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# A data-driven optimization-based approach for siting and sizing of electric taxi charging stations

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## Abstract

This paper presents a data-driven optimization-based approach to allocate chargers for battery electric vehicle (BEV) taxis throughout a city with the objective of minimizing the infrastructure investment. To account for charging congestion, an  $M/M/x/s$  queueing model is adopted to estimate the probability of BEV taxis being charged at their dwell places. By means of regression and logarithmic transformation, the charger allocation problem is formulated as an integer linear program (ILP), which can be solved efficiently using Gurobi solver. The proposed method is applied using large-scale GPS trajectory data collected from the taxi fleet of Changsha, China. The key findings from the results include the following: (1) the dwell pattern of the taxi fleet determines the siting of charging stations; (2) by providing waiting spots, in addition to charging spots, the utilization of chargers increases and the number of required chargers at each site decreases; and (3) the tradeoff between installing more chargers versus providing more waiting spaces can be quantified by the cost ratio of chargers and parking spots.

## Keywords

Electric taxis, Charging infrastructure planning, GPS trajectory data, Integer programming, Queueing model

## Disciplines

Civil Engineering | Transportation Engineering

## Comments

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4

## 1 1. Introduction

2 Replacing conventional gasoline vehicles (CGVs) with alternative fuel vehicles, such as battery electric  
3 vehicles (BEVs), offers an appealing chance to reduce greenhouse gas (GHG) emissions and other  
4 harmful pollutants in highly populated urban areas (Buekers et al., 2014; Yuksel and Michalek, 2015).  
5 However, the fear that the vehicle has insufficient range to reach the destination, referred to as range  
6 anxiety, has been shown to be a significant obstacle to market acceptance of BEVs (Neubauer and  
7 Wood, 2014; Rauh et al., 2014). One way to mitigate range anxiety is through the deployment of public  
8 charging infrastructure, but high costs of equipment and installation limit the coverage of the charger  
9 network (Agenbroad and Holland, 2014; Peterson and Michalek, 2013; Schroeder and Traber, 2012).  
10 Thus, it is vital to place and size new charging stations based on charging demands, so as to best utilize  
11 limited resources.

12 Various facility location models have been proposed to optimize the layout of hydrogen or gas  
13 refueling stations (Aikens, 1985; Church and Velle, 1974; Kuby et al., 2009; Nicholas, Handy, and  
14 Sperling, 2004; Wang and Lin, 2009). But most of the existing facility location models cannot be  
15 applied to optimizing the location of charging stations for the following two reasons: (1) The placement  
16 and sizing of hydrogen and gas stations are usually under certain restrictive conditions for  
17 environmental, health, and safety reasons (Kuby et al., 2009), while the conditions for siting chargers  
18 are more flexible. It is common to install chargers in parking garages, curbside, and in surface parking  
19 lots, so that BEVs can be charged at their dwell places, such as home, work places, and commercial  
20 places (Pearre et al., 2011; Tian et al., 2014; Zou et al., 2016). (2) Unlike CGVs, it generally takes a  
21 long time to charge BEVs, ranging from 30 minutes to several hours, resulting in a long waiting time  
22 for incoming BEVs if all the chargers are occupied (Foley et al., 2010; Li et al., 2015). Waiting for  
23 charging is usually an unpleasant experience for BEV drivers. Drivers might have taken a detour and  
24 traveled some extra distance to reach the charging station due to limited accessibility, creating an even  
25 worse user experience. Detours and waiting times for charging hinder the adoption of BEVs. If BEVs  
26 can be charged during their dwell time without behavioral changes—that is, if the time between two  
27 consecutive trips can be utilized for charging—consumers are more likely to adopt the BEV technology.

28 In this paper, we investigate the following problem: given daily travel and dwell patterns (including  
29 dwell locations, dwell time, and arrival rates) of a fleet of taxis, determine where to deploy new charging  
30 stations and how many chargers to install at each station. The objective is to minimize the overall  
31 infrastructure investment while satisfying the charging demand. Dwell patterns of potential BEVs are  
32 derived from trajectories of over 7,910 taxis whose travel activities were recorded for one week. In  
33 particular, to account for charging congestion, an  $M/M/x/s$  queueing model is adopted to estimate  
34 the waiting time and the fraction of customers who are turned away. The probability that electric taxis  
35 can be charged at their dwell places during the day is considered as the model constraint, and the  
36 objective is to minimize the total cost of building charging stations and installing chargers. In general,  
37 optimizing the performance of queue systems is a difficult problem because of the nonlinear relationship  
38 of the performance metrics as functions of the arrival and service rates, and the computational time  
39 increases exponentially with the size of the problem (Bertsimas et al., 1994; Mung et al., 2002). In the  
40 proposed charger location problem, hundreds of charging stations need to be sited in the city, and  
41 multiple chargers need to be assigned to each station. Thus, solving the nonlinear mixed integer program  
42 is computationally demanding. An approximation method is presented to transform the formulation to  
43 an integer linear program (ILP), which can be solved efficiently. In summary, the main contributions  
44 of this paper include the following: (1) develop queueing models to describe charging congestion based  
45 on taxis' dwell patterns observed from GPS tracked trajectory data; (2) formulate the charger allocation  
46 problem as an integer linear program, considering charging congestion phenomenon; and (3) investigate  
47 the tradeoff between installing more chargers versus providing more waiting spaces and the impact of  
48 charger power on waiting time.

1 The rest of the paper is organized as follows. Section 2 presents an overview of related work,  
2 followed by problem statement and assumptions in Section 3. The optimization model and the integer  
3 program formulation are presented in Section 4. Results and conclusions are summarized in Section 5  
4 and 6, respectively.

## 5 **2. Literature review**

6 To help policy makers and investors efficiently allocate charging resources, various mathematical  
7 models have been developed for charging infrastructure planning. For example, [Frade et al. \(2011\)](#) used  
8 a maximal covering model to optimize the location of charging stations in Lisbon, Portugal, with the  
9 objective of maximizing demand coverage. [Feng et al. \(2012\)](#) proposed a method for charging station  
10 planning using weighted Voronoi diagram. Based on the quantified cost for detour charging, the road  
11 network of planning area is partitioned by Voronoi diagram so as to minimize the users' travel cost. [He  
12 et al. \(2016\)](#) compared three classic facility location models and found the p-median model was more  
13 effective than the set covering model and the maximal covering location model for locating electric  
14 vehicles (EV) charging facilities. [Li et al. \(2016\)](#) developed a multi-period multipath charging station  
15 location model that captured the dynamics in the topological structure of network, formulated the model  
16 as a mixed integer program, and solved it by genetic algorithm. [He et al. \(2015\)](#) formulated the problem  
17 of optimally locating public charging stations within a budget limit as a bi-level mathematical program  
18 and solved the problem using a genetic algorithm. [Guo et al. \(2016\)](#) established a network-based multi-  
19 agent optimization model for planning fast charging stations that simultaneously captured the selfish  
20 behaviors of individual investors and travelers and their interactions. The model was solved based on  
21 variational convergence theorems. [Ghamami et al. \(2016\)](#) developed a mixed integer program with  
22 nonlinear constraint to configure charging infrastructure along highway corridors. The model minimizes  
23 the total system cost and considers the realistic patterns of O–D demands and flow-dependent charging  
24 delay. Based on a tour-based equilibrium framework, [Wang et al. \(2016\)](#) considered a special EV  
25 network composed of fixed routes for an electric bus fleet and optimized the deployment of recharging  
26 stations and recharging schedule so as to ensure an electric bus can be charged when it stops within a  
27 pre-specified duration.

28 Travel survey data offer an insight into the charging demand and are usually taken as an indicator  
29 of the deployment of charging infrastructure. With regard to the studies based on travel survey, [Chen  
30 et al. \(2013\)](#) formulated a mixed integer programming problem to optimally allocate a constrained  
31 number of charging stations based on the parking information from over 30,000 personal-trip records  
32 collected by household travel survey in Seattle, Washington, United States. Assisted by three spatial  
33 data sets, including the National Household Travel Survey (NHTS) data, [Aultman-Hall et al. \(2012\)](#)  
34 identified optimal charging locations in the rural areas in Vermont, United States. In comparison with  
35 traditional travel survey data, Global Positioning Systems (GPS) tracked travel survey data contain  
36 more accurate information about trip length, dwell place, and dwell duration of each vehicle, thus  
37 providing a way to estimate the spatial and temporal distribution of charging demand. Individual GPS  
38 tracked trajectory data, collected from conventional gasoline vehicles and representing real world travel  
39 activities, have been used to site public charging stations in previous studies. In [Dong et al. \(2014\)](#) a  
40 charger location optimization problem considering daily travel activity constraints was proposed based  
41 on GPS-based travel survey data collected in the greater Seattle metropolitan area.

