2013

Improving network traffic flow reliability through dynamic anticipatory tolls

Jing Dong
Iowa State University, jingdong@iastate.edu

Hani Mahmassani
Northwestern University

Abstract
This paper proposes an anticipatory reliability pricing concept and investigates the potential effectiveness of implementing time-varying and state-dependent reliability tolls in tandem with real-time traffic estimation and prediction systems. Travel reliability, related to the flow breakdown probability, is conveyed to drivers in the form of a dynamic toll, for consideration in their route selection. In addition, to manage the traffic proactively a rolling horizon approach is used to generate dynamic tolls based on predicted traffic conditions. The experimental results show that anticipatory reliability tolls could divert travellers away from actual or likely bottlenecks that experience unstable flow, and thus contribute to reducing congestion on toll roads and increasing system utilisation.

Keywords
dynamic pricing, flow breakdown, reliability, real-time traffic estimation and prediction

Improving network traffic flow reliability through dynamic anticipatory tolls

Jing Dong [a], Hani Mahmassani [b]

[a] *Department of Civil, Construction and Environmental Engineering, Iowa State University, Ames, IA, USA*

[b] *Transportation Center, Northwestern University, Evanston, IL, USA* (Email: masmah@northwestern.edu)

Contact Information for the corresponding author

Dr. Jing Dong

Assistant Professor

350 Town Engineering Building

Iowa State University

Ames, Iowa 50011

Phone: (515) 294-3957

Fax: (515) 294-8216

E-mail: jingdong@iastate.edu

To cite this article: Jing Dong & Hani Mahmassani (2013) Improving network traffic flow reliability through dynamic anticipatory tolls, Transportmetrica B: Transport Dynamics, 1:3, 226-236, DOI: 10.1080/21680566.2013.856776
To link to this article: **http://dx.doi.org/10.1080/21680566.2013.856776**

Improving network traffic flow reliability through dynamic anticipatory tolls

This paper proposes an anticipatory reliability pricing concept and investigates the potential effectiveness of implementing time-varying and state-dependent reliability tolls in tandem with real-time traffic estimation and prediction systems. Travel reliability, related to the flow breakdown probability, is conveyed to drivers in the form of a dynamic toll, for consideration in their route selection. In addition, to manage the traffic proactively a rolling horizon approach is used to generate dynamic tolls based on predicted traffic conditions. The experimental results show that anticipatory reliability tolls could divert travelers away from actual or likely bottlenecks that experience unstable flow, and thus contribute to reducing congestion on toll roads and increasing system utilization.

Keywords: dynamic pricing; flow breakdown; reliability; real-time traffic estimation and prediction.

Introduction

Travel time reliability has been recognized as a primary consideration in travelers' route and departure time choices, in addition to the travel time itself (Noland and Polak, 2002; Brownstone and Small, 2005; Fosgerau and Karlström, 2010; Jiang et al., 2011; Vovsha et al., 2012). Travel time reliability could be quantified using a variety of measures, including the standard deviation of the travel time, the so-called buffer time, the difference between the 90th and 10th percentiles of the travel time distribution, and the probability that a trip can be successfully completed within a specified time interval (Nicholson et al., 2003; Dong et al., 2006; Tu et al., 2007; Higatani et al. 2009). These measures, which are generally based on day-to-day travel time variation or theoretical distributions, have limited ability to capture the effect of non-recurrent congestion, and hence are not particularly appropriate for real-time traffic operations.

A different travel reliability measure that recognizes the probabilistic nature of flow breakdown and relates to the prevailing flow rate on the facility of interest, was previously proposed by the authors (Dong and Mahmassani, 2009a). The flow breakdown reliability measure captures the collective effects of the drivers on the road and is based on a relation that could be empirically realized through specification and parameter estimation using common detector data. Accordingly, it provides an approach for predicting flow and travel time reliability in real-time, for possible dissemination to the traveling public. Recognizing that flow breakdown might occur at a wide range of flow levels, it is desirable to minimize the flow breakdown likelihood by keeping flow levels on such facilities below the levels at which the onset of breakdown increases rapidly in likelihood. This may be implemented by providing travelers with travel time reliability information (as part of Advanced Traveler Information Systems). Alternatively, when monetary tolls may be charged, the reliability measure can be reflected in dynamically varying tolls.

Anticipatory pricing strategies take advantage of predicted traffic conditions, rather than prevailing and/or historical conditions, in setting time-varying link tolls along a freeway corridor to maintain level of service (LOS) targets and avoid traffic flow breakdown on toll links or value-priced lanes (Dong et al., 2011). The anticipatory toll generator, intended to operate in tandem with real-time traffic estimation and prediction systems (TrEPS), utilizes prediction in conjunction with sensor measurements in setting dynamic prices. This paper introduces the reliability toll concept and investigates the potential effectiveness of utilizing predicted reliability measures for efficiently determining time-varying and state-dependent tolls so as to maintain high LOS on the toll facilities as well as in the network overall. Anticipatory reliability tolls help induce travelers to select socially preferred paths—by shifting travelers from less reliable (higher likelihood of breakdown) to more reliable paths, thereby decreasing the probability of breakdown for those paths already near the critical state. This behavior could help avoid or delay the onset of traffic breakdown, so as to benefit the system, although the traveler is still assumed to choose a path maximizing his/her own travel utility.

