Logistic Depot Planning under Repair and Maintenance Cost Uncertainties under Changing Climate

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Abstract
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Keywords
Logistic Depot Planning, Repair and Maintenance Cost Volatility, Changing Climate, Stochastic Optimal Control, Lattice

Disciplines
Operational Research | Transportation Engineering

Comments
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Zhuoyi Zhao[*], John Jackman[**], and K. Jo Min[***]

Iowa State University, USA.

Abstract

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Introduction

Nowadays, global warming has resulted visible changes to the infrastructure due to its impact on the thawing of the permafrost regions. Because of the corresponding effects, such as ice melts, sea level rises, the buildings in Arctic regions suffer significant weakening of their foundations. Meanwhile, this arouses the problem of increasing repair and maintenance (R&M) cost which, typically, fluctuates substantially over time. Consequently, critical decisions need to be made. For example, what is the most economically rational strategic planning for logistic depots? When is the optimal time to implement such strategic planning?

Under such framework, the research problem we aim to solve is, given the R&M cost uncertainty, what is the optimal time for an airport in the Arctic region to be re-located? The approach to be used

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*** Associate Professor & Associate Chair for Undergraduate Education, Department of Industrial and Manufacturing Systems Engineering, Iowa State University. Correspondence concerning this article should be addressed to K. Jo Min, Iowa State University. Contact: jomin@iastate.edu.
is binomial lattice model discretized from geometric Brownian motion (GBM) process based on the stochastic optimal control theory.

The rest of this article is organized as follows. Related literature on the repair and maintenance of logistic depots and real option approach is introduced in the next section. In the modelling section, binomial lattices mimicking the evolution of airport R&M cost before and after re-location are constructed separately. The options that the decision makers have are examined, and the lattices are valued to find the economic strategy and the optimal time to re-locate the airport. A numerical study is conducted to demonstrate our findings. Finally, conclusions and future research are presented.

Literature Review

Repair and Maintenance of Logistic Depots

Airport in Arctic Regions. Airfield pavements inspection is conducted every three years and the condition is measured by the average pavement condition index (PCI). Typically, runway rehabilitation (e.g., a pavement overlay) is required when PCI drops below 70, and reconstruction is in need when PCI falls to 55-60 (Anderson, 2019). Permafrost degradation and increase in the active permafrost layer undermine the stability of paved airport runways in the Canadian (and likely Alaskan) north (Mills & Andrey, 2002). In fact, every year, Alaska Department of Transportation and Public Facilities (DOT&PF), the largest airport owner in the United States, spends $34M on operations and maintenance on its 240 rural airports (Anderson, 2019). To adjust to the changing climate, the transportation infrastructure maintenance costs in Alaska is expected to increase to an even higher level (Smith & Levasseur, 2002).

Other adaption options to climate change for airport would be major refurbishment (with a life span of 10-20 years), and reconstruction or major upgrade (with a life span of 50 years) (Auld, Maclver, & Klaassen, 2006). Moreover, considering the uncertainty of climate change in the future, Dobes (2010) suggested three alternatives for airport runways, i.e., to build a longer runway immediately, to purchase additional land for a runway extension but to wait until temperatures increased significantly before undertaking its construction, as well as to purchase a financial option to buy the land if temperatures rise by a specific date in the future.

Furthermore, the changing coastlines and rising sea levels might lead to the relocation of airport runways in the long term (Potter, 2002). According to Northwest Arctic Borough (2019), Alaska DOT&PF has been cooperating with the Federal Aviation Administration (FAA) to re-locate the Noatak Airport in the Village of Noatak from perspective of safety, reliability and cost-effectiveness. In the long run, the re-location of airport appears to be the most efficient way, and thus, it will be the focus of this paper.

Harbor Maintenance Trust Fund (HMTF) Expenditures. Harbor Maintenance Trust Fund (HMTF) functions as appropriation from the Congress for harbor dredging (Frittelli, 2010). The HMTF balance (in million $) from 1988 to 2011 are plotted in Figure 1, where HMTF expenditure is colored in red.

