Trade Policy under Asymmetric Information

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Trade Policy under Asymmetric Information

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Abstract

We consider optimal trade policy for a large country with private information. We show that the optimal tariff leads to a signaling equilibrium with higher tariffs and lower welfare than under complete information, whereas the optimal import quota replicates the complete information equilibrium and thus is superior to the tariff. We also show that, with the tariff, the country may be better off being uninformed. Finally, we show that if the importing nation cannot commit to its tariff, the use of futures contracts together with the dynamically consistent tariff leads to the same equilibrium as under complete information with commitment.
Trade Policy under Asymmetric Information

I. Introduction

One of the most prolific literatures in international economics concerns the relationship between tariffs and quotas¹. As the literature shows, while the two tools can be equivalent in a deterministic, competitive setting, there are a number of scenarios under which they differ. One such circumstance is when firms possess market power, since the residual demand (or supply) curve firms face differs under the two instruments. Another case in which the two tools differ is in the presence of uncertainty. If the policy-maker faces uncertainty when she sets the policy tool, and if the tool cannot be made contingent upon the random variable, then the two policies give rise to different probability distributions of outcomes. While a normative comparison of the two tools cannot be made without specifying the economic model and the reason that commercial policy is being used, the general consensus is probably that tariffs are superior to quotas. The superiority of tariffs, in most cases, arises because price and quantity decisions can respond to the realization of the random variable more fully under a tariff than under a quota.

What has not been widely discussed is the fact that the level of the policy tool may be influenced by private information. For example, consider a government that can impose import restrictions on a good (e.g., oil), and assume the optimal level of these restrictions depends upon government reserves. Assume that under full information the optimal tariff decreases (or the import quota decreases) as oil reserves increase. If all parties know the oil reserves, the government can set the policy tool without considering how it affects expectations concerning reserves, and thus the two policy instruments will be equivalent (assuming all the usual caveats apply). However, if oil reserves are private information, exporters may use the level of the tariff or quota to make inferences concerning these reserves. In this case, the importing government, when setting the tariff (or quota), will take into account not only the objective being pursued in using this instrument, but also the impact of the level of the instrument on exporter’s beliefs.

Thus, either the tariff or the quota may serve as a signal about the level of reserves (also called the type) of the importing country. However, the impact on exporting nations of the actual type of the importing country will differ under the two policy regimes and thus the two policy tools will give rise to different equilibria. With the increased role of governments in

¹ The comparison of price-based tools and quantity-based tools is certainly not restricted to the international economics literature. One of the more contemporary areas where this comparison is also germane is in the environmental literature where taxes and command and control regulations are compared.
modern economies, and especially with the growing importance of Communist, or formerly
Communist countries (e.g., China and Russia) in the world trading system, it should be apparent
that this discussion is more than academic. It is clear that some governments, at the same time
they are either setting trade policy or entering into direct agreements with foreign buyers or
suppliers, also have private information which would be relevant in forecasting price (and other
aggregates). One need look no further than commodity markets to see that announcements of
past purchases (or lack of purchases) by foreign countries can affect prices. Furthermore, even
in the case of a market-oriented country like Canada, the Wheat Board may simultaneously
possess government powers for affecting trade plus private information (such as wheat stocks).
Moreover, the strong push toward tariffication in recent GATT (and WTO) negotiations is
largely predicated upon the belief that the impact of tariffs is more transparent than that of
quotas. Thus, it would seem important to compare the informational implications of each tool.

Collie and Hviid (1994) is one of the first attempts to analyze the signaling role of tariffs.
They use a partial equilibrium model in which a nation imports from a monopolist supplier, and
the domestic government uses import tariffs to recapture some of the monopoly rents. However,
since the government possesses private information about domestic demand, the tariff may signal
the type of consumer to the foreign monopolist. As a result of this signaling effect, the tariff will
be higher and the country will be worse off than under complete information. Note that in their
model the tariff is not the first best instrument under complete information.

In this paper we continue the analysis of the signaling role of different trade policy
instruments. Since any normative comparison of policy instruments must provide a rationale for
the use of policy, we work within the context of a large country that uses the tariff, or quota, to
improve its terms of trade and domestic welfare. The basic model is the standard two good
international trade model, where one large country (US) uses trade policy to improve welfare,
whereas all other countries (ROW) pursue free trade. The specific model we use is simplified
enough to be tractable, yet rich enough to demonstrate equivalence of tariffs and quotas under
full information, and the inferiority of quotas to tariffs under uncertainty. Similarly to Collie
and Hviid (1994), we show that the tariff in the signaling equilibrium of the asymmetric
information game exceeds that under full information and that both exporting and importing
nations are worse off than under complete information. We also show that if the policy-active
government uses a quota the resulting outcome is the same as under full information. Thus,
quotas are superior to tariffs in an environment where the policy-active government has private information. However, it is well known that quotas have been largely abolished under GATT agreements. For this reason, in addition to comparing tariffs and quotas we consider other instruments that may be employed by the policy-active government to signal reserves.

If the policy-active government can postpone commitment to a tariff rate it may have an incentive to wait until after the foreign producers have made their production decisions. Under asymmetric information, when the government in the importing country sets its final tariff after production decisions, the welfare in the importing country may improve compared to a situation when the tariff is set before the production decisions, but the welfare must be lower than in the situation of commitment under full information. Obviously, if the importing nation could make its private information public and could precommit to a tariff rate before production decisions are made, then the outcome would mimic the equilibrium under full information. However, since it is very unlikely that the private information is hard (that is, its revelation cannot be manipulated) this outcome is not feasible. Under these circumstances the following question is important: When the information is asymmetric and when quotas cannot be used, is there a policy instrument that can restore the importing nation’s welfare to the level it has under full information? It turns out that if the importing nation can sell forward contracts and can set its tariff after production decisions are made, then the answer to this question is positive. Welfare in both the exporting and the importing nations improves as a result of the ability to sell forward.

The rest of this paper is organized as follows. In the next section we briefly describe the model and the sequence in which decisions are made. We also solve for the full information equilibrium, assuming the US uses commercial policy to maximize domestic welfare. In section 3 we assume that there is uncertainty concerning US reserves of the import good, that the distribution of reserves is common knowledge, and that the US must choose its policy instrument before anybody learns reserves. In this setting we replicate the standard conclusion that tariffs dominate quotas under uncertainty. In section 4, we assume that the US government learns its level of reserves before setting policy, but that other countries – when making production decisions – know only the prior distribution of reserves and the actual tariff (or quota) chosen by the importing country. The results in this section concerning the tariff parallel those in Collie and Hviid (1994), but we also show that the quota is superior to the tariff. In section 5, we consider the same asymmetric information setting but allow the domestic government to engage
in forward transactions. As with the tariff (and quota), the amount purchased forward serves as a signal to producers in exporting countries. The ability to purchase forward allows the importing country to achieve the same welfare as in the case when producers in the exporting countries are completely informed about the level of reserves. We conclude with a discussion concerning the implications of the paper and possible extensions.

