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Mohammad Rahdar
St. Ambrose University

Lizhi Wang
Iowa State University, lzwang@iastate.edu

Guiping Hu
Iowa State University, gphu@iastate.edu

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Keywords

Inventory control, Uncertain demand, Uncertain lead time, Tri-level optimization model, Supply chain management

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A Tri-Level Optimization Model for Inventory Control with Uncertain Demand and Lead Time

Mohammad Rahdar[a,*], Lizhi Wang[b], Guiping Hu[b]

[a] *Department of Engineering and Physics, St. Ambrose University, Davenport, IA 52803*
[b] *Department of Industrial and Manufacturing Systems Engineering, Iowa State University, Ames, IA 50011*

Abstract

We propose an inventory control model for an uncapacitated warehouse in a manufacturing facility under demand and lead time uncertainty. The objective is to make ordering decisions to minimize the total system cost. We introduce a two-stage tri-level optimization model with a rolling horizon to address the uncertain demand and lead time regardless of their underlying distributions. In addition, an exact algorithm is designed to solve the model. We compare this model in a case study with three decision-making strategies: optimistic, moderate, and pessimistic. Our computational results suggest that the performances of these models are either consistently inferior or highly sensitive to cost parameters (such as holding cost and shortage cost), whereas the new tri-level optimization model almost always results in the lowest total cost in all parameter settings.

Keywords: Inventory control, Uncertain demand, Uncertain lead time, Tri-level optimization model, Supply chain management

1. Introduction

Uncertainty along a supply chain network is ubiquitous; it may arise for the arrival of raw materials or it may appear over customer demands. Since

*Corresponding author
Email addresses: rahdar@sau.edu (Mohammad Rahdar), lzwang@iastate.edu (Lizhi Wang), gphu@iastate.edu (Guiping Hu)

the stakeholders along the supply chain are interconnected, inventory systems are often complicated concerning uncertainty and variability. Several studies [1, 2, 3, 4] have mentioned that there are typically three sources of uncertainty in a supply chain: suppliers, manufacturing, and customers. Supplier uncertainty leads to variability in lead time and customer uncertainty appears in order time or quantity, both of which would cause unexpected costs.

Regarding to analytical approaches, models of inventory control can be classified into four types: (i) deterministic, (ii) stochastic demand and fixed lead time, (iii) fixed demand and stochastic lead time, and (iv) stochastic demand and lead time. Most studies on inventory control systems focused on deterministic models or addressing uncertainty from either the demand or supply side. [5] and [6] proposed a model with a central warehouse and several retailers to estimate the optimal reorder point when the demand was uncertain. [7] studied a supply chain including a manufacturer, a distributor, and a retailer with an uncertain demand to minimize the total system cost.[8] introduced a capacitated lot-sizing problem under stochastic demands. In addition, [9] considered a two-level supply chain with one warehouse and multiple retailers and assumed that retailers faced independent Poisson demand processes. Moreover, in the model proposed by [10], demand rate for perishable products was a random variable following a normal distribution. On the other hand, significant research has been also done to address the uncertainty of lead time. [11] proposed a model to minimize the total cost of an integrated vendor-buyer supply chain when the lead time is stochastic. Furthermore, [12] assumed that the lead time was an independent random variable from a normal distribution. [13] developed an inventory model where the lead time was a random variable which followed either normal or exponential distributions. Another approach of considering lead time was described by [14], who developed a finite time horizon inventory model with interval-valued lead time. Few studies have been devoted to addressing uncertainty from both suppliers and customers. However, both sources of uncertainty and their interactions could have convoluted implications to the entire supply chain. In this paper, we propose a new inventory control model that takes into account both lead time uncertainty and demand variability.

It has been shown that if the probabilistic description of randomness is available, stochastic programming is an effective tool to address uncertainty, but this information is not always available in real applications [15, 16]. As reported by [17], supply chain models with stochastic parameters can be classified into two main approaches, probabilistic approach and scenario approach. When there is probability information about uncertain parameters, the parameters can be considered as random variables in the probabilistic approach. Otherwise, uncertainty can be characterized by defining a set of scenarios, which represents a number of potential future states [17]. This paper presents a novel method for multi-period decision-making problems with uncertainty, which balances the curse of dimensionality and the robustness of the solution.

We introduce an inventory control model for a warehouse in a manufacturing facility, which orders one part to make one product. Although in reality manufacturers use multiple parts to produce multiple products, there are realistic circumstances where our assumption is reasonable. The one product assumption is a common one [18, 19, 20, 21, 22] since the production lines of multiple products are usually separate and independent. Furthermore, many manufacturers order parts in the unit of kits, which contain an aggregated set of components and parts needed for the manufacture of a particular assembly of product. The goal is to define the order policy to minimize system costs. Demand and lead time are uncertain parameters, and the probability distributions are unknown. The only available information is that uncertain parameters are independent random variables that can take some values from their intervals. The assumption on unknown distributions of demand and lead time is motivated by the observations of real world demand and lead time dynamics, where the demand distribution is constantly changing and sensitive to unpredictable events, news, advertisement, and emerging competitors, and lead time distribution also varies depending on weather and time of the year. Therefore, historical distributions cannot be used as a reliable prediction of the future demand and lead time distributions. Similar assumptions have been made in many other studies, such as [23, 24]. In addition, the shortage is allowed and fully backlogged. The objective is to determine the time and size of orders, such that the total cost, which consists of order, inventory holding, and shortage costs, is minimized. Since uncertain demand is observed in each period and the exact lead time is realized when the order arrives, it is a multi-stage decision-making problem and it suffers from the curse of dimensionality. Many researchers [25, 26, 27] work on alleviating the curse of dimensionality but we propose a new method in the concept of our problem to approximate the decision-making problem and reduce the curse of dimensionality by developing a two-stage tri-level optimization model. This simplified model is solved in a rolling horizon framework. Under this approach, the first stage decisions are implemented; then, the next planning horizon is planned with updated information [28]. There is a large body of literature [29, 30, 31, 32] on simulation-based optimization methods to improve the performance of an inventory system under uncertainty, which [32] use the rolling horizon approach to determine the safety stock level when demand is an uncertain parameter.

The contributions of this paper are as follows: First, unlike the most previously proposed models, we take into account uncertainty on both demand and supply sides. Second, we propose a new tri-level optimization model for the inventory control problem. It is an approximation of the multi-stage decision-making problem, which suffers from the curse of dimensionality, to keep computational tractability. Third, we design an exact algorithm for the tri-level optimization model to efficiently search for the worst-case scenario in the scenario space.

The remainder of the paper is organized as follows: in Section 2, detailed problem formulation is discussed. Section 3 is devoted to algorithm development. Section 4 presents the experimental results and sensitivity analysis. Finally, the conclusion with a summary is reported in Section 5.

