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Disciplines
Agribusiness | Econometrics | Economic Theory | Finance | Growth and Development

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TIME VARYING DISCOUNT RATES AND RENT-PRICE RATIOS

IN FARMLAND MARKETS

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June 1990
ABSTRACT

This paper constructs a version of Campbell and Shiller's dividend-price ratio model in order to study the consistency of farmland price behavior with the implications of a present value formulation that accounts for time-varying discount rates. The model imposes testable restrictions on the joint behavior of rent-price ratios and a linear combination of the ex-post required rate of return and rent growth rates. The restrictions are found to be inconsistent with annual Iowa farmland price and rent movements for the 1926-1986 sample period.
1. Introduction

The recent boom and subsequent bust in farmland prices has generated renewed interest in the economic determination of farmland prices. Particular interest has focused on the consistency of observed farmland price movements with rational asset pricing models derived from the Efficient Markets Hypothesis (EMH). According to the EMH, the real price of a real or financial asset should not deviate systematically from its fundamental value, which can be expressed as the discounted present-value of the expected price of the asset at the end of a given holding period plus the discounted expected present-value of net real payments (i.e., rent payments in the case of farmland) obtained from ownership of the asset over that holding period. Systematic deviations of the asset's real price from its fundamental value imply the existence of unexploited profit opportunities in that market and have adverse implications for efficient resource allocations.

Alston and Melichar assumed a constant real discount rate and a constant expected growth rate for real rents. In this case, the theory implies that the real rent-price ratio should be approximately constant over time and equal to the difference between the discount rate and the mean growth rate of rents. Their analyses of rent and price time series confirmed this implication in the sense that they found strong correlations between average growth rates of prices and rents.

Falk (1989, 1991) maintained the assumption of a constant discount rate but allowed real rents to vary systematically over time. He applied a variety of tests developed in the financial economics literature to formally test the implications of the EMH using annual Iowa farmland price and rent data over the 1921-1986 sample period. Although his analysis confirmed the high degree of correlation between rent and price movements, the implications of the EMH were
uniformly rejected by these tests. Intuitively, it is not hard to reconcile the high correlation between rents and prices with the apparent inconsistency of farmland price movements with the constant expected returns version of the EMH. Under the assumption of constant expected returns and in the absence of an explosive rational price bubble, the EMH implies that current farmland price should be equal to the expected present value of current and future rents. In the absence of uncertainty about future rents, this means that current farmland price would be a moving-average of actual rents, which suggests that prices should be relatively unresponsive to rent movements in any single period. Thus, the high correlation between current price and current rent suggests a tendency of price to overreact to rent movements. Shiller (1981) exploited this implication of the present value model in his seminal work on excess volatility of stock market prices.

Featherstone and Baker studied the behavior of annual U.S. farm sector asset values over the 1910 to 1985 sample period allowing for systematic variations in returns (asset income) and discount rates (commercial paper rates). Their strategy was to assume that asset values, discount rates, and returns have a vector autoregressive representation, which they used to evaluate simulated dynamic responses of asset values to shocks in asset values, discount rates, and returns. They concluded that asset values have tended to overreact, relative to the implications of the EMH, to each of the three types of shocks, with asset values moving further and further away from their fundamental values for approximately six years before beginning to revert toward their fundamental values. Although Featherstone and Baker do not impose a constant discount rate or a constant rent growth rate, their conclusions rely primarily on relatively informal innovation accounting exercises rather than on formal statistical tests.
of the restrictions the theory imposes on the parameters of the vector autoregression. Furthermore, as Hamilton and Whiteman have shown in a related context, omission of relevant variables in the VAR can easily lead these sorts of procedures to detect spurious systematic deviations of asset prices from their fundamental values.

The purpose of this paper is to derive and formally test the restrictions implied by the EMH for farmland price and rent time series, allowing for systematic variations in rents and discount rates. The analytical framework and empirical strategy are based on Campbell and Shiller’s (1988) dividend-ratio model, which they used to study annual U.S. stock market price and dividend movements. The analysis will be applied to the Iowa farmland price and rent data used by Falk (1989,1991).

The remainder of the paper is organized as follows. Section 2 summarizes Campbell and Shiller’s dividend-ratio model and its testable implications. The data are described in Section 3. The empirical results are presented in Section 4 and discussed in Section 5. A summary of the paper and its main conclusions are provided in Section 6.

2. The Theoretical Model and its Implications

Let \( P_t \) denote the real price per acre of farmland at the start of period \( t \) (or, equivalently, at the end of period \( t-1 \)) and let \( D_t \) denote the real per acre rent payment to the landowner during period \( t \). Rent payments are assumed to be paid in a lump sum at the end of period \( t \). The realized gross real return on an acre of land held throughout period \( t \) is \( H_t \), where

\[
(1) \quad H_t = \frac{P_{t+1} + D_t}{P_t}.
\]

It follows that the realized log gross real return, \( h_t \), is given by
(2) \( h_t = \log(P_{t+1} + D_t) - \log P_t \),

which is a nonlinear function of prices and rents. Campbell and Shiller (1988) derive the following linear approximation of \( h_t \) based on a first-order Taylor expansion:

(3) \( h_t \approx \xi_t - k + \rho p_{t+1} + (1-\rho)d_t - P_t \),

where \( p_t \) and \( d_t \) are the natural logs of \( P_t \) and \( D_t \), respectively. The constants \( k \) and \( \rho \) emerge from the approximation procedure such that \( \rho \) is close to but less than one. If \( \delta_t \) is defined as the log rent-price ratio \( d_{t-1} - p_t \), then according to (3) an equivalent approximation of \( h_t \) is

(4) \( h_t \approx \xi_t - k + \delta_t - \rho \delta_{t+1} + \Delta d_t \).

Solving the first-order difference equation (4) recursively in the forward direction and imposing the terminal condition \( \lim_{i \to \infty} \rho^i \delta_{t+1} = 0 \), we obtain a representation of the approximate log rent-price ratio in terms of discounted future returns and dividend growth rates, i.e.,

(5) \( \delta_t \approx \sum_{j=0}^{\infty} \rho^j [h_{t+j} - d_{t+j}] - k/(1-\rho) \).

