Flavor changing chronomagnetic moments: a Monte Carlo study of the top quark

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Flavor changing chronomagnetic moments: 
A Monte Carlo study of the top quark

by

Matthias Daniel Hosch

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We have studied direct top quark production and single top quark production through anomalous flavor changing chromomagnetic moments. We find that it is possible to detect anomalous flavor changing chromomagnetic moments of the top quark using direct top quark production if the charm quark coupling strength is larger than $\kappa_c/\Lambda > 0.062\text{ TeV}^{-1}$ at Run 2 of the Tevatron, or larger than $\kappa_c/\Lambda > 0.0084\text{ TeV}^{-1}$ at the LHC, or if the up quark coupling is larger than $\kappa_u/\Lambda > 0.019\text{ TeV}^{-1}$ at the Tevatron or larger than $\kappa_u/\Lambda > 0.0033\text{ TeV}^{-1}$ at the LHC. Single top quark production cannot measure couplings as small, but may still be of use since it has a related process within the standard model, which will be studied. A deviation of this measurement from the standard model may point toward flavor changing couplings of this type.
1 INTRODUCTION

Why do We Study Particle Physics?

Particle physics has recently come under attack from politicians concerned about the large budgets the particle accelerators require, and from the public, who see no benefits to offset the huge expense. Why then do we who are involved in the study of high energy interactions continue to feel that the projects are worthwhile?

In spite of the image of a huge pork barrel associated with particle physics, there are tangible benefits to studying these interactions. The engineering challenges involved in building such large accelerators are immense. Huge super-conducting magnets needed to be researched, designed, and built for the Super-conducting Super Collider (SSC). Even though the collider was never completed, much of the research was done. The area around the collision point in any collider is highly radioactive, and electronics capable of withstanding this environment are needed. Even computer science gets a boost, as huge amounts of data must be taken, processed, and stored by computer systems.

Although the side benefits of particle physics research are significant, the scientists do not study the field for this reason. While other branches of physics are going about the useful and important work of examining and describing such things as super-conductors,
atomic nuclei, and galaxies, particle physicists are continuing the grandest tradition of physics. We are seeking the final theory, the one hypothesis which, in its simple and elegant form, contains all of physics. While others study complexity, we search for simplicity. This quest is what draws many, including myself, to this field.

Why then do we need particle accelerators to study particle physics? Can’t this work be done another way? In short the answer to this question is no. we cannot study particle physics without particle accelerators. A well known principle of quantum mechanics, known as the de Broglie relation, along with the rules of wave motion, prohibits such study. Particle physics is partially concerned with the discovery of new forms of matter, and the forces that join them together. Atoms and molecules combine to make up everyday matter. Protons, neutrons, and electrons combine to form atoms. Quarks combine to form protons and neutrons. Each step further brings smaller and smaller particles to go along with greater and greater simplicity.

The de Broglie relation asserts that all matter moves in a wavelike motion, and that the momentum (and hence the energy) of a particle is a function of the wavelength of this motion,

$$p = \frac{h}{\lambda}.$$  (1.1)

Now, it is a general result of wave optics, especially diffraction, that a wave is not diffracted by a particle that is smaller than its wavelength. Stated another way, the diffraction of a wave is not affected by structures smaller than its wavelength. We may treat a straightedge as precisely straight when we diffract visible light past its edge, but if we use a shorter wavelength of light, we must inspect that straightedge for defects,
perhaps even arising from the atoms that make it up. Therefore, if we wish to examine extremely small particles, we need extremely small wavelengths for our probes. Since a particle's momentum is inversely proportional to its wavelength, we need extremely high momenta to examine these structures.

For example, when viewed with visible light, normal matter appears as if it were a continuous solid. Only when we view matter with a higher energy probe (such as an electron microscope) are atoms resolved. When viewed with still higher energy, such as alpha particles from nuclear decay, the nucleus becomes visible. Still higher energies allow us to look inside the nucleus to see the protons and neutrons. And, at even higher energies, we see the quarks that make up the protons and neutrons.

By increasing the energy, we have also discovered new forms of matter, that do not exist at lower energies. Along the way, we have discovered muons and pions and lambda particles that we never knew existed before we increased the energy of our probes. These new particles have pointed the way towards new symmetries, and our theories have become ever more elegant as a result.

As we probe even higher energies, will we see some new form of matter that makes up quarks? Will we see some unexpected particles which will make clear what is now obscured? One thing is clear, at every step along the way, our theory will become ever more powerful and elegant, bringing us closer to the dream of a final theory\textsuperscript{1}.

\textsuperscript{1}Apologies to Steven Weinberg for stealing his beautiful phrase
Phenomenological Lagrangians

The purpose behind phenomenological Lagrangians is to parameterize, in a model independent way, some or all of the low energy behavior of a higher energy theory. In order to do this, we write down all of the possible interactions (operators) of known particles and assign to each a form factor. In principle, there are an infinite number of such interactions as the number of initial and final state particles are not fixed. The dimension of an operator is the sum of the mass dimensions of all of the factors in the operator. Since a Lagrangian must be dimension four\(^2\), the form factor assigned to a higher dimension operator must itself contain mass dimensions. This mass dimension (often denoted by \(\Lambda\)) is usually factored out of the form factor and is often assumed to be the scale at which new physics explicitly appears in the Lagrangian.

It is commonly assumed that the so called "higher dimension" operators are small, as the operator is a function of some coupling strength, \(\alpha\) divided by \(\Lambda\). This coupling should be small compared to \(\Lambda\) or the phenomenological theory may have theoretical problems, such as nonunitarity. Therefore, any above dimension six or so are usually omitted. The following is an example of a term which might be included in a phenomenological Lagrangian:

\[
L_{\text{phenom}} = \frac{\alpha(q^2)}{\Lambda^2 \epsilon} \bar{c}\gamma^\mu c \bar{b}\gamma_\mu b. \tag{1.2}
\]

The form factors, in principle, may be calculated from a higher energy theory (one whose new particles have mass \(> \Lambda\)) via loop corrections at low energy, or may be

\(^{2}\text{We often confuse the Lagrangian and the Lagrangian density. See Appendix A for details about the mass dimension of the Lagrangian.}\)
used as they are to calculate observables as a function of these form factors. Since we have no clear indication of what the nature of the theory at higher energy is, it is the latter use that we are mostly concerned with. Generally, even removing the higher dimension operators from the phenomenological Lagrangian leaves far too many degrees of freedom to be useful as an experimental tool. Often, many of the remaining form factors are removed by considering only those that obey a certain symmetry, or interact with certain particles. For example, one could consider only those form factors which leave CP invariant and involve the top quark. In addition, the form factors are sometimes assumed to be independent of the energy scale of the process.

The phenomenological Lagrangian written down in this manner is known to be non-renormalizable. Since it is widely believed that any theory representing reality must be renormalizable, phenomenological Lagrangians are not considered to be the final theories in themselves, but are merely low energy approximations of some unknown high energy theory. Nevertheless, it is possible to calculate higher order corrections using phenomenological Lagrangians. Because of the non-renormalizability, the ultraviolet poles in the matrix element do not cancel out. As a non-rigorous approximation, we may replace those poles with an ultraviolet cutoff, commonly assumed to be proportional to $\ln(\frac{\Lambda}{\mu})$, where $\Lambda$ is the scale of new physics, and $\mu$ is the mass scale of the process being examined. The result, while it cannot be considered a rigorous prediction, may be used as an estimate of the first order correction.
A Model for Flavor Changing Chromomagnetic Top Quark Moments

The form of the phenomenological Lagrangian that we used for our study was motivated by several factors. First, and most important, was the recent discovery of the top quark\cite{1,2} at a mass of 175 GeV. This mass is significant for two reasons. First, it is 35 times larger than the bottom quark, which is its weak isospin partner and the next heaviest quark. No other weak isospin doublet has such a huge mass splitting. Second, 175 GeV is almost exactly equal to $\frac{v}{\sqrt{2}}$, where $v$ is the vacuum expectation value of the Higgs boson. The mass of the top quark is related to the electroweak symmetry breaking scale, indicating that new top quark physics may be related to the Higgs potential.

In the SM the mass of the fermions is given by $m_i = \frac{1}{\sqrt{2}} \lambda_i v$, where $\lambda_i$ is the coupling strength of the Higgs boson to the fermion. For the top quark then, this coupling is nearly equal to one, $\lambda_{\text{top}} \approx 1$. Why is the top quark alone in having this significant mass? Its significance makes the top quark sector an ideal place for beyond the standard model corrections to appear. We have decided to limit the operators in our phenomenological model to those which contain the top quark.

Beyond this requirement, we needed to choose an operator whose contribution would not be dwarfed by the processes of the standard model. Flavor changing neutral currents arising from higher order corrections of the standard model are extremely small\cite{3}, and cannot be observed at any planned or existing collider. A measurement of flavor changing neutral currents is thus a clear signal of physics beyond the standard model.
An additional bonus is given by the fact that the backgrounds to the processes which could measure this type of vertex must mimic the top quark. Given the large mass, this is not an easy thing to do, and thus the background will be significantly reduced.

There are four different neutral currents in the standard model, corresponding to exchange of a photon, a $Z^0$ boson, a gluon, and a Higgs boson. Others are considering flavor changing couplings to the photon[7, 13], the $Z^0$ boson[8, 11, 13], and the Higgs boson[6, 9]. We have chosen to focus on the flavor changing couplings of the gluon to the top quark. Others have also worked on this coupling[10], including several of my collaborators[5].

We further limited the form of our operators by insisting that they be gauge invariant and CP conserving. The gauge invariance leaves QCD unbroken, and the CP odd terms do not significantly enhance the cross section. Once all of these requirements are met, we are left with two possible operators:

\[
\frac{\kappa_u}{\Lambda} g_s \bar{u} \sigma^{\mu \nu} \lambda^a \frac{\lambda^a}{2} t G_{\mu \nu}^a + H.c., \\
\frac{\kappa_c}{\Lambda} g_s \bar{c} \sigma^{\mu \nu} \lambda^a \frac{\lambda^a}{2} t G_{\mu \nu}^a + H.c. ,
\]

where $\Lambda$ is the new physics scale, $\kappa_c$ and $\kappa_u$ define the strengths of the couplings, $G_{\mu \nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - ig_s f^{abc} G_\mu^b G_\nu^c$ is the gauge field tensor of the gluon, and $g_s$ is the strong coupling constant.

As we describe in Appendix A, the procedure for finding the Feynman rules is equivalent to taking the derivative of the interaction Lagrangian with respect to the fields. So, we can write the Feynman rules for our phenomenological Lagrangian as follows:
There has been some work done on this Lagrangian, mostly for the charm quark terms, finding limits upon the coupling parameter through the non-observation of top quark decays into a charm quark or gluon [5], and from the fact that higher order processes such as $b \to s\gamma$ [4] do not deviate from the standard model. In Ref. [5] the authors derive a limit by examining the measured branching fraction, $BF(t \to bW)$, and assuming that anything that does not result in a $b$ quark and a $W$ boson results in a charm quark and a gluon through our coupling. The $1\sigma$ lower limit to $BF(t \to bW)$ then implies an upper limit to $BF(t \to cg)$, giving an upper limit to the coupling parameter $\frac{\kappa_c}{\Lambda} \leq 0.95 \text{ TeV}^{-1}$. Though they didn't consider a $t \to ug$ decay, it is relatively easy
to insert into the limit. Examining the formula for $t \rightarrow cg$ decay, we see that simply replacing $\kappa_c$ by $\sqrt{\kappa_c^2 + \kappa_g^2}$ gives the proper contribution from $t \rightarrow ug$. Thus, from top quark decay, we can place the limit $\frac{\sqrt{\kappa_c^2 + \kappa_g^2}}{\Lambda} \leq 0.95$ TeV$^{-1}$.

In addition to the limits that have already been placed, there are several new signals that can be explored. The most intuitive is to search directly for $t \rightarrow cg$ and $t \rightarrow ug$ in $t\bar{t}$ production. This signal suffers from the relatively small $t\bar{t}$ cross section. Without a large sample of top quarks, this process is of limited utility. However, CERN's Large Hadron Collider (LHC) may produce enough $t\bar{t}$ pairs to make this a useful process to study.

A second possibility is the production of single top quarks. This process proceeds through four separate channels: $q\bar{q} \rightarrow t\bar{c}$, $gg \rightarrow t\bar{c}$, $cg \rightarrow tg$, and $cq \rightarrow tq$. There are also equivalent processes involving the up quark. At first glance, it would seem that the processes involving the charm quark in the initial state are negligible compared to the other processes because of the fact that the fraction of charm quarks in the proton is so small, compared to the valence quarks and gluons. However, this turns out not to be the case. When the Feynmann diagrams are written down, one notices that the processes with initial state charm quarks involve a massless $t$-channel gluon exchange. This exchange is well known to significantly increase the cross section of these processes. Therefore, we cannot neglect these processes, and when such a process involves an up quark in the initial state it actually dominates the other contributions. (Since the up quark is a valence quark, a large percentage of the proton is composed of up quarks.)

In ref. [10] the authors have studied this possibility. They have limited themselves
to the study of the charm quark vertex only, and also have limited themselves to the Tevatron only. In our study of this subject, presented in Chapter 3, we will extend their results to include the up quark coupling, and also to include Run 2 at the Tevatron (which has a slightly higher energy and a much higher luminosity than Run 1) and the LHC. The major difference between the two works is the fact that we considered all of the possible contributions to the single top quark production cross section, while the authors of ref. [10] only considered the \( q\bar{q} \rightarrow t\bar{c} \) channel. They justified this by placing a cut upon \( M_{tc} \), which is equivalent to a cut on \( \sqrt{s} \), in order to reduce the background from other processes. For large values of this cut, there are many more valence quarks in the proton than gluons and sea quarks, and they concluded that the other processes could not significantly contribute to the cross section. We do not make this assumption, and we will argue for a completely different set of cuts. Nevertheless, the authors claim that their cuts would discover this vertex if \( \frac{\sqrt{s}}{\Lambda} > 0.4 \text{ TeV}^{-1} \).

