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Some Conceptual and Scaling Evaluations of Snowmelt Events Forced by Warm Soil

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Abstract
Snowfall occasionally occurs over bare soil with high thermal storage in its upper layer. Quantification and generalization of the potential impact of the thermal storage on episodic snowmelt is evaluated using a scaling approach and assuming negligible net thermal flux at the snow cover top. Soil thermal flux contribution to snowmelt is found to be affected significantly by the level of soil wetness. It is shown that, for a soil temperature of 10°C prior to the snowfall, the contribution of wet soil thermal flux is significant within the first 12 h when compared with intense surface moist enthalpy flux or solar radiation. Implications of these results to modeling of snowmelt using coupled soil–atmosphere models are elaborated.

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NOTES AND CORRESPONDENCE

Some Conceptual and Scaling Evaluations of Snowmelt Events Forced by Warm Soil

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ABSTRACT

Snowfall occasionally occurs over bare soil with high thermal storage in its upper layer. Quantification and generalization of the potential impact of the thermal storage on episodic snowmelt is evaluated using a scaling approach and assuming negligible net thermal flux at the snow cover top. Soil thermal flux contribution to snowmelt is found to be affected significantly by the level of soil wetness. It is shown that, for a soil temperature of 10°C prior to the snowfall, the contribution of wet soil thermal flux is significant within the first 12 h when compared with intense surface moist enthalpy flux or solar radiation. Implications of these results to modeling of snowmelt using coupled soil–atmosphere models are elaborated.

1. Introduction

In many midlatitude locations, snowfall occurs occasionally over relatively warm soil. These situations are most typical with episodic snowfall during autumn and spring, when solar irradiance is relatively high and thus soil thermal storage is relatively large when considering its effect on snowmelt. Likewise they may occur in winter in southern latitudes affected by snowfall. In midlatitudes, such situations may also occur following unseasonably continuous warm and sunny weather during winter. During autumn and spring, regional bare soil occurs widely in midlatitudes, thus soil–snow thermal interaction in the above situations is of practical significance. Research attention has been given to the significance of soil fluxes on seasonal snow cover ablation (e.g., Kuzmin 1961; Gray and Male 1981; Marks and Dozier 1992; Cline 1997; among others). For short, episodic events of snowfall over warm soil that are discussed in this note, soil thermal flux exchange with the snow layer is likely to be vastly larger in comparison with the flux for prolonged seasonal snow cover. Evaluation of soil flux effects on snowmelt over warm soils should be of interest from a hydrological point of view. Furthermore, the increasing trend of incorporating snow models in numerical weather prediction models suggests these situations might be important, particularly following the formation of relatively shallow snow cover over warm soil. For example, predicted daytime shelter temperatures are affected considerably by the existence of snow cover; however, the persistence of an episodic shallow snow cover might be highly dependent on the soil heat storage.

Although an episodic warm soil effect on snowmelt is well known, apparently there has not been an effort to quantify the potential contribution to snowmelt that might result. It is the objective of this note to provide such evaluation. This evaluation can be performed using a numerical model but is constrained to specific cases. On the other hand, a scaling approach provides simplified expressions to generalize the effect. The scaling approach was adopted to infer a range of magnitude for soil thermal flux and snowmelt in the above events. Implications for snowmelt prediction by regional coupled atmosphere–snow models are suggested.

2. Formulation

In this note's evaluations, soil temperature in the layer involving snowmelt is assumed to be much warmer than 0°C prior to the snowfall. Under clear sky, vertical profiles of soil temperature acquire a diurnal, radiationally forced variation in the upper soil layer. For scaling purposes, we assume an intermediate situation with iso-

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thermal soil temperature in the upper soil layer. Also we consider a rapid transition from a clear-sky, warm environment to a snowfall situation. Following the snowfall, mulching of the upper soil layer by snow causes an immediate cooling of the soil skin temperature to 0°C. To isolate the contribution of the soil flux, we assume that the air temperature is ~0°C, the background wind is weak, and skies are overcast, so that thermal fluxes at the top of the snow cover (i.e., the sensible, latent, and radiative fluxes) are likely to be small. Also it is assumed that no rainfall occurs. The upward thermal soil flux from the warm soil is therefore the main thermal energy source for snowmelt. The temporal and time-integrated magnitude of soil thermal flux is estimated in the following way.