2. The Model

We use the simplest general equilibrium model capable of clarifying the roles tariffs or quotas may play as signaling devices. We assume there are two goods, the numeraire good, \( x \), and a second good, \( y \). Furthermore, we assume there is one policy-active large country (the U.S.) that imports good \( y \) and exports good \( x \). There is a collection of small identical countries that pursue free trade, and these can be aggregated into a single country (ROW). Agents within each country have identical quasi-linear preferences given by:

\[
U = c_x + \left( \frac{A c_x}{\gamma} - \left( \frac{c^2_x}{2\gamma} \right) \right); \quad \bar{U} = \bar{c}_x + \left( \frac{A \bar{c}_x}{\beta} - \left( \frac{c^2_x}{2\beta} \right) \right),
\]

where \((c_x, c_y), (\bar{c}_x, \bar{c}_y)\) denote the consumption vectors in the US and ROW, respectively (the “bar” over the variable denotes the ROW). Let \((e_x, 0)\) and \((\bar{e}_x, 0)\) denote the endowment vector of a private agent in the US and ROW, respectively. In addition to private endowments, we assume that the US government has endowments (reserves) of good \( y \), and we denote these (on a per capita basis) by \( R \). Thus, aggregate per capita endowments in the US are \((e_x, R)\). As we discuss later, producers in the exporting countries may not know the value of \( R \) in the initial stages of the game. Finally, we assume that no production takes place in the US but that in ROW good \( y \) can be produced using inputs of the numeraire:\(^2\)

\[
(q_y^2 / 2\delta) - \bar{q}_x \leq 0,
\]

---

\(^2\) Assuming no production occurs in the US is a simplifying assumption. The main issue that arises if there were US production is, when the US government has private information not available to ROW, what is the information set of US producers? The structure used here makes US agents completely passive and bypasses that issue.
where $\bar{q}_x, \bar{r}_x$ denotes output of good $y$ and input of good $x$, respectively, in ROW. Finally, for simplicity, we normalize the number of agents in each country to one.

The demands and indirect utility function for the preferences given in (1) are:

$$(3) \quad c^*_y = A - \gamma p_y; \quad V^* = I + \left(\frac{(A - \gamma p_y)^2}{2\gamma}\right); \quad \bar{c}_y = A - \beta \bar{p}_y; \quad \bar{V}^* = \bar{I} + \left(\frac{(A - \beta \bar{p}_y)^2}{2\beta}\right)$$

where $(I, \bar{I})$ denote income in each country. Foreign income consists of the value of endowments plus the net profits from production, whereas domestic per capita income consists of the value of endowments plus tariff (or quota) revenue:

$$(4) I = e_x + p_y R + (p_y - \bar{p}_y) m_y; \quad m_y = (c_y - R); \quad \bar{I} = \bar{c}_x + (\bar{p}_y \bar{q}_y - \left[\bar{q}_y^2 / 2\delta\right]) .$$

In (4), $m_y$ denotes US imports of good $y$, and hence $(p_y - \bar{p}_y) m_y$ is the tariff revenue.

The case of full information serves as a useful benchmark. For this reason, we start with the scenario where the sequence of decisions is the following. In the initial stage, the value of US reserves $R$ is learned by all agents. In the following stage, the US government irrevocably sets its trade policy. Foreign producers make production decisions after observing the level of the US trade policy instrument. Finally, trade and consumption decisions are made and implemented.

Throughout, we assume production decisions are made before consumption decisions. We also assume for now that the government can commit to its trade policy, so that no time consistency issues arise. The more important point concerns when the value of $R$ is discovered and by whom. In this section we assume the value of $R$ is known to all agents at the beginning of the game, so there is no uncertainty and no signaling content in trade policy. In the next section we consider uncertainty, in which the true value of $R$ is unknown to all at the beginning, and is discovered simultaneously after trade policy and production decisions are made. In sections 4 and 5, where we discuss signaling issues, $R$ will be learned by the government at stage 1, but will not be revealed to other agents until after production decisions are made.

Since information is complete, when foreign producers make production decisions they can perfectly forecast price and, hence, their supply rule and foreign income are given by:
Using (3), the import demand and export supply equations can be written as:

\[ \bar{y}_y = (\delta + \beta) \bar{p}_y - \bar{A}; \quad m_y = (A - R) - \gamma p_y \]  

Closing the model requires specifying the level of the trade policy instrument (tariff or quota). As is well-known, the two instruments are identical in this case. Letting \( t \) denote the specific tariff, so that \( p_y = t + \bar{p}_y \), equilibrium prices, and trade flows are:

\[ \bar{p}_y = \frac{A + \bar{A} - R - \gamma t}{\gamma + \sigma}; \quad p_y = \frac{A + \bar{A} - R + \sigma t}{\gamma + \sigma}; \quad \bar{y}_y = \frac{\sigma (A - R) - \gamma (\bar{A} + \sigma t)}{\gamma + \sigma}, \]

where \( \sigma \equiv \delta + \beta \). We assume \( \{\sigma (A - R) - \gamma \bar{A}\} \geq 0 \) for all possible values of \( R \). Substituting (7) back into equation (3), using (4)-(7), gives indirect utility for the US in terms of the tariff:

\[ V(t) = e_x + \frac{(M + R)^2}{2\gamma} + \frac{R(A + \bar{A} - R)}{\gamma + \sigma} + M(1 - \phi)t - \frac{\gamma \phi(2 - \phi)t^2}{2}, \]

where \( M \equiv \phi(A - R) - (1 - \phi)\bar{A} \) and \( \phi \equiv \frac{\sigma}{(\gamma + \sigma)} \).

The US government chooses tariff \( t \) to maximize the indirect utility function \( V(t) \). Thus, the optimal tariff, equilibrium prices, imports and welfare are given by:

\[ t^*(R) = \frac{M}{(\gamma + \sigma)\phi(2 - \phi)} > 0; \quad m_y^*(R) = \frac{M}{2 - \phi}; \quad \bar{p}_y(R) = \frac{\phi(A - R) + \bar{A}}{\phi(2 - \phi)(\gamma + \sigma)}; \]
\[ p_y(R) = \frac{2(A - R) + \bar{A}}{(2 - \phi)(\gamma + \sigma)}; \quad V^*(R) = e_x + \frac{(M + R)^2}{2\gamma} + \frac{R(A + \bar{A} - R)}{\gamma + \sigma} + \frac{M^2(1 - \phi)}{2(\gamma + \sigma)\phi(2 - \phi)}. \]

\(^3\) We focus on interior solutions and assume the level of income is sufficient to guarantee both goods are consumed. In the US all government revenue, both from endowments \( (R) \) and from trade restrictions, is rebated to consumers.
Note that the solution possesses the following properties: (i) the optimal specific tariff is a monotonically decreasing function of government reserves \( R \); (ii) optimal imports (exports) – and hence the optimal quota – are also a decreasing function of government reserves; and (iii) the world and domestic price are monotonically decreasing functions of government reserves. Also, the equilibrium does not depend on whether an import tariff or import quota is used.