Problem statement

Travel time unreliability could be due to a variety of factors, some endogenous and others exogenous to the traffic stream. Our concern is mainly focused on unreliability due to inherent properties of the traffic stream, especially congestion phenomena. Several empirical studies have documented that flow breakdown does not necessarily occur at the same location, nor at the same prevailing flow level (Persaud, 1998; Evans et al., 2001). To model the probabilistic nature of traffic breakdown, previous studies have treated the facility "capacity" or pre-breakdown flow rate as a random variable, with traffic breakdown occurring when the traffic demand exceeds the random capacity (Brilon et al., 2005; Dong and Mahmassani, 2009a,b). This results in a probability of breakdown occurring at a given flow (demand) level.

To avoid terminological confusion and remain consistent with the traditional (engineering) definition of highway capacity (as the maximum flow rate that can reasonably be expected to pass through a section of a roadway), we view the flow rate observed immediately before traffic breaks down, that is, pre-breakdown flow rate, as a random variable. The probability distribution function of the pre-breakdown flow rates has been calibrated to follow the Weibull distribution based on data samples from freeway sections in California, USA (Dong and Mahmassani, 2009b; Kim et al., 2010) and Germany (Brilon et al., 2005). The pre-breakdown flow distribution function expresses the probability that traffic breaks down in the next time interval (for a given time discretization).

$$F(q) = 1 - e^{-(\frac{q}{\sigma})^s} \quad (1)$$

$F(\cdot)$ = probability distribution function

q = pre-breakdown flow rate

σ = scale parameter

s = shape parameter

The anticipatory reliability pricing strategy is intended to mitigate the unreliability resulting from traffic interactions as the system approaches congested states. By efficiently and dynamically managing traffic through reliability tolls, flow breakdown may be avoided or alleviated. Specifically, given historical and real-time traffic measurements from road sensors, the problem is to find reliability toll values and to incorporate the dynamic pricing information in travelers' route choice decision.

Methodology

Reliability toll

Inspired by the marginal external cost based Pigouvian toll in static congestion pricing models (Walters, 1961), a reliability toll is proposed from a control-theoretic perspective, intended for online operations. The concept of reliability toll is illustrated in Figure 1. The pre-breakdown flow rate is viewed as a random variable that follows the Weibull distribution. The travel time and flow rate relation is represented by the backward bending curve. In order to normalize the data, the ratio of travel time to free-flow travel time, referred to as travel time index, is plotted instead of travel time. The travel time index and flow rate relation is represented by two monotonic functions: (1)

when the traffic is uncongested, travel time index is a non-decreasing function of flow rates; and (2) under congested traffic conditions, a decreasing function is used, as a longer travel time is experienced at a lower flow rate (corresponding to heavier congestion). The uncongested and congested regimes are divided at the point where the maximum flow rate is achieved. As travel time remains near-constant or increases only mildly with the flow rate under uncongested traffic conditions, compared to the backward bending portion of the travel time curve, a fixed free flow travel time is used to approximate uncongested travel time in this analysis.

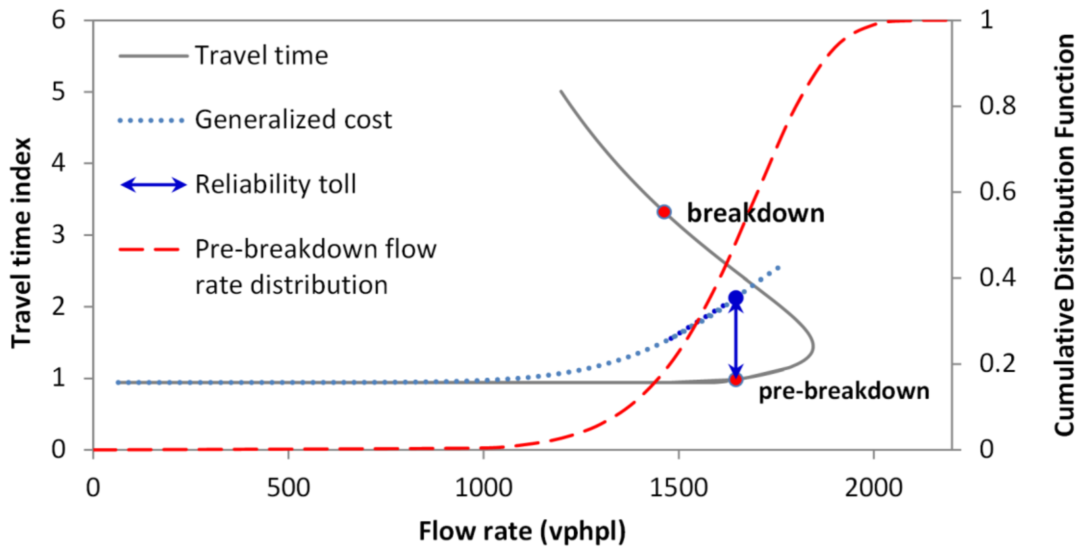

Figure 1. Reliability toll.

