long-term unemployment. In contrast to our framework, all jobs in his model only last for one period, and there are no costs to labor market participation. While his analysis provides a number of interesting implications for aggregate labor market outcomes, it does not address the important interactions between wages at each stage of the lifecycle, age-targeted labor market policies (such as public pension programs), and retirement decisions.

[3] The paper that is closest in spirit to our paper is Sala-i-Martin (1996). In his setup, the old are assumed to be less productive than the young. Moreover, since there are spillovers in the production technology resulting from the average level of labor productivity, the old lower the productivity of the young in the economy. Social security helps induce the old to pull out of the labor force, thereby raising the average level of labor productivity in the economy and promoting economic growth. In contrast, we abstract away from possible differences between young and old workers, except for their naturally different positions along the lifecycle and their labor market experiences. In our setting, gross output from a match with either a young or a newly employed old worker is the same.

requiring a long employment history as a prerequisite for participation, pension programs raise the future value of current employment thereby inducing the young to work for less (in exchange for future transfer payments). They also raise the option value to not working for the old and eligible. In this manner, pension programs effectively transfer income away from the young and towards the elderly and eligible. In our setup, therefore, cross-cohort redistribution can take place *indirectly* via firms in the form of higher (lower) wages for the old (young) in addition to the more standard pay-as-you-go transfers. The potential substitutability of one form of redistribution for another is an important, yet neglected dimension in the debate about social security reform currently in progress in all OECD countries.

The plan for the rest of the paper is as follows. In the next section, we outline the model environment, specify the timeline of events, describe the various search-related costs, and compute payoffs to firms and workers. In Section 3, we compute wages and discuss the properties of the wage function for young workers, especially its connection to pension benefits. Section 4 defines an equilibrium in our model and describes a result on existence and uniqueness. As a benchmark for the effects of social security and induced retirement, Section 5 outlines an equilibrium in which there are no public pensions and all workers participate in the labor market. Section 6 establishes the "positive" aspect of our analysis by demonstrating that economies can obtain higher welfare under public pension programs that cause retirement to occur. Section 7 contains some concluding remarks. Proofs of important results are contained in the appendices.

2 The Model

2.1 Environment

Consider an economy consisting of an infinite discrete sequence of two-period lived overlapping generations populated by two types of agents, workers and firms. There is no population growth. In each period, there are workers of two different ages – the young (with measure $\frac{1}{2}$) and the old.

At birth, all workers are jobless. Old workers may be in one of three possible states:

the long-term unemployed (those who did not find jobs when young), displaced (they were employed while young, but have involuntarily lost their job; see discussion below), or employed.[4] All workers are risk-neutral. There are no saving instruments. Firms produce a homogeneous consumption good each period using labor as the sole factor of production. Production is the result of pairwise matching between one worker and a firm. Firms are infinitely-lived with a total population of measure $\mathcal{F}$ in each period. They each have access to the same technology and seek to maximize the present discounted stream of revenues net of all costs. Workers and firms share the same discount factor $\beta \in (0, 1)$.

2.2 Time line

The time line is as follows. At the start of each period, the labor market opens. At that time, jobless workers, be they old or young (the new born agents), choose whether to search for vacancies or not. If they decide to search, they incur a search cost, s, which is expressed in terms of disutility of search. As described in Pissarides (2000), s represents the imputed value of leisure in terms of output (utility). On the other side of the market, firms make the decision whether to pay some upfront costs (described below) and enter the labor market to look for employees. Each firm may employ at most one worker. Let U (F_v) denote the total mass of unemployed workers (unfilled vacancies) at the start of a period.

A stochastic matching technology connects all job seekers with open vacancies. The technology does not discriminate on the basis of age, and therefore, *any* job seeker (old or young) faces the *same* (endogenous) probability α of getting matched with a vacancy.[5] Once the labor market opens, firms and workers have at most one opportunity to meet and match. At the end of any period, the employment relationship between a worker and a firm ends involuntarily with a given probability b.[6] Put differently, a given match lasts

[4] Following Pissarides (1992), we refer to those who did not find jobs when young as the long-term unemployed. Since displaced workers are individuals who found job matches when young, but incurred a job separation, we can also refer to them as 'separated' workers. Hence, we use the terms 'displaced' and 'separated' interchangeably.

[5] Our matching structure bears many similarities to Pissarides (1992). As in his framework, workers and firms may make at most one job contact each period, and the probabilities of matching are the same for each type of worker irrespective of age (i.e., we also assume a *non-discriminating* matching technology).

[6] All job separations in the model are exogenous and outside of the worker's influence. In this sense, b is a measure of the frequency of involuntary job separations, and therefore, parameterizes the degree of job security. See Gottschalk and Moffitt (1999) for related discussion.

for a minimum (maximum) length of one (two) period(s).

At the beginning of the period, an old worker finds himself in one of three possible employment categories: employed [attached to a match from the previous period with probability $\alpha(1-b)$], unemployed [with probability $(1-\alpha)$], or displaced (working when young, but lost the job with probability αb).[7] On-the-job search is disallowed by our assumption regarding timing of labor market openings. For future reference, note that the long-term unemployed, unlike displaced workers, have no prior history of labor force attachment. This will create a distinction between them if governmental transfer payments are contingent on their employment history.[8] At the end of the period, young employed workers learn their employment status for the following date (i.e., whether their current match survives to the next period or gets dissolved); at this time, old workers die.

2.3 The Labor Market

As discussed in the introduction, public pension programs in many countries aim to induce retirement by the elderly so as to alleviate unemployment among the young. In this paper, we focus solely on the role played by public pension programs in encouraging the *jobless* elderly to withdraw from the labor market.[9] In many European countries, for example, workers can collect early retirement benefits after an involuntary separation (see

[7] Long term job attachment is an important feature of labor market behavior. For example, 34% of U.S. male workers aged 25 and over had worked for their current employer for 10 years or more in February 2000; for workers aged 55-64, 28% had worked for their current employer 20 years or more. In addition, Hall (1982) finds that after a job has lasted 5 years, the probability that it will eventually last 20 years or more in all rises to close to 0.5 among workers in their early thirties. These data imply that tenure with a firm can be quite long. The low frequency nature of our overlapping generations setup is well-suited to capture this aspect of the labor market. It bears emphasis that job turnover in our framework is entirely involuntary.

[8] This is one of the benefits of our deterministic, discrete-time model. Since each worker receives only one job contact each period, it is very easy to trace an old worker's employment status to his employment history. The linkages between eligibility for transfer payments (such as social security) and a worker's prior labor market history are clearly important, yet often ignored in models of the labor market.

[9] Displacement is an important route towards retirement in many OECD countries. For example, Chan and Stevens (2002) show that displacement increases the probability of retirement in the U.S. labor market. Specifically, they emphasize that this may be due to the costs of job search and loss of firm-specific human capital. O'Leary and Wandner (2000) conclude that while less than 10% of displaced workers under the age of 55 permanently exit the labor force, more than 25% between the ages of 55 and 64 and almost half of workers over the age of 65 opt for retirement instead of searching for alternative sources of employment upon displacement. Diamond and Hausman (1984) also discuss how job loss among older workers leads to retirement.

Gruber and Wise (1999) and especially, Boldrin, Jimenez-Martin, and Peracchi, 1999, among others). In France, the "contrat de solidarité" recognizes the "double need to encourage 55-59 year-old workers to stop work and to bring young workers into the labor market, as rising youth unemployment was a growing concern to society as a whole." A precondition to receiving unemployment benefits for people over the age of 55 is that they stop "seeking employment".[10]

We formally motivate these ideas in a setting where an individual's position along the lifecycle affects his opportunities in the labor market. Furthermore, the participation decisions of all workers have general equilibrium implications through their impact on the number of job vacancies created by firms. En route to studying the possible desirability of policies that affect labor market participation by the elderly, we analyze a setting where a particular subset of workers chooses to retire. In particular, we consider the general equilibrium consequences of public policies that encourage displaced (separated) workers to withdraw from the labor market.