2. Model formulation

2.1. Problem statement

We consider an uncapacitated warehouse for a single item in a manufacturing facility. The demand and lead time are both uncertain. Decisions are made over an indefinite discrete time period to minimize the order, inventory, and shortage costs. We assume that shortage is fully backlogged, demand and orders come at the beginning of the decision period, and the manager has full information about the demand, current inventory/shortage, and order arrival status to make an order decision for that period.

For modeling purposes, we label the current period as period 1 and we impose a finite planning horizon $\{1, 2, \ldots, T\}$. The solution from this model can be applied in a rolling horizon manner, in which the model is solved in each decision period with updated information and only the order decision for the current period is actually executed. This process is illustrated in Figure 1. The decision-making model $P(\tau)$ has a planning horizon from period τ to $\tau + T - 1$. After solving the decision-making model $P(\tau)$, and determining the order policy, we divide the decision of the planning horizon into two parts: the decision of the first period, $\{\tau\}$, and the decision of the second period and afterward, $\{\tau + 1, \ldots, \tau + T - 1\}$. Order policy of period τ is implemented and τ is increased by 1, the initial parameters of the next planning horizon are updated, and the model is run again. Therefore, the decision of periods $\{\tau + 1, \ldots, \tau + T - 1\}$ may reschedule in the next planning horizon. Solid lines in Figure 1 indicate the fixed decisions.

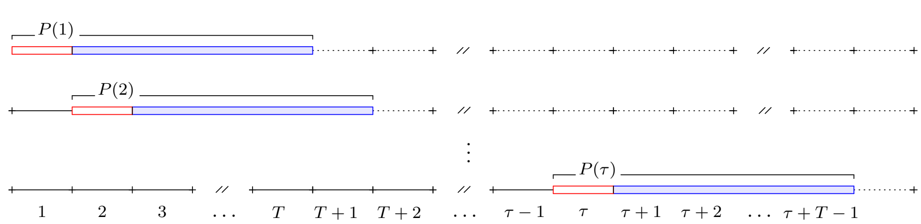

Figure 1: Rolling horizon approach

The fidelity of the aforementioned planning model largely depends on the planning horizon parameter T. From a computational tractability perspective, due to the well-known curse of dimensionality [33], multi-stage decision-making models with $T \geq 3$ are notoriously hard to solve. From a practical perspective, however, models with such a small planning horizon are systemically short-sighted and may yield solutions that are too myopic to be practically useful. Our proposed approach is a tri-level optimization model that represents a compromise between these two competing perspectives. In the remainder of the section, we first give the deterministic version of the planning model in Section 2.2 and then introduce the tri-level optimization model in Section 2.3.

2.2. Deterministic model

Consider a simplified version of the inventory control model where the demand and lead time in all periods are assumed to be constant and known. As such, the multi-stage decision-making problem reduces to a deterministic single stage optimization model.

Table 1 includes the notations used in formulating the deterministic model. It is worth noting that the random lead time is represented by a set of binary parameters $\delta_{k,t}, \forall k, t$, indicating whether or not the order made in period k arrives by period t. For example, if the lead time of an order made in period 3 is 4, then $\delta_{3,4} = \delta_{3,5} = \delta_{3,6} = 0$ and $\delta_{3,t} = 1, \forall t \in \{7, 8, \ldots T\}$.

The deterministic inventory control model is given in (1a)-(1d). The objective of the model is to minimize the total cost over the planning horizon. The four cost terms in (1a) are the variable order cost, fixed order cost, inventory holding cost, and shortage cost, respectively. Equation (1b) calculates the inventory level at the end of period t. The four terms on the right-hand-side of Constraint (1b) are, respectively, the initial inventory at period 0, the total amount of ordered items that arrive by period t, the amount of shortage at period t, and the total amount of demand that is served between periods 1 and t. Constraint (1c) ensures that a fixed order cost is incurred if at least one item is ordered in that period. The supports of the decision variables are defined in

Table 1: Notation in the deterministic model

Decision variables	
$q_t \in \mathbb{Z}^+$	Number of batches ordered in period t, $\forall t \in \{1, 2, \ldots, T\}$
$I_t \in \mathbb{Z}^+$	Inventory level in period t, $\forall t \in \{1, 2, \ldots, T\}$
$g_t \in \mathbb{Z}^+$	Shortage amount in period t, $\forall t \in \{1, 2, \ldots, T\}$
$v_t \in \{0, 1\}$	Indicating whether an order is placed in period t ($v_t = 1$) or not ($v_t = 0$), $\forall t \in \{1, 2, \ldots, T\}$
Parameters	
c	Variable order cost
f	Fixed order cost
h	Inventory holding cost
p	Shortage cost
T	Number of periods in the planning horizon
M	A sufficiently large positive number (big-M)
μ	Order batch size
I_0	Initial inventory level at the beginning the planning horizon
K	Number of periods before the planning horizon with orders on the way
q_k	Number of batches ordered in period k, $\forall k \in \{1 - K, \ldots, -1, 0\}$ before the planning horizon
$\hat{d}_t$	Assumed demand of period t, $\forall t \in \{1, \ldots, T\}$
$\hat{\delta}_{k,t}$	Assumed order arrival status, indicating whether ($\hat{\delta}_{k,t} = 1$) or not ($\hat{\delta}_{k,t} = 0$) the order made in period k arrives by period t, $\forall k \in \{1 - K, \ldots, t - 1\}, \forall t \in \{k + 1, \ldots, T\}$

Constraint (1d).

$$\min \quad \zeta = c\mu \sum_{t=1}^{T} q_t + f \sum_{t=1}^{T} v_t + h \sum_{t=1}^{T} I_t + p \sum_{t=1}^{T} g_t \tag{1a}$$

$$\text{s.t.} \quad I_t = I_0 + \sum_{k=1-K}^{t-1} \mu q_k \hat{\delta}_{k,t} + g_t - \sum_{i=1}^{t} \hat{d}_i \qquad t \in \{1, 2, \ldots, T\} \tag{1b}$$

$$q_t \leq M v_t \qquad t \in \{1, 2, \ldots, T\} \tag{1c}$$

$$q_t, I_t, g_t \in \mathbb{Z}^+; v_t \in \{0, 1\} \qquad t \in \{1, 2, \ldots, T\} \tag{1d}$$

2.3. Tri-level optimization model

Relaxing the simplifying assumptions on perfect information of demand and lead time results in a multi-stage decision-making problem, in which uncertain demand is observed in each period but the exact lead time is not realized until when the order arrives. We propose a two-stage tri-level optimization model to approximate the multi-stage decision-making problem and to alleviate its curse of dimensionality. The first stage refers to the first period of the planning horizon, whereas all the remaining periods are aggregated into the second stage; a similar modeling approach has been suggested by [33]. As such, after the first stage decision has been made, all uncertain parameters for period 2 and beyond are assumed to be observable, and thus, the second stage becomes a deterministic problem. We further assume that the first stage will take a pessimistic view of uncertainty and anticipate the worst-case scenario for the second stage. Therefore, the two-stage decision-making model is formulated as a tri-level optimization model, in which the upper level makes the first stage decision, the middle level identifies the worst-case scenario given the first stage decision, and the lower level makes the second stage decision given the first stage decision and under the worst-case scenario. The first period is solved with the top level, but it does not ignore further periods demands. In fact, the top level anticipates the worst scenario of demand and lead time (identified by the middle level) as well as how the further periods would respond to it (calculated by the bottom level) and then makes its decisions accordingly. This simplified model may become more appropriate in a rolling horizon framework [34], in which the tri-level model is solved in every period with updated information, but only the first stage decisions are implemented.