Equation (5) holds definitionally, subject only to the approximation error and the terminal condition. To transform it into a theoretical model with testable implications for actual data, we assume that the ex-ante required gross real rate of return in the land market during period \( t \) is equal to the ex-ante gross real rate of return on a competing asset or portfolio, subject to a constant multiplicative risk premium. That is, we assume

(6) \( E_t H_t = CE_t(1+r_t) \)

where \( C \) is a constant risk premium, \( r_t \) is the ex-post net real rate of return on an appropriately chosen competing asset and \( E_t \) denotes the market's rationally formed forecast conditioned on its information set at the start of period \( t \).
Using the approximation that $\log(1+x) \approx x$ for small values of $x$, we can rewrite (6) as

(7)  \[ E_t h_t = E_t r_t + c \]

where $c = \log(C)$. Taking expectations of both sides of (5) conditional on the information set $I_t$ and assuming that $\delta_t$ is in that information set, we obtain the rent-price ratio model

(8)  \[ \delta_t \approx E_t \sum_{j=0}^{\infty} \rho^j [r_{t+j} - \Delta d_{t+j}] + (c-k)/(1-\rho). \]

Thus, aside from a constant, the rent-price ratio model explains the log rent-price ratio, $d_{t-1}p_t$, in terms of the expected present value of all future "growth-adjusted discount rates," $r_{t+j} - \Delta d_{t+j}$. The model implies that the current rent-price ratio will be relatively high when expected future growth rates in rents are relatively low and/or when expected future discount rates are relatively high.

Testable implications of the model

Let $J_t$ denote the set containing current and past rent-price ratios and past growth-adjusted discount rates, i.e., $J_t = \{ \delta_{t-s}, r_{t-1-s} - \Delta d_{t-1-s}, s \geq 0 \}$. Assume that $J_t$, which is observable by the econometrician, is contained in the market's information set $I_t$. It follows from (8) and an application of the law of iterated expectations that an alternative representation of the log rent-price ratio is

(9)  \[ \delta_t \approx \mathbb{E}( \sum_{j=0}^{\infty} \rho^j [r_{t+j} - \Delta d_{t+j}] \mid J_t ) + (c-k)/(1-\rho). \]

It is interesting to note that the right-hand-sides of (8) and (9) must be equal even though they are constructed from potentially different information sets. This results, as Shiller (1989, p.163) explains, because both information sets
are constrained to include $\delta_t$, which is a sufficient statistic for market participants' information about the present value of future growth-adjusted discount rates.

Equation (9) provides the basis for the empirical tests of the rent-price ratio model based only on time series observations of $p_t$, $r_t$, and $d_t$. This equation will be used to test whether excess returns in the farmland market are predictable based on the information set $J_t$. If they are, then in contrast to the implications of the Efficient Markets Hypothesis, they must also be predictable based on the market's full information set $I_t$. Of course, failure to uncover predictable excess returns based on the information set $J_t$ does not imply that excess returns are unpredictable based on the information set $I_t$.

Next, assume that $\delta_t$ and $r_{t-1} - \Delta d_{t-1}$ are jointly covariance stationary processes which have the p-th order vector autoregressive representation

$$
\delta_t = C_{11}(L)\delta_{t-1} + C_{12}(L)(r_{t-2} - \Delta d_{t-2}) + u_{1t}
$$

$$
r_{t-1} - \Delta d_{t-1} = C_{21}(L)\delta_{t-1} + C_{22}(L)(r_{t-2} - \Delta d_{t-2}) + u_{2t}
$$

where $C_{ij}(L) = C_{ij}L + \ldots + C_{ijp}L^p$, for $i, j = 1, 2$; $L$ is the lag operator defined by $L^s x_t = x_{t-s}$, for all integer $s$; and $u_{it}$ is a zero-mean, constant variance, and serially uncorrelated process for $i = 1, 2$ with $E(u_{1t}u_{2t}) = \sigma_{12}$ for all $t$. Notice that (10) assumes that $\delta_t$ and $r_t - \Delta d_t$ are zero-mean processes. That is, we are omitting the constant term that appears in (9). There is no loss in generality in making this assumption since the theory does not impose any restrictions on these means. The empirical analysis will be conducted using demeaned versions of these two time series.

Finally, it will be convenient to introduce the companion form of the VAR:

$$
z_t = A\delta_{t-1} + v_t
$$

where $z_t = [\delta_t, \delta_{t-1}, \ldots, \delta_{t-p+1}, r_{t-1} - \Delta d_{t-1}, r_{t-2} - \Delta d_{t-2}, \ldots, r_{t-p} - \Delta d_{t-p}]'$; $v_t$ is a
2px1 vector whose first element is $u_{1t}$, whose $p+1$ element is $u_{2t}$, and whose other elements are zero; and A is the 2px2p companion matrix of the VAR, i.e.,

$$\begin{bmatrix}
C_{111} & C_{211} & \cdots & C_{p11} & C_{112} & C_{212} & \cdots & C_{p12} \\
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
0 & 1 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
\end{bmatrix}$$

$$A = \begin{bmatrix}
C_{121} & C_{221} & \cdots & C_{p21} & C_{122} & C_{222} & \cdots & C_{p22} \\
0 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
\end{bmatrix}$$

Campbell and Shiller (1988) derive the following testable implications of the rent-price ratio model. First, suppose that market participants find it useful to forecast current and future growth-adjusted discount rates using more information than simply its own history. This additional information should, according to (8), be reflected in the current log rent-price ratio since that ratio is an optimal forecast of current and future growth-adjusted discount rates based upon the market's full information set. In this case, $\delta_t$ should be found to Granger-cause the $r_{t-1} - \Delta d_{t-1}$ process in their bivariate vector autoregressive representation. That is, at least some of the elements of $C_{21}(L)$ should be nonzero.

Second, the rent-price ratio model implies the existence of sets of cross-equation restrictions on the VAR coefficients, which are most conveniently
expressed in terms of the companion form of the VAR. It can be shown [see Shiller (1989), pp. 163-4] that (9) and (11) imply that

$$e_1'(I - \rho^iA^i) = e_2'A(I - \rho A)^{-1}(I - \rho^iA^i)$$

for \( i = 1,2,\ldots \), where \( I \) is the 2p\times2p identity matrix, \( e_1 \) is a 2p\times1 vector whose first element is one and whose other elements are 0, and \( e_2 \) is a 2p\times1 vector whose \( p+1 \) element is one and whose other elements are 0. We are especially interested in the special cases where \( i = 1 \) and \( i = \infty \). When \( i = 1 \), these restrictions reduce to the set of 2p linear restrictions

$$e_1'(I - \rho A) - e_2'A = 0,$$

which have the economic interpretation that one-period excess returns are unpredictable on the basis of \( J_t \). If \( i = \infty \), the restrictions (12) reduce to the set of 2p nonlinear restrictions

$$e_1' = e_2'A(I - \rho A)^{-1}$$

since, by virtue of the stationarity of \( z_t \) and the restriction that \( \rho \) is positive and less than one, \( I - \rho^iA^i \) converges to \( I \) as \( i \) approaches infinity. The economic interpretation of (14) is that the log rent-price ratio \( \delta_t \) is equal to \( \delta_t' \), the unrestricted VAR forecast of the present value of expected current and future expected growth-adjusted discount rates.

