NUMERICAL CALCULATION OF DIFFRACTION COEFFICIENTS IN ANISOTROPIC MEDIA

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INTRODUCTION

Ultrasonic inspection is used to detect and size crack-like defects in pressure vessels and pipework used in the nuclear industry. Reliable inspection can only be achieved if the inspection technique is understood, is optimised and subsequently applied correctly. Austenitic steels are used because of their corrosion resistance and toughness. Welds and centrifugally cast materials tend to crystallise with grains larger than the ultrasonic wavelength required to achieve the desired resolution in the inspection and thus appear anisotropic. Since the grains in a weld grow along the varying, directions of maximum heat flux during cooling, the welds are inhomogeneous as well as anisotropic. We wish to understand the ultrasonic signals scattered by cracks in such inhomogeneous anisotropic materials. To calculate large numbers of cases we would like to use a relatively efficient tool: (ray tracing) and wish to incorporate the diffraction and reflection which occurs at the defect through the use of diffraction or scattering coefficients.

Analytic calculations of these for cracks in arbitrary orientations relative to the grain structure are not generally available, although one set by Norris and Achenbach has been published for a transversely isotropic material [1,2]. An alternative approach is to solve the wave propagation and scattering numerically and to use the data so generated to construct diffraction coefficients. We discuss our attempts to do this with limited success and highlight some of the reasons why this is difficult.

METHOD

For inhomogeneous anisotropic materials, only two numerical methods seem appropriate: finite difference or finite element solution of the equation of motion:
\[ \rho(\mathbf{r}) \ \ddot{u}_i(\mathbf{r}) = \left[ C_{i\alpha}(\mathbf{r})u_{\alpha,j}(\mathbf{r}) \right]_j \]  

(1)

where \( u_i(\mathbf{r}) \) is the \( i \)th component of displacement at position \( \mathbf{r} \), \( \rho(\mathbf{r}) \) the local density, \( C_{i\alpha}(\mathbf{r}) \) the local elastic constants and \( u_{\alpha,\beta} \) denotes \( \partial u_\alpha / \partial x_\beta \), \( \ddot{u}_i \) denotes \( \partial^2 u_i / \partial t^2 \) and the summation convention applies. Temple [3] developed a 3-D finite difference computer program to solve this equation in materials of arbitrary inhomogeneity and anisotropy. Displacements are calculated on a discrete, regular, mesh at discrete times for an incident finite width, single cycle pulse of the desired wave mode. All wave interactions and mode conversions, such as all types of permissible surface waves on the stress-free surface of the crack, are automatically included.

For each angle of incidence on the crack, two finite difference calculations are carried out for the particular crack orientation and anisotropic elastic constants. One of the calculations is for wave propagation in a material without a defect and the other includes the crack and hence all scattering effects. Detector points are nominated to lie on the surface of a sphere centred on the midpoint of the crack edge. These detectors record the displacements at each timestep to give A-scans. The detector which lies on the axis of the incident beam also records the incident signal in the case when no crack is present. The detectors are labelled by two angles \( \theta \) and \( \phi \) defined in figure 1.

The first difficulty for an anisotropic material is the fact that phase and group velocity directions are not necessarily parallel. Energy therefore arrives at a detector along the group velocity direction. A time harmonic wave of frequency \( \omega \) with displacements \( \mathbf{u} \) and amplitude \( A_p \) is given by

\[ u_p = A_p \exp \ i \omega (m \cdot \mathbf{r} - t) \]  

(2)

Figure 1. Angles used to describe diffraction at a crack.
where \( \mathbf{m} \) is the slowness vector proportional to the inverse of the phase velocity. It satisfies equation (1) provided [4] that the Green-Christoffel equation:

\[
\left[ C_{ijkl} n_i n_j - \rho V_p^2 \delta_{kl} \right] = 0
\]

(3)
is satisfied. Here \( n_i \) are the components of a unit vector in the direction of \( \mathbf{m} \), the eigenvalues \( V_p \) are the allowed phase velocities. The eigenvectors \( \mathbf{e} \) associated with each eigenvalue can be used to express the group velocity \( V_g \) as [5].

\[
V_g = \frac{C_{ijkl} \mathbf{e} e_i m_j}{\rho}
\]

(4)

Given \( V_g \), we must invert relationships 3 and 4 to obtain a set of three phase velocities and associated eigenvectors \( \mathbf{e}_p (p = 1, 2, 3) \) for each mode: quasi-P, quasi-SV, quasi-SH. This can be done but not necessarily uniquely. The displacements at each detector are then projected on to the appropriate polarization eigenvectors for the detector. This gives A-scans resolved into mode types.

