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Welfare Maximization, Product Pricing, and Market Allocation in the Gasoline and Additives Market

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Abstract
Some energy Policy analyses focus on the effects of large price changes in international markets. Multi-sector econometric models (Broadman and Hogan) or Computable General Equilibrium models (Uri; Kemfert and Welsch; Breuss and Steininger) help to evaluate the overall consequences for a country's energy sector and macro economy. Mathematical programming models also remain useful, especially when environmental or performance constraints limit production choices in the energy sector. But sector models are typically extensions of firm problems, in that the objective function concerns processor profits or costs, input prices are given, and product demands are taken as inelastic (Vlachou, Basso, and Andrikopoulos; Manne). Under these assumptions, consumer price adjustments are synomous with firm cost adjustments. For better understanding of markets and price relationships in the presence of environmental regulation, inelastic facor supplies and price responsive demands should be taken into account.

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Some energy Policy analyses focus on the effects of large price changes in international markets. Multi-sector econometric models (Broadman and Hogan) or Computable General Equilibrium models (Uri; Kemfert and Welsch; Breuss and Steininger) help to evaluate the overall consequences for a country's energy sector and macro economy. Mathematical programming models also remain useful, especially when environmental or performance constraints limit production choices in the energy sector. But sector models are typically extensions of firm problems, in that the objective function concerns processor profits or costs, input prices are given, and product demands are taken as inelastic (Vlachou, Basso, and Andrikopoulos; Manne). Under these assumptions, consumer price adjustments are synomous with firm cost adjustments. For better understanding of markets and price relationships in the presence of environmental regulation, inelastic facor supplies and price responsive demands should be taken into account.

Mathematical programming models of markets have also been a mainstay in applied economic research. These models exploit the fact that the equilibrium of a perfectly competitive market or sector is implied by maximum welfare allocations (Samuelson). One advantage of spatial equilibrium models is estimation of market entry and exit prices (Takayama and Judge). Similarly, sector models evaluate the competitiveness of value-added enterprises such as processing sectors in a market setting (McCarl and Spreen, Takayama and Judge (1964)).

In some cases where constraints are imposed by a government policy, programming models suggest that markets maximize the welfare that can be obtained with the policy in place.

For instance, Bawden and Takayama/Judge have shown that welfare maximization, constrained by trade policy, is consistent with market equilibrium; many of the situations encountered in international trade have been considered, including the fixed import duty, the variable levy, the fixed export subsidy, and the fixed import quota. Cox and Chavez show the equivalence of welfare maximization and market outcomes, when welfare is constrained by a government sanctioned system of price wedges and the net extractions of taxes (contributions of subsidies) are subtracted from (added to) sector welfare. Research on the relevance of programming models for situations where the market must perform in harmony with a government policy, however, is limited.

This paper demonstrates the usefulness of programming models for markets where quality and environmental restrictions impinge on market outcomes. Quality standards are an increasingly common form of market intervention, as a means of ensuring product performance, food safety, and environmental compatibility. The case of fuels and additive markets, where agricultural products are becoming important, is emphasized in this paper. First, a model of the consumer demand for gasoline, the production and blending of the intermediate products (additives and refinery gasoline), and the demand/supply for inputs to gasoline production (petroleum, natural gas and byproducts, and biomass) is discussed. Second, a welfare function and a quality restriction on the octane of gasoline is specified for the fuel sector. The first order conditions for this problem are shown consistent with a competitive market and the effects of the quality constraint on market pricing of gasoline is discussed. Third, a three good numerical example is presented to illustrate the tractability of the programming problem, and to indicate the effects of quality restriction on market pricing.

Factor-Product Relationships in the Fuel and Additives Market

The main material and product flows in the gasoline complex are shown in chart 1. Starting with the factor inputs on the LHS, crude petroleum (Q_o) is the main input into the refinery. Several types of refinery gasoline (Q_r) are produced and then blended into automobile fuel (Q_s). Each type of refinery gasoline has unique qualities; some perform well (e.g., have high octane) in a gasoline engine but burn dirty from an environmental viewpoint; some have moderate performance characteristics and burn clean; some have marginal performance characteristics and still burn dirty. Gasoline additives (Q_p) are produced because they have more desirable performance and/or environmental properties. The additives are blended into motor gasoline, sometimes at slightly higher cost than other gasoline components, to improve the characteristics of refinery gasoline. Several input chemicals (Q_I) are used in the production of additives. Many of the input chemicals are byproducts of natural gas production. Others can be produced directly from natural gas. The supply of some input chemicals is also supplemented by the byproducts of petroleum refining. Biomass (including corn) is also an input for one gasoline additive. In chart 1, the natural gas and biomass blocks are shown by dotted lines because these factors are not explicitly included in the model.

Supply and Demand

Three sets of supply and Demand functions are needed to specify a maximization problem and market model: consumer demand, processing supply (marginal cost), and factor supply.

Consumers require different gasoline grades according to the performance characteristics of their automobile. The price-dependent demand function for grade i is

$$P_{si} = \alpha_{si} - \beta_{si} Q_{si} .$$

No substitution between grades is assumed because technology defines an appropriate quality.