Based on the unrestricted estimates of the VAR (10), Wald tests can be applied to formally evaluate the validity of (13) and (14). Notice that (13) and (14) are algebraically equivalent restrictions, i.e., (13) can be derived from (14) by post-multiplying both sides of (14) by \( I - \rho A \). Despite their algebraic equality, it is well-known that in finite samples Wald test statistics are not invariant with respect to nonlinear transformations of restrictions and so it is useful to test both sets of restrictions. In addition, the recent financial economics literature has shown that economically significant predictable excess
returns over short holding periods can be difficult to detect statistically.\(^1\) Thus, the nonlinear restrictions (14), in contrast to the linear restrictions (13), may stand a better chance of detecting persistent excess returns in this market.

3. The Data

To test the rent-price ratio model, time series data are required for the nominal price per acre of farmland, nominal rent per acre of farmland, and the nominal ex-post discount rate, whose expectation multiplied by a constant risk premium is assumed to determine the ex-ante required rate of return in the farmland market. This paper will use the annual Iowa farmland price and rent data previously used and described by Falk (1989,1991). These data are available since 1921, although for reasons that will be explained below this study will focus on the 1926-1986 sample period. Featherstone and Baker used farm asset value and farm asset income data, which are available since 1910. Despite the slightly longer sample period over which these data are available, it seems preferable to use actual farmland prices and rents rather than the approximations implied by the more general measures. The relative homogeneity of Iowa farmland and its historically active rental market provide additional advantages to this data set.

Although the appropriate measure of the nominal discount rate is not obvious, Featherstone and Baker (p.536) note that "[A] short-term risk-free series of interest rates like T-bills would be ideal to use to calculate an ex-post real interest rate series." Since Treasury bill rates do not extend as far back as 1910, Featherstone and Baker used commercial paper rates. Time series data do exist since 1926 and these data will be used in this paper. In order to take advantage of these data, the sample period will begin in 1926. Closing the
sample period in 1986 facilitates more direct comparisons between the results from this study and the results reported in Falk's earlier studies (which used 1921-1986 and 1922-1986 sample periods) and with Campbell and Shiller's (1988) results for the stock market (which used the 1926-1986 sample period for New York Stock Exchange index data). The 1926-1986 sample period does account for the two major cycles in farmland prices during this century. That is it accounts for the large decline in farmland prices during the depression of the 1930's and it accounts for the boom in land prices during the 1970's followed by the bust of the early 1980's. Although not reported in the paper, the analysis was also performed over the 1926-1972 sample period to see whether the results are primarily being driven by the most recent cycle in land prices and rents. The conclusions drawn in this paper turn out to be independent of which of two sample periods is considered.

Notice that although the theoretical model and its implications are based on real prices, real rents, and real discount rates, these variables are important only to the extent that they determine the log rent-price ratio and the growth-adjusted discount rate in (9) and (10). It follows that deflating the (end of) period t-1 nominal rent and (beginning of) period t nominal price by the same price deflator implies the same log rent-price ratio as the undeflated log rent-price ratio. Similarly in correcting the nominal period t discount rate and rent growth rate for inflation and then subtracting the implied real rent growth rate from the implied real discount rate to obtain the real growth-adjusted discount rate, the inflation measure will drop out. Consequently, the rent-price ratio model does not require that nominal prices, rents, and discount rates be converted into real form, avoiding controversy regarding the appropriate choice of a price index. However, we will also be interested in analyzing the
special case of a constant expected required real rate of return, i.e., the case
where $E_t r_{t+s} = r$ for all $t$ and for all $s \geq 0$, where $r$ is some constant. In this
case, the growth adjusted discount rate, $r_t - \Delta d_t$, is replaced by simply minus
the growth rate of real rents, $-\Delta d_t$, which does depend upon how $d_t$ is deflated.
For this case, nominal year $t$ rents are deflated by the January of year $t+1$
Consumer Price Index.

Table 1 provides a brief description of the data and data sources. Table
2 reports various summary statistics for these data. Two key maintained
assumptions of the rent-price ratio model are that the log rent-price ratio and
the growth-adjusted discount rate processes are covariance stationary. Notice
from Table 2 that the sample mean growth rate of farmland prices (2.94% per year)
is smaller than the sample mean growth rate of rents (4.02% per year). The null
hypothesis that the log rent-price ratio has tended to increase over time as a
random walk with drift was tested against the alternative hypothesis of
covariance stationarity by a Phillips-Perron unit root test and it can be
rejected at the 10% significance level. A similar test applied to the growth-
adjusted discount rate time series rejects the unit root null against the
stationarity alternative at the one percent level.2 These tests provide some
empirical support for the maintained hypotheses of the rent-price ratio model
that $\delta_t$ and $r_t - \Delta d_t$ are stationary processes.

4. Empirical Results

The first step in the test strategy is to estimate the vector
autoregression (10). Given the lag length $p$, the VAR coefficients can estimated
by applying ordinary least squares to each of the two equations in (10). As noted
previously, the theory does not restrict the unconditional means of the log rent-
price ratio, $\delta_t$, or the growth-adjusted discount rate, $r_t - \Delta d_t$, and so their sample means were removed prior to the estimation of the VARs. Furthermore, in order to evaluate the role of allowing for a time-varying discount rate in the model, we also consider the special case of a constant required rate of return. This amounts to imposing the restriction that $E_t r_{t+s} = r$ for all $t$ and for all $s \geq 0$, or equivalently, since we will be working in deviations from means, the constant discount rate case amounts to setting $r_t = 0$ for all $t$. This case also requires, as noted above and in contrast to the time-varying discount rate case, that $\Delta d_t$ be measured as the difference in the log of real (as opposed to nominal) rents.

The implications of the log rent-price ratio model will be considered for three alternative lag lengths of the VAR: $p = 1, 2, \text{ and } 4$. Table 3 provides the results of applying Sims' Chi-square test to compare the fits of the VAR for various values of $p$. These results suggest that for both versions of the VAR there is a statistically significant improvement in the explanatory power of the VAR in increasing the lag length from one to two. The evidence is less compelling with regard to the desirability of further increases in the lag length. The summaries of the estimated VARs given in Table 4 provide additional insight into the results of the lag length tests. There are only marginal gains in the explanatory power of the $\delta_t$ equation with increases in the lag length beyond one. However, the explanatory power of the $r_{t-1} - \Delta d_{t-1}$ equation increases substantially in moving from one lag to two lags for both versions of the model, although additional increases in the lag length add little to the explanatory power of this equation.