Before considering the signaling role of trade policy, we briefly review what happens if uncertainty is present when government trade policy and foreign production decisions are made.

3. The Optimal Tariff and Quota under Uncertainty

In this section we modify the information structure and sequence of actions as follows:

Stage 1: The US government irrevocably sets trade policy, given the common knowledge concerning the distribution of reserves;
Stage 2: Foreign producers make production decisions;
Stage 3: The true value of reserves is revealed to all parties;
Stage 4: Trade and consumption decisions are made and implemented.

We assume \( R \) is distributed on the interval \( [R, \bar{R}] \subset \mathcal{R} \). The non-equivalence of tariffs and quotas with this information structure is well-known. The sole purpose of this section is to motivate the next section.

We simultaneously consider both the tariff and the quota equilibria. Denote the specific tariff by \( t \) and the quota by \( L \). Given the levels of the trade instrument, foreign production and reserves, equilibrium world and domestic prices and imports for each case are given by:

\[
\bar{p}_y^* (L) = \frac{L + \bar{A} - \bar{q}_y}{\beta}; \quad p_q^* (L, R) = \frac{A - L - R}{\gamma}; \quad m_q^* (L) = L
\]

\[
\bar{p}_y^* (t, R) = \frac{A + \bar{A} - R - \bar{q}_y - \gamma t}{\gamma + \beta}; \quad p_q^* (t, R) = \left( \bar{p}_y^* + t \right); \quad m_q^* (t, R) = \frac{\beta (A - R) - \gamma (\bar{A} - \bar{q}_y + \beta t)}{(\gamma + \beta)}
\]
where the superscript “\(q\)” in (10) refers to the case of a quota whereas the superscript “\(t\)” in (11) refers to the case of a specific tariff\(^4\). Note that world prices do not depend on realized reserves under the quota; thus, foreign producers do not need to know the distribution of reserves (or their true value) to forecast world price. Under the tariff, world prices depend on realized reserves and the tariff, and thus foreign output depends on beliefs concerning the distribution of \(R\). In this section, since trade policy is chosen before \(R\) is known, the tariff does not modify the producers’ beliefs, but in the next section this will no longer be the case.

Due to risk-neutrality and the model’s quadratic structure, output depends on expected prices: \(\bar{q}_y = \delta \bar{p}_y^e\), where \(\bar{p}_y^e\) is the expected price conditional on information available when production decisions are made. For a quota, there is no uncertainty as expected and realized world prices are the same; in the case of a tariff, due to the linear structure of the price forecast, the expected price depends only upon expected reserves. Thus, solving for each case we have:

\[
\begin{align*}
(12) \quad \bar{q}_y^q &= \frac{\delta (L + A)}{\sigma}; \quad \bar{p}_y^q = \frac{(L + A)}{\sigma}; \quad p_y^q (L, R) = \frac{A - L - R}{\gamma}; \\
(13) \quad \bar{p}_y^t &= \frac{A + A - R^c - \gamma t}{\gamma + \sigma}; \quad \bar{q}_y^t = \delta \bar{p}_y^e; \quad \bar{p}_y^t = \bar{p}_y^e - \frac{\varepsilon}{(\gamma + \beta)}; \quad p_y^t = \bar{p}_y^e + t - \frac{\varepsilon}{(\gamma + \beta)},
\end{align*}
\]

where \(R^c \equiv E(R)\) and \(e \equiv (R - R^c)\).

For each case, the optimal tariff (or quota) is found by substituting the equilibrium price relations and import revenue back into the indirect utility function and maximizing expected utility over the policy instrument. Since the first order conditions are linear, the solution for each case depends only upon the expected value of reserves, and certainty equivalence holds for the policy instruments (i.e., expected imports under the optimal tariff equal the optimal quota, and the expected tariff-equivalent under the optimal quota equals the optimal tariff). Let \(t^q = p_y^q - \bar{p}_y^q = \frac{\sigma (A - L - R) - \gamma (L + A)}{\gamma \sigma}\) denote the tariff-equivalent under the quota. Performing the optimization yields the following:

\(^4\) It is well-known that *ad valorem* and specific tariffs are not equivalent under uncertainty. We consider only the specific tariff and quota cases.
\[ L' = \frac{M^e}{(2 - \phi)}; \quad E(V'^e) = V'^e(R^e) - \frac{Var(R)}{2\gamma}; \quad t'^e(R) = t'^e - \frac{(R - R^e)}{\gamma}; \]

\[ t'^e = \frac{M^e}{(\gamma + \sigma)\phi(2 - \phi)}; \quad E(V') = V'^e(R^e) - \frac{2(\gamma + \beta)Var(R)}{(\gamma + \beta)^2}; \quad m'_e(R) = L' - \frac{\beta(R - R^e)}{(\gamma + \beta)}; \]

where \( M^e = \left[ \phi(A - R^e) - (1 - \phi)A \right] \), \( V'^e(R^e) \) denotes maximized US utility when \( R \equiv R^e \) (from equation (9)) and \( Var(R) \) is the variance of \( R \). As is well-known, the tariff yields strictly higher expected utility \( (\beta > 0) \). Although not shown here, it is clear that the exporting countries are also better off in the case of the tariff than the quota due to the (strict) convexity of the indirect utility function in (export) price. The conclusion follows since expected price is the same under the two regimes, but export price is deterministic under the quota, but stochastic under the tariff.

### 4. Tariffs and Quotas as Signaling Devices

We now consider the signaling role of tariffs and quotas under asymmetric information. In particular, the information structure and the timing of decisions are as follows:

**Stage 1:** The value of \( R \) is learned by the government but is not revealed to foreign agents\(^5\).

**Stage 2:** The US government irrevocably sets trade policy (tariff or quota).

**Stage 3:** Foreign producers make production decisions based upon their prior beliefs concerning \( R \) and the observed level of the trade policy instrument.

**Stage 4:** The true value of \( R \) is revealed to all agents.

**Stage 5:** Trade and consumption decisions are made and implemented.

This decision structure differs from the case of uncertainty as the government learns its true reserves before setting policy, and it differs from the full information case as foreign producers do not learn the true value of \( R \) until after production decisions are made. It is these differences that result in the US government’s trade policy being a potential signal of reserves.

Consider first the optimal quota. As previously noted, under a quota foreign prices depend only upon the quota level and not upon the true level of reserves. Thus, it is irrelevant to

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\(^5\) Since US private agents are completely passive in the model it is immaterial when they learn the true value of \( R \).
foreign producers whether the quota accurately reveals true reserves. Even if the quota fully reveals the level of reserves, the equilibrium is unaffected by the fact the quota serves as a signal. Since foreign production under the quota does not depend on US reserves, the US government has no incentive to misrepresent its type. Thus, the equilibrium under the quota is exactly the same as when both parties are initially informed about the true level of reserves. This equilibrium is given by (9), with the optimal quota given by: \( \frac{L^*(R)}{2} = \frac{M}{(2 - \phi)} \).