In terms of deriving the endogenous labor market participation decisions of all workers (in particular, old workers), we adopt the following algorithm. We first condition on a set of strategies where all separated workers have chosen to withdraw from the labor market by accepting retirement benefits rather than incurring the costs of job search. We then study how public pensions must be designed in order to support the conjectured steady-state equilibrium. We proceed by verifying that a separated worker is better off choosing to collect pension benefits rather than searching for a job. This is the algorithm we adopt in order to endogenize labor force participation for every type of worker at each stage of the lifecycle.

2.4 Costs

Firms incur sunk costs of posting vacancies, denoted by a. Once they have incurred this cost and searched for workers, all firms are equally likely to find a worker. The probability that

[10] In Britain, the Job Release Scheme which ran between 1977 and 1988, "specifically encouraged older workers to stand down to make way for younger ones. Once out of employment, changes to the unemployment benefit regime in 1983 removed the requirement for men over 60 to look for work, encouraging them to see themselves as retired." For more details, see the OECD (1995) study on "The Labor Market and Older Workers".

a vacancy finds a worker is θ (to be determined in equilibrium below). The probabilities of meeting a given type of worker, however, will depend on the proportion of each type in the labor market. In our conjectured equilibrium, only the young and the long-term unemployed actively search for jobs. While the total measure of unemployed workers is U, the total measures of the young and long-term unemployed are u_y and u_o. The probability of finding a young unemployed worker is $\theta\tilde{u}_y$, where $\tilde{u}_y \equiv \frac{u_y}{U}$. Similarly, the probability that a vacancy locates a long-term unemployed worker is $\theta\tilde{u}_o$. The next lemma reports these population proportions for future use.

Lemma 1

$$U = \frac{1 + (1-\alpha)}{2}$$

$$\tilde{u}_y \equiv \frac{u_y}{U} = \frac{1}{1 + (1-\alpha)}$$

$$\tilde{u}_o \equiv \frac{u_o}{U} = \frac{(1-\alpha)}{1 + (1-\alpha)}$$

An important point to note here is that the population proportions are all endogenous variables since they depend on α. An implication of this is that policies aimed at altering the age-composition of the labor force also change these proportions, and thereby affect the probability with which firms encounter workers of other age groups. This general equilibrium effect is at the heart of our analysis.

Following the insights of Oi (1962) and Hutchens (1986), we posit that there are costs which must be incurred at the beginning of an employment relationship. We refer to these as "hiring" costs, and denote them as h. Let the exogenously-determined market value of the firm's output be normalized to 1. Matches with new hires require the firm and the worker to incur the costs of "hiring and training" so that the net output from new matches is $(1-h)$ while net output from a match with an old, retained worker is 1.[11] Under this interpretation, one may view h as a cost that is incurred each time a firm makes a new

[11] Note that our framework differs from the standard search-theoretic model with ex-ante heterogeneity. Although one may view the old, retained workers in our setup as "high" types and the displaced and the long-term unemployed workers as "low" types, the probability of becoming a "high" type is endogenous. This is an important distinguishing feature of our model. In particular, as we demonstrate below, the chance of becoming a "high" type will be crucially affected by policy.

hire. Alternatively, h may proxy a productivity differential between new and old matches. In the latter sense, one may also interpret h as a parameter which reflects the importance of firm-specific human capital. Firms therefore derive higher net revenues from employing workers with longer expected tenure.

The wage rate(s) for the different types of workers are determined (see below) in accordance with the protocols of Nash bargaining. As shown there, the presence of age-targeted labor market policies and the aforementioned accumulation of firm-specific human capital will cause the wages of workers (with different employment histories) to vary.

2.5 Specification of Labor Market Policies

We incorporate various aspects of real-world age-specific labor market policies, such as public pension programs and long-term old-age unemployment insurance programs, into our model. These take a particularly simple and stylized form. Old workers, currently or previously employed, are eligible for transfer payments from the government. As is common in many countries, these payments are tied to a worker's prior attachment to the labor market. In that vein, we assume that an individual who worked when young is potentially eligible for a fixed lump sum benefit of B_0.

We also allow for aspects of earnings reductions, as observed in many programs, in our framework. We capture the notion of an "earnings test" by asserting that workers who work when old receive only a fraction δ of benefits due to them.[12]'[13] For example, suppose that an old worker who retained her job from a previous match receives a wage of w_o^e. Then gross of pension benefits, he obtains total income in the amount $w_o^e + \delta B_0$. Since we conjecture that displaced workers choose to retire, in equilibrium, they earn total income B_0. The long-term unemployed are not eligible for benefits since they have no documented

[12] The "earnings test" that was applied in the United States until 2000 could be described as follows. In 1999, a worker age 62 to 65 could earn up to $9,600 without the loss of any benefits, then benefits were reduced $1 for each $2 of earnings above this amount; for workers age 65 to 69, the earnings test floor was $15,500 and benefits were reduced at a rate of $1 for each $3 in earnings. Although our framework is not suited to capture the specific features of various versions of the earnings test, we can consider its implications, more broadly defined, for retirement behavior and wages of older workers.

[13] In our specification, the higher the value of δ, the lower is the implicit tax rate on elderly work. Gruber and Wise (1999) find that while this tax is relatively low in the United States (around 20%), it is much higher in a number of European countries (as much as 80%).

history of labor force attachment.[14]

In order to study the desirability (or lack thereof) of public pension programs that induce workers of certain ages and employment histories to withdraw from the labor market, we will henceforth construct the model under the conjecture that public policies successfully induce *only* the separated workers to leave the labor market, and collect B_0. In our analysis below, we will provide a set of sufficient conditions under which this conjectured equilibrium exists.[15]

2.6 Workers' Payoffs

Let J_y denote the expected lifetime utility accruing to a worker who decides to search when young, J_o^e the expected utility of an old worker who begins the period employed and continues his employment, J_o^u the expected utility of an old worker who did not get matched when young and is back in the labor market seeking employment, and J_o^s the expected utility of an old separated (displaced) worker. Then, it is easy to see that

$$J_y = -s + \alpha \left[w_y + (1-b)\beta J_o^e + b\beta J_o^s\right] + (1-\alpha)\beta \max\{0, J_o^u\} \tag{1}$$

$$J_o^u = -s + \alpha w_o^u; \quad J_o^e = w_o^e + \delta B_o \tag{2}$$

$$J_o^s = B_o \tag{3}$$

It is instructive to explore the economic interpretation of eq.(1), as the explanations of the other value functions follow straightforwardly. A young worker seeking employment incurs an upfront cost s. Upon entering the labor market, he gets matched with a firm with probability α. In that case, he gets a wage w_y and also the expected discounted continuation payoffs from possible employment and separation the following period. If he is unsuccessful in finding a job, he will find himself in the state of being a long-term unemployed worker.

[14] For now, we ignore issues relating to funding of these programs. The consequent analytical tractability allows us to explicitly endogenize the labor market participation patterns on the basis of the design of public pension programs and formally prove existence of the conjectured steady-state equilibrium. In Section 6 , we study the effects of pension programs under a balanced budget constraint.

[15] Pissarides (1976), in an infinite-horizon model with sequential search, also studies the choice of labor market participation. He derives the optimal number of times individuals will choose to search for jobs before becoming "discouraged" and withdrawing from the labor market. However, he does not consider the role of the lifecycle in his analysis. His model also does not address how labor market participation is influenced by pension or labor market policies.