Fixed proportions production processes are adequate for this problem. Most additive processes combine two or more chemicals to make a third chemical, so they might be referred to as ‘constructive’ processes. Hence the supply (marginal cost) function for additives processing is stated in terms of the production of the additive output. The processing supply curve for additive i is

$$P^{*}_{pi} = \alpha_{pi} + \beta_{pi} Q_{pi}$$

The marginal cost function includes wages and utilities but does not include the cost of material inputs. The cost of the input chemicals, expressed on a per unit *output* basis, must be added to the processing to obtain the marginal cost of additive production. In the case of constant marginal cost, the slope of the above supply function is zero ($\beta_{pi} = 0$).

A refinery breaks a petroleum molecule into many smaller molecules, making several types of refinery gasoline. So the refinery might be referred to as a destructive process. Here, the supply (marginal cost) for petroleum processing is stated in terms of the crude petroleum *input* as follows

$$P^{*}_{op} = \alpha_{o} + \beta_{o} Q_{o}$$

Again, this marginal cost function does includes wages and utilities but excludes costs for material inputs. The refinery is actually a collection of fixed proportion production process (Gary and Handwerk). Most of the choices concerning the product mix coming from a barrel of oil are set in the long-run period when the configuration of fixed proportion production processes is chosen. The remaining choices in short-run allocation decisions, the allocation of intermediate gas-oils for gasoline or diesel production, the sale or internal use of residual fuel oil, and the proportion of kerosine in the gasoline mix do not vary widely from year to year. Hence, the fixed proportions assumption is a good first approximation.

Most of the factor supply curves for additive inputs are likely upward sloping because they are the byproducts of natural gas production. Further, domestic production is supplemented by imports. The price-dependent supply for additive input i is

$$P_{\ell i} = \alpha_{\ell i} + \beta_{\ell i} Q_{\ell i}$$

For the moment, assume that the factor supply curves include one biomass based input for the production of one additive. A processing complex for ethanol and a supply curve for crude petroleum will be important for large simulation models of this sector. For now, they just add complexity without changing the basic relationships developed here. Also, the price of crude petroleum (P_o) is taken as exogenous.

Maximizing Welfare

Sector welfare is consumer surplus less the operating and material costs associated with processing. The objective function states consumer welfare as the area under the product demand curves. Processing costs are given by the area under the appropriate processing supply function–processing costs for several additives processes and the refinery sector are given below. Factor costs for additive inputs are also given by the area under the appropriate supply function. Finally, expenditures defined by price times quantity are used for crude petroleum, since the petroleum price is exogenous to the refinery sector.

The lagrangian for the maximization problem also includes several constraints on sector welfare. Most of these constraints are (judicially stated) supply-utilization identities for the markets in the gasoline and additives sector. To illustrate performance and environmental constraints on the market, an octane constraint on gasoline grades is also included. The lagrangian for a three-dimensional example is given in equation (1): there are three gasoline products (regular, midgrade, premium), three additive inputs (isobutane, propylene, and corn), three additive processes (MTBE, alkylates and ethanol), and three types of refinery gasoline (catalytic cracker, reformer and coker). Further generalization is possible, but is more difficult to see the development of properties for this maximization problem.

The groups of supply-utilization identities are identified by the greek letter for the corresponding lagrange multiplier. All of these constraints follow the inequality convention of programming models of markets, that supply equals or exceeds demand. The constraints with λ state that the supply of a factor for additive production plus the factor supply derived as a byproduct of petroleum production equals or exceeds the demand for the factor. Further, the production of additive (Q_{pi}) i times the input requirement of factor j in the production of additive i (r_{ij}) defines the demand for a particular factor arising from a given additive.

The other supply constraints concern blending. The variable Z_{ij} indicates the amount of gasoline component i used in product j. There are six gasoline components in the example. Components 1, 2, and 3 are gasoline additives. Components 4, 5, and 6 are types of refinery gasoline. The equations with ϕ_i state that the supply of gasoline components in a particular grade of fuel equals or exceeds the demand for that grade of fuel. The equations with μ_i state that the supply of additive i equals or exceeds the demand for that additive in all grades of fuel. The equations with θ_i state that the supply of each type of refinery gasoline equals or exceeds the demand for that type of refinery gasoline across all fuel grades. Notice that the subscript for θ corresponds to the origin (first) index for the variable Z. Blending is specified as a costless activity here but, as the reader can verify, the main results are unchanged if a constant blending cost term for is added for each Z_{ij}.

