ULTRASONIC FIELD PATTERNS AND BROADBAND IMAGING - SIMULATIONS IN TRANSVERSELY ISOTROPIC MEDIA

M. Spies
Fraunhofer-Institute for Nondestructive Testing (IZFP)
6600 Saarbrucken, Germany

P. Fellinger, U. Schleichert, K. J. Langenberg
Department of Electrical Engineering, University of Kassel
3500 Kassel, Germany

INTRODUCTION

Thorough application of NDE - and imaging techniques in anisotropic media has to overcome those inherent problems like beam splitting, beam distortion and deviation between wave propagation direction and energy flow, known as beam skewing. A theory of elastic wave propagation in transversely isotropic media which properly accounts for these effects has been presented previously [1], an overview has been presented at last year’s conference [2]. Analytic expressions have been derived characterizing the propagation of Gaussian Wave Packets (GWPs) in these media thus making possible the simulation of real pulse propagation. In order to provide information from a practical point of view the theory has been evaluated to yield field patterns of GWPs in unidirectional graphite-epoxy as well as centrifugally cast stainless steel. The plane wave spectral decomposition of Green’s dyadic and triadic functions, also presented last year, has been used to derive the algorithm of forward-backward-propagation of elastic wavefields yielding a simple solution to the inverse problem, the so-called elastodynamic holography. With this basic imaging technique, field distributions of GWPs are propagated forth and back; for comparison, the conventional isotropic algorithm [3] is also applied, resulting in field distributions which differ both in intensity and position. Finally the simulations, being evaluated in frequency domain, are performed at multiple frequencies thus in principle making up a mode-matched FT-SAFT [4], neglecting the vector scattering amplitudes.

WAVE EQUATION FOR TRANSVERSELY ISOTROPIC MEDIA

The wave equation for the displacement vector \( \mathbf{u} \) reads [5]

\[
(\nabla \cdot \mathbf{C} \cdot \nabla) \mathbf{u} + \rho \omega^2 \mathbf{u} = -\mathbf{f},
\]

where \( \rho \) is the mass density, \( \mathbf{f} \) accounts for the volume force density and \( \omega \) denotes the circular frequency, if we assume a time dependence \( \sim e^{-j\omega t} \). The elastic stiffness tensor for the transversely isotropic case is given by

\[
\mathbf{C}(a) = \lambda_\perp \mathbf{I} + \mu_\perp \left[ (\mathbf{I})^{1324} + (\mathbf{I})^{1342} \right] + (\nu - \lambda_\perp) \left[ \mathbf{I} \mathbf{a a} + \mathbf{a a I} \right] \\
+ (\mu_{\parallel} - \mu_\perp) \left[ (\mathbf{I} \mathbf{a a})^{1324} + (\mathbf{a a I})^{1324} + (\mathbf{I} \mathbf{a a})^{1342} + (\mathbf{a a I})^{1342} \right] \\
+ \left[ \lambda_\perp + 2\mu_\perp + \lambda_{\parallel} + 2\mu_{\parallel} - 2(\nu + 2\mu_\parallel) \right] \mathbf{a a a a},
\]

(2)
Table 1. Elastic constants (in GPa) and mass density (in g/cm³) of transversely isotropic materials (fiber direction || x-axis).

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>$C_{11}$</td>
<td>145.8</td>
<td>238.0</td>
</tr>
<tr>
<td>$C_{33}$</td>
<td>13.5</td>
<td>246.0</td>
</tr>
<tr>
<td>$C_{44}$</td>
<td>3.4</td>
<td>63.0</td>
</tr>
<tr>
<td>$C_{66}$</td>
<td>6.8</td>
<td>135.0</td>
</tr>
<tr>
<td>$C_{13}$</td>
<td>10.2</td>
<td>120.0</td>
</tr>
<tr>
<td>$\rho$</td>
<td>1.6</td>
<td>8.1</td>
</tr>
</tbody>
</table>

Figure 1. Orientation of the cartesian coordinate system with regard to the fiber axis ($\hat{K}$ designates wave propagation direction).

where the unit vector $a$ points into the fiber direction; thus it lies in the x-y plane, since this is the interesting case under concern (Fig. 1). The upper indicial notation indicates pertinent index change in the tetrads, $\mathbf{I}$ is the dyadic idemfactor; $\lambda_{ij}$, $\mu_{ij}$ are the elastic constants corresponding to $C_{11}$, $C_{33}$, $C_{44}$, $C_{66}$ and $C_{13}$, respectively, in Voigt-notation (fibers || x-axis). For the materials considered in this paper, unidirectional graphite-epoxy (GE) and centrifugally cast stainless steel (CCSS), the respective elastic constants and the mass density are given in Table 1.