Table 4 also reports the results of Granger-causality tests. Recall that one of the implications of the model is that in the vector autoregressive
representation of $\delta_t$ and $r_{t-1}^{\Delta d_{t-1}}$, $\delta_t$ should be found to Granger-cause the $r_{t-1}^{\Delta d_{t-1}}$ process, unless market participants can forecast current and future growth-adjusted discount rates exactly based on its own history. The failure of $\delta_t$ to Granger-cause $r_{t-1}^{\Delta d_{t-1}}$ is equivalent to the condition that the coefficients on lagged $\delta$'s in the $r_{t-1}^{\Delta d_{t-1}}$ equation are jointly equal to zero, which can be tested by an F-test based on OLS estimates of that equation. The results in Table 4 indicate that the null hypothesis of no causality can easily be rejected for the second and fourth-order VARs, for both the constant and time-varying return models, though it cannot be rejected for the first-order VAR in either case.

Next, we consider the cross-equation restrictions that the rent-price ratio imposes on the VAR coefficients, beginning with the linear form of these restrictions as given by equation (13). Recall that these restrictions correspond to the condition that single-period excess returns are unpredictable based on current and past log rent-price ratios and past growth-adjusted discount rates. Campbell and Shiller (1988) show that a Wald test of these restrictions is numerically equivalent to an F-test that the coefficients in the regression of the approximate excess return $\xi_t - r_t$ on the $p$ lagged values of $\delta_t$ and $r_{t-1}^{\Delta d_{t-1}}$ are jointly equal to zero, where $\xi_t$ was defined in equations (3) and (4). Table 5 contains a summary of these regressions and the corresponding F-tests under the assumption that the parameter $\rho$, required to calculate $\xi_t$ from the data, is equal to .946. According to these results, the linear restrictions are easily rejected for all values of $p$ and for both versions of the model, except for the constant expected returns version of the model augmented by a first-order VAR in which case the restrictions can be rejected only at approximately the 17 percent significance level. In addition to these
regressions, we also considered whether actual single period excess returns, \( h_t - r_t \), have been predictable based on observed log rent-price ratios and growth-adjusted discount rates. The regression results appear to be insensitive to whether actual or approximate excess returns are used to form the dependent variable, which provides some positive evidence regarding the quality of the approximation (3) upon which the log rent-price ratio model is based.

Finally, we consider the set of nonlinear cross-equation restrictions (14). These restrictions imply that the theoretical log rent-price ratio \( \delta_t' \), which is the expected present value of current and future growth-adjusted discount rates conditional upon the information set \( J_t \), is equal to the observed log rent-price ratio, \( \delta_t \). Table 6 compares the behavior of \( \delta_t \) with the behavior of the estimated \( \delta_t' \) series in several ways. It specifies the ratio of their estimated standard deviations and their estimated contemporaneous cross-correlation, which should both be equal to one, sampling error aside, if the rent-price ratio model accurately describes the behavior of \( \delta_t \). In addition a formal test of the equality of \( \delta_t \) and \( \delta_t' \) is conducted by applying a Wald test to test the restrictions (14).

For the second and fourth-order VARs and for the constant and time-varying expected real return versions of the model, the results are quite similar. Although the ratio of the standard deviations of \( \delta_t \) and \( \delta_t' \) is very close to one in the time-varying expected return cases but only about one-half in the constant expected return cases, their contemporaneous correlation is nearly -1 and, not surprisingly, the formal Wald test easily rejects their equality. The constant expected returns version of the model augmented with a first-order VAR appears to perform best in regard to the relationship between \( \delta_t' \) and \( \delta_t \) since their correlation is estimated to be .966 and the Wald test implies that the null
hypothesis of equality cannot be rejected at the 20 percent level, even though the standard deviation of the estimated $\delta_t$ series is less than half of the standard deviation of the actual log rent-price ratio. The time-varying expected real return version of the model augmented by a first-order VAR obtains some support from the Wald test, which can reject the model at about the 10 percent marginal significance level, but it performs poorly based on the other two criteria.

5. Discussion

The main results from the previous section can be summarized as follows. For second and higher-order VARs and for the constant and time-varying expected returns versions of the model, the relatively weak implication of the model that current and past values of the rent-price ratio add useful information with respect to forecasting current and future growth-adjusted discount rates seems to be consistent with the data. However, in each of these cases the stronger implications of the model cannot be supported by the data. Single-period excess returns appear to be predictable based on current and past rent-price ratios and past growth-adjusted discount rates and, therefore, would be predictable based on any larger information set. Furthermore, there is very strong negative correlation between the actual and theoretical rent-price ratios with the equality of these measures being easily rejected on the basis of a nonlinear Wald test. These results are consistent with the previous results reported by Falk (1991) for this data set.

The results obtained on the basis of the first-order VAR are quite different. In this case, the evidence of the predictability of single-period excess returns is much weaker although, in contrast to the results for higher-
order VARs, the first-order VAR does not support the model's implication that there should be useful information in the current and past rent-price ratios with respect to forecasts of current and future growth-adjusted discount rates. The discrepancies between the actual and theoretical rent-price ratios are less severe in this case than they are for the higher-order VARs. In fact, in the case of the constant expected returns version of the model augmented by a first-order VAR, the correlation between the actual and theoretical rent-price ratios is .966 and the nonlinear Wald test cannot easily reject at conventional significance levels the restrictions which imply the equality of these two ratios over time.

In their study of stock market price and dividend behavior over the 1926-1986 sample period, Campbell and Shiller (1988) found qualitatively similar results. That is, tests of the predictability of single-period excess (approximate) returns were rejected at the ten percent level regardless of the VAR lag length considered (1, 3, or 5) and regardless of whether a constant expected return or time-varying expected return (based on T-bill returns) was assumed. The nonlinear Wald test rejected the equality of the actual and theoretical rent-price ratios at or above the .005 level in all these cases. The ratio of the standard deviations of $\delta_t'$ and $\delta_t$ ranged from .29 to .544, which are of the same order of magnitude as we found for the constant expected returns version of the model but substantially lower than the range of values reported in this study for the time-varying expected return case.