Octane equations are included to illustrate the effects of performance or environmental constraints. These equations, indicated by the lagrange multiplier ψ_i, are also a supply-demand identity. They state that the supply of octane equals or exceeds the demand for octane. O_i indicates the octane content of a particular gasoline component and K_j indicates the octane performance standard. An conventional form for this quality constraint in firm lp models, which states that recipe shares of gasoline components equals or exceeds the quality standard, can be obtained by dividing both sides of the equation by Q_{sj}.

$$
\begin{aligned}
\mathcal{L} = & \sum_{i=1}^{3} (\alpha_{si} Q_{si} - \frac{\beta_{si}}{2} Q_{si}^2) - \sum_{i=1}^{3} (\alpha_{pi} Q_{pi} + \frac{\beta_{pi}}{2} Q_{pi}^2) - \sum_{i=1}^{3} (\alpha_{\ell i} Q_{\ell i} + \frac{\beta_{\ell i}}{2} Q_{\ell i}^2) - (\alpha_o Q_o + \frac{\beta_o}{2} Q_o^2) - P_o Q_o \\
& + \lambda_1 [Q_{\ell 1} - (r_{11} Q_{p1} + r_{12} Q_{p2} + r_{13} Q_{p3}) + Q_0 x_1] \\
& + \lambda_2 [Q_{\ell 2} - (r_{21} Q_{p1} + r_{22} Q_{p2} + r_{23} Q_{p3}) + Q_0 x_1] \\
& + \lambda_3 [Q_{\ell 3} - (r_{31} Q_{p1} + r_{32} Q_{p2} + r_{33} Q_{p3}) + Q_0 x_1] \\
& + \mu_1 [Q_{p1} - (Z_{11} + Z_{12} + Z_{13})] \\
& + \mu_2 [Q_{p2} - (Z_{21} + Z_{22} + Z_{23})] \\
& + \mu_3 [Q_{p3} - (Z_{31} + Z_{32} + Z_{33})] \\
& + \theta_4 [Q_o y_1 - (Z_{41} + Z_{42} + Z_{43})] \\
& + \theta_5 [Q_o y_2 - (Z_{51} + Z_{52} + Z_{53})] \\
& + \theta_6 [Q_o y_3 - (Z_{61} + Z_{62} + Z_{63})] \\
& + \phi_1 [(Z_{11} + Z_{21} + Z_{31} + Z_{41} + Z_{51} + Z_{61}) - Q_{s1}] \\
& + \phi_2 [(Z_{12} + Z_{22} + Z_{32} + Z_{42} + Z_{52} + Z_{62}) - Q_{s2}] \\
& + \phi_3 [(Z_{13} + Z_{23} + Z_{33} + Z_{43} + Z_{53} + Z_{63}) - Q_{s3}] \\
& + \psi_1 [(O_1 Z_{11} + O_2 Z_{21} + O_3 Z_{31} + O_4 Z_{41} + O_5 Z_{51} + O_6 Z_{61}) - K_1 Q_{s1}] \\
& + \psi_2 [(O_1 Z_{12} + O_2 Z_{22} + O_3 Z_{32} + O_4 Z_{42} + O_5 Z_{52} + O_6 Z_{62}) - K_2 Q_{s2}]
\end{aligned}
$$

$$+ \quad \psi_3 [(O_1Z_{13} + O_2Z_{23} + O_3Z_{33} + O_4Z_{43} + O_5Z_{53} + O_6Z_{63}) - K_3Q_{s3}]$$

First Order Conditions

The first order conditions are derivatives with respect to quantity variables and lagrange multipliers that are determined in the market and processing system. The first order conditions for the three dimensional example are given below. Equations (a) through (e) are obtained by differentiating quantity variables. Equations (f) through (j) are supply-demand identities obtained by differentiating with respect to a lagrange multiplier for a constraint. The identities are stated as inequalities that allow supply to be greater than or equal to demand.

A. $$\frac{\partial \mathcal{L}}{\partial Q_{si}} = (\alpha_{si} - \beta_{si}Q_{si}) - \phi_i - \psi_i K_i \geq 0; \; i = 1,2,3$$

B. $$\frac{\partial \mathcal{L}}{\partial Q_{\pi}} = -(\alpha_{pi} - \beta_{pi}Q_{pi}) - \mu_i - (\lambda_1 r_{1i} + \lambda_2 r_{2i} + \lambda_3 r_{3i}) \geq 0$$

C. $$\frac{\partial \mathcal{L}}{\partial Q_{\ell i}} = -(\alpha_{\ell i} - \beta_{\ell i}Q_{\ell i}) + \lambda_1 \geq 0$$

D. $$\frac{\partial \mathcal{L}}{\partial Q_{o}} = -(\alpha_o - \beta_o Q_o) + (\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3) + (\theta_4 y_1 + \theta_5 y_2 + \theta_6 y_3) - P_o \geq 0$$

E. $$\frac{\partial \mathcal{L}}{\partial Z_{ij}} = -\mu_i + \phi_j + \psi_j O_i \geq 0 \qquad i = 1,2,3$$

$$= -\theta_i + \phi_j + \psi_j O_i \geq 0 \qquad i = 4,5,6$$

F. $$\frac{\partial \mathcal{L}}{\partial \lambda_i} = Q_{\ell i} + Q_o x_i - (r_{i1}Q_{p1} + r_{i2}Q_{p2} + r_{i3}Q_{p3}) \geq 0 \qquad i = 1,2,3$$

G. $$\frac{\partial \mathcal{L}}{\partial \mu_i} = Q_{pi} - (Z_{i1} + Z_{i2} + Z_{i3}) \geq 0 \quad ; \; i = 1,2,3$$

H. $$\frac{\partial \mathcal{L}}{\partial \theta_i} = Q_o y_o - (Z_{i1} + Z_{i2} + Z_{i3}) \geq 0 \quad ; \; i = 4,5,6$$

I. $$\frac{\partial \mathcal{L}}{\partial \phi_i} = (Z_{1i} + Z_{2i} + Z_{3i} + Z_{4i} + Z_{5i} + Z_{6i}) - Q_{si} \geq 0 \quad ; \; i = 1,2,3$$

J. $$\frac{\partial \mathcal{L}}{\partial \psi_i} = O_1Z_{1i} + O_2Z_{2i} + O_3Z_{3i} + O_4Z_{4i} + O_5Z_{5i} + O_6Z_{6i} - K_iQ_{si} \geq 0 \quad ; \; i = 1,2,3$$

Market Equilibrium

In the present case it turns out that market equilibrium results from welfare maximization with a quality constraint. Some dimensions of this result are apparent after straightforward inspection of first order conditions. Other aspects are more involved.