Using (2) three different types of solutions of (1) have been obtained [1,2]: plane wave solutions (SH, qSV, qP) and Gaussian Wave Packets (via paraxial hf-approximation) for the homogeneous wave equation ($f = 0$) as well as a solution via Green’s functions for the inhomogeneous case ($f \neq 0$). In the following the GWP-approach will be used to simulate real pulse propagation as well as to provide synthetic data, which will serve to check the validity and demonstrate the effectiveness of the holographic imaging algorithm described below.

ULTRASONIC FIELD PATTERNS VIA GWP-EVALUATION

The GWP introduced by Norris [7] as a high frequency solution of the equation of motion (1) is in the form of a plane wave modulated by a Gaussian envelope in all directions.
Figure 2. Evaluation and representation of GWP-field patterns. The pulse parameters used are \( f_0 = 5 \) MHz carrier frequency, half width 10 mm, half length 1.5 mm.

about the pulse center \( \bar{R}(0) = 0 \). At time \( t \geq 0 \), the pulse center has propagated to \( \bar{R}(t) = \bar{R}(0) + t \vec{c} \) where \( \vec{c} \) is the group velocity. The pulse is then defined by

\[
u(\bar{R}, t) = U_0 \hat{u} \left[ \frac{\det M(t)}{\det M(0)} \right]^{1/2} \exp \left[ j \omega \left( \frac{1}{2} (\bar{R} - \bar{R}) \cdot M(t) : (\bar{R} - \bar{R}) \right) \right], \tag{3}
\]

where \( U_0 \) is the initial amplitude, \( \hat{u} \) the polarization vector, \( \omega \) the slowness and \( M \) the so-called envelope matrix \([7,2]\). For the case of transversely isotropic media with arbitrary fiber orientation in the \( x-y \) plane the analytical representations of these quantities have been derived and listed in detail in \([1]\).

To obtain the respective field patterns the numerical evaluation of Eqn. (3) is performed in the \( x-z \) plane in the following way (Figure 2). The envelope of the GWP has its largest extension along a line thru the center of the pulse, perpendicular to the wave propagation direction \( \hat{k} \). The amplitude of the displacement vector is calculated along this line for subsequent time steps as the packet moves thru the material and is plotted as indicated in Figure 2 (right). In the resulting representations \( \hat{k} \)-direction is perpendicularly downward, whereas the amplitude pattern is aligned along the direction of energy flow \( \hat{c} \). Figure 3 shows the field patterns in CCSS for three angles of incidence for a qSV-GWP (fiber direction \( \parallel x \)-axis), clearly displaying the effects of beam splitting, distortion and - skewing. In the case of \( \theta = 30^\circ \) a considerably antisymmetric pattern results with also a remarkable skewing effect, the patterns for \( \theta = 0^\circ \) and \( 90^\circ \) appear principally different with a splitting into two major parts in the first and a relatively small distortion in the second case. Similar results are obtained for a qP-packet in GE, Figure 4a)-c) show the patterns for \( \theta = 90^\circ \), \( 85^\circ \) and \( 82^\circ \) (fiber direction \( \parallel x \)-axis). In Figure 4d)-f) the fiber direction is varied from \( \alpha = 30^\circ \) to \( \alpha = 90^\circ \) for \( \theta = 85^\circ \), where a smaller depth is reached as compared to the case \( \alpha = 0^\circ \), since the GWP moves out of the observed \( x-z \) plane. (Figs. 3 and 4 are shown in logarithmic scale.)

\[ ^1 \text{Apart from the oscillation-structured appearance the field patterns obtained for CCSS using the GWP-model compare quite well with those obtained by Rose et al. [6] for the same material using an approximate point-source-synthesis-model (see also [1]).} \]
Figure 3. GWP-field patterns (qSV) in CCSS: angle of incidence a.) $\theta = 0^\circ$, b.) $\theta = 30^\circ$, c.) $\theta = 90^\circ$ (fiber direction $\alpha = 0^\circ$).