From (1), it is also clear that the value of a job to a young worker is much more than just the current wage. Because jobs are potentially durable (long-lasting), a match today bestows certain continuation privileges to the worker, a fact that will play a prominent role during the wage-bargaining phase.[16]

2.7 Payoffs to firms

Firms begin each period in one of two possible states. They may currently have a vacancy, or they may be matched with an old worker from a previous employment relationship. Letting Π_v (Π_f) be the expected lifetime profits of a firm that has an unfilled (filled) vacancy at the beginning of the period, the following equations describe the associated expected present discounted profits of a firm in each state:

$$\begin{aligned} \Pi_v &= -a + \theta\tilde{u}_y \{(1-h-w_y) + (1-b)\beta\Pi_f + b\beta\Pi_v\} \\ &\quad +\theta\tilde{u}_o \{[1-h-w_o^u] + \beta\Pi_v\} + (1-\theta)\beta\Pi_v \end{aligned} \tag{4}$$

$$\Pi_f = (1-w_o^e) + \beta\Pi_v \tag{5}$$

As indicated above, if the firm is currently matched with an old worker, it will have a vacancy next period it if incurs the cost a. Note that the firm does not face any hiring costs if the employment relationship from the previous period is retained. Also note that firms take the proportions, $\tilde{u}_y$ and $\tilde{u}_o$, as given when deciding whether to enter the labor market.

The following closed form expression for the steady state payoff to entry will be of considerable use below:

$$\Pi_v = \frac{-a + \theta\tilde{u}_y(1-h-w_y) + \theta\tilde{u}_y(1-b)\beta(1-w_o^e) + \theta\tilde{u}_o(1-h-w_o^u)}{\left[1-\theta\tilde{u}_y(1-b)\beta^2 - \theta\tilde{u}_y b\beta - \theta\tilde{u}_o\beta - (1-\theta)\beta\right]}. \tag{6}$$

[16] Davidson, Martin, and Matusz (1994) demonstrate how the durability of jobs results in a social surplus when workers have finite lives in an infinite-horizon economy. However, unlike their paper, we embed this idea into an overlapping generations framework. In addition, we explicitly introduce important features of pension programs which reinforce the role of employment beyond current compensation thereby leading to intergenerational income redistribution.

2.8 Matching

Unemployed workers and unfilled vacancies are brought together each period through a stochastic matching technology. The matching technology describes the total number of matches, $m = \mu\mathcal{M}(U, F_v)$, that are formed at the beginning of each period, depending on the total masses of unemployed workers and unfilled vacancies. Since α represents the probability that an unemployed worker will find any vacancy in the time period and θ is the probability that any unfilled vacancy will find an unemployed worker, it follows that the total number of workers who find employment $(\alpha \cdot U)$ must equal the total number of firms that filled their vacancies $(\theta \cdot F_v)$: $\alpha \cdot U = \theta \cdot F_v$. It is important to note that α and θ are determined in equilibrium, and that both workers and firms take them as given when making their decisions. Noting that $m = \theta \cdot F_v$, we have

$$\alpha U = \theta F_v = m = \mu\mathcal{M}(U, F_v) \tag{7}$$

the *matching condition*. It is standard to assume that the matching technology takes the Cobb-Douglas form: $m = \mu(U)^{1-\phi}(F_v)^{\phi}$ where $\phi \in [0, 1]$. Noting that $\theta F_v = \mu(U)^{1-\phi}(F_v)^{\phi}$, it follows that $\theta = \left[\mu\left(\frac{U}{F_v}\right)\right]^{1-\phi}$. An increase in either the number of unemployed workers or unfilled vacancies increases the number of matches each period, but at a decreasing rate. Ceteris paribus, more matches occur when μ is higher.

For analytical tractability, in all of the algebra we report below, we will assume $\phi = 1$. Then, $\theta = \mu \leq 1$ obtains. In fact, it is easiest to conduct our analysis (and obtain closed form solutions to various endogenous variables) for the case where $\theta = \mu = 1$. This implies that $\mathcal{M}(U, F_v) = F_v$. Vacancies always find a worker, but workers may find a vacancy only with a probability $\alpha \in (0, 1)$ that will be determined below.[17] In such an economy, the congestion problems facing unemployed workers are severe. Below, we will remark on how our results depend on the extent of congestion problems encountered by both workers and vacancies by studying numerical examples with $\phi < 1$.

[17] We only study equilibria in which there are more unemployed workers than vacancies. Hence, we have both $U > F_v$ and $\alpha < 1$ in our analysis.

3 Bargaining and Wage Determination

The friction inbuilt into the job-firm matching process creates the possibility that a firm may remain unproductive or a worker may remain unemployed in any period. Firms and workers must therefore weigh the implications of finding themselves in these states and their outside options when bargaining over their share of current and future surplus produced. Two important things deserve mention here. First, the outside options available to workers are crucially affected by policy, and second, these outside options are dependent on past employment history *and* on one's position in the lifecycle. Below, we will demonstrate the powerful implications of this last observation. To foreshadow, we will establish the presence of a "skewness" in bargaining position towards the young, and the role played by pension programs in "undoing" some of the resultant inequities.

3.0.1 Wage functions

We now turn to the determination of the wage offer functions for both young and old workers. Matches between workers and unfilled vacancies leads to a surplus that is to be divided between the worker and the firm. Nash bargaining dictates that the total match surplus be shared by the firm and the worker; principally for analytical tractability, we assume symmetric Nash bargaining. For an old worker with an unbroken employment relationship from the previous period, the gain from the match is $[w_o^e + \delta B_0] - B_0$. The corresponding gain to the firm is $(1 - w_o^e + \beta\Pi_v) - \beta\Pi_v = 1 - w_o^e$.[18] Then, Nash bargaining implies

$$w_o^e = \frac{1 + (1 - \delta)\, B_o}{2} \tag{8}$$

Analogously, it follows that the wages to a long-term unemployed worker (one who has no history of labor force attachment) is given by

$$w_o^u = \frac{(1 - h)}{2}. \tag{9}$$

[18] We assume that even when a match survives on to the second period, wages are determined by a fresh process of bargaining at the start of the second period. Also note that the outside option, due to the timing assumptions in the model, is discounted.

Finally, we turn to the wage determination for a young worker. The gains from trade for the firm are given by

$$1 - h - w_y + \beta\left[b\Pi_v + (1-b)\Pi_f + \Pi_v\right] - \beta\Pi_v \tag{10}$$

while the young worker's surplus from finding employment is given by

$$w_y + \beta\left[bJ_o^s + (1-b)J_o^e - J_o^u\right] \tag{11}$$

The analytical expression for the wages accruing to a young worker is described in the following lemma.

Lemma 2 *a) Under the assumption of linear matching ($\phi = 1$), the expression for the wage function for the young is given by*

$$2w_y = (1-h)\left(1 + \frac{\beta\alpha}{2}\right) - \beta\left(B_0 + s\right) \tag{12}$$

b) If $(1-h) > \beta\left(B_0 + s\right)$, then $w_y > 0$.

Note that part b) of Lemma 2 is a *sufficient* condition for young workers to earn positive wages, since it holds for any possible value of α.

Ceteris paribus, higher search costs s, reduce the wages to the young by reducing the option value to waiting and searching in the future. It also follows that, ceteris paribus, a higher pension benefit B_0, reduce the wages to the young, an issue to which we now turn.

