Valuation of an Option to Build a Power Plant in a Transmission Network under Demand Uncertainty

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Abstract
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Disciplines
Energy Systems | Operational Research | Risk Analysis

Comments
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Abstract— In this paper, we investigate how to value an option to build a power plant when electricity demand fluctuates over time. Towards this aim, we first construct a transmission network, and obtain locational marginal prices for the network buses utilizing optimal power flow. Next, we construct a lattice model under the assumption that the demand fluctuation over time is represented by a geometric Brownian motion. Based on this demand lattice, we derive the economic consequences of costs to a bus with and without a power plant in a risk neutral world. These in turn will lead to the computation of the value of an option to build a power plant. This value of the option will be useful for the electric power planning as the bus with a higher value of this option indicates that the community in this bus is demonstrating a higher degree of potential need for such a power plant.

I. INTRODUCTION

Whether to build a new power plant at a community or transmit from another community to meet its demand is a significant decision for generation and transmission planners as such a decision has a significant consequence on labor and capital requirements as well as the entire transmission network. This paper aims to address this issue by showing how to value an option to build such a power plant for the transmission network when demand is uncertain.

Exploiting the key characteristics of real options that are often used for large-scale, irreversible investment decisions providing managerial flexibility under uncertainties [1] & [2], our model assumes that the change in demand follows a geometric Brownian motion (GBM) process. Using a resulting lattice model, as well as the optimal power flow model resulting in locational marginal prices for the communities connected in a transmission network, we show how the economic value of an option to build a power plant can be determined.

II. BACKGROUND

Optimal Power Flow (OPF) has been widely used in system planning [3] where the goal is to fulfill the customer demand while minimizing the cost [4]. In this paper, we use a small angle (DC) approximation called DCOPF in [3]. We note that electric power operations are uncertain and often volatile. For example, the recent power outage in Texas led to a residential electricity bill that is as high as $17,000 [5] & [6]. The locational marginal price for some communities remained near $9,000 for several days [7]. With this backdrop, in our paper, we model uncertain power demand as GBM [8]. Moreover, as in Kucuksayacigil and Min [9], the continuous GBM process is discretized as a lattice (see e.g., Cox et al. [10]).

The real options analysis for transmission planning has been used in previous studies. Abadie and Chamorro [11] worked with a binodal transmission network where the decision was adding a power line between two nodes. Osthues et al. [12] proposed a real options analysis method to evaluate expansion decisions using a multi-node system.

In what follows, for a small transmission network, we show how DCOPF leads to locational marginal prices. Then, we show how the volatility in demand is modeled in a binomial lattice. Based on these two steps, we then perform real option analysis to value the option to build a power plant.

III. PROPOSED IDEA

We present a 3-node network as above, which can be considered as a simplified version of an example in Chapter 8 of Wood et al., (2014) [13]. We note that there are three generators over Bus 1, 2 and 3. The marginal cost of generator 1, 2 and 3 is ($7.92/MWh when it is built), $7.85/MWh and $7.97/MWh, respectively. The physical transmission limit of transmission line P12 is 210MW. There are consumption centers at all the buses, and the total demand load is 850 MW.

![Figure: 1 - node network (case 1)]
We now consider the first case, where Bus 1 has no generator and the demand at this bus is satisfied by Generators 2 and/or 3. In the second case, we will add a generator at Bus 1 and the total demand will be met by the combination of all three generators. The resulting power flow and total generation cost of both cases will be compared.

To obtain the LMPs at different nodes, the DCOPF is solved. Specifically,

\[ MC_i = \text{marginal cost of node } i \]
\[ G_i = \text{generator at node } i \]
\[ \theta_i = \text{phase angle for node } i \]
\[ P_{ij} = \text{Power-flow in line } i-j \]
\[ P_{\text{load}i} = \text{demand load at node } i \]

**Objective Function (first case):** minimize \( (MC_2 * G_2) + (MC_3 * G_3) \)

Decision variables are \( G_1, G_2, G_3 \) and \( \theta_1, \theta_2, \theta_3, \) and \( P_{12}, P_{13}, P_{23} \) and \( P_{\text{load}1}, P_{\text{load}2}, P_{\text{load}3} \) and \( P_{\text{total} \text{load}} \)