The tri-level optimization model addresses two limitations of the stochastic programming model. First, the distributional information is not available or reliable for estimating the demand and lead time for the future; the proposed model is a more applicable approach here because it does not rely on probabilities information. Second, the two-stage decision-making problem is an under-estimation of the true cost of the multi-stage problem by assuming that all uncertainty outcomes will become observable in the second period. Although the tri-level model also uses a two-stage framework and underestimates the true cost, the worst-case scenario consideration offsets this effect. Also, the definition of the worst-case is adjustable to accommodate different risk tolerances of decision makers. The determination of the first stage decisions is a trade-off between a pessimistic anticipation of the worst-case scenario and an optimistic assumption of perfect information throughout the rest of the planning horizon.

The tri-level optimization model is developed using notations defined in Table 2. The assumption is that demands and lead times are uncertain, but we know the lower and upper bounds of these uncertain parameters, which are time dependent and independent of each other. It should be noted that for $t \in \{2, \ldots, T\}$, demand and order arrival status were defined as parameters in Table 1, but they become the middle level decision variables in the tri-level optimization model. The objective of the middle level is to identify a scenario that will result in the highest cost to the bottom level. The middle level of the model does not represent the decision of a person; rather, it reflects the essence of robust optimization, which is to identify the worst-case scenario so that the top level can make appropriate decisions to hedge against such scenario.

Table 2: Notation in the tri-level model

Decision variables for the upper level	
$q_1 \in \mathbb{Z}^+$	Number of batches ordered in period 1
$I_1 \in \mathbb{Z}^+$	Inventory level in period 1

$g_1 \in \mathbb{Z}^+$	Shortage amount in period 1
$v_1 \in \{0, 1\}$	Indicating whether an order is placed in period 1 ($v_1 = 1$) or not ($v_1 = 0$)
x	Aggregated upper level decision variables, $x = [q_1, I_1, g_1, v_1]^\top$

Decision variables for the middle level

$d_t \in \mathbb{Z}^+$	Demand of period $t, \forall t \in \{2, \ldots, T\}$
$\delta_{k,t} \in \{0, 1\}$	Order arrival status, indicating whether ($\delta_{k,t} = 1$) or not ($\delta_{k,t} = 0$) the order made in period $k, \forall k \in \{1-K, \ldots, T-1\}$ arrives by period $t, \forall t \in \{k+1, \ldots, T\}$
y	Aggregated middle level decision variables, $y = [d_2, \ldots, d_T, \delta_{1-K,2}, \ldots, \delta_{1,2}, \delta_{1-K,3}, \ldots, \delta_{2,3}, \ldots, \delta_{T-1,T}]^\top$

Decision variables for the lower level

$q_t \in \mathbb{Z}^+$	Number of batches ordered in period t, $\forall t \in \{2, 3, \ldots, T\}$
$I_t \in \mathbb{Z}^+$	Inventory level in period t, $\forall t \in \{2, 3, \ldots, T\}$
$g_t \in \mathbb{Z}^+$	Shortage amount in period t, $\forall t \in \{2, 3, \ldots, T\}$
$v_t \in \{0, 1\}$	Indicating whether an order is placed in period t ($v_t = 1$) or not ($v_t = 0$), $\forall t \in \{2, 3, \ldots, T\}$
z	Aggregated lower level decision variables $z = [q_2, \ldots, q_T, I_2, \ldots, I_T, g_2, \ldots, g_T, v_2, \ldots, v_T]^\top$

Parameters

c	Variable order cost
f	Fixed order cost
h	Inventory holding cost
p	Shortage cost
T	Number of periods in the planning horizon
M	A sufficiently large positive number (big-M)
K	Number of periods before the planning horizon with orders on the way
μ	Order batch size
I_0	Initial inventory level at the beginning the planning horizon

l^{D}	Lower bound of demand
u^{D}	Upper bound of demand
l^{L}	Lower bound of lead time
u^{L}	Upper bound of lead time
$\tilde{d}_1$	Observed demand of period 1
$\tilde{\delta}_{k,1}$	Observed order arrival status, indicating whether ($\tilde{\delta}_{k,1} = 1$) or not ($\tilde{\delta}_{k,1} = 0$) the order made in period $k, \forall k \in \{1-K, \ldots, -1, 0\}$ arrives by period 1.
c_1	Aggregated objective function coefficients of the first stage decisions, $c_1 = [c\mu, f, h, p]^\top$
c_2	Aggregated objective function coefficients of the second stage decisions, $c_2 = [c\mu, \ldots, c\mu, f, \ldots, f, h, \ldots, h, p, \ldots, p]^\top$

Using the notations of aggregated decision variables and parameters, we formulate the tri-level optimization model as follows.

$$\min_{x \in \mathcal{X}} \left\{ c_1^\top x + \max_{y \in \mathcal{Y}(x)} \left\{ \min_{z \in \mathcal{Z}(x,y)} \quad c_2^\top z \right\} \right\} \tag{2}$$

Here, the lower level solves a deterministic problem, $\min_{z \in \mathcal{Z}(x,y)} c_2^\top z$, to minimize the total cost for periods 2 to T given the first stage order decision, x, made at the upper level and the worst-case scenario, y, identified by the middle level. The feasible set $\mathcal{Z}(x, y)$ is defined as

$$\mathcal{Z}(x,y) = \left\{ \begin{array}{lll} z: & I_t = I_0 + \sum_{k=1-K}^{t-1} \ \mu q_k \delta_{k,t} + g_t - \sum_{i=1}^{t} d_i & \forall t \in \{2, 3, \ldots, T\} \\ & q_t \leq M v_t & \forall t \in \{2, 3, \ldots, T\} \\ & q_t, I_t, g_t \in \mathbb{Z}^+, v_t \in \{0, 1\} & \forall t \in \{2, 3, \ldots, T\} \end{array} \right\}. \tag{3}$$

Notice that the term $\sum_{k=1-K}^{t-1} \ \mu q_k \delta_{k,t}$ is nonlinear, since both q_k and $\delta_{k,t}$ are part of decision variables z and y, respectively. We will linearize this term in Section 3.