Next, consider the incentive structure under the tariff\(^6\). The US government sets a tariff \( t(R) \) that may be a function of its private information \( R \). After observing the tariff, foreign producers make an inference about the true level of US reserves. We let \( r(t) \) denote the updated beliefs of foreign producers about US reserves. If foreign producers can perfectly infer the level of reserves \( R \), then we will say that the equilibrium is separating.

Under a tariff, realized foreign price depends on the tariff, the level of foreign production, and the true value of reserves. Thus, actual reserves (potentially) affect foreign production decisions in two ways – through the tariff, which is known when production decisions are made, and through the realized price – which is not known, but may be inferred from the tariff.

We solve backward to determine the equilibrium. Given production levels, the tariff and the realized value of reserves, foreign prices are given by equation (11). When foreign producers make their output decision they use their forecast of the future foreign price

\[
\bar{q}_y = \delta E(\bar{p}_y | t) = \delta \left\{ \left( A + \bar{A} - r(t) - \bar{q}_y - \gamma t \right) / (\gamma + \beta) \right\}
\]

where \( E(\bar{p}_y | t) \) is the expected foreign price conditional on the announced tariff rate. Note that the tariff has two distinct effects on this output decision: (1) the direct effect on foreign prices; and (2) the inferential effect, captured by \( r(t) \), as the announced tariff rate affects inferences made by foreign producers about reserves, and hence world price. Simplifying (16) yields foreign output, foreign price, and trade flows, given tariffs, beliefs and realized reserves:

\[
\bar{q}_y = \delta \left\{ \frac{A + \bar{A} - r(t) - \gamma t}{\gamma + \sigma} \right\}; \quad \bar{p}_y (t, R) = \left( \frac{A + \bar{A} - R - \gamma t}{\gamma + \sigma} \right) - s\Delta; \quad m_y = M - \gamma \phi t + \gamma \Delta,
\]

\(^6\) We consider only a specific tariff. With an ad valorem tariff there may not be a separating equilibrium; e.g., in our model, if \( \bar{A} = 0, \beta = 0 \), so there is no foreign consumption of \( y \), the optimal ad valorem tariff is independent of \( R \).
where \( \Delta = R - r(t) \), \( \alpha \equiv \frac{\delta}{\delta + \beta} \); \( s = \frac{\alpha\phi}{(1-\alpha\phi)(\gamma + \sigma)} \) and \( M \) is as defined earlier. The term \( \alpha \in [0,1] \) reflects the fraction of the slope of the export supply curve that is due to the price responsiveness of production. From (17) we see that, ceteris paribus, forecasts of lower reserves lead to more foreign output and hence –due to this increased output - a lower world price. By comparing (17) to equation (7), we see that equilibrium imports for this case differ from the full information case only by the term \( (\gamma\Delta) \) on the right hand side of (17). Thus, in a separating equilibrium imports, as a function of \( t \), will be the same as under full information (though actual imports will differ if the tariff differs). Note that the responsiveness of exports (imports) to the tariff, as viewed by the importing nation, will differ from the perfect foresight case if the government believes that the foreign producers’ forecast of \( R \) depends on the tariff rate.

Finally, substituting all the preceding into the US’ indirect utility function gives US utility as a function of actual reserves, the tariff rate, and the belief function:

\[
V(t) = e_x + p_y R + tm_y + \left[ (A - \gamma p_y)^2 \right]^{\frac{1}{2\gamma}}; \quad p_y = \bar{p}_y + t
\]

where \( (m_y, \bar{p}_y) \) are given in (17). This expression differs from the full information case only in that prices and imports depend upon \( \Delta \), the forecasting error made by producers. In a separating equilibrium (that is, when there is no forecasting error) realized prices, quantity and hence utility will be the same as under full information if the tariff is the same.

Rewriting (18) as a function of the tariff \( t \), the foreign output level \( \bar{q}_y \), and reserves \( R \), we obtain the following expression for the indirect utility of a representative domestic agent:

\[
\bar{V}(R, \bar{q}_y, t) = b_0 + b_1 R + b_2 \bar{q}_y + b_3 t + b_4 R^2 + b_5 \bar{q}_y^2 + b_6 t^2 + b_7 R \bar{q}_y + b_8 R t + b_9 \bar{q}_y t.
\]

The US government’s problem of setting trade policy is a dynamic game where foreign producers maximize expected profits and the US government’s objective function is given by (19). In this game, the importing nation of any type wants to persuade foreign producers that it has low reserves. Since, under full information the tariff rate is inversely related to the level of reserves, a high tariff may signal a low level of reserves. If the US government with relatively
low reserves sets the same tariff rate as under full information, then types with larger reserves may want to mimic the low reserve government’s behavior. Thus, to separate from these types the low reserve type may want to increase its tariff above the full information level (thus making imitation more costly). In other words, we intuitively expect that, similarly to Collie and Hviid (1994), a fully separating equilibrium might exist in which tariffs are monotonically declining in reserves and tariff levels are, in general, higher than in the full information setting. To support our intuition we solve for the sequential equilibrium of our game, which is defined as follows.

**Definition:** A sequential equilibrium consists of strategies \( t(R) \) and \( \bar{q}_y(t) \) and beliefs \( r(t) \) such that:

(i) The importing government chooses \( t(R) \) to maximize the ex ante indirect utility function \( \bar{V}(R, \bar{q}_y(t), t) \) for all \( R \);

(ii) Foreign producers choose output that maximizes their expected profits given beliefs \( r(\cdot) \).

This output level is given by \( \bar{q}_y(t) = \delta \left( \frac{A + \bar{A} - r(t) - \gamma t}{\gamma + \sigma} \right) \); and

(iii) Foreign producers’ beliefs \( r(\cdot) \) on the equilibrium path are determined by Bayes’ rule given the prior probability over the importing government’s types and the importing government’s equilibrium strategy \( t(R) \).

Let \( V(R, r, t) \) denote the indirect utility of the representative domestic agent when the domestic government’s true type is \( R \), foreign producers’ inference about its type is \( r \) and tariff \( t \) is chosen. Substituting the expression for \( \bar{q}_y(t) \) into \( \bar{V}(R, \bar{q}_y(t), t) \) it is easy to verify that

\[
V(R, r, t) = a_0 + a_1 R + a_2 r + a_3 t + a_4 R^2 + a_5 r^2 + a_6 t^2 + a_7 R r + a_8 R t + a_9 r t .
\]

Let \( t^*(R) \) denote the optimal tariff when foreign producers are completely informed about domestic reserves and the domestic government can precommit to its trade policy.

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7 The definitions of parameters \( b_0, \ldots, b_9 \) are given in Appendix 2.