Assuming constant thermal diffusivity k_d with depth, the soil thermal conduction equation can be written as

$$\frac{\partial T}{\partial t} = k_d \frac{\partial^2 T}{\partial z^2}, \tag{1}$$

where T is the soil temperature, z is depth increasing downward from the surface, and t is time. The initial and boundary conditions are

$$\begin{aligned}
&T(z = 0,\, t \geq 0) = 0°\mathrm{C}; && \text{at the soil top}\\
&T(z > 0,\, t = 0) = T_o && \\
&T(z = z_b,\, t \geq 0) = T_o; && \text{at large depth in the soil } (z_b \rightarrow \infty).
\end{aligned}$$

The solution to (1) following the method of Carslaw and Jaeger (1950) is

$$T(z, t) = T_o\left\{\mathrm{erf}\left[\frac{z}{2(k_d t)^{1/2}}\right]\right\}. \tag{2}$$

Expanding the erf into a Taylor series around $z = 0$ yields for the surface soil thermal flux F_t at time t:

$$F_t = k_c \frac{\partial T}{\partial z} = k_c \frac{T_o}{(\pi k_d t)^{1/2}}, \tag{3}$$

where k_c is the soil thermal conductivity. Note that $F_t \rightarrow \infty$ when $t \rightarrow 0$; however, the time-integrated soil flux at time t, $\hat{F}_t$, acquires finite value, which is given by

$$\hat{F}_t = k_c \frac{1.13}{(k_d)^{1/2}} T_o t^{1/2}. \tag{4}$$

Note also that soil thermal conductivity increases with volumetric soil wetness. The thermal diffusivity increases with wetness increase for low soil volumetric wetness but typically decreases somewhat in moderately to highly wet soils (e.g., Hillel 1982).

Equation (4) can be written alternatively as

$$\hat{F}_t = 1.13 I T_o t^{1/2}, \tag{5}$$

where $I = (\rho_s C_s k_c)^{1/2}$ is the soil thermal inertia (ρ_s and C_s are the soil density and specific heat, respectively). The cumulative soil thermal flux is thus proportional to I, T_o, and $t^{1/2}$.

Assuming snow cover at 0°C and that the net thermal flux at the snow layer top is 0, then the corresponding snow amount melted in terms of snow water equivalent (SWE) is given by

$$\mathrm{SWE} = \frac{\hat{F}_t}{L_i \rho_w} \quad \text{(m)}, \tag{6}$$

where L_i is latent heat of fusion for ice (~3.33×10^5 J kg^{-1}), and ρ_w (=1000 kg m^{-3}) is water density.

Infiltration of snowmelt water (at 0°C) into the warmer nonsaturated soil generates a thermal sink whose magnitude we estimate here. Assuming that all melted snow water infiltrates the soil, then the integrated thermal flux $\hat{F}_{wt}$ associated with this process is constrained by

$$\hat{F}_{wt} < 0.5 \mathrm{SWE} \rho_w C_w T_o, \tag{7}$$

where C_w (=4186 J kg^{-1} K^{-1}) is the specific heat of water, and T_o is given in degrees Celsius. Substituting (6) into (7) yields

$$\hat{F}_{wt} < 0.006 T_o \hat{F}_t. \tag{8}$$

Thus, as long as the snowmelt is forced only by the soil thermal flux, $\hat{F}_{wt}$ is at least an order of magnitude smaller than $\hat{F}_t$ and can be neglected.