The middle level observes the order decision, x, made at the upper level and solves a bilevel optimization model, $\max_{y \in \mathcal{Y}(x)} \left\{ \min_{z \in \mathcal{Z}(x,y)} \quad c_2^\top z \right\}$, to identify the worst-case scenario, anticipating the response of the lower level. The feasible set $\mathcal{Y}(x)$ is defined as

$$\mathcal{Y}(x) = \left\{ \begin{array}{lll} y: & \tilde{\delta}_{k,1} \leq \delta_{k,2} & \forall k \in \{1-K, \ldots, 0\} \\ & \delta_{k,t} \leq \delta_{k,t+1} & \forall k \in \{1-K, \ldots, T-2\}, \forall t \in \{\max\{k+1, 2\}, \ldots, T-1\} \\ & l^{\mathrm{D}} \leq d_t \leq u^{\mathrm{D}} & \forall t \in \{2, 3, \ldots, T\} \\ & l^{\mathrm{L}} \leq 1 + \sum_{t=k+1}^{T} (1 - \delta_{k,t}) \leq u^{\mathrm{L}} & \forall k \in \{1-K, \ldots, T-1\} \\ & d_t \in \mathbb{Z}^+ & \forall t \in \{2, 3, \ldots, T\} \\ & \delta_{k,t} \in \{0, 1\} & \forall k \in \{1-K, \ldots, T-1\}, \forall t \in \{\max\{k+1, 2\}, \ldots, T\} \end{array} \right\}.$$

The first and second constraints ensure that once an order arrives in period t, all subsequent status variables must be set as $\delta_{k,\tau} = 1, \forall \tau \geq t$. The third and fourth constraints set the lower and upper bounds for demand and lead time in the second stage periods, respectively.

The upper level solves the tri-level optimization model (2), which minimizes the combined cost terms for period 1, $c_1^\top x$, and for the rest of the planning horizon, $c_2^\top z$, anticipating the response from the middle and lower levels. The feasible set $\mathcal{X}$ is defined as

$$\mathcal{X} = \left\{ \begin{array}{ll} x: & I_1 = I_0 + \sum_{k=1-K}^{0} \mu q_k \tilde{\delta}_{k,1} + g_1 - \tilde{d}_1 \\ & q_1 \leq M v_1 \\ & q_1, I_1, g_1 \in \mathbb{Z}^+, v_1 \in \{0, 1\} \end{array} \right\}.$$

2.4. Stochastic programming model

Stochastic programming is a common approach to formulate optimization problems that involve uncertainty and have different time restriction of decisions. We also develop a two-stage stochastic programming model to compare with the tri-level optimization model. Many researchers [35, 36, 37] developed stochastic programming models to improve the performance of inventory systems. In the stochastic programming model, the problem is formulated over a finite set of scenarios, s, each with an associated probability. The first-stage decisions are made based on the expected value of the second-stage decisions. We applied this approach in the context of the tri-level model; the formulation of the stochastic programming model is as follows.

$$
\begin{aligned}
\min \quad & \zeta = c\mu q_1 + f v_1 + h I_1 + p g_1 && \text{(4a)}\\
& + \mathbf{E}\left[c\mu \sum_{t=2}^{T} q_t^s + f \sum_{t=2}^{T} v_t^s + h \sum_{t=2}^{T} I_t^s + p \sum_{t=2}^{T} g_t^s \right] && \\
\text{s.t.} \quad & I_1 = I_0 + \sum_{k=1-K}^{0} \mu q_k \hat{\delta}_{k,1} + g_1 - \hat{d}_1 && \text{(4b)}\\
& q_1 \leq M v_1 && \text{(4c)}\\
& I_t^s = I_0 + \sum_{k=1-K}^{t-1} \mu q_k^s \hat{\delta}_{k,t}^s + g_t^s - \sum_{i=1}^{t} \hat{d}_i^s \qquad t \in \{2,3,\ldots,T\}, \forall s && \text{(4d)}\\
& q_t^s \leq M v_t^s \qquad t \in \{2,3,\ldots,T\}, \forall s && \text{(4e)}\\
& q_1, I_1, g_1 \in \mathbb{Z}^+; v_1 \in \{0,1\} && \text{(4f)}\\
& q_t^s, I_t^s, g_t^s \in \mathbb{Z}^+; v_t^s \in \{0,1\} \qquad t \in \{1,2,\ldots,T\}, \forall s && \text{(4g)}
\end{aligned}
$$

3. Algorithm design

We define $\tilde{\mathcal{Y}} = \bigcup_{x \in \mathcal{X}} \mathcal{Y}(x)$ and let $\{y^i : \forall i \in \mathcal{I}\}$ denote all the elements in set $\tilde{\mathcal{Y}}$, where $\mathcal{I}$ is the set of superscripts for y^i with $|\mathcal{I}| = |\tilde{\mathcal{Y}}|$. Then model (2) is equivalent to

$$
\min_{x,z,\xi} \left\{ c_1^\top x + \xi : x \in \mathcal{X}; \xi \geq c_2^\top z^i, z^i \in \mathcal{Z}(x, y^i), \forall i \in \mathcal{I} \right\}. \tag{5}
$$

Here, instead of treating the worst-case scenario y as a decision variable for the middle level, we consider all possible scenarios of $y^i, \forall i \in \mathcal{I}$ as given parameters and define a response variable z^i for each possible scenario y^i. The constraints $\xi \geq c_2^\top z^i, \forall i \in \mathcal{I}$ and the objective function $c_1^\top x + \xi$ ensure that only the worst-case scenario cost is being minimized. As such, the middle level is eliminated, and the upper and lower levels merge into one single level optimization model (5). This reformulation is challenged by the potentially enormous number of additional decision variables z^i and constraints, which may make it computationally intractable.

We propose an exact algorithm for model (2) by using the reformulation (5) and overcoming the challenges with its dimensions. The steps of the algorithm are described in Algorithm 1. The idea is to solve model (5) with a small subset $\hat{\mathcal{Y}} \subseteq \tilde{\mathcal{Y}}$ of scenarios, which is a relaxation of (5), and iteratively add new scenarios. Such scenarios are generated in line 10 by solving the middle and lower levels with fixed upper level decisions from the relaxation solution. The resulting bilevel model either confirms the optimality of the upper level decision or yields a worst-case scenario that will be included in $\hat{\mathcal{Y}}$ in the next iteration.

