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Masatoshi Uehara
Harvard University

Jae Kwang Kim
Iowa State University, jkim@iastate.edu

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Keywords

Profile maximum likelihood estimation, Calibration estimation, Nonignorable nonresponse, Semiparametric response model, Instrumental variable

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Semiparametric response model with nonignorable nonresponse

MASATOSHI UEHARA

Department of Statistics, Harvard University
uehara_m@g.harvard.edu

JAE KWANG KIM

Department of Statistics, Iowa State University
jkim@iastate.edu

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Abstract

How to deal with nonignorable response is often a challenging problem encountered in statistical analysis with missing data. Parametric model assumption for the response mechanism is often made and there is no way to validate the model assumption with missing data. We consider a semiparametric response model that relaxes the parametric model assumption in the response mechanism. Two types of efficient estimators, profile maximum likelihood estimator and profile calibration estimator, are proposed and their asymptotic properties are investigated. Two extensive simulation studies are used to compare with some existing methods. We present an application of our method using Korean Labor and Income Panel Survey data.

Key Words: Profile maximum likelihood estimation; Calibration estimation; Nonignorable nonresponse; Semiparametric response model; Instrumental variable.

1 Introduction

Statistical analysis with missing data is an area under extensive research in recent years. To analyze partially missing data, the first step is to understand the response mechanism that causes missingness in the data. If the missingness for the study variable is conditionally independent of that variable, conditional on the other auxiliary variables, the response mechanism is called missing at random or ignorable, in the sense of Rubin (1976). Otherwise, the response mechanism is called missing not at random or nonignorable. It is more challenging to handle nonignorable nonresponse because the assumed response model cannot be verified from the observed study variables only and the response model may not be identifiable. Some identification assumptions are often made for the response model for the sake of making a valid inference from the incomplete data. Furthermore, the parametric model approach is known to be sensitive to the failure of the assumed parametric model (Kenward, 1998). To obtain a robust result, it is desirable to make the weakest possible model assumptions on the response mechanism. Kim and Shao (2013) contains a comprehensive review of the methods for parameter estimation under nonignorable nonresponse.

Instead of making parametric model assumptions for the response mechanism, we consider a semiparametric response model which allows more flexibility for the response mechanism. The semiparametric response model was first considered in Kim and Yu (2011), but the proposed method requires a validation sample for estimating model parameters. Shao and Wang (2016) also considered the same semiparametric model and proposed a parameter estimation method based on a calibration approach (Kott and Chang, 2008).

The proposed method of Shao and Wang (2016) is not necessarily efficient. We consider more efficient parameter estimation methods under the same semiparametric response model.

We consider the following goals to meet in this paper. First of all, by taking the profile maximum likelihood estimation and profile calibration approach, we propose more efficient estimators under the semiparametric response model than the previous proposed estimator in Shao and Wang (2016). Next, we also propose a more efficient estimator for mean functional than the inverse probability weighted estimator used in Shao and Wang (2016). However, the proposed method can be computationally heavy as it involves another nonparametric estimator for the outcome model in addition to the nonparametric part in the response model. To solve this problem, we propose an approach using a parametric working model for the outcome model in the same spirit of the generalized estimating equation (Liang and Zeger, 1986). Under such approach, when the working model is well-specified, the asymptotic variance form is not changed compared with an estimator when using a nonparametric model for the outcome model. Additionally, even when the working model is mis-specified, the estimators for the response probability and mean functional are still consistent.

The paper is organized as follows. In Section 2, the basic setup and the model are introduced. In Section 3, the proposed method for estimating the parameters in the semiparametric response model is presented. In Section 4, estimation of mean functional using the proposed method is discussed. In Section 5, results from two extensive simulation studies are presented. In Section 6, we present an application of the proposed method using Korean

Labor and Income Panel Survey data. Some concluding remarks are made in Section 7.