Interestingly, they too found the strongest support for the model in the correlation of $\delta_t'$ and $\delta_t$ associated with the first-order VAR: .995 for the constant expected returns version of the model and 1.000 for the time-varying expected returns version of the model. Increasing the order of the VAR reduced these correlations substantially. For the fifth-order VAR the correlations they
report are -.089 (constant expected returns) and -.353 (time-varying expected returns). To explain the discrepancies between the results from using a first-order VAR rather than a higher-order VAR, Campbell and Shiller note that they too found the fit of the VAR, particularly the equation for the growth-adjusted discount rate, to improve considerably as the lag length of the VAR increased beyond one, with less significant improvements in the fit of either equation in the VAR with additional increases in the lag length. Furthermore, they note that in their data set, $\Delta d_{t-1}$ is not persistent or smooth (a conclusion which appears warranted in our study as well based on the results of the first-order VAR in the constant expected returns case) so that "$\delta_t'$ does not put a large weight on it and instead moves closely with $\delta_t$" in this case. Consequently, they suggest that the results associated with the first-order VAR can be explained as artifacts of the information set assumed by the first-order VAR which generates spurious correlation between $\delta_t$ and $\delta_t'$.

We conclude that the empirical results presented in this paper provide little evidence to support the plausibility of the rent-price ratio model, with or without the assumption of time-varying expected returns, as an explanation of Iowa farmland price and rent relationships. The results are broadly consistent with the results reported by Campbell and Shiller (1988) in their study of the model's consistency with annual U.S. stock market price and dividend relationships over the same sample period. The results are also consistent with Falk's (1989,1991) studies of the constant expected returns version of the present value model. In all of these cases, not only are the empirical implications of the theoretical model strongly rejected, but there is evidence that the rejection is attributable to negative correlation between an observed linear combination of actual asset income and actual asset price and the
corresponding linear combination of actual asset income and the fundamental value of the asset. 6

There are at least two areas of concern with regard to the appropriate interpretation of the results presented in this paper. The first is that the validity of the statistical methods used in this paper are based on large sample theory. Monte Carlo evidence presented by Campbell and Shiller (1989) does suggest that the procedures used in this paper have a tendency to reject the restrictions of the rent-price ratio model too often in finite samples. However, it seems unlikely, based on that evidence, that finite sample biases can easily explain the extreme rejections reported in this paper.

Second, the results may be sensitive to the approximation errors embedded in the rent-price ratio model (9) as a consequence of the approximation (3). Campbell and Shiller (1988) offer evidence which suggests that the approximation errors are quite small for their data sets. However, the quality of the approximation for the Iowa farmland market data needs to be considered. The results reported in Table 5 offer some evidence that the approximation errors in this data set are not too severe. We conclude this section by offering two additional types of evidence that support this contention.

In the first part of Table 7, we compare some summary statistics for the exact nominal return series, \( h_t \), the approximate nominal return series, \( \xi_t \), and the approximation error, \( h_t - \xi_t \), where \( \xi_t \) was constructed from (3) setting \( \rho \) equal to .946 and setting \( k \) equal to .21. The sample means and standard deviations of \( h_t \) and \( \xi_t \) are virtually the same and their contemporaneous sample correlation is virtually equal to one. The average approximation error is very close to zero, although that error is positively correlated with the true return. It is possible that period-by-period approximation errors are small but they tend
to accumulate over time into more meaningful errors. The second part of Table 7 presents some evidence in this regard. It summarizes the results obtained from computing an approximate log rent-price ratio, $\delta_t^A$, on the basis of equation (5) subject to the terminal condition that in 1986 the approximation held exactly, and setting $\rho$ and $k$ equal to .946 and .21, respectively. Following Campbell and Shiller (1988), we then compare the behavior of $\delta_t$ and $\delta_t^A$ for only the first half of the sample period in order to reduce the effects of the terminal condition on the behavior of $\delta_t^A$. Again, there is little evidence that approximation errors are likely to explain our results.

6. Conclusion

Falk (1989, 1991) applied a battery of formal statistical tests to evaluate the constant expected returns version of the present value model of farmland price determination. Using annual Iowa farmland price and rent data over the 1921-1986 sample period, he concluded that the implications of this version of the present value model cannot be reconciled with the behavior of Iowa farmland prices. Falk (1991) suggests that the failure of this model is reflected in the predictability of single-period returns and a strong tendency for Iowa farmland prices to rise (fall) in response to changes in rents which signal a fall (rise) in the fundamental value of that land.

These studies leave open the question of whether generalizations of the present value model which allow for time-varying expected returns can be reconciled with farmland price behavior. Campbell and Shiller (1988) developed a dividend-price ratio model to consider this generalization with respect to stock market price determination. This paper has applied the dividend-price ratio model to Iowa farmland data. The major implications of the dividend-price ratio
model for Iowa farmland prices and rents are strongly rejected, with the results being insensitive to whether we allow for time-varying expected returns or not. In either case, the results indicate that single-period excess returns in the Iowa farmland market are predictable and that Iowa farmland price has a strong tendency to move in the wrong direction (relative to its fundamental value) in response to movements in rents. That is, the results are consistent with the results reported by Falk (1991). They are also consistent with, though somewhat more extreme than, the results reported by Campbell and Shiller (1988) in their study of annual stock market price movements.

If the failure of the present value model of farmland prices and the strong negative correlation between observed and fundamental land values are accepted as stylized facts then the object of future research seems clear. Recent developments in the financial economics literature may suggest fruitful directions in which to proceed. At the same time, there appear to be important qualitative similarities between the results obtained in studies of farmland markets and those obtained in related studies of the stock market. These similarities may be useful to economists in obtaining a better understanding of the speculative forces generally at work in asset markets.
FOOTNOTES

1 See, for example, Summers.

2 Phillips-Perron Z-statistics based on fourth-order Newey-West corrections were the basis for the unit root tests. These tests implied that the hypothesis of a unit root in the logged nominal price and logged nominal rent series could not be rejected against the alternative of trend stationarity at the ten percent level. The null hypothesis of a unit root in the growth rate of nominal prices and nominal rents could be rejected against the alternative of covariance stationarity at the one percent level. As noted in the text, the null hypothesis of a unit root in the log rent-price ratio and the null hypothesis of a unit root in the growth-adjusted discount rate could be rejected against the covariance stationarity alternative at the ten and one percent levels, respectively. In analyzing the constant expected returns version of the model, we require that the real rent series is difference stationary. The Phillips-Perron unit root tests indicated that the hypothesis of a unit root in the logged real rent series could not be rejected against the trend stationary alternative. However, a unit root in the growth rate of real rents could be rejected against the covariance stationary alternative at the one percent level.