3. Evaluations

a. *Soil temperature modification by snow cover*

Figure 1 provides the analytic derivation of the soil temperature after 12 (Fig. 1a) and 24 h (Fig. 1b) from the onset of snow cover [based on (2), with T_o = 10°C and the soil types given in Table 1]. Because the computed temperature is dependent linearly on T_o, the presented results can be used to infer profiles for any value of T_o. The two basic soil types selected (sand and clay) represent contrasting extremes in soil textures. Different values of soil volumetric wetness were considered: 1) for sand, values of 0%, 20%, and 40%, which correspond to near-permanent wilting point, moderately wet, and nearly saturated soil, respectively; and 2) for clay, values of 20% and 40%, which correspond to near-permanent wilting point and highly wet soil, respectively. The evaluations are done under the assumption that the snow cover temperature is 0°C. The high-volumetric-wetness soil (40%) and the moderate soil volumetric wetness (20%) show almost identical profiles of temperature. For the dry sand case, the temperature drop is noticeably less penetrative downward when compared with the wet sand soils.

b. *Characteristic impact of subsurface thermal storage on snowmelt*

Illustrative $\hat{F}_t$ patterns were computed using (4) for the soils given in Table 1, assuming T_o = 10°C. The

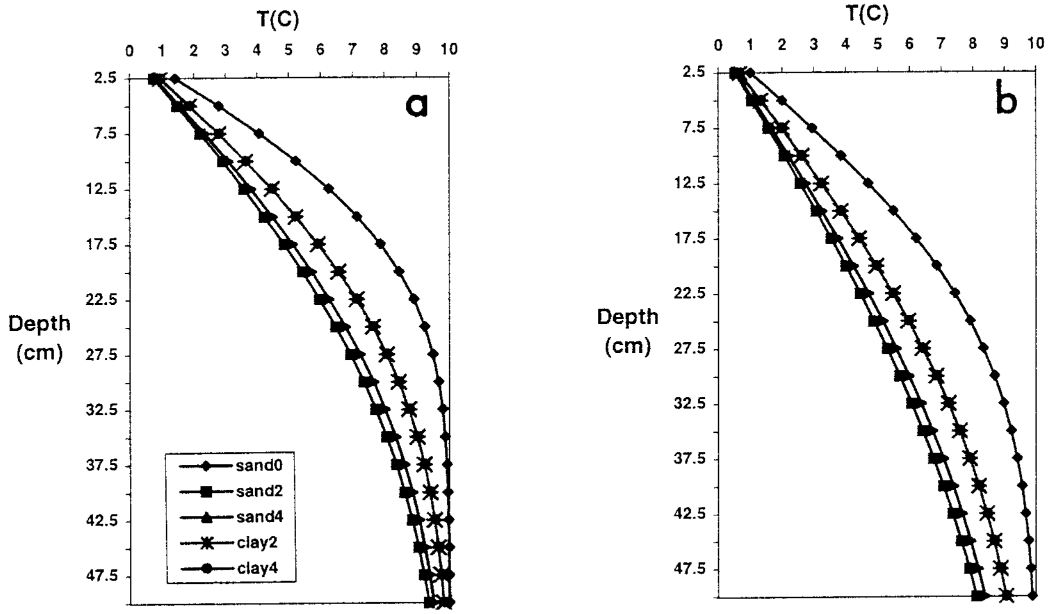

FIG. 1. Computed soil temperature profiles during snowmelt for the five soil situations listed in Table 1. The numbers in the soil-type legend indicate soil volumetric wetness (%) multiplied by 0.1. Initial profile is isothermal with T_o = 10°C. (a) After 12 h; (b) after 24 h. (Note that curves clay2 and clay4 overlap.)