At a certain pre-breakdown flow rate level, traffic could maintain its flow state in the next time interval or transition to a congested state with a lower flow rate and a longer associated delay. The probability that traffic can sustain at flow rate q is represented by the survival function $S(q)$, defined as:

$$S(q) = 1 - F(q) \tag{2}$$

Thus, at a given flow rate the expected link travel time is expressed as a combination of travel times under uncongested and congested traffic conditions, weighted by the appropriate probabilities.

$$E(t|q) = t_0 \cdot S(q) + t_c(q) \cdot (1 - S(q)) = t_0 + t'(q) \cdot (1 - S(q)) \tag{3}$$

where

$E(t|q)$ = expected link travel time at flow rate q

$t_c(q)$ = travel time under congested traffic conditions, which is flow dependent

t_0 = travel time under uncongested traffic conditions

$t'(q)$ = difference between congested and uncongested travel times, that is, $t'(q) = t_c(q) - t_0$

The term $t'(q) \cdot (1 - S(q))$ in Equation (3) corresponds to the breakdown-induced expected delay, and can be communicated to the travelers in the form of dynamic tolls. The reliability toll is computed considering the probability of breakdown and the delay likely to be experienced by the traveler conditional upon the occurrence of breakdown. The value of travel time parameter could be used to convert the toll from time to monetary cost, i.e.:

$$\beta = \alpha_{VOT} \cdot t'(q) \cdot (1 - S(q)) \tag{4}$$

β = reliability toll

α_{VOT} = value of travel time

The generalized cost, a combination of travel time and link toll, is assumed to govern travelers' route choice decision. For clarity, and with no loss of generality, vehicle operating cost is not considered in the present study.

$$GC = \alpha_{VOT} \cdot t_0 + \beta = \alpha_{VOT}(t_0 + t'(q) \cdot (1 - S(q)) \quad (5)$$

GC = generalized cost

Note that charging a reliability toll equal to the expected delay taking breakdown probability into account is equivalent to considering the expected travel time as the generalized cost. As shown in Figure 1, when the flow rate is low, the reliability toll is close to zero and the generalized cost is determined by the uncongested regime travel time. When traffic reaches higher flow rates while still flowing, the reliability toll plays an important role in diverting travelers away from the likely breakdown links. As a result, if travelers choose their routes according to the reliability tolls the overall network performance might be improved. In fact, the numerical experiment, presented in the next section, suggests a system wide improvement in terms of lower average travel times. However, it is unclear how close the solution may be to a system optimum and whether improvement in network traffic conditions is guaranteed or not.

Anticipatory pricing strategy

Pricing strategies that merely react to measured flow (or occupancy) levels may result in charging users high prices to use facilities where breakdown has already occurred, and stop-and-go conditions are prevailing. Though high prices might be justified in such circumstances in order to recover traffic flow more quickly, this means that customers who have may have paid the maximum allowed toll charges may in fact be experiencing unacceptably poor traffic conditions. The principal idea of an anticipatory pricing strategy is to set the price at a sufficiently high level before, and not after the onset of breakdown, taking advantage of using prediction in conjunction with sensor measurements (Dong et al., 2011). As shown in Figure 2, the traffic prediction model captures the evolution of traffic demand and flows in the network over the near future. Within the prediction horizon, travelers are assigned to the least generalized cost paths based on the prevailing travel time and reliability toll values, resulting in predicted traffic flow reliability information to be supplied to the anticipatory toll generator. The anticipatory reliability tolls can therefore be calculated and implemented in real time.

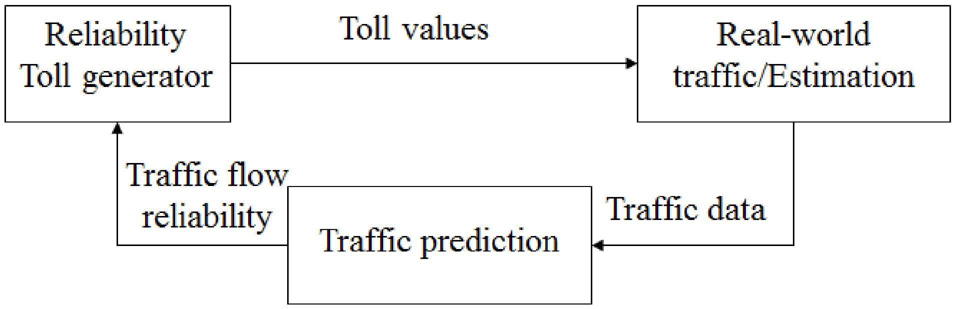

Figure 2. Anticipatory reliability pricing framework

Rolling horizon implementation

A closed-loop rolling horizon method is adopted to support the real-time application of the anticipatory pricing scheme. The rolling horizon procedure is one of the most common methods employed in practice for generating and implementing solutions to dynamic programming problems (Baker, 1977). As shown in Figure 3, the entire planning horizon is subdivided into several overlapping stages, each of which contains h units. Within each stage (prediction horizon) anticipatory information for the entire stage is solved but implemented only for the roll period, that is, the first l units of the stage. The horizon is then rolled forward to obtain the next stage. This procedure is repeated until the end of the planning horizon. The basic idea behind the rolling horizon approach is that vehicles currently assigned to the network will not be influenced by vehicles assigned far into the future as the currently assigned vehicles will probably be out of the network by that time. The current network state obtained in the estimation model (and the real world) is sent to the prediction model as the starting point, or initial conditions. of state prediction (indicated by the solid arrows in Figure 3). The toll values calculated in the prediction model are transferred to the estimation model and implemented in the actual system, as represented by the dashed arrows.