2 Basic setup

Let (X, Y) be a vector of random variables from F , completely unspecified, and Y is subject to missingness. We are interested in estimating $\mu = E(Y)$ from the n independent observations of $(x_i, \delta_i, \delta_i y_i)$, $i = 1, \dots, n$, where δ_i is the response indicator function of y_i , that is, $\delta_i = 1$ if y_i is observed and $\delta_i = 0$ otherwise. If the response mechanism is ignorable in the sense that

$$P(\delta = 1 \mid x, y) = P(\delta = 1 \mid x), \quad (1)$$

then parameter estimation in the model $f(y \mid x)$ can be made without an explicit model assumption for the response mechanism. If assumption (1) is believed to be unrealistic, then we often make a strong model assumption on $P(\delta = 1 \mid x, y)$, such as

$$P(\delta = 1 \mid x, y) = \pi(x, y; \phi) \quad (2)$$

for some known function $\pi(\cdot) \in (0, 1]$ with unknown parameter ϕ . For example, the logistic regression model

$$\pi(x, y; \phi) = \frac{\exp(\phi_0 + \phi_1 x + \phi_2 y)}{1 + \exp(\phi_0 + \phi_1 x + \phi_2 y)} \quad (3)$$

can be used in the response model (2). Estimating the model parameter in (2) is challenging because the model can be non-identifiable. A sufficient condition for model identification is to assume that $x = (x_1, x_2)$ and x_2 is conditionally independent of δ given (x_1, y) . Variable x_2 is called nonresponse instrumental variable (Wang et al., 2014).

Under the correct specification of the parametric response model and instrumental variable assumption, several methods are available for estimating the parameter ϕ . There are mainly four approaches: (1) maximum likelihood estimation approach (Riddles et al., 2016; Morikawa et al., 2017), (2) calibration approach based on Generalized Method of Moments (Chang and Kott, 2008; Shao and Wang, 2016; Morikawa and Kim, 2017), (3) empirical likelihood approach (Qin et al., 2002; Tang et al., 2014), (4) pseudo likelihood approach (Tang et al., 2003; Zhao and Shao, 2015). Note that (2) and (3) are essentially the same (Morikawa and Kim, 2018; Qin, 2017). Elaborating on the first and the second approaches, we propose estimation methods under the following semiparametric response model, which is a more general class than the parametric response model. The semiparametric response model is represented as

$$\pi(x, y) = \frac{\exp\{g(x) - \gamma y\}}{1 + \exp\{g(x) - \gamma y\}}, \quad (4)$$

where $g(\cdot)$ is completely unspecified. This model was firstly introduced in Kim and Yu (2011).

There are two things to note about this model. First, this is a more flexible model compared with commonly used parametric logistic models such as (3). Secondly, it is also viewed as a natural extension of the nonparametric response model in the ignorable case (Hirano et al., 2003) by fixing $\gamma = 0$. Although this semiparametric response model has the above good properties, it was required that γ is either known or estimable from a validation sample. Shao and Wang (2016) introduced an estimation method without using a validation sample by assuming the existence of x_2 in $x = (x_1, x_2)$, which is the nonresponse instrumental variable. Under this assumption, $g(x)$ becomes a

function of x_1 . We also assume this assumption for handling the identification problem of γ .

3 Estimation under semiparametric response model

Under the semiparametric response model (4) and nonresponse instrumental variable assumption, we propose new methods for estimating $g(x_1)$ and γ . We denote the true $g(x_1)$ as $g^*(x_1)$ and the true γ as γ^* for clarity. We also write the evaluating function of γ and g at γ^* and g^* as $\cdot|_{\gamma^*}$ and $\cdot|_{g^*}$.

Under the semiparametric response model in (4), we can obtain

$$E\left\{\frac{\delta}{\pi(X_1, Y)} - 1 \mid X_1 = x_1\right\} |_{g^*, \gamma^*} = 0,$$

which leads to

$$\exp\{g_\gamma(x_1)\} = \frac{E\{\delta \exp(\gamma Y) \mid x_1\}}{E\{1 - \delta \mid x_1\}}, \quad \exp\{g^*(x_1)\} = \exp\{g_\gamma(x_1)\} |_{\gamma^*}.$$

For fixed γ , we can use nonparametric methods to estimate $\exp\{g_\gamma(x_1)\}$. More specifically, when the sample space is continuous, we can use a Kernel regression estimator

$$\exp\{\hat{g}_\gamma(x_1)\} = \frac{\sum_{i=1}^n \delta_i \exp(\gamma y_i) K_h(x_1 - x_{1i})}{\sum_{i=1}^n (1 - \delta_i) K_h(x_1 - x_{1i})},$$

where K_h is a kernel with bandwidth h .

