Exchange rate uncertainty, futures markets, and foreign direct investment

Hongmo Sung
Iowa State University

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Exchange rate uncertainty, futures markets, and foreign direct investment

by

Hongmo Sung

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Graduate College
Iowa State University

This is to certify that the doctoral dissertation of

Hongmo Sung

has met the dissertation requirements of Iowa State University

Signature was redacted for privacy.

Major Professor

Signature was redacted for privacy.

For the Major Program

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For the Graduate College
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CHAPTER I. INTRODUCTION

A. Background

Recently, the WTO was organized. It is expected that the organization may play a role to force countries to be more open. In a more open world, capital and technology are more likely to move across countries, and so firms in each country will have to be more and more concerned about producing and selling abroad. Generally, international trade decisions are made under uncertain conditions in foreign exchange. A change in exchange rate can affect international firms' profits through their production, trade, and investment. In addition, the major currencies have been very volatile in the last decades. Therefore, the firms producing at their foreign plants or selling abroad increasingly need to manage the volatility of exchange rates.

Figure 1 shows the fluctuation of exchange rates in the U.S. dollar per major currencies from 1968 to 1993. The Bretton Woods international monetary system, the fixed exchange rate system formalized in 1945 to provide a stable monetary framework for international trade, worked reasonably well until the U.S. dollar became significantly overvalued in 1970 due to the large balance of payments deficits of the U.S. Most major countries temporarily adopted floating exchange rate system after the devaluation of the U.S. dollar in 1971, and many other countries adopted floating exchange rate system after the second devaluation of the U.S. dollar in 1973. Afterwards the foreign exchange rates have freely fluctuated over time, and foreign currency futures were introduced in 1972 at
Figure 1. Trends of the U.S. dollar per major currencies normalized from 1968 to 1993.¹

the Chicago Mercantile Exchange with the sharp increase in exchange rate volatility resulting from the widespread adoption of floating exchange rates.

Furthermore, there may be also price or demand uncertainty for both imported and exported goods. Since many internationally traded goods do not have futures, forward, or options markets, firms cannot hedge uncertain prices in those markets. In this situation, they may want to hedge the risks in the futures, forward, or option markets through foreign exchange. For these reasons, the multinational firms have increasingly used hedging instruments to protect against exchange rate risk.² Therefore, the introduction of foreign exchange futures markets provides a good opportunity to hedge exchange rate risk. In this sense, analyzing the implications of hedging instruments to reduce exchange

¹ Major currencies are France franc (FR), Germany deutschmark (GE), Japanese yen (JA), Canada dollar (CA), Italy lira (IT), and U.K. pound (UK). They are normalized in the following way,

\[ x = \frac{X - \text{mean}}{\text{standard deviation}} \]

where \( x \) is the normalized exchange rate (dollar/foreign currency) and \( X \) is the nominal exchange rate.

² See Kawai and Zilcha (1986), and Broll and Wahl (1992).
rate risk can be a major concern to multinational firms, which produce and sell in foreign
countries.

A more open world provides multinational firms more chance to obtain flexibility
in production by allowing them to open foreign plants. Thus, investment decisions of
multinational firms become more important in this circumstance. Cost structures of both
domestic and foreign plants and the variability of the exchange rate are key components in
determining their foreign direct investment decisions. In addition, in most major
industries, firms are not just price-takers, but they have monopolistic power in some
degree in the domestic market and even in the world market. Thus, studying an
imperfectly competitive market seems to be more realistic.

B. Purpose of the study

The purpose of the study is to analyze the production, hedging, and investment
decision behavior of multinational firm (MNF) which produces and sells in domestic and
foreign markets under exchange rate uncertainty, with monopolistic power in both
markets. Assuming that some decisions are made under uncertainty, while other decisions
are made after the resolution of uncertainty, I examine how the variability of the exchange
rate influences the composition of production and the level of total output of the firm. I
also examine how the firm’s welfare changes under risk, and then investigate how these
effects change when transportation costs exist.

Since the volatility of the exchange rate makes the firm’s profit unstable, it may use
hedging instrument(s) to reduce risk. If the firm uses some hedging instrument(s), the
effects of the uncertainty may change. Therefore, assuming that futures markets are available, I will investigate how the effects change and what the optimal futures position is. I expect that perfect hedging is not attained by using futures markets alone because the assumption I made on the sequence of decisions makes the indirect profit function nonlinear in the random exchange rate, which does not allow perfect hedging with single hedging instrument. Since these effects are subject to the firm's attitude toward risk, I analyze them for risk neutrality and for risk aversion. I also study the issue of foreign direct investment (FDI) in the supply side. Before the firm produces abroad, it will examine whether opening a foreign plant as well as a domestic plant is more profitable than opening a home plant only. Once the firm decides to open a foreign plant, it needs to invest in the foreign country to build a plant. Thus, FDI is positioned in the center of concern when we consider the MNF. I will investigate how the firm makes its foreign direct investment decision when total output is fixed and when total output is price responsive. Finally I will examine how the variability of the exchange rate affects both firms' welfare if the MNF faces a local competitor in the foreign market.

Previous research is reviewed in chapter II. In chapter III, the effect of uncertainty on production is examined in the absence and presence of transportation costs. In chapter IV, I study how the presence of futures markets affects the allocation of production for the monopoly after I reexamine the separation theorem. I also characterize the optimal futures position in this chapter. The issue of foreign direct investment is analyzed in chapter V.

\footnote{See Moschini and Lapan (1992)}
CHAPTER II. LITERATURE REVIEW

Recently there have been many contributions to the theory of international firms under stochastic exchange rates. How exchange rate risk affects the volume of trade has been examined in various situations. The role of hedging instruments also has been analyzed in previous research. Ethier (1973) and Baron (1976) analyzed the effect of uncertain exchange rate on the output and international trade decisions, and the effect of forward exchange on international trade. It was shown that a risk averse firm’s production decision is independent of the firm’s attitude toward risk and its subjective probability of the future spot exchange rate in the presence of a forward exchange market. This property, known as the separation theorem, has been extended in several ways.

Kawai and Zilcha (1986) examined the trade behavior of a risk averse firm in the presence of exchange rate and commodity price uncertainties. They showed that the more risk averse firm produces less when the profit functions are monotonically increasing in the joint-product of two random variables. They also verified the separation theorem and showed that the optimal forward-futures contracting is a full double-hedge, assuming that the forward foreign exchange market is unbiased and the forward foreign exchange and commodity futures markets are jointly unbiased. Moreover, they investigated the role of the existence of both forward exchange and commodity futures markets in comparison to

---

4 In their paper, unbiased forward market is characterized by \( E[R] = R_f \), and "jointly unbiased" forward-futures markets are characterized by \( E[RP] = R_f P_f \) where \( R \) is the random exchange rate, \( P \) is the price of the commodity, \( R_f \) is the forward exchange rate, and \( P_f \) is the futures price of the commodity.
the case where only one or no market is available. They found that the existence of forward exchange and commodity futures markets increases the volume of international trade. Paroush and Wolf (1986) reexamined the properties of separation and full hedging even in the presence of basis risk under commodity price uncertainty when forward and futures markets are available. They showed that when the agent hedges in the futures markets with basis risk, her or his production decisions are risk free due to additional hedging opportunities in the forward market. Wolf (1995) examined a competitive firm's import, production, and hedging decisions under input price and exchange rate uncertainties in a flexible exchange rate regime. He found that risk aversion and the variance of the exchange rate have a negative impact on imports, while their effect on hedging decision depends on the nature of the equilibrium price structure - on whether one faces a contango or backwardation - as well as on the initial magnitude of hedging. He also showed that imports of the input and hedging are lower under both input price and exchange rate uncertainty than under just exchange rate uncertainty if the input price in the first case is not smaller than that in the second case. Unlike most of the other research in this area, Katz (1984) examined the separation theorem under the imperfectly competitive forward market, and showed that the firm's production decision is not independent of the parameters affecting the firm under uncertainty.

Zilcha and Eldor (1991) studied the behavior of a competitive risk-averse firm which faces uncertain exchange rate in a multiperiod framework. They showed that the firm's optimal capital/labor ratio declines in all periods in the presence of unbiased forward markets and these ratios are independent of the utility function and its subjective beliefs.
However, in terms of the absolute level of capital and labor, the usual separation theorem does not hold since the employment of the variable input in the future periods, and thus its production, remains uncertain. In addition, they showed that in the unbiased forward exchange market the firms tend to overhedge in both time periods when the exchange rates are positively correlated over time. Donoso (1995) investigated how introducing a perfectly competitive and possibly biased forward currency market affects the export and hedging decisions of a risk-averse exporting firm in a multiperiod framework under exchange rate risk. He found that the separation between exporting and hedging decisions holds at all time periods in a perfectly competitive forward currency market. Therefore, exporting decisions are independent of the distribution of the stochastic spot exchange rates, implying that exchange rate stabilizing policies will have no impact on exporting decisions in the presence of a forward currency market. He also showed that the introduction of a biased forward currency market does not always lead to an increase in the volume of exports. This result is most substantially different from the others in terms of an unbiased forward currency market.

Most papers in these studies assumed that there exists a competitive, risk-averse firm. However, some papers, such as Eldor and Zilcha (1987) and Broll and Zilcha (1992), analyzed an imperfectly competitive commodity market. I will describe these articles in detail.

Eldor and Zilcha (1987) examined a price discriminating firm, producing only in the home country and always exporting, which is a monopoly in the domestic market but a price-taker on the foreign market under exchange rate uncertainty. Since they assumed
that the production decision is made under uncertainty, but the sales decision is made after the exchange rate is known, the profit function in their model is nonlinear in the exchange rate. They showed that exchange rate uncertainty reduces the optimal output of the price discriminating firm, and that the optimal output and export decreases with the increase in the firm's risk aversion. In the presence of forward foreign exchange markets, they showed that the optimal forward hedge of the price discriminating firm is lower than that of a competitive firm. They also verified that the separation theorem holds even for a price discriminating firm.

Broll and Zilcha (1992) analyzed the implications of foreign exchange futures markets in the context of a risk-averse multinational firm with monopoly power in domestic and foreign markets, which produces and sells in both markets, under exchange rate uncertainty. Assuming that all decisions are made before exchange rate uncertainty is resolved, they investigated the effects of exchange rate uncertainty and the role of futures markets on the international production, sales, and direct investment. Given the von Neumann-Morgenstern utility, the firm chooses the levels of production, sales, and futures contracts in both countries in order to maximize its expected utility of profits. Assuming that foreign currency futures markets are available, the profit function is described as

\[ \Pi = R(y) + e\overline{R}(x + \overline{x} - y) - C(x) - e\overline{C}(\overline{x}) + z(e_f - e) \]

where \( \overline{y} = x + \overline{x} - y \) is sales in the foreign market, \( x \) is the output, \( y \) is domestic sales, \( R(y) \) is the domestic revenue function, \( C(x) \) is the cost function, \( z \) is the amount of futures currency commitment, \( e_f \) is the forward exchange rate which is given, and \( e \) is the
stochastic spot exchange rate whose distribution function is known. In the absence of any hedging instrument, they found that exchange rate uncertainty leads to lower domestic production and higher foreign production, and higher domestic sales and lower foreign sales. They also investigated the hedging behavior of the firm and the separation theorem in the presence of futures markets. They showed that the optimal levels of the allocation of outputs and sales of the firm are independent of its attitude towards risk and the distribution function of the random exchange rate, implying that the separation theorem holds for this model. For the unbiased case \( e_f = \bar{e} \), the firm hedges such that the profits are totally independent of the realization of the exchange rate. However, they found that if the forward exchange rate is biased \( e_f \neq \bar{e} \), the hedging behavior depends on the attitude towards risk and the distribution function of the random exchange rate. The firm hedges more (less) than its net revenue in the foreign market as \( e_f > \bar{e} \) (as \( e_f < \bar{e} \)).

Furthermore, they showed that perfect hedging with futures market can be made under the single source of risk from the exchange rate because the assumption made on the sequence of decisions leads to a linear profit function in the random exchange rate.

However, if there exists multiple sources of risk, the interaction among the sources of risk may lead to a nonlinear profit function in random variable. Another possibility for a nonlinear profit function can arise from the sequence of decisions. In contrast to their assumption that all decisions are made before exchange rate uncertainty is resolved, if some decisions are made after the resolution of uncertainty, then the profit function would be nonlinear in the random variable. It may change the effect of uncertainty on the firm's
production decision. Furthermore, with nonlinear profit functions, using futures market alone as a hedging instrument will not lead to perfect hedging. In such cases, the separation theorem may not hold.

Moschini and Lapan (1992) showed that perfect hedging by using futures market alone is not possible under output price risk because the sequence of decisions made in their model leads to a nonlinear profit function in random price. They assumed that the quasi-fixed input decision of a competitive firm is made under price risk, but output decision is made after output price uncertainty is resolved. With this assumption, they verified that the optimal quasi-fixed input level is larger under risk if the shadow price function of the quasi-fixed input is convex in output price. They also showed that the optimal futures hedge is a short position equal to the expected output with unbiased futures prices for the quadratic profit function. In addition, they showed that the optimal level of the quasi-fixed input is not affected by expectations about the uncertain price but depends on the known futures price when the shadow price of the quasi-fixed input is linear in output price, implying that the separation theorem holds.

Table 1 summarizes the differences on assumptions and results among these papers. The model in my study differs from Moschini and Lapan (1992) in the sense that they considered a price-taking firm in a closed economy. I assume that under exchange rate uncertainty, a firm produces in the foreign plant as well as in the domestic plant, and sells with a monopolistic power in both markets. It also differs from Eldor and Zilcha (1987) who examined a price discriminating firm, producing only in the home country and selling as a monopoly in the domestic market but a price-taker in the foreign market.
Table 1. Summary of previous research

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<td>Results</td>
<td>• Lower optimal output under risk. • The separation theorem holds. • Lower hedging level than a competitive firm.</td>
<td>• Lower domestic and higher foreign productions. • The separation theorem holds. • Full hedges in the unbiased futures market.</td>
<td>• Short position of the optimal futures hedge. • Perfect hedging with futures market alone is not possible. • The separation theorem holds.</td>
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</table>

Therefore, in their paper, there is no issue about the allocation of production and foreign direct investment. The timing of decisions is a very important component in my model. I assume that some decisions are made under uncertainty, while other decisions are made after the resolution of uncertainty. This sequence of decisions makes my model different from Broll and Zilcha (1992). Since the assumptions in my model differ from the previous studies in various ways, the issues considered, the approach to solve them, and the results
will also be different. Furthermore, I examine some issues not considered in these previous studies, such as the existence of transportation costs and the issue of a strategic advantage in the FDI decision.

I also review previous research for the issue of direct investment, particularly those papers which studied the effect of the variability in the exchange rate on the FDI decision. Goldberg and Kolstad (1995) have examined the implications of exchange rate variability for foreign direct investment flows for a MNF in a two-period model. They assumed that the firm chooses its domestic and foreign capacities under exchange rate and foreign demand uncertainties, and produces at capacity only for foreign sales after the resolution of uncertainty. This assumption implies that capacity and production decisions are essentially the same, and are made in the first period. They also made the assumption on the cost structure as follows. In order to eliminate the ability of the producers to buy the option of channeling production \textit{ex post} to the more profitable plant, all factors in production are fixed. In addition, capacity costs are equal to 1 per unit of domestic output, and equal to $e$ per unit of output abroad so that $e$ can be interpreted as the ratio of foreign production cost to domestic production cost where $e$ is the exchange rate. They showed that a risk neutral firm is indifferent regarding the location of production facilities. They also argued that under risk aversion, it is desirable to locate all production in the foreign country with the positive correlation between exchange rate and foreign demand shocks, but it is desirable to have some domestic production with the negative correlation. In the presence of exchange rate uncertainty only, however, it is always desirable to locate some production abroad. An interesting result of their findings is that an increase in the
variability of exchange rates leads to the larger share of FDI, but the change in the absolute level of FDI is ambiguous.

The sequence of decisions made in my model differs from theirs in the sense that I assume that the investment decision is made before the resolution of the exchange rate, but production decision is made after the exchange rate is known. In addition, their cost structure differs from that in my model. There is no fixed cost in their model, while the model in my study has fixed costs. Thus, the firm in their model has no economy of scale. Because of the difference of the assumptions mentioned above, I expect that the results may differ from theirs.

Bailey and Tavlas (1991) reviewed the arguments of proponents of managed exchange rates who believe that variable exchange rates impede investment. George Zis (1989) and IMF (1984) insisted that short-term volatility impedes direct investment, and John Williamson argued that long-term misalignment is harmful to direct investment. Also, Paul Krugman argued that exchange rate instability, which comes from reasonable market responses to change in policies and underlying conditions and from failures in the international financial markets, makes firms cautious and unwilling to change their production and pricing decisions, inhibiting direct investment. Bailey and Tavlas (1991) did not agree with the arguments of proponents of managed exchange rate. After they investigated the effects of the exchange rate variability (short-term volatility and long-term misalignment) on direct investment, they concluded that the effect of exchange rate risk on direct investment is ambiguous because increased risk may lead to a reduction of trade and
domestic investment and to an increase in foreign direct investment. They also supported their argument with their empirical investigation.

Cushman (1985) analyzed the effects of real exchange rate risk and expectations on direct investment for four different cases, depending upon where to buy inputs and produce, where to finance capital acquisitions, and where to sell output; (i) foreign production and sale using foreign inputs financed at home or abroad, (ii) foreign production and sale with capital financed domestically and exported intermediate good to the foreign subsidiary, (iii) domestic production and sale with imported intermediate good from a foreign subsidiary whose capital is financed at home, (iv) domestic and foreign production but foreign sale only with capital purchased and financed at home or capital purchased abroad but financed at home. Assuming that an international firm makes its input decisions under exchange rate uncertainty, he found that the direct effect of risk is to lower foreign capital cost and thus to increase foreign direct investment. However, when the costs of other inputs are also affected, induced productivity changes or output price changes may offset the direct effect, reducing foreign direct investment.

Even though these theoretical studies found that the total effect of exchange rate uncertainty on the level of FDI is not clearly determined, there are some empirical studies showing that increases in exchange rate risk are positively and significantly related with foreign direct investment flows for some of the data collected.