![Figure 1. HMTF Balance](image)

**Figure 1.** HMTF balance (in million $) (Frittelli, 2010)
By converting the data from plot to numeric values in Table 1 using WebPlotDigitizer (Rohatgi, 2019), and re-plotting HMFT expenditure data over time (Figure 2), it can be observed that, HMFT expenditure has a positive growth rate in the long term and fluctuates over time.

Table 1

<table>
<thead>
<tr>
<th>Year</th>
<th>HMTF expenditure</th>
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<tbody>
<tr>
<td>0</td>
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</tr>
<tr>
<td>1</td>
<td>192.9825</td>
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<tr>
<td>2</td>
<td>192.9825</td>
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<td>3</td>
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Figure 2. HMFT expenditure from 1988 to 2011

**Real Options Approach**

Real options approach has been widely applied in areas such as investment under uncertainty, supply chain and logistics, innovation and technology as it incorporates the flexibility the decision makers face in many operating decisions (Trigeorgis & Tsekrekos, 2018). Specifically, in terms of the application of real options in infrastructure, fruitful literature can be found in the field of parking garage, toll road, highway, real estate, hospital, power plant, airport, etc. (Martins, Marques, & Cruz, 2013). In 2003, Smit analyzed the optional and strategic features of European airport expansion using real options
theory and game theory. Chambers (2007) conducted a comprehensive study on the uncertainty in
airport design and investigated the airport flexible design strategies using a real options approach.
Kwakkel, Walker, & Marchau (2010) investigated the airport strategic planning problem incorporating
the uncertain factors such as fuel price, new type of aircraft, the liberalization and privatization of
airlines and airports. In 2014, Park, Kim, & Kim estimated the value of wait option for the improvement
investment on drainage system under the flood damage uncertainty using binomial model.

Modelling
To start, the R&M cost of an existing airport in the Arctic region at year t, $dC_1 t$ (unit: $) is modeled
as a GBM process.

\[ dC_1 t = \alpha_1 C_t dt + \sigma_1 dC_t z_t \]  

(1)

where $\alpha_1$ is the growth rate of the R&M cost of the existing airport (unit: % per year), while $\sigma_1$ is the
volatility of the R&M cost of the existing airport (unit: % per square root of year). $dt$ is the increment
of time, while $dz$ is the increment of a standard Wiener process $z_t$. That is,

\[ dz_t = \varepsilon_t \sqrt{dt} \text{ where } \varepsilon_t \sim N(0, 1). \]

Similarly, the R&M cost of a new airport after re-location at year t, $dC_2 t$ (unit: $) is assumed to
follow a GBM process as well with different parameters.

\[ dC_2 t = \alpha_2 C_t dt + \sigma_2 dC_t z_t \]  

(2)

where $\alpha_2$ is the growth rate of the R&M cost of the new airport (unit: % per year), while $\sigma_1$ is the
volatility of the R&M cost of the new airport (unit: % per square root of year). $dt$ and $dz$ have the
same meaning as it in $dC_2 t$.

The parameters in both GBM process are different is because there is no proportional relationship
between the two R&M costs due to lack of evidence. Furthermore, the new airport should be embedded
with more reinforced infrastructure that has longer life cycle. Therefore, the annual R&M cost is
expected to be reduced and to have a lower volatility. As a result, there is no closed form solution
when solving the optimal threshold of R&M cost, and that invites us to implement the binomial lattice
model to obtain the solution.

The continuous R&M cost evolution can be discretized using binomial lattice model, where the up
multiplier $u$, the down multiplier $d$, and the risk neutral probability $q$ are defined as follows (White,
2016).

\[ u = e^{\sigma \sqrt{\Delta t}} \]  

(3)

\[ d = e^{-\sigma \sqrt{\Delta t}} \]  

(4)

\[ q = \frac{1+\rho-d}{u-d} \]  

(5)

where $\Delta t$ is the time interval, and $\rho$ is the annual discount rate for money (unit: %).