42 Since it is common for taxis to install GPS devices for the purpose of navigation and operational  
43 monitoring, taxi trajectories become a major data source to examine the market potential of BEV taxis  
44 ([Baert and Kort, 2013](#); [Chrysostomou et al., 2016](#); [Tian et al., 2014](#); [Wang et al., 2015](#); [Yang et al.,  
45 2016](#)) and optimize the siting of public charging stations ([Tu et al., 2016](#), [Shen et al., 2016](#)). Taxi fleet  
46 has some desirable features for deploying plug-in electric vehicles. Fuel cost savings are significant, as  
47 taxis are driven heavily; thus, the payback period tends to be shorter. [Cai et al. \(2014\)](#) demonstrated the  
48 potential public charging stations by extracting public parking “hotspots” from taxi trajectory data in

1 Beijing, China. This research was expanded by [Shahraki et al. \(2015\)](#), in which an optimization model  
2 was developed to determine optimal charger allocation, with the objective of maximizing electrified  
3 fleet vehicle miles traveled (VMT) of plug-in hybrid electric vehicles (PHEVs). Based on an event-  
4 based simulation, [Sellmair and Hamacher \(2014\)](#) proposed an algorithm to optimize the number of  
5 charging stations per taxi stand based on real world driving patterns of conventional taxis in Munich,  
6 Germany. The objective was to maximize economic benefit of the entire system including BEV taxi  
7 drivers and charging station investors. In [Jung et al. \(2014\)](#) a bi-level simulation-optimization model  
8 was proposed to allocate chargers for a fleet of 600 shared-taxis in Seoul, Korea, with an objective of  
9 minimizing the queue delay. [Ahn and Yeo \(2015\)](#) proposed an Estimating the Required Density of EV  
10 Charging (ERDEC) stations model to estimate the optimal density of charging stations aiming at  
11 minimizing the range anxiety based on taxi trajectories in Daejeon City, Korea, which was a pioneering  
12 work considering charging queueing. Using the real-world BEV taxi trajectory data collected from  
13 Shenzhen, China, [Li et al. \(2015\)](#) built an optimization framework to find the optimal locations to site  
14 stations and the optimal assignment of chargers, which also took charging congestion into consideration.  
15 The objective was to minimize the average time to find the charging stations and the waiting time for  
16 an available charger.

17 In summary, the optimal layout of charging stations is mainly generated using two types of  
18 approaches: (1) Without pre-defined candidate sites, the charging station location optimization is  
19 considered as a set covering problem based on the configuration of the road network in the study area  
20 (e.g., [Frade et al., 2011](#), [Feng et al., 2012](#), [Ahn and Yeo, 2015](#)). The drawback of this approach is that  
21 it may be impracticable to install chargers at certain locations. (2) Given a set of candidate sites,  
22 including existing charging, gas, or hydrogen refueling stations, new stations or chargers are assigned  
23 to the study area with a limited budget (e.g. [Cai et al., 2014](#), [Jung et al., 2014](#), [Li et al., 2015](#)). As  
24 mentioned above, the distribution characteristics of gas and hydrogen stations are different from those  
25 of charging stations. Moreover, in the early stage of the BEV market, charging stations are scarce across  
26 the city. With more BEVs on the road, more chargers will be scattered all over the city, and the model  
27 complexity is increased. The reason our research focused on dwell charging is that, first, the dwell  
28 places are supposed to have desired space for parking, which can be equipped with chargers; second,  
29 the average dwell time can be considered as the indicator of charger power, i.e., fast chargers are  
30 preferred by the stations with shorter dwell time and vice versa.

31 Charging congestion is another concern of this paper. The waiting time may be significant if all  
32 the chargers are occupied during the peak hours, especially at the popular dwell sites. [Hosseini and  
33 MirHassani \(2015\)](#) developed a recharging station location model with queue considering capacity,  
34 recharging time, and waiting time, and solved the problem using a heuristic algorithm. In the context  
35 of predicting EVs' charging demand and its impacts on power grid, the charging congestion effect is  
36 modeled using queueing theory. In [Ghamami et al. \(2016\)](#) the average waiting time for charging was  
37 computed using deterministic queueing theory. The most widely-adopted queueing model is  $M/M/s$ ,  
38 assuming that both BEV arrival rate  $\lambda$  and charging service rate  $\mu$  follow Poisson distributions and  
39 that a vehicle will join the queue no matter how long the queue is ([Akbari and Fernando, 2015](#); [Bae and  
40 Kwasinski, 2012](#); [Qiu et al., 2013](#); [Said et al., 2013](#); [Said, 2015](#)). To account for limited waiting spaces,  
41 [Fan et al. \(2015\)](#) applied an  $M/G/x/s$  queueing model, which assumes a general service time  
42 distribution and at most  $s$  BEVs can be accommodated at a station. In this paper, an  $M/M/x/s$   
43 queueing model is adopted, which assumes Poisson-distributed arrival and service rates with  $x$   
44 chargers and  $s - x$  waiting spaces.

45 Due to the computational complexity of larger-scale network optimization, little research has been  
46 done to study the problem of siting and sizing charging stations simultaneously considering charging  
47 congestion. This paper presents a data-driven optimization-based approach for charging infrastructure  
48 planning using extensive vehicle activity data.



### 1 3. Problem statement and assumptions

#### 2 3.1. Problem statement

3 Given a set of candidate sites for installing public charging stations  $\mathbf{J} = \{1, 2, \dots, N\}$  that are favorable  
4 dwell places of taxi drivers, a set of BEVs  $\mathbf{I} = \{1, 2, \dots, M\}$  that need to be charged at public charging  
5 stations at least once a day, and the probabilities that BEVs can be charged during dwell time without  
6 travel pattern changes, the problem is to find optimal locations of charging stations and optimal  
7 assignment of chargers to each station, so as to minimize the total investment of public charging  
8 infrastructure.

9 The charging demand is estimated based on vehicles' dwell patterns extracted from GPS  
10 trajectories of taxis in Changsha, China, collected from 0:00 on October 8, 2015, to 23:59 on October  
11 14, 2015 (local time). By the end of 2014, there were 7,957 taxis operating in Changsha. It is required  
12 by the local government that GPS devices are installed on all taxis for monitoring purpose. Thus, the  
13 dataset includes the entire taxi fleet in Changsha, with some missing data due to data collection and  
14 transmission errors. A GPS signal is captured roughly every 10 seconds for each taxi. The data include  
15 time-stamped location (i.e., longitude and latitude), spot speed, azimuth, and operational status (i.e.,  
16 empty or occupied). All trajectories were cleaned by removing invalid points caused by data recording  
17 or transmission errors.

18 The study area is partitioned into a number of equal size cells first. Each cell has a quadrature edge  
19 of  $0.005^\circ$  latitude and longitude, approximately equivalent to  $0.5 \times 0.5 \text{ km}^2$ . If the GPS records indicate  
20 a vehicle dwelling in the same cell for more than 20 minutes, a dwell event of BEV  $i$  in cell  $j$  is  
21 labeled. The study area (i.e.,  $27^\circ \sim 29^\circ \text{ N}$ ,  $111^\circ \sim 115^\circ \text{ E}$ ) is divided into 320,000 non-overlapping square  
22 cells. For each cell, the number of dwell events is counted. There are 2,460 valid cells that have records  
23 of taxis parked during the one-week period. It is possible that some of the dwell locations are drivers'  
24 home locations, for which the number of daily dwell events is relatively low. Fig. 1 presents the  
25 frequency distribution of the number of daily dwell events per cell. 72.7% of the valid cells have less  
26 than 5 daily dwell events. It is comparatively uneconomical to install public charging stations in places  
27 that are not attractive to taxi drivers. Thus, only the popular cells, where on average at least 5 dwell  
28 events occurred per day, are considered as candidates for installing public charging stations. Fig. 2  
29 shows the spatial distribution of valid cells. There are 666 cells with a daily arrival rate no less than 5  
30 veh/day on average, and these are selected as the candidate sites. The selected cells are mostly located  
31 in the densely populated area of the inner city. The most popular dwell site is located near Huanghua  
32 Airport and far from downtown. It has 10,777 dwell events weekly, with an average frequency of nearly  
33 64 per hour. A charging queue and congestion problem might be observed if the taxi fleet is replaced  
34 by BEVs.

35 **Place Fig. 1 about here**

36 **Place Fig. 2 about here**

37 A taxi with complete whole-day consecutive trip records is considered a valid sample. Identified  
38 from GPS trajectories, there are 53,092 valid taxis with 185,404 dwell events occurring at the selected  
39 666 cells over the one-week period. During the dwell time, taxi drivers usually have a meal, change  
40 shift, or refuel the vehicle. Fig. 3 shows the number of dwell sites at which one driver would dwell in a  
41 day. 15.86% of the taxis dwell at the same sites during the day. For taxis that dwell at multiple sites, if  
42 they are turned away at one site, it is possible for them to be charged at the next dwell location. Thus,  
43 the allocation of chargers can be optimized based on taxis' dwell patterns.

44 **Place Fig. 3 about here**

#### 45 3.2. Assumptions

### 1 3.2.1. Queueing theory for charging system

2 When a driver arrives at a charging station and finds all chargers are occupied, he/she can either wait  
3 or decide not to charge at this location. Assume that, at each charging station, the arrival rate,  $\lambda$ , follows  
4 a Poisson distribution and the service time,  $1/\mu$ , follows an exponential distribution. The waiting line  
5 priority rule of the system is first-come, first-served. Hence, the charging congestion problem can be  
6 formulated as the Markovian queueing system with a finite number of chargers,  $x$ , and a finite capacity,  
7  $s$ , symbolically denoted by  $M/M/x/s$ .