3.0.2 Discussion of the wage function for the young

Suppose for the moment that all public pension programs are absent, i.e., $B_0 = 0$. In this case, using (8)-(9), we have:

$$w_o^u = \frac{(1-h)}{2}, \quad w_o^e = \frac{1}{2}$$

Also, using (11), the young worker's surplus from finding employment is given by:

$$w_y + \beta(1-b)w_o^e - \beta J_o^u.$$

and, using (10), the firm's surplus (assuming free entry) from hiring the young worker is given by:

$$1 - h - w_y + \beta(1-b)\,(1 - w_o^e)\,.$$

Equating, we get

$$\begin{aligned} w_y &= \frac{(1-h)}{2} + \frac{1}{2}\beta(1-b)\,(1-w_o^e) - \frac{1}{2}\beta(1-b)w_o^e + \frac{\beta}{2}J_o^u && (13)\\ &= \frac{(1-h)}{2} + \frac{\beta}{2}J_o^u > w_o^u && (14) \end{aligned}$$

A young worker can expect that his job will last beyond the current period. The wage function for young workers reflects this via the fact that the value of employment this period is more than just the current wage. It is now apparent that inequities in bargaining strength over the lifecycle arise, *purely* because of agents' positions on the lifecycle. Young workers, who have the option of searching for jobs when old, will have a higher threat point in negotiating over wages than old workers (who have no such outside option).[19]

We are now in a position to isolate a key social function played by pension programs towards reducing the aforementioned inequity. To see this, recall that young workers (by virtue of the fact that they likely have a period ahead of them) have a higher bargaining position than the old. Also a fundamental eligibility criterion for receiving pensions when old is a history of labor force attachment. Employment when young therefore raises the worker's expected net income in the future. The threat point of a young worker (arising from their position in the lifecycle) is therefore partially reduced because the firm is aware that having a job today implies current (and future) benefits to the employee; the firm naturally extracts part of that surplus. It is in this sense that public pensions help redistribute bargaining strength from young to old workers, raising the wages for the old and

[19] It is important to note here that past private earnings do not affect a worker's *current* bargaining strength because of our earlier assumption ruling out any form of asset accumulation or saving.

eligible and reducing the wages of the young.[20,21]

The above discussion was based entirely on "ceteris paribus" arguments, since α was held constant throughout the discussion. We now turn to the determination of α along with all other endogenous variables.

4 Equilibrium

4.1 Definition and Existence

We focus exclusively on time-invariant equilibria. This will allow us to investigate the properties of long-run equilibria in the labor market. A steady-state equilibrium with no labor market participation by the displaced (separated) workers is formally defined below.

DEFINITION *A steady-state equilibrium with no labor market participation by displaced workers* [an *"induced retirement equilibrium"*] *consists of wage functions* w_y, w_o^e, and w_o^u [*defined in* (8), (9), and (12)], *policy parameters,* B_0 and δ, *and a quadruple* (α, θ, U, F_v) *satisfying the following conditions: (i) Symmetric Nash bargaining; (ii) (Unrestricted Entry for firms):* $\Pi_v = 0$ *; (iii) (Steady-State):* $\alpha U = \theta F_v = \mu M(U, F_v)$, *with linear matching,* $\theta = 1$, *and (iv) the labor market participation/non-participation constraints hold:* $J_o^u > 0, J_y > \beta J_o^u, J_o^e > B_o$ and the displaced worker constraint holds (see below).

4.2 Labor Market Participation Conditions

As stated in the definition of the equilibrium, we impose a pattern of labor market participation across workers of different age groups and employment histories and then state

[20] Black (1987) also finds that social security affects age-earnings profiles. In his model, workers would rather receive private pension payments than wages as a result of social security taxes. As workers become older, they switch from pension payments to wages since the returns from pension savings would be lower. Therefore, social security tends to generate upward-sloping age earnings profiles. In his work, the retirement date is exogenous (he does not explore the early retirement incentives in the social security system). In addition, there is no unemployment in his model.

[21] In contrast to symmetric Nash bargaining, we could posit that agents' bargaining weights vary across the lifecycle. If old individuals receive higher weights than young workers, young agents' outside options would have less impact on wage determination. Consequently, there would be less need for intergenerational redistribution.

conditions under which this pattern emerges as an equilibrium. In particular, we study a steady-state equilibrium in which old workers who have experienced job loss during the course of their careers choose to accept their pension benefits and withdraw, rather than incur the costs of job search. Old individuals who have retained their jobs continue working since they have higher productivity than when they were initially employed. In contrast, the long-term unemployed with no access to pension benefits choose to look for jobs. We also provide conditions to ensure that the young actively search for jobs.

We begin with a discussion of the participation conditions for old workers who have retained jobs from their youth. In order for them to continue working, we must have

$$J_o^e = w_o^e + \delta B_o > B_o. \tag{15}$$

which using (8) reduces to $1 > (1-\delta) B_o$, a sufficient condition for which is $1 > B_o$.

The next step is to find conditions under which displaced workers choose to accept pension benefits and retire rather than incur the costs of job search. If a separated worker chooses to accept pension benefits, his expected utility is: $J_o^s = B_o$. However, the decision to withdraw from the labor force must yield higher expected utility. Therefore, the following condition must hold:

$$B_o > -s + \alpha w^s + \alpha\delta B_o + (1-\alpha) B_o \tag{16}$$

Under the assumption that an individual displaced worker chooses to search for a job (an 'individual' deviation), with probability α the displaced worker would be able to obtain employment and would earn total income $(w^s + \delta B_o)$. In this event, the wage he would earn is given by:

$$w^s = \frac{(1-h) + (1-\delta) B_o}{2} \tag{17}$$

Alternatively, if unable to find employment, the worker would still be able to collect pension benefits. Using (17) in (16), it follows that policy- induced withdrawal by displaced workers occurs if $(1-h) > (1-\delta) B_o$ and

$$\alpha < \frac{2s}{[(1-h) - (1-\delta) B_o]} \tag{18}$$

holds. This provides an upper-bound for α.

In contrast, for the long-term unemployed to search for jobs, we require that:

$$J_o^u = -s + \alpha w_o^u > 0$$

Using (9), this condition may be rewritten to provide a minimal value for α for which the long-term unemployed remain active in the labor market:

$$\alpha > \frac{2s}{(1-h)} \tag{19}$$

Obviously, if the search costs are too high, the long-term unemployed would be better off choosing not to search for jobs. A quick comparison of (18) and (19) reveals the following insight regarding the earnings test.

Lemma 3 *For an induced retirement equilibrium to exist, it is necessary but not sufficient that there be an earnings penalty, i.e.,* $\delta < 1$ *must obtain.*

If $\delta = 1$, an induced retirement equilibrium does not exist. Finally, in order for young workers to search for jobs, the expected utility of participation when young must exceed the value of waiting and looking for a job when old. This implies that:

$$J_y = -s + \alpha\left[w_y + (1-b)\beta J_o^e + b\beta J_o^s\right] + (1-\alpha)\beta J_o^u > \beta J_o^u \tag{20}$$

The following lemma provides conditions on α such that young workers choose to actively search in the labor market.

Lemma 4 *a) Suppose that the displaced worker constraint is satisfied and* $(1-h) > (B_0 + s)$*. In addition, let* $b_1 \equiv [(1-h) - \beta(B_0+s)]$*. If* $\alpha w_y(\alpha) > s$*, then (20) is satisfied and young workers will choose to search for jobs, which obtains when*

$$\alpha > \alpha_L \equiv \frac{-b_1 + \sqrt[2]{b_1^2 + 4\beta(1-h)s}}{\beta(1-h)} > 0 \tag{21}$$

b) $\alpha_L > \frac{2s}{(1-h)}$

To summarize, a valid induced retirement equilibrium value of α under linear matching must satisfy (18), (19), and (21); additionally, the level of benefits B_0 must satisfy $(1-h) > \max\left[(1-\delta)B_o, \beta(B_0+s)\right]$. Henceforth we will maintain the assumption:

Assumption 1 *a)*

$$(1-h) > \max\left[(1-\delta)B_o, \beta\left(B_0+s\right)\right] \tag{22}$$

b)

$$3\left(1-h\right) > 4a \tag{23}$$

4.3 Equilibrium Entry Condition

Firms enter the labor market in search of employees until all profit opportunities from new jobs are driven to zero. This "free-entry condition" dictates that the expected present value of future profits attributable to filling the marginal vacancy must equal the cost of vacancy-posting and hiring the next worker. Utilizing the wage functions described above, along with $\Pi_v = 0$ [see (6)], we have

$$\left(\frac{a}{\theta}\right)\frac{1}{\tilde{u}_y} = (1-h-w_y) + (1-b)\beta\left(1-w_o^e\right) + \left(\frac{\tilde{u}_o}{\tilde{u}_y}\right)(1-h-w_o^u) \tag{24}$$

Then, setting $\theta = 1$, candidate equilibrium values of α are derived from (24) using Lemma 1, (8), (9), and (12). One of the major benefits of using a simple matching technology like ours is that closed-form solutions to (24) can be analytically derived.[22]

The following proposition describes the conditions required for existence of an induced retirement equilibrium.