Algorithm 1 Algorithm of solving the tri-level model (2)

1: Inputs: $\mathcal{X}, \tilde{\mathcal{Y}}$, and $\mathcal{Z}(x,y), \forall x \in \mathcal{X}, y \in \tilde{\mathcal{Y}}$
2: Initialize $(x^*, y^*, z^*) = \emptyset, \zeta^{\mathrm{L}} = -\infty, \zeta^{\mathrm{U}} = \infty$
3: Identify a set $\hat{\mathcal{Y}}$ such that $\emptyset \subset \hat{\mathcal{Y}} \subseteq \tilde{\mathcal{Y}}$ and define $\hat{\mathcal{I}} = \{i : \forall y^i \in \hat{\mathcal{Y}}\}$
4: **while** $\zeta^{\mathrm{L}} < \zeta^{\mathrm{U}}$ **do**
5: Solve the following Master problem
$\mathcal{M}(\hat{\mathcal{I}})$: $\min_{x,z,\xi} \left\{ c_1^\top x + \xi : x \in \mathcal{X}; \xi \geq c_2^\top z^i, z^i \in \mathcal{Z}(x, y^i), \forall i \in \hat{\mathcal{I}} \right\}$
6: **if** infeasible **then**
7: Return model (2) is infeasible
8: **else**
9: Let $(\hat{x}, \hat{\xi})$ denote the corresponding components of an optimal solution
10: Solve the following Subproblem $\mathcal{S}(\hat{x})$: $\max\limits_{y \in \mathcal{Y}(\hat{x})} \left\{ \min\limits_{z \in \mathcal{Z}(\hat{x}, y)} c_2^\top z \right\}$ and let $(\hat{y}, \hat{z})$ denote an optimal solution
11: Update $\zeta^{\mathrm{L}} \leftarrow c_1^\top \hat{x} + \hat{\xi}$, $\zeta^{\mathrm{U}} \leftarrow \max\{\zeta^{\mathrm{U}}, c_1^\top \hat{x} + c_2^\top \hat{z}\}$, $\hat{\mathcal{Y}} \leftarrow \hat{\mathcal{Y}} \cup \{\hat{y}\}$, and $\hat{\mathcal{I}} \leftarrow \{i : \forall y^i \in \hat{\mathcal{Y}}\}$
12: **end if**
13: **end while**
14: Return $x^* = \hat{x}, y^* = \hat{y}, z^* = \hat{z}$

Since y and z are treated as variables in the Subproblem $\mathcal{S}(\hat{x})$, the multiplication of q_k and $\delta_{k,t}$ introduces nonlinearity to the set $\mathcal{Z}(\hat{x}, y)$, which was defined in (3). To linearize the set $\mathcal{Z}(\hat{x}, y)$, we introduce new variables $u_{k,t} =$

$q_k\delta_{k,t}, \forall k \in \{2,...,T-1\}, t \in \{k+1,...,T\}$. Accordingly, we add four new sets of constraints. Variable $u_{k,t}$ is equal to q_k if the order made in period k arrives by period t; otherwise, it is zero. The linearized set $\mathcal{Z}(\hat{x}, y)$, denoted as $\tilde{\mathcal{Z}}(\hat{x}, y)$, is defined as follows.

$$\tilde{\mathcal{Z}}(\hat{x}, y) = \left\{ \begin{array}{lll} z: & I_t = I_0 + \sum_{k=1-K}^{1} \mu q_k \delta_{k,t} + \sum_{k=2}^{t-1} \mu u_{k,t} + g_t - \sum_{i=1}^{t} d_i & t \in \{2,3,\ldots,T\} \\ & q_t \leq M v_t & t \in \{2,3,\ldots,T\} \\ & u_{k,t} \geq q_k - M(1-\delta_{k,t}) & k \in \{2,\ldots,T-1\}, t \in \{k+1,\ldots,T\} \\ & u_{k,t} \leq M\delta_{k,t} & k \in \{2,\ldots,T-1\}, t \in \{k+1,\ldots,T\} \\ & u_{k,t} \leq q_k & k \in \{2,\ldots,T-1\}, t \in \{k+1,\ldots,T\} \\ & u_{k,t} \geq 0 & k \in \{2,\ldots,T-1\}, t \in \{k+1,\ldots,T\} \\ & q_t, I_t, g_t \in \mathbb{Z}^+, v_t \in \{0,1\} & t \in \{2,3,\ldots,T\} \end{array} \right\}$$

The resulting Subproblem $\mathcal{S}(\hat{x})$ is a bi-level integer linear programming problem, which can be solved by existing algorithms such as [38]. The algorithm is able to find the optimal solution to model (2) in no more than $(|\mathcal{X}|+1)$ iterations, which is a finite number since $\mathcal{X}$ is a finite set. For all $i \in \{1,\ldots,|\mathcal{X}|+1\}$, let $\hat{x}^i$ denote the solution from line 9 in the ith iteration, then there must exist $1 \leq j < k \leq |\mathcal{X}|+1$ such that $\hat{x}^j = \hat{x}^k$.

4. Computational experiments

We conducted an experiment to test and compare the performances of the tri-level optimization model, stochastic programming model, and three decision-making strategies, which we will refer to as optimistic, moderate, and pessimistic models. We also run a deterministic model with the perfect information of demand and lead time, which we call it the perfect model, to estimate the performance of other five models in different cost parameter settings. In this section, first, we explain how the computational experiments are set up and then, we present the experimental results and sensitivity analysis.

4.1. Simulation setup

The planning horizon is $T = 5$ periods and the simulation run is $\mathcal{T}$ periods, which is different for each instance. We ran each model $\mathcal{T}$ times through the simulation experiment from $\tau = 1$ to $\tau = \mathcal{T}$. Order policy of period τ is implemented, the total cost of period τ is saved, and τ is increased by 1 to run the model again. Each box, $P(\tau)$, in Figure 2 represents a decision-making model for period τ, which has a planning horizon of $\{\tau, \tau+1, \ldots, \tau+T-1\}$. The downward arrows into the box represent observed realizations of uncertain demand, $\tilde{d}_\tau$, and order arrival status, $\{\tilde{\delta}_{\tau-K,\tau}, \tilde{\delta}_{\tau-K+1,\tau}, \ldots, \tilde{\delta}_{\tau-1,\tau}\}$. Here, the binary uncertainty parameter $\tilde{\delta}_{k,\tau}$ indicates whether ($\tilde{\delta}_{k,\tau} = 1$) or not ($\tilde{\delta}_{k,\tau} = 0$) the items that were ordered in period k arrive in or before period τ. The horizontal arrows into the box $P(\tau)$ represent decisions made in the previous period $\tau - 1$, including the inventory level $I_{\tau-1}$, shortage level $g_{\tau-1}$, and the order decisions made in the past K periods, $\{q_{\tau-K}, q_{\tau-K+1}, \ldots, q_{\tau-1}\}$, where K is the upper bound on the uncertain lead time. These previously made decisions are used as parameters in $P(\tau)$.