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Broll and Zilcha (1992) investigated the effects of the presence of futures market on FDI that increases demand for the MNF product, such as advertisement. They proved that the introduction of an unbiased currency futures market leads to a higher FDI affecting the demand side rather than the technology in the foreign country, and also showed that FDI moves directly with the futures price. However, the firm in my study invests in supply side to reduce potential costs, such as building new plants in foreign country.
I consider a risk-averse multinational firm which produces and sells a homogeneous commodity in domestic and foreign markets under exchange rate uncertainty. The firm has monopolistic power in both markets. Production decisions are made when the exchange rate is not known, while the sales decisions are made after the exchange rate is known with certainty. This sequence of decisions is critical in the analysis of the issues examined in this study because it makes the profit function nonlinear in the stochastic exchange rate. The firm chooses the optimal level of sales in both markets to maximize its profit after the exchange rate uncertainty is resolved, and the sales decision problem can be written as

$$\max_{Y, \bar{Y}} \Pi = \prod(Y)Y + e\overline{\prod}(\bar{Y})\bar{Y} - C(q) - e\overline{C}(\bar{q}),$$

s.t.

$$Y + \bar{Y} \leq q + \bar{q}$$

where \(\prod(.)\) is the inverse demand function, \(Y\) is the amount of domestic sales, \(e\) is the exchange rate measured as the home currency units per foreign currency unit, \(q\) is the amount of domestic production, and "—" denotes the corresponding symbol for the foreign country. The cost functions in both countries, \(C(.)\) and \(\overline{C}(.).\), are assumed to be different and assumed to have positive and nondecreasing marginal costs; \(C'(.)\), \(\overline{C}'(.) > 0\),
and \( C^*(.) \), \( \bar{C}^*(.) \geq 0 \). Assuming that all outputs are sold (i.e., \( Y + \bar{Y} = q + \bar{q} \)), the maximization problem for the optimal sales decision can be rewritten as

\[
\max_{Y, \bar{Y}} \Pi = R(Y) + e\bar{R} (Q - Y) - C(q) - e\bar{C}(\bar{q})
\]

where \( \bar{Y} = Q - Y \), \( R(Y) \equiv P(Y)Y \), and \( \bar{R}(\bar{Y}) \equiv \bar{P}(\bar{Y})\bar{Y} \).

The revenue functions, \( R(Y) \) and \( \bar{R}(\bar{Y}) \), are assumed to have positive and nonincreasing marginal revenues, \( R'(Y), \bar{R}'(\bar{Y}) > 0 \), and \( R'(Y), \bar{R}'(\bar{Y}) \leq 0 \). For positive values of sales in both markets, the first order condition of this problem is

\[
R'(Y) - e\bar{R}'(Q - Y) = 0.
\] (3-1)

This implies that marginal revenues must be equalized between the domestic and foreign sales at the optimum. From equation (3-1), the optimal levels of sales are obtained as functions of \( Q \) and \( e \),

\[
Y^* = Y^*(Q, e)
\]

\[
\bar{Y}^* = \bar{Y}^*(Q, e).
\]

Note that there is no risk in this optimal sales decision. Equation (3-1) provides the relationship between sales and the exchange rate. Totally differentiating equation (3-1) provides the following expression;

\[
\frac{\partial Y^*}{\partial e} = R'(R^* + \bar{R}^*)^{-1} < 0.
\]

The optimal domestic sales is negatively related to the exchange rate. It also can be determined that the optimal foreign sales is positively related to the exchange rate.\(^6\)

\(^6\)From \( Q = Y(Q, e) + \bar{Y}(Q, e) \), \( \frac{\partial Y}{\partial e} = -\frac{\partial \bar{Y}}{\partial e} \)
Using the optimal levels of sales, the production decision problem is solved under exchange rate uncertainty. At this stage of analysis, it is assumed that hedging instruments are not available. Using the von Neumann-Morgenstern utility where $U' > 0$ and $U' < 0$, this problem can be written as

$$\max_{q, \bar{q}} E[U(\Pi)]$$

s.t.

$$\Pi = R(Y^*) + eR(Q - Y^*) - C(q) - eC(\bar{q}),$$

$$Y^* = Y^*(Q, e), \quad \bar{Y}^* = \bar{Y}^*(Q, e), \quad \text{and} \quad Q = q + \bar{q}.$$ 

It is assumed that the distribution of the exchange rate, $e$, is known. For positive values of $q$ and $\bar{q}$, the first order conditions for this maximization problem can be derived as

$$\frac{\partial E[U(\Pi)]}{\partial q} = E[U' \cdot (R^* \frac{\partial Y^*}{\partial Q} + eR^* \cdot (1 - \frac{\partial Y^*}{\partial Q}) - C')] = 0$$

$$\frac{\partial E[U(\Pi)]}{\partial \bar{q}} = E[U' \cdot (\bar{R}^* \frac{\partial \bar{Y}^*}{\partial Q} + e\bar{R}^* \cdot (1 - \frac{\partial \bar{Y}^*}{\partial Q}) - e\bar{C}')] = 0.$$ 

Using equation (3-1), these can be simplified as

$$\frac{\partial E[U(\Pi)]}{\partial q} = E[U' \cdot (eR' - C')] = 0 \quad (3-2)$$

$$\frac{\partial E[U(\Pi)]}{\partial \bar{q}} = E[U' \cdot (e\bar{R}' - e\bar{C}')] = 0. \quad (3-3)$$

I investigate the effect of exchange rate uncertainty on output decision in the absence of transportation costs in the first two sections of this chapter, and then examine the effect in the presence of transportation costs in the last section. As I reviewed in chapter 2, Broll and Zilcha (1992) showed that a risk averse firm with linear profit
function produces less at home and more abroad under exchange rate uncertainty in the absence of hedging instrument. The profit function in their model is linear in the exchange rate because they assumed that all decisions are made before the resolution of exchange rate uncertainty, which is the only source of risk. However, as mentioned earlier, the sequence of decisions assumed in my model makes the profit function nonlinear in the exchange rate. Although the sequence of decisions in my model is different from that in the linear model, the effects of the uncertainty on the allocation of outputs between the domestic and foreign markets for a risk averse firm are the same as those in the linear model, holding total output constant. For a risk neutral firm, however, uncertainty has no effect on the allocation of outputs.

I also analyze the effect on total output for a risk neutral firm, and verify that the risk neutral firm is better off under uncertainty, regardless of the optimal level of total output. The effect on total output depends on the type of demand functions. The firm's total production is lower under uncertainty with linear demand, but higher with constant elasticity demand.

On the other hand, if there exist transportation costs, the firm's production decisions are changed. A risk neutral firm will, in general, produce more at home under uncertainty than under certainty in the presence of transportation costs, while uncertainty has no effect on the allocation of production in the absence of transportation costs. Also, the firm is likely to trade less in the presence of transportation costs.
A. Absence of transportation costs

Assuming that there exists no transportation cost, I analyze the effect of the exchange rate uncertainty on production in this section. I first examine this effect on the allocation of outputs for the risk neutral firm and for the risk averse firm, and then on the level of total outputs for the risk neutral firm. Finally I analyze how uncertainty affects the risk neutral firm's welfare.

1. The effect on the allocation of outputs

As I mentioned before, a risk averse firm produces less in the domestic plant and more in the foreign plant under uncertainty, given total output unchanged. However, a risk neutral firm makes the same allocation of outputs under uncertainty as under certainty. In the absence of transportation cost, I show this effect for risk aversion first, and then prove it for risk neutrality. The effect for a risk averse firm can be verified in the following manner.

Using equations (3-1), (3-2), and (3-3), we obtain
\[ C'(q^*)E[U'] = \bar{C}'(\bar{q}^*)E[U'e] \]

where \( q^* \) and \( \bar{q}^* \) are the optimal allocation of production between the domestic and foreign markets under uncertainty. Given the profit function,
\[ \Pi = R(Y) + e\bar{R}(\bar{Y}) - C(q) - e\bar{C}(\bar{q}) \], I also derive \( \bar{C}'E[U'e] < \bar{e}\bar{C}'E[U'] \) where
Together with equation (3-4), this inequality provides

$$C'E[U'] < \bar{e}C'E[U'].$$

Since the marginal utility is assumed to be positive, this can be reduced to

$$C'(q^*) < \bar{e}C'(q^*)$$

(3-5)

for the uncertainty case. For the deterministic case, equation (3-4) becomes

$$C'(q^*) = \bar{e}C'(q^*)$$

(3-6)

where the superscript $\text{c}$ denotes the optimal outputs under certainty. I can now prove the following proposition.

PROPOSITION 3-1. Holding the level of total production unchanged, exchange rate uncertainty results in lower domestic production and higher foreign production for the risk averse firm, $q^*(Q) < q^*(Q)$ and $\bar{q}^*(Q) > \bar{q}^*(Q)$.

PROOF. Let's assume $q^* < q^{'*}$. Equations (3-5) and (3-6) together with $q^* < q^{'*}$ imply

$$C'(q^*) < \bar{e}C'(q^{'*}) \leq \bar{e}C'(q^{'*}) = C'(q^{'*})$$

given $C' > 0$. However, $C'(q^*) < C'(q^{'*})$ implies $q^* < q^{'*}$, given $C' > 0$. Under the assumption that $Q$ is fixed, that contradicts the assumption, $q^* < q^{'*}$. It proves $q^* < q^{'*}$ and $\bar{q}^* > \bar{q}^{'*}$. Q.E.D.

This proposition shows that a risk averse firm produces less in the domestic plant and more in the foreign plant under exchange rate uncertainty, given unchanged total

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7 Assume that foreign net revenue is positive for all $\epsilon > 0$, $\frac{\partial \Pi}{\partial \epsilon} = \bar{R}(\bar{Y}) - \bar{C}(\bar{q}) > 0$. Using

$$\frac{\partial \Pi}{\partial \epsilon} > 0$$

and the assumption of $U^* < 0$, I obtain $\frac{\partial U^*}{\partial \epsilon} = U^* \frac{\partial \Pi}{\partial \epsilon} < 0$, and thus $\text{cov}(U^*, \epsilon) < 0$. Then I get $E[U\epsilon] < (E[U^*])(E[\epsilon])$ from $\text{cov}(U^*, \epsilon) = E[U\epsilon] - (E[U^*])(E[\epsilon]) < 0$. Therefore, $\bar{C}'E[U\epsilon] < \bar{e}C'E[U^*]$ where $\bar{C}' > 0$. 

---
output. It implies that the firm moves some of its production from the home plant to the foreign plant to avoid exchange rate risk. When the firm engages in international trade, it faces risk from the variability of the exchange rate which makes its profit unstable. In order to avoid risk and to make the profit more stable, the risk averse firm may want to increase the foreign production, instead of producing at home and exporting.

However, a risk neutral firm produces the same amounts of output at home and abroad under uncertainty as under certainty. The variability of the exchange rate does not affect the levels of the domestic and foreign production. This can be easily shown as follows. Equation (3-4) reduces to

\[ C'(q^*) = \bar{C}'(\bar{q}^*) \]  

for a risk neutral firm, while equation (3-6) becomes \( C'(q^c) = \bar{C}'(\bar{q}^c) \) for the deterministic case. These two equations are exactly the same. Since \( q^* \) and \( q^c \) solves the same equation, given total output constant, they must be the same level. Therefore, \( q^* = q^c \) and \( \bar{q}^* = \bar{q}^c \), given total output unchanged.

2. The effect on total output

In this section, I analyze how the uncertainty affects the level of total output for the risk neutral case \( (U'' = 0) \) in the absence of transportation cost. The effect on total output depends on the type of demand in this case. I analytically show this effect for some specific types of demand when the firm is risk neutral.
The optimal solutions for sales and the allocation of outputs can be derived by solving the first order conditions of the sales and production decision problems.\(^8\)

Substituting equation (3-1) into equation (3-2), I obtain the following equation,

\[
E[U'\cdot (R'(Y) - C'(q))] = 0.
\]

For risk neutrality, this can be rewritten as

\[
E[R'(Y*(Q,e)) - C'(q*(Q,e))] = 0, \tag{3-8}
\]

after substituting the optimal solutions for sales and the domestic output back into the equation. The optimal total output under uncertainty, \(Q^*\), solves equation (3-8). Let's define a function \(J\) as

\[
J(Q) \equiv \frac{\mathbb{E}[\Pi(Q,e)]}{\partial Q} = E[R'(Y*(Q,e)) - C'(q*(Q,e))]. \tag{3-9}
\]

Then, the evaluation of the function \(J(Q)\) at \(Q^*\) equals zero by the definition of the maximization problem,

\[
J(Q^*) = E[R'(Y*(Q,e)) - C'(q*(Q,e))] = 0.
\]

In order to compare \(Q^*\) to the output level under certainty, \(Q^c\), I evaluate the function \(J(Q)\) at \(Q^c\);

---

\(^8\) For the risk neutral firm, the first order conditions of the production decision problem becomes

\[
E[eR'(Y) - C'(q)] = 0 \quad \text{and} \quad E[eR'(Y) - eC'(q)] = 0.
\]

It can be reduced to

\[
E[C'(q) - eC'(q)] = 0 \quad \text{or} \quad C'(q) - eC'(Q - q) = 0.
\]

Thus, the optimal allocation of outputs is a function of total output and the mean of the exchange rate \((\bar{e}\)\), \(q^* = q^*(Q,\bar{e})\) and \(\bar{q}^* = \bar{q}^*(Q,\bar{e})\).

\(^9\) The indirect objective function of the production decision problem becomes a function of \(Q, \bar{e}\), and the distribution of \(e\). Using the envelope theorem, the first derivative of the indirect expected profit function with respect to total output turns out to be

\[
\frac{\partial \mathbb{E}[\Pi(Q,e)]}{\partial Q} = E[R'(Y(Q,e)) - C'(q(Q,\bar{e}))].
\]
\[
J(Q^e) = E[R'(Y^*(Q^e,e)) - C'(q^*(Q^e,e))] = E[R'(Y^*(Q^e,e)) - R'(Y^*(Q^e,e))]
\]
because \( R'(Y^*(Q^e,e)) = C'(q^*(Q^e,e)) \) for the deterministic case. The optimal levels of sales and the domestic production of the risk neutral firm, \( Y^*(Q^*,e) \) and \( C'(q^*(Q^*,e)) \), are determined by the same rules as under certainty, \( Y^*(Q^*,e) \) and \( C'(q^*(Q^*,e)) \), respectively. Since \( \frac{\partial J}{\partial Q} = \frac{\partial^2 E[\Pi(Q,e)]}{\partial Q^2} < 0 \) by the second order condition of the production decision problem,\(^{10}\) the following statement can be obtained,

\[
Q^e < Q^* \text{ as } J(Q^e) < J(Q^*).
\]
The evaluation of \( J(Q) \) at \( Q^e \) provides

\[
J(Q^e) < 0 \text{ as } E[R'(Y(Q^e,e))] < R'(Y(Q^e,e)), \text{ i.e.,}
\]

\[
E[R'(Y(Q^e,e))] = R'(Y(Q^e,e)) \text{ as } \frac{\partial^2 R}{\partial e^2} > 0
\]
by Jensen's inequality. Thus the sign of the second derivative of the domestic marginal revenue with respect to the exchange rate must be determined to compare \( Q^e \) with \( Q^* \), i.e., \( Q^* < Q^e \) as \( \frac{\partial^2 R}{\partial e^2} < 0 \). By taking the first derivative of the domestic marginal revenue function, \( R'(Y(Q^e,e)) \), with respect to the exchange rate, we can obtain

\[
\frac{\partial R'}{\partial e} = \frac{\partial^2 R'}{\partial Y \partial e} Y = R^* \overline{R}'(R^* + e\overline{R}^*)^{-1}
\]
where \( \frac{dY}{de} = \overline{R}'(R^* + e\overline{R}^*)^{-1} < 0 \). Then the second derivative is obtained as

\[
\frac{\partial^2 E[\Pi(Q,e)]}{\partial Q^2} = E[R^* \frac{\partial^2}{\partial Q^2} - C^* \frac{\partial}{\partial Q}] < 0
\]
\[
\frac{\partial^2 R'}{\partial e^2} = R^* \left( \frac{\partial^2 \gamma}{\partial e^2} \right) + R^* \frac{\partial^2 \gamma}{\partial e^2}
\]

\[
= (R^* + eR^*)^2 \frac{\partial^2 \gamma}{\partial e^2} [e \tilde{R}' \cdot (\tilde{R}' + R') - 2R' \tilde{R}' \cdot (R^* + eR^*)].
\]

where \[
\frac{\partial^2 \gamma}{\partial e^2} = -(R^* + eR^*)^{-1} \frac{\partial \tilde{R}}{\partial e} (\tilde{R} + R + (R^* - eR^*) \frac{\partial \tilde{R}}{\partial e}).
\]

The sign of the second derivative of the domestic marginal revenue with respect to the exchange rate depends on the shape of the marginal revenue functions, \( R' \) and \( \tilde{R}' \), given the prior assumptions that the marginal revenues are positive but nonincreasing, \( R'(Y) > 0, \tilde{R}'(Y) > 0, R'(Y) \leq 0 \), and \( \tilde{R}'(Y) \leq 0 \). Thus, the second derivative of the domestic marginal revenue with respect to the exchange rate cannot be generally signed without using a specific function for demand. If the marginal revenue functions are nonconvex, \( R^*, \tilde{R}^* \leq 0 \), the second derivative \( \frac{\partial^2 R'}{\partial e^2} \) will be negative and thus total output will be less under uncertainty than under certainty. However, if the marginal revenue functions are convex, \( R^*, \tilde{R}^* > 0 \), then the sign of the second derivative is ambiguous, and so is the effect on total output. Specifically, I examine this effect for two different types of demands, linear demands and constant elasticity demands. The effect of uncertainty on total output for these particular cases can be described as in the following proposition.

**Proposition 3-2.** If the demand functions in both markets are linear, then the risk neutral firm produces less under uncertainty than under certainty, \( Q^c > Q^* \). However, if the demand functions in both markets have constant elasticity, then the firm produces more under uncertainty than under certainty, \( Q^c < Q^* \).
PROOF. With the linear demand functions in both markets, we can simplify the second derivative of the domestic marginal revenue with respect to the exchange rate as

\[
\frac{\partial^2 R'}{\partial \varepsilon^2} = -2R^*R'R'\cdot(R^* + eR^*)^{-2}.
\]

This expression is negative, \(\frac{\partial^2 R'}{\partial \varepsilon^2} < 0\). Thus, as argued previously, \(J(Q^*) < 0\) and hence \(Q^* < Q^c\), given \(\frac{\partial J}{\partial Q} < 0\).

On the other hand, when the demand functions have constant elasticity, the second derivative of the domestic marginal revenue with respect to the exchange rate is positive.

Inverse demand functions for the domestic and foreign markets can be expressed as

\[
P = \alpha Y^{-\beta} \quad \text{and} \quad \overline{P} = \overline{\alpha} Y^{-\overline{\beta}}
\]

where \(\alpha, \overline{\alpha} > 0\), and \(0 < \beta, \overline{\beta} < 1\).