Proposition 1 *a) The unique solution to (24) in terms of* α, *is given by*

$$\alpha \equiv \alpha\left(B_0;\delta\right) = 2\left\{\frac{\beta s + \beta(1-b) + \beta B_0\left[1-\left(1-b\right)\left(1-\delta\right)\right] + 2\left[(1-h)-2a\right]}{\left[(1-h)\left(2+\beta\right)-4a\right]}\right\}. \tag{25}$$

b) Suppose Assumption 1 holds. Then, for α *in (25) to be part of an induced retirement equilibrium with* $\theta = 1$, *(18), (19), and (21) must hold or, more compactly, the condition*

$$\alpha \in \left\{\alpha_L, \min\left(\frac{2s}{\left[(1-h)-(1-\delta)B_o\right]}, 1\right)\right\}$$

[22] As stated above, each worker experiences the same probability of finding a job vacancy. This assumes strict enforcement of age discrimination laws. Alternatively, one could consider that firms sort across different types of workers. In this manner, there would effectively be two different labor markets: a market for young workers and a separate market for old. Under equilibrium entry, firms earn the same expected net profits. However, more vacancies would be posted in the young labor market.

holds.[23]

4.4 Partial equilibrium effects of increasing the generosity of benefits

As discussed in Mulligan and Sala-i-Martin (2004), many OECD countries have munificent pension programs that induce the jobless elderly to retire and make way for the young. In this section, we establish the effects of varying the generosity of pension programs within an induced retirement equilibrium. Assuming that an equilibrium exists, we are able to analytically derive the effects of pension benefits on employment and the age-composition of the labor force. We begin by reporting the results of some comparative static exercises conducted with respect to B_0. It bears emphasis here that all the upcoming results in this subsection assume away any issues relating to funding of B_0 and are hence to be understood as being "partial equilibrium" in nature. As Section 6 will demonstrate, these insights are robust to settings in which pension benefits are funded endogenously by payroll taxes. The principal benefit of the "partial equilibrium" perspective is that it allows us to derive a number of interesting clean results analytically.

Proposition 2 *Under Assumption 1, an increase in B_0 raises the probability of finding employment. In particular, we have:*

$$\frac{\partial \alpha}{\partial B_0} = \frac{2\beta\left[1-(1-b)(1-\delta)\right]}{\left[(1-h)(2+\beta)-4a\right]} > 0. \tag{26}$$

An increase in B_0 reduces the wages of the young and at the same time raises the wages of old workers with jobs. Since the young constitute the bulk of the jobseekers, this raises the benefit from firm entry, and more firm entry makes it easier for any given worker to find a vacancy thereby raising the employment rate. Therefore, we refer to this transmission channel of pension programs as the "vacancy creation effect." On the face of it, Proposition 2 is a formal statement of the type of argument governments use to defend generous pension programs ostensibly intended to free up jobs for the young.

[23] For a generic θ, it is easily checked that (25) is given by

$$\alpha = \frac{2}{(1-h)(2+\beta)-\frac{4a}{\theta}}\left[2\left(1-h-\frac{2a}{\theta}\right)+(1-(1-b)(1-\delta))\beta B_0+\beta s+\beta(1-b)\right]$$

which we use later when doing numerical computations with a non-linear matching technology.

An immediate consequence of Proposition 2 is the following corollary.

Corollary 1 *An increase in B_0 changes the age-composition of the labor force via*

$$\frac{\partial \tilde{u}_y}{\partial B_0} = -\frac{1}{(\tilde{u}_y)^2}\left(-\frac{\partial \alpha}{\partial B_0}\right) > 0$$

$$\frac{\partial \tilde{u}_o}{\partial B_0} = -\frac{1}{\left[1+\frac{1}{(1-\alpha)}\right]^2}\frac{-1}{(1-\alpha)^2}(-)\frac{\partial \alpha}{\partial B_0} < 0$$

As established by Proposition 2, higher pension benefits increase the probability that any worker is able to obtain employment. In particular, since there will be less workers who are unable to find jobs when young, the pool of the long-term unemployed will be smaller. Consequently, more generous pension benefits raise the probability with which firms are likely to encounter a young worker. As we describe below, from examining (8), (9), and (12), this "vacancy creation effect" has implications for age-earnings profiles in the economy:

Proposition 3 *An increase in B_0 raises the wages to the old and employed and has no effect on the wages of the never-before-employed. The effect on young wages is ambiguous.*

The effect on w_y can be seen from the expression for w_y, reproduced here for convenience:

$$2w_y = (1-h)\left(1+\frac{\alpha\beta}{2}\right) - \beta\,(B_0+s)$$

On the one hand, a higher B_0 raises α which serves to raise w_y [the "vacancy creation effect" i.e., workers can find jobs more easily and hence their "price" must go up], but on the other hand, a higher B_0 serves to reduces w_y [this is the "bargaining strength redistribution effect" that was discussed in Section 3]. The net impact is ambiguous and depends on the relative strength of the two aforementioned effects. In particular, the vacancy creation effect somewhat compromises the income redistributive goal of social security.

Under the linear matching technology, the effect of an additional vacancy on the probability of finding of a job can be substantial. This may even cause the wages of the young to rise with benefits. Numerical computations confirm that with a small degree of non-linearity ($\phi < 1$) in the matching function, the "vacancy creation effect" (an indirect influence) is muted and dominated by the direct "bargaining strength redistribution effect". Intuitively, the latter effect will dominate the vacancy creation effect as long as firms also encounter congestion problems in the labor market, i.e., there is some degree of diminishing returns to the addition of another vacancy.[24]

Example 1 *Let* $s = 0.15$, $a = 0.2$, $h = 0.4$, $\delta = 0.2$, $\beta = 0.95$, $\mu = 0.4$, $B_0 = 0.2$, $b = 0.2$, *and* $\phi = 0.7$. *For this parametric specification,* $\alpha = 0.61867$, *and for this value of* α, *all other conditions outlined in the definition of the induced retirement equilibrium hold. Using this configuration, it is apparent from Figure 1 that raising the level of benefits lowers the wages to the young and raises the wages to the old and eligible, thereby accomplishing cross-cohort income redistribution.*

This, in some sense, is a major punchline of the paper. In the model, pension programs raise the incomes of the old with jobs and tend to reduce the incomes of the young, thereby engineering an intergenerational income distribution towards the elderly. The novelty of our paper lies in the fact that we can demonstrate the presence of such intergenerational income distribution in the complete absence of any equity or political economy concerns.

5 The absence of policy

In this section, as a benchmark for considering the effects of induced retirement, we briefly outline the environment in the absence of any policy intervention. Since much of the basic structure of the economy remains the same, we choose to minimize detailed discussion of the analysis.

We start by revisiting the value functions describing the expected lifetime utility of workers.

$$J_y = -s + \alpha \left[w_y + (1-b)\beta J_o^e + b\beta J_o^u\right] + (1-\alpha)\beta J_o^u; \quad J_o^e = w_o^e; \quad J_o^u = -s + \alpha w_o^u$$

Recall that in the absence of policy, the old separated are indistinguishable from the old

[24] See Bhattacharya, Mulligan, and Reed (2003) for further discussion.

never-before-employed, and can be lumped into the single category of jobless elderly. The value functions for the firms are the same as before.