Direct inspection of equation c suggests that the marginal cost of inputs to additive production (the price dependent supply equation) equal to the marginal benefit (λ_i) from using another unit of the resource. Hence, the marginal value of additive production (μ_i) equals the sum of marginal processing costs from the processing supply function plus factor costs (sum of input requirements times input prices) from equations b.

Similarly, equation d states that the marginal cost of petroleum processing equals the net marginal revenue. The marginal cost comes directly from the petroleum processing equation. The revenues from byproduct processing are the sum of byproduct yields (x_i) times the price of the corresponding factor (λ_i) since the inputs of additive production are the byproducts of the refinery. Finally the sum of refinery gasoline yields (y_i) times the lagrange multiplier (θ_i) represent revenues from processing a unit of petroleum because the multiplier θ_i is the marginal increase in welfare (processor profit) from another unit of refinery gas type i.

For product pricing, consider a profit maximizing firm that buys refinery gas and additives as inputs, and then blends them. This marketing/blending firm maximizes profits subject to several of the same constraints present in the maximum multane lagrangian. Specifically, supply conditions for additives (μ), refinery gas (θ), and consumer gasoline blending (ϕ), and the quality constraint (ψ) must all be satisfied. The constrained profit function is the revenues from sale of consumer grade gasoline less input expenditures for additives and refinery gasoline:

$$
\begin{aligned}
\pi = \; & (P_{s1}Q_{s1} + P_{s2}Q_{s2} + P_{s3}Q_{s3}) \\
& - (P_{p1}Q_{p1} + P_{p2}Q_{p2} + P_{p3}Q_{p3}) \\
& - (P_{r1}Q_{r1} + P_{r2}Q_{r2} + P_{r3}Q_{r3}) \\
& + \mu_1 [Q_{p1} - (Z_{11} + Z_{12} + Z_{13})] \\
& + \mu_2 [Q_{p2} - (Z_{21} + Z_{22} + Z_{23})] \\
& + \mu_3 [Q_{p3} - (Z_{31} + Z_{32} + Z_{33})] \\
& + \theta_4 [Q_{r1} - (Z_{41} + Z_{42} + Z_{43})] \\
& + \theta_5 [Q_{r2} - (Z_{51} + Z_{52} + Z_{53})] \\
& + \theta_6 [Q_{r3} - (Z_{61} + Z_{62} + Z_{63})] \\
& + \phi_1 [(Z_{11} + Z_{21} + Z_{31} + Z_{41} + Z_{51} + Z_{61}) - Q_{s1}] \\
& + \phi_2 [(Z_{12} + Z_{22} + Z_{32} + Z_{42} + Z_{52} + Z_{62}) - Q_{s2}] \\
& + \phi_3 [(Z_{13} + Z_{23} + Z_{33} + Z_{43} + Z_{53} + Z_{63}) - Q_{s3}] \\
& + \psi_1 [(O_1Z_{11} + O_2Z_{21} + O_3Z_{31} + O_4Z_{41} + O_5Z_{51} + O_6Z_{61}) - K_1Q_{s1}] \\
& + \psi_2 [(O_1Z_{12} + O_2Z_{22} + O_3Z_{32} + O_4Z_{42} + O_5Z_{52} + O_6Z_{62}) - K_2Q_{s2}] \\
& + \psi_3 [(O_1Z_{13} + O_2Z_{23} + O_3Z_{33} + O_4Z_{43} + O_5Z_{53} + O_6Z_{63}) - K_3Q_{s3}]
\end{aligned}
$$

The marketer's choice variables are: Q_{s1}, Q_{s2}, Q_{s3}, Q_{p1}, Q_{p2}, Q_{p3}, Q_{r1}, Q_{r2}, Q_{r3}, and Z_{ij}.