8 If  $s$  is equal to the number of chargers  $x$ , that is, an arriving BEV leaves the system without  
9 waiting for service if all chargers are busy, we denote it as  $M/M/x/x$  (i.e., no waiting spaces). If  $s$   
10 is greater than  $x$ , the arriving BEV will be rejected if the maximum system capacity is reached;  
11 otherwise, it will wait in line for service. We denote this queueing system as  $M/M/x/K$ , where  $K$   
12 is the maximum number of customers that can be accommodated in the system (i.e., number of chargers  
13 plus number of waiting spaces). The maximum queue size,  $K - x$ , can be considered as the number of  
14 parking spaces provided for customers waiting for a charger. It is assumed that, for the  $M/M/x/K$   
15 queueing system, every five chargers are equipped with one parking spot for waiting. Thus, the system  
16 capacity can be computed by Eq. (1), where the parameter  $\delta$  is set as 5.

$$17 \quad K = x + \lceil x / \delta \rceil \quad (1)$$

### 18 3.2.2. Service time

19 Service time, or charging time, varies based on the type of chargers. In addition, BEV drivers might not  
20 move their vehicles until they finish their dwell activities even if the battery is fully charged. Therefore,  
21 the average service times at cell  $j$ , denoted as  $\bar{t}_j$  (unit: day), are estimated in two ways: (1) Drivers  
22 are assumed to move their vehicles when they depart; that is, the service time is estimated based on the  
23 dwell time obtained from the trajectory data. The service rate, denoted as  $\mu_{0j}$  (unit: veh/day), is  
24 computed as  $\mu_{0j} = 1/\bar{t}_j$ . (2) The service time is determined by the charging power. It is noted that  
25 not all the charging power can be directly transferred to battery energy, and a portion of the power is  
26 lost during the charging process. Let  $\alpha$  denote the charging efficiency. In this paper, we set  $\alpha$  as 1.3 (Nie  
27 and Ghamami, 2013). For example, if the input charging power is 104 kW, the effective charging power  
28 is 80 kW. Assuming the battery capacity is 40 kWh, it takes 30 minutes to fully charge the battery if  
29 using an effective charging power of 80 kW, and it takes 180 minutes with an effective charging power  
30 of 13.3 kW. Six effective charging powers are taken into account: 80, 40, 26.7, 20, 16, and 13.3 kW.  
31 Accordingly, the average service time of each station is assumed as 30, 60, 90, 120, 150, and 180  
32 minutes, and the service rate  $\mu$  is 48, 24, 16, 12, 9.6, and 8 vehicles per day, respectively. Since we  
33 do not estimate initial state of charge (SOC) of BEVs when they arrive at the stations, regardless of  
34 what the SOC is after charging, in case (1) BEVs are assumed to be unplugged and removed from the  
35 charger when their dwell time runs out, and in case (2) BEVs will be unplugged as soon as the assumed  
36 service time runs out.

### 37 3.2.3. Charging demand

38 The sites where taxis frequently dwell are likely to have ample parking spaces to install chargers and  
39 thus are selected as candidate sites. The number of average daily dwell events occurring at these  
40 locations can be derived from the GPS trajectory data. In this paper, the average daily charging demand  
41 (i.e., daily arrival rate) of cell  $j$ , denoted as  $\lambda_j$  (unit: veh/day), is assumed to be the average number  
42 of daily dwell events occurring in the cell. At an early market with a small number of BEV taxis on the  
43 road, the charging demand is likely less than the number of dwell events. As the number of BEVs on  
44 the road increases, the demand for dwell charging increases too. In particular, Beijing has recently

1 announced that all internal combustion engine (ICE) taxis will be replaced by BEVs by 2020 (DRC of  
 2 Beijing, 2016), in which case all the dwell taxis might use the chargers. Furthermore, if the chargers are  
 3 open to private BEVs and other commercial BEVs, the charging demand could exceed the number of  
 4 daily taxi dwell events. Predicting the future charging demand for each cell is beyond the scope of this  
 5 research. Since taxi dwell patterns represent the spatial distribution attributes of charging demand, for  
 6 simplicity we assume the charging demand  $\lambda_j$  follows the taxis arrival patterns at the charging  
 7 stations.

## 8 4. Methodology

### 9 4.1. Formulation

#### 10 4.1.1. Charging reject rate

11 As mentioned above, valid BEV samples are supposed to dwell at least at one candidate site during the  
 12 day. The factors indicating that BEVs are being charged successfully during their dwell events include  
 13 (1) chargers are installed at the dwell places, and (2) chargers or waiting spaces are available when  
 14 BEVs arrive.

15 Cell  $j$  is assumed as the dwell place for BEV  $i$ . Once BEV  $i$  arrives at cell  $j$ , the probability  
 16 that neither chargers nor waiting spaces are available is denoted as  $R_j$  ( $R_j \in [0, 1]$ ). The probability  
 17 of BEV  $i$  being turned away at cell  $j$  is considered as  $r_{ij}$  ( $r_{ij} \in [0, 1]$ ), where  $r_{ij} = R_j$ . Obviously,  
 18 if BEV  $i$  does not have the opportunity to park at cell  $j$  during the day, the reject rate  $r_{ij}$  equals 1.  
 19 The relationship between  $r_{ij}$  and  $R_j$  can be written by Eq. (2)

$$20 \quad r_{ij} = R_j^{h_{ij}} \quad (2)$$

21 where  $h_{ij}$  is a binary variable indicating whether BEV  $i$  dwells at cell  $j$ . If it does,  $h_{ij} = 1$ ;  
 22 otherwise,  $h_{ij} = 0$ .

23 Define  $W_i$  as the daily reject rate of BEV  $i$ , which is the multiplication of reject probabilities  
 24 that BEV  $i$  is turned away at cell  $j$  ( $\forall j \in \mathbf{J}$ ) (Eq. (3)). The probability of BEV  $i$  being charged  
 25 at least once in a day during its dwell events is then  $1 - W_i$ .

$$26 \quad W_i = \prod_{j=1}^N r_{ij} \quad (3)$$

27 The tradeoff between BEVs daily reject rate and charger network coverage is as follows. If  $W_i = 1$ ,  
 28 it is impossible for BEV  $i$  to take the dwell time for charging during the day because there is no  
 29 charging station wherever it dwells. Therefore, it has no choice but to detour for charging, which  
 30 increases BEV drivers' cost. As to BEV taxis, detour charging not only leads to extra travel distance  
 31 but also reduces the daily operating time and decreases taxi drivers' revenue. If  $W_i = 0$ , wherever BEV  
 32  $i$  dwells, it can always be charged because of adequate charger network coverage, which, of course,  
 33 requires enormous infrastructure investment. The daily reject rate  $W_i$  can be considered as a measure  
 34 of service quality of charger coverage and can be calculated based on queueing theory.

Denote  $p_s$  as the probability of a full system in which prospective BEV drivers are turned away. It is widely known as Erlang's loss formula and determined by the utilization ratio  $\rho_j = \lambda_j / \mu_j$  and the number of chargers  $x_j$ . The probability  $p_s$  for an  $M/M/x/s$  system is given by Gross (2008)

$$p_s = \frac{\rho^s}{x!x^{s-x}} p_0 \quad (4)$$

where  $p_0$  represents the probability that no customers are in the system, and it equals

$$p_0 = \begin{cases} \left( \sum_{n=0}^{x-1} \frac{\rho^n}{n!} + \frac{\rho^x (1 - \rho^{s-x+1})}{x!(1 - \rho_s)} \right)^{-1}, & \rho_s \neq 1 \\ \left( \sum_{n=0}^{x-1} \frac{\rho^n}{n!} + \frac{\rho^x}{x!} (s - x + 1) \right)^{-1}, & \rho_s = 1 \end{cases} \quad (5)$$

where  $\rho$  is the ratio of arrival rate  $\lambda$  and service rate  $\mu$ , and  $\rho_s$  is the average utilization of the system

$$\rho_s = \frac{\rho}{x} = \frac{\lambda}{x\mu} \quad (6)$$

Thus, the reject rate at cell  $j$  (namely,  $R_j$ ) can be expressed as

$$R_j = \begin{cases} p_s(x_j, \rho_j) & x_j > 0 \\ 1 & x_j = 0 \end{cases} \quad (7)$$

The daily reject rate  $W_i$  can be calculated using Eq. (2)–(7). Define  $r_{\max}$  as the maximum allowable daily reject rate for each BEV (i.e., the probability that BEVs cannot be charged at any of the dwell places during the day). The service quality constraint  $W_i \leq r_{\max}$  ( $\forall i \in \mathbf{I}$ ) should be satisfied when optimize the siting and sizing of charging stations.

#### 4.1.2. Optimization model

Let binary variable  $y_j$  denote the deployment configuration for cell  $j$ . When  $y_j = 1$ , at least one charger is installed at cell  $j$ ; otherwise, no charging station is installed at cell  $j$ . Let integer variable  $x_j$  denote the number of chargers installed at cell  $j$ . The infrastructure cost includes a fixed cost of deploying a new charging station (denoted as  $V$ , unit: yuan per station) and the unit cost of a charger (denoted as  $C$ , unit: yuan per charger). The charger cost  $C$  varies greatly depending on the type of chargers. Single-port chargers with AC Level 2 capabilities cost ¥ 6500–13,000 excluding installation and potential electrical upgrades in order to provide the appropriate outlet near the EV parking spot, while the cost of DC fast chargers can be as high as ¥ 65,000–260,000 depending on the features and brands (Delucchi et al., 2013; Schroeder and Traber, 2012). The fixed cost of deploying a new charging station  $V$  includes the installation cost and permit fee. Unlike home stations, where hardware is the dominant cost, installation is the major contributor to public station cost (60% to 80% of total) (Agenbroad and Holland, 2014). To minimize the investment, the total cost can be expressed as

$$\min J(x, y) = C \sum_{j=1}^N x_j + V \sum_{j=1}^N y_j \quad (8)$$

subject to

$$W_i \leq r_{\max}, \quad \forall i \in \mathbf{I} \quad (9)$$

$$x_j \geq 0, \text{ integer}, \quad \forall j \in \mathbf{J} \quad (10)$$

$$y_j = \begin{cases} 1 & x_j > 0 \\ 0 & x_j = 0 \end{cases} \quad (11)$$

## 4.2. Solution method

Optimizing the performance of queueing systems is a difficult task because of the nonlinearity of the performance metrics as functions of the arrival and service rates. In general, the computational time scales exponentially with the problem size. It is computationally demanding to solve the proposed model with 666 queueing systems in the network. Note that the reject probability of an  $M/M/x/s$  system is a posynomial function, which has convexity properties (Mung et al., 2002). After a logarithmic transformation, a global optimum solution can be found efficiently. Therefore, the optimization model is reformulated into an integer linear program (ILP).