It is clear that the wage functions for old jobless workers and retained individuals are the same as in the previous section since they do not depend on the probability of finding a job. In contrast, the wage paid to the young is however different. Even though the gains from trade to the firm from hiring a young worker remain the same as before, a young worker's surplus from finding employment is now given by

$$w_y + b\beta J_o^u + (1-b)\beta J_o^e - \beta J_o^u = w_y + \beta(1-b)s - \beta(1-b)\alpha w_o^u + (1-b)\beta w_o^e$$

In particular, the surplus reflects that young workers will choose to search for jobs when they become old since induced retirement does not occur. Under symmetric bargaining, and free entry, it can be shown that

$$w_y^{np} = \frac{(1-h)}{2}\left[1 + \frac{\beta(1-b)\alpha}{2}\right] - \frac{\beta(1-b)s}{2} \tag{27}$$

where the superscript "*np*" signifies "no policy".

The most crucial difference between the environment with and without policy is in the nature of the equilibrium. As we have discussed earlier, the case with policy focuses on an equilibrium in which pension benefits successfully induce the old and separated to withdraw from the labor force. The appropriate comparison is with a setting without policy intervention in which *every* jobless worker is in the labor force.

DEFINITION *A steady-state equilibrium without policy intervention and with labor market participation from all workers consists of wage functions* w_o^u, w_o^e, and w_y [*defined in* (8), (9), and (27)], *and a quadruple* (α, θ, U, F_v) *satisfying the following conditions: (i) Symmetric Nash bargaining; (ii) (Unrestricted Entry for firms):* $\Pi_v = 0$*; (iii) (Steady-State):* $\alpha U = \theta F_v = \mu\mathcal{M}(U, F_v)$*, with* $\theta^{np} = 1$*, and (iv) the labor market participation constraints hold:* $J_o^u > 0, J_y > \beta J_o^u$, and $J_o^e > 0$.

As before, firms enter until $\Pi_v^* = 0$. Analogous to the equilibrium entry condition derived in the model with policy, it can be checked that

$$\alpha^{np} = \frac{4(1-h) - 8a + 2(1-b)(1+s)\beta}{(1-b)\left[(1-h)\beta + 2(1-h) - 4a\right]} \tag{28}$$

We conclude this section with an important result.

Proposition 4 *Young workers always earn higher wages in the absence of induced retirement. That is,* $w_y^{np} > w_y$ *where* w_y *is defined in eq. (12).*

This is the crux of the income redistribution argument. Pension policies, by their very nature, raise the future value of employment, and thereby reduce wages to the young. In the absence of such policies, as Proposition 4 indicates, the wages of the young are relatively high. Since they form the bulk of the job seekers, ceteris paribus, a higher wage to the young adversely affects firm entry, and possibly reduces aggregate worker welfare.

Furthermore, by encouraging old displaced workers to retire, social security programs can play an important role in improving the allocation of workers to jobs since they allow young workers to more easily find jobs and accumulate firm-specific human capital. In what follows below, we first aim to demonstrate our efficiency rationale for public pensions and induced retirement. In order to accomplish this objective, we explicitly introduce a government budget constraint into our framework so that pensions are funded within the economy. Specifically, we present a setting where payroll taxes imposed on both firms and workers are used to pay for pension benefits. Section 6 below establishes that public pensions through induced retirement can lead to higher welfare than when public pensions are absent.

6 Are pension programs welfare enhancing?

The principal point of this paper is to argue that pension programs, through their effect on the wage structure, their inducement to pull the old displaced workers out of the labor market, and thereby encourage firms to create more job vacancies, can improve the operation of the labor market and might therefore be desirable on efficiency grounds alone (abstracting from the more standard equity and political economy motives). To that end, before discussing the overall effects of pension programs and their interactions with labor market conditions, we first seek to demonstrate that endogenously funded pension programs and publicly induced retirement can lead to higher welfare than having no pension program at all. Below we sketch a version of our model that introduces payroll taxes on workers and firms which are then used to pay the old separated to stay away from the labor market. We compute aggregate welfare (defined below) for this economy and compare it to aggregate welfare for the economy described in Section 5.

Much of the analysis set forth above will remain valid in this section. To begin with, the value functions for workers of different types are given by

$$J_y = -s + \alpha\left[(1-\tau)w_y + (1-b)\beta J_o^e + b\beta J_o^s\right] + (1-\alpha)\beta J_o^u$$

$$J_o^u = -s + \alpha(1-\tau)w_o^u, \qquad J_o^e = \left[(1-\tau)w_o^e + (1-\tau)\delta B_o\right],$$

and, under the conjectured equilibrium that displaced workers do not search,

$$J_o^s = (1-\tau)B_o$$

where τ is the common tax rate on wage and benefit income. In addition, in the steady-state we observe:

$$\Pi_v = -a + \theta\tilde{u}_y\left\{[1-h-(1+\tau)w_y] + (1-b)\beta\Pi_f + b\beta\Pi_v\right\} + \theta\tilde{u}_o\left\{[1-h-(1+\tau)w_o^u] + \beta\Pi_v\right\} + (1-\theta)\beta\Pi_v$$

$$\Pi_f = 1-(1+\tau)w_o^e + \beta\Pi_v$$

It is easily verified that

$$\Pi_v = \frac{-a + \theta\tilde{u}_y\left[1-h-(1+\tau)w_y\right] + \theta\tilde{u}_y(1-b)\beta\left[1-(1+\tau)w_o^e\right] + \theta\tilde{u}_o\left[1-h-(1+\tau)w_o^u\right]}{\left[1-\theta\tilde{u}_y(1-b)\beta^2 - \theta\tilde{u}_y(1-b)b\beta^2 - \theta\tilde{u}_o\beta - (1-\theta)\beta\right]}$$

Using the process of wage determination analogous to the one described in Section 3 above, it can be shown that

$$w_o^u = \frac{(1-h)}{2}; \qquad w_o^e = \frac{1+(1-\delta)(1-\tau)B_o}{2} \tag{29}$$

$$2w_y = \left[(1-h) + (1-b)\beta\left\{1-(1+\tau)w_o^e\right\}\right] \tag{30}$$
$$-\left[\beta b(1-\tau)B_o + (1-b)\beta\left\{(1-\tau)w_o^e + \delta(1-\tau)B_o\right\} - \alpha(1-\tau)w_o^u\right]$$

Utilizing the wage functions described above, along with $\Pi_v = 0$, we have the same equilibrium entry condition as in (24) given by

$$\frac{a}{\theta}\left(\frac{1}{\tilde{u}_y}\right) = \left[1-h-(1+\tau)w_y\right] + (1-b)\beta\left[1-(1+\tau)w_o^e\right] + \left(\frac{\tilde{u}_o}{\tilde{u}_y}\right)\left[1-h-(1+\tau)w_o^u\right] \tag{31}$$

What remains for us to describe is the government budget constraint. The payroll taxes paid by firms are given by

$$F_v \theta \tilde{u}_o \tau w_o^u + F_v \theta \tilde{u}_y \tau w_y + F_f \tau w_o^e$$

since some job vacancies will be filled by the long-term unemployed and others by the young. In addition, some taxes will be paid by firms with retained workers from prior established employment relationships. In contrast, taxes paid out by all the workers are given by

$$\frac{\alpha \tau w_y}{2} + \frac{\alpha(1-b)\tau\left(w_o^e + \delta B_o\right)}{2} + \alpha u_o \tau w_o^u + \frac{\alpha b}{2}\tau B_o$$

The expenditure by the government on workers is given by

$$\frac{\alpha(1-b)}{2}\delta B_o + \frac{\alpha b}{2} B_o$$

We assume that the government balances its budget. It is also easy to verify that the

$$F_f = \frac{\alpha(1-b)}{2}; \qquad F_v = \frac{\alpha(1-b)(2-\alpha)}{\theta\left[2(1-b)+\alpha(1-\alpha)\right]}; \qquad \mathcal{F} = \frac{\frac{\alpha(1-b)}{2}}{\theta\left[1 - \frac{b}{2-\alpha} - \frac{1-\alpha}{2}\right]} + \frac{\alpha(1-b)}{2}$$

For completeness sake, we define an equilibrium below.