Considering the demerit of binomial lattice model, i.e., computation complexity, a three-phase
binomial lattice model is constructed to demonstrate the evolution of airport R&M cost before and
after re-location (Figure 3).
For planning purposes, the following assumptions are proposed.

**Assumption 1:** The re-location fee is paid by the decision maker at the time when the re-location decision is made.

**Assumption 2:** The re-location process will take one time period to complete after the re-location decision is made.

**Assumption 3:** The R&M cost at the decision-making time follows the cost evolution before re-location, and that afterwards will follow the cost evolution after re-location.

Let $C_{t_i}$ denote the R&M cost at time $t_i (i=1,2,3)$. $R$ is the re-location fee ($R=0$ if no re-location occurs). For instance, if the decision maker decides to re-locate an airport at time $t_i$, a re-location fee $R$ will be paid by the decision maker at time $t_i$. The re-location will take the period from time $t_i$ to time $t_{i+1}$ to complete. $C_{t_i}$ follows the cost evolution before re-location, while $C_{t_{i+1}}$ follows the cost evolution after re-location.

In a three-phase binomial lattice model, the decision maker can opt to re-locate the airport either at time 1 or at time 2 with certain re-location strategies. The exhaustive and mutually exclusive option sets the decision maker has are as listed below, followed by the corresponding valuation equation. The valuation of lattice is measured by the sum of Net Present Value (NPV) of R&M costs at each time point plus the discounted re-location fee if there is an airport re-location.

**Option 1.** At time 1, decide to re-locate (Figure 4).

\[ V_{\text{option 1}} = C_1 + (1 + \rho)^{-1}C_2 + (1 + \rho)^{-2}[q_2u_2C_2 + (1 - q_2)d_2C_2] + R \] (6)

**Option 2.** At time 1, decide not to re-locate; at time 2, decide to relocate regardless of the R&M level at time 2 (Figure 5).
Figure 5. R&M cost lattice for option 2

\[ V_{\text{option } 2} = C_1 + q_1[(1 + \rho)^{-1}u_1C_1 + (1 + \rho)^{-2}C_2] + (1 - q_1)[(1 + \rho)^{-1}d_1C_1 + (1 + \rho)^{-2}C_2] + (1 + \rho)^{-1}R \]  

(7)

**Option 3.** At time 1, decide not to re-locate; at time 2, decide to relocate only when R&M cost increases from \( C_1 \) to \( u_1C_1 \), i.e., no actions will be taken if R&M cost decreases from \( C_1 \) to \( d_1C_1 \) (Figure 6).

![Diagram for Option 3](image)

**Figure 6.** R&M cost lattice for option 3

\[ V_{\text{option } 3} = C_1 + q_1[(1 + \rho)^{-1}u_1C_1 + (1 + \rho)^{-2}C_2] + (1 - q_1)[(1 + \rho)^{-1}d_1C_1 + (1 + \rho)^{-2}[q_1u_1d_1C_1 + (1 - q_1)d_1^2C_1]] + (1 + \rho)^{-1}R \]  

(8)

**Option 4.** At time 1, decide not to re-locate; at time 2, decide to relocate only when R&M cost decreases from \( C_1 \) to \( d_1C_1 \), i.e., no actions will be taken if R&M cost increases from \( C_1 \) to \( u_1C_1 \) (Figure 7).

![Diagram for Option 4](image)

**Figure 7.** R&M cost lattice for option 4
Option 5. At time 1, decide not to re-locate; at time 2, decide not to relocate regardless of the R&M level at time 2 (Figure 8).

\[
V_{\text{option 5}} = C_1 + q_1 \{ (1 + \rho)^{-1} u_1 C_1 + (1 + \rho)^{-2} [q_1 u_1^2 C_1 + (1 - q_1) u_1 d_1 C_1] \} + \\
(1 - q_1) \{ (1 + \rho)^{-1} d_1 C_1 + (1 + \rho)^{-2} C_2 \} + (1 + \rho)^{-1} R 
\]

(9)

**Numerical Study**

In this section, we conduct a numerical study to illustrate how to utilize our model to identify the optimal re-location time. The hypothetical parameter values used in the numerical example are listed in Table 2 and Table 3.