With these specific demand functions, I obtain the following expressions,

\[
R' = \alpha(1 - \beta)Y^{-\beta}, \quad R^* = -\frac{\beta}{Y} R', \quad R^* = \frac{\beta(1 + \beta)}{Y^2} R',
\]

\[
\overline{R}' = \overline{\alpha}(1 - \overline{\beta})Y^{-\overline{\beta}} = \frac{1}{e} R', \quad \overline{R}^* = -\frac{\overline{\beta}}{eY} R', \quad \text{and} \quad \overline{R}^* = \frac{\overline{\beta}(1 + \overline{\beta})}{eY^2} R'.
\]

We can see that the marginal revenue functions are convex, \(R^*, \overline{R}^* > 0\). Using these expressions, the second derivative can be written and signed as

\[
\frac{\partial^2 R'}{\partial \varepsilon^2} = \Omega \left[ \overline{\beta}(1 - \beta)Y + \overline{\beta}(1 - \overline{\beta})Y^2 \right] > 0
\]

where \(\Omega = e^{-1} \beta (R')^2 \overline{R}'Y^{-2} \overline{Y} (\overline{\beta}Y + \overline{\beta}Y)^{-1}(R^* + eR^*)^{-2} > 0\). The positive second derivative, \(\frac{\partial^2 R'}{\partial \varepsilon^2} > 0\), implies \(J(Q^c) > 0\) and hence \(Q^c < Q^*\), given \(\frac{\partial J}{\partial Q} < 0\). Q.E.D.
After all, the risk neutral firm produces less under uncertainty if the marginal revenue functions are non-convex, but more if the demand functions are constantly elastic. Otherwise, the effect on total output is ambiguous.

3. Firm’s welfare

Regardless of the optimal level of total output, it will be shown that the welfare of the risk neutral firm is greater under uncertainty than under certainty, because the indirect profit function is convex in the exchange rate. That is, its expected profit with exchange rate risk, \( E[\Pi^*(Q^*, e)] \), is greater than its profit with certainty, \( \Pi^c(Q^*, \bar{e}) \), where the functions are described as

\[
E[\Pi^*(Q^*, e)] = E[R(Y^*(Q^*, e)) + e\bar{R}(Q^*-Y^*(Q^*, e)) - C(q^*) - e\bar{C}(Q^*-q^*)],
\]

\[
\Pi^c(Q^*, \bar{e}) = R(Y^c(Q^*, \bar{e})) + \bar{e}\bar{R}(Q^c - Y^c(Q^*, \bar{e})) - C(q^c) - \bar{e}\bar{C}(Q^c - q^c).
\]

This result can be stated by the following proposition.

PROPOSITION 3-3. Exchange rate risk always increases the expected profit of the risk neutral firm.

PROOF. Evaluating the expression for the expected profit of the risk neutral firm at \( Q^c \), we obtain

\[
E[\Pi^*(Q^c, e)] = E[R(Y^*(Q^c, e)) + e\bar{R}(Q^c - Y^*(Q^c, e)) - C(q^c) - e\bar{C}(Q^c - q^c)]
\]

where \( q^* (Q^c) = q^c \). By the definition of maximization problem, \( Q^* \) provides the maximum level of the expected profit. Thus, the evaluation of the expected profit at \( Q^c \) cannot be greater than the evaluation at \( Q^* \),
\[ E[\Pi^*(Q^*,e)] \geq E[\Pi^*(Q^c,e)]. \]

Also, I obtain \( E[\Pi^*(Q^*,e)] \geq E[\Pi^*(Q^c,\bar{e})] \) as by Jensen's inequality.

Therefore, the comparison depends on the sign of the second derivative of the indirect profit function with respect to the exchange rate. Using the envelope theorem, I derive

\[ \frac{\partial \Pi}{\partial e} = R - C \quad \text{and} \quad \frac{\partial^2 \Pi}{\partial e^2} = -\overline{R} \frac{\partial Y}{\partial e} > 0 \]

where \( \frac{\partial Y}{\partial e} = \overline{R} \cdot (R^* + \overline{R}^*)^{-1} \). Since the indirect profit function is convex in the exchange rate, I conclude that the risk neutral firm is always better off with risk,

\[ E[\Pi^*(Q^*,e)] > \Pi^c(Q^c,\bar{e}). \quad \text{Q.E.D.} \]

B. Existence of transportation costs

I have assumed that there is no transportation cost in the previous analysis. I relax this assumption and investigate the firm's trade and production behaviors. With the existence of transportation costs, the firm has additional costs when it engages in trade. Since the costs make trade expensive, the volume of trade is likely to shrink. I examine how the existence of transportation costs affects the firm's trade and output decisions in the presence of exchange rate risk. Again, it is assumed that there is no hedging instrument. Also, the sequence of the firm's decisions is assumed to be the same as before. The existence of transportation costs alters the firm's profit function to

\[ \Pi = R(Y) + e\overline{R}(\overline{Y}) - C(q) - e\overline{C}(\overline{q}) - tX - \overline{tX} \]
where \( Y = q - X + \bar{X} \), \( \bar{Y} = q + X - \bar{X} \), \( Q = q + q \), \( t \) is a transportation cost per unit of product traded, \( X \) is the volume of product exported to the foreign market, and \( \bar{X} \) is the volume of product imported into the domestic market (\( X \geq 0 \) and \( \bar{X} \geq 0 \)). Following similar procedures as before, the sales decision problem in the existence of transportation cost can be described as

\[
\max_{X, \bar{X}} \, \Pi = R(q - X + \bar{X}) + eR(Q - q + X - \bar{X}) - C(q) - e\bar{C}(Q - q) - tX - t\bar{X}
\]

s.t.

\( X \geq 0 \), \( \bar{X} \geq 0 \).

Since exports and imports are non-negative, there might be corner solutions. The first order conditions of this problem can be written as

\[
\frac{\partial \Pi}{\partial X} = -R'(q - X + \bar{X}) + e\bar{R}'(Q - q + X - \bar{X}) - t \leq 0
\]

\[
\frac{\partial \Pi}{\partial \bar{X}} = R'(q - X + \bar{X}) - eR'(Q - q + X - \bar{X}) - t \leq 0.
\]

In order to investigate the trade behavior of the firm, the first order conditions are evaluated at \( X = \bar{X} = 0 \),

\[
\frac{\partial \Pi}{\partial X} \bigg|_{X = \bar{X} = 0} = -R'(q) + e\bar{R}'(Q - q) - t,
\]

\[
\frac{\partial \Pi}{\partial \bar{X}} \bigg|_{X = \bar{X} = 0} = R'(q) - eR'(Q - q) - t.
\]
Note that if \( \frac{\partial \Pi}{\partial X} \bigg|_{X=\bar{X}=0} \geq 0 \), then \( \frac{\partial \Pi}{\partial X} \bigg|_{X=\bar{X}=0} < 0 \) for \( t > 0 \), and if \( \frac{\partial \Pi}{\partial X} \bigg|_{X=\bar{X}=0} \geq 0 \), then \( \frac{\partial \Pi}{\partial X} \bigg|_{X=\bar{X}=0} < 0 \) for \( t > 0 \). Thus, there are three possible cases that might be optimal,

(i) \( \frac{\partial \Pi}{\partial X} \bigg|_{X=\bar{X}=0} > 0 \) and \( \frac{\partial \Pi}{\partial X} \bigg|_{X=\bar{X}=0} \leq 0 \),
(ii) \( \frac{\partial \Pi}{\partial X} \bigg|_{X=\bar{X}=0} \leq 0 \) and \( \frac{\partial \Pi}{\partial X} \bigg|_{X=\bar{X}=0} > 0 \), and
(iii) \( \frac{\partial \Pi}{\partial X} \bigg|_{X=\bar{X}=0} \leq 0 \) and \( \frac{\partial \Pi}{\partial X} \bigg|_{X=\bar{X}=0} \leq 0 \).

These cases can be summarized as follows;

(i) If \( \frac{\partial \Pi}{\partial X} \bigg|_{X=\bar{X}=0} > 0 \), \( R'(q) - e\bar{R}'(Q-q) < -t \), then \( X > 0 \) and \( \bar{X} = 0 \), and
\[
X \text{ solves } R'(q - X) - e\bar{R}'(Q-q + X) + t = 0.
\]

(ii) If \( \frac{\partial \Pi}{\partial X} \bigg|_{X=\bar{X}=0} > 0 \), \( R'(q) - e\bar{R}'(Q-q) > t \), then \( X = 0 \) and \( \bar{X} > 0 \), and
\[
\bar{X} \text{ solves } R'(q + \bar{X}) - e\bar{R}'(Q-q - \bar{X}) - t = 0.
\]

(iii) If \(-t \leq R'(q) - e\bar{R}'(Q-q) \leq t \), then \( X = \bar{X} = 0 \).

From these equations, the optimal level of exports and imports can be expressed as a function of \( Q, q, e, \) and \( t \), \( X^* = X^*(Q,q,e,t) \) and \( \bar{X}^* = \bar{X}^*(Q,q,e,t) \). These first order conditions imply that the firm engages in one-way trade; that is, exporting and importing at the same time \( (X > 0 \text{ and } \bar{X} > 0) \) are not optimal in this model. Using these optimal solutions for trade, the firm's production decision problem can be written as
\[
\max_q E[U(\Pi)]
\]
s.t.
\[
\Pi = R(q - X^* + \bar{X}^*) + e\bar{R}(Q - q + X^* - \bar{X}^*) - C(q) - e\bar{C}(Q - q) - tX^* - t\bar{X}^*.
\]
\[
X^* = X^*(Q, q, e, t) \text{ and } \bar{X}^* = \bar{X}^*(Q, q, e, t).
\]

For positive value of the domestic production, the first order condition of this problem is expressed as
\[
\frac{dE[U(\Pi)]}{dq} = E[U'(R'(q - \bar{X}^* + X^*) - e\bar{R}'(Q - q + X^* - \bar{X}^*) - C'(q) + e\bar{C}'(Q - q))] = 0 \quad (3-9)
\]
by using the envelope theorem. The optimal level of the domestic production \(q^*\) is determined by solving equation (3-9), and the optimal level of the foreign production \(q^*\) is determined in the equation \(\bar{q}^* = Q - q^*\), given total output.

1. The effect on trade behavior

It is expected that the existence of transportation costs discourages the firm from engaging in trade because transportation cost raises the costs of trade. The presence of transportation cost provides less opportunity for the firm to trade because it costs more to the firm. The first order conditions of the sales decision problem provide that

- no export but positive import if \(R'(q) - e\bar{R}'(Q - q) > t\),
- no trade if \(-t \leq R'(q) - e\bar{R}'(Q - q) \leq t\),
- positive export but no import if \(R'(q) - e\bar{R}'(Q - q) < -t\).
By rewriting the first and third cases above, I obtain the followings;

\[ X = 0 \leq \bar{X} \text{ and } \bar{X} \text{ solves } R'(q + \bar{X}) - e\bar{R}'(Q - q - \bar{X}) - t = 0 \text{ if } e < e_1, \quad (3-10) \]

\[ X \geq 0 = \bar{X} \text{ and } \bar{X} \text{ solves } R'(q - \bar{X}) - e\bar{R}'(Q - q + \bar{X}) + t = 0 \text{ if } e > e_2 \quad (3-11) \]

where \( e_1(q) \) is defined as the value of \( e \) at \( \bar{X} = 0 \) satisfying equation (3-10),

\[ e_1(q) = \frac{R'(q) - t}{\bar{R}'(Q-q)} \]

and \( e_2(q) \) as the value of \( e \) at \( X = 0 \) satisfying equation (3-11),

\[ e_2(q) = \frac{R'(q) + t}{\bar{R}'(Q-q)} \]

Then, I compare \( e_1 \) with \( e_2 \) to ensure that \( e_2 \) is greater than \( e_1 \) for all positive transportation costs, \( e_2 - e_1 = \frac{2t}{\bar{R}'(Q-q)} > 0 \). This result provides that the firm imports (exports) if the domestic (foreign) marginal revenue is greater than the foreign (domestic) marginal revenue after compensating transportation cost, \( e < e_1 \) (\( e > e_2 \)). However, the firm would not trade if neither marginal revenues can compensate for transportation cost, i.e., if the difference between the domestic and foreign marginal revenues is less than transportation cost (\( e_1 < e < e_2 \)).

If there exists no transportation cost, equations (3-10) and (3-11) becomes

\[ X = 0 \leq \bar{X} \text{ and } \bar{X} \text{ solves } R'(q + \bar{X}) - e\bar{R}'(Q - q - \bar{X}) = 0 \text{ if } e < e_0, \quad (3-12) \]

\[ X \geq 0 = \bar{X} \text{ and } \bar{X} \text{ solves } R'(q - \bar{X}) - e\bar{R}'(Q - q + \bar{X}) = 0 \text{ if } e > e_0 \quad (3-13) \]

where \( e_0(q) \) is the value of \( e \) at \( X = \bar{X} = 0 \) in the absence of transportation cost,

\[ e_0(q) = \frac{R'(q)}{\bar{R}'(Q-q)} \]. The firm will import (not trade, or export) if the domestic marginal revenue is greater than (equal to, or less than) the foreign marginal revenue,

\[ 11 \text{ } e_1 \text{ satisfies } R'(q) - e\bar{R}'(Q - q) = t, \text{ and } e_2 \text{ satisfies } R'(q) - e\bar{R}'(Q - q) = -t \]
Figure 2. Relationships between trade and the exchange rate

\[ e < e_0 \ (e = e_0, \text{ or } e > e_0) \]. These relationships are represented in Figure 2. The line (3-10) and (3-11) represent the relationship between trade and the exchange rate in the presence of transportation costs, while the line (3-12) and (3-13) explain it in its absence. As we can see in Figure 2, the bottoms of the line (3-12) and (3-13) meet each other at \( e_0 \). There is no trade only at \( e_0 \) in the absence of transportation costs because the domestic and foreign marginal revenues are equalized at \( e_0 \). In the presence of transportation costs, however, the firm will import (not trade, or export) if \( e < e_i \ (e_i \leq e \leq e_2, \text{ or } e > e_2) \). It indicates that the introduction of transportation costs changes the firm’s trade behavior in the following manner. In the absence of transportation cost, the firm maximizes its profit by trading except when \( e = e_0 \). The firm has no incentive to trade only at the point of \( e_0 \) that provides that the domestic and foreign marginal revenues are the same. However, there exists a domain on \( e \) at which the firm has no incentive to trade in the presence of transportation costs, \( e_i < e < e_2 \), where \( e_i < e_0 < e_2 \). In this domain, the firm does not
want to trade because transportation costs are greater than the benefit from trade.

Therefore, the firm is likely to trade less in the presence of transportation costs.

2. The effect on the allocation of outputs

In section A of this chapter, I have studied how uncertainty affects the allocation of outputs in the absence of transportation costs. Without transportation costs, holding total output constant, a risk neutral firm produces the same amounts at home under uncertainty as under certainty, while a risk averse firm expands foreign production under uncertainty. Also, we can see how a change in the value of transportation costs affects the allocation of production under certainty. Under certainty, if the firm exports to the foreign country (i.e., \( X > 0 \)), it produces less at home and more abroad when transportation costs increase. We can see this by examining the first order condition of the production decision problem.\(^\text{12}\)

By totally differentiating the first order condition for the case of exporting under certainty, we obtain \( \frac{dq}{dt} = -\frac{1}{C^* + \bar{e} C^*} < 0 \). This indicates that the existence of

\(^{12}\) In the deterministic case, the first order conditions and the firm's trade behavior can be described as follows. From the first order conditions of the sales decision problem,

\[
X = 0 \leq \bar{X} \text{ and } \bar{X} \text{ solves } R'(q + \bar{X}) - \bar{e} \bar{R}'(Q - q - \bar{X}) - t = 0 \text{ if } R'(q) - \bar{e} \bar{R}'(Q - q) > t \text{ (} \bar{e} < e_1 \text{)},
\]

\[
X \geq 0 = \bar{X} \text{ and } X \text{ solves } R'(q - X) - \bar{e} \bar{R}'(Q - q + X) + t = 0 \text{ if } R'(q) - \bar{e} \bar{R}'(Q - q) < -t \text{ (} \bar{e} > e_2 \text{)},
\]

\[
X = \bar{X} = 0 \text{ if } -t < R'(q) - \bar{e} \bar{R}'(Q - q) < t \text{ (} e_1 < \bar{e} < e_2 \text{)}.
\]

And the first order condition of the output decision problem is \( \frac{\partial R}{\partial q} = R' - \bar{e} \bar{R}' + C^* + \bar{e} C^* = 0 \).

Using the first order condition of the sales decision problem, it can be written as

\[
C'(q) - \bar{e} \bar{C}'(Q - q) = t \quad \text{ when } \bar{e} < e_1,
\]

\[
C'(q) - \bar{e} \bar{C}'(Q - q) = -t \quad \text{ when } \bar{e} > e_2.
\]

\[
R'(q) - \bar{e} \bar{R}'(Q - q) - C'(q) + \bar{e} \bar{C}'(Q - q) = 0 \quad \text{ when } e_1 < \bar{e} < e_2
\]

where \( e_1 \) and \( e_2 \) are functions of total output, \( e_1(Q) \) and \( e_2(Q) \).
transportation costs reduces the domestic production in the deterministic case because the firm does not want to pay more when it exports.