Figure 4. GWP-field patterns (qP) in GE: angle of incidence a.) $\theta = 90^\circ$, b.) $\theta = 85^\circ$, c.) $\theta = 82^\circ$ (fiber direction $\alpha = 0^\circ$); fiber direction d.) $\alpha = 30^\circ$, e.) $\alpha = 60^\circ$, f.) $\alpha = 90^\circ$ (angle of incidence $\theta = 85^\circ$). Notice the increased scale in x-direction in c.).
ELASTODYNAMIC HOLOGRAPHY FOR TRANSVERSELY ISOTROPIC MEDIA

The plane wave spectral decomposition of the dyadic and triadic Green functions in the transversely isotropic case [2] according to

\[ G(\mathbf{x}, \mathbf{y}, z, \omega) = \sum_{\alpha} G^\alpha_{\mathbf{x}} e^{izK_{2\alpha}}, \quad \tilde{G}(\mathbf{x}, \mathbf{y}, z, \omega) = \sum_{\alpha} \tilde{G}^\alpha_{\mathbf{x}} e^{izK_{2\alpha}} \]

(4)

with \( \tilde{G}^\alpha_{\mathbf{x}} \) and \( \tilde{G}^\alpha_{\mathbf{y}} \) (\( \alpha = SH, qSV, qP \)) explicitly given in [1], has been used to derive a simple solution to the inverse problem in form of the elastodynamic version of Rayleigh-Sommerfeld-Holography [1], in a way similar to the isotropic case [3].

Using the 2D-Fourier-transformed distribution \( \tilde{u}_0 (K_x, K_y, z = 0, \omega) \) of the displacement detected at the material’s surface, the displacement vector at any plane \( z < 0 \) in the material can be calculated - using FFTs - from

\[ u(x, y, z \leq 0, \omega) = \left( \frac{1}{2\pi} \right)^2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{u}_0 (K_x, K_y, z = 0, \omega) \times \]

\[ \times \left\{ A_{xH} e^{izK_{2SH}} u(K^2_{SH} - K_x^2 - K_y^2) + A_{xSV} e^{izK_{2qSV}} u(K^2_{SV} - K_x^2 - K_y^2) + A_{xP} e^{izK_{2qP}} u(K^2_{qP} - K_x^2 - K_y^2) \right\} e^{i(zK_x + yK_y)} dK_x dK_y, \]

(5)

where \( u(K^2_x - K^2_y - K^2_{z}) \) designates the unit step function. In this propagation-backpropagation algorithm the separation dyads \( \hat{A} \) are of fundamental importance, since they split the total displacement distribution detected at \( z = 0 \) into the respective SH-, qSV- and qP parts. Their explicit expressions - again as a function of fiber direction \( \hat{g} \) - examples for their graphical representation as well as the evaluation of their name-giving properties can be found in [1].

The effectiveness of the propagation-backpropagation algorithm for the isotropic case has been checked and confirmed in [3] using simulated scattering data obtained for horizontal and aligned cracks, especially emphasizing the superiority of the elastodynamic vector algorithm against its scalar version. It therefore suffices to show the validity of the transversely isotropic algorithm (5). This is done by performing a numerical evaluation for the two-dimensional case \( (K_x, z) \) according to Figure 5. A SH-GWP \( (\theta = 82^\circ) \) is launched in GE \( (\alpha = 30^\circ) \) at \( t = 0.1 \text{s} \) and \( z = 0 \) mm to provide \( \tilde{u}_0 (K_x, z = 0, \omega_0) \) detected at \( z = 0' = -10 \text{mm} \) (count of sample: x-FFT 512, t-FFT 256) where \( \omega_0 \) is chosen equal to the carrier frequency of the pulse \( \omega_0 = 2\pi \cdot 5 \text{MHz} \). Using (5) this displacement distribution is then propagated - down to \( z = -30 \text{mm} \) - and backpropagated. The results of these procedures given in Figure 6a-b), where the sum of the three displacement components is plotted in linear scale, are identical, confirming the correctness of the algorithm (5). The isotropic backpropagation from \( z = -30 \text{mm} \) to \( z = -10 \text{mm} \), performed for comparison, yields a distribution which differs both in intensity and position (Fig. 6c), clearly emphasizing the need for the “proper” algorithm.