### 4.2.1. Regression

Assuming the reject rate  $R_j$  is a function of the number of chargers  $x_j$ , a series of exponential regression models is set up in the form of  $R_j = a_j e^{b_j x_j + c_j}$  for the purpose of simplifying the computation of reject rate  $R_j$  ( $\forall j \in \mathbf{J}$ ). The regression coefficients  $a_j$ ,  $b_j$ , and  $c_j$  vary with system utilization  $\rho_j$ . In the case study, since 666 cells are selected as the candidate sites for deploying charging stations, 666 exponential regression models are developed to quantify the effects of charger counts upon the reject rate. Table 1 lists the statistics of regression coefficients. Regardless of the utilization  $\rho_j$ , R-squared values are all above 0.935, which indicates the exponential regression formation is a suitable approximation of the relationship between dependent variables  $R_j$  and independent variables  $x_j$ . Fig. 4 plots the regression results of two example cells under the assumption of  $M/M/x/x$  queueing system. The system utilization ratios (i.e.,  $\rho_j = \lambda_j / \mu_{0j}$ ) of Cell 308 and Cell 111 are 0.34 and 73.82, respectively. Cell 308, located at Guzhang Park along the waterfront, is a newly developed area. This less popular site generates 15 dwell events per day on average. Cell 111, located near Huanghua Airport, is one of the most popular dwell sites and generates 1,540 dwell events daily on average. Obviously, the busier the station is, the more chargers need to be installed so as to satisfy the charging demand.

Place Table 1 about here

Place Fig. 4 about here

### 4.2.2. Logarithmic transformation

Using the regression models, Eq. (3) can be written as a monomial where variables and coefficients are positive real numbers, and all of the exponents are real numbers:

$$W_i = \prod_{j=1}^N (a_j e^{b_j x_j + c_j})^{h_{ij}} \quad (12)$$

The constraint Eq. (9) can be replaced with its logarithm form as

$$\sum_{j=1}^N b_j h_{ij} x_j \leq \ln r_{\max} - \ln \left( \prod_{j=1}^N (a_j e^{c_j})^{h_{ij}} \right) \quad (13)$$

Since when  $x_j = 0$ , no chargers are installed at cell  $j$ , and all the arrival BEVs will be turned away, the reject rate of cell  $j$ ,  $R_j$ , should equal 1. However, because the regression model is an approximation of the queuing model, there is no guarantee that when  $x_j = 0$ ,  $R_j = 1$ . In Eq. (13) we use an approximation that  $a_j e^{c_j} \approx 1$  and, therefore,  $\ln \left( \prod_{j=1}^N (a_j e^{c_j})^{h_{ij}} \right) = 0$ . Hence, constraint Eq. (13) is simplified as Eq. (14)

$$\sum_{j=1}^N b_j h_{ij} x_j \leq \ln r_{\max} \quad (14)$$

An interesting observation from Table 1 is that the average values of coefficients  $b_j$  gradually increase with decreasing service rates, while coefficients  $a_j$  and  $c_j$  are roughly unchanged. This supports the approximation in Eq. (14) that only coefficients  $b_j$  are included as inequality constraints. As expected, by increasing the number of waiting spaces in  $M/M/x/K$  system, fewer chargers are required in comparison with the  $M/M/x/x$  system. Thus, it is observed that the average values of  $b_j$  in  $M/M/x/K$  systems are smaller than those in  $M/M/x/x$  systems. It is speculated that  $b_j$  and  $x_j$  are positively correlated. For example, based on the assumption of  $\mu = \mu_0$  (i.e., when dwell time is considered as the service time), the average value of  $b_j$  is close to that when  $\mu = 12$  veh/day. Specifically, it is because the average dwell time in different candidate sites is about 109.9 minutes and, accordingly, the average system service rate  $\mu_0$  (i.e.,  $\mu_0 = (\sum_j \mu_{0j}) / |\mathbf{J}|$ ) is equal to 13.1 veh/day. The similar system utilization leads to similar configuration of charger layout, which will be further illustrated in Section 5.

Since constraint Eq. (11) is a logic constraint where  $y_j$  is binary and  $x_j$  is a positive integer, a Big-M reformulation is used to convert it into an internal mixed-integer problem. The reformulation is illustrated in Eq. (15), where  $\bar{M}$  is chosen as a sufficiently large value. If  $y_j$  is 0 (false),  $x_j$  is guaranteed to be 0; otherwise,  $x_j$  is unconstrained.

$$x_j \leq \bar{M} y_j \quad (15)$$

#### 4.2.3. Integer programming solver

The optimization model is formulated in a standard form of ILP as follows:

$$\min J(x, y) = C \sum_{j=1}^N x_j + V \sum_{j=1}^N y_j$$

1 subject to

$$2 \quad \sum_{j=1}^N b_j h_{ij} x_j \leq \ln r_{\max}, \quad \forall i \in \mathbf{I}$$

$$3 \quad x_j \leq \bar{M} y_j, \quad \forall j \in \mathbf{J}$$

$$4 \quad x_j \geq 0, \text{ integer}, \quad \forall j \in \mathbf{J}$$

$$5 \quad 0 \leq y_j \leq 1, \text{ integer}, \quad \forall j \in \mathbf{J}$$

6 Since there are 666 candidate cells (i.e.  $|\mathbf{J}|=666$ ) and 53,092 valid taxi samples (i.e.  $|\mathbf{I}|=53,092$ ),  
7 the formulation leads to a large-scale ILP problem with 55,756 constraints (i.e.,  $|\mathbf{I}|+4|\mathbf{J}|$ ) and 1,332  
8 (i.e.,  $2|\mathbf{J}|$ ) decision variables. The problem is solved using MATLAB R2015b (MathWorks, 2015) and  
9 YALMIP (Lofberg, 2004) as a modeling language and Gurobi 6.5.1 (Gurobi Optimization, Inc., 2016)  
10 as the solver. The time spent solving such an optimization problem is about 0.67 seconds running on a  
11 personal workstation with 3.50 GHz CPU and 16GB of RAM.

## 12 5. Results

### 13 5.1. Charging station siting

14 Fig. 5 shows the optimized layout of charging stations in Changsha (the map is zoomed to the inner  
15 city). The red cells represent locations where charging stations should be installed, and the black cells  
16 represent the candidate sites which are not selected. There are 35 black cells that are excluded from the  
17 candidate sites. Various scenarios were tested with the combination of  $r_{\max}$  ranging from 5% to 25%  
18 and ratios of  $C$  and  $V$  ranging from 1:0 to 1:10. When  $C/V$  equals 1:0, there is no fixed cost  
19 associated with a new charging station. Under ideal circumstances, proper electric outlets are already  
20 available and investors do not have to pay for installation and the permit fee. However, in most cases,  
21 the fixed cost  $V$  can hardly be avoided and varies greatly depending on the type of charger and location  
22 of the charging station. For example, if DC fast chargers are deployed, the fixed cost is likely to be  
23 higher because of the transformer upgrades. But no matter how the scenario changes, the configurations  
24 of excluded sites remain the same. It might be because the dwell pattern  $\mathbf{H} = [h_{ij}]$ , derived from the  
25 GPS dataset, plays a determining role in charging station siting. In the most extreme case, if all the  
26 candidate sites have at least one BEV taxi that only dwells once during the day, none of candidate sites  
27 will be excluded.

28 **Place Fig. 5 about here**

29 Table 2 presents the summary statistics of the selected and excluded cells. Compared with the  
30 selected candidates, the excluded cells have lower arrival rates and shorter dwell times, resulting in  
31 smaller system utilization. Additionally, the selected cells are not the only charging opportunity for any  
32 BEV taxis; the BEVs that dwell there can be charged at other candidate sites. As mentioned in section  
33 4.2.2, we assume there is a positive correlation between the coefficients  $b$  and number of chargers.  
34 Obviously, the coefficients  $b$  of excluded cells are much smaller than those of selected cells.

35 **Place Table 2 about here**

### 36 5.2. Charging station sizing

37 Fig. 6 compares total numbers of chargers needed under different reject rate requirements and charging  
38 powers. By increasing the service rate (i.e., increasing the charging power) or decreasing the reject rate  
39 requirement, a lower number of chargers is required to satisfy the charging demand. As mentioned

1 earlier, the assumptions of  $\mu = \mu_0$  and  $\mu = 12$  veh/day result in similar service rates; thus, the  
 2 number of required chargers is close in these two scenarios. For a BEV with a range of 200 km and an  
 3 electricity consumption rate of 0.2 kWh/km (e.g., BYD E6), AC Level 2 chargers with an effective  
 4 charging power of 20 kW are recommended, assuming the BEVs stay plugged in until their dwell time  
 5 runs out. Since it takes about 2 hours to fully charge BEV-200km with AC Level 2 chargers, the service  
 6 rate  $\mu$  is about 12 veh/day. The other observation is that, when using a lower power charger, the  
 7 curve declines more rapidly with looser reject rate requirements. This is because according to the  
 8 constraint Eq. (14), the descent rate is determined by the coefficient  $b$ , which is much smaller at a low  
 9 service rate than when fast chargers are adopted.