Now, I investigate how the existence of transportation costs changes the firm's production behavior under uncertainty. It will be shown that given total output unchanged, a risk neutral firm produces more domestically under uncertainty than under certainty when transportation costs exist, assuming that the firm exports under certainty. However, whether a risk averse firm produces more or less domestically under uncertainty in the existence of transportation costs is not unambiguously determined. The effect of the existence of transportation costs on the allocation of outputs is investigated by examining the first order condition of the output decision problem. By using the integration expressions, equation (3-9) becomes

\[
\frac{\mathcal{E}[U(\Pi)]}{\partial \eta} = \left[ \int_{\eta}^{\eta} U' \cdot (t - C'(q^*) + e\bar{C}'(Q - q^*)) g(e)de \right]
+ \left[ \int_{\eta}^{\eta} U' \cdot (R'(q^*) - e\bar{R}'(Q - q^*) - C'(q^*) + e\bar{C}'(Q - q^*)) g(e)de \right]
+ \left[ \int_{\eta}^{\eta} U' \cdot (-t - C'(q^*) + e\bar{C}'(Q - q^*)) g(e)de \right]^{13}
\]

13 Using the first order conditions of the sales decision problems,
\[
\frac{\mathcal{E}[U(\Pi)]}{\partial \eta} = \int_{0}^{\eta} U' \cdot (R' - e\bar{R}' - C' + e\bar{C}') g(e)de
+ \int_{0}^{\eta} U' \cdot (R' - e\bar{R}' - C' + e\bar{C}') g(e)de
+ \int_{0}^{\eta} U' \cdot (t - C'(q^*) + e\bar{C}'(Q - q^*)) g(e)de
+ \int_{0}^{\eta} U' \cdot (R'(q^*) - e\bar{R}'(Q - q^*) - C'(q^*) + e\bar{C}'(Q - q^*)) g(e)de
+ \int_{0}^{\eta} U' \cdot (-t - C'(q^*) + e\bar{C}'(Q - q^*)) g(e)de.
\]
where $q^*$ is the optimal level of the domestic output under uncertainty and $g(e)$ is the density function of the exchange rate. The first order conditions of the sales decision problem are used to get equation (3-14). Since $q^*$ solves equation (3-9), the evaluation of 

$$\frac{\partial E[U(\Omega)\mid q]}{\partial q} \Bigg|_{q^*} = 0$$

at $q = q^*$ is equal to zero. In order to compare $q^*$ with $q^e$, I also evaluate

$$\frac{\partial E[U(\Omega)\mid q]}{\partial q} \Bigg|_{q^e} = 0$$

where $q^e$ is the optimal level of the domestic output under certainty,

$$\frac{\partial E[U(\Omega)\mid q]}{\partial q} \Bigg|_{q^e} = \left[ \int_{0}^{\infty} \left[ U'(\cdot) \cdot (t - C'(q^e) + e\bar{C}'(Q - q^e)) \right] g(e) \, de \right] + \left[ \int_{e(q^e)}^{\infty} \left[ U'(\cdot) \cdot (R'(q^e) - e\bar{R}'(Q - q^e) - C'(q^e) + e\bar{C}'(Q - q^e)) \right] g(e) \, de \right] + \left[ \int_{0}^{e(q^e)} \left[ U'(\cdot) \cdot (-t - C'(q^e) + e\bar{C}'(Q - q^e)) \right] g(e) \, de \right].$$

I examine the effect of the existence of transportation costs on the allocation of outputs when the risk neutral firm exports to foreign country under certainty ($\bar{e} > e_2(q^e)$). The first order condition of the production decision problem in the deterministic case is $C'(q^e) - \bar{e}\bar{C}'(Q - q^e) = -t$ when the firm exports. Substituting $C'(q^e) = \bar{e}\bar{C}'(Q - q^e) - t$ and assuming risk neutrality, it can be written as

\[14\] The result for the case of $\bar{e} < e_1(q^e)$ is symmetric with the case of $\bar{e} > e_2(q^e)$. The firm exports to foreign country as $\bar{e} > e_2(q^e)$, while the firm imports from foreign country as $\bar{e} < e_1(q^e)$. The latter can be considered as that the foreign plant of the firm exports from the foreign country to the home country. For the case of $e_1 < \bar{e} < e_2$, transportation costs are high enough to make no trade. Thus, the effect of uncertainty on the allocation of outputs is ambiguous in this case.

\[15\] See footnote 12.
\[
\frac{\partial \mathcal{E}(\Pi)}{\partial q} \bigg|_{q'} = \int_{0}^{\infty} \left[ 2t + (e-\overline{e})\overline{C}(Q-q^c) \right] g(e)de \\
+ \int_{e_{r}(q^c)}^{\infty} \left[ R'(q^c) - e\overline{R}'(Q-q^c) + t + (e-\overline{e})\overline{C}'(Q-q^c) \right] g(e)de \\
+ \int_{e_{l}(q^c)}^{\infty} \left[ (e-\overline{e})\overline{C}'(Q-q^c) \right] g(e)de \\
= 2t \int_{0}^{\infty} g(e)de + \int_{e_{l}(q^c)}^{\infty} \left[ R'(q^c) - e\overline{R}'(Q-q^c) + t \right] g(e)de \\
+ \overline{C}' \int_{0}^{\infty} (e-\overline{e})g(e)de.
\]

Assuming that \( g(e) > 0 \) for some \( e < \bar{e} \) (i.e., assuming that there is some \( e \) that will lead to no trade), this evaluation is positive because the first term is non-negative, the second term is positive, and the third term is zero.\(^{16}\) Therefore, the evaluation of \( \frac{\partial \mathcal{E}(\Pi)}{\partial q} \) at \( q = q^c \) is greater than that at \( q = q^* \), \( \frac{\partial \mathcal{E}(\Pi)}{\partial q} \bigg|_{q} > 0 = \frac{\partial \mathcal{E}(\Pi)}{\partial q} \bigg|_{q^*} \). Assuming that the second order condition of the output decision problem is satisfied, this implies \( q^* > q^c \).

Thus, if the risk neutral firm exports under certainty (\( \bar{e} > e_2 \)), it will produce more at the domestic plant under uncertainty than under certainty when transportation costs exist, provided that there is some positive possibility the exchange rate will be such that, for low \( e \), it will not be optimal to export \textit{ex post}. This result can be compared with that in the absence of transportation costs. As shown in section A of this chapter, in the absence of transportation costs, the risk neutral firm produces the same amount at the domestic plant.

\(^{16}\) Since \(-t < R'(q^c) - e\overline{R}'(Q-q^c) < t \) or \( 0 < R'(q^c) - e\overline{R}'(Q-q^c) + t < 2t \) in this domain, the second term is positive. And the third term is zero because
\[
\int_{0}^{\infty} (e-\overline{e})g(e)de = \int_{0}^{\infty} eg(e)de - \overline{e} \int_{0}^{\infty} g(e)de = \int_{0}^{\infty} eg(e)de - \overline{e} - \overline{e} = 0.
\]
under uncertainty as under certainty, given total output constant. This is because the random exchange rate does not affect the optimal allocation of production of the risk neutral firm. Technically, the optimal levels of the allocation of production under uncertainty and under certainty are determined by the same rule for the risk neutral firm.

In contrast to that, uncertainty leads the risk neutral exporting firm to expand the domestic production comparing with the deterministic case in the existence of transportation costs.

If the firm is risk averse, the evaluation at \( q = q^* \) becomes

\[
\frac{\partial \mathbb{E}[U(T(q))]}{\partial q} \bigg|_{q^*} = \left[ \tau \left( q^* \right) \left( U' \cdot (2t + (e - \bar{e})C'(Q - q^*)) \right) g(e) \right] de
\]

\[
+ \int_{q^*}^{q^*} \left[ U' \cdot (R'(q^*) - e\bar{R}'(Q - q^*) + t + (e - \bar{e})C'(Q - q^*)) \right] g(e) de
\]

\[
+ \int_{0}^{q} \left[ U' \cdot (e - \bar{e})C'(Q - q^*) \right] g(e) de
\]

\[
= 2t \int_{0}^{q} U'(e) g(e) de + \int_{q^*}^{q} \left[ U' \cdot (R'(q^*) - e\bar{R}'(Q - q^*) + t) \right] g(e) de
\]

\[
+ C \int_{q^*}^{q} \left[ U' \cdot (e - \bar{e}) \right] g(e) de.
\]

Since we cannot unambiguously sign this expression, the effect of the uncertainty on the allocation of output is ambiguous. The risk averse firm may produce more or less in the domestic plant under uncertainty in the presence of transportation costs.

In summary, the existence of transportation cost changes the firm's production decision as well as its trade behavior. The existence of transportation costs discourages a firm from engaging in trade because it costs more to the firm. Given total output unchanged, a risk neutral firm produces more at the domestic plant under uncertainty than under certainty in the presence of transportation costs, while it produces the same amount
in the domestic plant under uncertainty as under certainty in the absence of transportation
costs. Even though the effect for a risk averse firm is ambiguous, we can see at least that
the firm’s production decision has changed, in comparison to the case of no transportation
cost in which a risk averse firm produces less in the domestic plant under uncertainty than
under certainty.
CHAPTER IV. HEDGING PROBLEM

When I studied the effect of exchange rate uncertainty in the previous chapter, I assumed that no hedging instrument was available. In this chapter, I introduce foreign exchange futures markets to give agents an opportunity to reduce risk from the volatility of the exchange rate. I examine whether the separation theorem holds in this particular circumstance. Also, I investigate how the presence of foreign exchange futures markets changes the effect of uncertainty on outputs, and examine how the risk averse firm's welfare changes when futures markets are available. Then, I study what the optimal amount of futures contracts is for a risk averse firm. The rule of the timing of sales and production decisions is the same as before. The firm chooses the levels of sales according to the same rule, \( R(Y) - e\overline{R}(Q - Y) = 0 \) and \( \overline{Y} = Q - Y \). However, the production decision problem is different because the profit function is changed. And the firm also chooses the optimal amount of futures contracts before the exchange rate is known. The production and hedging decision problems of the risk averse firm is described as

\[
\max_{q, \overline{q}, h} \quad E[U(\Pi)]
\]

s.t.

\[
\Pi = \int R(Y^*) + e\overline{R}(Q - Y^*) - C(q) - e\overline{C}(\overline{q}) + h(e_f - e),
\]

\[
Y^* = Y^*(Q, e), \quad \overline{Y}^* = \overline{Y}^*(Q, e), \quad \text{and} \quad Q = q + \overline{q}
\]
where $h$ is the amount of futures currency sold, and $e_f$ is the deterministic futures exchange rate. We can derive the first order conditions of this problem as equations (3-2), (3-3), and

$$\frac{\partial E[U(\Pi)]}{\partial h} = E[U'(e_f - e)] = 0.$$ (4-1)

The optimal allocation of production in the presence of futures markets is determined by solving equations (3-2), (3-3), and (4-1) simultaneously, given total output, $Q = q^f + q^\bar{f}$ where $q^f$ and $q^\bar{f}$ are the optimal domestic and foreign outputs in the presence of futures markets, respectively. Reducing equations (3-2), (3-3), and (4-1), I obtain

$$C'(q^f) = e_f C'(q^\bar{f})$$ (4-2)

This rule is the same as that in the deterministic case if the deterministic futures exchange rate is unbiased, $e_f = \bar{e}$. Thus, the optimal allocation of outputs in the presence of unbiased futures markets must be the same as that in the deterministic case.

A. Separation theorem

The separation theorem states that a risk averse firm's production decision is independent of the firm's attitude toward risk and its subjective probability of the exchange rate in the presence of futures markets. Many previous studies have shown that the separation theorem holds in various cases where the profit function is linear in the random variable; under multiple sources of uncertainty (Kwai and Zilcha (1986)), in the case of an unbiased price with the presence of basis risk (Paroush and Wolf (1986)), for a monopolistic multinational firm (Broll and Zilcha (1992)), in the multiperiod framework
with a perfectly competitive forward market (Donoso (1995)). However, some other studies have proved that the separation theorem does not hold in some cases; under an imperfectly competitive forward market (Katz (1984)), for the absolute levels of capital and labor in the presence of unbiased forward market even though it holds for the optimal capital-labor ratio in the multiperiod framework (Zilcha and Eldor (1991)). One interesting paper (Eldor and Zilcha (1987)) showed that the separation theorem holds for a price discriminating firm even with a nonlinear profit function. However, in my study, I expect that it holds for the optimal allocation of production in the presence of futures markets, but not hold for the optimal total output. As mentioned in chapter 2, a major difference of my model from theirs is that the multinational firm in my study produces both in the home and foreign countries with monopolistic power in both markets, while the firm in their model, producing only in the home country, is a monopolist in the home market but a price-taker in the foreign market.

In my model, the optimal allocation of output is independent of the firm's attitude toward risk and its subjective probability of the exchange rate, but the optimal total output depends on the type of utility function and the probability beliefs about the exchange rate when futures foreign exchange markets are available. That is, the separation theorem holds for the optimal allocation of production but not for the optimal level of total output. It can be shown as follows. Plugging \( \bar{q} = Q - q' \) into equation (4-2), I obtain

\[
C'(q') = e_{\bar{C}}(Q - q').
\]
From this equation, the optimal domestic output is determined as a function of fixed total output and the deterministic futures exchange rate only. Therefore, $q^f$ and $\bar{q}^f$ do not depend on the type of utility function and the probability distribution of the exchange rate, $q^f = q^f(Q,e_f)$ and $\bar{q}^f = Q - q^f(Q,e_f)$, implying that the separation theorem holds.

Relaxing the assumption of fixed total output, the optimal level of total output, $Q^f$, is determined in equation (3-2) or (3-3) after plugging $q^f = q^f(Q,e_f)$ and $\bar{q}^f = Q - q^f(Q,e_f)$ back into those equations. Then, $Q^f$ solves the equation

$$E[U'(W)R'(Y^*(Q,e))] - C'(q^f(Q,e_f))E[U'(W)] = 0$$

where $\Pi^f = R(Y^*) + e\bar{R}(Q - Y^*) - C(q^f) - e\bar{C}(\bar{q}^f) + h(e_f - e)$,

$Y^* = Y^*(Q,e), \bar{Y}^* = \bar{Y}^*(Q,e), q^f = q^f(Q,e_f)$, and $\bar{q}^f = \bar{q}^f(Q,e_f)$.

In this equation, we can see that $Q^f$ depends on the utility function and the probability distribution of the exchange rate. Therefore, I conclude that the separation theorem holds for the optimal allocation of outputs, but not for the optimal level of total output in this nonlinear profit model.

B. Presence of futures markets

When a currency futures market is introduced, agents have more opportunity to reduce risk from the volatility of the exchange rate since the firm can hedge against exchange rate risk on its exports in foreign exchange futures markets. This opportunity may lead the firm to produce more at home and export more, holding other things
constant, because one of the major reasons for an exporting firm to produce abroad is to avoid risk from the exchange rate variability.

In this section, I will examine the effect of the presence of futures markets on the allocation of outputs, and also study how the effect of uncertainty on total output is changed if foreign exchange futures markets exist. I also analyze how the risk averse firm's welfare changes when futures markets are available. It will be demonstrated that the risk averse firm produces more domestically in the presence of futures markets than in the absence of those markets except for the upward biased case. I also show that the risk averse firm produces less under uncertainty than under certainty in the presence of futures markets when the marginal revenue functions are non-convex, \( R^\prime, \overline{R^\prime} \leq 0 \), or more when the demand functions are constantly elastic. In addition, I verify that the risk averse firm is better off with risk in the presence of unbiased foreign exchange futures markets.

1. The effect on the allocation of outputs

It was shown in chapter 3 that a risk averse firm produces less at home under uncertainty than under certainty, \( q^* < q^\circ \). Now I examine how the presence of futures markets changes this effect for the risk averse firm.

Given total output fixed, the following expression can be derived from equation (4-2), using the assumption of positive marginal cost,

\[
C'(q^f) \leq \overline{\epsilon C'}(q^f) \text{ as } e_r \leq \overline{\epsilon}.
\] (4-3)
Since we cannot directly compare \( q^f \) with \( q^* \), I examine the impact of the existence of futures markets for unbiased and biased cases separately where \( q^*(q^*) \) and \( q^f(q^f) \) are the optimal domestic (foreign) outputs without and with futures markets.

For the unbiased case, \( e_f = \bar{e} \), equation (4-3) becomes \( C'(q^f) = \bar{e}C'(q^f) \) and we can think of \( q^f \) as \( q^* \) because they are determined by the same rule. Then, I obtain exactly the same result as the one obtained in section A of chapter 3, using the same technique. That is, \( q^* < q^f \) and \( q^* > q^f \) can be derived from equation (3-5) and \( C'(q^f) = \bar{e}C'(q^f) \), given total output, \( Q = q^* + q^* = q^f + q^f \). The risk averse firm produces more at home in the presence of unbiased futures markets than in the absence of them, given unchanged total output.

On the other hand, for the biased case, the effect of uncertainty is changed in a different way. In order to compare \( q^f \) with \( q^* \), I will show that a higher deterministic futures exchange rate leads to higher domestic production in the presence of futures markets, \( \frac{dq^f}{de_f} > 0 \). The comparison depends on how the futures exchange rate is biased, upward \( (e_f < \bar{e}) \) or downward \( (e_f > \bar{e}) \). Equation (4-2) tells us that when \( e_f \) increases, \( C'(q^f) \) must increase to satisfy the equation, and thus \( q^f \) increases with the assumption of positive and increasing marginal costs. This can be mathematically shown with equation (4-2), \( C'(q^f) - e_f\bar{C'}(Q - q^f) = 0 \). Given total output constant, total differentiation of equation (4-2) provides
\[ C'\dot{d}q^f + e_f \ddot{C}^* \dot{d}q^f - \ddot{C}'de_f = 0 \text{ or} \quad \frac{dq^f}{de_f} = \frac{\ddot{C}'}{C^* + e_f \ddot{C}^*} > 0. \]

This implies that the deterministic futures exchange rate and domestic production in the presence of futures markets move together. This result can now proceed to explain the impact of the presence of futures market on outputs for the biased case.

When the futures exchange rate is downward biased (or contango), \( e_f > \bar{e} \), \( q^f \) increases because of a larger \( e_f \) than that in the unbiased case (\( e_f = \bar{e} \)), holding total output constant. That is, for the case of contango, the firm produces more domestically in the presence of futures markets, \( q^* < q^f \) and \( \bar{q}^* > \bar{q}^f \). However, when the futures exchange rate is upward biased (or normal backwardation), \( e_f < \bar{e} \), \( q^f \) decreases because of a smaller \( e_f \) than that in the unbiased case. Since this effect offsets the initial increase in domestic production due to the presence of unbiased futures markets, the impact of the presence of futures markets is indefinite, depending on how much \( q^f \) decreases with a smaller \( e_f \). Therefore, we can conclude that the optimal level of domestic production of the firm in the presence of futures markets will be higher than it is in the absence of futures markets except for the upward biased case.

2. The effect on total output

I have showed that exchange rate uncertainty differently affects the composition of outputs for the risk averse firm when currency futures markets exist. In this section, I study the effect on total output in the presence of currency futures markets. A risk averse
firm produces more or less under uncertainty in the presence of futures markets, depending on the curvature of the marginal revenue functions. That is, the introduction of futures markets may alter the effect of uncertainty on total output.

PROPOSITION 4-1. Given unbiased futures markets, the effect of the presence of futures markets on total output depends on the curvature of the domestic marginal revenue curve in the exchange rate, i.e., \( Q^* \leq Q^f \) as \( \frac{\partial^2 R^*}{\partial q^2} < 0 \). If the marginal revenue curves are non-convex, \( R^*, \overline{R}^* \leq 0 \), then \( Q^f < Q^* \). However, if the marginal revenue curves are strictly convex, \( R^*, \overline{R}^* > 0 \), then the effect on total output is ambiguous. Specifically,

(a) if the demand functions are linear, then \( Q^f < Q^* \).