8 \( r(\cdot) \) is a probability distribution over \([R, \bar{R}]\).

9 The definition of parameters \( a_0, \ldots, a_9 \) in equation (20) are given in Appendix 2.
instrument. From Section 2, we have that \( t^*(R) = \left[ M(R)/\sigma(2-\phi) \right] \). In Appendix 1 we prove the following lemma\(^{10}\).

**Lemma 1:** In any separating equilibrium of the game, \( t(R) = t^*(R) \).

The intuition behind the lemma is straightforward. The worst beliefs, from the US government’s perspective, that foreign producers may have about US reserves, both on and off the equilibrium path, is given by \( R \). Hence, a deviation from \( t^*(R) \) cannot be credibly punished.

Mailath (1987) has identified conditions on the signaling parties’ utility function that ensure uniqueness of the separating equilibrium. Using his results, the proof of the following proposition is found in Appendix 1:

**Proposition 1:** Suppose that the equilibrium triplet \( (t(\cdot), q(\cdot), r(\cdot)) \) is fully separating, and suppose that the initial value condition \( t(R) = t^*(R) \) holds (Lemma 1). Then

(i) \( t(R) \) is continuous and strictly decreasing on \([R, R]\). It is differentiable on \((R, R)\) and satisfies the differential equation

\[
\frac{dt}{dR} = \left( \frac{\delta}{(\gamma - \phi)} \right) \left( \frac{\phi(A - R) - (1 - \phi)A + \gamma(1 - \phi)t}{\phi(A - R) - (1 - \phi)A - \sigma(2 - \phi)t} \right) = \left( \frac{\delta}{(\gamma + \beta)} \right) \left( \frac{M + \gamma(1 - \phi)t}{M - \sigma(2 - \phi)t} \right).
\]

(ii) The solution to the differential equation (21) coupled with the initial value condition \( t(R) = t^*(R) \) is unique.

Thus, the signaling game has a unique separating equilibrium given by (21) together with the initial value condition \( t(R) = t^*(R) \). Although \( t(R) \) is the unique separating equilibrium it is not the unique sequential equilibrium of the signaling game, as the game also has pooling equilibria where some types pool at the same tariff rate. However, only the unique separating equilibrium survives the universal divinity criterion of Banks and Sobel (1987). The universal divinity criterion applied to our game implies that if the foreign producers observe out-of-equilibrium

\(^{10}\) All proofs are given in Appendix 1.
tariff rates their posterior beliefs place positive probability only on types that are most likely to deviate from the equilibrium. The proof that the unique sequential equilibrium satisfying the universal divinity criterion is separating is similar to the proof of a similar result in the game of investment and rate regulation in Besanko and Spulber (1992). For this reason, we omit details of the proof and only outline its structure. As a first step in the proof, one can show that the tariff schedule \( t(R) \) is non-increasing in \( R \) for any sequential equilibrium satisfying the universal divinity criterion. Using this result, we can demonstrate that at any sequential equilibrium satisfying the universal divinity criterion, the foreign exporters put probability one on the type \( R = \tilde{R} \) for off-the-equilibrium path tariff rates \( t < t(\tilde{R}) \). Also, for off-the-equilibrium path tariff rates \( t > t(\tilde{R}) \), the foreign exporters assess probability one to the event \( R = \hat{R} \). Finally, these results can be used to prove that the unique sequential equilibrium satisfying universal divinity criterion is fully separating.

The following proposition verifies our intuition on the relationship between the tariffs under full and incomplete information. Proof is provided in Appendix 1.

**Proposition 2:** The tariff \( t(R) \) in the unique separating equilibrium is strictly greater than the full-information tariff \( t^*(R) \) for all \( R \in \left[ \bar{R}, \tilde{R} \right] \).

Equilibrium tariffs are higher under asymmetric information since the government has an incentive to misrepresent its type. The more price-responsive is foreign output, the greater is the government’s incentive to misrepresent its type. The higher tariff implies that both importing and exporting nations are worse off than under full information. Thus, the signaling equilibrium is *Pareto inferior* to the full information equilibrium and hence to the quota.

**Corollary:** In the asymmetric information game described here, all parties prefer quotas to tariffs.

The result that tariffs are higher in the signaling equilibrium is similar, in spirit and intuition, to the problem that arises with an inability to commit to tariffs. In the latter case, governments have an incentive to revise tariffs after production decisions are made; since foreign
producers anticipate this, they reduce output, leading to an equilibrium that is Pareto inferior to the commitment equilibrium.

From the preceding it is not clear how the importing nation is affected by knowing its true reserves before choosing its tariff – i.e., how from an ex ante perspective expected welfare in the case of asymmetric information compares to that under uncertainty. Clearly, when a quota is used, expected welfare under asymmetric information is higher than under uncertainty since the quota equilibrium replicates the full information equilibrium.

In general, the equilibrium signaling tariff is non-linear and expected utility for the two cases is not readily compared. However, for the special case in which the smallest tariff is zero \( t^\beta (\bar{R}) = 0 \), the tariff under signaling is linear in reserves. For this case the tariff rules are\(^{11}\):

\[
(22) \quad t^\beta = \left( \frac{M(R)}{\sigma(2 - \phi)} \right); \quad t^u = \left( \frac{M(R^*)}{\sigma(2 - \phi)} \right); \quad t^{si} = \sigma t^\beta;
\]

\[
\sigma_i = \left[ \left( 1 + \xi \right) + \sqrt{(1 + \xi)^2 + 4\xi \theta} \right] / 2 > 1; \quad \xi \equiv \left( \alpha(1 - \phi)/(1 - \alpha \phi) \right); \quad \theta = \left( \phi(2 - \phi)/(1 - \phi)^2 \right)
\]

where the superscripts refer to full information, uncertainty, and the signaling equilibrium, respectively. For this case realized welfare under the signaling equilibrium is given by:

\[
(23) \quad V^*(R) = e + \left( \frac{M + R}{2 \gamma} + \frac{R(A + \bar{A} - R)}{\gamma + \sigma} + \frac{M^2 (1 - \phi) \sigma_i (2 - \sigma_i)}{2(\gamma + \sigma) \phi(2 - \phi)} \right)
\]

Clearly, \( E(V^\beta) > \text{Max} \big\{ E(V^u), E(V^{si}) \big\} \); comparing \( E(V^u) \) and \( E(V^{si}) \) yields, after simplification:

\[
(24) \quad E[V^{si} - V^u] = \left[ \frac{(1 - \phi)^2 (2\sigma_i - \sigma_i^2)}{2\sigma(2 - \phi)} + \frac{\delta(\beta + \phi)(\gamma + \beta))}{(\gamma + \beta)^2} \right] \text{Var}(R) - \frac{(M^\ast)^2 (1 - \phi)(\sigma_i - 1)^2}{2\sigma(2 - \phi)}
\]