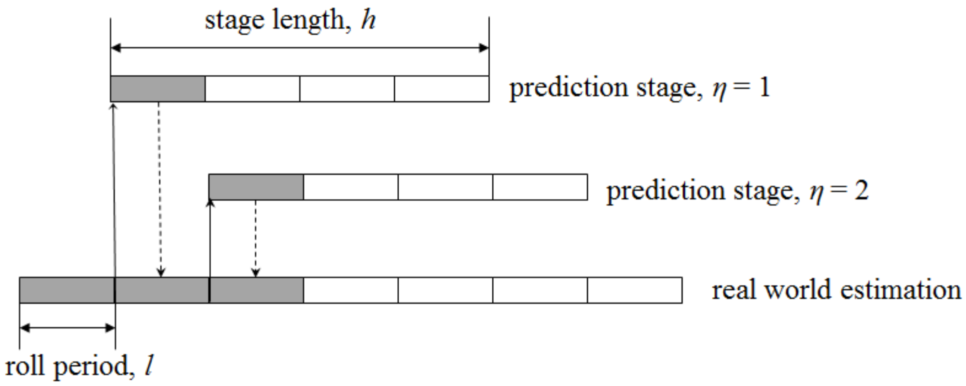

Figure 3 The closed-loop rolling horizon framework

The following quantities are defined in Figure 3:

h = prediction horizon length in terms of time intervals

l = roll period in terms of time intervals

η = stage number, η = 1, ..., $\left\lfloor \frac{T}{l} \right\rfloor$

T = planning horizon in terms of time intervals

The reliability toll is calculated based on the predicted flow rates within the prediction stage. Note that flow rates in one time interval are often correlated with those in the previous intervals. This is captured by simulating flow propagation on the traffic network for the length of the prediction stage, thereby capturing the underlying traffic physics. Whether breakdown probabilities in these time intervals should reflect a spatio-temporal correlation pattern of breakdown occurrence or not calls for further empirical investigation that is beyond the scope of the present paper. Such a pattern would be captured through more elaborate multivariate cdf's. For ease of mathematical manipulation and clarity of exposition, assume that the breakdown occurrences in the time intervals within the prediction horizon are independent. The probability of flow breakdown (in at least one of the time intervals over the prediction horizon) can be calculated as follows:

$$\tilde{p}_\eta = 1 - \prod_{i=1}^{h} S(\tilde{q}_{i\eta}) = 1 - e^{\sum_{i=1}^{h} -(\frac{\tilde{q}_{i\eta}}{\sigma})^s} \tag{6}$$

$\tilde{p}_\eta$ = probability of flow breakdown in prediction stage η

$\tilde{q}_{i\eta}$ = predicted flow rate for the *i*-th interval in prediction stage η

Assuming that the value of travel time is known and the extra delay is represented as a function of the prevailing (pre-breakdown) flow rate, the corresponding reliability toll can therefore be computed as:

$$\beta_\eta = \alpha_{VOT} \cdot t'(q) \cdot \tilde{p}_\eta \tag{7}$$

Note that the anticipatory reliability toll, β_η, is generated based on the prevailing (observed) flow rate (i.e. q) and the predicted flow rates in stage η (i.e. the probability of flow breakdown $\tilde{p}_\eta$ is a function of $\tilde{q}_{i\eta}$, $i = 1, \cdots, h$, as shown in Equation (6)).

Reliability toll generation in an online predictive system

A real-time traffic estimation and prediction system (TrEPS) is an essential methodology to enable implementation and evaluation of online traffic management, as it can incorporate field observations and traffic measures, as well as estimated and predicted network states. As a deployable real-time system, TrEPS recognizes the fact that origin-destination (OD) demand information and network conditions can only be reliably available for a short period of time in the future. New OD desires are being continuously estimated and corrected using the stream of actual observations from different data sources. Based on the updated OD demand, a new network state is predicted at every stage. In particular, DYNASMART-X (Mahmassani et al., 1998; Mahmassani and Zhou, 2005) is adopted to implement the anticipatory pricing strategy for the purpose of managing the traffic on toll facilities. DYNASMART-X uses a simulation-based dynamic traffic assignment approach for real-time traffic estimation and prediction. As illustrated in Figure 4, the time-varying flow rates on each link are predicted over the prediction horizon (along with the link travel time information), based on which anticipatory reliability tolls can be computed and transferred to the state estimation module. Note that the proposed solution algorithm, based on rolling horizon implementation, may fluctuate and may not converge to a unique and exact solution.