One anomaly that emerged in the application of these tests is that the null hypothesis of a unit root in the nominal interest rate could not be rejected against the trend stationarity or covariance stationarity alternatives at the ten percent level. If, however, the growth-rate of nominal rents is stationary while nominal interest rates have a unit root then, then the growth-adjusted discount rate should be nonstationary because of a unit root. This anomaly also
emerged in Campbell and Shiller's (1988) unit root tests.

The nature of the linear approximation (3) suggests choosing \( \rho \) equal to \( \exp(g-h) \) and \( k \) equal to \( -\log(\rho) - (1-\rho)\delta \), where \( h \) is the mean of the realized log gross real returns, \( h_t \); \( g \) is the mean of real rent growth rates; and \( \delta \) is the mean log rent-price ratio. Sample means of \( h_t \), \( d_t \) - \( d_{t-1} \), and \( \delta_t \) were used to estimate \( \rho \) and \( k \). Note that for this purpose, nominal land prices and rents need to be deflated by a price index. We chose to deflate the nominal land price at the start of period \( t \) and the nominal rent at the end of period \( t-1 \) by the January of year \( t \) Consumer Price Index (Ibbotson Associates). The implied estimates of \( \rho \) and \( k \) are .946 and .21, respectively. For most of our purposes, only the value of \( \rho \) is of interest. Values of \( \rho \) equal to .9 and .975 were considered and the results were very robust to these variations.

\( \delta_t' \) can be computed from the data and the estimated VAR coefficients by solving the right-hand-side of (9) subject to (11) to obtain \( \delta_t' = e_2'(I-\rho A)^{-1}z_t \).

The Wald statistic was computed as follows. Let \( \text{vec}(c) \) denote the 4p x 1 column vector of the estimated VAR coefficients, i.e.,

\[
\text{vec}(c) = [c_{111} \ldots c_{p11} \ldots c_{122} \ldots c_{p22}]' 
\]

where \( c_{ijk} \) denotes the OLS estimate of \( C_{ijk} \), for \( i = 1, \ldots, p \) and \( j, k = 1, 2 \). Let \( V \) denote the 4p x 4p estimated variance-covariance matrix of \( \text{vec}(c) \), which was evaluated by applying White's Heteroskedasticity-Consistent Covariance Matrix Estimator. Let \( x \) denote \( e_1' - e_2'(I-\rho A)^{-1} \), evaluated at \( c \). That is, \( x \) denotes the 2p x 1 vector of estimated deviations from (14). Finally, let \( dx/d\text{vec}(c) \) denote the 2p x 4p matrix of partial derivatives of the elements of \( x \) with respect to the
elements of vec(c), which were evaluated numerically. Then the Wald statistic, which is distributed as a Chi-square with 2p degrees of freedom under the null hypothesis (14), is W where

\[ W = x' \left( \frac{1}{dx/d\text{vec}(c)} \right) V \left( \frac{dx/d\text{vec}(c)}{} \right)' \frac{1}{x} \]

Falk (1991) found the sample correlation of \( P_t - \theta D_t \) and \( P'_t - \theta D'_t \) over the 1921-1986 sample period to be -.86, where \( P_t \) and \( D_t \) are the real price and rent per acre of Iowa farmland in year \( t \); \( \theta \), which is defined as the reciprocal of the assumed constant real discount rate, was set equal to 17.75 (implying an annual discount rate of 5.6 percent); and \( P'_t \) is the price series implied by the estimated theoretical model. Campbell and Shiller (1987) found a sample correlation of -.46 for this linear combination with \( P_t \) and \( D_t \) denoting current and once-lagged real year \( t \) stock market price and dividends, respectively, and the annual discount rate set to 8.2 percent.
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________________________. "The Dividend-Ratio Model and Small Sample Bias:

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University, July 1989.

______. "Formally Testing the Present Value Model of Farmland Prices."

Featherstone, A.M. and T.G. Baker. "An Examination of Farm Sector Real Asset


Melichar, E. "Capital Gains versus Current Income in the Farming Sector."


Shiller, R.J. "Do Stock Prices Move Too Much to Be Justified by Subsequent


TABLE 1
DESCRIPTION OF DATA SET

Nominal land price: Price per acre of Iowa farmland at the start of year $t$, $$/acre; United States Department of Agriculture’s Farm Real Estate Value, by state, series (1925-1949) spliced with Iowa State University’s Extension Service’s Iowa Land Value series (1950-1985). Each price was pushed forward one period to obtain the $P_t$ series since, at least with respect to the Iowa State University data, the reported price represents an end-of-the-year price. 1926-1986.

Nominal rent: Gross annual cash rent per acre of Iowa farmland during year $t$, $$/acre/year; Iowa Agricultural Statistics Service, published by the USDA’s Economic Research Service. 1926-1986.


TABLE 2  
SUMMARY STATISTICS FOR LAND MARKET DATA  
SAMPLE PERIOD: 1926-1986

<table>
<thead>
<tr>
<th>Series</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_t - P_{t-1}$</td>
<td>0.0294</td>
<td>0.122</td>
</tr>
<tr>
<td>$d_t - d_{t-1}$</td>
<td>0.0402</td>
<td>0.086</td>
</tr>
<tr>
<td>$r_t$</td>
<td>0.0351</td>
<td>0.034</td>
</tr>
<tr>
<td>$\delta_t$</td>
<td>-2.786</td>
<td>0.147</td>
</tr>
</tbody>
</table>

Note: All variables in this table are nominal annual measures. $p_t$ is the log stock price, $d_t$ is the log rent, $r_t$ is the nominal discount rate, and $\delta_t$ is the log rent-price ratio $d_{t-1} - p_t$. 
TABLE 3
TESTING VAR LAG LENGTHS

General Representation of the VAR:

\[ \delta_t = c_{11}(L)\delta_t + c_{12}(L)(r_{t-1-\Delta d_{t-1}}) + u_{1t} \]
\[ r_{t-1-\Delta d_{t-1}} = c_{21}(L)\delta_t + c_{22}(L)(r_{t-1-\Delta d_{t-1}}) + u_{2t} \]

where \( c_{ij}(L) = c_{ij,1}L + \ldots + c_{ij,p}L^p \), \( i,j = 1,2 \).