\(^{11}\)Let the equilibrium tariff under signaling be: \( t^{si}(M) = \mu(M) t^\beta \), where \( t^\beta \) is the full information tariff. In general, \( \mu(M) \) is given by: \( \left( (\sigma_i - \mu(M))/(\sigma_i - 1) \right)^\pi \left( (\mu - \sigma_i)/(1 - \sigma_i) \right)^{(1 - \pi)} = (M/M). \) \( \sigma_i, \sigma_2 \) are the roots of: \( x^2 - (1 + \xi)x - \xi \theta = 0 \), and \( \xi, \theta \) are defined in \( (22) \); thus, \( \sigma_i > 1, \sigma_2 < 0 \). \( M \) is the value of \( M \) at the highest level of reserves \( M = \phi(A - \bar{R}) - (1 - \phi)\bar{A} \), and \( \pi \equiv ((\sigma_2 - 1)/(\sigma_1 - \sigma_2)) \in [0,1] \).
where \( \sigma_1 > 1 \) since tariffs are higher under the signaling equilibrium than under full information. If \( \sigma_1 \) is close to one (\( \alpha \) close to zero) and/or the \( \text{Var}(R) \) is large relative to the expected value of \( M \), then the signaling equilibrium will yield higher expected utility than the case of uncertainty since the costs due to signaling will be relatively low. However, in other cases the signaling equilibrium yields lower expected utility for the informed agent, implying that the information concerning reserves has a negative (expected) value for the policy-active importing nation.

Finally, note that the country may be worse off in the signaling equilibrium than under free trade – as can also happen if commitment is not feasible. Realized home welfare under free trade is given by (23), with \( \sigma_1 = 0 \). Thus:

Proposition 3. If \( \sigma_1 > 2 \) the signaling equilibrium leads to lower welfare than free trade\(^{12}\).

5. Time-consistency of Trade Policy and Forward Contracts

In this section we study what happens if the US government cannot irrevocably commit its trade policy before production decisions are made. We consider only two informational environments: the full information case and the case in which the US government has superior information. It is well known (see Lapan (1988a), Maskin and Newberry (1990)) that in the full information case both the importing and exporting countries are worse off due to this inability to precommit to announced trade policy. In particular, if the US government can revise its announced policy after foreign producers have made their production decisions and before the consumption and trade decisions have been made, the ex ante optimal tariff (derived in Section 2) is not time-consistent. The time-consistent solution (i.e., when foreign producers’ forecast of the future tariff rate is correct) entails a lower level of imports and a higher tariff rate than in the case when the policy-active government can precommit. On the other hand, under asymmetric information the ability of the importing government to revise its announced trade policy after production decisions may benefit both the importing and the exporting nations\(^{13}\).

\(^{12}\) \( \sigma_1 \geq 2 \) implies: \( \alpha \geq \left[ \frac{2(1-\phi)}{(2-\phi^2)} \right] \). For example, if the slope of the domestic import demand schedules equals the slope of the foreign export supply schedule (\( \phi = .5 \)), then \( \alpha \geq (4/7) \) suffices.

\(^{13}\) If the domestic government cannot precommit to its tariff before production decisions are made and if it does not have any other policy instruments (e.g., production or consumption taxes and/or subsidies) and cannot offer forward
In what follows we assume that the policy-active government sets its final tariff after foreign exporters’ production decisions. Lapan (1988b) demonstrates, under complete information, that if the policy-active government can use forward contracts an ex ante optimal solution can be restored. It is interesting to consider the role of forward contracts when the trading parties are asymmetrically informed. In this case, forward contracts have an additional role of serving as a signal to foreign producers. For this reason, we consider the following sequence of decisions to investigate the role of forward contracts as signaling devices.

**Stage 1:** The level of US reserves $R$ is learned by the government but is not revealed to foreign agents.

**Stage 2:** The US government decides on the number of forward contracts to be bought forward.

**Stage 3:** Foreign producers make production decisions based upon their prior beliefs concerning $R$ and the actual number of contracts purchased forward.

**Stage 4:** The US government sets its tariff.

**Stage 5:** The true value of $R$ is revealed to all agents.

**Stage 6:** Trade and consumption decisions are made and implemented.

Let $F$ denote the number of forward contracts bought by the US government and let $p^F_y$ denote the price specified in the contracts. The government’s choice of the number of forward contracts $F(R)$ is a function of its private information $R$. After observing $F$, foreign producers make an inference about the true level of $R$. We let $r(F)$ denote the updated beliefs of foreign producers about US reserves. Similarly to the previous section, we solve the game backward.

Domestic per capita income is given by

\[
I = \epsilon_x + p_y R + \left(p_y - \bar{p}_y\right)m_y + \left(\bar{p}_y - \bar{p}^F_y\right)F.
\]

(25)

Note that in any separating equilibrium of the game the forward price $\bar{p}^F_y$ is equal to the spot price $\bar{p}_y$. Otherwise, one of the trading parties would be unwilling to trade forward. Given contracts, the ex post tariff is given by $t^c(R,R^c) = \frac{1}{d}\left(\left(\beta + \delta\right)A - \gamma A\right) - \frac{d}{\beta + 2\gamma}R - \frac{\gamma \delta}{\beta + 2\gamma}R^c$, where “d” is defined below equation (30). It can be verified that under many parameterizations the domestic government prefers to set its tariff after production decisions are made.
the predetermined output levels, the tariff, and the realized value of \( R \), foreign prices are given by (11): 
\[
\tilde{p}_y(t, R) = \frac{A + \tilde{A} - R - \tilde{q}_y - \gamma t}{\gamma + \beta}.
\]
Substituting this expression into the US’ utility function and optimizing over the tariff rate we get the ex post optimal tariff rate as a function of contracts traded forward, the actual reserves and the foreign producers’ output:

\[
t = \left[ \left( \beta(A - R) - \gamma(\tilde{A} - \tilde{q}_y) - (\beta + \gamma)F \right) \right] / (\beta(\beta + 2\gamma))
\]  (26)

When foreign producers make their output decision they use their forecast of the future tariff and foreign price

\[
\tilde{q}_y = \delta \left\{ (A + \tilde{A} - r(F) - \tilde{q}_y - \gamma t) / (\gamma + \beta) \right\}
\]  (27)

where \( t \) is the tariff rate in (26). Solving (27) for \( \tilde{q}_y \), we obtain

\[
\tilde{q}_y = \delta \left\{ (\beta(A - r(F)) + (\beta + \gamma)\tilde{A} + \gamma F) / (\beta(\beta + 2\gamma) + \delta(\beta + \gamma)) \right\}
\]  (28)

Similarly to the tariff in the preceding section, forward contracts have two effects on the production decisions of foreign exporters, the direct and the inferential effects. Substituting (28) into (26) we obtain the ex post optimal tariff as a function of the domestic government’s true type \( R \), foreign producers’ beliefs \( r \) and the number of forward contracts, \( F \):

\[
t = \frac{1}{\beta(\beta + 2\gamma) + \delta(\beta + \gamma)} \left\{ (\beta + \delta) A - \beta(\beta + 2\gamma) + \delta(\beta + \gamma) \right\} \frac{\delta}{\beta + 2\gamma} \frac{R - \delta}{\beta + 2\gamma} r(F) - \gamma \tilde{A} - (\beta + \gamma + \delta) F \} \}
\]  (29)

When performing the above manipulations we implicitly assumed that the US government knows foreign producers’ beliefs, foreign producers know that the US government knows, and so on ad infinitum. As is well known, any sequential equilibrium should satisfy this requirement.