Another possibility is to examine direct top quark production. In single top quark production, the top quark was always accompanied in the final state by another non-top quark jet. In direct top quark production, this associated jet is absent. This process proceeds through a charm quark and a gluon in the initial state combining directly to produce a top quark, \( cg \rightarrow t \). There is also a process involving the up quark in the initial state. Even though the \( cg \rightarrow t \) process suffers from the same lack of charm quarks in the proton as in the single top quark processes, direct top quark production is an order \( \alpha_s(\frac{\sqrt{s}}{\Lambda})^2 \) process, where as single top quark production is order \( \alpha_s^2(\frac{\sqrt{s}}{\Lambda})^2 \). The lack of the additional \( \alpha_s \) enhances direct top quark production relative to single top quark
production by a factor of $\approx 10$. This more than compensates for the lack of charm quarks in the proton. This possibility is studied in Chapter 2.

This coupling would also affect $t\bar{t}$ production through diagrams like $c\bar{c} \rightarrow t\bar{t}$ with a $t$ or $u$ channel gluon. This process suffers the same lack of charm quarks in the proton as the other process, but it must suffer it twice, making it unlikely that this process would ever be seen. There are diagrams like $u\bar{u} \rightarrow t\bar{t}$ with a $t$ or $u$ channel gluon. These are much more likely. However, when one considers that they are of order $\alpha^2(\frac{\Lambda}{\Lambda})^4$, it becomes clear that these processes are unlikely to probe the small values of $\frac{\alpha}{\Lambda}$ that can be probed with other techniques.

One last process that is interesting to us is double top quark production, proceeding through a process like $cc \rightarrow tt$. This process has no equivalent in the standard model, since it relies on flavor changing neutral currents to produce it. The corresponding diagram with up quarks in the initial state is more likely to be seen. This process also has the extremely rare signature of two top quarks in the final state, rather than a top quark and an anti-top quark. Again though, the cross section depends upon $(\frac{\alpha}{\Lambda})^4$. This suppression is so significant that even this otherwise promising signal is of no use in finding a limit to $\frac{\alpha}{\Lambda}$.

**Dissertation Organization**

Chapter 1 is this introduction, which includes a short introduction to particle physics, a brief description of phenomenological Lagrangians, and a description of our model. Chapter 2 is a paper published in Phys. Rev. D56, 5725 on the subject of direct top
quark production. Chapter 3 is a paper, prepared for submission to Physics Review D, on single top quark production. Chapter 4 contains the conclusions. I have also included an appendix on field theory, and an appendix discussing the Monte Carlo techniques used in our study.

References


2 DIRECT TOP QUARK PRODUCTION AT HADRON COLLIDERS AS A PROBE OF NEW PHYSICS


M. Hosch, K. Whisnant, and B.-L. Young

Abstract

We examine the effect of an anomalous flavor changing chromomagnetic moment which allows direct top quark production (two partons combining into an unaccompanied single top quark in the s-channel) at hadron colliders. We consider both t-c-g and t-u-g couplings. We find that the anomalous charm quark coupling parameter $\kappa_c/\Lambda$ can be measured down to .06 TeV$^{-1}$ (.009 TeV$^{-1}$) at the Tevatron with the Main Injector upgrade (LHC). The anomalous up quark coupling parameter $\kappa_u/\Lambda$ can be measured to .02 TeV$^{-1}$ (.003 TeV$^{-1}$) at the Tevatron (LHC).

Introduction

With the discovery of the top quark [1, 2], the long anticipated completion of the fermion sector of the standard model has been achieved. Its unexpected large mass
in comparison with the other known fermions suggests that the top quark may play a unique role in probing new physics, and has prompted both theorists and experimenters alike to search for anomalous couplings involving the top quark. On the experimental side, the CDF [3, 4] and D0 [5] collaborations have begun to explore the physics of top quark rare decays [3]. On the theoretical side, a systematic examination of anomalous top quark interactions, in a model independent way, has been actively undertaken[6, 7].

One possible set of anomalous interactions for the top quark is given by the flavor-changing chromo-magnetic operators:

\[
\frac{\kappa_u}{\Lambda} g_s \bar{u} \sigma^{\mu\nu} \frac{\lambda^a}{2} t G^a_{\mu\nu} + h.c., \tag{2.1}
\]

and

\[
\frac{\kappa_c}{\Lambda} g_s \bar{c} \sigma^{\mu\nu} \frac{\lambda^a}{2} t G^a_{\mu\nu} + h.c. \tag{2.2}
\]

where \( \Lambda \) is the new physics scale, \( \kappa_c \) and \( \kappa_u \) define the strengths of the couplings, \( G^a_{\mu\nu} \) is the gauge field tensor of the gluon, and \( g_s \) is the strong coupling constant. The investigation of these couplings is well motivated. Although these operators can be induced in the standard model through higher order loops, their effects are too small to be observable[8]. Therefore, any observed signal indicating these types of couplings is direct evidence for physics beyond the standard model.

It has been argued that the couplings in Eqs. (1) and (2) may be significant in many extensions to the standard model, such as supersymmetry (SUSY) or other models with multiple Higgs doublets [8, 9, 10, 11], models with new dynamical interactions of the top quark[12], and models where the top quark has a composite[13] or soliton[14] structure.
In particular, Ref. [10] suggests that the supersymmetric contributions to a t-c-g vertex may be large enough to measure at a future hadron collider.

T. Han et. al.[15] have placed a limit on the top-charm-gluon coupling strength, $\kappa_c$, by examining the decay of the top quark into a charm quark and a gluon. They find that an upper limit on $\kappa_c/\Lambda$ of $0.43(0.65) \, \text{TeV}^{-1}$ with(without) b-tagging for 200 pb$^{-1}$ of data can be measured at the Tevatron. If the c and u jets are not distinguished, their result applies equally well to $\kappa_u/\Lambda$, if one uses the up quark coupling alone. or to the sum, added in quadrature, when both are considered.

In this paper, we will examine these operators in a model independent way using direct top quark production at the Fermilab Tevatron and at the CERN LHC. In this scenario, a charm (or up) quark and a gluon from the colliding hadrons combine immediately to form an s-channel top quark, which then decays. The production of a single, unaccompanied top or anti-top quark is very small in the standard model. We will take as our signal only the case where the top quark decays to a b quark and a W boson. While the $t \to cg$ (or ug) decay will occur in the presence of the anomalous couplings given in Eqs. (1) and (2), it is smaller than the $t \to bW$ decay for $\kappa/\Lambda \lesssim 0.75 \, \text{TeV}^{-1}$, and will have a negligible branching ratio for $\kappa/\Lambda \lesssim 0.2 \, \text{TeV}^{-1}$. Given the existing upper bound of the anomalous coupling mentioned earlier [15], $t \to bW$ will be the dominant decay mode of the top quark. Since the W boson decay into a charged lepton (electron or muon) and its corresponding neutrino has an identifiable signature, we consider only the $t \to bW \to b\ell\nu_l$ decay for our signal. With the decays so chosen, we find that the backgrounds are manageable, as will be discussed in detail later.
Figure 2.1  Feynmann diagram for direct top quark production and subsequent decay into $b\ell\nu_l$.

**Direct Top Quark Production**

We have calculated tree level cross sections for direct top quark production, $p\bar{p} \rightarrow t \rightarrow bW^+ \rightarrow b\ell^+\nu_l$, using the flavor-changing chromomagnetic moments in Eqs. (1) and (2) (see Fig. 2.1). The $\ell^+$ in this process is either a positron or an anti-muon, and $\nu_l$ is its corresponding neutrino. We also included direct anti-top quark production in our calculation ($p\bar{p} \rightarrow \bar{t} \rightarrow \bar{b}W^- \rightarrow \bar{b}l^-\bar{\nu}_l$). The parton cross section for direct top (or anti-top) quark production is given by:

$$d\sigma = \frac{1}{4(4\pi)^5} \frac{\hat{s} - M_{l,\nu_l}^2}{\hat{s}^2} |\mathcal{M}|^2 d\Omega_b d\Omega_\ell dM_{l,\nu_l}^2,$$  \hspace{2cm} (2.3)

where the spin averaged squared matrix element is

$$|\mathcal{M}|^2 = \frac{256\pi^3 \alpha^2 \alpha_s \kappa_{c(u)}^2}{3\sin^4 \theta_W \Lambda^2} \frac{\hat{s}(p_b \cdot p_{\nu_l}) \left[ \hat{s} (q_{c(u)} \cdot p_\ell) + m_\ell^2 (q_\ell \cdot p_\ell) \right]}{\left( \hat{s} - m_t^2 \right) \left( \left( M_{l,\nu_l}^2 - M_W^2 \right)^2 + M_W^4 \Gamma_W^2 \right)},$$  \hspace{2cm} (2.4)

$p_{b,l,\nu_l}$ are the 4-momenta of the outgoing b quark, lepton, and neutrino respectively, $q_{c(u),g}$ are the 4-momenta of the incoming charm(up) quark and gluon, $\Gamma_W$ is the decay...
width of the $W$ boson,

$$\Gamma_t = \Gamma_{t \to bW} \left[ 1 + \frac{128 M_W^2 \alpha_s}{3\alpha_2 \left(1 - \frac{M_W^2}{m_t^2}\right)^2 \left(1 + \frac{2M_W^2}{m_t^2}\right)} \left(\frac{\kappa_c^2 + \kappa_u^2}{\Lambda^2}\right) \right]$$  \hspace{1cm} (2.5)$$

is the decay width of the top quark, including the anomalous contribution for $t \to cg$ (or $t \to ug$), $\Gamma_{t \to bW}$ is the standard model top quark decay width to a $b$ quark and $W$ boson,

$$M_{l\nu}^2 \equiv (p_l + p_{\nu})^2$$  \hspace{1cm} (2.6)$$
is the invariant mass squared, not necessarily on shell, of the $W$ boson, and $\sqrt{s}$ is the parton center of mass energy.

As mentioned earlier, we considered only the case which has a charged lepton (muon or electron) in the final state, to identify the $W$ boson. Compared to the hadronic decay mode of the $W$, the background for these processes is smaller and the signal is not as hard to identify. In order to examine the kinematics of the decay products, we calculated the full three body phase space for the process, using the Breit-Wigner propagators to broaden the top quark and $W$ boson distributions. Figure 2.2 shows the cross section at the Tevatron and LHC as a function of $\kappa/\Lambda$. In the top quark decay width, we included an additional term arising from $t \to c(u)g$, as shown in Eq. (5). This term is proportional to $|\kappa/\Lambda|^2$ and contributes significantly to the top quark width only if $\kappa/\Lambda \approx 0.2\,\text{TeV}^{-1}$. One can see the effect of the additional channel for top quark decay, which decreases the $t \to bW$ branching ratio and causes a noticeable deviation from quadratic behavior for $\kappa/\Lambda \approx 0.2\,\text{TeV}^{-1}$. (For $\kappa/\Lambda \approx 0.2$ there is a deviation from the straight line that one would expect on a log-log plot if the cross section scaled quadratically.)
Figure 2.2  Direct top quark cross section vs. \( \kappa/\Lambda \) at Run 2 of the Tevatron and the LHC. The cross sections for Run 1 of the Tevatron are barely distinguishable from Run 2, and are not shown here.

We calculated the \( p\bar{p} \) (for the Tevatron) and \( pp \) (for the LHC) cross sections for direct top quark production with the MRSA structure functions [16]. We have also examined the effect of using the CTEQ3M [17] structure functions. The difference between the two sets of structure functions is small. Several distributions were calculated, including the transverse momenta, the pseudorapidities, the jet separation, from the lepton, and the reconstructed \( \sqrt{s} \).

In order to reduce the \( W + 1 \text{ jet} \) background, we made a series of cuts, which we will call the basic cuts, on the kinematic distributions. They are:

\[
\begin{align*}
\mathbf{p_T}(b,l,\nu_l) & \geq 25 \text{ GeV} , \\
\eta_b & \leq 2.0 ,
\end{align*}
\]
\[ \eta \leq 3.0, \tag{2.9} \]
\[ \Delta R \geq 0.4, \tag{2.10} \]

where \( \eta_{b,l} \) are the pseudorapidities, \( \Delta R \equiv \sqrt{(\eta_b - \eta_l)^2 + (\phi_b - \phi_l)^2} \) is the separation between the b jet and the charged lepton in the detector, and \( \phi_{b,l} \) are the azimuthal angles. We also assumed a Gaussian smearing of the energy of the final state particles, given by:

\[ \Delta E/E = 30%/\sqrt{E} \mp 1\%, \text{ for leptons,} \tag{2.11} \]
\[ = 80%/\sqrt{E} \mp 5\%, \text{ for hadrons,} \tag{2.12} \]

where \( \mp \) indicates that the energy dependent and independent terms are added in quadrature.

To enhance the signal relative to the background, we want to make cuts on \( \sqrt{s} \), which should be sharply peaked at \( m_t \) for the signal. To experimentally determine \( \sqrt{s} \), one must reconstruct \( p_t = p_b + p_l + p_\nu \). The neutrino is not observed, but its transverse momentum can be deduced from the missing transverse momentum. The longitudinal component of the neutrino momentum is determined by setting \( M_{l,\nu} = M_W \) in Eq. (6), and is given by:

\[ p_{L}^{\nu} = \frac{\chi p_L \pm \sqrt{p_T^2 (\chi^2 - p_T^2 p_{T,\nu}^2)}}{p_T^2}, \tag{2.13} \]

where

\[ \chi = \frac{M_W^2}{2} + p_T \cdot p_{T,\nu}^2, \tag{2.14} \]

and \( p_L \) and \( p_T \) refer to the longitudinal and transverse momenta respectively. Note that there is a two fold ambiguity in this determination. We chose the solution which would
best reconstruct the mass of the top quark. In some rare cases, the quantity under the square root in Eq. (13) is negative due to the smearing discussed above. When this happened, we set this square root to zero, and used the corresponding result for the neutrino longitudinal momentum.