**Generator load balance equality constraint:**
\[ P_{\text{total} \text{load}} - (G_1 + G_2) = 0 \]
\[ P_{\text{total} \text{load}} = P_{\text{load}1} + P_{\text{load}2} + P_{\text{load}3} \]
\[ P_{\text{load}1} = 200 \text{MW}, \quad P_{\text{load}2} = 550 \text{MW}, \quad P_{\text{load}3} = 100 \text{MW} \]

**Nodal power balance constraints:**
\[ [B_x] \theta_i = P_i - P_{\text{load}i} \text{ where } \theta_i \text{ is in radians and } [B_x] \text{ is in per unit.} \]
To keep the values of \( P_i - P_{\text{load}i} \) in MW, we shall need to multiply \([B_x] \theta_i\) or simply the values in \([B_x]\) matrix by 100 to convert the power from per unit to MW with an MVA system base of 100 MVA. Therefore,
\[ 100 * [B_x] \theta_i = P_i - P_{\text{load}i} \]

Where, \([B_x] = \)

\[
\begin{pmatrix}
(1/x_{ij}+1/x_{ji}) & -1/x_{ij} & -1/x_{ji} \\
-1/x_{ij} & (1/x_{ij}+1/x_{ji}) & -1/x_{ji} \\
-1/x_{ji} & -1/x_{ji} & (1/x_{ji}+1/x_{ij})
\end{pmatrix}
\]

\([B_x] \) is the susceptance matrix with \( x_{ij} \) components. \( \theta_i \) is the phase angle for node \( i \).

\[ \theta_i = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} \text{ where } B_{ij} \text{ is the susceptance of the branch } i \text{ to } j \]
given by:
\[ B_{ij} = (-1/x_{ij}) \]
\[ x_{ij} = \text{reactance between line } i \text{ and } j \]

Reactance for line 1-2, 1-3 and 2-3 is 0.1 ohms, 0.125 ohms and 0.2 ohms, respectively.

**Power-flow in each branch:**
\[ P_{ij} = 100 * B_{ij}(\theta_i - \theta_j) = 100 * (\theta_i - \theta_j)/x_{ij} \]

**Transmission limit constraints:**
\[ 100 * (\theta_i - \theta_j)/x_{ij} \leq P_{ij\text{max}} \]
\[ 100 * (\theta_i - \theta_j)/x_{ij} \leq P_{ij\text{max}} \]

We note that there will be additional non-negativity constraints on some variables such as \( 0 \leq G_1, G_2, G_3 \). After determining the locational marginal price for 3 buses from the mathematical model, demand growth can be analyzed using a binomial lattice approach. This approach has been proven successful in modelling the uncertainties.

**Demand lattice:**

As in [4] & [10], the change in some quantity \( S \) is determined by multiplication factors “\( u \)” and “\( d \)”. It goes up or down with risk neutral probabilities \( q \) and \( 1-q \).

![Figure 2 - Demand Lattice](image)

\[ u = e^{\sigma \sqrt{\Delta t}} \]
\[ S_u = u \times S \]
\[ d = e^{-\sigma \sqrt{\Delta t}} \]
\[ S_d = d \times S \]

The values of \( S_u \) and \( S_d \) can be determined from the above-mentioned equations, where \( \sigma \) is the volatility of the process \( S \), and \( \Delta t \) is the time step in the lattice. As in White [4], we use a risk-free rate and assume a continuous compounding return. Risk-neutral probabilities are probabilities of possible future outcomes which have been adjusted for risk. Risk neutral approach assumes that the decision maker is indifferent about the risk [4]. In the above example, if the network is congested due to a transmission limit constraint, one viable option is adding one power generator to the network. For this problem, our first case is proceeding without any additional generator, and the second case is adding a generator at the bus with the highest LMP.
**OPF and LMP:**

We use the Excel solver to calculate the values for our DCOPF model. Solving the model using “simplex LP” function and setting the decision variables as generator dispatch, phase angles and subject to the constraints mentioned above, we obtain the DC optimal power flow. The sensitivity report of our model gives us the LMP values as the shadow prices. We can also verify the values from our excel model using the superposition method given by D. S. Kirschen et al. [14].