Sims' Lag Length Test:

I. Constant Expected Real Returns

<table>
<thead>
<tr>
<th>Lag Comparison</th>
<th>Sims Chi-Square Statistic (DF)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p=2 ) vs. ( p=1 )</td>
<td>22.34 (4)</td>
<td>.0002</td>
</tr>
<tr>
<td>( p=3 ) vs. ( p=2 )</td>
<td>9.76 (4)</td>
<td>.045</td>
</tr>
<tr>
<td>( p=4 ) vs. ( p=3 )</td>
<td>5.65 (4)</td>
<td>.227</td>
</tr>
<tr>
<td>( p=5 ) vs. ( p=3 )</td>
<td>9.29 (8)</td>
<td>.320</td>
</tr>
</tbody>
</table>

II. Time-Varying Expected Real Returns

<table>
<thead>
<tr>
<th>Lag Comparison</th>
<th>Sims Chi-Square Statistic (DF)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p=2 ) vs. ( p=1 )</td>
<td>29.06 (4)</td>
<td>.00001</td>
</tr>
<tr>
<td>( p=3 ) vs. ( p=2 )</td>
<td>5.97 (4)</td>
<td>.201</td>
</tr>
<tr>
<td>( p=4 ) vs. ( p=2 )</td>
<td>10.89 (8)</td>
<td>.208</td>
</tr>
</tbody>
</table>

Notes: Each series was demeaned prior to estimation of the VARs. Sims’ Chi-square statistic is \( (T-k)[\ln(\text{det}(V_{r})) - \ln(\text{det}(V_{u}))] \), where \( T \) is the common effective sample size used to estimate the restricted and unrestricted VARs, \( k \) is equal to the number of coefficients estimated per equation in the unrestricted model, \( \ln(\text{det}(V_{r})) \) is the natural log of the determinant of the estimated contemporaneous covariance matrix of the disturbances in the restricted model, and \( \ln(\text{det}(V_{u})) \) is the corresponding value computed from the unrestricted model. Under the null hypothesis, the statistic is asymptotically distributed as a chi-square with \( m \) degrees of freedom, where \( m \) is the total number of restrictions on the system.
TABLE 4
SUMMARY OF THE ESTIMATED VARS

I. Constant Expected Real Returns

A. First-Order VAR

\[ \delta_t = .9383 \delta_{t-1} + .231 \Delta \delta_{t-2}, \ R^2 = .70 \]
\[ (.0823) \quad (.145) \]

\[ \Delta \delta_{t-1} = -.0432 \delta_{t-1} + .115 \Delta \delta_{t-2}, \ R^2 = .016 \]
\[ (.0786) \quad (.138) \]

H0: \ \delta_{t-1} does not Granger-cause \ \delta_t, \ p-value of F-statistic = .117.
H0: \ \delta_t does not Granger-cause \ \Delta \delta_{t-1}, \ p-value of F-statistic = .585.

B. Second-Order VAR

\[ \delta_t = (1.298 - .434L)\delta_{t-1} - (.284 - .0150L)\Delta \delta_{t-2}, \ R^2 = .74 \]
\[ (.145) \quad (.149) \quad (.140) \quad (.142) \]

\[ \Delta \delta_{t-1} = (-.449 + .492L)\delta_{t-1} + (.192 - .198L)\Delta \delta_{t-2}, \ R^2 = .21 \]
\[ (.134) \quad (.137) \quad (.129) \quad (.131) \]

H0: \ \Delta \delta_{t-1} does not Granger-cause \ \delta_t, \ p-value of F-statistic = .135.
H0: \ \delta_t does not Granger-cause \ \Delta \delta_{t-1}, \ p-value of F-statistic = .00274.

C. Fourth-Order VAR

\[ \delta_t = (1.049 + .0433L + .0642L^2 - .468L^3)\delta_{t-1} \]
\[ (.159) \quad (.246) \quad (.254) \quad (.194) \]

\[ - (.0491 - .0537L + .1230L^2 - .0864L^3)\Delta \delta_{t-2}, \ R^2 = .78 \]
\[ (.159) \quad (.152) \quad (.145) \quad (.144) \]

\[ \Delta \delta_{t-1} = (-.398 + .230L + .322L^2 - .144L^3)\delta_{t-1} \]
\[ (.154) \quad (.238) \quad (.245) \quad (.188) \]

\[ + (.101 - .0974L - .267L^2 + .0590L^3)\Delta \delta_{t-2}, \ R^2 = .29 \]
\[ (.149) \quad (.157) \quad (.144) \quad (.127) \]

H0: \ \Delta \delta_{t-1} does not Granger-cause \ \delta_t, \ p-value of F-statistic = .895.
H0: \ \delta_t does not Granger-cause \ \Delta \delta_{t-1}, \ p-value of F-statistic = .0074.
II. Time-Varying Expected Real Returns

A. First-Order VAR

\[
\delta_t = 0.943 \delta_{t-1} + 0.328 (r_{t-2} - \Delta d_{t-2}), \quad R^2 = 0.72
\]

\[
r_{t-1} - \Delta d_{t-1} = 0.00135 \delta_{t-1} + 0.386 (r_{t-2} - \Delta d_{t-2}), \quad R^2 = 0.14
\]

H₀: \( r_{t-1}-\Delta d_{t-1} \) does not Granger-cause \( \delta_t \), p-value of F-statistic = 0.014.

H₀: \( \delta_t \) does not Granger-cause \( r_{t-1}-\Delta d_{t-1} \), p-value of F-statistic = 0.987.

B. Second-Order VAR

\[
\delta_t = (1.225 - 0.350L)\delta_{t-1} + (0.281 + 0.050L)(r_{t-2} - \Delta d_{t-2}), \quad R^2 = 0.75
\]

\[
r_{t-1} - \Delta d_{t-1} = (0.540 - 0.653L)\delta_{t-1} + (0.374 - 0.106L)(r_{t-2} - \Delta d_{t-2}), \quad R^2 = 0.42
\]

H₀: \( r_{t-1}-\Delta d_{t-1} \) does not Granger-cause \( \delta_t \), p-value of F-statistic = 0.065.

H₀: \( \delta_t \) does not Granger-cause \( r_{t-1}-\Delta d_{t-1} \), p-value of F-statistic = 0.00003.

C. Fourth-Order VAR

\[
\delta_t = (1.036 + 0.0622L - 0.0109L^2 - 0.390L^3)\delta_{t-1}
\]

\[
+ (0.0509 - 0.0151L + 0.0549L^2 + 0.0628L^3)(r_{t-2} - \Delta d_{t-2}), \quad R^2 = 0.78
\]

\[
r_{t-1} - \Delta d_{t-1} = (0.506 - 0.465L - 0.204L^2 + 0.0116L^3)\delta_{t-1}
\]

\[
+ (0.295 - 0.0320L - 0.124L^2 + 0.0464L^3)(r_{t-2} - \Delta d_{t-2}), \quad R^2 = 0.44
\]

H₀: \( r_{t-1}-\Delta d_{t-1} \) does not Granger-cause \( \delta_t \), p-value of F-statistic = 0.96.