**Background Calculation**

The main source of background to the direct top quark production is \( p\bar{p} \rightarrow W + 1 \text{ jet} \). Another background process is standard model single top quark production when the associated jets are not observed. Examining the data presented in Ref. [18], we conclude that single top quark production is less than 1% of the \( W + 1 \) jet background when b-tagging is not used. When b-tagging reduces the \( W + 1 \) jet background by a factor of 100, the single top quark background may be as large as 20% of the total background. However, since the discovery limit on \( \kappa/\Lambda \) scales as \( B^{-\frac{1}{2}} \) where \( B \) is the number of background events, a 20% change in the background affects the discovery limit by only 5%. We therefore ignore this background.

We used the VECBOS Monte Carlo [20] to calculate the cross section for the \( W + 1 \) jet background. We modified the program to produce the same distributions that were calculated for the signal, and applied the same basic cuts used in the signal calculation, Eqs. (7-10). To determine additional cuts which optimize the discovery limits on \( \kappa/\Lambda \), we examined the kinematic distributions in \( \sqrt{s}, p_T, \eta, \) and \( \Delta R \). We found that three distributions, \( \sqrt{s}, p_Tb, \) and \( \eta_l \), were most useful in isolating the signal from the background. These are shown in Figs. 2.3, 2.4, and 2.5, with the charm quark in the initial state and
Figure 2.3 \( \sqrt{s} \) distributions for the (a) basic and (b) optimized cuts without b-tagging at the upgraded Tevatron. The solid line represents the direct top quark production \((\kappa_c/\Lambda = 0.2 \text{ TeV}^{-1})\). The dotted line is one thousandth of the \( W + 1 \) jet background.

\( \kappa_c/\Lambda = 0.2 \) TeV for the upgraded Tevatron. The solid lines represent direct top quark production, and the dashed lines represent the \( W + 1 \) jet background divided by 1000.

The cuts were optimized for each of four cases: Run 1 at the Tevatron with pp collisions at \( \sqrt{s} = 1.8 \) TeV and 100 pb\(^{-1}\) of data per detector, Run 2 with \( \sqrt{s} = 2.0 \) TeV and 2 fb\(^{-1}\), Run 3 with 2.0 TeV and 30 fb\(^{-1}\), and the LHC with pp collisions at 14 TeV and 10 fb\(^{-1}\). The optimized cuts are shown in Table 2.1. The corresponding distributions with the up quark in the initial state are not shown; they have the same shape as for the charm quark, but are a factor of ten larger in magnitude, due to the much larger size of the valence up quark distribution in the initial state.

To further reduce the background, we assumed that silicon vertex tagging of the b
Figure 2.4 $p_T^b$ distributions for the (a) basic and (b) optimized cuts without b-tagging at the upgraded Tevatron. The solid line represents the direct top quark production ($\kappa_c/\Lambda = 0.2$ TeV$^{-1}$). The dotted line is one thousandth of the $W + 1$ jet background.

Table 2.1 Optimized cuts for direct top quark production.

<table>
<thead>
<tr>
<th></th>
<th>$E_C.M.$</th>
<th>$(p_T^b)_{\text{min}}$</th>
<th>$(\sqrt{s})_{\text{min}}$</th>
<th>$(\sqrt{s})_{\text{max}}$</th>
<th>$(\eta)_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run 1</td>
<td>1.8 TeV</td>
<td>35 GeV</td>
<td>155 GeV</td>
<td>205 GeV</td>
<td>1.8</td>
</tr>
<tr>
<td>Run 2</td>
<td>2.0 TeV</td>
<td>45 GeV</td>
<td>160 GeV</td>
<td>205 GeV</td>
<td>1.0</td>
</tr>
<tr>
<td>Run 3</td>
<td>2.0 TeV</td>
<td>45 GeV</td>
<td>160 GeV</td>
<td>205 GeV</td>
<td>1.0</td>
</tr>
<tr>
<td>LHC</td>
<td>14.0 TeV</td>
<td>35 GeV</td>
<td>165 GeV</td>
<td>195 GeV</td>
<td>1.0</td>
</tr>
</tbody>
</table>
Figure 2.5  $\eta_t$ distributions for the (a) basic and (b) optimized cuts without b-tagging at the upgraded Tevatron. The solid line represents the direct top quark production ($\kappa_c/\Lambda = 0.2 \text{ TeV}^{-1}$). The dotted line is one thousandth of the $W + 1$ jet background.
jet would be available, with 36% efficiency at Run 1 of the Tevatron, and 60% at Runs 2 and 3, and at the LHC. In addition, we assumed that 1% of all non-\(b\) quark jets would be mistagged as \(b\) quark jets.

When \(b\)-tagging is present, if the jet produced is mistaken as a \(b\) jet, it remains a part of the background. The background can be reduced by a factor of 100 if the \(W + 1\) jet sample does not include a significant fraction of \(b\) quarks in the final state. It is possible to estimate the fraction of \(b\) quarks in the \(W + 1\) jet sample by taking the ratio \(|V_{cb}|^2/|V_{ud}|^2\) and multiplying by the ratio of the distribution fraction of charm quarks to up quarks in the proton, \(.2 - .005(.7 - .05)\), in the momentum fraction region where most of the events occur for the Tevatron (LHC). We estimate that the fraction of \(b\) quark jets in the \(W + 1\) jet background is less than \(.03%(.12%)\), much less than the anticipated mistagging rate of 1%. We therefore ignore the possibility of having \(b\) quarks in the \(W + 1\) jet sample. Including \(b\)-tagging does not significantly affect the optimized cuts.

**Results and Discussion**

We can use the results of the signal and background calculations to determine the minimum value of \(\kappa_c/\Lambda\) or \(\kappa_u/\Lambda\) observable at hadron colliders. Assuming Poisson statistics, the number of signal events \((S)\) required for discovery of a signal at the 95% confidence level is:

\[
\frac{S}{\sqrt{S + B}} \geq 3, \tag{2.15}
\]
where B is the number of background events obtained by multiplying the background cross section by the luminosity and dividing by 100 if b-tagging is present. The luminosity, background cross section, and signal cross section needed for discovery of anomalous flavor changing couplings is given in Table 2.2. The discovery limits may then be determined by comparing the signal calculation for a given $\kappa/\Lambda$ to the signal needed, which can be obtained from Table 2.2. These discovery limits are shown in Table 2.3.

Because the charm and up quarks are in the initial state, their contributions to direct top quark production cannot be distinguished. A plot of the discovery limit when both $\kappa_c$ and $\kappa_u$ are assumed to be nonzero is show in Fig. 2.6.

The results quoted in this paper all use the MRSA structure functions. When using the CTEQ3M structure functions, the direct top quark cross section increases by 15% when the charm quark coupling is used, corresponding to a 7% decrease in the discovery limit for $\kappa_c/\Lambda$. This is primarily due to a larger charm quark density in the proton with the CTEQ3M structure functions. The $W + 1$ jet cross section does not change significantly, nor does the direct top quark cross section when the up quark coupling is

<table>
<thead>
<tr>
<th>Run</th>
<th>$\sqrt{s}$ (TeV)</th>
<th>Luminosity ($fb^{-1}$)</th>
<th>Background ($fb$)</th>
<th>$W/o$ b-tag ($fb$)</th>
<th>$W/ b$-tag ($fb$)</th>
</tr>
</thead>
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<tr>
<td>Run 1</td>
<td>1.8</td>
<td>0.1</td>
<td>19400</td>
<td>1370</td>
<td>190</td>
</tr>
<tr>
<td>Run 2</td>
<td>2.0</td>
<td>2</td>
<td>13000</td>
<td>245</td>
<td>27</td>
</tr>
<tr>
<td>Run 3</td>
<td>2.0</td>
<td>30</td>
<td>13000</td>
<td>63</td>
<td>6.4</td>
</tr>
<tr>
<td>LHC</td>
<td>14</td>
<td>10</td>
<td>79000</td>
<td>267</td>
<td>27</td>
</tr>
</tbody>
</table>

Table 2.2 Signal needed for the discovery of anomalous t-c-g and t-u-g couplings at the Tevatron and LHC at 95% confidence level. The background cross sections use the optimized cuts described in Table 2.1.
Table 2.3  Discovery limits on $\kappa_c/\Lambda$ (with $\kappa_u = 0$) and $\kappa_u/\Lambda$ (with $\kappa_c = 0$) at the Tevatron and LHC. The results are reported in TeV$^{-1}$.

<table>
<thead>
<tr>
<th>b tagging?</th>
<th>1.8 TeV</th>
<th>2 TeV</th>
<th>14 TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.1 fb$^{-1}$</td>
<td>2 fb$^{-1}$</td>
<td>30 fb$^{-1}$</td>
</tr>
<tr>
<td>charm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>no</td>
<td>.38</td>
<td>.14</td>
<td>.073</td>
</tr>
<tr>
<td>yes</td>
<td>.22</td>
<td>.062</td>
<td>.030</td>
</tr>
<tr>
<td>u quark</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>no</td>
<td>.096</td>
<td>.045</td>
<td>.023</td>
</tr>
<tr>
<td>yes</td>
<td>.058</td>
<td>.019</td>
<td>.0094</td>
</tr>
</tbody>
</table>

Figure 2.6  Discovery limits for $\kappa_c$ and $\kappa_u$ for each of the colliders considered, for $\Lambda = 1$ TeV.
used. This difference reflects our lack of understanding of the charm quark distribution in the proton. Ultimately, this effect will be part of the theoretical uncertainty to the measured value of $\kappa_c$.

We considered cases with and without b-tagging for each of the possibilities in Table 2.3. With the exception of Run 1 at the Tevatron, b-tagging improved the discovery limit on $\kappa/\Lambda$ by 2.0 — 2.5 times. However, for the data from Run 1 at the Tevatron, b-tagging improves the discovery limit by only 40%. This is mostly due to less efficient b-tagging, and to the smaller number of events available with a lower luminosity.

In some single top quark production processes, there are regions of overlap between, for example, $2 \rightarrow 1$ subprocesses and $2 \rightarrow 2$ subprocesses. In particular, we worried about an overlap between the direct top quark production and the gluon fusion diagram in which one of the gluons is dissociated into a $c\bar{c}$ pair, and the $c$ combines with the other gluon to produce a top quark. Care must be taken with these processes to avoid double counting. A systematic method exists for calculating a subtraction term which solves this difficulty[18, 19]. The effect of the double counting is most significant if the initial state particles are massive. In the case of direct top quark production due to anomalous t-c-g or t-u-g couplings, the initial state particles are light enough that this does not significantly affect the overall cross section. We have therefore ignored this effect in our calculation.

Although the background due to single top quark production (a top quark with an associated jet) is small in the SM, there exists also the possibility for single top quark production with the anomalous t-c-g (or t-u-g) coupling [21], e.g. via $q\bar{q} \rightarrow t\bar{c}$ ($q\bar{q} \rightarrow t\bar{u}$).
If the jet associated with the top quark is not seen, this would enhance the direct top quark signal due to the anomalous coupling. Therefore, the discovery limits quoted in Table 2.3 are conservative estimates of the level to which $\kappa/\Lambda$ may be probed. A full treatment of single top quark production due to the anomalous t-c-g and t-u-g couplings will be considered elsewhere.

In conclusion, we have calculated the discovery limits for the anomalous chromomagnetic couplings t-c-g and t-u-g in hadron colliders using direct production of an s-channel top quark. We conservatively estimate that an anomalous charm quark coupling can be detected down to $\kappa_c/\Lambda = 0.06 \text{ TeV}^{-1}$ at Run 2 of the Tevatron, and $0.009 \text{ TeV}^{-1}$ at the LHC. The cross section for the anomalous up quark coupling is larger, and we can measure $\kappa_u/\Lambda$ down to $0.02 \text{ TeV}^{-1}$ at Run 2 of the Tevatron, and $0.003 \text{ TeV}^{-1}$ at the LHC. The discovery limits for the upgraded Tevatron are approximately two (six) times better than those obtained in Ref. [15] for $\kappa_c/\Lambda$ ($\kappa_u/\Lambda$). The relative size of the direct top quark production and the anomalous top quark decay rate will help to differentiate the t-c-g and the t-u-g couplings.

Finally, we note that, in Ref [10], the authors found that electroweak-like corrections in a supersymmetric model can give $\text{Br}(t \rightarrow cg)$ as large as $1 \times 10^{-5}$ for the most favorable combinations of the parameters. In terms of our anomalous coupling parameter, this corresponds to $\kappa_c/\Lambda = 0.0033$. If supersymmetry is the only source for the anomalous t-c-g coupling, our calculations therefore indicate that future improvements at the LHC will be needed to make this a detectable signal, unless QCD-like corrections [9] further enhance the SUSY contributions, as discussed in Ref. [10].
Acknowledgments

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3 ANOMALOUS FLAVOR CHANGING COUPLINGS AND SINGLE TOP QUARK PRODUCTION AT HADRON COLLIDERS

A paper to be submitted to Phys. Rev. D.
T. Han, M. Hosch, K. Whisnant, B.-L. Young, and X. Zhang

Abstract

If there is new physics associated with the top quark, a possible avenue for its realization is the anomalous coupling of the top quark to gluons. In particular anomalous flavor changing couplings have a very clear signal that is not available in the standard model. We use single top quark production to limit the strength of the possible anomalous coupling. In particular, we find that the anomalous coupling strength for a top-charm-gluon coupling strength may be measured to $\kappa_c/\Lambda = 0.092\text{TeV}^{-1}$ at Run 2 of the Tevatron with $2\text{ fb}^{-1}$ of data and $\kappa_c/\Lambda = 0.013\text{TeV}^{-1}$ at the LHC with $2\text{ fb}^{-1}$ of data. Similarly, the anomalous top-up quark-gluon may be measured to $\kappa_c/\Lambda = 0.026\text{TeV}^{-1}$ at the Tevatron and $\kappa_c/\Lambda = 0.0061\text{TeV}^{-1}$ at the LHC.
Introduction

Since the discovery of the top quark at the Tevatron[1, 2], there has been considerable interest in exploring the properties of the top quark. Its unusually large mass makes it an ideal place in which to probe for beyond the standard model physics, and has already given rise to considerable exploration of anomalous couplings of the top quark. A systematic examination of these couplings is currently being undertaken[3, 4, 5, 6, 7].