The first order conditions for the profit maximizing marketer/blender are:

A′. $$\frac{\partial \pi}{\partial Q_{si}} = P_{si} - \phi_i - \psi_i K_i \geq 0 \; ; \quad i = 1,2,3$$

B′. $$\frac{\partial \pi}{\partial Q_{\pi}} = P_{pi} + \mu_i \geq 0 \; ; \quad i = 1,2,3$$

C′. $$\frac{\partial \pi}{\partial Q_{ri}} = P_{ri} + \theta_{i+3} \geq 0 \; ; \quad i = 4,5,6$$

E′. $$\frac{\partial \pi}{\partial Z_{ij}} = -\mu_i + \phi_j + \psi_j O_i \geq 0 \quad ; \; i = 1,2,3$$

$$-\theta_i + \phi_j + \psi_j O_i \geq 0 \quad ; \; i = 4,5,6$$

These conditions are the same as the welfare maximizing conditions. In fact, equation (A′) is identical to equation (A), except that (A′) contains one explicit price variable, while (A) contains the quantity dependent price function. Both versions state that the marginal benefit (price) of consumer gasoline equals or exceeds the marginal cost of producing another unit of consumer's gasoline. In turn, the marginal cost consists of two components: the marginal value (ϕ_i) of another unit of blended gasoline supply and a quality adjustment that reflects the value of another unit of octane supplied (ψ_i * K_i). Next, equations (E) and (E′) state that the marginal value of another unit of consumer gasoline equals the marginal cost of the corresponding additive or refinery gasoline less a correction for the value of the octane provided by additive or refinery gas i (ψ_i * O_i). Equations (B′) and (C′) merely assign a market price variable to a lagrange multiplier.

In the case where the quality constraint is not binding ($\psi_i = 0$), the production and consumer price revert to a more familiar form. Specifically, equations (A) and (A′) state that the marginal benefit or price of another unit of consumer gasoline equals the marginal cost (ϕ_i). Meanwhile, equations (E) and (E′) state that the marginal cost of additive or refinery gasoline i equals the marginal cost of blended consumer gasoline of grade i. That is, the consumer price of all gasoline grades are equal, and all additives and all types of refinery gasoline have the same price. Hence, the welfare maximization problem gives a market equilibrium, regardless of whether a quality restriction is present.

Product Price and Quality Relationships

To investigate the price and quality relationships implied by market equilibrium, look at the marketer/blender margin between the sales price of blended gasoline and the purchase price for additives or refinery gasoline. Begin with the equation (A) for grade j of blended consumer gasoline when the octane constraint is binding:

$$P_{sj} = \phi_j + \psi_j K_j$$

The equation (E) for additive type i is:

$$\phi_j = P_{pi} - \psi_j O_i$$

Substituting yields and rearranging yields the marketing price relationship between consumer prices for gasoline and the prices paid for additives:

$$P_{sj} + \psi_j (O_i - K_j) = P_{pi}$$

The second term on the left hand side of the above equation defines a price premium (discount) that is paid for an additive i that has an octane above (below) the standard for grade j (K_j). For instance, $O_i = 113$ for ethanol and $K_j = 87$ for regular gasoline. If ψ_j = \$.01/octane.gallon, then the ethanol price (P_{pj}) will exceed the price of regular gasoline (P_{sj}) by \$.52/gallon. A similar price relationship for the types of refinery gasoline can also be developed. The price for a low octane refinery gasoline, like coker gas, would have a price below blended consumer gasoline. Usually, grade standards are associated with product performance or environmental attributes. Then the multiplier ψ_j represents a cost associated with producing quality. But standards are sometimes arbitrary. In this context the shadow value times octane differential represents the subsidy equivalent of the quality restriction for inputs having above average amounts of the restricted attribute. Similarly, the discount for below average inputs represents a tax associated with the quality restriction.

An Example

An example helps to demonstrate tractability of the programming problem and to illustrate operation of markets with a quality constraint. Modify a trade example from p. 143 of Takayama and Judge (1971). Three importing country demand equations from their example are used for consumer gasoline demand in the new example. Similarly, three exporting country supply equations from their example are used as equations of factor supply for the additives industry. Also, processing supply functions for additives were added. The reference point values of the processing functions corresponds to the magnitude of transport costs in the trade example. Then, a petroleum price and processing function that were lower than the other factor supplies was included, and the intercepts of factor supply equations were adjusted so that all factors and processes were used in the baseline solution. The assumed parameter values for supply and demand functions are given in appendix table a.

The processing technology and quality assumptions correspond roughly to the actual refinery and additives processes. The yields of refinery gasoline from petroleum roughly correspond to yields of coker gasoline, catalytic cracker gasoline, and reformer gasoline. The octane assumptions, low, medium, and high correspond to the same processes. The refinery yields of inputs for additives processing roughly correspond to the yields of iso-butane, butylene, and propylene from oil. There are three additives processes. The first process uses equal amounts of factor 1 and factor 2. The second additive process uses equal amounts of all three factors. The third Addie process uses only input three. Roughly, the first two processes correspond to the production of alkylates or polymer gasoline. The third process corresponds to iso-octane or perhaps ethanol. The assumed octane values also correspond to alkylates, polymers, and ethanol, respectively. The assumed parameter values for process yields, octane content, and octane standards are given in appendix b.

The results of two simulations are shown in table 1. The right column shows the results of the market simulation without the octane constraint. The left column shows the market simulation when the quality constraint is imposed. The content of the simulations is interesting because it depicts the situation in the additives and refinery gasoline competition; the additives have more scarce and higher quality attributes than the refinery qualities. Further, the average quality of the refinery is below the performance standard set in the constraint.