When the sample space is discrete, we can use

$$\exp\{\hat{g}_\gamma(x_1)\} = \frac{\sum_{i=1}^n \delta_i \exp(\gamma y_i) \mathbf{I}(x_1 = x_{1i})}{\sum_{i=1}^n (1 - \delta_i) \mathbf{I}(x_1 = x_{1i})},$$

where $I(\cdot)$ denotes an indicator function. For the convenience of notation, whether the sample space is continuous or not, we denote them as

$$\exp\{\hat{g}_\gamma(x_1)\} = \frac{\tilde{E}\{\delta \exp(\gamma Y) \mid x_1\}}{\tilde{E}\{1 - \delta \mid x_1\}}.$$

Then, as discussed in Shao and Wang (2016), the profile response probability is obtained:

$$\pi_p(x_{1i}, y_i; \gamma) = \frac{\exp\{\hat{g}_\gamma(x_{1i}) - \gamma y_i\}}{1 + \exp\{\hat{g}_\gamma(x_{1i}) - \gamma y_i\}}.$$

Using the above profile response probability, we take two approaches for the estimation of γ : (1) maximum likelihood estimation approach, (2) calibration approach. In both cases, the idea is to use $\pi_p(x)$ in the objective function, which is considered to be suitable when $g(x_1)$ is known, in the same spirit of the profile likelihood approach in Murphy and Van Der Vaart (2000). Note that once γ is estimated, we can use $\pi_p(x_i, y_i; \hat{\gamma})$ as the estimated profile response probability. Here, we first explain the maximum likelihood estimation approach and the calibration approach later.

3.1 Maximum likelihood estimation approach

To compute the maximum likelihood estimator in the missing data setting, we usually take two directions: (1) maximizing observed likelihood, (2) solving mean score equation. We start with the second direction. In appendix, we show that the estimator derived from the second perspective can also be interpreted from the first perspective.

When $g(x_1)$ is known, the observed score equation is

$$0 = \sum_{i=1}^n \left[\delta_i \left\{ 1 - \pi(x_{1i}, y_i; g^*, \gamma) \right\} y_i - (1 - \delta_i) E_0 \left\{ \pi(X, Y; g^*, \gamma) Y \mid x_i \right\} \right],$$

where $E_0[\cdot]$ denotes the expectation with respect to the distribution of y given x among $\delta = 0$. By replacing $g(x_1)$ with $\hat{g}_\gamma(x)$ and $E_0[\pi y \mid x_i]$ with a nonparametric estimator, the estimator $\hat{\gamma}_{p\text{-score}}$ is defined as the solution to

$$0 = \sum_{i=1}^n \left[\delta_i \left\{ 1 - \pi_p(x_{1i}, y_i; \gamma) \right\} y_i - (1 - \delta_i) \frac{\tilde{E}\{\delta \exp(\gamma Y) \pi_p(X_1, Y; \gamma) Y \mid x_i\}}{\tilde{E}\{\delta \exp(\gamma Y) \mid x_i\}} \right]. \quad (5)$$

This is based on the following relationship (Kim and Yu, 2011):

$$E_0\{e(X, Y) \mid x\} = \frac{E_1\{\exp(\gamma Y)e(X, Y) \mid x\}}{E_1\{\exp(\gamma Y) \mid x\}},$$

where $e(x, y)$ is any function of (x, y) .

We now derive asymptotic results of the estimator obtained from (5). By imposing technical conditions, we can ensure consistency by following Theorem 5.11 in van der Vaart (2002). For detail, see appendix. We obtain the following lemma as a first step. This lemma is also important in the next subsection for developing a new estimator from the calibration approach.