One promising avenue for this exploration is through single top quark production. Its unique signal (one top quark jet accompanied by a non-top quark jet) implies that background processes will be small, and a small signal will easily be seen. In Standard Model single top quark production[16] the accompanying jet will almost always be a bottom quark jet, allowing us to reduce the already small background through the use of b vertex tagging. In flavor changing neutral current theories, the accompanying jet need not come from a bottom quark. We can therefore use b-tagging, along with the kinematic distributions to separate the Standard Model processes from the non-standard model processes.

A possible set of anomalous interactions is given by the flavor-changing chromo-magnetic operators:

\[ \frac{\kappa_u}{\Lambda} g_\ast \bar{u} \sigma^{\mu\nu} \lambda^a \frac{\lambda^b}{2} t G^a_{\mu\nu} + h.c., \] (3.1)

and

\[ \frac{\kappa_c}{\Lambda} g_\ast \bar{c} \sigma^{\mu\nu} \lambda^a \frac{\lambda^b}{2} t G^a_{\mu\nu} + h.c., \] (3.2)

where \( \Lambda \) is the new physics scale, \( \kappa_c \) and \( \kappa_u \) define the strengths of the couplings, \( G^a_{\mu\nu} \) is...
the gauge field tensor of the gluon, and $g_s$ is the strong coupling constant. The search for flavor-changing neutral interactions is well motivated. Although such interactions can be produced by higher order terms in the Lagrangian, the effect is too small to be observable. Any signal indicating these types of couplings is therefore evidence of physics beyond the standard model.

It has been suggested that couplings of this type may be large in many extensions to the standard model, especially in models with multiple Higgs doublets such as supersymmetry[8, 9, 10, 11]. Models with new dynamical interactions of the top quark[12] and models where the top quark has a composite[13] or soliton[14] structure also contribute significantly to this coupling.

In this paper, we will examine the effect of these operators upon single top quark production at the Tevatron and LHC. We will consider only those processes which lead to a lone top (or anti-top) quark and one associated jet. There are four different subprocesses which lead to this final state, depending upon the initial state of the system. They are: $q\bar{q} \rightarrow t\bar{c}$, $gg \rightarrow t\bar{c}$, $cq(\bar{q}) \rightarrow t(q\bar{q})$, and $cq \rightarrow tg$. Associated processes which replace the charm quark with the up quark are also considered, as well as single anti-top quark production. While the $t \rightarrow c(u)g$ decay will occur with our anomalous couplings, it becomes negligible when $\kappa/\Lambda$ is smaller than about 0.2 TeV$^{-1}$. Since it will be possible to probe to this limit, and since the $t \rightarrow bW \rightarrow bl\nu_l$ provides a much more identifiable signal, we will choose $p\bar{p} \rightarrow t + j \rightarrow bl\nu_l + j$ to be our signal.

A limit on the size of the anomalous coupling parameter $\kappa/\Lambda$ can be placed by examining the branching ratio of the top quark, as measured at CDF and D0[15]. Since
the Standard Model top quark decays into a W boson and a b quark nearly 100% of the
time, a deviation of the branching ratio from unity may indicate the presence of new
physics. By looking for $t \rightarrow c(u)g$ decays, the coupling parameter $\kappa/\Lambda$ may be measured
down to $0.43 \text{ TeV}^{-1}$.

A search for direct top quark production (a top quark jet without an accompanying
jet) may also place a more stringent limit upon the size of the anomalous coupling
parameter[17]. This procedure relies on its extremely rare signal, and on the large
fraction of gluons in the initial state to boost its signal relative to the background. The
up quark operator has the additional bonus that its other initial state particle is a valence
quark. Combined with b-tagging (of the top quark decay products) this provides nearly
ideal conditions for measuring the anomalous coupling parameter. Using this process,
$\kappa_c/\Lambda$ can be measured to $0.062 \text{ TeV}^{-1}$ at Run 2 of the Tevatron with $2 \text{ fb}^{-1}$ of data, and
to $0.0084 \text{ TeV}^{-1}$ at the LHC with $10 \text{ fb}^{-1}$ of data. $\kappa_u/\Lambda$ can be measured to $0.019 \text{ TeV}^{-1}$
at the Tevatron, and to $0.0033 \text{ TeV}^{-1}$ at the LHC.

In ref.[18], the authors studied the effect of the anomalous top-charm-gluon coupling
on single top quark production at the Tevatron. They found that $\kappa_c/\Lambda$ can be measured
to $0.4 \text{ TeV}^{-1}$ at Run 1 of the Tevatron using this process. We will revisit their prediction
and extend it to the anomalous up quark coupling and also examine both couplings at
the LHC.
Single Top Quark Production

We have calculated the tree level cross sections for single top quark production at hadron colliders using the flavor changing chromomagnetic couplings described above. We also included single anti-top quark production in our calculation. There are four separate processes leading to single top quark production with flavor changing moments, \( q\bar{q} \rightarrow t\bar{c}, \quad gg \rightarrow t\bar{c}, \quad qc \rightarrow qt(\bar{q}c \rightarrow qt), \) and \( gc \rightarrow gt. \) The differential cross sections for these processes are long and are not very illuminating. They will not be presented here.

We considered only the case where the top quark decays into a b quark and W boson, and the W decays into an electron or muon and its neutrino. The other possible decay modes \( (t \rightarrow cg \text{ or } t \rightarrow bW \rightarrow bq\bar{q}) \) are harder to identify, and have larger backgrounds associated with them. We assumed that the top quark was on mass shell when we calculated the decay process. The cross section at the Tevatron and at the LHC is shown in figure 3.1 as a function of \( \kappa/\Lambda. \) In the branching ratio of the top quark to \( b + W, \) we included an additional piece arising from \( t \rightarrow cg. \) This term is proportional to \( |\kappa/\Lambda|^2 \) and appreciably affect the branching ratio only if \( \kappa/\Lambda \gtrsim 0.2. \)

We calculated the \( p\bar{p} \) (for the Tevatron) and \( pp \) (for the LHC) cross sections using the MRSA structure functions[19]. Several distributions were calculated, including transverse momenta, pseudorapidities, jet-jet and jet-lepton separations, the invariant mass of the b quark and W boson \( (M_{b,W}), \) and the center of mass energy \( (\sqrt{s}). \)

In order to reduce the background, we made a series of cuts on the kinematic distri-
Figure 3.1  Single top quark production cross sections vs. \( \kappa/\Lambda \) at Run 2 of the Tevatron and at the LHC. The solid and short dashed lines are for the charm quark and the up quark at Run 1 of the Tevatron respectively. The long dashed and dash-dotted lines are for the charm and up quarks at the LHC. Run 2 of the Tevatron was not plotted, as it is nearly indistinguishable from the cross sections for Run 1.
butions. These we will call the basic cuts. They are:

$$p_T(b,j,l,\nu) \geq 15 \text{ GeV} \quad (3.3)$$

$$\eta(b,j,l) \leq 2.5 \quad (3.4)$$

$$\Delta R(jj,jl) \geq 0.4 \quad (3.5)$$

where $\eta_{b,j}$ are the pseudorapidities, $\Delta R_{12} \equiv \sqrt{(\eta_1 - \eta_2)^2 + (\phi_1 - \phi_2)^2}$ is the separation between two of the tracks in the detector, and $\phi_{1,2}$ are the azimuthal angles. $\Delta R_{jj}$ is the separation between the $b$ jet and the associated non-$b$ jet, and $\Delta R_{jl}$ is the larger of the separation between the $b$ jet and the lepton and the separation between the associated non-$b$ jet and the lepton. We also assumed a Gaussian smearing of the energy of the final state particles, given by:

$$\Delta E/E = 30\% / \sqrt{E} \oplus 1\%, \text{ for leptons} \quad (3.6)$$

$$= 80\% / \sqrt{E} \oplus 5\%, \text{ for hadrons} \quad (3.7)$$

where $\oplus$ indicates that the energy dependent and independent terms are added in quadrature.

To further enhance the signal relative to the background, we need to make cuts on $M_{b,W}$, which should be sharply peaked at $m_t$ for the signal. To experimentally determine $M_{b,W}$, one must reconstruct $p_t = p_b + p_l + p_{\nu}$. The $b$ jet is identified in this case by $b$ vertex tagging. The neutrino is not observed, but its transverse momentum can be deduced from the missing transverse momentum. The longitudinal component of the neutrino momentum is determined by setting $M_{l,\nu} = M_W$, and is given by:

$$p_T^{\nu} = \frac{xp_L^i \pm \sqrt{p_L^2(x^2 - p_T^2 p_T^{\nu})}}{p_T^2} \ , \quad (3.8)$$
where

\[ \chi = \frac{M_W^2}{2} + p_T^L \cdot \vec{p}_T^e, \]  

(3.9)

and \( p_L \) and \( p_T \) refer to the longitudinal and transverse momenta respectively. Note that there is a two fold ambiguity in this determination. We chose the solution in which \( M_{bW} \) is closest to the mass of the top quark. If we did not have the option to vertex tag the b quark there would be an additional two fold ambiguity due to the uncertainty in determining which of the two quark momenta to use for \( p_b \) when determining \( M_{bW} \).

In some rare cases, the quantity under the square root in Eq. (8) is negative due to the smearing discussed previously. When this happened, we set this square root to zero, and used the corresponding result for the neutrino longitudinal momentum. This process artificially inserts a peak in the background at \( M_{bW} = m_t \), due to choosing the result closest to \( m_t \) without regard to its correctness.

**Background Calculation**

The major source of background to flavor changing single top quark production is \( p\bar{p} \rightarrow W + 2 \text{ jets} \). In standard model single top quark production, this background is hugely suppressed (by a factor of \( \approx 10000 \)) by requiring a double b tag on the signal. Because of the flavor changing aspect of our signal, there is only a single b quark present, and we may only enforce a single b tag on our signal. This leads to a much smaller suppression of \( W + 2 \text{ jets} \) (a factor of \( \approx 50 \)). Standard model single top quark production and \( Wb\bar{b} \) production are also important background processes.

We used the VECBOS Monte Carlo[20] to calculate the cross section to \( W + 2 \text{ jets} \)
and $W + bb$ background. VECBOS was not able to choose which of the two jets should be tagged as the $b$ quark, or to form the $M_{b,W}$ distribution variable (which we found to be critical to our analysis), so we modified the program to make the same cuts as used in the signal calculation, and also to produce distributions of the variables mentioned earlier. Our own Monte Carlo was used to calculate the standard model single top quark production.

To isolate the signal from the background, we examined the the kinematic distributions in $\sqrt{s}, M_{b,W}, p_T, \eta, \text{and} \Delta R$. We found four of the variables, $M_{b,W}, p_T, \Delta R_{jj}, \text{and} \Delta R_{jl}$ to be especially useful in determining additional cuts which optimize the discovery limit in $\kappa/\Lambda$. These distributions are shown in figures 3.2-3.5.

We optimized the cuts for each of four cases. Runs 1, 2, and 3 at the Tevatron, and the LHC. Run 1 at the Tevatron is a $pp$ collider with center of mass energy $E_{cm} = 1.8$ TeV and an integrated luminosity of 100 pb$^{-1}$ per detector. Run 2 will have $E_{cm} = 2$ TeV and 2 fb$^{-1}$ of data. Run 3 might have $E_{cm} = 2$ TeV and 30 fb$^{-1}$ of data. The LHC will be a $pp$ collider at $E_{cm} = 14$ TeV, and will have about 10 fb$^{-1}$ of data. The optimized cuts that we found are shown in Table 3.1.

To further reduce the background, we assumed that silicon vertex tagging of the $b$ quark would be available, with an efficiency of 36% at Run 1 of the Tevatron, and 60% at Runs 2 and 3 and at the LHC. We also assumed that 1% of non-$b$ quarks would be mistagged as $b$ quarks in all of the experiments. The actual mistagging rate may be significantly less than this, but is difficult to calculate. It may be measured in the usual way, by examining the difference between tagged and untagged cross sections of some
Figure 3.2 \( M_{bW} \) distributions after optimized cuts at Run 1 of the Tevatron. b-tagging has been included. The solid line represents the sum of all of the background processes, the long dashed line is the sum of the up quark processes, and the short dashed line is the sum of the charm quark processes.
Figure 3.3 $p_T$ distributions after optimized cuts at Run 1 of the Tevatron. b-tagging has been included. The solid line represents the sum of all of the background processes, the long dashed line is the sum of the up quark processes, and the short dashed line is the sum of the charm quark processes.
Figure 3.4  $\Delta R_{jj}$ distributions after optimized cuts at Run 1 of the Tevatron. b-tagging has been included. The solid line represents the sum of all of the background processes, the long dashed line is the sum of the up quark processes, and the short dashed line is the sum of the charm quark processes.
Figure 3.5 $\Delta R_{ji}$ distributions after optimized cuts at Run 1 of the Tevatron. b-tagging has been included. The solid line represents the sum of all of the background processes, the long dashed line is the sum of the up quark processes, and the short dashed line is the sum of the charm quark processes.
Table 3.1  Optimized cuts for the discovery of $s$ with single top quark production. For the LHC there are different cuts depending on whether we are optimizing for the charm quark or the up quark.