To extract the modulus of the diffracted signals, we take the Fourier transform of each of the signals and divide the modulus of the diffracted signal by the modulus of the incident signal, frequency by frequency. This yields a modulus which should not be dependent on frequency, but may be because various length scales have been introduced such as a finite crack width and beam width.

The frequency dependent phase is obtained for the incident wave and each of the diffracted signals. The low frequency values, excluding zero frequency, are fitted to a straight line and the zero frequency intercept obtained. Subtraction of the values for the incident and diffracted A-scans yields the phase difference.

To demonstrate that this processing works as expected, some artificial data was set up, exactly similar to the results of the finite difference calculations. The artificial data represented a compression wave incident and diffracted with an amplitude profile given by

\[
A(\theta, \phi) = (0.1 + 0.9 \sin^2 \phi) \left[ 0.1 + 0.9 \sin^2 \theta / 2 \right]
\]

(5)

and a phase \( \varphi(\theta, \phi) \) satisfying for all \( \phi \)

\[
\varphi(\theta, \phi) = \begin{cases} 
30^\circ & 0 \leq \theta \leq 90^\circ \\
70^\circ & 90^\circ < \theta \leq 180^\circ \\
110^\circ & 180^\circ < \theta \leq 270^\circ \\
150^\circ & 270^\circ < \theta < 360^\circ 
\end{cases}
\]

(6)

The results recovered are shown in figures 2 and 3.

These demonstrate that the processing works when the complications of anisotropic materials are absent.

Taking a full, 3-D time dependent calculation for a surface piston source modelled as normal displacements with a single cycle of sine wave at a specified frequency, with the source at 90° to the crack and incident propagation along the z-axis for a material with elastic constants given in table 1, and carrying out the whole processing described above yields the results shown in figures 4 and 5 for the modulus and phase respectively.
Figure 2. Recovery of a specified amplitude profile for artificial data in isotropic material.

Figure 3. Recovery of a specified phase profile for artificial data in an isotropic material.
Table 1. Elastic constants used for anisotropic material

\begin{align*}
C_{11} &= C_{22} = 94.5 \times 10^9 \text{ Nm}^{-2} \\
C_{12} &= 80.5 \times 10^9 \text{ Nm}^{-2} \\
C_{13} &= C_{23} = 21.0 \times 10^9 \text{ Nm}^{-2} \\
C_{33} &= 42.0 \times 10^9 \text{ Nm}^{-2} \\
C_{44} &= C_{55} = 10.5 \times 10^9 \text{ Nm}^{-2} \\
C_{66} &= 7.0 \times 10^9 \text{ Nm}^{-2} \\
\rho &= 2.71 \times 10^3 \text{ kgm}^{-3}
\end{align*}

The modulus values have been smoothed with a sixth order surface polynomial and are interpretable as diffraction coefficients whereas the phase values are a mess. Some of the possible reasons for this are: the phase comes from a \( \tan^{-1} \) operation which returns values between \(-\pi\) and \(+\pi\), and therefore arbitrary jumps of around \(2\pi\) are possible; the phase is sensitive to the projection of displacements on to the allowed polarization vectors and these in turn depend on being able to invert the group velocity directions to get accurate phase velocity directions (this is non-unique but not at the number of angles giving difficulties in the plots). Norris and Achenbach [1] calculated pulse-echo responses for several source angles. Our calculation, at \( \phi = 90^\circ, \theta = 90^\circ \), corresponds to a caustic in the analytical calculation for both isotropic and this anisotropic material, so it is not possible to compare the two cases directly.

Figure 4. Modulus of a compression wave diffraction coefficient for a compression wave incident at \( \theta = 90^\circ, \phi = 90^\circ \) in an anisotropic material with elastic constants given in table 1.
DISCUSSION

Apart from the difficulties of finite width cracks and finite width ultrasonic beams, both of which introduce frequency dependent effects, the physics of wave propagation along certain directions in anisotropic media leads to other difficulties. For example, along certain directions there are three phase velocities for each mode giving rise to pulses which separate into distinct parts and which decay with distance according to $r^{-5/6}$ for cuspidal edges of the group velocity surface or as $r^{1/2}$ along directions corresponding to conical points [6]. Such behaviour has been seen in numerical calculations [7].

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REFERENCES