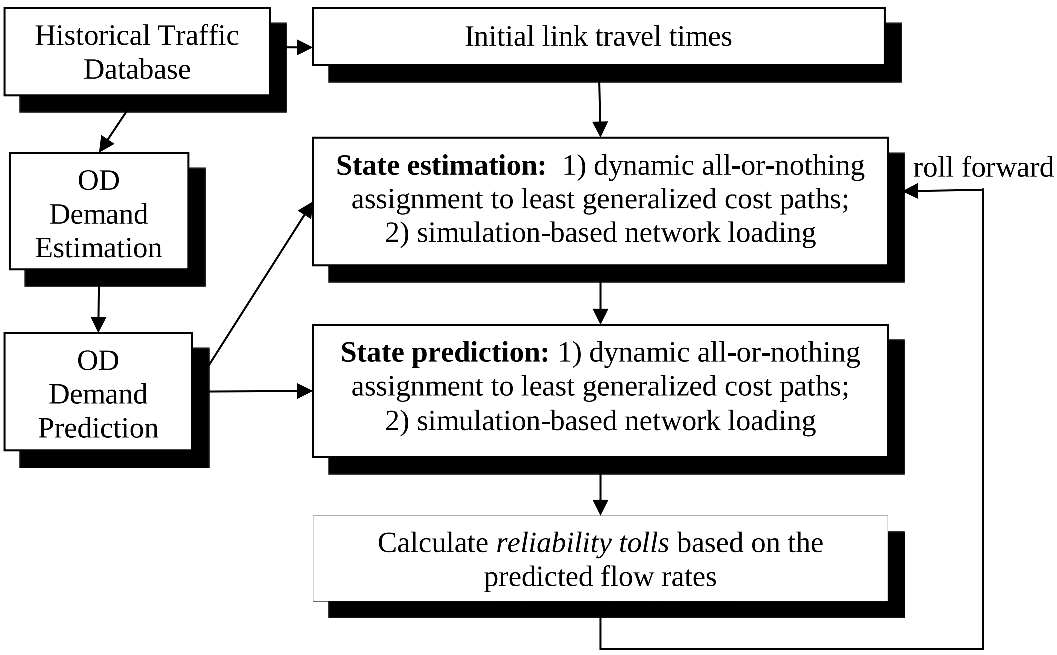

Figure 4. Anticipatory reliability toll generation in TrEPS

The algorithm steps are as follows.

Step 0: Initialization. Run state estimation for the first roll period $t = 1, \cdots, l$. Vehicles are assigned to the least generalized cost paths based on the prevailing travel time upon their departure.

Step 1: Transfer current network states from state estimation module to state prediction module.

Step 2: Run state prediction for the stage length $t = \eta \cdot l + 1, \cdots\cdots, \eta \cdot l + h$. Vehicles are assigned to the least generalized cost paths based on the prevailing travel time and the reliability toll information upon their departure.

Step 3: At the end of the prediction horizon calculate anticipatory reliability tolls based on the predicted time-dependent flow rates and the prevailing flow rate (see Equation (7)). Transfer time-dependent travel times and the reliability tolls to state estimation module. Calculate time-dependent least generalized cost path for each origin, destination and departure time interval.

Step 4: In state estimation module, advance one roll period $\eta = \eta + 1$. Assign newly generated vehicles to the predictive time-dependent least generalized cost paths and reroute existing vehicles to up-to-date (predictive) least generalized cost paths according to bounded rationality, that is, switch to a new path if the improvement in the remaining trip time exceeds the indifference band. Run state estimation for the roll period $t = (\eta - 1) \cdot l + 1, \cdots\cdots, \eta \cdot l$.

Step 5: If the end of the planning horizon is not reached, go back to Step 1; otherwise terminate.

Numerical experiments

Simulation experiment setting

To demonstrate the impact of anticipatory reliability pricing on network performance, simulation experiments are conducted using the Irvine test bed network (see Figure 5). The Irvine network includes two interstate freeways, namely the I-5 and the I-405, as well as part of the state highway 133. The rest of the network consists of arterials and ramps. This network has 326 nodes, 626 links and 61 traffic analysis zones. Detector coverage of the freeways in the network is substantial, and allows calibration of the traffic flow model and the breakdown probability distribution function for each freeway link. Calibration details are described in Dong and Mahmassani (2009a, 2009b) and Kim et al. (2010). The morning peak 7-9AM is chosen as the study period. In order to eliminate the boundary effects and reflect real world traffic conditions, a 15-minute warm-up period (i.e. 6:45-7:00 AM) and a 45-minute clearance time (i.e. 9:00-9:45 AM) are also included in the experiments. However, only the vehicles departing between 7AM and 9AM are of interest, while other vehicles are treated as background traffic. The rolling horizon approach is implemented using a stage length of 30 minutes and a roll period is of 5 minutes.

Figure 5. Irvine testbed network

Travelers are assumed to have access to real-time information, both pre-trip and en route, so they can choose or switch to the up-to-date best path that minimizes their generalized travel cost. In particular, upon their departure, travelers will choose the least generalized cost paths based on the prevailing travel time and reliability toll. During their trip, when predictive information is available, travelers will evaluate their options en-route, and possibly switch to a new path if the generalized cost savings exceed an indifference band, reflecting boundedly-rational behavior (Mahmassani and Stephan, 1988; Mahmassani and Jayakrishnan, 1991). Two scenarios are evaluated: the “no toll” scenario assumes that no road pricing is deployed and travelers select the least time path; users under the “reliability toll” scenario will consider the dynamic pricing information in their route choice, in addition to travel time, and choose the path with the least generalized cost.