<table>
<thead>
<tr>
<th></th>
<th>$M_{bw,min}$</th>
<th>$M_{bw,max}$</th>
<th>$pT_{b,min}$</th>
<th>$\Delta R_{jj,min}$</th>
<th>$\Delta R_{ji,min}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tevatron, Run 1</td>
<td>150 GeV</td>
<td>200 GeV</td>
<td>35 GeV</td>
<td>1.0</td>
<td>0.4</td>
</tr>
<tr>
<td>Tevatron, Run 2</td>
<td>150 GeV</td>
<td>200 GeV</td>
<td>35 GeV</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Tevatron, Run 3</td>
<td>150 GeV</td>
<td>200 GeV</td>
<td>35 GeV</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>LHC (charm)</td>
<td>145 GeV</td>
<td>205 GeV</td>
<td>35 GeV</td>
<td>1.5</td>
<td>1.0</td>
</tr>
<tr>
<td>LHC (up)</td>
<td>150 GeV</td>
<td>200 GeV</td>
<td>30 GeV</td>
<td>1.5</td>
<td>0.4</td>
</tr>
</tbody>
</table>

well known, but unrelated signal, and applied to other calculations.

Results and Discussion

We may use the results of the signal and background calculation to determine the minimum values of $\kappa_c/\Lambda$ and $\kappa_u/\Lambda$ that can be observed at 95% confidence level at the different colliders. Since the signal is quadratic in $\kappa/\Lambda$, and since we have calculated the signal for $\kappa/\Lambda = 0.2 \text{TeV}^{-1}$, the minimum value of $\kappa/\Lambda$ is given by:

$$\frac{\kappa}{\Lambda} = 0.2 \text{ TeV}^{-1} \sqrt{\frac{9}{2}(1 + \sqrt{1 + \frac{4}{9} L\sigma_b})}{L\sigma_0}$$  \hspace{1cm} (3.10)

where $L$ is the integrated luminosity available, $\sigma_b$ is the total cross section for all of the background processes after b-tagging, and $\sigma_0$ is the cross section for the signal processes, evaluated at $\kappa/\Lambda = 0.2 \text{TeV}^{-1}$. These discovery limits are shown in Table 3.2.

The charm and up quarks have, so far, been treated separately, as if only one of the couplings could exist at the same time. This, of course, is not the case. If the couplings do exist together, we may simply add the cross sections of the two different couplings together, since we have treated them in exactly the same manner (except for the LHC,
Table 3.2  The discovery limits on $\kappa_c/\Lambda$ and $\kappa_u/\Lambda$ for each of the collider options discussed in the text. In both the charm and up quark cases, we assumed that the coupling of the other type did not exist.

<table>
<thead>
<tr>
<th></th>
<th>Run 1</th>
<th>Run 2</th>
<th>Run 3</th>
<th>LHC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{cm}$ (TeV)</td>
<td>1.8</td>
<td>2.0</td>
<td>2.0</td>
<td>14.0</td>
</tr>
<tr>
<td>$\mathcal{L}$ (fb$^{-1}$)</td>
<td>.1</td>
<td>2</td>
<td>30</td>
<td>10</td>
</tr>
<tr>
<td>$\kappa_c/\Lambda$ (TeV$^{-1}$)</td>
<td>.31</td>
<td>.092</td>
<td>.046</td>
<td>.013</td>
</tr>
<tr>
<td>$\kappa_u/\Lambda$ (TeV$^{-1}$)</td>
<td>.082</td>
<td>.026</td>
<td>.013</td>
<td>.0061</td>
</tr>
</tbody>
</table>

where we must calculate the charm and up quark contributions using the same cuts).

A plot of their discovery limits, when considered together, is shown in Fig. 3.6. Indeed, barring charm quark tagging, there is no way that we could separated the two processes. Even with charm quark tagging, we could only tag the processes with the charm quark in the final state ($q\bar{q} \rightarrow t\bar{c}$ and $gg \rightarrow t\bar{c}$), which are intrinsically smaller than the other available processes. Given the low efficiency of charm quark tagging, the advantage of this method is dubious at best.

In a previous paper[17], we found the discovery limits for direct top quark production (a single top quark, without any other associated jet). The limits we found were more strict than those we find here, and thus the discovery of our anomalous coupling would be easiest with direct top quark production. However, the processes presented here have an important competing process in the standard model. Much more time will be devoted to discovering and studying standard model single top quark production than direct top quark production. Thus, single top quark production is more likely to actually be the discovery channel for our anomalous couplings, should they exist at a strength
Figure 3.6  Discovery limits for $\kappa_c/\Lambda$ vs. $\kappa_u/\Lambda$ for each of the colliders considered. The solid, short dashed, and long dashed lines are at Runs 1, 2, and 3 at the Tevatron respectively. The dash-dotted line is at the LHC.
which can be detected. The number of b-tags is an important difference between the standard model and anomalous single top quark processes, though. An effort to study single top quark production with only one b-tag would be required.

In ref[18] the authors studied single top quark production through only one channel, $q\bar{q} \rightarrow t\bar{c}$. We have found this to be the least important of all of the channels. While it would seem that the presence of initial state valence quarks ought to make this the dominant process, the massless t-channel exchange of a gluon in the $cg \rightarrow tg$ process more than makes up for the lack of initial state valence quarks, and it becomes the most important process. Each of the other processes, $cq \rightarrow tq$ and $gg \rightarrow t\bar{c}$, also have massless, or nearly massless, t-channel exchanges increasing their parton cross sections. We also note that the authors of ref[18] used a cut on the center of mass energy, $\sqrt{s} > 300$ GeV, in order to reduce the background relative to the signal. Because of the dominance of the massless t-channel exchanges, the center of mass energy is now peaked at lower values, and this is no longer a useful cut.

Acknowledgments

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4 CONCLUSIONS

We have used the production of top quarks to explore the anomalous flavor changing neutral current discussed in chapter 1. Direct top quark production yields the tightest constraint on the coupling parameter. Single top quark production is also useful because it resembles the standard model top quark processes, so that our couplings may be discovered by experimenters looking at standard model single top quark production. The coupling parameters that can be measured are shown in Table 4.1.

One of the major concerns with this coupling is how to disentangle the contributions from the up quark and the charm quark. Because single top quark production includes diagrams with a different initial state than direct top quark production, we ought to be able to measure both direct and single top quark production, which have different initial

Table 4.1 The discovery limit for $\kappa/\Lambda$ for both single top quark production and direct top quark production. Values are shown for each of the colliders considered.

<table>
<thead>
<tr>
<th></th>
<th>Direct Top</th>
<th>Single Top</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\kappa_c/\Lambda$ (TeV$^{-1}$)</td>
<td>$\kappa_u/\Lambda$ (TeV$^{-1}$)</td>
</tr>
<tr>
<td>Tevatron, Run 1</td>
<td>.22</td>
<td>.058</td>
</tr>
<tr>
<td>Tevatron, Run 2</td>
<td>.062</td>
<td>.019</td>
</tr>
<tr>
<td>Tevatron, Run 3</td>
<td>.030</td>
<td>.0094</td>
</tr>
<tr>
<td>LHC</td>
<td>.0084</td>
<td>.0033</td>
</tr>
</tbody>
</table>
states, and use the difference in these cross sections to separate the contributions of the different quarks. This turns out not to be possible, because the primary contribution to the single top quark cross section comes from $c(u)g \rightarrow tg$, which has the exact same initial state as direct top quark production.

It will be possible to disentangle the contributions if the couplings turn out to be large enough to measure their strength from top quark decays, $t \rightarrow c(u)g[1]$. In both single and direct top quark production, the up quark coupling produces about ten times as many events as the charm quark coupling, due to the valence up quark in the initial state. However, in top quark decays, the charm and up quarks are on an equal footing, and the top quark will decay into a charm or an up quark based solely upon their respective coupling strengths. Comparing the coupling strengths from these two different processes against what we expect from Monte Carlo simulations will allows us to solve for both $\kappa_c/\Lambda$ and $\kappa_u/\Lambda$ simultaneously.

It was briefly mentioned in the direct top quark chapter that the couplings we reported were conservative estimates of the real couplings we expect to find. This is because single top quark production can look exactly like direct top quark production when the associated jet gets lost in the detector. This happens when the jet is collinear to the beam, and goes down the beam pipe, or when the jet is soft, and doesn’t have enough energy to be reconstructed. Because both single top quark production and direct top quark production are functions of the coupling strength, this process is part of the signal, not part of the background. However, the calculation of this contribution is nontrivial. An examination of the single top quark diagrams shows that several of
them contain a t channel gluon exchange, which is known to be divergent when the jet is collinear. There are also several soft infrared singularities.

In the single top quark calculation, we regulated these singularities by requiring a $p_T$ cut on all of the jets. This kept us well away from the singular region of the calculation. If we wish to calculate the single top quark contribution to the direct top quark cross section, we must enforce the opposite condition, the $p_T$ of the associated jet must be less than some cut. Therefore, we must deal with the various singularities in the calculation. This can only be done by calculating the next to leading order diagrams to direct top quark production, and using the result to cancel the infrared singularities in the single top quark calculation. Because the phenomenological coupling is not renormalizable, this procedure leaves us with ultraviolet divergences, which must then be dealt with. Generally this is done by cutting off the integral at the mass scale $\Lambda$, where new physics alters the Lagrangian, making our coupling no longer valid.

A final concern is about real models of this coupling. In the standard model, this coupling is very small, of order $\kappa_c/\Lambda \sim 10^{-5}$ TeV$^{-1}$.[2]. Two Higgs doublet models increase this, depending upon the parameters of the model to $\kappa_c/\Lambda \lesssim 10^{-3}$ TeV$^{-1}$.[2]. Supersymmetric models can enhance this coupling still further to $\kappa_c/\Lambda \lesssim 10^{-2}$ TeV$^{-1}$ for the most favorable combination of the parameters[3, 4]. Couplings of this size can be probed at the LHC. These coupling strengths all come from calculations of rare top quark decays, where the $q^2$ of the gluon is identically zero. However, in a real model, the derived coupling $\kappa/\Lambda$ would be a function of $q^2$. Therefore, production mechanisms, where $q^2 \neq 0$, might have an intrinsically larger coupling than decay mechanisms, where
\( q^2 = 0 \). In our calculation, we have completely ignored any substructure that the coupling might have and replaced it with a constant. The coupling strength is the same for production and decay mechanisms. We see that, though the decay rates do not bode well for measurement of \( \kappa/\Lambda \), this does not mean that a large coupling cannot be created by real models.

In summary, it is possible to detect anomalous flavor changing chromomagnetic moments of the top quark using direct top quark production if the charm quark coupling strength is larger than \( \kappa_c/\Lambda > 0.062 \text{ TeV}^{-1} \) at Run 2 of the Tevatron, or larger than \( \kappa_c/\Lambda > 0.0084 \text{ TeV}^{-1} \) at the LHC, or if the up quark coupling is larger than \( \kappa_u/\Lambda > 0.019 \text{ TeV}^{-1} \) at the Tevatron or larger than \( \kappa_u/\Lambda > 0.0033 \text{ TeV}^{-1} \) at the LHC. Single top quark production cannot measure couplings as small, but may still be of use since it has a related process within the standard model. Even if the couplings are not measured, placing an upper limit upon the coupling strength will rule out a region of phase space for such models as supersymmetry.

References


APPENDIX A QUANTUM FIELD THEORY

Given a specific model, how do we construct our theory? The simple answer is that we apply quantum field theory to a Lagrangian. A Lagrangian is a function of position and velocity that contains the physics we are trying to study. The standard model Lagrangian contains almost all of the physics we know of today. The one exception, gravity, is not a part of the standard model because we have not yet discovered how to quantize it. I will say more about the standard model later. Other Lagrangians also exist. Some contain a subset of the standard model (e.g., Quantum Electrodynamics which is the theory of electricity and magnetism), some contain new physics that we wish to examine (e.g., Supersymmetry which contains a symmetry that may or may not exist), or even physics that we know does not exist (e.g., $\sigma^4$ theory which is a simple theory we use for teaching purposes, among other things).

Quantum field theory is fully relativistic quantum mechanics, and prescribes how to proceed from the Lagrangian to observable quantities such as the decay rate of a particle or its scattering cross section.
Classical Lagrangians

In classical (non-quantum) theory, the Lagrangian is a single function describing the motion of a particle through space and time. The Lagrangian is written as the kinetic energy minus the potential energy, \( L = T - V \). The classical action is then written as \( S = \int_{t_i}^{t_f} L \, dt \). Acting according to the principle of least action, we minimize \( S \) and the Euler-Lagrange equations result.

\[
\frac{\partial L}{\partial q_i} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} = 0
\]  

(A.1)

where \( q_i(t) \) is the generalized position of the particle, and \( \dot{q}_i(t) = \frac{d}{dt} q_i(t) \) is its generalized velocity. The Euler-Lagrange equations are the equations of motion of the system described by the Lagrangian.

Classical field theories, such as Classical Electrodynamics, can also be written in terms of the Lagrangian formalism. In this case the fields are functions of position, as well as time, so we define the Lagrangian density, \( \mathcal{L} \), as \( S = \int V \, \mathcal{L}(\phi, \phi) \, d^4x \). Here, \( \phi \) is the \( i \)th field and \( \phi = \frac{\partial}{\partial x^\mu} \phi \) is its derivative. The action has dimension \( \frac{M L^1}{T} \) where \( M \) is a mass dimension (e.g. kilograms), \( L \) is a length dimension (e.g. meters) and \( T \) is a time dimension (e.g. seconds). It is common in particle physics to set \( c = \hbar = 1 \), relating the all dimensions to an energy dimension. Using these relations, \( M \rightarrow \text{MeV} \), \( T \rightarrow \text{MeV}^{-1} \), and \( L \rightarrow \text{MeV}^{-1} \). Thus, the action turns out to be dimensionless when we set these constants to one. Examining the integrals defining the Lagrangian, we see that the Lagrangian is dimension one. That is it contains one power of MeV. The Lagrangian density is dimension four.
Minimizing the action again leads to the Euler-Lagrange equations:

\[ \frac{\partial L}{\partial \phi_i} - \frac{\partial}{\partial x^\mu} \frac{\partial L}{\partial \phi_{i,\mu}} = 0 \]  \hspace{1cm} (A.2)

For field theory, the Euler-Lagrange equations are the field equations of the theory. I will often omit the word density and refer to the Lagrangian density as the Lagrangian.