Table 2

<table>
<thead>
<tr>
<th>Common parameter values</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount rate for money (%)</td>
<td>$\rho = 0.05$</td>
</tr>
<tr>
<td>Time interval (year)</td>
<td>$t = 1$</td>
</tr>
<tr>
<td>Re-location fee ($)</td>
<td>$R = 1,000,000$</td>
</tr>
</tbody>
</table>
Table 3

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Before re-location</th>
<th>After re-location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volatility of R&amp;M cost (%)</td>
<td>$\sigma_1 = 0.20$</td>
<td>$\sigma_2 = 0.10$</td>
</tr>
<tr>
<td>Up multiplier</td>
<td>$u_1 = 1.221$</td>
<td>$u_2 = 1.105$</td>
</tr>
<tr>
<td>Down multiplier</td>
<td>$d_1 = 0.819$</td>
<td>$d_2 = 0.905$</td>
</tr>
<tr>
<td>Probability for an increase</td>
<td>$q_1 = 0.575$</td>
<td>$q_2 = 0.725$</td>
</tr>
<tr>
<td>Probability for a decrease</td>
<td>$1 - q_1 = 0.425$</td>
<td>$1 - q_2 = 0.275$</td>
</tr>
<tr>
<td>Initial R&amp;M cost ($)</td>
<td>$C_1 = 120,000$</td>
<td>$C_2 = 100,000$</td>
</tr>
</tbody>
</table>

The R&M cost lattice before and after re-location are presented in Figure 9.

![Figure 9. Numerical example for R&M cost lattice before and after re-location](image)

Sequentially, the R&M cost lattice for each of the five options are calculated in Figure 10 – Figure 14.

Option 1

![Figure 10. Numerical example for R&M cost lattice for option 1](image)
Using equation (6) – (10), the value of lattice for the five options can be obtained, and the results are summarized in Table 4.
Table 4

Value of lattice for each option

<table>
<thead>
<tr>
<th>Option</th>
<th>R&amp;M cost before or after re-location</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Time 1</td>
<td>Time 2</td>
</tr>
<tr>
<td>1</td>
<td>Before</td>
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<td>3</td>
<td>Before</td>
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<tr>
<td>4</td>
<td>Before</td>
<td>Before</td>
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<tr>
<td>5</td>
<td>Before</td>
<td>Before</td>
</tr>
</tbody>
</table>

In this three-phase lattice model, the 5th option (airport will never be re-located) turns out to optimal. However, this decision might not be economically rational in the long run when more phases are included in the lattices. This is because as the time goes by, the R&M cost saved by the airport re-location accumulates, and it might exceed the one-time expense on airport re-location at certain time points in the future.

Additionally, suppose the airport re-location has to happen, the most economic strategy for re-location is to decide at time 2 and re-location will be completed by time 3, regardless of the R&M cost level at time 2. The optimal time to re-locate is time 2.

**Conclusion**

In summary, this paper examines the strategy, e.g., the optimal time, for airport re-location in Arctic regions given the uncertain R&M cost using binomial lattice. We start with modeling R&M cost as GBM process followed by discretization. We then derive the mutually exclusive option sets that the decision maker has and propose the valuation of lattice for each option. The most economic strategy can be selected by minimizing the value of cost, and the optimal time to re-locate will be identified accordingly. We note that the economically rational strategy and the optimal time for airport re-location depend on the parameter values such as the volatility of R&M cost, airport re-location fee, discount rate for money.