10 **Place Fig. 6 about here**

11 The charger utilization (denoted by  $\eta$ ) is defined as the ratio of the number of average occupied  
 12 chargers and all chargers in the system. Fig. 7 presents the results of charger utilization given different  
 13 scenarios. The value of  $\eta$  increases along with the increment of maximum allowable reject rate and  
 14 the decrement of charging power. For the same maximum allowable reject rate, the queuing system of  
 15  $M/M/x/K$  requires significantly fewer chargers than the  $M/M/x/x$  system requires. On average,  
 16 the number of chargers is reduced by 26.7% if fast chargers (i.e.,  $\mu = 48$ ) are deployed and is reduced  
 17 by 13.1% if slow chargers (i.e.,  $\mu = 8$ ) are installed. As waiting space is provided, the value of  $\eta$   
 18 increases by 42.6% and 21.9%, respectively, with  $\mu = 48$  and  $\mu = 8$ .

19 **Place Fig. 7 about here**

20 Fig. 8 presents the charger spatial distribution when  $\mu = \mu_0$  and  $r_{\max} = 5\%$ . The mid-sized  
 21 stations where  $5 \leq x < 20$  are most preferred by  $M/M/x/x$  system, while the small-sized stations  
 22 with fewer than 5 chargers are most favorable for  $M/M/x/K$  system. One super station located near  
 23 the airport is observed that requires 117 chargers with the  $M/M/x/x$  system and 110 chargers and 22  
 24 waiting spaces with the  $M/M/x/K$  system. On average 1,540 dwell events occurred every day at that  
 25 candidate site, and the service rate was 17.07 veh/day. Pragmatically speaking, not all the BEV taxis  
 26 will charge their batteries during dwell time. However, taking account of the charging demand from  
 27 private BEVs or other commercial BEVs that travel a long distance to get to the airport, we believe the  
 28 counted dwell events provide an insight into the real state of charging demand for this area. If the  
 29 chargers are replaced by more powerful chargers, e.g., with charging power of 80 kW, the service rate  
 30 rises to 48 veh/day, and the number of chargers can be reduced to 44 units with the  $M/M/x/x$  system  
 31 or 40 units with the  $M/M/x/K$  system.

32 **Place Fig. 8 about here**

### 33 5.3. Waiting spots and waiting time

34 As can be seen from Fig. 6, the  $M/M/x/K$  system shows its advantage in fewer chargers, although it  
 35 generates waiting time and needs more parking spaces. Fig. 9 shows the average waiting times when  
 36 additional parking spots for waiting are available. When  $r_{\max} = 5\%$ , since there is a plentiful supply of  
 37 chargers to satisfy the strict reject rate requirement, the average waiting time is less than 2.5 minutes,  
 38 which is probably tolerable for most BEV drivers. The most powerful chargers with the service rate of  
 39  $\mu = 48$  perform best, with the average waiting times ranging from 2.1 minutes (with  $r_{\max} = 5\%$ ) to 5.9  
 40 minutes (with  $r_{\max} = 25\%$ ). As the maximum allowable reject rate goes up, the average waiting time  
 41 increases, especially for low power chargers.

42 **Place Fig. 9 about here**



1 In the  $M/M/x/K$  system we assume every 5five chargers equipped with one parking spot for  
 2 waiting. Undoubtedly, one charger occupies one parking spot. Let  $n_c$  denote the number of parking  
 3 spots used for charging,  $n_w$  denote the number of parking spots used for waiting, and  $n_{sum} = n_c + n_w$   
 4 denote the sum of required parking spots. We compare the value of  $n_{sum}$  between  $M/M/x/x$  and  
 5  $M/M/x/K$  systems and present their difference in Fig. 10. From the results we can see that, despite  
 6 the  $M/M/x/K$  system reducing the number of chargers (i.e.,  $n_c^{xx} - n_c^{xk}$ ), it requires more parking  
 7 spots (i.e.,  $n_{sum}^{xk} - n_{sum}^{xx}$ ). The waiting space of the system with the most powerful chargers increases  
 8 drastically, from 44 (with  $r_{max}=5\%$ ) to 423 (with  $r_{max}=25\%$ ), while the fluctuation of slow chargers  
 9 is not significant. This happens because, with the slow chargers, waiting spots are occupied for longer  
 10 times compared to fast charging systems and are not influenced by the reject rate. Due to the higher  
 11 turnover rate of waiting spots in fast charging systems, especially when fewer chargers are required  
 12 with  $r_{max}=25\%$ , more waiting space helps to reduce the possibility of BEVs being turned away.

13 **Place Fig. 10 about here**

14 The allocation problem for charging facilities is different from that for gas or hydrogen stations.  
 15 For charging facilities, the question is whether or not the facility should be equipped with more chargers  
 16 or with enough parking spots for waiting. In general, it depends on the price of one charger (denoted by  
 17  $C_c$ ) and the price of one parking spot (denoted by  $C_p$ ), and the values of  $C_c$  and  $C_p$  vary based  
 18 on the charger type and the location of stations, respectively. To investigate this question, we define  
 19  $\beta$  as the ratio of  $C_c$  and  $C_p$  and  $\beta_0$  as the trade-off parameter where

$$20 \quad \beta_0 = \frac{n_{sum}^{xk} - n_{sum}^{xx}}{n_c^{xx} - n_c^{xk}} = \frac{n_w^{xk}}{n_c^{xx} - n_c^{xk}} - 1 \quad (16)$$

21 If  $\beta > \beta_0$ , that is

$$22 \quad \beta = \frac{C_c}{C_p} > \frac{n_{sum}^{xk} - n_{sum}^{xx}}{n_c^{xx} - n_c^{xk}} \quad (17)$$

23 it costs more to install chargers than to provide waiting space given the same required reject rate;  
 24 otherwise, deploying more chargers is preferred. Table 3 lists the values of  $\beta_0$  under different  
 25 scenarios. In particular, we assume one parking spot costs ¥ 40,000 during a charger's life-cycle,  
 26 chargers associated with  $\mu=48$  cost ¥ 120,000, and chargers associated with  $\mu=8$  cost ¥ 4,000.  
 27 Hence, it is suggested to provide more waiting spots when fast chargers are deployed since  $\beta=300\%$   
 28 and  $\beta_0$  is smaller than 200% when  $r_{max} \leq 25\%$ . If installing chargers with low charging power and  
 29 setting  $r_{max}$  as 25%,  $\beta$  is equal to 100% thereof, and it is more economical to increase the number  
 30 of chargers.

31 **Place Table 3 about here**

## 6. Conclusions

This paper presents a data-driven optimization model to allocate charging stations and chargers throughout a city with the objective of minimizing overall investment. The proposed approach takes vehicles' dwell pattern as input and the probability of BEVs being charged during their dwell time as constraints. Charging congestion is taken into consideration and formulated using queueing theory. By means of regression and logarithmic transformation, the optimization model is transformed into an ILP problem and solved by Gurobi solver efficiently. The key findings from the results include the following: (1) The dwell pattern of the taxi fleet determines the siting of charging stations, and 35 out of 666 candidate sites do not have to install chargers after optimization. (2) When waiting space is offered, the utilization of chargers can be improved and the number of chargers can be reduced by 13.1% to 26.7%, compared to charging stations with no waiting space. However, it will require more parking spots and increase users' waiting time. (3) The tradeoff between installing more chargers versus providing more waiting spaces depends on the cost ratio of chargers and parking spots, which varies with the charger power and required reject rate as well. For 20 kW chargers, in order to satisfy at least 95% of the charging demand, it is more economical to install more chargers instead of providing more waiting spaces when the price of chargers is less than 23% of the cost of parking spots.

The main caveat of the proposed approach is that it does not account for the SOC when BEVs arrive at charging stations. BEVs with high level of SOC may not charge at their dwell place. Also, depending on the type of chargers and SOC, charging time may vary for different vehicles. Another aspect that needs further research is the penetration rate of BEVs. The dataset tested in this paper captures only the charging demand of potential BEV taxis, while trajectory data collected from other kinds of private or commercial BEVs can be used to better estimate the charging demand and further support the charging infrastructure planning for Changsha. Moreover, due to the lack of land use information, the space limitation of charging and waiting areas at each site is not considered in the model formulation. The proposed model could be further improved if information about the cost of acquiring the land is available.

Despite these caveats, the methodology presented in this paper provides a tool for infrastructure providers, city planners, and other stakeholders to determine where charging stations should be located and how many chargers and waiting spaces need to be assigned.

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2 **Fig. 1.** Frequency distribution of the number of daily dwell events per cell.

3 **Fig. 2.** Arrival rate distribution (unit: veh/day).

4 **Fig. 3.** Distribution of the number of daily dwell sites for each taxi.