**Quantization**

Quantization proceeds most simply in the Hamiltonian formalism. To connect Hamiltonians with Lagrangians, define the momentum conjugate to \( \phi_i \):

\[ \pi_i(x) = \frac{\partial L(\phi_i, \phi_{i,\mu})}{\partial \phi_i} \]  \hspace{1cm} (A.3)

We can then define the Hamiltonian density:

\[ \mathcal{H}(\pi_i, \phi_i) = \sum_i \pi_i \dot{\phi}_i - L(\phi_i, \phi_{i,\mu}) \]  \hspace{1cm} (A.4)

Once we have written the Hamiltonian in terms of the conjugate momenta and position, \( \pi_i(x, t) \) and \( \phi_i(x, t) \) respectively, we can easily quantize the theory by assuming the momenta and position are Heisenberg operators and writing the commutation relation for them:

\[ [\phi_i(x, t), \pi_j(x', t)] = i\hbar \delta_{ij} \delta(x - x') \]  \hspace{1cm} (A.5)

\[ [\phi_i(x, t), \phi_j(x', t)] = [\pi_i(x, t), \pi_j(x', t)] = 0 \]  \hspace{1cm} (A.6)

These relations are valid for integer spin bosons. To represent the half-integer spin fermions, we simply replace the commutation relations in the equations above with equivalent anti-commutation relations.
A fully quantized theory must include the concept of particle creation and destruction. To include these in our Hamiltonian, we write the field in terms of an expansion of the free fields:

$$\phi_i(x, 0) = \frac{1}{(2\pi)^3} \int d^3k \ a_i(k) \alpha_i(k)e^{ik\cdot x} + \beta_i(k)a_i^\dagger(k)e^{-ik\cdot x}$$  \hspace{1cm} (A.7)

where $a_i^\dagger(k)$ creates a particle of type $i$ and momentum $k$, $a_i(k)$ destroys a particle of type $i$ and momentum $k$, and $\alpha_i$ and $\beta_i$ are normalization constants.

**The Klein Gordon Field**

As an example of quantization, consider the Klein Gordon field, which represents a spin-zero neutral boson. Just as the Schrödinger equation in non-relativistic quantum mechanics was derived from the form of the non-relativistic energy, $E = \frac{p^2}{2m} + V(x)$, by replacing the energy and momenta with the operators, $p \rightarrow i\hbar \nabla$ and $E \rightarrow i\hbar \frac{\partial}{\partial t}$, we may derive the Klein-Gordon field by using the same replacements in the fully relativistic equation, $E^2 - p^2 - m^2 = 0$, leading to the Klein-Gordon equation:

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2 - m^2\right)\phi(x) = 0$$  \hspace{1cm} (A.8)

where for convenience, we have adjusted our units by setting $\hbar = c = 1$.

The Lagrangian for the Klein Gordon field is:

$$\mathcal{L} = \frac{1}{2}(\partial_{\mu}\phi \partial^{\mu}\phi - m^2\phi^2)$$  \hspace{1cm} (A.9)

Examining the Euler-Lagrange equations for the Klein-Gordon Lagrangian, we return to the Klein-Gordon equation:

$$\left(\partial_{\mu} \partial^{\mu} - m^2\right)\phi = 0$$  \hspace{1cm} (A.10)
where we have written the derivatives in fully covariant notation to show that this equation is indeed relativistically invariant. The conjugate momentum of the Klein Gordon field is:

$$\pi = \frac{\partial L}{\partial \phi} = \phi$$  \hspace{1cm} (A.11)$$

So then, we can write the Hamiltonian for the Klein-Gordon Field:

$$\mathcal{H} = \phi^2 + (\nabla \phi)^2 + m^2 \phi^2$$  \hspace{1cm} (A.12)$$

Note that the Hamiltonian is not a relativistic invariant. Upon second quantization, the field is written as:

$$\phi(x) = \int \frac{d^3 k}{\sqrt{2 \omega_k}} \left( a(k)e^{ikx} + a^\dagger(k)e^{-ikx} \right)$$  \hspace{1cm} (A.13)$$

where \( \omega_k = \sqrt{k^2 + m^2} \) is the energy of the particle. Here, \( a(k) \) destroys a particle of momentum \( k \) and \( a^\dagger(k) \) creates a particle of momentum \( k \).

From the definition of \( \phi \), we can find the second quantized version of the conjugate momentum:

$$\pi(x) = \dot{\phi}(x) = i \int d^3 k \sqrt{\frac{2}{\omega_k}} \left( a(k)e^{ikx} - a^\dagger(k)e^{-ikx} \right)$$  \hspace{1cm} (A.14)$$

From the commutation relations for \( \phi \) and \( \pi \), we can derive commutation relations for \( a \) and \( a^\dagger \).

$$[a(k), a(k')] = 0, \quad [a^\dagger(k), a^\dagger(k')] = 0, \quad [a^\dagger(k), a(k')] = \delta(k - k')$$  \hspace{1cm} (A.15)$$

From here, we can define the ground state, \( |0\rangle \), as the state for which \( a(k)|0\rangle = 0 \) for all \( k \). We can build up a set of orthonormal basis states, representing every possible combination of momenta. This is done by applying the creation operator for a particle
of momentum \( k \), \( a^\dagger(k) \), to the ground state:

\[
|k\rangle = a^\dagger(k)|0\rangle \quad (A.16)
\]

States with more than one particle may be defined by successively applying the creation operator.

The quantization of the fields allow us to define constants of the motion, such as the Energy, \( H \), and the momentum \( \mathbf{P} \) in terms of the number operator \( N = a^\dagger a \).

\[
H = \int d^3x \left[ \frac{1}{2} \dot{\phi}^2 + (\nabla \phi)^2 + m^2 \phi^2 \right] \quad (A.17)
\]

\[
= \frac{1}{(2\pi)^3} \int d^3k \ \omega_k [a^\dagger(k)a(k) + \frac{1}{2}] \quad (A.18)
\]

\[
\mathbf{P} = -\int d^3x \ \dot{\phi} \nabla \phi \quad (A.19)
\]

\[
= \frac{1}{(2\pi)^3} \int d^3k \ k[a^\dagger(k)a(k) + \frac{1}{2}] \quad (A.20)
\]

The second term in both of these constants is infinite, prompting us to remember that a constant may be freely subtracted from the energy and momenta. Since the offending term, the \( \frac{1}{2} \), in the equations above resulted from commuting the operators to arrive in the correct form, we define the normal order of the operators by writing them with all of the creation operators to the left of the destruction operators, and assuming that the commutators all vanish.

\[
N(aa^\dagger a) = aa^\dagger a := a^\dagger aa \quad (A.21)
\]

We then redefine the Hamiltonian and others as the normal ordering of its operators:

\[
H = \int d^3x \left[ \frac{1}{2} \dot{\phi}^2 + (\nabla \phi)^2 + m^2 \phi^2 \right] : \quad (A.22)
\]

\[
= \frac{1}{(2\pi)^3} \int d^3k \ \omega_k [a^\dagger(k)a(k)] \quad (A.23)
\]
Normal ordering will become important later as we derive observable quantities from the Hamiltonian.

The Dirac Equation

In order to describe fermions, we use the Dirac Equation. This equation was developed in an attempt to write down a relativistic first order equation. \( \gamma^0 E - \gamma \cdot p = m \). Squaring both sides of this equation should recover the relativistic energy, \( E^2 - p^2 = m^2 \).

It is easily seen that the \( \gamma \)'s in this equation are not simple numbers, or even complex numbers. They must therefore be operators, and it is possible that they do not commute with each other. Taking care not to commute \( \gamma \)'s, we square the Dirac equation:

\[
(\gamma^0)^2 E^2 - (\gamma^0 \gamma^i + \gamma^i \gamma^0) E p_i + \gamma^i \gamma^j p_i p_j = m^2 = E^2 - p^2
\]  

(A.26)

This equation immediately implies several relations between the \( \gamma \)'s:

\[
(\gamma^0)^2 = 1, \quad \{\gamma^0, \gamma^i\} = 0, \quad \gamma^i \gamma^j = -\delta^{ij} 1
\]  

(A.27)

Which can be summed up by assuming that the \( \gamma \)'s form a Lorentz vector, and writing:

\[
\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}
\]  

(A.28)

The simplest matrices which can represent the \( \gamma \) operators are \( 4 \times 4 \).

We can now write down the Dirac equation:

\[
(i\gamma^\mu \partial_\mu - m1)\psi(x) = 0
\]  

(A.29)
Since we now must write the fields $\psi$ as $1 \times 4$ matrices, we have a Hermitian conjugate to the Dirac equation:

$$\psi^\dagger(x)(-i(\gamma^\mu)^\dagger \overleftarrow{\partial}_\mu - m\mathbf{1}) = 0 \quad (A.30)$$

where the backwards arrow over the derivative indicates that it acts to the left rather than to the right, as is traditional. From the fact that we require quantum mechanics to be Hermitian, we can find the Hermitian conjugate of $\gamma^\mu$ to be $(\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0$. Defining $\bar{\psi} = \psi^\dagger\gamma^0$, we can write Eq.(A.30) as:

$$\bar{\psi}(x)(i(\gamma^\mu) \overleftarrow{\partial}_\mu + m\mathbf{1}) = 0 \quad (A.31)$$

Both equations may be derived from the Dirac Lagrangian:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi \quad (A.32)$$

Because the $\psi$'s contain 4 elements, they easily represent both spin-1/2 particles and their anti-particles at the same time.

Second Quantization proceeds by writing the fields in terms of creation and annihilation operators:

$$\psi(x) = \sum_r \int d^3k \left( c_r(k)u_r(k)e^{-ik\cdot x} + d_r^\dagger(k)v_r(k)e^{ik\cdot x} \right) \quad (A.33)$$

In this equation, $c_r(k)$ destroys a particle of momentum $k$ and spin $r$, $d_r^\dagger(k)$ creates an anti-particle momentum $k$ and spin $r$, and $u_r(k)$ and $v_r(k)$ are the spinors for the particle and anti-particle respectively. From this equation, we can derive $\bar{\psi}$:

$$\bar{\psi}(x) = \sum_r \int d^3k \left( c_r^\dagger(k)\bar{u}_r(k)e^{ik\cdot x} + d_r(k)\bar{v}_r(k)e^{-ik\cdot x} \right) \quad (A.34)$$
Here, \( c^\dagger \) creates a particle, and \( d \) destroys an anti-particle.

As before, we can write relations between the creation and annihilation operators. For the case of fermions, however, we assume that they anti-commute rather than commute:

\[
\{ c^\dagger(k), c(k') \} = \{ d^\dagger(k), d(k') \} = \delta(k-k')
\]  \hspace{1cm} (A.35)

All other combinations of the creation and destruction operators anti-commute.

Again, we define the ground state, \( |0\rangle \), such that \( c_r(k)|0\rangle = d_r(k)|0\rangle = 0 \) for all \( k \) and \( r \). Again, we can build particle states by applying the particle creation operator to the ground state, \( |k, r\rangle = c^\dagger_r(k)|0\rangle \). Similarly, we build anti-particle states by applying the anti-particle creation operator, \( d^\dagger_r(k) \). Since the combination \( c^\dagger_r(k)c^\dagger_r(k) = 0 \), no two fermions with the same quantum numbers can exist, which leads immediately to Fermi-Dirac statistics. Practically, the fermions are somewhat localized, so what we really say is that no two fermions with the same quantum numbers can exist in the same region of space-time. Note that this arises from the anti-commuting property of the fermion operators. No such thing can be said about the bosons, which have commuting operators.

As in the case of the Klein Gordon equations, we define the normal order of the creation and destruction operators, but with a slight difference. Instead of using the commutation relations to put the operators in the proper order, for fermions, we must use the anti-commutation relations, which we then assume to vanish. This means that when we interchange two fermions, a negative sign appears.

\[
N(cc^\dagger c) =: c^\dagger c := -c^\dagger cc
\]  \hspace{1cm} (A.36)
Interactions

So far, we have written down Lagrangians which describe the propagation of particles through free space. However, we know that particles do not exist “in a vacuum”, they interact with the other particles around them. How then do we include these factors in the Lagrangians? Notice that the free Lagrangians only include two fields in each term. Since each field either creates or destroys a particle, we can view these two field Lagrangians as creating a particle at point A and destroying it at point B. What comes between these two points is the free field propagation of the particle. Suppose, however, that some term in the Lagrangian includes a third field. Then the Lagrangian may, for instance, create a particle at point A and destroy particles at points B and C. Where did the additional particle come from? There must have been an interaction somewhere between these three points which created the additional particle.

One of the most powerful tools we have for introducing interactions into our Lagrangians is the gauge principle. For example, it is noticed that the Dirac Lagrangian is invariant under a global phase transformation, $\psi \rightarrow \psi' = e^{i\sigma} \psi$. (where $\bar{\psi} \rightarrow \bar{\psi}' = \bar{\psi} e^{-i\alpha}$. the two exponents cancel out, leaving the Lagrangian exactly the same.) What if, instead of a global $U(1)$ transformation, we made a local phase transformation, where the exponent, $\alpha$, depends upon the $x$ variable? Then, instead of recovering the original Lagrangian, we are left with a total derivative as an additional piece.

$$\mathcal{L} \rightarrow \mathcal{L}' = \bar{\psi} e^{-i\alpha(x)} (\gamma^\mu \partial_\mu - m) e^{i\alpha(x)} \psi \quad (A.37)$$

$$= \bar{\psi} (\gamma^\mu \partial_\mu - m + i[\gamma^\mu \partial_\mu \alpha(x)]) \psi \quad (A.38)$$
To return the Lagrangian to an invariant form, we recall that a gauge field is invariant when you add a total derivative. Therefore, we add a gauge field to the Lagrangian.

\[ \mathcal{L} = \bar{\psi}(\gamma^\mu(\partial_\mu + ieA_\mu) - m)\psi \]

When we perform the local phase transformation, the total derivative piece that is left over is absorbed into the gauge field, thus leaving the Lagrangian invariant.