H₀: \( \delta_t \) does not Granger-cause \( r_{t-1}-\Delta d_{t-1} \), p-value of F-statistic = 0.0004.

Note: \( \delta_t \) is the log of the rent-price ratio, \( r_t \) is the nominal annual discount rate, and \( d_t \) is the log rent (which is deflated by the t+1 January CPI for the constant expected real returns version of the model). L is the lag operator.
TABLE 5

REGRESSIONS OF ACTUAL AND APPROXIMATE RETURNS ON INFORMATION

I. Constant Expected Real Return Model

\[ y_t = \alpha_0 + (\beta_0 + \beta_1 L + \ldots + \beta_{p-1} L^{p-1}) \delta_t + (\theta_0 + \theta_1 L + \ldots + \theta_{p-1} L^{p-1}) \Delta d_{t-1} + u_t \]

\[ H_0: \beta_i = 0 \text{ and } \theta_i = 0, i = 0, 1, \ldots, p-1. \]

II. Time-Varying Expected Real Return Model

\[ y_t = \alpha_0 + (\beta_0 + \beta_1 L + \ldots + \beta_{p-1} L^{p-1}) \delta_t + (\delta_0 + \delta_1 L + \ldots + \delta_{p-1} L^{p-1}) (r_{t-1} - \Delta d_{t-1}) + u_t \]

\[ H_0: \beta_i = 0 \text{ and } \delta_i = 0, i = 0, 1, \ldots, p-1. \]

<table>
<thead>
<tr>
<th>Model</th>
<th>(x_t)</th>
<th>(R^2)</th>
<th>F-statistic</th>
<th>p-value of F-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>(h_t)</td>
<td>1</td>
<td>.06</td>
<td>1.90</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>.37</td>
<td>7.77</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>.51</td>
<td>6.02</td>
</tr>
<tr>
<td></td>
<td>(\varepsilon_t)</td>
<td>1</td>
<td>.06</td>
<td>1.85</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>.37</td>
<td>7.77</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>.51</td>
<td>6.03</td>
</tr>
<tr>
<td>II</td>
<td>(h_t - r_t)</td>
<td>1</td>
<td>.28</td>
<td>10.45</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>.55</td>
<td>15.76</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>.61</td>
<td>9.06</td>
</tr>
<tr>
<td></td>
<td>(\varepsilon_t - r_t)</td>
<td>1</td>
<td>.27</td>
<td>10.34</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>.55</td>
<td>15.69</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>.61</td>
<td>9.04</td>
</tr>
</tbody>
</table>

Note: \(\delta_t\) is the log rent-price ratio, \(d_{t-1} - p_t\), where \(p_t\) is the nominal price of land at the start of period \(t\), \(D_t\) is the nominal rent in period \(t\), and \(p_t\) and \(d_t\) are their natural logs, respectively. \(r_t\) is the nominal discount rate in period \(t\). \(h_t\) is the actual log one-period nominal return, calculated as \(\log(P_{t+1} + D_t) - \log P_t\). \(\varepsilon_t\) is the approximation to \(h_t\), which aside from a constant, is calculated as \(\rho P_{t+1} + (1-\rho)d_t - p_t\), where \(\rho\) was set equal to .946. For the constant expected real return version of the model, \(D_t\) was divided by the January of year \(t+1\) CPI to calculate \(\Delta d_t\).
TABLE 6
TESTING THE RESTRICTIONS ON THE RENT-PRICE RATIO

I. Constant Expected Real Return Model

<table>
<thead>
<tr>
<th>VAR Lag Length</th>
<th>$\sigma(\delta')/\sigma(\delta)$</th>
<th>$\text{correlation}(\delta, \delta')$</th>
<th>$H_0: \delta' = \delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.497</td>
<td>.966</td>
<td>3.04 (0.2185)</td>
</tr>
<tr>
<td>2</td>
<td>.486</td>
<td>-.892</td>
<td>73.77 (0.0000000)</td>
</tr>
<tr>
<td>4</td>
<td>.543</td>
<td>-.872</td>
<td>108.96 (0.0000000)</td>
</tr>
</tbody>
</table>

II. Time-Varying Expected Real Return Model

<table>
<thead>
<tr>
<th>VAR Lag Length</th>
<th>$\sigma(\delta')/\sigma(\delta)$</th>
<th>$\text{correlation}(\delta, \delta')$</th>
<th>$H_0: \delta' = \delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.356</td>
<td>.148</td>
<td>4.62 (0.099)</td>
</tr>
<tr>
<td>2</td>
<td>1.016</td>
<td>-.963</td>
<td>55.54 (0.0000000)</td>
</tr>
<tr>
<td>4</td>
<td>.935</td>
<td>-.957</td>
<td>75.94 (0.0000000)</td>
</tr>
</tbody>
</table>

Note: $\delta$ is the log rent-price ratio and $\delta'$ is the unrestricted forecast of the present value of future growth-adjusted discount rates based upon the theoretical dividend-ratio model and derived from the estimated VARs described in Table 5. The null hypothesis that $\delta_t' = \delta_t$ was tested by applying a nonlinear Wald-test (p-values appear below the test statistic) using numerical derivatives and White's heteroskedasticity-consistent covariance matrix estimator.
| I. Comparison of the exact return, $h_t$, and the approximate return, $\epsilon_t$. |
|-------------------|-------------------|-------------------|
|                   | $h_t$             | $\epsilon_t$      | $\epsilon_t - h_t$ |
| mean              | 0.0899            | 0.0890            | -0.00088           |
| standard deviation| 0.119             | 0.120             | 0.0015             |
| correlation with exact return | 1.000 | 0.99993 | 0.374 |

| II. Comparison of actual log rent-price ratio, $\delta_t$, to the approximate log rent-price ratio, $\delta_t^A$ |
|-------------------|-------------------|-------------------|
|                   | $\delta_t$        | $\delta_t^A$      | $\delta_t^A - \delta_t$ |
| mean              | -2.816            | -2.804            | 0.0117              |
| standard deviation| 0.094             | 0.094             | 0.0018              |
| correlation with $\delta_t$ | 1.000 | 0.9998 | -0.031 |

**Note:** In part II, $\delta_t^A$ was computed recursively for the sample period 1927-1986 according to

$$\delta_{t+1}^A = \frac{1}{\rho}[k + \delta_t^A + \Delta d_t - h_t]$$

using the terminal condition $\delta_{1986}^A = \delta_{1986}$ and setting $k = .21$. The summary statistics in part II were computed only for the first 30 years of the sample.