Now, look at how the quality standard affects the market. First, note that the price and production of two additives increases. Also note that the total additive output increases (table 2). Hence the price and quantity of all inputs to additive production also increase. In contrast, the consumption of petroleum decreases and the production of all types of refinery gas also falls. However, the decline in output of refinery gasoline is smaller than the increase in output of gasoline additives(table 2). Hence, the overall supply of gasoline to consumers increases and the price for all grades of consumer gasoline falls.

The Processing and Marketing Costs and returns are shown in table 3. Three broad categories of activities are organized. Notice and the expenditures on raw materials and processing costs match intermediate product sales within one-half of one percent. Similarly, the intermediate product sales match the revenues from sales of final consumer products. Hence, the estimates show the processing and marketing system doing business at cost, which one would expect in a competitive system.

Extensions

Constrained welfare maximization is still consistent with market equilibrium under a more general set of policy assumptions that apply to the fuel and additives markets. Other types of quality constraints and fuel sector fiscal policies are particularly relevant. Additional quality constraints, such as vapor, oxygen and benzene content give constrained welfare maximization conditions that are similar to equations (A)−(J) that have been discussed, except that a sum of shadow values of quality constraints, instead of one shadow value, enter the first order conditions.

Nonlinear constraints also give similar equilibrium conditions, except that the partial derivative of a quality parameter with respect to a fuel type replaces the attribute concentration of the fuel type in equilibrium conditions.

Two important fiscal policies in the gasoline fuel sector are the federal excise tax on gasoline and the rebate for using ethanol blends. Welfare maximization again turns out to be consistent with market equilibrium, when the net revenue that the public sector extracts from the gasoline and additives sector is subtracted from sector welfare. First, consider the excise tax. Suppose the tax on consumer gasoline of grade i is T_{si}. Then the government's revenue for grade i gasoline is $T_{si} * Q_{si}$.

Second, retailers receive a rebate of S $ for each gallon of grade i gasoline they sell that is a 10% ethanol blend. Further, a prorated subsidy is given for gasoline that contains less than a 10% blend. Suppose Z_{3i} is the quantity of ethanol blended into gasoline of grade i. Then the subsidy per gallon of gasoline is

$$(Z_{3i}/Q_{si})\,(1/.1)\,S$$

Notice that the full subsidy is given when Z_{3i}/Q_{si} equals 0.1 and is prorated proportionately otherwise. The government expenditure, or more precisely the excise tax loss, associated with the blending credit is

$$\frac{Z_{31}}{0.1\,Q_{31}} S\,Q_{si} = 10\,S\,Z_{31}$$

which suggests that the blending credit is equivalent to a subsidy on additive 3 that is ten times the rebate level in the gasoline market. Hence, the government's net revenue extraction from the fuel sector is the excise tax less the ethanol blending rebate, added across gasoline grades.

$$E = \sum_{i=1}^{3} T_{si} Q_{si} - 10\,S \sum_{i=1}^{3} Z_{3i}$$

When the revenue extraction is subtracted from the sector welfare function in the lagrangian, the revised first order conditions give standard results for taxes and subsidies on market prices. For instance, the term T_{si} is subtracted from the LHS of the first order condition A. This gives the standard result for an excise tax in the marketplace, namely, that the producer price plus the excise tax (and quality adjustment) equals the consumer price. Similarly, the LHS of equation E for additive 3 now add the term 10 S. Combining equation A and additive 3's equation E, gives the revised marketing margin from additive 3 to gasoline grade i

$$P_{si} = \mu_3 - (10S - T_{si}) - \psi_i(o_3 - K_i)$$

Since additive 3 has the blending credit, the producer price of additive three will likely be above the consumer gasoline price by (ten times) the amount that the blending credit exceeds the excise tax collections. The price adjustment for additive qualities remains as before.

Summary and Conclusions

The markets for gasoline fuel factors, processing and consumption are amenable to mathematical programming models. It was shown that markets provide the best possible outcome, in the sense of maximizing sector welfare, in the presence of the performance and environmental constraints the characterize the gasoline fuel industry. For fiscal measures such as excise taxes and ethanol blending credits, the sector welfare function must be reduced to account for the government extraction of revenue from the sector. Then the Net welfare maximizing conditions are still the market equilibrium conditions. This means that programming models can anticipate the competitiveness of processes in the marketplace and evaluate policy changes with reasonable data requirements. The constrained welfare maximization equivalence to market outcomes in the presence of government policies is very robust. In fact, it seems to suggest a new Second Best Principle for markets that must function next to an intervening government: the market will provide the highest welfare for the sector, given the constraints imposed by government policies.

The example problem is tractable. Solutions were routinely obtained without convergence problems and the results had magnitudes that corresponded roughly to Takayama and Judge trade example.

The results also shed some light on how quality regulations function in the market. Quality regulations provide a price incentive to expand output of additives and refinery gasolines with the desired attributes. They also reduce the price and output of gasoline components with less desirable attributes. As the example suggests, then it is possible that a consumer price decrease can accompany a quality restriction when the elasticity of supply for additives is large enough to offset the declining output of refinery gasoline.

The conventional view holds that a quality and environmental standard increases a firm's costs at fixed input price, and therefore raises consumers' prices by a corresponding amount. The results of this paper show that market-level effects, such as adjusting factor prices, will offset or maybe even reverse the conclusions of the firm-level analysis.