Results and discussion

Traffic conditions on a representative freeway link (I-405N Jeffrey section) are examined and compared for the two scenarios. Figure 6 shows the time-varying traffic density on the link, as well as the dynamic toll values. When no toll is applied in the network, the density could reach or be close to the jam density, which corresponds to the low speed (traffic breakdown) condition shown in Figure 7. Imposing the reliability toll contributes to delaying the onset of breakdown and alleviating its extent.

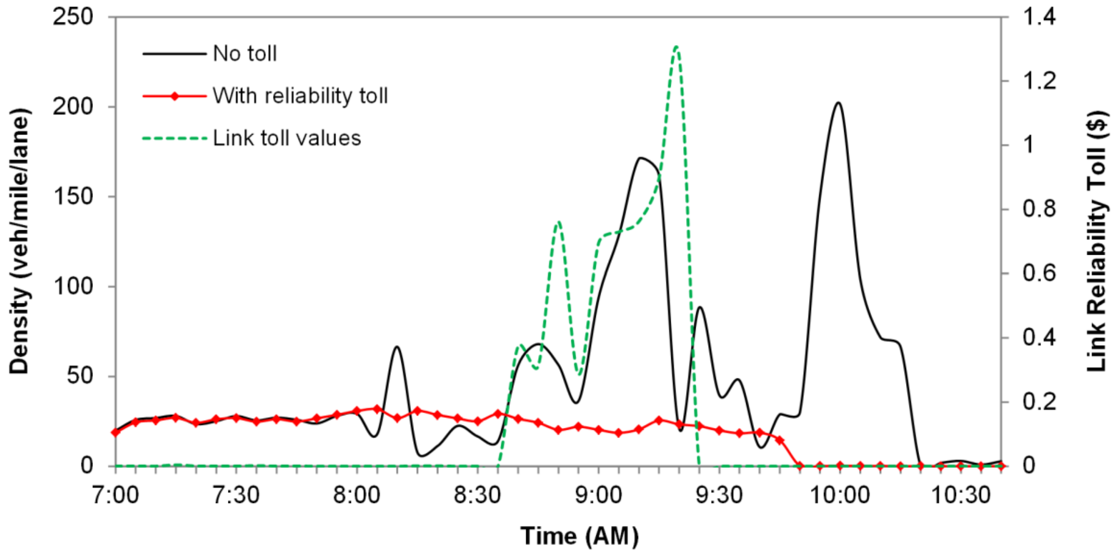

Figure 6. Time-varying link density comparison

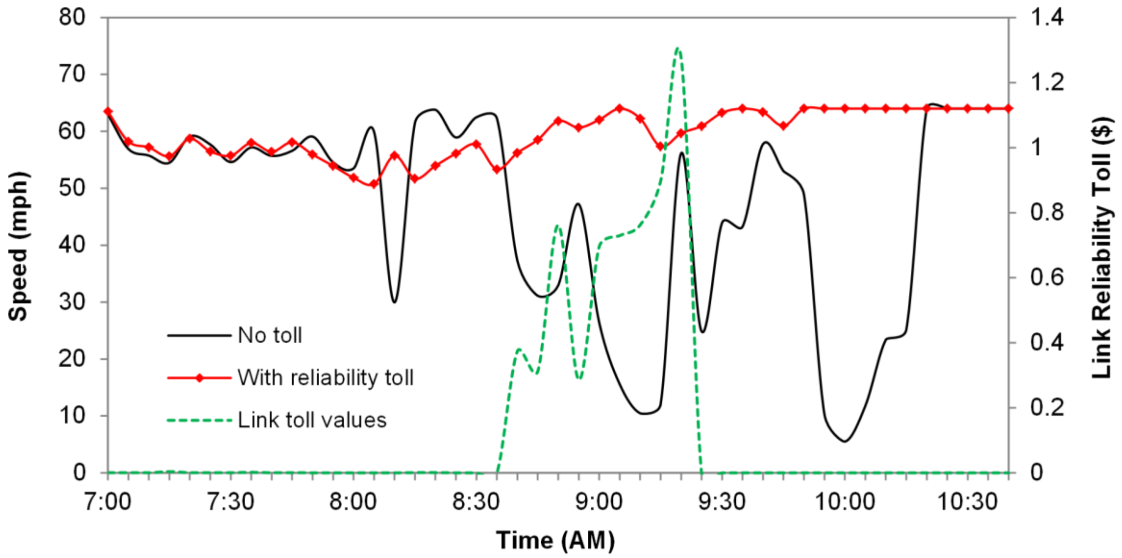

Figure 7. Time-varying link speed comparison

Figure 8 compares accumulated flow rates on the link. More vehicles travel through the link under the "reliability toll" scenario, indicating an increase in freeway utilization.