5 **Fig. 4.** Regression results of sample candidate sites, assuming  $M/M/x/x$  queueing system. (a) Cell  
6 308:  $R_j = 0.449e^{-1.419x_j+0.800}$ , R-squared = 0.999, RMSE=0.016,  $\rho_j = 0.34$ ; (b) Cell 111:  $R_j = 0.234e^{-0.026x_j+1.579}$ ,  
7 R-squared = 0.951, RMSE=0.072,  $\rho_j=73.82$ .

8 **Fig. 5.** Optimized layout of charging stations.

9 **Fig. 6.** Overall number of chargers. (a)  $M/M/x/x$ ; (b)  $M/M/x/K$ .

10 **Fig. 7.** Charger utilization. (a)  $M/M/x/x$ ; (b)  $M/M/x/K$ .

11 **Fig. 8.** Charger distribution. (a)  $M/M/x/x$ ,  $\mu = \mu_0$ ,  $r_{\max} = 5\%$ ; (b)  $M/M/x/K$ ,  $\mu = \mu_0$ ,  $r_{\max} = 5\%$ .

12 **Fig. 9.** Average waiting time.

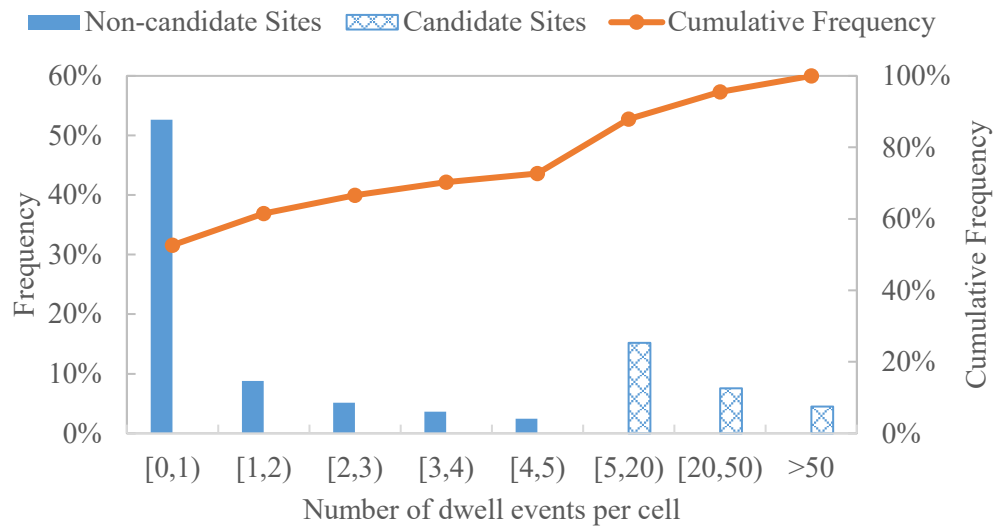
13 **Fig. 10.** Increased parking spots of  $M/M/x/K$  system.

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- 2 **Table 1** Characteristic of regression coefficients.
- 3 **Table 2** Statistics of selected and excluded cells attributes.
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- 5

1 **Figures**

2



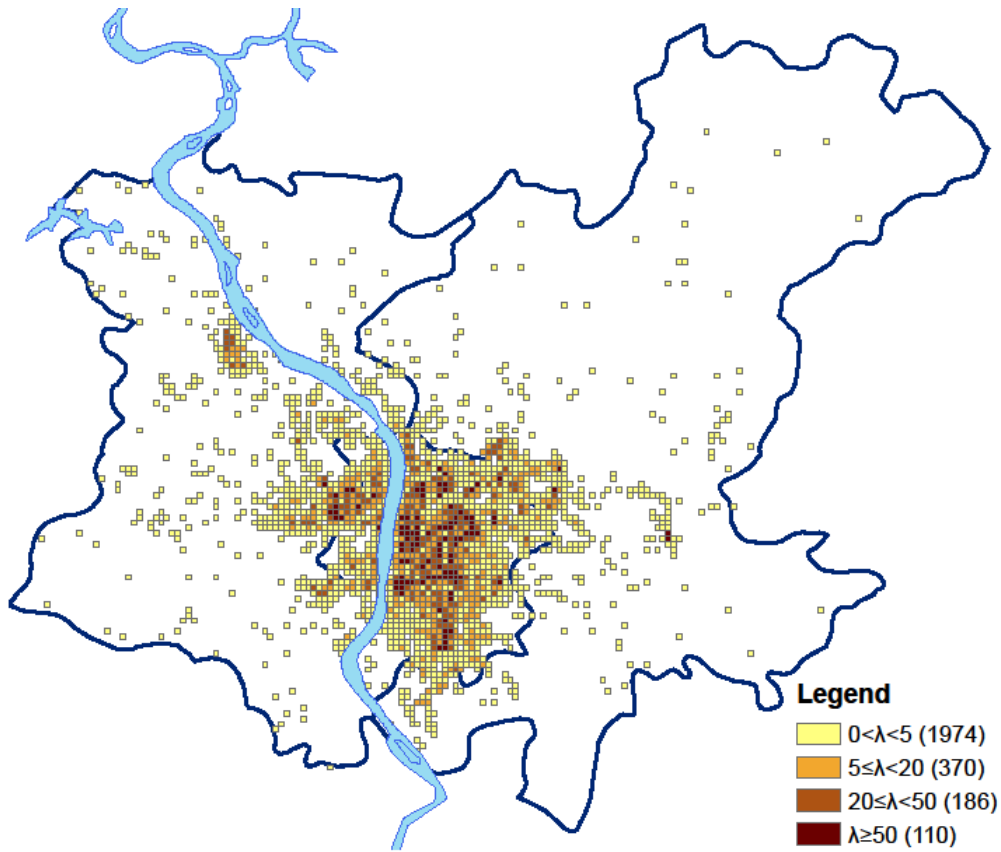
3

4 **Fig. 1.** Frequency distribution of the number of daily dwell events per cell.

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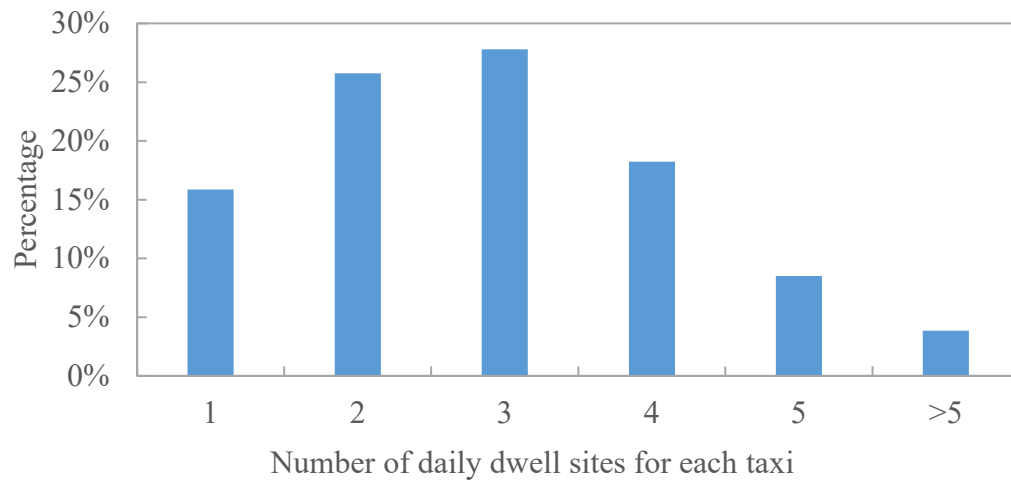


2

3 **Fig. 2.** Arrival rate distribution (unit: veh/day).

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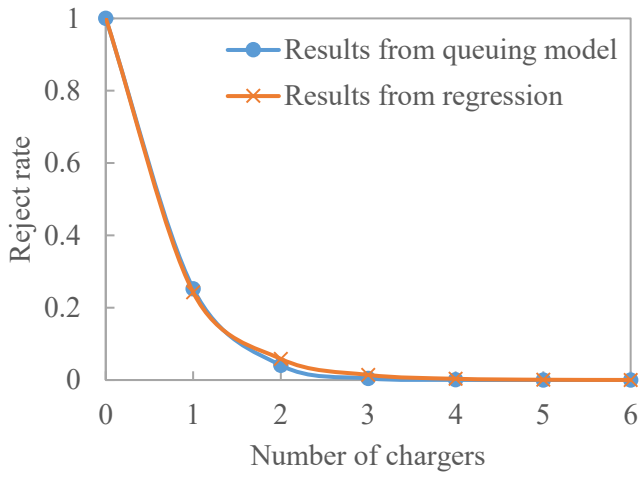
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**Fig. 3.** Distribution of the number of daily dwell sites for each taxi.