Finally, substituting all the preceding into the US’ indirect utility function gives US utility as a function of actual reserves, the number of forward contracts, and the belief function:

\[
V(R, r, F) = c_0 + c_1 R + c_2 r + c_3 F + c_4 R^2 + c_5 r^2 + c_6 F^2 + c_7 Rr + c_8 RF + c_9 r F,
\]  (30)

where \( d \equiv \beta(\beta + 2\gamma) + \delta(\beta + \gamma) \), and the parameters \( c_0, \ldots, c_9 \) are defined in Appendix 2.
The strategic interaction between the US government and the foreign producers is a signaling game, where the foreign producers’ output rule, given the beliefs, is given by (28) and the US government maximizes the indirect utility function of its representative agent. As in the preceding section, the US government wants to convince foreign producers that it has low reserves. Under full information, the number of forward contracts \( F^*(R) \) offered by the US government is given by \( F^*(R) = \left( \alpha M(R)/(2 - \phi) \right) \). The resulting tariff rate and the welfare of importing and exporting nations are the same as in the full information case of Section 2. Note also that the number of forward contracts is inversely related to the level of reserves. Thus, a large number of forward contracts may signal a low level of reserves.

We want to investigate whether in the asymmetric information setting a separating equilibrium exists and how it is related to the full information forward contracts and tariff. It is straightforward to verify that the indirect utility function in (30) satisfies properties (1)-(5) from Mailath (1987). However, it does not satisfy the single-crossing property. More precisely, \( \left[ (\partial V/\partial r)/(\partial V/\partial F) \right] = -(\beta/\gamma) \) which is invariant to changes in \( R \). Moreover, the initial value condition \( F(\bar{R}) = F^*(\bar{R}) \) is not satisfied. The solution to the differential equation\(^{14}\) 

\[
(\frac{dF}{dR}) = (\beta/\gamma),
\]

given the initial value condition, is not an equilibrium since type \( \bar{R} \) has an incentive to deviate from its strategy. Thus, the results from Mailath (1987) do not apply to this case. The separating equilibrium of the game is found by differentiating\(^{15}\) (30) with respect to \( F \) and equating the derivative to 0. The resulting equation has two solutions. The first is the solution \( (\frac{dF}{dR}) = (\beta/\gamma) \), but the second-order conditions are not satisfied at this solution. The second solution is the full information level of forward contracts \( F^*(R) \). It is straightforward to verify that this strategy constitutes a part of a sequential equilibrium. Substituting for \( F \) in (29) one can easily verify the optimal tariff is equal to the optimal tariff under full information (equation (9) in Section 2). Thus, when the US government can purchase forward and has an ability to revise its tariff after production decisions are made, the unique separating equilibrium outcome is the same as under full information (Section 2).

\(^{14}\) The solution to this differential equation when the initial value condition is satisfied is given by 
\[
F(R) = \frac{1}{\beta + 2\gamma + \delta} \left[ \delta(A - \gamma A) - \frac{A}{\gamma} \frac{d}{dR} \right] + \frac{\beta}{\gamma} R,
\]

which is the same as under complete information.
Proposition 4. Under asymmetric information and assuming the policy active government cannot precommit to its tariff, the use of forward contracts leads to a sequential equilibrium that replicates the full information commitment equilibrium.

6. Conclusion

It is a fundamental premise of international negotiations that tariffs are superior to quotas as trade restrictions. One reason for this belief is that tariffs are thought to be more transparent. We have shown that, in the context of asymmetric information, tariffs are inferior to quotas. The informational content of tariffs induces governments with private information to use higher tariffs in order to signal their type. The resulting tariff equilibrium is more restrictive than the quota equilibrium since, with quotas, foreign exporters do not care about the type of the foreign government, once trade volumes are known. Since the quota’s signaling role is unimportant, it can support the full information equilibrium. We have also seen that, with tariffs and asymmetric information, the inability to precommit tariffs does not necessarily lower welfare. In fact, we have shown that – as in the full information case – the use of forward contracts with the dynamically consistent tariff supports the full information commitment equilibrium.

Our results are somewhat analogous to the old literature on the relative ability of tariffs and quotas to protect countries from domestic, or foreign, disturbances. In our paper, since the “disturbance” (i.e., the private information) is internal, the quota effectively isolates foreign countries from this disturbance, and therefore the signaling aspect of the quota is unimportant to foreigners. Naturally, this raises the question of how these instruments compare when the private information possessed by the US government concerns foreign disturbances.

A final question that occurs is how the presence of asymmetric information affects the conclusion that tariff negotiations can, in conjunction with market access commitments, lead to efficient outcomes that do not require contracting on domestic policy (as, for example, in Bagwell and Staiger (2001)). Since market access commitments, like quotas, are (implicit) contracts on trade volumes, the combination of tariff and quantitative commitments, which are used to avoid contracting on domestic policies, may have to be rethought in situations with asymmetric information.

15 Note that that foreign producers’ beliefs \( r \) are a function of forward contracts \( F \).
Appendix 1

Proof of Lemma 1: Suppose that \( t(R) \neq t^*(R) \) is the equilibrium strategy of type-\( R \) government. The equilibrium payoff is given by \( V(R,R,t(R)) \). Consider a deviation of the government to a strategy \( t^*(R) \). The payoff for this strategy is given by

\[
V(R,r(t^*(R)),t^*(R)) = a_0 + a_1 R + a_2 r(t^*(R)) + a_3 t^*(R) + a_4 \overline{R}^2 + a_5 \left[ r(t^*(R)) \right]^2 + a_6 \left[ t^*(R) \right]^2 + a_7 R r(t^*(R)) + a_8 \overline{R} t^*(R) + a_9 r(t^*(R)) t^*(R)
\]

In what follows we replace \( r(t^*(R)) \) by \( r \) to simplify our notation. Thus, the gain to this deviation is given by

\[
\Delta = a_2 r + a_3 r^2 + a_4 r \overline{R} + a_5 t^*(R) + a_6 \overline{R} t^*(R) + a_7 t(R) + a_8 \overline{R} t(R) + a_9 r t(R) + a_6 \left[ t(R) \right]^2
\]