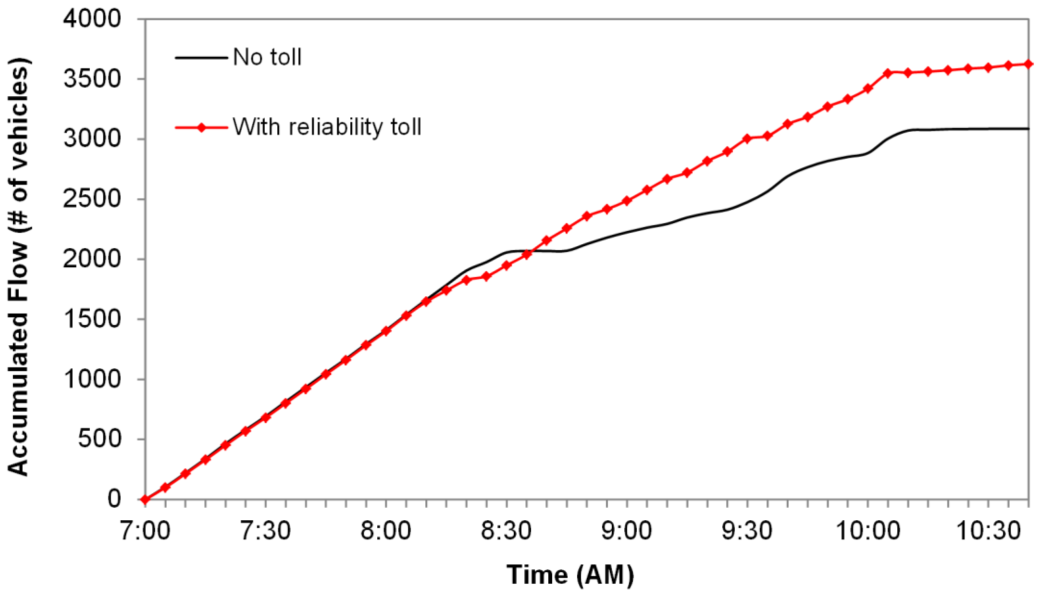

Figure 8. Accumulated flow rate comparison

To evaluate the network performance, the average experienced travel times of vehicles departing in different time intervals are compared and shown in Figure 9. Significant time savings are observed when reliability tolls are charged.

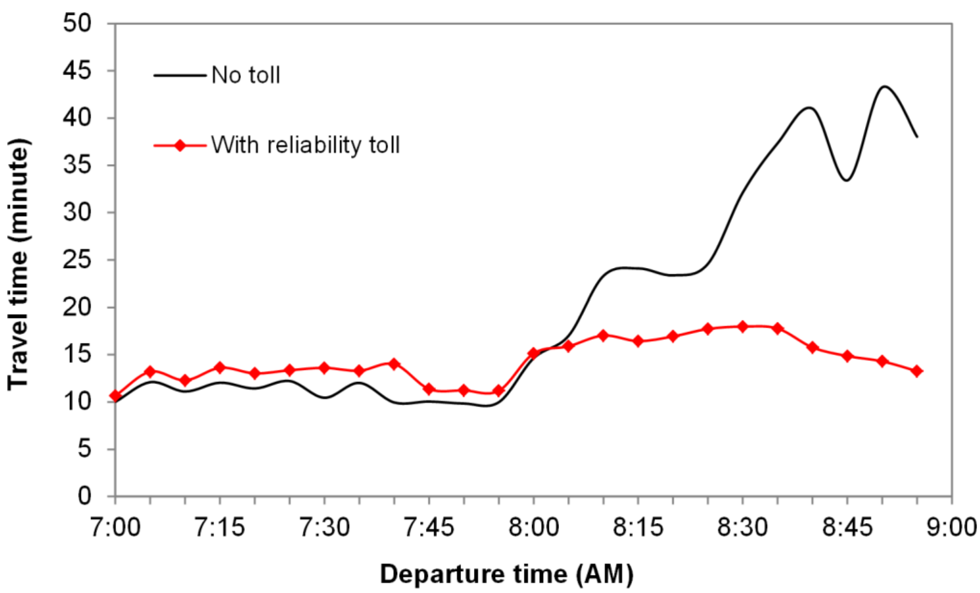

Figure 9. Average travel time comparison

Conclusion

This paper introduces the concept of a reliability toll that reflects the expected potential delay caused by a probabilistic traffic breakdown phenomenon. Such reliability pricing strategy is implemented in connection with a real-time traffic estimation and prediction framework. Simulation experiments are conducted using an actual network, to demonstrate the effectiveness of the reliability pricing strategy in the context of real-time route choice. The experimental results show that applying anticipatory reliability tolls contributes to reducing congestion on freeways and increasing system utilization, by diverting travelers away from actual or likely bottlenecks that experience unstable flow. As such, anticipatory reliability tolls could play an important role in the range of dynamic pricing approaches, recently reviewed in de Palma and Lindsey (2011), available to manage demand on transportation networks.

The main caveat of this study is that reliability tolls for each link are determined by dynamic link flow rates using the survival function. Although flow propagation along links is captured in the simulation-based network loading process, the toll is generated based on the flow rates of the particular link. Nevertheless, flow rates and other traffic measures on adjoining sections might improve breakdown prediction and thus produce a more effective toll charge. In future work, traffic conditions on adjacent links could be considered in the reliability toll generator.

References

Baker, K. R. (1977). An experimental study of the effectiveness of rolling schedules in production planning. *Decision Sciences*, 8(1), 19-27.