DEFINITION *A steady-state equilibrium with no labor market participation by displaced workers and internally funded pensions consists of wage functions* w_o^u, w_o^e, and w_y [*defined in* (29), and (30)], *and a quadruple* (α, θ, U, F_v) *satisfying the following conditions: (i) Symmetric Nash bargaining; (ii) (Unrestricted Entry for firms):* $\Pi_v = 0$*; (iii) (Steady-State):* $\alpha U = \theta F_v = \mu \mathcal{M}(U, F_v)$*, (iv) the labor market participation constraints hold:* $J_o^u > 0, J_y > \beta J_o^u$, and $J_o^e > (1-\tau)B_o$ *and the discouraged worker constraint holds,*[25] *and v) the government's budget is balanced.*

[25] In this case, the discouraged worker constraint requires

$$(1-\tau)B_o > -s + \alpha\left[(1-\tau)w^s + \delta\left(1-\tau\right)B_o\right] + (1-\alpha)(1-\tau)B_o \tag{32}$$

where

$$w^s = \frac{1 - h + (1-\delta)(1-\tau)B_o}{2}$$

We choose a population-based average of expected lifetime utility of each group of workers as our welfare criterion. In particular, we adopt the following measure of social welfare as our welfare criterion:[26]

$$W \equiv W(B_0, \delta) = \frac{1}{2}J_y + \frac{1}{2}\alpha\beta(1-b)J_o^e + \frac{1}{2}\alpha\beta b J_o^s + \frac{1}{2}\beta(1-\alpha)J_o^u$$

The task ahead is to compare aggregate welfare in the presence and absence of policy. In the presence of (internally-funded) policy, the old and separated stay out of the labor market. In the absence of such policy intervention, every worker participates in the labor market but there are no pension payments or taxes. The question for us is: is aggregate welfare higher when no pension programs are present and all workers remain active in the labor force?

As is readily apparent, a number of non-linearities enter the model with the introduction of the government budget constraint especially when benefits are funded by distortionary taxes. Therefore, we use numerical computations for the general case of non-linear matching to illustrate our reasoning.

Example 2 *Let* $s = 0.15$, $a = 0.2$, $\mu = 0.4$, $h = 0.25$, $\phi = 0.5$, $B_0 = 0$, $\beta = 0.9$, *and* $b = 0.2$. *Under this parametric specification, pension programs are not present, and it can be checked that all workers stay active in the labor market. The aggregate welfare in this case is* 0.2095. *Now consider an otherwise identical parametric specification except that* B_0 *is allowed to go from* 0.25 *to* 0.4 *and* $\delta = 0.8$. *By the government budget constraint, it follows that* τ *varies from* 0.108781 *to* 0.17043. *For this specification, it can be verified that all the conditions defined in the definition of equilibrium in this section are satisfied. As illustrated in Figure 2, aggregate welfare under induced retirement is higher than when there are no public pension benefits and retirement does not occur. Importantly, the probability of employment* (α) *is higher and wages to the young are lower under policy than in the absence of pension programs.*

Note that policy-induced retirement occurs as long as pension benefits are sufficiently generous and the implicit tax on elderly work is sufficiently high ($\delta = 0.8 < 1$). Once

[26] See Davidson et. al. (1994) for a similar welfare criterion.

pension benefits are equal to 0.25, induced retirement occurs. In particular, we observe that publicly induced retirement generates higher welfare since the vertical intercept in Figure 2 is equal to 0.247. As mentioned in the above example, welfare in the absence of pension programs is 0.2095.

7 Conclusion

Most countries have large public pension programs. Traditionally, these programs have been used to induce retirement by the elderly in order to free up jobs for the young and to redistribute income across generations. This paper provides an efficiency rationale for the inter-generational income redistribution focus of such programs in a framework which explicitly accounts for the role of the lifecycle in the labor market. It develops a model of the labor market characterized by search and matching frictions and embeds it into an overlapping generations framework. In our model, public pension programs alter the age composition of the labor force by inducing the jobless elderly to retire in exchange for pension benefits. By requiring a long history of labor market attachment in order to receive benefits, these programs raise the future value of current employment for the young. In turn, this raises the future value of current employment which alters the bargaining positions of agents and effectively redistributes income from the young to the old. In addition, depending on the design of the pension program, we show that the redistribution can take place directly via the government (explicit transfer payments) or indirectly via firms in the form of higher wages.

We believe that careful general equilibrium analysis of the underlying issues can shed important light and offer some guidance to policymakers. In this regard, we have ventured to study the efficiency and desirability of publicly-funded pension programs within the context of a dynamic general equilibrium model. In order to consider how age-targeted labor market policies such as social security should be designed in light of the ongoing trend towards an increasingly older population, we adopted the OG setup because it allows a natural and explicit separation of the workforce into young and old workers. The framework captures an important inter-generational conflict between the young and old since, in the model, these groups concurrently compete for the same jobs; additionally, the bargaining positions of the two during wage negotiations are different due to their different stages along the lifecycle. Moreover, the OG structure is naturally conducive to studying pension programs that tie in with the lifecycle and other "low frequency" aspects of the labor market, such as, long job tenure, and the accumulation of firm-specific human capital.

We used the search framework in the labor market for three important reasons. First, it allows us to endogenize both the supply side (through labor market participation choices) and the demand side (via endogenous creation of vacancies) of the labor market, a clear departure from the "lump-of-labor" line of thought in which there is a fixed stock of job vacancies. Importantly, we see that the amount of job creation responds to the design of pension programs through their impact on age earnings profiles in the economy.

Second, the retirement literature suggests that social security programs are designed to reduce labor market congestion problems for the young. The diminished prospects for job search are also a prominent factor in the labor market participation decisions of older workers. In this regard, we argue that a model with undirected search is appropriate since it allows us to demonstrate how labor market congestion contributes to potential intergenerational conflicts in the labor market. If firms were perfectly able to discriminate on the basis of age, this would imply that there are not any intergenerational congestion difficulties between workers and therefore, there would be little role for policy-induced retirement.

Finally, the decentralized notion of wage bargaining used in our framework allows us to study the effects of public pension programs on wage determination at each stage of the lifecycle. This is especially important given the fact that most real-world pension benefits are generally related in some way to the number of years worked and tend to increase with lifetime earnings. In this context, an important new effect that we identify is the role of social security in redistributing agents' outside options over the lifecycle. In our setup, younger (longer tenure) workers have the option of waiting while older (equally productive but with shorter tenure) workers do not. This inequity translates into high wages for the young, escalating labor costs (since young workers constitute the largest pool of the unemployed from which firms will have to find workers), and reduces firm entry. Positive replacement rates, raise the lifetime value of working when young and thereby reduces this inefficiency. It is true that the effect we isolate is especially strong in our two-period model but the basic qualitative insight definitely extends to a model in which different sets of workers have different expected tenures with firms.