(b) if the demand functions are constant elasticity, then \( Q^f > Q^* \).

From equations (3-1), (3-2), and (3-3), I derive the equation

\[
E[U'(R(Y) - C'(q))] = 0.
\]

After plugging the optimal solutions for sales and the domestic output in the presence of futures markets back into the equation, this becomes

\[
\frac{\partial E[U(TY) \mid q]}{\partial Q} = E[U'(R(Y * (Q, e)) - C'(q^f (Q, e_f)))] = 0.
\] (4-4)

Then the optimal level of total output in the presence of futures markets, \( Q^f \), solves this equation. In order to compare \( Q^f \) with \( Q^* \), I evaluate equation (4-4) at \( Q^e \) where \( Q^e \) is the optimal level of total output under certainty. Assuming that the deterministic futures exchange rate is unbiased, \( e_f = \bar{e} \), the evaluation at \( Q^e \) can be expressed as

\[
\frac{\partial E[U(TY) \mid q]}{\partial Q} \bigg|_{q^e} = E[U'(R(Y(Q^e, e_f)) - R'(Y(Q^e, e)))].
\]
using the first order condition of the production problem in the deterministic case,

\[ R'(Y(Q^*,e)) = C'(q(Q^*,e)) \]. Then we can conclude that

\[ Q^* \preceq Q' \text{ as } \frac{\partial \mathbb{E}[U(\Pi)]}{\partial Q} \bigg|_{Q^*} \geq 0 = \frac{\partial \mathbb{E}[U(\Pi)]}{\partial Q} \bigg|_{Q'}, \]

assuming that the second order condition of total output decision problem is satisfied.

Now define a function \( H(e) = R'(Y(Q^*,e)) \). Using Taylor series expansion, the function \( H(e) \) can be expanded around the mean of the exchange rate as

\[ H(e) = H(\bar{e}) + H_s(\bar{e}) \cdot (e - \bar{e}) + \frac{1}{2} H_{ss}(\bar{e})(e - \bar{e})^2 \text{ where } \bar{e} \in [\bar{e}, e]. \]

Then, the evaluation at \( Q^* \) can be written as

\[ \frac{\partial \mathbb{E}[U(\Pi)]}{\partial Q} \bigg|_{Q^*} = \frac{1}{2} E[U'H_{ss}(\bar{e}) \cdot (e - \bar{e})^2].^{17} \]

Therefore, \( \frac{\partial \mathbb{E}[U(\Pi)]}{\partial Q} \bigg|_{Q^*} \geq 0 \) as \( H_{ss}(\bar{e}) > 0 \), and thus \( Q^* \preceq Q' \) as \( H_{ss} > 0 \). It proves

\[ \frac{\partial^2 R'}{\partial e^2} \geq 0, \text{ given } H_{ss}(e) = \frac{\partial^2 R'(Y(Q^*,e))}{\partial e^2}. \]

---

\[^{17} \frac{\partial \mathbb{E}[U(\Pi)]}{\partial Q} \bigg|_{Q^*} = E[U' \cdot (H(e) - H(\bar{e}))] = E[U' \cdot (H(\bar{e}) + H_s(\bar{e}) \cdot (e - \bar{e}) + \frac{1}{2} H_{ss}(\bar{e}) \cdot (e - \bar{e})^2 - H(\bar{e}))] = E[U' \cdot (e - \bar{e})]H_s(\bar{e}) + \frac{1}{2} E[U'H_{ss}(\bar{e}) \cdot (e - \bar{e})^2] = E[U' \cdot (e - e_f)]H_s(\bar{e}) + \frac{1}{2} E[U'H_{ss}(\bar{e}) \cdot (e - \bar{e})^2] = \frac{1}{2} E[U'H_{ss}(\bar{e}) \cdot (e - \bar{e})^2] \]
This turns out to be the same result as derived in section B of chapter 3. As shown there, the second derivative of the domestic marginal revenue function with respect to the exchange rate is described as

\[
\frac{\partial^2 R'}{\partial e^2} = (R^* + eR^*)^{-2} \frac{\partial R'}{\partial e} \left[ eR' \cdot (R^*R^* + R^*\bar{R}^*) - 2R^*\bar{R}^* \cdot (R^* + eR^*) \right].
\]

The sign of this expression depends on the curvature of marginal revenue functions, \(R^*\) and \(\bar{R}^*\). If the marginal revenue curves are non-convex, the second derivative of the domestic marginal revenue with respect to the exchange rate will be negative, and thus the optimal level of total output will be less under uncertainty in the presence of futures markets than under certainty. For example, if the demand functions are linear, then the marginal revenue functions are also linear, \(R^* = \bar{R}^* = 0\), and the second derivative becomes negative. Therefore, the firm produces less under uncertainty in the presence of futures markets than under certainty. On the other hand, if the marginal revenue curves are strictly convex, then the second derivative can be positive, negative, or equal to zero, and so the effect on total output is ambiguous. However, if the demand functions are constant elasticity, then the second derivative eventually turns out to be positive, \(\frac{\partial^2 R'}{\partial e^2} > 0\), even though the marginal revenue functions are strictly convex. Therefore, the optimal level of total output in the presence of futures markets is greater under uncertainty than under certainty with constant elasticity demands.
3. Firm's welfare

Assuming that the futures exchange rate is unbiased, the risk averse firm hedging in futures markets is better off under uncertainty than under certainty, regardless of the optimal level of total output. In other words, its expected utility of the profit with futures markets ($\Pi'$) under exchange rate risk, $E[U(\Pi'(Y'(Q',e),h*))]$, is greater than its utility of the profit with certainty ($\Pi^c$), $U(\Pi^c(Y^c(Q^c,e),\tilde{e}))$, where $h^*$ is the optimal number of futures contracts sold, $Q'$ and $Q^c$ are the optimal levels of total output in the presence of futures markets and under certainty, respectively. The indirect profit functions are described as

$$
\Pi'(Y(Q',e),e,h^*) = R(Y(Q',e))+eR(Q' - Y(Q',e))
- C(q') - e\bar{C}(Q' - q') + (e_f - e)h^*
$$

$$
\Pi^c(Y(Q^c,e),\tilde{e}) = R(Y(Q^c,e)) + e\bar{R}(Q^c - Y(Q^c,e)) - C(q^c) - e\bar{C}(Q^c - q^c)
$$

PROPOSITION 4-2. Assuming an unbiased futures market ($e_f = \tilde{e}$), the risk averse firm hedging in futures markets benefits from uncertainty, even though perfect hedging is not feasible.

PROOF. It can be verified by showing that $E[U(\Pi'(Y(Q',e),e,h*))]$ is greater than $U(\Pi^c(Y(Q^c,e),\tilde{e}))$. Suppose the risk averse firm chooses $Q = Q^c$ and $h = \hat{h}$ where

$$
\hat{h} = \bar{R}(Q^c - Y(Q^c,e)) - \bar{C}(Q^c - q^c),
$$

which represent output and net foreign revenue for the deterministic case. Then, realized profits in the presence of exchange rate risk are

$$
\Pi'(Y(Q',e),e,\hat{h}) = R(Y(Q',e)) + e\bar{R}(Q' - Y(Q',e)) - C(q') - e\bar{C}(Q' - q')
+ (e_f - e)(\bar{R}(Q^c - Y(Q^c,e)) - \bar{C}(Q^c - q^c)).
$$
Hence, $\Pi' (Y(Q^e, e), e, \hat{h}) - \Pi^e (Y(Q^e, \bar{e}), \bar{e})$

$$= [R(Y(Q^e, e)) + eR(Q^e - Y(Q^e, e))] - [R(Y(Q^e, \bar{e})) + eR(Q^e - Y(Q^e, \bar{e}))]$$

$$+(e_f - \bar{e})(R(Q^e - Y(Q^e, \bar{e})) - \bar{C}(Q^e - q^e)).$$

Assuming $e_f = \bar{e}$, it implies $\Pi' (Y(Q^e, e), e, \hat{h}) > \Pi^e (Y(Q^e, \bar{e}), \bar{e})$ for all $e \neq \bar{e}$ since

$$[R(Y(Q^e, e)) + eR(Q^e - Y(Q^e, e))] > [R(Y(Q^e, \bar{e})) + eR(Q^e - Y(Q^e, \bar{e}))]$$

and $Y(Q^e, e) \neq Y(Q^e, \bar{e})$ for all $e \neq \bar{e}$ by the definition of maximization problem,

$$[R(Y(Q^e, e)) + eR(Q^e - Y(Q^e, e))] = \max_Y [R(Y) + eR(Q^e - Y)].$$

Therefore,

$$U(\Pi' (Y(Q^e, e), e, \hat{h})) > U(\Pi^e (Y(Q^e, \bar{e}), \bar{e}))$$

and thus

$$E[U(\Pi' (Y(Q^e, e), e, \hat{h}))] > E[U(\Pi^e (Y(Q^e, \bar{e}), \bar{e}))].$$

In addition, it is also true that

$$E[U(\Pi' (Y(Q^e, e), e, h^*))] \geq E[U(\Pi' (Y(Q^e, e), e, \hat{h}))]$$

by the definition of maximization problem,

$$E[U(\Pi' (Y(Q^e, e), e, h^*))] = \max_{q, Q, h} E[U(\Pi(Y(Q, e), e, h))]$$

for all $e$

where $\Pi(Y(Q, e), e, h) = R(Y(Q, e)) + e\bar{R}(Q - Y(Q, e)) - C(q) - e\bar{C}(Q - q) + (e_f - e) h$.

This proves that $E[U(\Pi' (Y(Q^e, e), e, h^*))] > U(\Pi^e (Y(Q^e, \bar{e}), \bar{e}))$ for all $e \neq \bar{e}$. Q.E.D.

Therefore, the risk averse firm is always better off with risk in the presence of unbiased futures markets, regardless of the optimal levels of total output and futures contracts sold.
C. Optimal futures position

I turn to the issue of what the optimal futures position is when other hedging instruments are not available. Again, it is not possible to perfectly hedge against risk with a single hedging instrument when the profit function is nonlinear in the random variable.\(^\text{18}\)

As mentioned earlier, the nonlinearity of the profit function may come from multiple sources of risk and/or the sequence of decisions. Since the assumption made on the sequence of decisions in this model makes the profit function nonlinear in the exchange rate, full hedging with futures contracts only is not attained here.

I assume that futures markets are unbiased \((e_f = \bar{e})\), and \(e = \bar{e} + \varepsilon\) where \(\varepsilon\) has a zero mean with a symmetric distribution \((f(\varepsilon) = f(-\varepsilon))\). In addition, foreign net revenue is assumed to be positive so that the first derivative of indirect profit function with respect to the exchange rate is positive, \(\frac{\partial \Pi}{\partial e} = \bar{R} - \bar{C} > 0\). Demand curves are assumed to be linear, implying that slopes of marginal revenue functions are constant, \(R^w = \bar{R}^w = 0\). Then, the following proposition can be derived.

PROPOSITION 4-3. The optimal futures position depends upon the shape of demand curves. With linear demands, the optimal futures position is short and less than the foreign net revenue of the deterministic case.

Defining \(G(Q,e) = \max_Y \{R(Y) + e\bar{R}(Q - Y)\}\), the profit function can be expressed as

\[
\Pi = G(Q,e) - C(q) - e\bar{C}(q) + h(e_f - e).
\]

The indirect revenue function is obtained as

\[^{18}\text{See Moschini and Lapan (1992, 1995).}\]
\[ G(Q,e) = R(Y^*(Q,e)) + e\bar{R}(Q - Y^*(Q,e)) \] after solving the sales decision problem.

Differentiating it with respect to the exchange rate, one obtains:

\[ G_e(Q,e) = \bar{R}(Q - Y^*(Q,e)), \quad G_{ee}(Q,e) = -(\bar{R})^2 (R^* + e\bar{R}^*)^{-1}, \quad \text{and} \]

\[ G_{ee}(Q,e) = 3(\bar{R})^2 \bar{R}^* (R^* + e\bar{R}^*)^{-2} + (\bar{R})^2 (R^* + e\bar{R}^*)^{-2} (R^* - e\bar{R}^*) \frac{\partial R}{\partial e}. \]

Using Taylor series expansion, the indirect revenue function can be expanded around the mean of the exchange rate as

\[ G(Q,e) = G(Q,\bar{e}) + G_e(Q,\bar{e}) e + \frac{1}{2} G_{ee}(Q,\bar{e}) e^2 + \frac{1}{6} G_{ee}(Q,e(\bar{e})) e^3 \quad \text{where} \quad e \in [\bar{e}, e]. \]

I use this expansion to describe the profit function as

\[ \Pi(e) = A + (B - h) e + \theta(e) \]

where \( A = G(Q,\bar{e}) - C(q) - \bar{C}(Q - q), \quad B = G_e(Q,\bar{e}) - \bar{C}(Q - q) = \bar{R}(Q - Y(Q,\bar{e})) - \bar{C}, \)

and \( \theta(e) = \frac{1}{2} G_{ee}(Q,\bar{e}) e^2 + \frac{1}{6} G_{ee}(Q,e(\bar{e})) e^3. \)

The first order condition of the hedging decision problem can be rewritten as

\[ \frac{\partial \Pi(e)}{\partial h} = -E[U'(\pi)] = \left[_{\bar{e}}^{e} [U'(\pi(-\pi))] \{\pi(\pi) - U'(\pi(\pi))] \} \right. \]
where $\Pi(\pm \varepsilon) = A - (B - h)\varepsilon + \theta(\pm \varepsilon)$ and $\Pi(-\varepsilon) = A - (B - h)\varepsilon + \theta(-\varepsilon)$. Evaluating this first order condition at $h = B$ yields;

$$\frac{\partial EU(\Pi(\varepsilon))}{\partial h} \bigg|_{h=B} = \int_{-\sigma_0}^{\sigma_0} \left[ U'(\Pi(-\varepsilon)) - U'(\Pi(\varepsilon)) \right] \sigma(\varepsilon) d\varepsilon$$

where $\Pi(\pm \varepsilon) = A + \theta(\pm \varepsilon)$ and $\Pi(-\varepsilon) = A + \theta(-\varepsilon)$.

**COROLLARY.** The optimal futures position can be described as follows:

$h^* > B$ as $G_{\varepsilon\varepsilon} > 0$, assuming that the second order condition of the production and hedging decision problem holds.

**PROOF.** Given $U^* < 0$, $[U'(\Pi(-\varepsilon)) - U'(\Pi(\varepsilon))] > 0$ as $[\Pi(-\varepsilon) - \Pi(+\varepsilon)] < 0$.

Since $\Pi(-\varepsilon) - \Pi(+\varepsilon) = \theta(-\varepsilon) - \theta(+\varepsilon) = -\frac{1}{6} \left[ G_{\varepsilon\varepsilon}(Q, e(\pm \varepsilon)) + G_{\varepsilon\varepsilon}(Q, e(-\varepsilon)) \right] \varepsilon^4$ for

$\varepsilon > 0$, $[\Pi(-\varepsilon) - \Pi(+\varepsilon)] > 0$ as $G_{\varepsilon\varepsilon} > 0$. Thus, $\frac{\partial EU(\Pi(\varepsilon))}{\partial h} \bigg|_{h=B} > 0$ as $G_{\varepsilon\varepsilon} > 0$. Given the second order condition, it proves $h^* > B$ as $G_{\varepsilon\varepsilon} > 0$. Q.E.D.

Since the sign of $G_{\varepsilon\varepsilon}$ depends on the second derivative of the marginal revenue functions, the optimal futures position cannot generally be determined without knowing the shape of demand curves. For the case of linear demands, $R^* = \bar{R} = 0$, we obtain $G_{\varepsilon\varepsilon}(Q, e) = 3(\bar{R}^*)^2 \bar{R}^*(R^* + e\bar{R}^*)^{-2} < 0$, and hence the optimal amount of futures foreign currency sold must be less than the foreign net revenue of the deterministic case, $h^* < B$. 
In order to show that the optimal futures position is short, I evaluate the first order condition of the hedging decision problem at $h = 0$. The profit function at $h = 0$ becomes

$$\Pi = R(Y^*(Q,e)) + eR(Q-Y^*(Q,e)) - C(q) - eC(q).$$

Given the assumption of the positive foreign net revenue, the evaluation at $h = 0$ becomes positive,

$$\frac{\partial E[U(\Pi(e))]}{\partial h} \bigg|_{h=0} = E[U'(\Pi)(e_f - e)] = E[U'(\Pi)(\bar{e} - e)] > 0.\text{20}$$

Together with the second order conditions, this implies that the optimal amount of futures foreign currency sold is greater than zero, $h^* > 0$.

Therefore, the optimal futures position is $0 < h^* < R(Q-Y(Q,e)) - \overline{C}$ with linear demands. Since the level of perfect hedging, $R(Q-Y(Q,e)) - \overline{C}$, is a function of the random exchange rate, complete hedging cannot be attained via futures contracts alone.

\text{20} \text{From } U^* < 0 \text{ and } \frac{\partial \Pi}{\partial \overline{e}} = \overline{R} - \overline{C} > 0 \text{ at } h = 0, \frac{\partial U'}{\partial \overline{e}} = U^* \frac{\partial \Pi}{\partial \overline{e}} < 0 \text{ at } h = 0, \text{ and thus}

$$\text{cov}(U',\overline{e}) < 0. \text{ Since } \text{cov}(U',\overline{e}) = E[U'(\overline{e})] - (E[U'])E[\overline{e}] = E[U'(\overline{e})] - E[U(\overline{e})] = E[U'(e - \overline{e})] < 0, E[U'(\overline{e} - e)] > 0 \text{ at } h = 0.$$
CHAPTER V. FOREIGN DIRECT INVESTMENT

In the previous chapters, I have assumed that the firm produces in both markets, implying that the firm has plants in both home and foreign countries. However, assuming that the firm decides where to open its plant at the beginning of the period, and then decides where and how much to produce after the resolution of uncertainty, it may not be profitable to open both plants, depending on which plant has lower marginal cost and which plant is cheaper to open. In order to examine the firm's investment decision, the assumption that the firm operates in both markets is relaxed in this chapter. The firm's decision about where to produce depends on the cost structure in each plant. Under certainty, assuming constant marginal costs, the firm will choose only one plant to open and produce there because it knows which plant is cheaper to open and operate. Under uncertainty, however, the exchange rate is unknown at the time the decision of where to open is made, and thus the firm does not know which plant will have the lower marginal cost \textit{ex post}. The firm will produce only at the plant with lower marginal cost, even though it may open both plants. This result follows because the firm knows which plant has lower marginal cost at the time of its production decision. Assuming that the marginal cost at both domestic and foreign plants are constant, I investigate how the variability of the exchange rate affects the firm's decision to open a foreign plant.