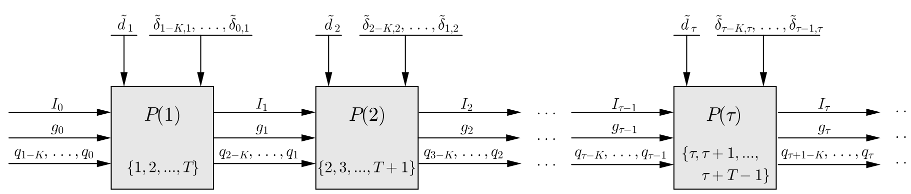

Figure 2: Planning horizons with length T in the simulation run

The five demand patterns used in the experiment are real world demand data from the Federal Reserve Bank of St. Louis and Census Bureau which is a part of U.S. Department of Commerce; the source of the demand data is reported in Table 3. We used the data for a time horizon of $\mathcal{T} + 2T - 1$ periods. We need T periods before the first planning horizon to be used as the historical data for the first planning horizons and $T - 1$ periods after the period $\mathcal{T}$ to have a complete planning horizon $P(\tau)$. The random lead times $\tilde{L}_k, \forall k \in \{-4, -3, -2, -1, 0, 1 \ldots, \mathcal{T} + 4\}$ were generated from a uniform distribution but never used directly in any of the models; rather, they were used to calculate the order arrival statuses $\tilde{\delta}_{k,t}$.

Since the ratio of h/p has a key role in inventory control models, we conducted a total of 5 sets of experiments with $h/p = \{0.1, 0.3, 0.5, 0.7, 0.9\}$ when $h = 5$ and $c = 1$ for each of the six models and 5 instances. We generated random values for lead times and used the demand data explained in Table 3, in which some of the data are in million dollars, so we converted them to the number of units through dividing them by one unit price. It is also assumed that there is no order on the way in period 1 but the initial inventory is enough to satisfy the demand of the first two periods.

Table 3: Experiment data

c	1
f	0
h	5
p	{5.6,7.1,10,16.7,50} to have the ratio $h/p = \{0.1, 0.3, 0.5, 0.7, 0.9\}$
T	5
μ	1
K	2
$\tilde{d}_t(1)$	Demand of instance 1 for period $t \in \{1, 2, ..., 491\}$ were used from [39].
$\tilde{d}_t(2)$	Demand of instance 2 for period $t \in \{1, 2, ..., 298\}$ were used from [40].
$\tilde{d}_t(3)$	Demand of instance 3 for period $t \in \{1, 2, ..., 238\}$ were used from [41].
$\tilde{d}_t(4)$	Demand of instance 4 for period $t \in \{1, 2, ..., 126\}$ were used from [42].
$\tilde{d}_t(5)$	Demand of instance 5 for period $t \in \{1, 2, ..., 51\}$ were used from [43].
$\tilde{L}_k$	Integer randomly generated from a uniform distribution within $[1, 2]$, representing the lead time for the order placed in period $k, \forall k \in \{-4, -3, -2, -1, 0, 1 \ldots, \mathcal{T} + 4\}$

The perfect model, optimistic, moderate, and pessimistic models use the same formulation (1) but with different assumptions about the data; the perfect model uses the real data and other three models use the historical data to predict the uncertain parameters demand and lead time. The optimistic model uses the minimum value of the last T periods, the moderate model uses the arithmetic mean, and the pessimistic model uses the maximum value of the historical data for the last T periods. The tri-level optimization model determines the uncertain demands and lead times in the middle level of the model; it needs the bounds of the demand and the lead time, which are estimated by using the historical data of the last T periods. The stochastic programming model uses the formulation (4). We generated 50 demand and lead time scenarios by bootstrapping the historical data, which were updated for each planning horizon. Demand data were collected from historical sales data of vehicles, computers, lumber, etc. and lead time data were generated from a uniform distribution. We did conduct a sensitivity analysis to make sure that its well parameterized. To identify an appropriate number of scenarios, we tested different numbers (from 20 to 150) and solved the stochastic programming model. We found that 50 scenarios gave a good trade-off between low computational time and reasonable representation of the scenario space.

4.2. Simulation results

Simulation results demonstrate that the tri-level model on average has lower total cost than the stochastic programming model and other three decision-making strategies for the different ratios of h/p. To conduct a sensitivity analysis, we ran all six models in different cost parameter settings for 5 instances and showed the results in Figure 3. This figure represents a combination of two cost parameters of h and p. Each column of graphs represents one instance and the first row shows the demand pattern on each instance. The next rows of graphs illustrate cost parameter settings for different ratio of h/p. The vertical axes of bar charts show the gap between each model and the perfect model and it is calculated by diving the solution of each model by the solution of the perfect model minus one. Therefore, if it is lower, means that the model has the solution closer to the perfect model. For example, for the graph in the middle (the third instance when $h/p = 0.5$), the solution of the tri-level optimization model is 1.39e7 and the solution of the perfect model is 0.85e7, so the bar chart value of the tri-level model in this graph is $(1.39e7/0.85e7) - 1 = 64\%$. The average gap between the five models (tri-level optimization, stochastic programming, optimistic, moderate, and pessimistic models) and the perfect model over all parameter settings and instances are 71.4%, 84.7%, 326.9%, 203.5%, and 79.5%, respectively.

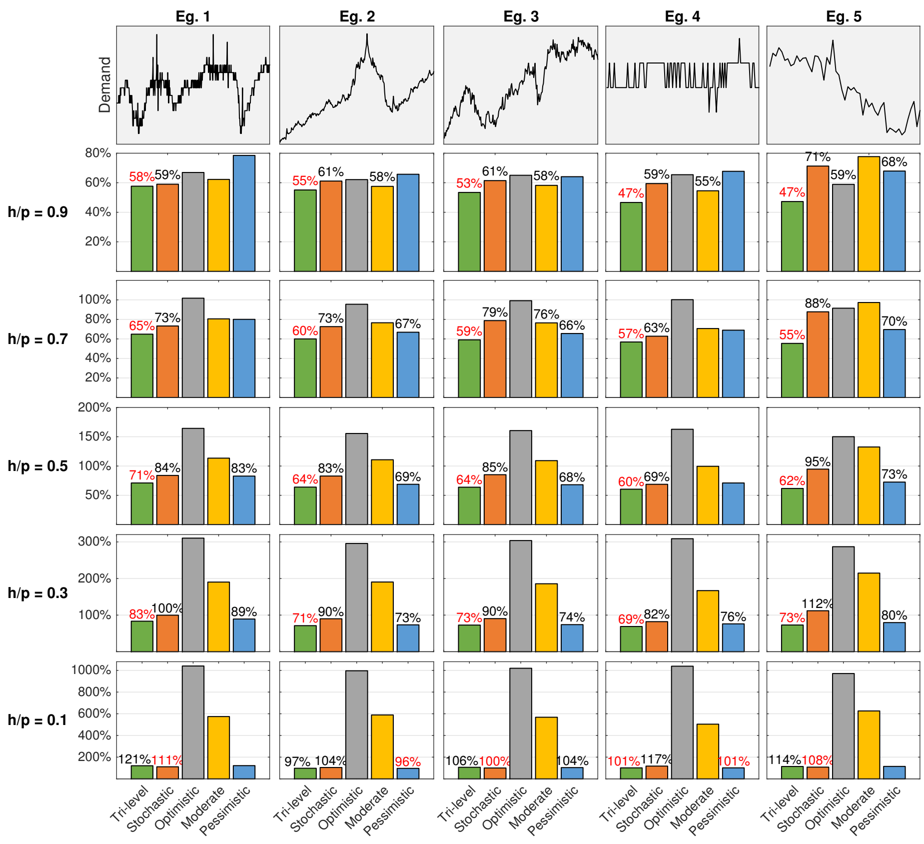

Figure 3: The comparison of tri-level optimization, stochastic programming, optimistic, moderate, and pessimistic models