accumulated surface soil flux $\hat{F}_t$ as a function of time (from the onset of snow cover) is shown in Fig. 2. For wet soils, $\hat{F}_t$ is larger by a factor of 3–4 when compared with dry sand. After 12 h, $\hat{F}_t$ for the highly wet soils reaches ~5 MJ, which, for a snow layer with density of 0.2 kg m^{-3}, implies a snowmelt of ~8 cm. After 24 h, the values of $\hat{F}_t$ are larger by a factor of 1.4, that is, ~7 MJ. For perspective, it is useful to compare $\hat{F}_t$ with the surface moist enthalpy flux producing snowmelt under strong warm advection. Following, for example, Leathers et al. (1998) and Baker et al. (1999), we assume a representative sensible heat flux of 150 W m^{-2} and latent heat flux of 100 W m^{-2} during a 12-h period. For this situation, the accumulated surface moist enthalpy flux is ~11 MJ. Thus, when snowfall occurs over highly wet and warm soil, soil thermal flux contribution to snowmelt is about half the magnitude of the surface moist enthalpy flux contribution under intense warm advection. In another comparison, the observed daily solar radiation on a horizontal surface during autumn and spring in the U.S. midlatitudes is about 20–25 MJ (U.S. Department of Energy 1981). With snow albedo of 0.7, the daily solar radiation absorbed by the snow is about 6–7.5 MJ, similar to the peak $\hat{F}_t$ values computed at 24 h. Last, consider the contribution of warm rainfall (droplets at temperature T_r) over snowcover (at 0°C). For rainfall amount P (cm), the energy available for melting snow is $\hat{H}_r = 4.186 \times 10^4 P T_r$ (J). Note that for saturated air, typically the case during a prolonged rainfall, the air temperature also is T_r. For relatively intense daily rainfall (P = 5 cm) and T_r = 10°C, $\hat{H}_r$ = 2.1 MJ. This value is somewhat higher than $\hat{F}_t$ in 24 h for dry sand soils but is only about one-third the magnitude of $\hat{F}_t$ for highly wet soils.

Figure 2 also presents the temporal depletion of SWE, which can be up to ~1.8 cm in 12 h and ~2.5 cm in 24 h for wet soils, whereas for dry sand soils, which have low thermal inertia, the melting is noticeably lower. Initially dry soil will become wet, particularly in the topsoil layer, because of water contribution from snowmelt. However, following Fig. 2, the contribution of snowmelt water to the soil volumetric wetness is rela-

TABLE 1. Thermal conductivity (k_c) and thermal diffusivity (k_d) of several representative soils [after Garratt (1992), Table A7]. Also given are the corresponding values of bare soil diurnal damping depth (z_{dd}) and soil thermal inertia (I). Here, θ is the soil volumetric wetness (%).

Soil type	k_c(W m^{-1} K^{-1})	k_d (m^2 s^{-1})	Z_{dd} (cm)	I (J m^{-2} s$^{-1/2}$ K^{-1})
Sand				
θ = 0	0.3	0.23 × 10^{-6}	8.0	620
θ = 20	1.9	0.84 × 10^{-6}	15.2	2076
θ = 40	2.2	0.74 × 10^{-6}	14.3	2552
Clay				
θ = 20	1.1	0.52 × 10^{-6}	12.0	1522
θ = 40	1.6	0.52 × 10^{-6}	12.0	2227

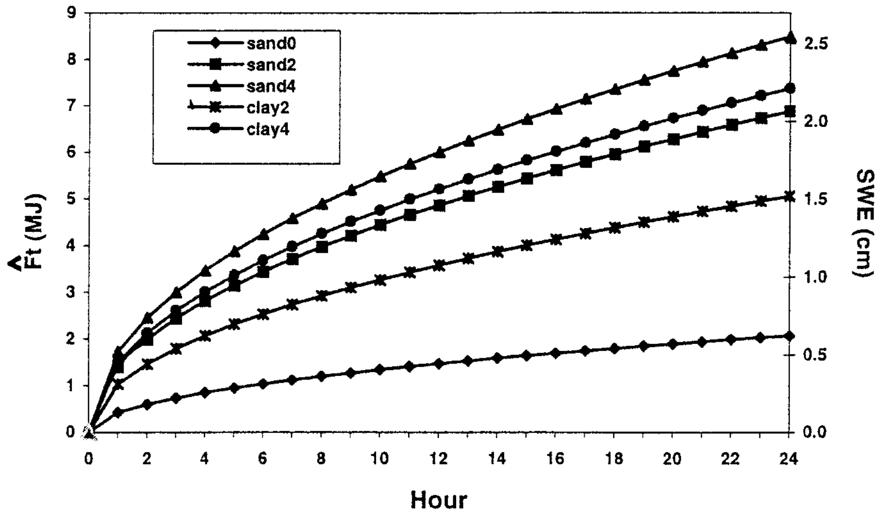

FIG. 2. Accumulated surface soil thermal flux ($\hat{F}_t$) vs time for snow-covered soils in Table 1, using (4) with T_o = 10°C. Also indicated is the SWE melt for these situations using (6).