In the deterministic case, the firm opens and produces only in the home (foreign) plant if \( c < \bar{c} \) and \( F \leq \bar{F} \) (\( c > \bar{c} \) and \( F \geq \bar{F} \)) where \( c \) and \( \bar{c} \) are constant marginal costs, \( \bar{c} \) (the mean of the exchange rate) is used for the actual exchange rate with
certainty, and $F$ and $\bar{F}$ are fixed costs for the investment in the home and foreign plants, respectively. Therefore, the firm opens and produces only in the home plant if $\hat{e} < \bar{e}$ and $F \leq \bar{e}\bar{F}$ where $\hat{e}$ is defined as the value of $e$ at which the marginal costs are equalized, $\hat{e} = \frac{c}{\bar{c}}$.

Under uncertainty, however, it is not clear where the firm will wish to open its plant(s) because which plant has lower marginal cost depends on the realized exchange rate. I assume that fixed costs are incurred at a known exchange rate and, for simplicity, $F \leq \bar{e}\bar{F}$. For example, the fixed costs can be licensing, certification, and other fixed fees firms incur to open a plant. All fixed costs are incurred before production, while the actual production decision is made after the exchange rate is known. Assuming that the actual exchange rate varies around its mean from $e_i$ to $e_u$ ($e_i < e < e_u$), the firm will choose to open only the home plant even under uncertainty if $\hat{e} < e_i$ and $F \leq \bar{e}\bar{F}$ since the foreign plant always has higher marginal cost in this case. However, if $e_i < \hat{e}$, the firm may want to pay the amount of $\bar{e}\bar{F}$ in order to have the flexibility in production that would allow it to produce in the foreign plant as well as the domestic plant, because the actual exchange rate may fall into the domain of $[e_i, \hat{e}]$, where the foreign plant is cheaper to operate. Figure 3 shows this situation. If this is the case, the firm may open both plants. Once the firm opens both plants, the firm will produce in the home plant with lower marginal cost $c$ when $e > \hat{e}$, and produce in the foreign plant with $\bar{e}\bar{c}$ when $e < \hat{e}$.

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21 $\bar{e}$ is used for the actual exchange rate with certainty at the time fixed costs are incurred.
22 If $\hat{e} < e_i$, then it is always true that $\hat{e} < e$ or $c < e\bar{c}$.
23 Foreign plant has lower marginal cost in the domain of $[e_i, \hat{e}]$, while home plant has lower marginal cost in the domain of $[\hat{e}, e_u]$. 
Figure 3. Relationship between marginal costs and exchange rate

The decision of how many plants to open thus depends on the net benefit from opening both plants. The firm will not open the foreign plant if the cost to open the foreign plant is greater than the expected benefit from the option of producing abroad. Thus, I study the firm's decision of opening its plants by examining the expected option-value \( E[V] \),

where \( V \) is the difference between the profits with both plants open \( (\Pi^b) \) and with only the home plant open \( (\Pi^h) \).

This issue will be analyzed for the case of \( c < \bar{e} \bar{c} \) (or \( \bar{e} < \bar{e} \) ) and \( F \leq \bar{e}\bar{F} \), assuming \( e_{1} < \bar{e} \).\(^{24}\) For convenience of the analysis, I first examine this issue when total output is held fixed, and then extend it to the case of price responsive total output. Finally, I investigate how exchange rate uncertainty affects the firm's investment behavior when it faces a competitor in the foreign market, and also analyze how its investment decision affects the foreign firm's welfare.

\(^{24}\) The case of \( c > \bar{e}\bar{c} \) and \( F \geq \bar{e}\bar{F} \) follows the same analysis.
A. Fixed total output

In this section, I study the firm's decision of opening its plants when total output is fixed, regardless of the level of price. I consider a firm that has contracts with distributors or wholesalers to supply a certain amount of the product for a certain period of time. The supply (output) of this firm does not vary with price during the period because the level of total output of the firm is predetermined. In this case, the firm decides where to open its plant under uncertainty, and decides where to produce under certainty. Since total output is assumed to be fixed, there is no decision about how much to produce. The firm's investment decision may vary with its attitude toward risk, and thus I investigate this issue for risk neutrality and for risk aversion separately.

1. Risk neutrality

If the risk neutral firm decides to use its option to open the foreign plant, it will pay the cost of opening the foreign plant and will have the flexibility in production to produce at the plant with lower marginal cost. Alternatively, the firm may open only the home plant and produce there with the assumption of \( c < \bar{c} \) (or \( \hat{c} < \bar{c} \)) and \( F \leq \bar{F} \).

Assuming that the firm sells only at home, the profits with both plants open (\( \Pi^b \)) and with only the home plant open (\( \Pi^h \)) can be described as

\[
\Pi^b = PQ - \min\{c, \bar{c}\}Q - F - \bar{F},
\]

\[
\Pi^h = PQ - cQ - F.
\]

---

25 While feasible, it will never be optimal to open only the foreign plant if \( c < \bar{c} \) and \( F \leq \bar{F} \), or if \( c \leq \bar{c} \) and \( F < \bar{F} \).
The cost of $\bar{e} F$ can be considered as the cost incurred to get flexibility in production.

The value of the option (V) providing a chance to produce at the plant with lower marginal cost can be expressed as the difference of $\Pi^b$ from $\Pi^h$,

$$V = \Pi^b - \Pi^h = cQ - \min\{c, e\bar{e}\} Q - \bar{e} F = \left(\max\{0, \hat{e} - e\} - \frac{\bar{e} F}{cQ}\right) cQ. \quad (5-1)$$

This value is represented graphically by "long put" in Figure 4. Figure 4 graphically shows that the value of opening the foreign plant is the difference between $\Pi^b$ and $\Pi^h$.

The expected option-value can be written as

$$E(V) = E(\Pi^b - \Pi^h) = E(\max\{0, \hat{e} - e\} - \frac{\bar{e} F}{cQ}) cQ. \quad (5-1)$$

This expression is the same as the expected net benefit from a long position of put options.

---

$^{26}$ $V = (c - \min\{c, e\bar{e}\}) Q - \bar{e} F = \left(\frac{c}{c} - \min\{\frac{c}{c}, e\}\right) cQ - \bar{e} F = (\hat{e} - \min\{\hat{e}, e\}) \bar{e} Q - \bar{e} F$

$= (\hat{e} + \max\{-\hat{e}, -e\}) \bar{e} Q - \bar{e} F = (\max\{\hat{e} - \hat{e}, \hat{e} - e\}) \bar{e} Q - \bar{e} F = (\max\{0, \hat{e} - e\} - \frac{\bar{e} F}{cQ}) \bar{e} Q$

$^{27}$ I call it "long put" because it has the same form as the value of put options purchased, not because it is the value of put options purchased.
where $\hat{e}$ can be considered as the strike price, $\frac{\varphi F}{cQ}$ as the price of the put option, and $cQ$ as the number of put options purchased. Thus, the value of providing a chance to produce at the plant with lower marginal cost has the same form as that of buying a put option on the exchange rate. Therefore, producing at the foreign plant provides the same effects as exercising a put option. The risk neutral firm wants to use the option to open the foreign plant if the expected option-value is positive, $E(V) > 0$ or $E(\max(0, \hat{e} - e)) > \frac{\varphi F}{cQ}$. The option to open both plants differs from a put option in the sense that the purpose of taking the option to open both plants is to take advantage from having a chance to produce at the plant with lower marginal cost, while agents may buy a put option to avoid a risk from an uncertain variable. The expected value of opening the foreign plant increases when the exchange rate is more volatile. It can be shown with a mean-preserving spread of the distribution of the exchange rate.

If a mean-preserving spread of the distribution of the exchange rate is employed, the variability of the exchange rate becomes riskier, and this leads to a higher expected option-value. Therefore, the probability that the firm takes the option to open the foreign plant gets higher when the exchange rate is more volatile. It can be analytically shown as follows. With the mean-preserving spread of the distribution, $e$ is replaced by $\bar{x} + \gamma (x - \bar{x})$ where $\gamma$ is the mean-preserving spread parameter, and $\bar{x} = \bar{e}$.

Using the integral expression, the expected option-value in equation (5-1) can be expressed as

$$
\bar{e} = E(e) = E(\bar{x} + \gamma (x - \bar{x})) = \bar{x}, \quad \text{and} \quad \text{var}(e) = E[(\bar{x} + \gamma (x - \bar{x})) - \bar{x}]^2 = \gamma^2 E[x - \bar{x}]^2 = \gamma^2 \text{var}(x).
$$

---

28 $\bar{e} = E(e) = E(\bar{x} + \gamma (x - \bar{x})) = \bar{x}, \quad \text{and} \quad \text{var}(e) = E[(\bar{x} + \gamma (x - \bar{x})) - \bar{x}]^2 = \gamma^2 E[x - \bar{x}]^2 = \gamma^2 \text{var}(x)$. 

\[ E(V) = E(\Pi^b - \Pi^h) = \mathcal{E}Q_{\tilde{\varepsilon}}^\varepsilon (\tilde{\varepsilon} - e)g(e)de - \bar{e}F. \]

For the mean-preserving spread, it turns out to be

\[ E(V) = \mathcal{E}Q_{\tilde{\varepsilon}}^\varepsilon \left[ \tilde{x} + \gamma (\tilde{x} - \bar{x}) - (\tilde{x} + \gamma (x - \bar{x})) \right] g(x)dx - \bar{e}F \]

\[ = \mathcal{E}Q_{\tilde{\varepsilon}}^\varepsilon \gamma (\tilde{x} - x)g(x)dx - \bar{e}F. \]  \(5-2\)

The derivative of the expected option-value with respect to \(\gamma\) can be derived from equation (5-2),

\[ \frac{\partial E(V)}{\partial \gamma} = \mathcal{E}Q_{\tilde{\varepsilon}}^\varepsilon (\tilde{x} - x)g(x)dx. \]

By signing it, we can see how the expected option-value varies with the mean-preserving spread parameter. This derivative is positive because \(\tilde{x}\) is greater than \(x\) over the domain integrated. Therefore, the value of retaining the flexibility in production increases when the variability of the exchange rate gets larger, and the firm is more likely to open both plants when the exchange rate is quite volatile.

2. Risk aversion

As described in the previous section, the profit with both plants open (\(\Pi^b\)) can be expressed as the profit with only the home plant open (\(\Pi^h\)) plus the term which has the same form as the value of put options purchased (\(W\)),

\[ \Pi^b = \Pi^h + W \]

---

\(29\) \(E(V) = E(cQ - \min[c, e\bar{e}]Q - \bar{e}F) = Q_{\tilde{\varepsilon}}^\varepsilon (c - e\bar{e})g(e)de + Q_{\tilde{\varepsilon}}^\varepsilon (c - c)g(e)de - \bar{e}F\)

\[ = \mathcal{E}Q_{\tilde{\varepsilon}}^\varepsilon (\tilde{\varepsilon} - e)g(e)de - \bar{e}F = \mathcal{E}Q_{\tilde{\varepsilon}}^\varepsilon (\tilde{\varepsilon} - e)g(e)de - \bar{e}F \]

\(30\) Since \(e = \bar{x} + \gamma (x - \bar{x}), \tilde{\varepsilon} = \bar{x} + \gamma (\tilde{x} - \bar{x})\).
where $W \equiv (\max\{0, \hat{e} - e\} - \frac{\bar{e}F}{\bar{Q}})\bar{Q}$. The risk averse firm opens both plants if $E[U(\Pi^b)] > U(\Pi^b)$, while the risk neutral firm uses the option to open both plants if $E[\Pi^b] > \Pi^b$. Note that $E[\Pi^b] > \Pi^b$ does not imply $E[U(\Pi^b)] > U(\Pi^b)$ for the risk averse firm. It is true for the risk averse firm that $E[U(\Pi^b)] < U(E[\Pi^b])$ by Jensen's inequality where $U^* < 0$ and $U(E[\Pi^b]) > U(\Pi^b)$ for the case of $E[\Pi^b] > \Pi^b$. Putting them together provides $E[U(\Pi^b)] > U(\Pi^b)$. Since, in this model, revenue is not subjected to exchange rate risk, the risk averse firm is less likely to open the foreign plant, even if $E[\Pi^b] > \Pi^b$. On the other hand, when the risk neutral firm is indifferent in opening the foreign plant, $E[\Pi^b] = \Pi^b$, the risk averse firm will not want to open the foreign plant because $E[U(\Pi^b)] < U(E[\Pi^b]) = U(\Pi^b)$ by Jensen's inequality.

Therefore, regardless of the expected benefit of the risk neutral firm from opening the foreign plant, doing so exposes it to potential exchange rate risk. To avoid this risk, the firm may want to sell put options. Suppose $E(\max\{0, \hat{e} - e\}) > \frac{\bar{e}F}{\bar{Q}}$ so that the risk neutral firm chooses to open the foreign plant, $E[\Pi^b] > \Pi^b$. If the risk averse firm sells put options with the strike price of $\hat{e}$, that are fairly priced at price $r$, \footnote{Fairly priced put options mean here that the expected value of a put option is equal to zero, $r = E(\max\{0, \hat{e} - e\})$.} $\Pi^b$ with put options sold $(\Pi^p)$ becomes

$$
\Pi^p = \Pi^b + (\max\{0, \hat{e} - e\} - \frac{\bar{e}F}{\bar{Q}})\bar{Q} + (r - \max\{0, \hat{e} - e\})z
$$
Figure 5. Role of short put options in the investment decision

The risk averse firm sells put options of $cQ$ at price $r$, and chooses to open the foreign plant if the risk neutral firm is better off by opening the foreign plant.

where $z$ is the number of put options sold. If the risk averse firm sells put options equal to the foreign exchange cost incurred when it produces abroad, i.e., $z = cQ$, then $\Pi^p$ has no risk, $\Pi^p = \Pi^h + (r - \frac{\tilde{e}}{cQ}) cQ$, and hence $E[U(\Pi^p)] = U(E[\Pi^p]) = U(\Pi^p)$. Thus, $\Pi^p > \Pi^h$ and $U(\Pi^p) > U(\Pi^h)$. As shown in Figure 5, exchange rate risk is offset by short put options, since these options eliminate all the risk occurred from the variability of the exchange rate. The assumptions that put options are fairly priced implies that put options do not affect expected profits and thus, with optimal risk hedging, risk attitudes do not matter. Therefore, when the risk neutral firm is better off by opening the foreign plant, the risk averse firm will also choose to open the foreign plant by selling put options that are fairly priced.
B. Price responsive total output

I have examined the firm's decision of opening its plants with total output fixed in the previous section. As mentioned in the previous section, this model may make sense for firms that have contracts with distributors or wholesalers for a certain period of time so that the firm's supply is not varied with price during that period. For many industries, however, the firm's production decision is likely to be affected by the price of the product. Returning to our assumption of a profit maximizing monopolist, I study the firm's decision of opening its plants when total output varies with price, and compare the results to those in the previous section.

As assumed earlier in this chapter, the exchange rate is known before the production decision is made, but it is unknown at the time of the decision of where to open plants. Remember that I assumed $c < e\bar{e}$ and $F < e\bar{F}$ so that the firm produces at home in the deterministic case. For simplicity of the analysis, I keep the assumption that the firm sells only at home. With price responsive total output, the profit functions become

$$\Pi^h(e) = \max_Q \left[ R(Q) - \min[\hat{e}, e] \bar{e}Q - F - e\bar{F} \right],$$

$$\Pi^d = \max_Q \left[ R(Q) - cQ - F \right].$$

When the firm opens the domestic plant only, the risk neutral firm chooses the optimal level of output by maximizing its profit with certainty, $\Pi^h$. The first order condition of this problem can be written as

$$R'(Q) - c = 0.$$
Note that, for this case, the optimal level of total output is independent of the exchange rate, and hence the firm is not exposed to exchange rate risk. This parallels the model of the previous section where total output was fixed.

On the other hand, when the firm opens both plants, the firm maximizes its profit, $\Pi^b$, by choosing the optimal level of output based upon the realized value of $e$. The first order condition of this problem can be written as

$$R'(Q) - c = 0 \quad \text{if } \hat{e} < e \quad (or \quad c < e\overline{e}),$$

$$R'(Q) - e\overline{e} = 0 \quad \text{if } \hat{e} > e \quad (or \quad c > e\overline{e}).$$

The optimal levels of total output and profit vary with the exchange rate when the firm chooses to open both plants, while they are independent of the exchange rate when the firm opens only the home plant. The optimal levels of total output for both cases are determined by the same rule and so are the same when the home plant is used (i.e., when $\hat{e} < e$). However, the optimal level of total output for the case when the foreign plant is used (i.e., when $\hat{e} > e$) depends on the exchange rate because it is a function of $e\overline{e}$,

$$Q^b = Q^b(e\overline{e}).$$

The slope of the line representing the relationship between the optimal level of total output and the exchange rate is derived by totally differentiating the first order condition of the production decision problem,

$$\frac{\partial Q^b}{\partial e} = \frac{\partial Q^h}{\partial e} = 0 \quad \text{if } \hat{e} < e, \quad \text{and} \quad \frac{\partial Q^b}{\partial e} = \frac{\overline{e}}{R^*} < 0 \quad \text{if } \hat{e} > e$$

where $Q^b$ and $Q^h$ are the optimal outputs when the firm opens both plants and when it opens only the home plant, respectively.
The optimal level of total output is negatively related to the exchange rate when \( \hat{e} > e \).