First, note that:

\[
a_2 r + a_3 r^2 + a_4 r \overline{R} - \left\{ a_2 R + a_3 \overline{R}^2 + a_7 \overline{R} \right\} = -\frac{\delta (r - \overline{R})}{(\beta + \gamma)(\beta + \gamma + \delta)} \left\{ 2(\beta + \gamma)(\beta + \delta)A - \gamma A \right\} - \delta \gamma (r - \overline{R}) - 2 \beta (\beta + \gamma + \delta) \overline{R}
\]

Thus, to prove that deviation is profitable it suffices to show that:

\[
a_1 t^*(R) + a_5 \overline{R} t^*(R) + a_6 \overline{R} t^*(R) + a_6 \left[ t^*(R) \right]^2 - \left\{ a_1 t(R) + a_5 \overline{R} t(R) + a_6 \left[ t(R) \right]^2 \right\}
\]

\[
\geq \left( a_1 + (a_6 + a_8) \overline{R} \right) \left[ t^*(R) - t(R) \right] + a_6 \left[ t^*(R) \right]^2 - \left[ t(R) \right]^2
\]

\[
= \frac{\gamma}{(\beta + \gamma + \delta)} \left\{ (\beta + \delta)(A - \overline{R}) - \gamma A \right\} t^*(R) - \frac{(\beta + \delta)(\beta + 2 \gamma + \delta)}{2} \left[ t^*(R) \right] - \left[ t(R) \right] > 0
\]

where the last inequality follows from the fact \( t^*(R) \) is the unique maximizer of

\[
-\frac{\gamma}{(\beta + \gamma + \delta)^2} \left\{ (\beta + \delta)(A - \overline{R}) - \gamma A \right\} - \frac{(\beta + \delta)(\beta + 2 \gamma + \delta)}{2} \left[ t^*(R) \right] - \left[ t(R) \right] < 0.
\]

Q.E.D.

Proof of Proposition 1: We use Theorem 2 and the corollary from Mailath (1987) to prove this proposition. For this purpose we only need to check that the assumptions used in these results hold (these assumptions correspond to conditions (1) – (5) in Mailath (1987)).

1) \( V(R,r,t) \) is a polynomial and, hence, twice continuously differentiable in its arguments;

\[
\frac{\partial V}{\partial r} = a_2 + 2a_3 r + a_4 R + a_5 t < 0
\]
(3) Type monotonicity: \( \frac{\partial^2 V}{\partial t \partial R} = a_8 < 0 \) for all \((R,r,t) \in [R,\overline{R}]^2 \times R_+\)

Note that the function \( V(R,r,t) \) is strictly concave in \( t \) \( \left( \frac{\partial^2 V}{\partial t^2} = a_6 < 0 \right. \) for all \((R,r,t) \in [R,\overline{R}]^2 \times R_+ \) \) and, hence, conditions of “strict” quasiconcavity (condition (4)) and boundedness (condition (5)) hold. \( Q.E.D. \)

**Proof of Proposition 2:** We know that \( \frac{dt(R)}{dR} < 0 \) and \( \frac{\partial V(R,R,t(R))}{\partial r} < 0 \). These two conditions imply that \( \frac{\partial V(R,R,t(R))}{\partial t} = l_0 + l_1R + l_2t(R) < 0 \). Rearranging this equation yields

\[
t(R) > \frac{(\beta + \delta)(A - R) - \gamma A}{(\beta + \delta)(\beta + 2\gamma + \delta)} = t^*(R) \quad \text{for all } R \in [R,\overline{R}]. \quad Q.E.D.
\]
Appendix 2: Parameters definition

Parameter definitions for equation (20):

\[ b_0 = e_x + \frac{A^2}{2\gamma} - \frac{A(A + A)}{(\beta + \gamma)} + \gamma(A + A)^2; \quad b_1 = \frac{(2\beta + \gamma)A + \beta A}{(\beta + \gamma)^2}; \quad b_2 = \frac{\beta A - \gamma A}{(\beta + \gamma)^2}; \]

\[ b_3 = -\frac{\gamma(\beta A - \gamma A)}{(\beta + \gamma)^2}; \quad b_4 = -\frac{2\beta + \gamma}{2(\beta + \gamma)^2}; \quad b_5 = \frac{\gamma}{2(\beta + \gamma)^2}; \quad b_6 = -\frac{\beta\gamma}{2(\beta + \gamma)^2}; \]

\[ b_7 = -\frac{\beta}{(\beta + \gamma)^2}; \quad b_8 = -\frac{\beta\gamma}{(\beta + \gamma)^2}; \quad b_9 = -\frac{\gamma^2}{(\beta + \gamma)^2}. \]

Parameter definitions, equation (21)

\[ a_0 = e_x + \frac{A^2}{2\gamma} - \frac{(A + A)((1 + \phi)A - (1 - \phi)A)}{2(\gamma + \sigma)}; \quad a_1 = \frac{(\beta + \gamma + \sigma)A + \beta A}{(\beta + \gamma)(\sigma + \gamma)}; \quad a_2 = -\frac{\delta(\phi A - (1 - \phi)A)}{(\beta + \gamma)(\sigma + \gamma)}; \]

\[ a_3 = -\frac{\phi A - (1 - \phi)A}{(\sigma + \gamma)}; \quad a_4 = -\frac{2\beta + \gamma}{2(\beta + \gamma)^2}; \quad a_5 = \frac{(1 - \phi)\gamma^2}{2(\beta + \gamma)^2(\sigma + \gamma)}; \quad a_6 = -\frac{\gamma\phi(2 - \phi)}{2}; \]

\[ a_7 = -\frac{\beta\phi}{(\beta + \gamma)(\sigma + \gamma)}; \quad a_8 = -\frac{\beta(1 - \phi)}{(\beta + \gamma)}; \quad a_9 = -(1 - \phi)^2\delta. \]

Parameter definitions, equation (30)

\[ c_0 = e_x + \frac{A^2}{2\gamma} + \frac{\beta}{d^2}[(\beta + \delta)A - \gamma A]^2 - \frac{\beta\gamma}{2d^2}[(2\beta + \delta)A + \beta A][2(\beta + \gamma + \delta)A - \gamma A]; \]

\[ c_1 = \frac{(2\beta + \delta)A + \beta A}{d}; \quad c_2 = -\frac{\beta\delta[(\beta + \delta)A - \gamma A]}{d^2}; \quad c_3 = \frac{\gamma\delta[(\beta + \delta)A - \gamma A]}{d}; \]

\[ c_4 = -\frac{1}{(\beta + 2\gamma)}; \quad c_5 = \frac{\beta\gamma^2}{2d^2(\beta + 2\gamma)}; \quad c_6 = -\frac{\gamma(\beta + \delta)(\beta + 2\gamma + \delta)}{2d^2}; \quad c_7 = \frac{\beta\delta}{d(\beta + 2\gamma)}; \]

\[ c_8 = -\frac{\beta + \delta}{d}; \quad c_9 = \frac{\beta(\beta + \delta)(\beta + 2\gamma + \delta)}{d^2}. \]
References