BROADBAND IMAGING BY MULTI-FREQUENCY HOLOGRAPHY

The simulation shown in Figure 6 - evaluated in frequency domain - has been performed at a single frequency. An application of (5) at multiple frequencies, which would make use of the additional information contained in the complete frequency spectrum \( \tilde{u}_0 (K_x, z = 0, \omega) \), can be motivated by considering the (isotropic) mode-matched FT-SAFT-algorithm given in [4]. With \( \tilde{u}_{\alpha}^{\text{hot}} \) being the displacement vector (Eqn. (5)) due to wave mode \( \beta \), generated by the scattering of the incident \( \alpha \)-mode at a defect, restriction to planar scatterers and therefore neglecting the vector scattering amplitudes yields the function \( \gamma(R) \) describing the scatterer’s surface according to...
\[ \gamma(\mathbf{R}) \simeq \int_{K} \left| u_{\alpha\beta}^{\text{med}} \right| e^{-i\mathbf{K} \cdot \mathbf{R}} \, dK, \]  

(6)

where \( s_\alpha = \omega^{-1} K_\alpha \) [1].

\[ \mathbf{K} = -K_\alpha \hat{\mathbf{R}}_\alpha + K_\beta \hat{\mathbf{R}}_\beta = -\omega \left( s_\alpha \hat{\mathbf{R}}_\alpha + s_\beta \hat{\mathbf{R}}_\beta \right). \]  

(7)

Here \( \hat{\mathbf{R}}_\alpha \) and \( \hat{\mathbf{R}}_\beta \) are the unit vectors in excitation and observation direction, respectively.

Equation (7) suggests to fill \( K \)-space in (6) by superimposing holograms obtained for different \( \omega \), providing an approximate image of the scattering surface. Since this multi-frequency holography is performed within a limited bandwidth, of course limited resolution is achieved.

To demonstrate the qualities of this imaging algorithm the following simulation, again using synthetic GWP-data is performed. A packet hits the interface in a two-layered GE-sample \((0^\circ/45^\circ, \text{each layer } 20 \text{ mm thick})\), the displacement vector due to the reflected (R) or the transmitted (T) GWPs, respectively, originating at the interface, is correspondingly detected at the upper or the lower surface to yield \( \mathbf{u}_{\alpha}(K, z = 0, \omega) \). This displacement distribution is then backpropagated into the material in order to get information about the scatterer. For an incident SH-GWP \((\theta = 90^\circ)\) the results are given in Figure 7, where again the sum of the three displacement components is shown in linear scale. Figure 7a) displays the backpropagated R-distribution where practically no information about the depth of the interface can be gained. However, in Figure 7b) where the superposition of only 9 holograms obtained in the frequency range \([4.84, 5.16] \text{ MHz}\) is plotted, the absolute maximum clearly indicates where the source of the reflected wavefields is located. As stated above increasing the number of frequencies yields a higher resolution, clearly indicating the location of the scattering surface at \( z = -20 \text{ mm} \) (Figure 7c),d)). Since the GWP has a limited extension, the image of the interface is obtained within a finite area.

**CONCLUSION**

The GWP-model allows a computationally quick evaluation of the field patterns of ultrasonic pulses in transversely isotropic media. The variation of pulse type, pulse parameters, angle of incidence and fiber direction can be used to check and optimize a respective experimental situation. The model also provides synthetic data, which have been used here to show the validity of the transversely isotropic holography algorithm as well as the effectiveness of its broadband evaluation for image reconstruction. The results obtained prove this multi-frequency holography to be an efficient imaging technique for planar scatterers in transversely isotropic materials. Future work will be concerned with the 3-D implementation of the algorithm as well as the evaluation of experimental data for GE-laminates, where in general the crucial defects to be detected are planar.

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**Figure 5.** Numerical evaluation of algorithm (5) using GWP-data.
Figure 6. Holograms calculated via a.) propagation, b.) backpropagation (both transversely isotropic), and c.) isotropic backpropagation using the initial displacement distribution generated by a SH-GWP ($\theta = 82^\circ, \alpha = 30^\circ$) according to Figure 5.

Figure 7. Backpropagation of reflected GWP-field distributions (incident SH-packet: $\theta = 90^\circ, \alpha = 0^\circ$): a.) single frequency hologram; coherent superposition of holograms obtained for b.) 9, c.) 33 and d.) 129 frequencies (scaling as shown in Figure 6.)
REFERENCES