We also show the results of the numerical experiment in another perspective to illustrate how much the total cost of the tri-level model is better or worse than other models. The relative performance of the tri-level model compared to the stochastic programming and three decision-making strategies is evaluated by the ratio $R = 100 \cdot (\text{mdl} - \text{tri})/\text{mdl}$, where "tri" is the average total cost of the tri-level model and "mdl" is the average total cost of the stochastic programming, optimistic, moderate, or pessimistic models over 5 instances. We plot and show the results regarding the performance ratio of total cost in Figure 4. When the performance ratio is positive, the tri-level model works better than the compared model; thus, a higher percentage means a higher relative performance of the tri-level model. For example, when $h/p = 0.3$, the average total cost of the tri-level model is better than the stochastic programming, optimistic, moderate, and pessimistic models by 10.6%, 56.7%, 39.8%, and 2.6%, respectively. The relative performance ratio is positive in all cases except the case when $h/p = 0.1$ in which the stochastic programming and the pessimistic model perform similarly to the tri-level optimization model. The optimistic model functions more effectively by increasing h/p because if h is high or p is low, it is better to have a lower inventory level and possibly more shortages. Conversely, the pessimistic model works more poorly when h/p is increased. It tends to have a higher inventory level by forecasting future demands and lead times as large as possible; thus, the total cost of this model is raised by increasing h/p.

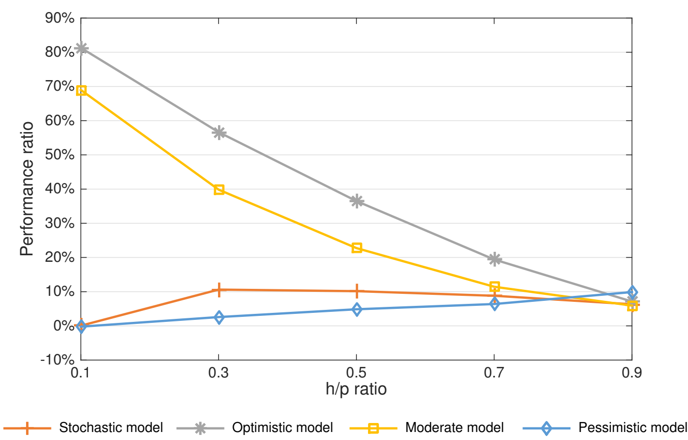

Figure 4: Impacts of holding and shortage costs on the relative performance ratio of the tri-level model (tri) compared to other models (mdl). The performance ratio: $R = 100 \cdot (\text{mdl} - \text{tri})/\text{mdl}$.

As mentioned in [44], some existing performance criteria in the literature are α, β, δ and γ service levels. We compared the β service level, which is fill rate, for the tri-level, stochastic programming, optimistic, moderate, and pessimistic models. Fill rate is the percentage of customer orders satisfied immediately from stock at hand. In general, it is improved by decreasing h/p ratio. The average fill rate of 5 instances for all models are shown in Figure 5. The fill rates of the optimistic, moderate and pessimistic models in all combinations of h and p are always equal to 72%, 83%, and 98%, respectively. When shortage cost is very high, that is $h/p = 0.1$, the fill rates of the tri-level optimization model and stochastic programming model are almost the same at 98%. However, by decreasing the shortage cost, the fill rate of the stochastic programming model drops to 85% while the fill rate of the tri-level optimization model reduces only by 2%.

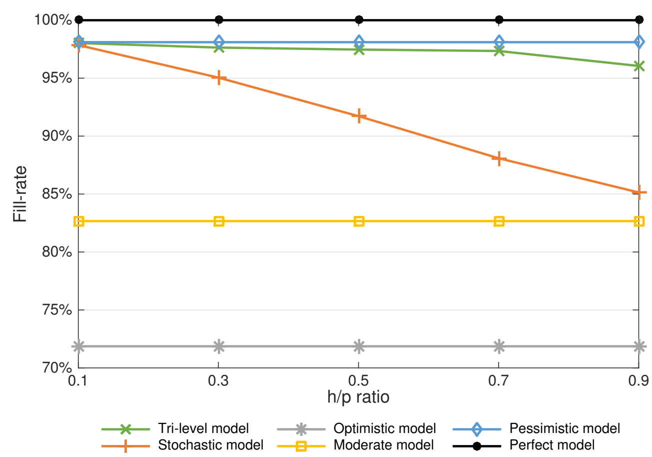

Figure 5: Impacts of holding and shortage costs on the fill-rate

The results of Figure 5 show that the tri-level optimization model does not appear to be as sensitive to the shortage cost as the stochastic model is, but it does respond to changes in shortage cost in a more subtle and efficient manner. To explain this observation, we broke down the total cost into the inventory holding and the shortage costs. The percentage of changes in inventory and shortage levels of all five examples when h/p ratio increases from 0.1 to 0.9 are summarized in Table 4. Positive and negative percentages indicate increase and decrease, respectively. Consider Example 1 in Table 4; when shortage cost decreases (the h/p ratio increases from 0.1 to 0.9), the tri-level model reduced the average inventory level per period (and the associated inventory cost) by 22% to take advantage of the reduced shortage cost. As a result, the fill-rate was reduced by 2%, and the average shortage level increased by 160% (from 0.34 to 0.90 units per period), but the shortage cost was reduced by 71% due to the dramatic drop in the shortage cost per unit. In contrast, the stochastic model's response to the reduced shortage cost was more dramatic. It reduced the inventory level by 51%, which reduced the fill-rate by 14% and caused a 961% increase in the shortage level (from 0.35 to 3.70 units per period) and 18% increase in the shortage cost. These changes reduce the total inventory holding and shortage cost of the tri-level optimization model and stochastic programming model by 36% and 31%, respectively. The last two rows of Table 4 report the average percentages over five examples.