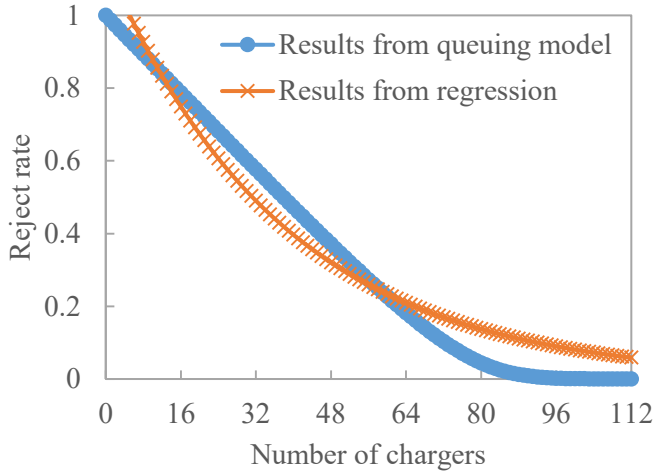
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3 (a) Cell 308:  $R_j = 0.449e^{-1.419x_j+0.800}$ , R-squared = 0.999, RMSE=0.016,  $\rho_j = 0.34$



4

5 (b) Cell 111:  $R_j = 0.234e^{-0.026x_j+1.579}$ , R-squared = 0.951, RMSE=0.072,  $\rho_j = 73.82$

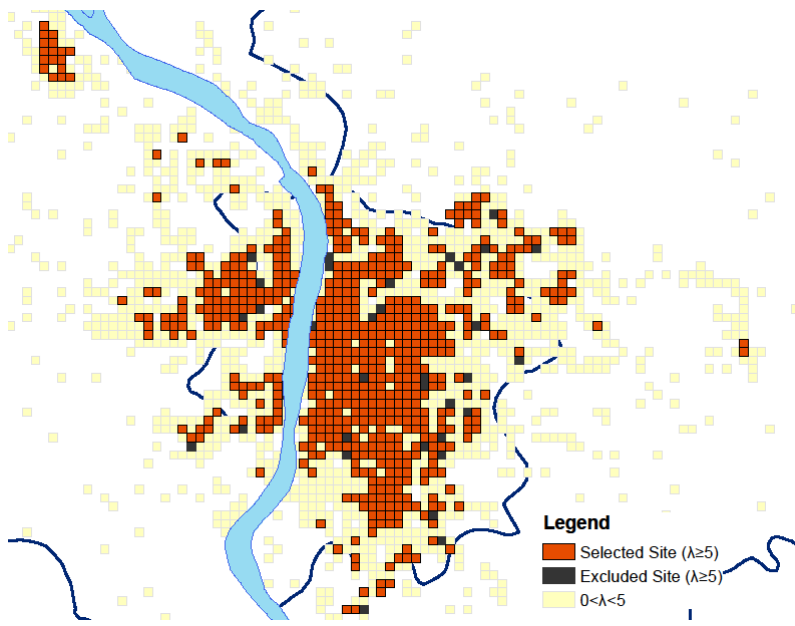
6 **Fig. 4.** Regression results of sample candidate sites, assuming  $M/M/x/x$  queueing system.

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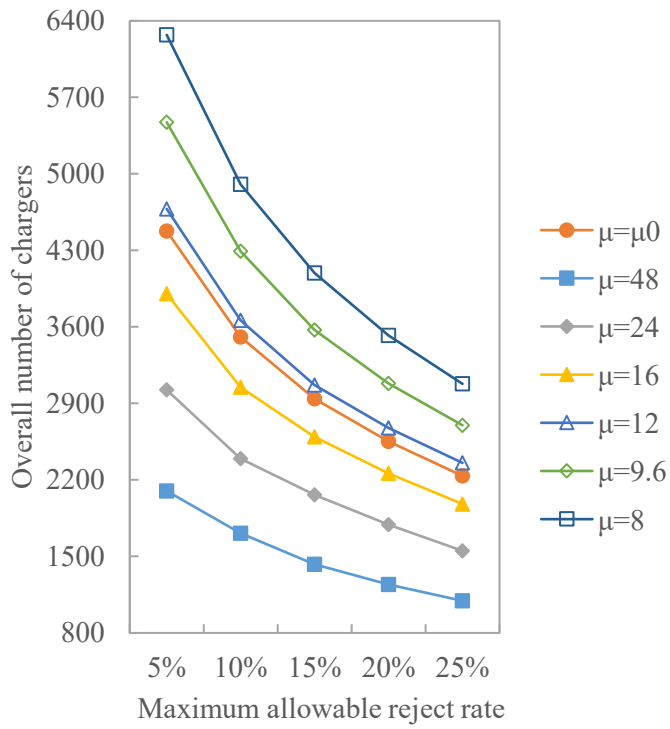
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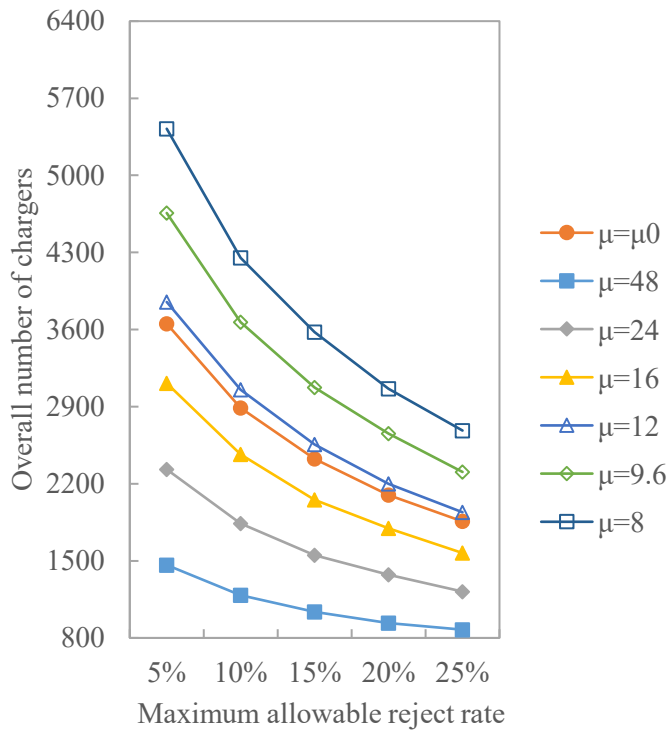
3 **Fig. 5.** Optimized layout of charging stations.

4



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2 (a)  $M/M/x/x$

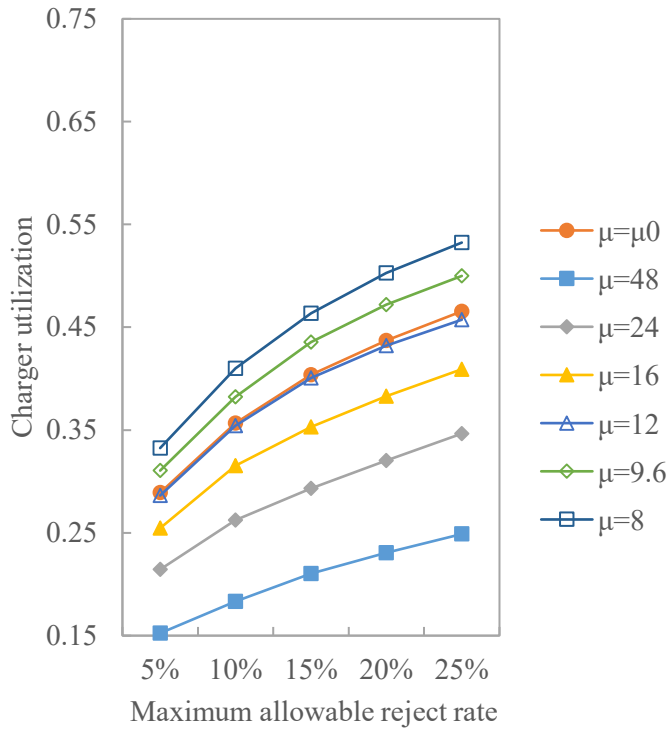


3

4 (b)  $M/M/x/K$

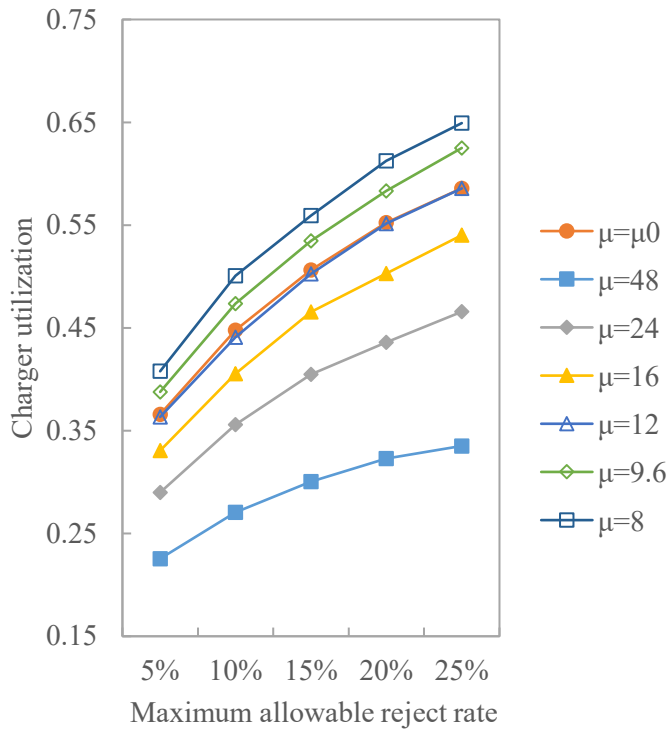
5 **Fig. 6.** Overall number of chargers.