This paper can be viewed as a preliminary exploration on the logistic depot planning under the R&M cost uncertainty under climate change. For future research, the influence of the aforementioned parameters on the re-location decision will be examined. For instance, under which circumstances, the airport should be re-located. Or when the discount rate for money decreases, should the decision maker re-locate the airport earlier or later? We will also incorporate the uncertainty of government policies by embedding a jump in diffusion process referring to Amin’s (1993) work. Furthermore, empirical approach will be implemented so that more phases can be included.
References


Appendix A

The estimation of parameters (i.e., volatility and growth rate) of the GBM process of HMTF expenditure is based on Dmouj (2006)’s work on stock price modelling. A stochastic process $S_t$ is considered to follow a GBM process when it satisfies the following the stochastic differential equation,

$$dS_t = \alpha S_t dt + \sigma S_t dB_t$$

where $B_t$ is a random walk process or Brownian motion with a drift.

$$B_t = \alpha t + \sigma \varepsilon \sqrt{t} \text{ where } \varepsilon \sim N(0, 1)$$

Suppose the initial value is $S_0$, the future values can be expressed as

$$S_t = S_0 e^{(\alpha - \frac{1}{2} \sigma^2) t + \sigma \varepsilon \sqrt{t}}$$

First, we denote $S_i$ as the HMTF expenditure at year $i$ (unit: million $). We assume the R&M cost at year 1988 as the initial value, $S_0 = 70.1754$.

1. $\tau$ is defined as the length of time interval between two consecutive measured periods whereas $\tau = t_i - t_{i-1}$, for $i = 1, 2, \ldots, n$ (n=23). In this case, $\tau = 1$ year.

2. Let $\alpha_i$ be the logarithm of the costs over the short time interval $\tau$, i.e.

$$\alpha_i = \ln S_i - \ln S_{i-1} = \ln \left( \frac{S_i}{S_{i-1}} \right) \text{ for } i = 1, 2, \ldots, n.$$ 

3. The unbiased estimator $\bar{\alpha}$ of the logarithm of the costs $\alpha_i$ is given by

$$\bar{\alpha} = \frac{1}{n} \sum_{i=1}^{n} \alpha_i.$$ 

4. The estimator of the of the standard deviation of the $\alpha_i$’s is given by

$$\nu = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (\alpha_i - \bar{\alpha})^2}.$$ 

5. $d(\ln S_i) = \ln S_i - \ln S_{i-1} = \ln \left( \frac{S_i}{S_{i-1}} \right) = \left( \alpha - \frac{1}{2} \sigma^2 \right) dt + \sigma \varepsilon \sqrt{dt}$

6. Since the estimator of the standard deviation of the yearly cost is equal to $\sigma \sqrt{\tau}$, it follows that $\sigma$ can be simply estimated by $\hat{\sigma} = \frac{\nu}{\sqrt{\tau}}$ with a standard error equal to $\frac{\hat{\sigma}}{\sqrt{2n}}$.

7. The growth rate is given by: $\bar{\alpha} = (\alpha - \frac{1}{2} \sigma^2) \tau$. Hence, $\hat{\alpha} = \bar{\alpha} + \frac{1}{2} \sigma^2$.

8. The estimated growth rate and volatility can be calculated using the following equations.

$$\begin{align*}
\hat{\sigma} &= \frac{\nu}{\sqrt{\tau}} = \sqrt{\frac{1}{(n-1)\tau} \sum_{i=1}^{n} (\alpha_i - \bar{\alpha})^2 \text{ with a standard error equal to } \frac{\hat{\sigma}}{\sqrt{2n}}} \\
\hat{\alpha} &= \bar{\alpha} + \frac{1}{2} \sigma^2
\end{align*}$$

We have $\hat{\sigma} = 0.3250$ with a standard error of 0.0479, and $\hat{\alpha} = 0.1604$. Therefore, the HMTF expenditure at a year is estimated as $S_i = 70.1754e^{(0.1604 - \frac{1}{2} \cdot 0.3250^2)i}$. 

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