In addition to the phase invariance described above, called the \( U(1) \) phase invariance, several other invariances, corresponding to new symmetries, can be studied. In particular, we have discovered an \( SU(2) \) symmetry describing the weak force of nuclear physics, and an \( SU(3) \) symmetry describing the strong nuclear interactions. Gauge invariance is an extremely powerful tool which has been essential in building our standard model of particle physics.

**The S Matrix Expansion**

To describe the interaction between the particles, we write the Schroedinger equation in the interaction picture:

\[ i\frac{d}{dt}|\Phi(t)\rangle = H_I(t)|\Phi(t)\rangle \]

where \( H = H_0 + H_I \). \( H_0 \) contains the free fields in the Hamiltonian, and \( H_I \) contains the interaction pieces. The time dependence of \( H_I \) is given by:

\[ H_I(t) = e^{iH_0(t-t_0)}H_I(t_0)e^{-iH_0(t-t_0)} \]

We now define the \( S \) matrix as connecting the states at \( t = -\infty \) and \( t = \infty \).

\[ |\Phi(\infty)\rangle = S|\Phi(-\infty)\rangle \]
The probability that a system is in the final state $|f\rangle$ after an interaction is given by:

$$|\langle f|\Phi(\infty)\rangle|^2 = |\langle f|S|i\rangle|^2 \equiv |S_{fi}|^2$$  \hspace{1cm} (A.43)

where we have defined $|i\rangle = |\Phi(-\infty)\rangle$. We can expand $|\Phi(\infty)\rangle$ about a complete set of states $|f\rangle$:

$$|\Phi(\infty)\rangle = \sum_f |f\rangle\langle f|\Phi(\infty)\rangle = \sum_f |f\rangle S_{fi}$$  \hspace{1cm} (A.44)

We must now solve the Schroedinger equation to find the final state $|\Phi(\infty)\rangle$. Turning Eq(A.40) into an integral equation, with initial state $|i\rangle$ we find:

$$|\Phi(t)\rangle = |i\rangle - i \int_{-\infty}^{t} dt_1 H_f(t_1)|\Phi(t_1)\rangle$$  \hspace{1cm} (A.45)

This equation can be solved by iteration:

$$|\Phi(t)\rangle = |i\rangle - i \int_{-\infty}^{t} dt_1 H_f(t_1)|i\rangle + (-i)^2 \int_{-\infty}^{t} dt_1 \int_{-\infty}^{t_1} dt_2 H_f(t_1)H_f(t_2)|\Phi(t_2)\rangle$$  \hspace{1cm} (A.46)

and so on, until we must evaluate an infinite number of integrations. However, if each successive integral is smaller than the one which precedes it, we can cut this series off before it becomes too hard to calculate. This is known as the perturbative expansion.

At time $t = \infty$ we then write the $S$ matrix as:

$$S = \sum_{n=0}^{\infty} (-i)^n \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{t_1} dt_2 \cdots \int_{-\infty}^{t_{n-1}} dt_n H_f(t_1)H_f(t_2)\cdots H_f(t_n)$$  \hspace{1cm} (A.47)

$$= \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \cdots \int_{-\infty}^{\infty} dt_n T\{H_f(t_1)H_f(t_2)\cdots H_f(t_n)\}$$  \hspace{1cm} (A.48)

where we have defined the time ordered product $T\{}$ such that the Hamiltonians are ordered so that the ones occurring at earlier times are to the right of those occurring at later times. We can generalize this expansion to use the Hamiltonian densities:

$$S = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int dx_1 \int dx_2 \cdots \int dx_n T\{H_f(x_1, t_1)H_f(x_1, t_2)\cdots H_f(x_1, t_n)\}$$  \hspace{1cm} (A.49)
Now that we have an expansion for the $S$ matrix in terms of the Hamiltonian density, we need to find the element $S_{fi}$ for a given initial and final state. Only certain terms in the expansion will contribute to $S_{fi}$ since it must contain creation operators to create the particles in $|f\rangle$ and destruction operators to destroy the particles in $|i\rangle$. It may also contain additional creation operators as long as there is a destruction operator to destroy the created particle. In other words, an intermediate, or virtual, particle may be created, as long as it is destroyed before we examine the final state. This process is much simpler if we don't have to write the intermediate states explicitly. This can be accomplished by noticing that, for $t_0 \neq t_1$:

$$T\{A(x_0)B(x_1)\} =: A(x_0)B(x_1) : + \langle 0|T\{A(x_0)B(x_1)\}|0\rangle$$

(A.50)

The second term $\langle 0|T\{A(x_0)B(x_1)\}|0\rangle$ is called the contraction of $A$ and $B$, and is zero unless one of the operators creates a particle which is destroyed by the other. G.C. Wick proved the generalization of this for an arbitrary number of operators. I will not write it down here, but essentially it boils down to taking the contraction of every possible set of operators which are not at equal time, followed by every possible set of two contractions, etc.

Now, the string of Hamiltonian densities in the expansion of the $S$ matrix contains a series of normal ordered operators:

$$T\{\mathcal{H}_i(x_1) \cdots \mathcal{H}_i(x_n)\} = T\{ :AB \cdots :z_1 \cdots :AB \cdots :z_n : \}$$

(A.51)

Wick's theorem can be used with this ordering, as long as operators which occur at the same time are not contracted.
The Cross Section

Now that we have a procedure to find $S_{fi}$, we need to find the observables for the theory. It is convenient to define the matrix element $M$ as:

$$S_{fi} = \delta_{fi} + (2\pi)^4 \delta^{(4)}(p_f - p_i) (-i M_{fi}) \prod_{f,i} \frac{1}{\sqrt{2E_{f,i}}}$$  \hspace{1cm} (A.52)

Note that $\delta_{fi}$ is the probability amplitude of nothing happening. R.P. Feynman noticed that there is a one to one correspondence between the matrix elements and a set of diagrams which can be used to represent the process pictorially, given a set of simple rules. Without going into great detail, we represent initial and final state fermions with a $u(p)$ or $\bar{u}(p)$, anti-fermions with $\nu(p)$ or $\bar{\nu}(p)$, and bosons with their polarization vectors, $\epsilon_\mu(q)$. For unobserved, internal lines, we include the appropriate propagator, and where three or more lines meet, we include the appropriate vertex factor. The propagator is just the Green's function for the free particle's equation of motion. The vertex factor can be deduced from the interaction piece of the Hamiltonian or Lagrangian, by taking the explicit derivatives in the Hamiltonian, figuring all of the possible permutations of identical field and stripping off the fields. For example, when the interaction Hamiltonian is written as:

$$\mathcal{H}_I = \bar{\psi}_e (-ieA^\mu \gamma_\mu) \psi_e$$  \hspace{1cm} (A.53)

the Feynman rule for the $e - e - \gamma$ vertex is:
This realization greatly simplifies the process of writing down the matrix elements for a given theory.

We can then write the probability of a transition from state $i$ to state $f$ in a volume of space $V$ as:

$$d\Gamma = V(2\pi)^{4}\delta^{(4)}(p_{f} - p_{i})|\mathcal{M}_{fi}|^{2} \prod_{f,i} \frac{1}{2E_{f}V} \prod_{f} \frac{V d^{3}p_{f}}{(2\pi)^{3}}$$

(A.55)

where $|\mathcal{M}_{fi}|^{2}$ is the sum (for final states) or average (for initial states) of $|\mathcal{M}_{fi}|^{2}$ for states which are not observed. For example, each spin state of the electron is a separate state, but if we are colliding unpolarized electron beams, then we must average over the unobserved spins of the incoming electrons.

The cross section, $d\sigma$, for two particles, A and B, colliding in the rest frame of particle B, may be defined as:

$$d\sigma = \frac{d\Gamma}{flux}$$

(A.56)

where the flux, written in the rest frame of particle B is written as $v_{A}/V$, where $v_{A} = \sqrt{(p_{A} \cdot p_{B})^{2} - m_{A}^{2}m_{B}^{2}}/2E_{A}m_{B}$ is the velocity of particle A. We can then write the two particle cross section:

$$d\sigma = \frac{(2\pi)^{4}\delta^{(4)}(p_{f} - p_{A} - p_{B})}{\sqrt{(p_{A} \cdot p_{B})^{2} - m_{A}^{2}m_{B}^{2}}} |\mathcal{M}_{fi}|^{2} \prod_{f} \frac{d^{3}p_{f}}{(2\pi)^{3}2E_{f}}$$

(A.57)
Note that all of the factors of $V$ have disappeared. Also, this equation is explicitly Lorentz covariant, allowing us to boost it into any frame we desire. In particular, the form of the equation is exactly the same in the center of momentum frame of the $A$ and $B$ particles.

The decay rate and the cross section will be our principle tools in comparing the results of the theory with experiment.

The Standard Model

No thesis in particle physics would be complete without a description of the standard model. The standard model is our current "best" description of the physical laws of the universe. One of the major components of the standard model are its three forces, the electromagnetic force, the weak nuclear force, and the strong nuclear force. The fourth known force, gravity, is not included in the standard model because, so far, it is not known how to write gravitation as a quantum theory. It is also the weakest of the forces, and is only a very minor perturbation to the other three forces. That is not to say that it is not important. The gravitational force will overwhelm all of the other forces at the Planck scale, and the laws of physics, as we know them, will break down at this scale.

In addition to the forces, the standard model contains six quarks and six leptons. The quarks are, in order from lightest to heaviest: the up quark ($u$), the down quark ($d$), the strange quark ($s$), the charm quark ($c$), the bottom quark ($b$), and the top quark ($t$). From these quarks, the familiar protons and neutrons are built, along with such exotic
particles as \( \Lambda \) particles and kaons. The leptons are: the electron (\( e \)) and its neutrino (\( \nu_e \)), the muon (\( \mu \)) and its neutrino (\( \nu_\mu \)), and the tau (\( \tau \)) and its neutrino (\( \nu_\tau \)). The neutrinos are thought to be massless.

There is a final particle, the Higgs boson which has not yet been discovered. The Higgs arises from the procedure used to insert mass into the standard model. The other particles were mostly inserted as they were discovered, although the existence of the charm and top quarks were inferred from certain symmetries existing in the standard model before they were discovered.

The electromagnetic force is described by a theory known as quantum electrodynamics or QED. QED is by far the most successful theory known to man. It is based on a \( U(1) \) phase invariance in the Dirac Lagrangian. The interaction piece of the Lagrangian is:

\[
\mathcal{L}_I = \sum_f e Q_f \bar{\psi}_f \gamma_\mu A^\mu \psi_f
\]

where \( f \) represents every fermion, both quarks and leptons mentioned above, and \( Q_f \) is the charge quantum number of the fermions. QED is especially useful as it is highly perturbative and it only contains a single gauge boson. For these and other reasons, many people like to use it as a simple theory with which they study the behavior of field theories in general.

The strong nuclear force is described by quantum chromodynamics, or QCD. QCD is motivated by observing that quarks may come in three "colors". If we write the quarks
as a three element matrix.

\[ \psi_q = \begin{pmatrix} \psi_{q,r} \\ \psi_{q,b} \\ \psi_{q,g} \end{pmatrix} \]  

(A.59)

where the \( r, b, \) and \( g \) indicate the color charges red, blue, and green, then there exists a local \( SU(3) \) phase invariance:

\[ \psi_q \rightarrow \psi'_q = e^{ig_3 T^a \alpha^a} \psi_q \]  

(A.60)

where the \( T^a \) are the eight generators of the \( SU(3) \) gauge group. Again, the interaction comes about through the gauge principle, assuming that the \( \alpha^a \)'s are a function of the position, and adding a field which transforms appropriately to keep the Lagrangian invariant. Of course, we wouldn't introduce such a symmetry if it weren't motivated by experimental considerations. Perhaps the best evidence for quarks coming three colors is given by the cross section \( e^+e^- \rightarrow q\bar{q} \). It is experimentally observed to be three time larger than it would be if the quarks were colorless. The factor of three is easily explained by the fact that, with color, there are three different types of quarks that can be produced, rather than just one if there was no color charge.

This theory is extremely difficult to work with because it is not so highly perturbative as QED. Also there is an added complication that the gluon, the equivalent of the photon in QED, carries a color charge, and thus it can interact with itself. These lead to all sorts of nastiness such as confinement and asymptotic freedom. Furthermore, the nature of confinement makes it impossible for a particle with a color charge, i.e. quarks and gluons, to exist alone. They are always accompanied by other quarks and gluons so that
the total of their color charge is always colorless.

The last of the fundamental forces, the weak nuclear force is motivated by the experimental observation of parity non-conservation in weak decays. This means that the weak nuclear force is not coupling in the same manner to different spins. We therefore assume a completely left handed coupling, and arrange the fermions with left handed spins in a doublet. This implies an $SU(2)$ symmetry, which we dub isospin. Only the left handed fermions have an isospin charge, the right handed spin states are left in isospin singlets. Incidentally, since the right-handed neutrinos have zero mass, zero charge, zero isospin, and zero color, they do not interact with any of the forces in the standard model. Therefore, we simply assume that such neutrinos do not exist. Even if they did, we could not detect them in any way.

Now, we arrive at a problem. The bosons which make up the weak interaction are experimentally known to be massive, and yet gauge symmetry of the Lagrangian does not admit the possibility of their having mass. This problem is solved by the introduction of a charged scalar isospin doublet:

$$\phi = \begin{pmatrix} \phi_1 + i \phi_2 \\ \sqrt{2} \\ \phi_3 + i \phi_4 \\ \sqrt{2} \end{pmatrix}$$

(A.61)

which has a scalar potential

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

(A.62)

If we choose $\mu^2 < 0$, this potential has the property that it is not minimized at $\phi^\dagger \phi = 0$, but at $\phi^\dagger \phi = -\frac{\mu^2}{2\lambda} = \frac{v^2}{2}$. This minimum can be satisfied in a number of ways by the fields in $\phi$, so we choose a point, called the vacuum, around which to expand the potential.
This expansion is known as spontaneous symmetry breaking, since what appeared to be a symmetry of the Lagrangian is no longer symmetric after expanding about the vacuum. Examination of the Lagrangian after spontaneous symmetry breaking indicates that we are left with a massive scalar particle, known as the Higgs boson, and three massless scalar particles, known as Goldstone bosons.