6



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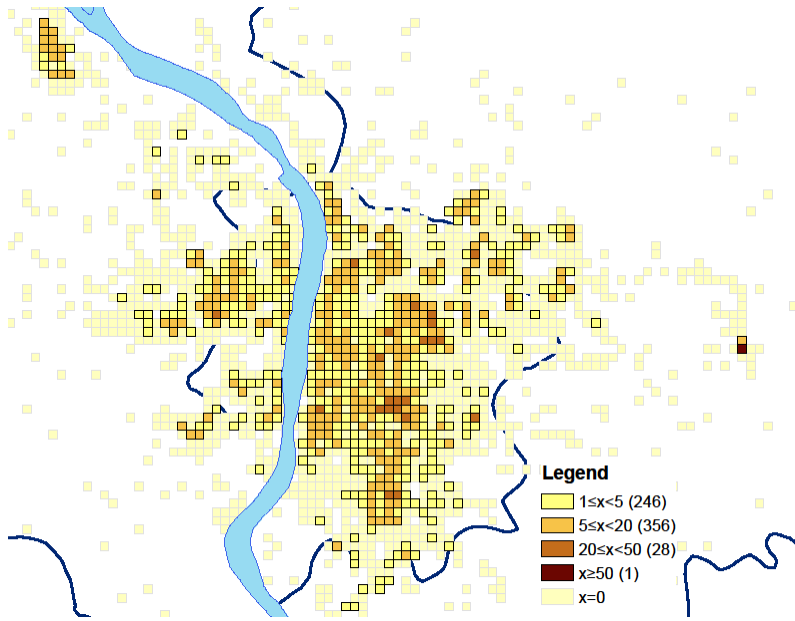
2 (a)  $M/M/x/x$



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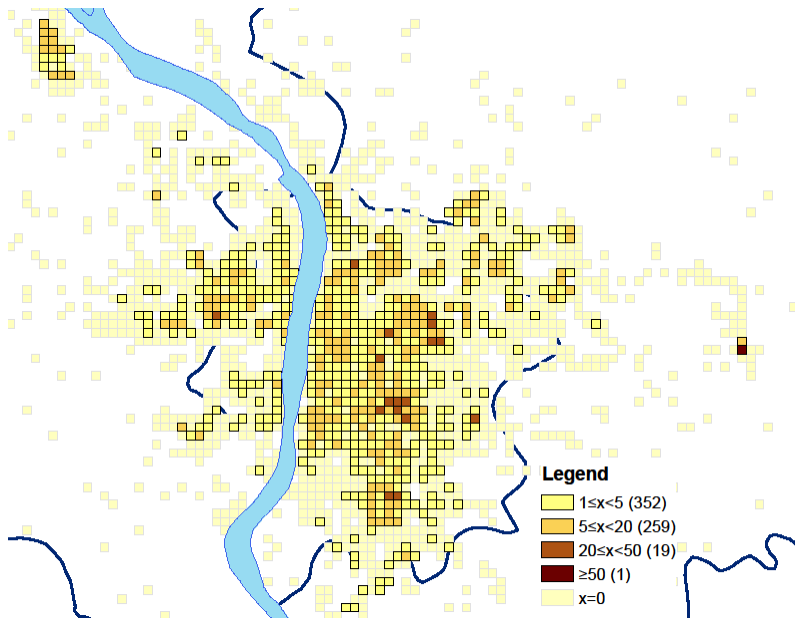
4 (b)  $M/M/x/K$

5 **Fig. 7.** Charger utilization.



1

2 (a)  $M/M/x/x$ ,  $\mu = \mu_0$ ,  $r_{\max} = 5\%$

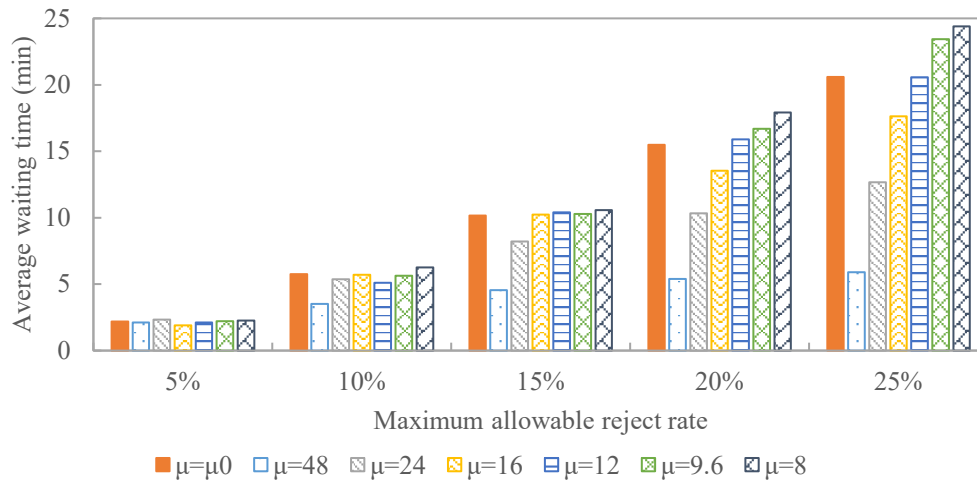


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4 (b)  $M/M/x/K$ ,  $\mu = \mu_0$ ,  $r_{\max} = 5\%$

5 **Fig. 8.** Charger distribution.

6

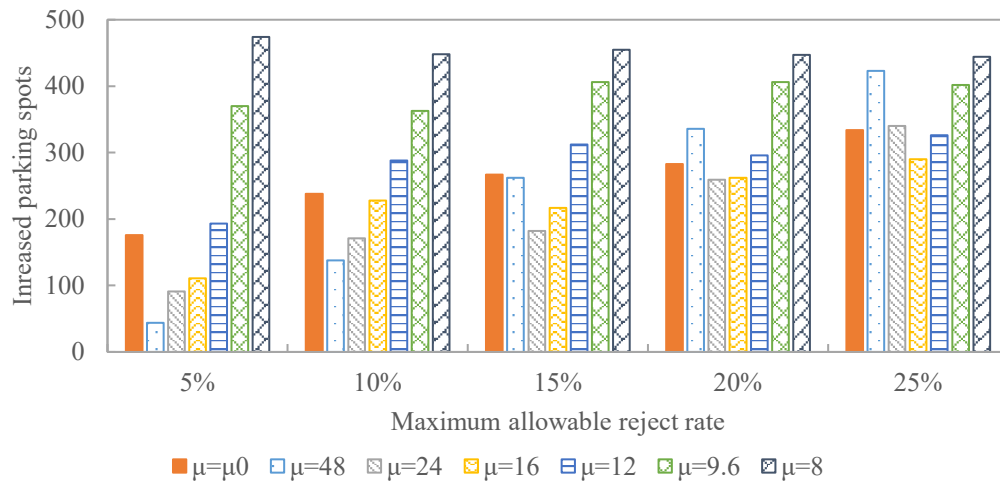


1  
2 **Fig. 9.** Average waiting time.

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2

3 **Fig. 10.** Increased parking spots of  $M/M/x/K$  system.

4

1 **Tables**

2 **Table 1**

3 Characteristic of regression coefficients.

<i>M/M/x/x</i>							
Service rate (veh/day)	Minimum value	Mean			Standard Deviation		
	R-squared	a	b	c	a	b	c
$\mu=\mu_0$	0.951	0.461	-0.728	0.803	0.032	0.403	0.058
$\mu=48$	0.962	0.452	-1.468	0.800	0.012	0.592	0.018
$\mu=24$	0.954	0.456	-1.045	0.801	0.013	0.467	0.016
$\mu=16$	0.950	0.458	-0.840	0.803	0.032	0.395	0.044
$\mu=12$	0.948	0.459	-0.714	0.807	0.037	0.348	0.070
$\mu=9.6$	0.947	0.467	-0.626	0.797	0.051	0.314	0.071
$\mu=8$	0.954	0.469	-0.560	0.799	0.060	0.287	0.090
<i>M/M/x/K</i>							
Service rate (veh/day)	Minimum value	Mean			Standard Deviation		
	R-squared	a	b	c	a	b	c
$\mu=\mu_0$	0.941	0.459	-1.107	0.801	0.035	0.789	0.048
$\mu=48$	0.956	0.450	-2.663	0.802	0.008	1.284	0.022
$\mu=24$	0.945	0.454	-1.750	0.800	0.026	0.973	0.030
$\mu=16$	0.940	0.456	-1.322	0.802	0.030	0.781	0.054
$\mu=12$	0.937	0.456	-1.068	0.806	0.026	0.651	0.059
$\mu=9.6$	0.939	0.459	-0.900	0.805	0.028	0.557	0.057
$\mu=8$	0.946	0.463	-0.780	0.804	0.055	0.486	0.086

4

5

1 **Table 2**

2 Statistics of selected and excluded cells attributes.

	$\lambda$		$\mu_0$		$\rho$		$b$	
	EC*	SC*	EC	SC	EC	SC	EC	SC
Mean	7.282	34.519	22.818	13.668	0.365	2.646	-1.479	-0.687
Median	5.714	16.000	20.739	10.943	0.338	1.423	-1.419	-0.641
Standard Deviation	4.237	79.757	10.658	8.138	0.187	4.352	0.363	0.363

3 \*EC: Excluded cells; SC: Selected cells.

4

1 **Table 3**

2 Values of trade-off parameters.

Maximum allowable reject rate Service rate (veh/day)	$r_{\max} = 5\%$	$r_{\max} = 10\%$	$r_{\max} = 15\%$	$r_{\max} = 20\%$	$r_{\max} = 25\%$
$\mu=48$	7%	26%	67%	109%	193%
$\mu=24$	13%	31%	36%	62%	103%
$\mu=16$	14%	39%	40%	57%	71%
$\mu=12$	23%	47%	61%	62%	79%
$\mu=9.6$	45%	58%	81%	95%	102%
$\mu=8$	56%	69%	88%	98%	112%

3