This relationship is shown in (a) of Figure 6. The optimal profit is obtained by using the optimal level of total output,

\[
\Pi^b(c) = R(Q^b(c)) - cQ^b(c) - F,
\]

\[
\Pi^b(c) = R(Q^b(c)) - cQ^b(c) - F - \bar{e}F \quad \text{if } \hat{e} < e,
\]

\[
\Pi^b(e\bar{e}) = R(Q^b(e\bar{e})) - e\bar{e}Q^b(e\bar{e}) - F - \bar{e}F \quad \text{if } \hat{e} > e
\]

where \( Q^b(c) = Q^b(c) \). Note that \( \Pi^b(c) = \Pi^b(c) - \bar{e}F \) and \( \Pi^b(e\bar{e}) > \Pi^b(c) - \bar{e}F \). The optimal profit is also negatively related to the exchange rate with the slope of \( -\bar{c}Q^b(e\bar{e}) \) when \( \hat{e} > e \). This slope is not constant because \( Q^b \) is a function of the exchange rate when \( \hat{e} > e \), while the slope of the profit function with fixed total output (\( \Pi^{bf}(e\bar{e}) \)) in the

---

32 \( \Pi^{bf} \) is profit with fixed total output when the firm opens both plants, \( \Pi^{bf} = PQ - e\bar{e}Q - F - \bar{e}F \) if \( \hat{e} > e \), and \( \Pi^{bf} = PQ - cQ - F - \bar{e}F \) if \( \hat{e} < e \). And \( \Pi^b \) is profit with price responsive total output when the firm opens both plants. \( \Pi^b \) is not less than \( \Pi^{bf} \), \( \Pi^b \geq \Pi^{bf} \), because \( \Pi^b \) is maximized with respect to \( Q \), while \( \Pi^{bf} \) is not and is just a function of predetermined total output.
domain of $\hat{e} > e$ is $-\epsilon Q$, which is constant. The relationships between the optimal profits and the exchange rate are shown in (b) of Figure 6. As shown there, the indirect profit function with price responsive total output is nonlinear in the exchange rate. The expected value of opening the foreign plant ($E[V] = E[\Pi^b - \Pi^h]$) can now be written as

$$E[V] = E_{\hat{e} < e}[R(\hat{Q}^b(e\hat{e})) - e\hat{e}Q^b(e\hat{e}) - (R(Q^h(c)) - cQ^h(c)) - eF] + E_{\hat{e} > e}[eF]$$

where $V = -\hat{e}F$ if $\hat{e} < e$,

$$V = R(\hat{Q}^b(e\hat{e})) - e\hat{e}Q^b(e\hat{e}) - (R(Q^h(c)) - cQ^h(c)) - \hat{e}F$$

if $\hat{e} > e$. The expression of $[\Pi^b - \Pi^h]$ is different from that in the case of fixed total output only when $\hat{e} > e$; in particular, the ability to adjust output increases the (expected) value of opening the foreign plant. The decision to open the foreign plant for the risk neutral firm depends on the sign of the expected option-value. If $E[V] > 0$, the risk neutral firm will have higher expected profit with both plants, and will therefore choose to open the foreign plant. Note, however, that for the risk averse firm, a put option will not provide a perfect hedge because profits are strictly convex (nonlinear) in $e$ for $\hat{e} > e$ due to the responsiveness of total output to the exchange rate.

It is possible to illustrate the expected option-value more clearly by employing a linear demand. Using the linear demand, $P(Q) = a - bQ$, the revenue function and the marginal revenue function are expressed as $R(Q) = aQ - bQ^2$ and $R'(Q) = a - 2bQ$. The

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33 The value of opening the foreign plant with fixed total output is $V = -\hat{e}F$ if $\hat{e} < e$, and $V = -e\hat{e}Q + cQ - \hat{e}F$ if $\hat{e} > e$. It is linear in the exchange rate.
first order condition of the production decision problem for the case of opening only the home plant then is
\[ a - 2bQ - c = 0. \]

From this equation, the optimal level of production can be calculated, \( Q^h = \frac{a-c}{2b} \), and thus the optimal profit, \( \Pi^h = \frac{(a-c)^2}{4b} - F \). Note that they are independent of the exchange rate.

On the other hand, when the firm uses its option to open the foreign plant, the first order condition of the production decision problem becomes
\[
\begin{align*}
    a - 2bQ - c &= 0 \text{ if } \hat{e} < e, \\
    a - 2bQ - e\bar{c} &= 0 \text{ if } \hat{e} > e.
\end{align*}
\]

The optimal levels of output and profit are thus
\[
\begin{align*}
    Q^b &= \frac{a-c}{2b} \text{ and } \Pi^b = \frac{(a-c)^2}{4b} - F - \bar{e}F^2 \text{ if } \hat{e} < e, \\
    Q^b &= \frac{a-e\bar{c}}{2b} \text{ and } \Pi^b = \frac{(a-e\bar{c})^2}{4b} - F - \bar{e}F^2 \text{ if } \hat{e} > e.
\end{align*}
\]

Then, the expected option-value, \( E[V] = E[\Pi^b - \Pi^h] \), can be calculated as
\[
E[V] = E \left[ \frac{(a-e\bar{c})^2 - (a-c)^2}{4b} - \bar{e}F^2 \right] + E \left[ -\bar{e}F \right]
= -\bar{e}F + \frac{e}{4b} \int_{\hat{e}}^l \left[ 2ac(1 - \frac{e}{\bar{e}}) - c^2 (1 - (\frac{e}{\bar{e}})^2) \right] g(e) de \quad (5-3)
\]
where \( \frac{e}{\bar{e}} < l \) in the domain integrated. Therefore, the risk neutral firm takes the option to open the foreign plant as well as the domestic plant if \( E[V] > 0 \).
Table 2. The expected values of opening the foreign plant

<table>
<thead>
<tr>
<th>( {\alpha, \beta} )</th>
<th>{8, 12}</th>
<th>{7, 13}</th>
<th>{6, 14}</th>
<th>{5, 15}</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E[V] )</td>
<td>-13.37</td>
<td>-3.4</td>
<td>7.71</td>
<td>19.53</td>
</tr>
</tbody>
</table>

In order to see how this expected value changes with the variability of the exchange rate, I perform a simple simulation. The values of the parameters used in this simulation are given by: \( \{\bar{\epsilon}, \alpha, b, c, \bar{\epsilon}, F, F'\} = \{10, 60, 1, 20, 2.2, 20, 2\} \). These specific values satisfy the assumptions made on this analysis, \( c < \bar{\epsilon}c \) (or \( \bar{\epsilon} < c \)) and \( F < eF' \).

Using an uniform distribution of the exchange rate,\(^{34}\) four different variabilities of the exchange rate are examined, \( \{\alpha, \beta\} = \{8, 12\}, \{7, 13\}, \{6, 14\}, \{5, 15\} \). Substituting these particular values into equation (5-3), we obtain the numerical values for the expected value of opening the foreign plant for each variability. These values are shown in Table 2. This table tells us that the expected value of opening the foreign plant increases with larger variability of the exchange rate. When \( \{\alpha, \beta\} = \{8, 12\} \) or \( \{7, 13\} \), the firm is likely to open only the home plant because the variability is not large enough to induce the firm to open the foreign plant. The firm, however, has more chance to get positive benefit from opening the foreign plant if the variability gets larger, for example, \( \{\alpha, \beta\} = \{6, 14\} \) or \( \{5, 15\} \). This is consistent with the result from the simulation performed in the next section.\(^{35}\) The difference between the cell (ii) and (iv) in all tables of Table 3 provides the

\(^{34}\) The uniform distribution of \( \epsilon \) can be written as \( g(\epsilon) = \frac{1}{\beta - \alpha} \) for \( \alpha < \epsilon < \beta \) and 0 elsewhere.

\(^{35}\) The numerical values for costs are slightly different in the two simulations. In this simulation, the values given to the costs satisfy \( c < \bar{\epsilon}c \) and \( F = \bar{e}F' \), while the values in the simulation of the next section satisfy \( c = \bar{\epsilon}c \) and \( F < \bar{e}F' \). Since both cases implies that the firm produces at home in the deterministic case, the qualitative results for both cases are the same.
expected option-value, $E[V]$, when there is no competitor. Therefore, we can conclude that the firm is more likely to open the foreign plant with larger variability of the exchange rate.

C. Strategic advantage

Under the assumption that the multinational firm (MNF) does not sell, or face any competitor in the foreign market, I have examined how its foreign direct investment decision is affected by the variability of the exchange rate. In order to study a more realistic case, I relax this assumption and assume that the MNF sells and faces a competitor in the foreign market. I will now investigate the case where the MNF sells in both domestic and foreign markets, while the local (foreign) firm sells only in its own market. Assuming that both firms are risk neutral, the MNF may or may not open the foreign plant, depending upon its expected profits, and produces at the plant with lower marginal cost, but the local firm produces only in its own country. I keep the same assumption on the sequence of decisions; both firms’ sales and production decisions are made after the resolution of the exchange rate uncertainty, but their investment decisions are made under uncertainty. I also assume that both firms move at the same time so that they play a Cournot-Nash game; that is, neither firm makes its decisions before or after its competitor makes decisions. In this simultaneous move game, even if the local firm produces and sells in the foreign market only, the optimal output and profit of the local firm are indirectly affected by exchange rate uncertainty through the optimal output of the MNF. Thus, the MNF may gain a strategic advantage by having a chance to open the
foreign plant. If uncertainty encourages the MNF to open the foreign plant, it may hurt the local firm, which has no such opportunity.

Assuming that all marginal costs are constant, the profit functions of the MNF can be described as

$$\Pi^* = (P(Y) - c)Y + (e\bar{P}(Y + \hat{Y}) - c)\bar{Y} - F,$$

$$\Pi^b = P(Y)Y + e\bar{P}(Y + \hat{Y})Y - \min[c, ec] (Y + \bar{Y}) - F - e\bar{F}$$

where $P(Y)$ is the inverse demand of the domestic market, $\bar{P}(Y + \hat{Y})$ is the inverse demand of the foreign market, and $\hat{Y}$ is the level of sales of the local firm. Similarly, the profit function of the local firm can be written as

$$\Pi^l = \bar{P}(Y + \hat{Y})\hat{Y} - d\hat{Y} - \bar{G}$$

where $d$ is the constant marginal cost of the local firm, and $\bar{G}$ is the fixed cost of the local firm. As in the previous section, I keep the assumption that the MNF opens and produces only at the domestic plant in the deterministic case, i.e., $c < ec$ and $F \leq e\bar{F}$.\(^{36}\)

Under uncertainty, the MNF may choose to open only the home plant or may open both plants, depending upon the variability of the exchange rate. The firms choose the level of sales to maximize their profits after the exchange rate is known. If the MNF opens only the home plant, assuming that all the sales are positive, the optimal conditions of each firm can be expressed as

\(^{36}\) In the deterministic case, the MNF is allowed to open both plants, but has no advantage in doing so if $c < ec$. Opening the foreign plant conveys no strategic advantage, *per se*, and thus opening only the home plant is always a dominant strategy.
\[
\frac{\partial \Pi^h}{\partial \mathcal{H}} = P(Y) + P'(Y)Y - c = 0, \quad (5-4)
\]
\[
\frac{\partial \Pi^h}{\partial \mathcal{H}} = eP(Y + \hat{Y}) + eP'Y - c = 0, \quad (5-5)
\]
\[
\frac{\partial \Pi^{lh}}{\partial \mathcal{H}} = \bar{P}(\bar{Y} + \hat{\bar{Y}}) + \bar{P}'\hat{\bar{Y}} - \bar{d} = 0 \quad (5-6)
\]

where the superscript \( h \) denotes the MNF and the superscript \( lh \) denotes the local firm when the MNF opens only the home plant. By solving these equations, we obtain the optimal solutions for sales, \( Y^h, \bar{Y}^h(e), \) and \( \hat{Y}^h(e) \). Note that the optimal domestic sales of the MNF is independent of the exchange rate, while exchange rate uncertainty affects the optimal sales of the local firm as well as the optimal foreign sales of the MNF.

On the other hand, if the MNF opens both plants, the optimal conditions for the MNF will be
\[
\frac{\partial \Pi^b}{\partial \mathcal{H}} = P(Y) + P'(Y)Y - \hat{c} = 0, \quad (5-7)
\]
\[
\frac{\partial \Pi^b}{\partial \mathcal{H}} = e\bar{P}(\bar{Y} + \hat{\bar{Y}}) + e\bar{P}'\bar{Y} - \hat{\bar{d}} = 0 \quad (5-8)
\]

where \( \hat{c} = \min\{c, e\bar{c}\} \) and the optimal condition of the local firm is the same as equation (5-6). Note that if \( c < e\bar{c} \), the conditions for the MNF are the same as those when the MNF opens only the home plant because \( \hat{c} = c \) if \( c < e\bar{c} \) (or \( \hat{e} < e \)), and thus the optimal sales are the same in both cases. However, if \( c > e\bar{c} \), then \( \hat{c} = e\bar{c} \) and the optimal sales in the home market \( (Y^b) \) depend on the exchange rate, while all sales in the foreign market \( (\bar{Y}^b, \hat{\bar{Y}}^b) \) are independent of the random exchange rate.
We can have some idea about the shape of the optimal sales in the exchange rate from total differentiation of the optimal conditions. For the case when the MNF opens only the home plant, by totally differentiating equations (5-5) and (5-6), we can obtain

\[
\frac{\partial \bar{R}^h}{\partial \epsilon} = -\frac{\bar{R}' \hat{R}^*}{e[(\bar{R}' - \bar{R})'(\bar{R}' - \hat{R})]} \\
\frac{\partial \bar{R}^h}{\partial \epsilon} = -\frac{\bar{R}' \hat{R}^*}{e[(\bar{R}' - \bar{R})'(\bar{R}' - \hat{R}) - \bar{R}' \hat{R}^*]}
\]

where \( R(Y) = P(Y)Y \), \( R(Y + Y) = P(Y + Y)Y \), and \( R(Y - Y) = P(Y + Y)Y \), whereas

\[
\frac{\partial Y^b}{\partial \epsilon} = 0.
\]

It is assumed that all the revenue functions have positive and decreasing marginal revenues, \( R', \bar{R}', \hat{R}' > 0 \) and \( R^*, \bar{R}^*, \hat{R}^* < 0 \). Since \(|\bar{R}'| > |\bar{R}^* - \hat{R}|\) and \(|\hat{R}'| > |\bar{R}^* - \hat{P}|\), and \( \bar{R}', \bar{R}^*, \hat{R}^* < 0 \), the denominator in \( \frac{\partial \bar{X}^h}{\partial \epsilon} \) is positive, and thus \( \frac{\partial \bar{X}^h}{\partial \epsilon} \) is positive. In a similar manner, the sign of \( \frac{\partial \hat{X}^h}{\partial \epsilon} \) is negative as long as \( |\hat{R}'| > |\hat{P}| \). Therefore, the optimal foreign sales of the MNF when it opens only the home plant is an increasing function in the exchange rate, and the optimal sales of the local firm in the same case is a decreasing function in the exchange rate. On the other hand, when the MNF opens both plants, how \( Y^b, \bar{Y}^b \), and \( \hat{Y}^b \) respond to the variability of the exchange rate depends on the domain. If \( c > e\hat{c} \), the MNF produces at the foreign subsidiary, and thus \( \bar{Y}^b \) and \( \hat{Y}^b \) are independent of the exchange rate, whereas \( Y^b \) depends on the exchange rate. From equation (5-7), we obtain

\[
\frac{\partial Y^b}{\partial \epsilon} = \frac{\hat{c}}{2P' + P^*Y^b} \text{ and thus } \frac{\partial^2 Y^b}{\partial \epsilon^2} = \frac{\hat{c}(3P' + P^*Y^b)}{(2P' + P^*Y^b)^2} \frac{\partial Y^b}{\partial \epsilon}.
\]
Since the denominator of $\frac{\partial R^b}{\partial e}$ is equal to the slope of the domestic marginal revenue of the MNF, $R^*$ (assumed to be negative), the sign of $\frac{\partial R^b}{\partial e}$ is negative. Therefore, if $c > e\bar{c}$, the optimal domestic sales of the MNF when it opens both plants is a decreasing function in the exchange rate where its shape depends on the shape of the domestic demand curve. However, if $c < e\bar{c}$, production takes in the home plant of the MNF and the impact of the exchange rate variability on production is the same as in the case when the MNF open only the home plant.

These relationships provide the implication for the production and sales behavior of the firms. The MNF sells more at home and abroad when it opens both plants than when it opens only the home plant if $c > e\bar{c}$, while it sells the same amounts at home and abroad for both cases if $c < e\bar{c}$. This implies that the MNF is expected to produce and sell more when it opens both plants. However, the local firm is expected to produce and sell less when the MNF opens both plants because $\hat{P}^b < \hat{P}^h$ if $c > e\bar{c}$ and $\hat{P}^b = \hat{P}^h$ if $c < e\bar{c}$.

We can see in Figure 7 how the optimal levels of sales of both firms for each case are related to the exchange rate where the curvature of the optimal sales are determined using linear demands as shown below.
Figure 7. Relationships between the optimal sales and the exchange rate

Using linear demands with the slopes of $-1, P = a - Y$ and $\bar{P} = \bar{a} - (\bar{Y} + \hat{Y})$, the optimal sales and the indirect profits when the MNF opens only the home plant are obtained as

$$
Y^h = \frac{a - c}{2}, \quad \bar{Y}^h(e) = \frac{\bar{a} + \bar{d} - 2\frac{c}{e}}{3}, \quad \hat{Y}^h(e) = \frac{\bar{a} - 2\bar{d} + \frac{c}{e}}{3},
$$

$$
\Pi^h = \frac{(a - c)^2}{4} + e\frac{(\bar{a} + \bar{d} - 2\frac{c}{e})^2}{9} - F, \quad \Pi^{h^*} = \frac{(\bar{a} - 2\bar{d} + \frac{c}{e})^2}{9} - \bar{G}.
$$

We can see their shapes by examining the first and the second derivatives with respect to the exchange rate;

$$
\frac{\partial Y^h}{\partial e} = \frac{2c}{3e^2} > 0, \quad \frac{\partial^2 Y^h}{\partial e^2} = -\frac{4c}{3e^3} < 0, \quad \frac{\partial \hat{Y}^h}{\partial e} = -\frac{c}{3e^2} < 0, \quad \frac{\partial^2 \hat{Y}^h}{\partial e^2} = \frac{2c}{3e^3} > 0,
$$

$$
\frac{\partial \Pi^h}{\partial e} = \frac{(\bar{a} + \bar{d} + 2\frac{c}{e})(\bar{a} + \bar{d} - 2\frac{c}{e})}{9} > 0, \quad \frac{\partial^2 \Pi^h}{\partial e^2} = \frac{8c^2}{9e^3} > 0,
$$

$$
\frac{\partial \Pi^{h^*}}{\partial e} = -\frac{2c(\bar{a} - 2\bar{d} + \frac{c}{e})}{9e^2} < 0, \quad \frac{\partial^2 \Pi^{h^*}}{\partial e^2} = \frac{2c(2\bar{a} - 4\bar{d} + 3\frac{c}{e})}{9e^3} > 0,
$$
assuming positive values of sales. Therefore, when it opens only the home plant, the
optimal foreign sales of the MNF are increasing and concave in the exchange rate, and the
optimal sales of the local firm are decreasing and convex, while the optimal domestic sales
of the MNF are independent of the exchange rate. The profits of the MNF are increasing
and convex in the exchange rate, and the profits of the local firm are decreasing and
convex.