Brilon, W., Geistefeldt, J. and Regler, M. (2005). Reliability of freeway traffic flow: a stochastic concept of capacity. In Mahmassani, H.S. (ed.) *Transportation and Traffic Theory. Flow, Dynamics and Human Interaction*. Elsevier, pp. 125-144.

Brownstone, D. and Small, K. (2005). Valuing time and reliability: assessing the evidence from road pricing demonstrations. *Transportation Research Part A* 39A(4), pp. 279–293.

De Palma, A. and R. Lindsey (2011). Traffic congestion pricing methodologies and technologies. *Transportation Research Part C: Emerging Technologies*, 19(6), pp. 1377-1399.

Dong, J., Lu, C.-C., and Mahmassani, H. S. (2006). How reliable is this route? Predictive travel time and reliability, for anticipatory traveler information systems. *Transportation Research Record*, 1980, 117-125.

Dong, J. and Mahmassani, H.S. (2009a) Flow breakdown, travel reliability and real-time information in route choice behavior. In Lam, W., S.C. Wong and H. Lo (eds.) *Proceedings of the 18th International Symposium on Transportation and Traffic Theory*, Springer, pp. 675-695.

Dong, J. and Mahmassani, H.S. (2009b) Flow breakdown and travel time reliability. *Transportation Research Record: Journal of the Transportation Research Board* 2124, pp. 203-212.

Dong, J., Mahmassani, H.S., Erdogan, S. and Lu, C.-C. (2011) State-dependent pricing for real-time freeway management: anticipatory versus reactive Strategies. *Transportation Research Part C: Emerging Technologies*, 19(4), pp. 644-657.

Evans, J. L., Elefteriadou, L., and Gautam, N. (2001). Probability of breakdown at freeway merges using Markov chains. *Transportation Research Part B*, 35, 237-254.

Fosgerau, M. and Karlström, A. (2010). The value of reliability. *Transportation Research Part B*, 44(1), pp. 38-49.

Higatani, A., Kitazawa, T., Tanabe, J., Suga, Y., Sekhar, R. and Asakura, Y. (2009) Empirical analysis of travel time reliability measures in Hanshin expressway network. *Journal of Intelligent Transportation Systems: Technology, Planning, and Operations*, 13(1), pp. 28-38.

Jiang, L., Mahmassani, H.S., and Zhang, K. (2011) Congestion Pricing, Heterogeneous Users, and Travel Time Reliability: Multi-criterion Dynamic User Equilibrium Model and Efficient Implementation for Large-Scale Networks. *Transportation Research Record: Journal of the Transportation Research Board* 2254, pp. 58-67.

Kim, J., Mahmassani, H.S. and Dong, J. (2010) Likelihood and Duration of Flow Breakdown Modeling the Effect of Weather. *Transportation Research Record: Journal of the Transportation Research Board* 2188, pp. 19-28.

Mahmassani, H. S., Hawas, Y., Abdelfatah, A., Abdelghani, K., Chiu, Y-C. and Kang, Y. (1998). DYNASMART-X, System Design Report, Volume I-III, *Technical Report* ST067-85-V(I)-V(III), Center for Transportation Research, the University of Texas at Austin.

Mahmassani H. S., Stephan D. G. (1988) Experimental Investigation of Route and Departure Time Choice Dynamics of Urban Commuters. *Transportation Research Record* 1203: 69-84.

Mahmassani H. S., Jayakrishnan R. (1991) System Performance and User Response Under Real-Time Information in a Congested Traffic Corridor. *Transportation Research Part A.* 25(5), 293–308.

Mahmassani, H. S., and Zhou, X. (2005). Transportation system intelligence: Performance measurement and real-time traffic estimation and prediction in a day-to-day learning framework. In E. Abed (Ed.) *Advances in Control, Communication Networks, and Transportation Systems*, In Honor of Pravin Varaiya. Boston: Birkhauser.

Nicholson, A., Schmocker, J., and Bell, M., Iida, Y. (2003) Assessing transportation reliability: Malevolence and user knowledge. In M.G.H. Bell and Yasunori Iida (Eds.), *The Network Reliability of Transport: Proceedings of the 1st International Symposium on Transportation Network Reliability*. Pergamon.

Noland, R. B., and Polak, J. W. (2002). Travel time variability: a review of theoretical and empirical issues. *Transport Reviews*, 122 (1), 39-54.

Persaud, B., Yagar, S., Brownlee, R. (1998) Exploration of the breakdown phenomenon in freeway traffic. *Transportation Research Record,* 1634, pp. 64-69.

Tu, H., J.W.C. van Lint, and H.J. van Zuylen, (2007) The impact of traffic flow on travel time variability of freeway corridors. *Transportation Research Record*, 1993, pp. 59-66.

Vovsha, P., R. Donnelly, M. Bradley, J. Bowman, HS Mahmassani, T. Adler, K. Small, D. Brownstone, K. Kockelman, J. Wolf, and F. Koppelman (2012) Improving our Understanding of How Highway Congestion and Price Affect Travel Demand. SHRP2 Report, Transportation Research Board, Washington, DC.

Walters, A.A., 1961. The theory and measurement of private and social cost of highway congestion. *Econometrica*, 29, 676–699.