Appendix

A Proof of Lemma 1

In a steady state, contribution to the unemployed pool come from two sources, young workers (measure 0.5) and never-before-employed workers, of measure $\frac{(1-\alpha)}{2}$. Then, it follows that

$$U = \frac{(1-\alpha)}{2} + \frac{1}{2} = \frac{1+(1-\alpha)}{2}$$

$$\tilde{u}_y \equiv \frac{u_y}{U} = \frac{1}{1+(1-\alpha)}$$

$$\tilde{u}_o \equiv \frac{u_o}{U} = \frac{(1-\alpha)}{1+(1-\alpha)}$$

B Proof of Lemma 2

a) Using (10), we can compute the gains from trade for the firm from hiring a young worker as:

$$1 - h - w_y + b\beta\Pi_v + (1-b)\beta\Pi_f + d\beta\Pi_v - \beta\Pi_v$$

which using (5) and rearrangement yields

$$= (1 - h - w_y) + (1-b)\beta\left(1 - w_o^e\right)$$

Using (11), and (1)-(3), the young worker's surplus from finding employment is given by:

$$\begin{aligned} & w_y + b\beta J_o^s + (1-b)\beta J_o^e - \beta J_o^u \\ = \; & w_y + b\beta B_o + (1-b)\beta w_o^e + (1-b)\beta\delta B_o + \beta s - \beta\alpha w_o^u \end{aligned}$$

and further to

$$= w_y + b\beta B_o + (1-b)\beta w_o^e + (1-b)\beta\delta B_o + \beta s - \frac{\beta\alpha\left(1-h\right)}{2}$$

Then, equating the gains from trade, we get

$$2w_y = (1-h) + (1-b)\beta\left(1 - 2w_o^e\right) - b\beta B_o - (1-b)\delta\beta B_o - \beta s + \frac{\beta\alpha\left(1-h\right)}{2}$$

which simplifies to

$$= (1-h) - \beta B_0\left[(1-b)\left(1-\delta\right) + b + (1-b)\delta\right] - \beta s + \frac{\beta\alpha\left(1-h\right)}{2}$$

Notice that $\left[(1-b)\left(1-\delta\right) + b + (1-b)\delta\right] = 1$. Then, we have

$$2w_y = (1-h)\left(1 + \frac{\alpha\beta}{2}\right) - \beta\left(B_0 + s\right).$$

b) obvious.

C Proof of Lemma 4

a) It is easily seen that, as long as the discouraged worker constraint holds, the young will choose to search as long as there are positive gains from entry within the period:

$$\alpha w_y(\alpha) > s$$

which reduces to

$$(1-h)\,\alpha^2 + 2[(1-h) - (B_0 + s)]\alpha - 4s > 0.$$

Solving the above condition for a value of α in which it holds with equality provides us with a minimal value for α. Note that if $(1-h) > (B_0+s)$, then we must use the positive root as the solution for α. Define α_L as the value of α which satisfies the condition (note, this is the same sufficient condition that was required for young wages to be positive). It is given by:

$$\alpha_L \equiv \frac{-[(1-h) - \beta\,(B_0+s)] + \sqrt[2]{[(1-h) - \beta\,(B_0+s)]^2 + 4\beta(1-h)s}}{\beta\,(1-h)}$$

which is always positive as long as there are some search costs to be incurred by workers.

b) We seek to find conditions where $\alpha_L > \frac{2s}{(1-h)}$. First, recall $b_1 \equiv [(1-h)-(B_0+s)]$.Thus,

$$\alpha_L \equiv \frac{-b_1 + \sqrt[2]{b_1^2 + 4\beta(1-h)s}}{\beta(1-h)} > \frac{2s}{(1-h)}$$

reduces to

$$-b_1 + \sqrt[2]{b_1^2 + 4\beta(1-h)s} > 2\beta s$$

and further to

$$\sqrt[2]{b_1^2 + 4\beta(1-h)s} > b_1 + 2\beta s$$

Next, squaring both sides yields:

$$b_1^2 + 4\beta(1-h)s > (b_1 + 2\beta s)^2$$

which upon simplifying obtains:

$$0 > -2\beta B_0 - \beta s$$

which always holds.

D Proof of Proposition 1

a) Using (24), and setting $\theta = 1$, we get

$$\left(\frac{a}{\tilde{u}_y}\right) = (1-h-w_y) + (1-b)\beta\,(1-w_o^e) + \left(\frac{\tilde{u}_o}{\tilde{u}_y}\right)(1-h-w_o^u)$$

We begin by substituting in the steady-state population conditions from Lemma 1. Then, we use (8), (9), and (12) to get

$$a\left[1+(1-\alpha)\right]=(1-h-w_y)+\beta(1-b)\left(1-\frac{1+(1-\delta)B_o}{2}\right)+(1-\alpha)\left(1-h-\frac{(1-h)}{2}\right).$$

Simplification yields

$$\begin{aligned}2a+2a(1-\alpha) &= (1-h)+\beta B_0+\beta s-\frac{\beta\alpha(1-h)}{2}\\ &\quad+\beta(1-b)-\beta(1-b)(1-\delta)B_o+(1-\alpha)(1-h)\end{aligned}$$

which further simplifies to

$$(1-\alpha)\left[2a-(1-h)\right]+\frac{\alpha\beta(1-h)}{2}=(1-h)+\beta s+\beta(1-b)+\beta B_0\left[1-(1-b)(1-\delta)\right]-2a$$

and finally to

$$\alpha=\frac{2(1-h)+2\beta s+2\beta(1-b)+2\beta B_0\left[1-(1-b)(1-\delta)\right]-4a-\left[4a-2(1-h)\right]}{\left[(1-h)(2+\beta)-4a\right]}$$

Notice that

$$2(1-h)-4a-\left[4a-2(1-h)\right]=4\left[(1-h)-2a\right].$$

Then, it follows that

$$\alpha=2\left\{\frac{\beta s+\beta(1-b)+\beta B_0\left[1-(1-b)(1-\delta)\right]+2\left[(1-h)-2a\right]}{\left[(1-h)(2+\beta)-4a\right]}\right\}$$

E Proof of Proposition 2

Recall that firms enter until $\Pi_v=0$, or until the revenue from entry equals the upfront cost a. Also recall that firms take $\tilde{u}_y$ and $\tilde{u}_o$ as given, and that w_o^u does not depend on B_0. Then the revenue from entry R can be written as

$$R=\tilde{u}_y(1-h-w_y)+\tilde{u}_y(1-b)\beta(1-w_o^e)+\tilde{u}_o(1-h-w_o^u)$$

Using (8), (9), and (12), we get

$$\begin{aligned}R &= \tilde{u}_y\left((1-h)-\frac{(1-h)}{2}\left(1+\frac{\alpha\beta}{2}\right)+\frac{\beta(B_0+s)}{2}\right)\\ &\quad+\tilde{u}_y(1-b)\beta\left(1-\frac{1+(1-\delta)B_o}{2}\right)+\tilde{u}_o(1-h-w_o^u)\end{aligned}$$

which simplifies ultimately to

$$\begin{aligned} R &= \tilde{u}_y \left[\frac{(1-h)}{2} - \frac{(1-h)}{2}\frac{\alpha\beta}{2} + \frac{(B_0+s)}{2} \right] \\ &\quad + \frac{\tilde{u}_y(1-b)\beta}{2}\left[1-(1-\delta)\,B_o\right] + \tilde{u}_o\,(1-h-w_o^u) \end{aligned}$$

It follows that

$$\begin{aligned} \frac{\partial R}{\partial B_o} &= \frac{\tilde{u}_y}{2} - \frac{\tilde{u}_y(1-b)\beta\,(1-\delta)}{2} \\ &= \frac{\tilde{u}_y}{2}\left[1-(1-b)\beta\,(1-\delta)\right] > 0 \end{aligned}$$

so, an increase in B_0 raises the benefit from entry but does not raise the cost. Hence, ceteris paribus, there will be more firm entry with higher B_0. The fact that more firm entry leads to a higher probability of employment follows immediately from differentiating (25) to get

$$\frac{\partial \alpha}{\partial B_0} > 0$$

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