Table 4: Changes in inventory/shortage levels/costs and fill-rate when h/p ratio increases from 0.1 to 0.9

		Inventory level	Shortage level	Inventory cost	Shortage cost	Total inventory and shortage cost	Fill-rate
Eg. 1	Tri-level	−22%	160%	−22%	−71%	−36%	−2%
	Stochastic	−51%	961%	−51%	18%	−31%	−14%
Eg. 2	Tri-level	−14%	175%	−14%	−69%	−27%	−2%
	Stochastic	−50%	1032%	−50%	26%	−27%	−15%
Eg. 3	Tri-level	−14%	126%	−14%	−75%	−32%	−2%
	Stochastic	−47%	1056%	−47%	28%	−25%	−14%
Eg. 4	Tri-level	−26%	200%	−26%	−67%	−35%	−3%
	Stochastic	−41%	631%	−41%	−19%	−32%	−13%
Eg. 5	Tri-level	−17%	51%	−17%	−83%	−38%	−1%
	Stochastic	−30%	762%	−30%	−4%	−22%	−9%
Average	Tri-level	−19%	143%	−19%	−73%	−34%	−2%
	Stochastic	−44%	888%	−44%	10%	−27%	−13%

To have a broader range of tested scenarios, we examined the performance of models for a time horizon of 30 periods and 250 repetitions. For this purpose, we used the first example demand data (491 periods) to pick demands of 39 consecutive periods for 250 repetitions. We generated 250 random numbers between 1 and 452 as the starting point to pick the demands of 39 periods $\{-4, -3, \ldots, 34\}$, of which only 30 periods in the middle $\{1, 2, \ldots, 30\}$ were used to test the models and measure their performances. Results of solving 250 repetitions demonstrate that the tri-level model on average has lower total cost than other four models for different combinations of holding and shortage costs. The sample probability distribution of total cost for 250 repetitions and different combinations of cost parameters are shown in Figure 6. The horizontal and vertical axes of each graph represent the total cost and the probability density, respectively. The three decision-making strategies (optimistic, moderate, and pessimistic models) have different performances in response to cost parameters. As can be seen from Figure 6, the optimistic and pessimistic models are sensitive to the h/p ratio. The performance of the pessimistic model is almost as good as that of the tri-level model when the h/p ratio is low but it deteriorates as the h/p ratio increases. In contrast, the performance of the optimistic model improves as the h/p ratio increases but it is always worse than the pessimistic model. The moderate model is almost always in between. When the h/p ratio is low, the stochastic programming model also performs similarly to the tri-level optimization model, which is identical to the outcome of Figures 3 and 4, but it is more sensitive to changes in the h/p ratio. The tri-level optimization model is not sensitive to the changes in the parameter setting and outperforms other models in almost all combinations of cost parameters.

Figure 6 shows that the performances of different models are similar when the holding cost is very low (compared with the shortage cost) because all the models decide to keep a high level of inventory and the timing and quantity of orders become less relevant. On the other hand, when the holding cost is very high, then the consequences of better decision-making on the timing and quantity of orders become much more critical. The proposed tri-level model is good at finding the right time to make orders, so it outperforms the other models when holding cost is relatively high.

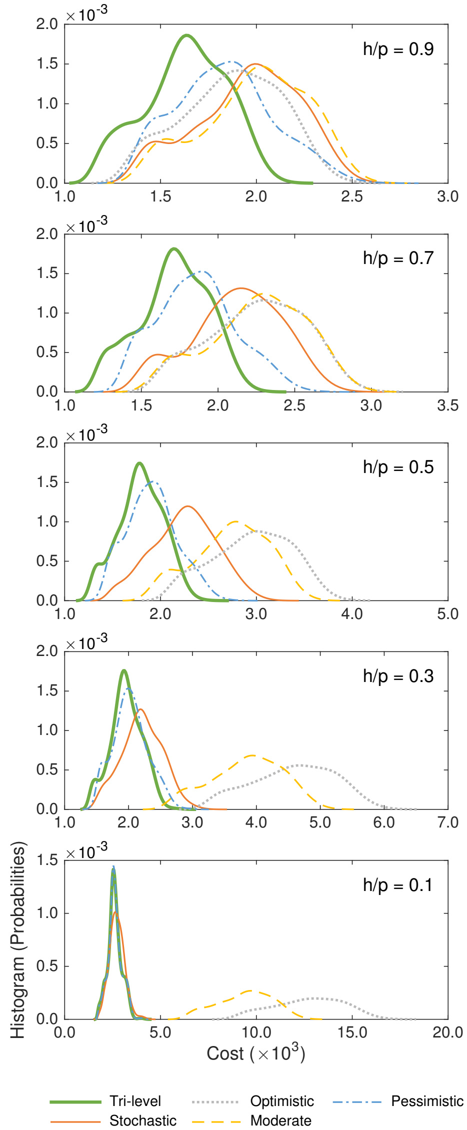

Figure 6: The sample probability distribution of the total costs for tri-level and four compared models with different h/p ratio

5. Conclusions

In this study, we propose a new approach to address uncertainty in a manufacturing facility which orders new items to satisfy demand. The demand and lead time are uncertain parameters, and shortages are fully backlogged. The objective is to make ordering decisions to minimize the total cost. This paper makes three contributions to the literature. First, we explicitly take into account two sources of uncertainty from both demand and lead time. Most previously proposed models focused on one of these two, but are still subject to significant uncertainty from the other source as well as the interactions of the two. Second, we propose a two-stage tri-level optimization model for the inventory control problem, which is a compromise between the accurate representation of the multi-stage decision-making under uncertainty nature of the problem and computational tractability. Third, we design an exact algorithm for the tri-level optimization model, which deploys a Benders decomposition framework to efficiently search for the worst-case scenario without enumerating the enormous scenario space.

The results suggest that the tri-level optimization model works more adaptively in response to the range of cost parameters. The performances of optimistic and pessimistic models are sensitive to the cost parameters, and the moderate model is almost always in between. The optimistic (pessimistic) model tends to have a higher shortage (inventory) level, so its performance is improved when h/p ratio is high (low). The stochastic programming model tries to achieve a trade-off between shortage and holding costs but its performance depends mainly on the historical data and distributional information. The proposed tri-level optimization model automatically adjusts its optimal ordering strategies according to the cost parameters and yields the lowest (or close to the lowest) total cost for all parameter settings. The results show that the tri-level optimization model outperforms the stochastic programming model in different cost parameter settings in terms of the total cost and fill-rate.

This study is subject to several limitations which suggest future research directions. For example, the proposed model assumes a single product made from a single part. Relaxing this assumption would require a more complicated model that reflects the uncertainty and interdependency of multiple parts on the demand and supply sides. In addition, we can include fixed and variable transportation costs in the model, where the decision maker has the option to ship certain parts or products together as a batch to save transportation cost.

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