tively small and would lead to at most a moderate increase in the soil thermal inertia of the soil layer that causes snowmelt. For example, based on Fig. 2, the 24-h SWE melt is ~0.6 cm in the dry sand soil case. Overall, the soil layer affected by sufficient moistening to alter noticeably the thermal inertia would be relatively thin in comparison with the soil layer contributing thermal energy to the snowmelt (see Fig. 3 discussed later). However, if the soil moistening is deep, its contribution to increasing soil thermal inertia in the affected soil layer may be at most moderate. Thus dry soil moistening from snowmelt would have typically only a secondary effect on increasing the magnitudes of $\hat{F}_t$ and snowmelt.

We may consider a more conservative scenario in which snowfall commences only at time Δt after a cold air mass enters the area. For example, passage of a cold front will cool the environment, but snowfall may not start immediately with the cold air passage. In this scenario, some depletion of soil thermal energy occurs before the soil is covered by snow. Suppose the background air temperature is ~0°C. Then the skin temperature before the snowfall is $\gtrsim$0°C [skin temperature of bare soil responds quickly to the outbreak of cold air (e.g., Mahrer and Segal 1985)]. Assuming an extreme situation in which the soil skin temperature in the pre-snowfall period drops to 0°C, we may then use (4) to estimate the energy depleted from the soil during the period Δt. The available soil flux for snowmelt $\hat{F}_{t*}$ is then given by

$$\hat{F}_{t*} = \hat{F}_t[1 - (\Delta t/t)^{1/2}]. \tag{9}$$

For t = 24 h and $\Delta t \lesssim 2$ h, $\hat{F}_{t*} \gtrsim 0.71\hat{F}_t$, indicating that depletion of the soil heat storage after the weather change but before the snowfall may be of secondary

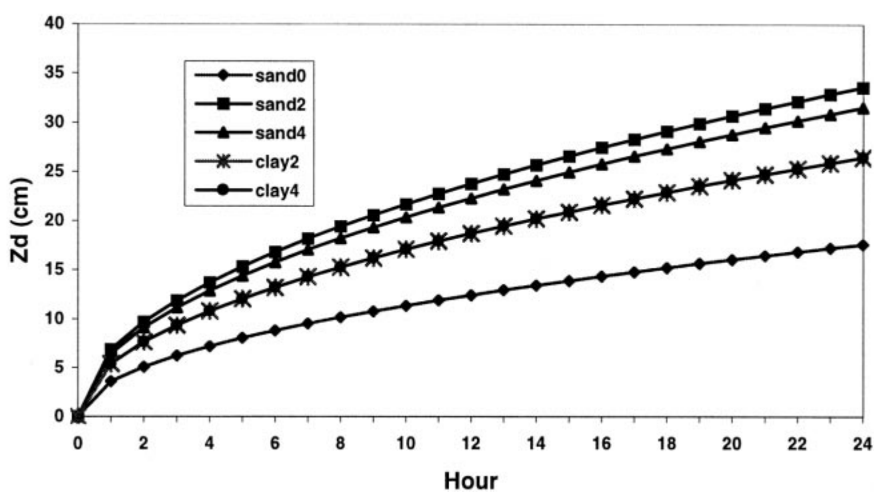

FIG. 3. The soil temperature damping depth (z_d) vs time, using (11) and the soils listed in Table 1. (Note that curves clay2 and clay4 overlap.)

importance. On the other hand, for $\Delta t = 6$ h, $\hat{F}_{t*} = 0.5\hat{F}_t$, suggesting a relatively large loss of soil heat before snowfall. When Δt is known, (9) provides guidance to the efficiency in using the soil heat for snowmelt under conditions less ideal than are assumed in deriving (4).

For meteorological numerical forecast models, it is difficult to initialize accurately the soil temperature profiles because of sparse temporal and spatial distribution of soil temperature observations. Even if the model ''recycles'' soil temperature from a previous forecast, it must encounter inaccuracies involved with determining physical properties of the soil. From (5), the bias in the simulated soil thermal flux under the conditions evaluated is linear to the errors in the initial soil temperature and the soil thermal inertia. Biases of up to a factor of 2 can be assumed in these two variables when used in a numerical forecast model. Thus the combined error in $\hat{F}_t$ can reach the extreme value of ~400%.