In the second case, the 3-bus network is modeled and solved for Figure 3. In this case, a generator at node 1 is added with marginal cost of $7.92/MWh. The resulting optimal power flow and LMP of all the buses will be compared with the first case.

**Demand Lattice:**

The following parameters with hypothetical values are used to construct the binomial lattice.

- Drift ($\mu$) = 15%/year
- Risk free discount rate ($r_f$) = 4.879% /year
- Volatility ($\sigma$) = 30%/year
- Time step ($\Delta t$) = 1 year
- Up-factor ($u$) = $e^{\sigma\sqrt{\Delta t}} = 1.35$
- Down factor ($d$) = $1/u = 0.741$
- Initial demand at bus 1 = 200 MW
- Total construction cost = $100,000

A binomial lattice can be constructed to depict demand evolution.

**Figure 4: Demand evolution lattice for Bus 1 (in MW)**

That is, we have a demand of 200 MW at the beginning of the modelling horizon and after one year, the demand can rise to 270 MW (or drop to 148.2 MW). Assuming continuous compounding, the risk neutral probability is given by,

$$q = e^{r_f} - d/u - d = 0.5074$$

**Economic Consequence Case 1:**

The first option is to proceed with Generator 2 and 3 to fulfill the future demand. Line 2-1 is constrained by a limit of 210 MW. Therefore, the line is congested and locational marginal price at bus 1 will be $8.045/MWh to fulfill the demand of 270 MW.

**Figure 5: Cost Lattice for Option 1 (in million $)**

So, the demand can still be fulfilled. But, due to high locational marginal price ($8.045/MWh), the cost to be paid by the community at Bus 1 will be

$$8,045 \times 8760 \times 270 = $19,028,034$$

That means, the community at bus 1 will pay $19.028 million for the whole year to fulfill the demand of 270 MW. The costs at other time points can be calculated similarly.

**Economic Consequence 2:**

A generator of 10 MW is installed at Bus 1. If the generator is added, the locational marginal price will be $7.92 after one year to satisfy 270 MW demand. Therefore, if demand goes up, the yearly expense by the community at Bus 1 will be $18.732 million, which is significantly lower than the first option due to the lower locational marginal price. Here, costs at other time points are similar to the first case due to the equal LMP.

**Net benefit and option value**

At the current state, when demand is 200 MW, the locational marginal price will be the same for two cases and therefore the
net benefit is zero for this state. It will also be true if the demand goes down (148.2 MW). But, if the demand is up (270 MW) after one year from the starting time, the associated costs for the first and second options are $19,028 million and $18,732 million respectively. Therefore, the net benefit for that time point is the difference between these costs which is $296,000.

We assume that the total construction cost of a power plant at Bus 1 is $100,000. Therefore, the “net benefit lattice after paying the total construction cost” can be attained by simply subtracting the total construction cost of $100,000 from net benefit.

![Figure 6: Net Benefit after paying construction cost (in $)](image)

The expected net benefit in the risk neutral world after one year from the starting period is:

\[ 196000 \times 0.5074 - \$100000 \times 0.4926 = \$50,190.4 \]

At the starting period, discounted expected net benefit in the risk neutral world is:

\[ 50190.4 \times e^{-0.04879} = \$47,800.4 \]

The numerical figure shows the worth of choosing option 2 (adding one generator at Bus 1) over option 1 (not adding any generator at Bus 1). This indicates that the value of option to build a power plant at Bus 1 is $47,800.40.

**IV. CONCLUSION**

In this paper, we have shown how the value of option to build a power plant can be calculated via a real options approach based on the concepts of optimal power flow and locational marginal price. Our approach can be expanded to address other critical option values such as the value of option to add a transmission line. Such a case will lead to interesting research question such as which option is of higher value between adding a generator vs. a line.

**REFERENCES**