On the other hand, the optimal sales and the indirect profits when the MNF opens
both plants are, if $c > e\bar{c}$:

$$
\begin{align*}
\gamma^b(e) &= \frac{a - ec}{2}, \quad \gamma^b = \frac{a + d - 2c}{3}, \quad \hat{\gamma}^b = \frac{a - 2d + c}{3}, \\
\Pi^b &= \frac{(a - ec)^2}{4} + \frac{e(a + d - 2c)^2}{9} - F - eF, \quad \Pi^b = \frac{(a - 2d + c)^2}{9} - \bar{G}.
\end{align*}
$$

The first and second derivatives of the optimal sales and the indirect profit of the MNF
with respect to the exchange rate for the case of $c > e\bar{c}$ can be shown to be

$$
\begin{align*}
\frac{\partial \gamma^b}{\partial e} &= -\frac{\bar{c}}{2} < 0, \quad \frac{\partial^2 \gamma^b}{\partial e^2} = 0, \\
\frac{\partial \Pi^b}{\partial e} &= -\frac{\bar{c}(a - ec)}{2} + \frac{(a + d - 2c)^2}{9}, \quad \frac{\partial^2 \Pi^b}{\partial e^2} = \frac{\bar{c}^2}{2} > 0.
\end{align*}
$$

In the case when it opens both plants, if $c > e\bar{c}$, the optimal domestic sales of the MNF are
decreasing and linear in the exchange rate, while the optimal sales of both firms in the
foreign market are independent of the exchange rate. The profits of the MNF are a
quadratic and convex function, and the profits of the local firm are independent of the
random exchange rate. However, for the case of $c < e\bar{c}$, the optimal sales and the profit
of the local firm will be the same as those when the MNF opens only the home plant, and the profit of the MNF will be $\Pi^h = \Pi^h - \bar{e} F$.

We can see this more clearly in Figure 8. In the case of $c > e\bar{c}$, the profit of the local firm is less when the MNF opens both plants than when the MNF opens only the home plant, while they are the same if $c < e\bar{c}$. This implies that the local firm is more likely to have lower profits when the MNF opens both plants because the expected profit of the local firm when the MNF opens both plants must be less than that when the MNF opens only the home plant. For the MNF, it is not clear in Figure 8 whether the expected profit of the MNF when it opens both plants is greater than that when it opens only the home plant. I will address this issue more in the simulation performed later in this section.

---

$37$ At $\hat{e}$, $\Pi^h$ is not necessarily differentiable because the marginal cost switches at that point, and thus it may be kinked at $\hat{e}$. Using linear demands, the slope of $\Pi^h$ at $\hat{e}$ can be written as follows:

$$\frac{\partial \Pi^h}{\partial e} \Bigg|_{\hat{e}} = \frac{2}{9} (\hat{a} + \hat{d} - 2\hat{c})^2$$

if $e < \hat{e}$, and

$$\frac{\partial \Pi^h}{\partial e} \Bigg|_{\hat{e}} = \frac{2}{9} (\hat{a} + \hat{d} + 2\hat{c})(\hat{a} + \hat{d} - 2\hat{c})$$

if $e > \hat{e}$. The slopes at $\hat{e}$ when $e < \hat{e}$ and when $e > \hat{e}$ are not necessarily the same. Also, the value of $e$ that minimizes $\Pi^h$ is not necessarily less than $\hat{e}$. It can be greater, equal to, or less than $\hat{e}$.  

Figure 8. Relationship between the profits and the exchange rate$^{37}$
Given linear demands, the following propositions describe how the variability of the exchange rate and the foreign direct investment decision of the MNF affect the firms' welfare.

**PROPOSITION 5-1.** Given linear demands, if the MNF opens only the home plant, both firms will benefit from exchange rate uncertainty.

Since the profits of both firms when the MNF opens only the home plant are convex in the random exchange rate, \( \frac{\partial^2 \Pi^h}{\partial e^2} > 0 \) and \( \frac{\partial^2 \Pi^f}{\partial e^2} > 0 \), we obtain \( E[\Pi^h(e)] > \Pi^h(\bar{e}) \) and \( E[\Pi^f(e)] > \Pi^f(\bar{e}) \), respectively, by Jensen's inequality. This implies that uncertainty makes both firms better off as long as the MNF does not open the foreign plant.

**PROPOSITION 5-2.** (i) Under uncertainty, opening the foreign plant by the MNF lowers the expected profit of the local firm. Furthermore, (ii) opening the foreign plant by the MNF makes the local firm worse off as the variability of the exchange rate increases.

**PROOF.** (i) It can be proved by showing that the expected profit of the local firm when the MNF opens the foreign plant is lower than that when the MNF opens only the home plant,

\[
E[\Pi^b - \Pi^h] = E[\Pi^h - \Pi^h] + E[i(\hat{\gamma}^b)^2 - (\hat{\gamma}^h)^2] = E[i(\hat{\gamma}^h)^2 - (\hat{\gamma}^h)^2] < 0
\]

since \( \hat{\gamma}^b < \hat{\gamma}^h \) if \( \hat{\gamma} > e \). Therefore, under uncertainty, the local firm's expected profits are lower if the MNF opens the foreign plant. Note that under certainty, the MNF's decision to open the foreign plant does not affect the local firm's profit, i.e., \( \Pi^b(\bar{e}) - \Pi^h(\bar{e}) = 0 \).
(ii) By employing a mean-preserving spread of the distribution of the exchange rate, it can be shown that the difference between the expected profits of the local firm when the MNF opens both plants and when the MNF opens only the home plant gets smaller (larger in absolute value because it is negative) as the variability of the exchange rate increases (i.e., as the mean-preserving spread parameter $\gamma$ increases). As done in the previous section, for the mean-preserving spread of the distribution, $e$ is replaced by $\bar{x} + \gamma(x - \bar{x})$. The difference between the expected profits can be written as

$$E[\Pi^b - \Pi^h] = \int_{-\infty}^{\infty} \left( \frac{\bar{a} - 2\bar{d}}{\bar{c}} + 1 \right)^2 - \left( \frac{\bar{a} - 2\bar{d}}{\bar{c}} + \frac{\hat{e}}{e} \right)^2 \left( \bar{x} + \gamma(x - \bar{x}) \right) g(x) dx.$$

The first derivative of this difference with respect to the mean-preserving spread parameter is described as

$$\frac{\partial}{\partial \gamma} \left[ E[\Pi^b - \Pi^h] \right] = \frac{\bar{c}^2}{9} \int_{-\infty}^{\infty} \left[ -2 \left( \frac{\bar{a} - 2\bar{d}}{\bar{c}} + \frac{\bar{x} + \gamma(x - \bar{x})}{\bar{x} + \gamma(x - \bar{x})} \right) \left( \frac{\bar{x} - \bar{x}}{(\bar{x} + \gamma(x - \bar{x}))^2} \right) \right] g(x) dx.$$

This expression is negative because $\hat{y}^h = \frac{\bar{a} - 2\bar{d} + \bar{c}}{3}$ is assumed to be positive, $\hat{x} - x > 0$ over the domain integrated, and $\bar{x} + \gamma(x - \bar{x}) > 0$. Q.E.D.
Therefore, the more volatile is the exchange rate, the more the local firm is hurt when the MNF opens the foreign plant. For a clearer analysis, I perform a simulation with linear demands in the model by examining several different variabilities of the exchange rate. Four cases for each situation are examined; (i) the MNF opens both plants and the local firm opens its plant, (ii) the MNF opens both plants and the local firm does not open, (iii) the MNF opens only the home plant and the local firm opens its plant, and (iv) the MNF opens only the home plant and the local firm does not open. I assume $c = \varepsilon F$ and $F < \varepsilon \bar{F}$ so that the MNF opens only the home plant in the deterministic case. Using linear demands, the indirect profit functions for each case can be described as follows.

Deterministic case:

(i) $\Pi^M_{NF} = \frac{(a-c)^2}{4} + \varepsilon \frac{(a+d - \frac{2c}{\bar{e}})^2}{9} - F - \varepsilon \bar{F}$ and $\Pi' = \frac{(a - 2d + \frac{c}{\bar{e}})^2}{9} - \bar{G}$

(ii) $\Pi^M_{NF} = \frac{(a-c)^2}{4} + \varepsilon \frac{(a - \frac{c}{\bar{e}})^2}{4} - F - \varepsilon \bar{F}$ and $\Pi' = 0$

(iii) $\Pi^M_{NF} = \frac{(a-c)^2}{4} + \varepsilon \frac{(a+d - \frac{2c}{\bar{e}})^2}{9} - F$ and $\Pi' = \frac{(a - 2d + \frac{c}{\bar{e}})^2}{9} - \bar{G}$

(iv) $\Pi^M_{NF} = \frac{(a-c)^2}{4} + \varepsilon \frac{(a - \frac{c}{\bar{e}})^2}{4} - F$ and $\Pi' = 0$
Uncertainty case:

(i) $\Pi^{\text{diff}} = \frac{(a - e\bar{c})^2}{4} + e\frac{(a + d - 2\bar{c})^2}{9} - F - \bar{e}\bar{F}$ and $\Pi' = \frac{(a - 2d + c)^2}{9} - \bar{G}$, if $c > e\bar{c}$

$\Pi^{\text{diff}} = (\Pi^{\text{diff}})^{\text{case (iii)}} - \bar{e}\bar{F}$, and $\Pi'$ is the same as in case (iii), if $c < e\bar{c}$

(ii) $\Pi^{\text{diff}} = \frac{(a - e\bar{c})^2}{4} + e\frac{(a - \bar{c})^2}{4} - F - \bar{e}\bar{F}$ and $\Pi' = 0$, if $c > e\bar{c}$

$\Pi^{\text{diff}} = (\Pi^{\text{diff}})^{\text{case (iii)}} - \bar{e}\bar{F}$, and $\Pi'$ is the same as in case (iv), if $c < e\bar{c}$

(iii) $\Pi^{\text{diff}} = \frac{(a - c)^2}{4} + e\frac{(a + d - 2\bar{c})^2}{9} - F$ and $\Pi' = \frac{(a - 2d + c)^2}{9} - \bar{G}$

(iv) $\Pi^{\text{diff}} = \frac{(a - c)^2}{4} + e\frac{(a - c)^2}{4} - F$ and $\Pi' = 0$

The values of the parameters used in this simulation are given by: \{e, a, c, \bar{c}, F, \bar{F}, \bar{a}, \bar{d}, \bar{G}\} = \{10, 60, 20, 2, 20, 2.2, 6, 2, 1.7\}. Using an uniform distribution for the exchange rate,\(^{38}\) I have examined three different variabilities of the exchange rate, \{x, \beta\} = \{9, 11\}, \{8, 12\}, \{7.9, 12.1\}. Using these particular numbers, I obtain the values of the profits for the deterministic case and the expected profits for the uncertainty case as in Table 3. This table provides specific examples for the implications of propositions (5-1) and (5-2). From the cell (iii) in all tables, we can see that both firms benefit from risk, and the benefits get larger as the variability of the exchange rate increases. However, the variability of the exchange rate in table (b) is not large enough to

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\(^{38}\) See footnote 34.
Table 3. Payoffs for different variabilities of the exchange rate

(a) Deterministic case

<table>
<thead>
<tr>
<th>MNF</th>
<th>Local firm</th>
<th>Open</th>
<th>Not open</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open both</td>
<td>(i) 375.78, 0.078</td>
<td>(ii) 398, 0</td>
<td></td>
</tr>
<tr>
<td>Open home</td>
<td>(iii) 397.78, 0.078</td>
<td>(iv) 420, 0</td>
<td></td>
</tr>
</tbody>
</table>

(b) When $\{\alpha, \beta\} = \{9, 11\}$,

<table>
<thead>
<tr>
<th>MNF</th>
<th>Local firm</th>
<th>Open</th>
<th>Not open</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open both</td>
<td>(i) 379.30, 0.037</td>
<td>(ii) 409.19, 0</td>
<td></td>
</tr>
<tr>
<td>Open home</td>
<td>(iii) 397.83, 0.085</td>
<td>(iv) 420.03, 0</td>
<td></td>
</tr>
</tbody>
</table>

(c) When $\{\alpha, \beta\} = \{8, 12\}$,

<table>
<thead>
<tr>
<th>MNF</th>
<th>Local firm</th>
<th>Open</th>
<th>Not open</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open both</td>
<td>(i) 398.32, 0.001</td>
<td>(ii) 420.73, 0</td>
<td></td>
</tr>
<tr>
<td>Open home</td>
<td>(iii) 398.02, 0.108</td>
<td>(iv) 420.14, 0</td>
<td></td>
</tr>
</tbody>
</table>

(d) When $\{\alpha, \beta\} = \{7.9, 12.1\}$,

<table>
<thead>
<tr>
<th>MNF</th>
<th>Local firm</th>
<th>Open</th>
<th>Not open</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open both</td>
<td>(i) 399.50, -0.002</td>
<td>(ii) 421.91, 0</td>
<td></td>
</tr>
<tr>
<td>Open home</td>
<td>(iii) 398.05, 0.111</td>
<td>(iv) 420.15, 0</td>
<td></td>
</tr>
</tbody>
</table>

make the MNF better off by opening the foreign plant (compare the cell (i) and (iii) in table (b)), even though risk makes the choice of opening the foreign plant more worthwhile than under certainty (compare the cell (i) in tables (a) and (b)). The dominant strategies in the deterministic case and under uncertainty with the small variability as in
table (b) are “Open home” for the MNF and “Open” for the local firm. Therefore, in these cases, the Nash equilibrium is that the MNF opens only the home plant and the local firm opens its plant, i.e., the cell (iii) in both tables (a) and (b). However, when the variability gets larger up to the level in table (c), the expected profits of the MNF are higher by opening the foreign plant than by opening only the home plant. Thus, the MNF has more incentive to open the foreign plant, reducing the expected profits of the local firm. As shown in the cell (i) of table (c), the expected profits of the local firm are close to zero with this variability. In spite of that, the dominant strategy of the local firm is still “Open”.

Note, however, that when the exchange rate variability gets larger (for example, the variability changes from table (b) to table (c)), the dominant strategy of the MNF changes from “Open home” to “Open both”. Therefore, the Nash equilibrium in table (c) is that the MNF opens the foreign plant and the local firm opens its plant. From tables (c) and (d), we can see that a tiny change in the variability makes the expected profits of the local firm negative if the MNF opens both plants, implying that the local firm may drop out from the industry. In this case (table (d)), given that opening the foreign plant is the dominant strategy of the MNF, the best strategy of the local firm is “Not open”. Thus the Nash equilibrium in table (d) is that the MNF opens the foreign plant and the local firm does not open. Furthermore, by comparing the cell (i) and (iii) in all tables, we can see that the larger variability of the exchange rate raises the expected value of opening the foreign plant by the MNF, $E[\pi^b(e) - \pi^h(e)]$, and raises the expected loss of the local firm when the MNF opens its foreign plant, $E[\pi^h(e) - \pi^a(e)]$. Therefore, the larger variability may induce the MNF to open the foreign plant, and so makes the local firm
worse off. Particularly, the cell (i) and (iii) of tables (a) and (b) shows that 
\[ E[\Pi^b(x)] - E[\Pi^f'(x)] \] is zero in the deterministic case, and 
\[ E[\Pi^b(x)] - E[\Pi^f'(x)] \] is negative under uncertainty, implying that opening the foreign plant by the MNF hurts the local firm even with a small variability of the exchange rate.

These situations are summarized in Figure 9. In domain I, both firms get benefits from risk. Their expected profits increase as the variability gets larger. However, if the variability is large enough to induce the MNF to open the foreign plant, the expected profits of the local firm decrease discontinuously due to the opening of the foreign plant by the MNF. Furthermore, as shown in domain II, an increase in the variability of the exchange rate decreases the expected profits of the local firm. Eventually its expected profits may become negative, as shown in domain III; the withdrawal of the local firm

\[ e = \bar{e} + \varepsilon. \]
increases the MNF's profits discontinuously. Therefore, providing the MNF a chance to
open the foreign plant as well as the domestic plant gives the MNF a potential strategic
advantage in competition with the local firm, and the variability of the exchange rate
induces the MNF to exercise this advantage.
CHAPTER VI. SUMMARY AND CONCLUSIONS

I analyzed the production, hedging, and investment behavior of a MNF under exchange rate uncertainty. The volatility of the exchange rate affects the firm's behavior in various ways, depending upon the situation in which the firm engages. The timing of decisions and the firm's attitude toward risk can be crucial components to determine the direction of this effect. I found that a risk averse firm produces less at home and more abroad under exchange rate uncertainty in the absence of hedging instruments for the nonlinear profit model, holding the level of total production unchanged. This result is the same as for the linear profit model (Broll and Zilcha (1992)). The effects of exchange rate uncertainty on total output of the risk neutral firm depends upon the shape of marginal revenue curves. Uncertainty leads to lower total output with linear demands, but higher with constant elasticity demands. Even though the firm produces more or less depending upon the curvature of demands, the risk neutral firm is always better off under risk. The effects on production were also examined when transportation costs exist. It is shown that uncertainty makes the risk neutral firm produce more at home, and discourages the firm from trading in the presence of transportation costs.

The study was extended to the case when foreign exchange futures markets are available. The separation theorem holds for the optimal allocation of outputs, but does not hold for the optimal level of total output in this nonlinear profit model. The availability of futures markets encourages the risk averse firm to produce more at home except in the upward biased case. Even though futures markets are available to reduce
risk, the risk averse firm may produce more or less, depending upon the curvature of the marginal revenues. The firm produces more under uncertainty than under certainty with linear demands, but less with constant elasticity demands. Regardless of the optimal level of total output, however, the risk averse firm benefits from risk in the presence of futures markets. In addition, the optimal futures position depends upon the shape of marginal revenue curves. For linear demands and unbiased futures markets, the optimal futures position is short, and full hedging with futures contract only is not attained because the indirect profit function is nonlinear in the random exchange rate.

The MNF currently producing at the home plant decides if it is more profitable to open the foreign plant by comparing its expected profits when it opens both plants with when it opens only the home plant. Regardless of the expected benefit of the risk neutral firm from opening the foreign plant, the risk averse firm is exposed to exchange rate risk. For fixed total output, the risk averse firm will open the foreign plant by selling put options that are fairly priced when the risk neutral firm benefits from opening the foreign plant. If total output is price responsive, the risk neutral firm is more likely to open the foreign plant than in the case of fixed total output because the profit is not less than that with fixed total output for every value of the exchange rate. For more reality, I investigated how uncertainty affects the firm's behavior if the MNF faces a competitor in the foreign market. Employing linear demands, the simulation I performed with the uniform distribution shows that if the MNF does not open the foreign plant, both firms benefit from risk. However, uncertainty raises the value of opening the foreign plant for the MNF, and thus induces the MNF to open it, reducing the expected profit of the local
firm under uncertainty. Consequently, the exchange rate variability may help the MNF drive the local firm out of business.
REFERENCES


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