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Table 1. The Effect of an Octane Standard on a Hypothetical Gasoline/Additive Sector

Variable Description (Example Classification)		**With Constraint**	**No Constraint**
	NSP	669.544	672.101
Product (Gasoline) Consumption (Q_s)			
(Regular)	S1	48.160	47.081
(Mid)	S2	23.892	23.540
(Premium)	S3	37.928	37.664
Processed Product (Additive) Production (Q_P)			
(MTBE)	P1	28.463	21.291
(Alkylate)	P2	10.318	19.428
(Ethanol)	P3	20.807	15.812
Factor (Nat. Gas/Chemical) Supply (Q_ℓ)			
(Butane)	L1	13.437	12.744
(Butylene)	L2	15.537	14.900
(Nat. Gas)	L3	20.955	18.967
Factor (Petroleum) Supply (Q_O)		172.518	167.975
Processed Product (Refinery) Production (Q_r)			
(Cat. Crack)	01	25.169	25.878
(Coker)	02	8.390	8.644
(Reformer)	03	16.780	17.252
Product (Gasoline) Consumer Price (P_s)			
	S1	15.184	15.292
	S2	15.222	15.292
	S3	15.229	15.292
Product (Gasoline) Supply Price (Marginal Cost) (ϕ_i)			
	S1	13.476	15.292
	S2	13.476	15.292
	S3	13.476	15.292
F.O.B. (Additive) Supply Price (μ_i or P_p)			
	P1	15.522	15.292
	P2	15.165	15.292
	P3	15.616	15.292
F.O.B. (Refinery Gas) Supply Price (θ_i or P_r)			
	01	14.790	15.292
	02	14.658	15.292
	03	15.165	15.292
Factor (Chem/net. Gas) Supply Price (λ_i)			
	L1	16.344	16.274
	L2	11.277	11.245
	L3	14.096	13.897
Processing, Additive, Price (P_p^*)			
	P1	1.712	1.532
	P2	1.258	1.486
	P3	1.520	1.396
Processing, Petroleum, Price			
	P_{op}^*	4.299	4.413
Value of Octane Constraint (Ψ_i)			
	S1	0.012	0
	S2	0.012	0
	S3	0.012	0

Table 2. Origin and Distribution of Refinery Gasoline and Additives to Gasoline by Grade

with quality constraint

from	to s1	s2	s3	total		
p1	8.247	8.799	11.417	28.463		
p2	0	10.318	0	10.318		
p3	8.041	0	12.766	20.807	additives	59.588
o1	15.075	4.775	5.345	25.195		
o2	0	0	8.399	8.399		
o3	16.797	0	0	16.797	refinery	50.391
total	48.16	23.892	37.927	109.979		

without quality constraint

from	to s1	s2	s3	total		
p1	15.567	0	5.724	21.291		
p2	14.635	0	4.792	19.427		
p3	12.827	0	2.984	15.811	additives	56.529
o1	0	10.988	14.89	25.878		
o2	4.051	4.575	0	8.626		
o3	0	7.981	9.274	17.255	refinery	51.759
total	47.08	23.544	37.664	108.288		

Table 3. Processing and Marketing Costs and Revenues

SELL FINAL CONSUMER PRODUCTS

P_s	Q_s	REVENUE
15.184	48.16	731.2614
15.222	23.892	363.684
15.229	37.92	577.4837
	SUM	1672.429

SELL INTERMEDIATE PRODUCTS (ADDITIVES AND REFINERY GAS)

P_p	Q_p	REVENUE
15.222	28.463	433.2638
15.165	10.318	156.4725
15.616	20.807	324.9221

P_r	Q_r	
14.79	24.169	357.4595
14.658	8.39	122.9806
15.165	16.78	254.4687
	SUM	1649.567

PURCHASE RAW MATERIALS AND PAY FOR PROCESSING COSTS

1	167.975	167.975

P^*_{op}	Q_o	REVENUE
4.299	167.975	722.1245

P^*_p	Q_p	
1.172	28.463	33.35864
1.258	10.318	12.98004
1.52	20.807	31.62664
	SUM	77.96532

P_ℓ	P_ℓ		
16.344	13.437	219.6143	
11.277	15.537	175.2107	
14.096	20.955	295.3817	
	SUM	690.2068	1658.272

Appendix Table A. Supply and Demand Parameters

Product (Gasoline) Demand:

$\alpha_{s1} = 20.0$ $\beta_{s1} = 0.1$
$\alpha_{s2} = 20.0$ $\beta_{s2} = 0.2$
$\alpha_{s3} = 20.0$ $\beta_{s3} = 0.125$

Processed (Additives) Services Supply:

$\alpha_{p1} = 1.0$ $\beta_{p1} = 0.025$
$\alpha_{p2} = 1.0$ $\beta_{p2} = 0.025$
$\alpha_{p3} = 1.0$ $\beta_{p3} = 0.025$

Processed (Oil) Services Supply:

$\alpha_{o} = 0.1$ $\beta_{o} = 0.025$ $P_{o} = 1.0$

Factor (Chemical) Supply:

$\alpha_{\ell 1} = 15.0$ $\beta_{\ell 1} = 0.1$
$\alpha_{\ell 2} = 15.0$ $\beta_{\ell 2} = 0.05$
$\alpha_{\ell 3} = 15.0$ $\beta_{\ell 3} = 0.1$

Appendix Table B. Technology and Quality Parameters

Refining gasoline yields per unit of oil processed

y_1 = 0.15
y_2 = 0.05
y_3 = 0.10

Refinery by product yields per unit of oil processed

x_1 = 0.025
x_2 = 0.0125
x_3 = 0.020

Requirement of input ℓi per unit output of additives type pi

$\ell 1$	0.5	0.33	0.0
$\ell 2$	0.5	0.33	0.0
$\ell 3$	0.0	0.34	1.0

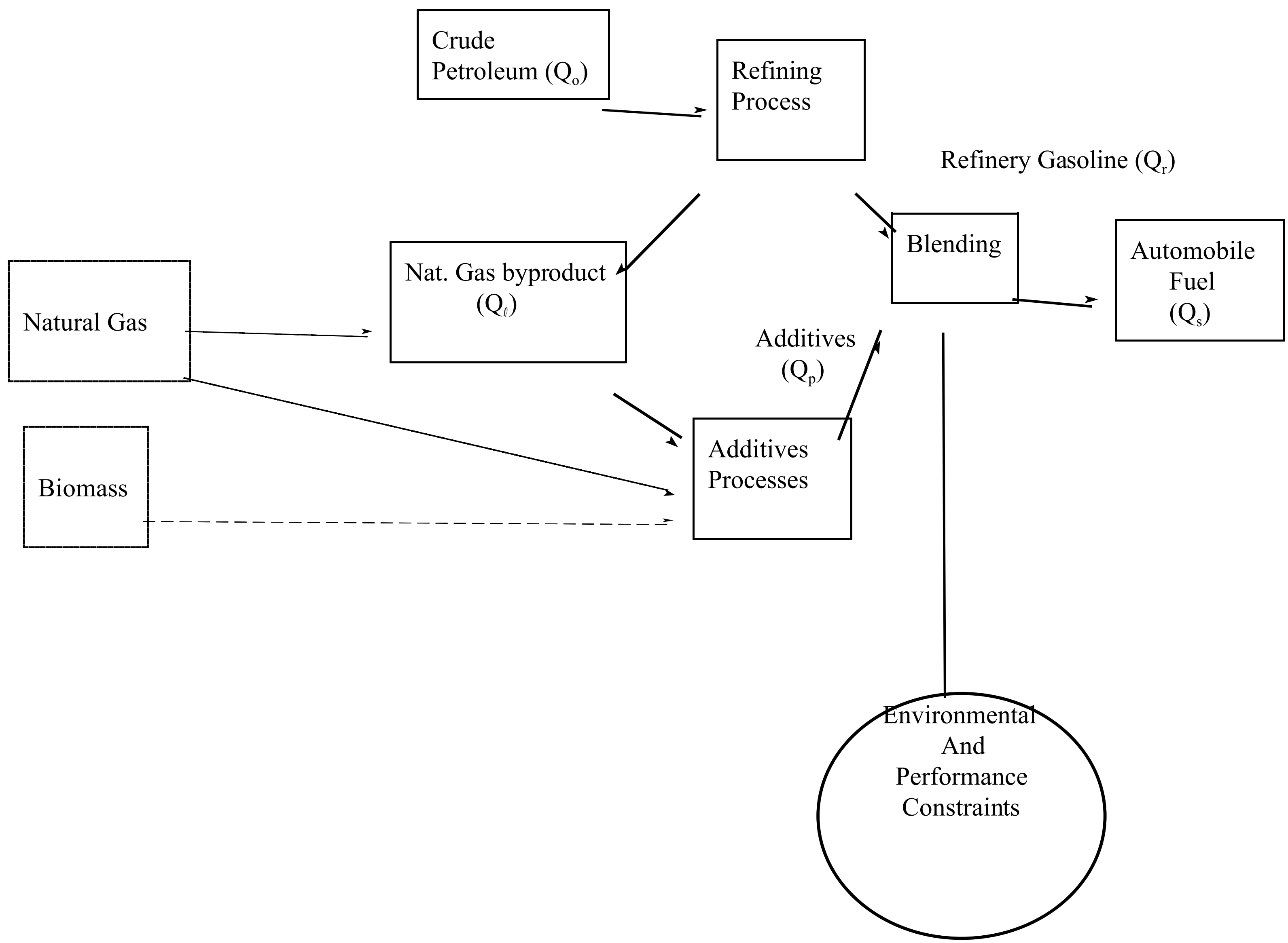
Octane standard by grade of gasoline

k_1 = 91
k_2 = 93
k_3 = 95

Octane content of gasoline components

01	70
02	63
03	90
P1	109
P2	90
P3	114

Chart 1. Factor-Product Relationships in the Gasoline/Additives Market.


Crude Petroleum (Q_o)
Refining Process
Refinery Gasoline (Q_r)
Blending
Automobile Fuel (Q_s)
Natural Gas
Nat. Gas byproduct (Q_ℓ)
Additives (Q_p)
Biomass
Additives Processes
Environmental And Performance Constraints