In situations associated with shallow snow cover, in which soil flux is the main contributor to melt, the above biases may affect the model-predicted time for final melt of the snow, which may have a noticeable effect on prediction of the shelter-level air temperature [cf. observational evaluations in Leathers et al. (1995)].

c. Damping depth for vertical temperature variation

Estimation of the characteristic depth for downward penetration of the soil temperature perturbation caused by snow cover, the ''damping depth,'' is useful within the scope of the current evaluation. It provides the characteristic depth of the soil layer contributing to the snowmelt.

The damping depth z_d is defined here as the depth below which $(T_o - T)/T_o \leq e^{-1}$. Thus, using (2), z_d is obtained by solving

$$\mathrm{erf}\left[\frac{z_d}{2(k_d t)^{1/2}}\right] = 0.63, \tag{10}$$

which upon evaluating the error function yields

$$z_d \cong 1.25(k_d t)^{1/2}. \tag{11}$$

Figure 3 depicts $z_d(t)$ for the soil conditions listed in Table 1. The depth z_d increases with soil wetness, because wet soil has larger k_d values. The peak value after 12 h is ~20 cm for the soils with intermediate and high volumetric wetness, and after 24 h it is ~30 cm.

The damping depth associated with the diurnal variation of bare soil temperature profile z_{dd} is given, following Hillel (1982), by

$$z_{\mathrm{dd}} = \left(\frac{k_d}{\pi} 86\,400\right)^{1/2}. \tag{12}$$

The computed z_{dd} values for the soils listed in Table 1 are somewhat smaller than the z_d values at 12 h; however, after 24 h, z_{dd} values are only one-half of z_d. This result indicates that the extraction of soil thermal heat energy under snow cover affects a deeper layer than that associated with nocturnal upward soil heat flux of bare soil.

For coupled atmosphere–snow–soil numerical weather prediction models, the depth of the imposed bottom soil layer may affect prediction of the surface soil flux at the interface with the snow layer. If the depth is smaller than z_d at the end of the simulation, temporal variation in the bottom boundary conditions for T should be included, but this information is unavailable. However, taking fixed T as the boundary condition can generate a spurious sink/source of heat. If, on the other hand, the bottom is deeper than z_d, reasonable prediction of the soil T can be obtained. It is likely that most existing coupled models have a deep-enough bottom soil layer. However, in some of these models, the soil module consists of two to three layers (e.g., force–restore approach or models with coarse soil vertical grid resolution), and the soil layer at the top typically is thin (~5–10 cm) while the bottom layer is much thicker. This situation is similar to adopting a bottom boundary that is too shallow, because the thick lower layer forms an effective boundary for short time simulations (<1 day). One might again obtain for this soil vertical grid resolution a spurious thermal soil flux at the snow–soil interface and a biased description of the snowmelt.

4. Conclusions

When abrupt snowfall occurs over a warm soil layer, the thermal storage of the upper soil layer may have noticeable effect on the snowmelt. It was shown in this note that the contribution of surface soil thermal fluxes to snowmelt may be significant even when compared with atmospheric moist enthalpy fluxes from strong warm advection over snow or with net clear-sky solar flux. Increased soil wetness is conducive to increased soil thermal flux and snowmelt. With a possible initial constant soil temperature of 10°C, the snowmelt may reach 1.8 cm of SWE in 12 h and ~2.5 cm in 24 h. Even if snowfall starts 1–2 h after cold front passage, the soil thermal flux contribution to snowmelt is not significantly changed. It was shown that, when a period of 12 h is considered, the characteristic soil depth contributing to snowmelt may be greater than ~20 cm, and it can extend to more than 30 cm after 24 h.

It was also found that, for coupled atmosphere–snow–soil numerical models with an isothermal initial soil temperature, the error in the simulated surface soil thermal flux is linearly related to the errors in the initial temperature and the specification of soil thermal inertia. Last, when the bottom boundary of the soil module is effectively too shallow (in comparison with the soil temperature damping depth), as might be the case in some atmospheric prediction models, spurious surface soil thermal flux would be simulated.

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