A slight detour is necessary here. Recall that I defined a massless $U(1)$ symmetry above called QED. I am here going to define another $U(1)$ symmetry, hypercharge, and an $SU(2)$ symmetry for the weak force. These symmetries are represented in the covariant derivative:

$$D_\mu = \partial_\mu - ig_1 \frac{Y}{2} B_\mu - ig_2 \frac{\tau^i}{2} W^i_\mu$$

(A.63)

Where $Y$ is the hypercharge of the particle in the interaction, and $\tau^i$ are the three generators of the $SU(2)$ symmetry. In their matrix representation, they are just the three Pauli spin matrices. Combining the Pauli spin matrices and the $W$ bosons, we find that there are two charged bosons, $W^\pm$, and a neutral boson, $W^0$. The $W^\pm$ couple a state of isospin $\frac{1}{2}$ to a corresponding state of isospin $\frac{-1}{2}$. We have grouped the particles into a series of isospin doublets:

$$\begin{pmatrix} \nu_e \\ e \\ u \\ d \\ \nu_\mu \\ \mu \\ c \\ s \\ \nu_\tau \\ \tau \\ t \\ b \end{pmatrix}$$

(A.64)

Now, the interaction between these forces and the other particles can be found by replacing the ordinary derivative in the Lagrangian by the covariant derivative:

$$\mathcal{L} = \bar{\psi}_f \gamma^\mu D_\mu \psi_f + (\partial_\mu \phi)^\dagger (\partial^\mu \phi)$$

(A.65)

$$\rightarrow \mathcal{L} = \bar{\psi}_f \gamma^\mu D_\mu \psi_f + (D_\mu \phi)^\dagger (D^\mu \phi)$$

(A.66)
After performing the spontaneous symmetry breaking, we find a surprise. The $W^0$ and $B$ bosons have changed, forming two neutral bosons, which we will call the $Z^0$ and $\gamma$. The $W^\pm$ and $Z^0$ bosons have gained a mass term, while the $\gamma$ has not. Also, the Goldstone bosons from above have vanished. Thus, we say that the $W^\pm$ and $Z$ bosons have "eaten" the Goldstone bosons and become massive. The remaining massless boson, $\gamma$, is just the photon of QED. This has the effect of unifying the weak nuclear and electromagnetic forces into a single theory, the electroweak theory.

Yet another complication in electroweak theory is the fact that the weak eigenstates, the states to which the weak vector bosons couple, are not the same as the physical mass eigenstates. This results in a mixing of states, e.g. the up quark may couple to the strange quark through the weak force, even though they are not in the same doublet. There is also an imaginary phase in the mixing matrix which results in CP violation.

The Higgs mechanism can also be used to add mass to the fermions in the standard model. We merely add a term to the Lagrangian of the form:

$$L_{\text{int}} = g_u (\bar{\psi}_L \phi \psi_{Ru} + \bar{\psi}_{Ru} \phi^\dagger \psi_L) + g_d (\bar{\psi}_L \phi \psi_{Rd} + \bar{\psi}_{Rd} \phi^\dagger \psi_L)$$  \hspace{0.5cm} (A.67)

where $\psi_L$ is the left handed doublet, $\psi_{Ru}$ is the right handed singlet of the up type fermions, and $\psi_{Rd}$ is the right handed singlet of the down type fermions. After spontaneous symmetry breaking, we are left with mass terms for each of the fermions, along with couplings of the fermions to the Higgs boson. This coupling strength is proportional to the mass of the fermion.

By combining all of the above effects, we produce the standard model of high energy particle physics. The particles that make up the standard model, and their quantum
numbers are shown in table A.1.

References

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1977)

[7] R.D. Field Applications of Perturbative QCD (Addison-Wesley Publishing Com-
pany, Inc., 1989)
Table A.1 The particles of the Standard model and their quantum numbers

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<th>Particle</th>
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<th>Weak Isospin</th>
<th>Hypercharge</th>
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<tr>
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APPENDIX B  MONTE CARLO METHODS

Basic Monte Carlo Theory

A Monte Carlo technique is any technique that uses random numbers to solve a problem. The solution to a problem may be represented as a parameter of some population. We then randomly generate the members of this population, and use statistics to make an estimate of the parameter we are studying. This estimate will be a function of the random numbers used to generate it. This procedure introduces properties into the solution which are sometimes quite good.

Sometimes, the problem to be solved already involves random processes. Such is usually the case in particle physics, and the Monte Carlo used is just a straightforward simulation of the process. Other times, the problem does not involve random numbers, and we must be more careful in applying the Monte Carlo method to the problem.

All Monte Carlo results are formally equivalent to integrations. Since the Monte Carlo result depends on the random numbers \((r_i)\) chosen, \(F = F(r_1, r_2, \ldots, r_n)\), is an unbiased estimator of the integral:

\[
I = \int_0^1 \cdots \int_0^1 F(x_1, \ldots, x_n) dx_1 \cdots dx_n \tag{B.1}
\]

as long as the random numbers are evenly distributed between 0 and 1.
A random number is a variable whose value cannot be predicted in advance. Even though the value of the variable cannot be predicted, its distribution, which gives the probability of finding a given value, may be well known. We define the probability density function as

\[ g(u)du = P[u < u' < u + du]. \] (B.2)

\( g(u) \) gives the probability of finding the random number, \( u' \), between \( u \) and \( u + du \). It is normalized so that the integral of \( g(u) \) over all \( u \) is 1. We can also define the integrated distribution function:

\[ G(u) = \int^{u}_{-\infty} g(x)dx \] (B.3)

\( G(u) \) is a monotonically non-decreasing function between 0 and 1. The expectation of some function \( f(u) \) is the average of the function,

\[ E(f) = \int f(u)dG(u) = \int f(u)g(u)du. \] (B.4)

The variance of the function is the average of the squared deviation of the function from its mean,

\[ V(f) = E( (f - E(f))^2 ) = \int (f(u) - E(f))^2dG(u). \] (B.5)

The square root of the variance is called the standard deviation.

Because the integration operator is linear, the expectation operator is also linear,

\[ E(cx + y) = cE(x) + E(y). \] (B.6)

However, the variance operator, being the result of two separate integrations is nonlinear,

\[ V(cx + y) = c^2V(x) + V(y) + 2cE[(y - E(y))(x - E(x))]. \] (B.7)
The last term in this equation is called the covariance between $x$ and $y$. If $x$ and $y$ are independent, then the covariance is zero.

Suppose we choose $n$ random numbers, $u_i$, evenly distributed between $a$ and $b$, and for each random number calculate the function $f(u_i)$. For large $n$, the average of the functions $f(u_i)$ will converge to the expectation of the function $f$.

$$\frac{1}{n} \sum_{i=1}^{n} f(u_i) \to \frac{1}{b-a} \int_{a}^{b} f(u)du$$ (B.8)

as long as $f(u)$ is integrable and is finite in the range $a$ to $b$. If $n$ approaches infinity, the convergence becomes complete, and the right hand side equals the left hand side of equation B.8. This result is known as the law of large numbers.

In real calculations, we cannot take an infinite number of random numbers, so we need to know what happens for finite values of $n$. The central limit theorem says that the sum of a large number of independent random numbers will be normally (Gaussian) distributed, no matter how the individual random number are distributed, if they have finite expectations and variances, and if enough random numbers are chosen. The Gaussian distribution is completely specified by its expectation and variance. Suppose we take $n$ independent random numbers $u_i$ with finite expectations $e_i$ and variances $v_i$. Then the sum $S = \sum u_i$ will have expectation $E(S) = \sum e_i$ and variance $V(S) = \sum v_i$, because the expectation operator is linear, and the variance operator is linear if the random numbers are chosen independently. Since $S$ is normally distributed, $E(S)$ and $V(S)$ are statements of probability for a given value of $S$.

To sum up, we have derived several properties of the Monte Carlo method. The Monte Carlo method involves replacing the integral $\int f(x)g(x)dx$ with the average of
the functions of a series of random numbers, $\frac{1}{n}\sum_{i}^{n} f(x_i)$, where the random numbers are distributed according to $g(x)$. If the variance of $f$ is finite, then the Monte Carlo estimate of the integral of $f$ converges to its true value as $n$ approaches infinity. The Monte Carlo estimate is normally distributed. The standard deviation of the Monte Carlo estimate is $\sqrt{V(f)}/\sqrt{n}$ for all $n$, even though this is only interesting for large values of $n$. In particular, we say that the Monte Carlo method converges as $1/\sqrt{n}$.

**Variance Reduction Techniques**

Why do we use Monte Carlo methods when numerical quadrature techniques converge much more quickly? For example, integration using Simpson's rule converges as $1/n^4$. One simple answer is that numerical quadratures converge so quickly in only one dimension. Using numerical quadrature, integrals in $d$ dimensions converge as the one dimensional convergence rate raised to the $1/d$ power, whereas Monte Carlo methods keep the same convergence rate. For example, in $d$ dimensions, Simpson's rule converges as $1/n^{4/d}$, whereas the Monte Carlo method still converges as $1/\sqrt{n}$. There are also limits to the minimum number of points where the function must be evaluated with numerical quadrature. These go like $n > m^d$ for numerical quadratures, where $m$ is the minimum number of points for a one dimensional integral. Monte Carlos may be evaluated with an arbitrary number of points.

Even so, we would like to increase the rate at which our Monte Carlo method converges. The uncertainty in the result is given by $\sqrt{V(f)/n}$, and we may reduce this either by increasing the number of points $n$, or by reducing the effective variance $V(f)$. 
There are several ways of reducing $V(f)$.

One of the easiest ways of increasing the convergence rate of the Monte Carlo method is by stratified sampling. Essentially, this amounts to choosing the random numbers more uniformly, which ought to reduce the variance because a large number of points may have been chosen where the function $f$ is large (or small). Because the integral we are approximating can be split into many different pieces, each integrating over a different piece of the phase space, we can similarly divide the Monte Carlo estimate into several pieces, and perform a weighted average of the individual pieces, where the weight is proportional to the phase space of the region divided by the number of points chosen within the region. If the regions and the number of points in each region are chosen carefully, there can be a dramatic reduction in the variance of the function. However, a poor choice may lead to an increase in the variance, so the function $f$ must be known in advance. However, uniform stratification, where each region is chosen to be the same size, and the same number of points are chosen in each region, never increases the variance, but may not reduce it by much either.

Importance sampling reduces the variance by choosing a large number of points where the function is the largest, and compensating for this by making the function smaller. This works because the variance of a function is small if the function never deviates much from its mean. Importance sampling reweights the function so that it is more nearly constant. Mathematically, this corresponds to a change of integration variables,

$$\int f(x)dx \rightarrow \int f(x)dG(x)/g(x) \quad (B.9)$$

Points are then chosen according to $G(x)$ rather than uniformly. Each value of $G$ is then
inverted to find $x$, and the function $f(x)/g(x)$ is evaluated. The relevant variance will then be $V(f/g)$, which will be small if $f$ and $g$ have nearly the same shape. There are drawbacks to this procedure though. The function $g(x)$ must be a probability distribution, analytically integrable, and its integral $G(x)$ must be analytically invertible. There are not many such functions. It is also unstable if the function $g$ has regions where it is small, leading to a large value of $f/g$.

Both of these methods require some knowledge of the function $f$ before Monte Carlo sampling takes place. This can be partially overcome by the use of adaptive methods, in which an initial run learns about the function, and then the main run uses these results to perform its integration.

**Random, Pseudorandom and Quasirandom Numbers**

In order to actually work with Monte Carlo methods, we need to realize that a computer cannot represent a random number. It is completely deterministic, and thus cannot generate a truly random number, which are completely unpredictable. A sequence of truly random numbers can only be generated by a random physical process, such as radioactive decay. Because computers are so fast these days, using a physical process to generate random numbers can be extremely difficult, and can destroy the efficiency of a calculation.

Therefore, we turn to the nearest thing to random numbers, which are pseudorandom numbers. These are numbers which are generated from a mathematical formula, and which are therefore completely deterministic, and not at all random. However, they are
supposed to be indistinguishable from truly random numbers. Then all of the results that we have derived above will be applicable. In theory, this is not generally possible, though in practice, it works well enough as long as the problem stays well away from the point where the generator breaks down.

The most widely used pseudo random generator is given by $r_i = a r_{i-1} \mod m$. For this generator, $m$ is usually chosen to be $2^d$ where $d$ is the number of bits that the computer uses to represent an integer. This generator is characterized by a period, the point at which the sequence of pseudorandom numbers begins to repeat itself, of length $m/4$.

Quasirandom numbers give up any pretense of being random. This is motivated by the realization that we do not really need a truly random sequence of numbers, and it is not generally possible anyway. It is more meaningful to attempt to find a sequence which has the required properties to give a good result.

One such quasirandom generator produces a sequence of $d$ dimensional vectors,

$$x_{ij} = i S_j \mod 1,$$

where $i$ runs from 1 to $d$, and $S_j$ is the square root of the $j^{th}$ prime number. This produces a quasirandom sequence with very good properties for large enough $n$. In order to improve these properties for small $n$, we use a shuffling technique, where each quasirandom number is put into a buffer. When a new number is needed, it is pulled from a pseudorandom spot in the buffer, and the next quasirandom number takes its place. This prevents the points from being poorly distributed.

Our Monte Carlo program directly simulates the high energy production process. In
addition to producing the cross section, it also produces a number of distributions, the cross section as a function of several of the parameters. It uses the quasi-random number generator described above, with an adaptive stratified sampling technique.

References


IMAGe EVALUATION
TEST TARGET (QA-3)