Lemma 3.1 *We define the right hand side of (5) as $t(\mathbf{w})$. Under certain conditions, we have*

$$\frac{1}{n}t(\mathbf{w}) = \frac{1}{n} \sum_{i=1}^n \left\{ \frac{\delta_i}{\pi_p(x_{1i}, y_i; \gamma)} - 1 \right\} \frac{E\{\delta \exp(\gamma Y) \pi_p(X, Y; \gamma) \mid x\}}{E\{\delta \exp(\gamma Y) \mid x\}} + o_p(n^{-1/2}).$$

Based on this lemma, we obtain the following result.

Theorem 1 *The asymptotic variance of $\hat{\gamma}_{p\text{-score}}$ is $n^{-1}A_1^{-1}B_1A_1^{-1}$ where*

$$A_1 = E \left[O(X, Y) \pi(X, Y) (Y - E_0[Y \mid X_1]) \{E_0(\pi Y \mid X) - E_0(\pi Y \mid X_1)\} \right] \Big|_{g^*, \gamma^*},$$

$$B_1 = E \left[O(X, Y) \{E_0(\pi Y \mid X) - E_0(\pi Y \mid X_1)\}^2 \right] \Big|_{g^*, \gamma^*},$$

and

$$O(X, Y) = \frac{1 - \pi(X, Y)}{\pi(X, Y)} = \exp \{ -g(X_1) + \gamma Y \}.$$

In practice, the estimator $\hat{\gamma}_{p-score}$ is unstable because of the double use of kernel estimators. Instead of using a kernel estimator for the calculation of the conditional expectation $E_0(\pi y \mid x_i)$, one can also use a model $f(y \mid x, \delta = 1; \beta)$ for a density y given x and $\delta = 1$ and apply fractional imputation (Kim, 2011). We can replace the conditional expectation term in (5) with

$$\frac{\sum_{j=1}^s \exp(\gamma y_{ij}) \pi_p(x_{1i}, y_{ij}; \gamma) y_{ij}}{\sum_{j=1}^s \exp(\gamma y_{ij})},$$

where $\{y_{ij}\}_{j=1}^s$ is a s -size sample obtained from the conditional density $f(y \mid x, \delta = 1; \hat{\beta})$ and $\hat{\beta}$ is the maximum likelihood estimator based on the observed samples. Throughout our work, we call the conditional distribution of Y given X and $\delta = 1$ as an outcome distribution and $f(y \mid x, \delta = 1; \beta)$ as a working parametric model. To avoid confusion, we write the estimator using this working parametric model as $\hat{\gamma}_{pw-score}$.

3.2 Calibration approach

Here, we consider the calibration approach (Chang and Kott, 2008). Based on this approach, when X_2 is a discrete random variable taking values from 1 to l , Shao and Wang (2016) introduced an estimator from the following moment conditions

$$\sum_{i=1}^n \left\{ \frac{\delta_i}{\pi_p(x_{1i}, y_i; \gamma)} - 1 \right\} v(x_{2i}) = 0, \quad (6)$$

where $v(x_2) = (\mathbf{I}(x_2 = 1), \dots, \mathbf{I}(x_2 = l))^T$, using Generalized Method of Moments (Hansen, 1982).

Although this estimator does not require a validation sample for estimating γ , its performance is unstable because of the poor choice of $v(x_{2i})$ in the

the moment condition. We consider a broader class of estimators based on the following moment conditions:

$$\sum_{i=1}^n \left\{ \frac{\delta_i}{\pi_p(x_{1i}, y_i; \gamma)} - 1 \right\} m(x_i; \gamma) = 0, \quad (7)$$

where $m(x; \gamma)$ is a function vector of x including parameter γ . We call this estimator $\hat{\gamma}_{p-gmm}$.

Here, the problem is how to choose the control variable $m(x; \gamma)$ in (7). We can easily deduce that the efficiency will increase as we increase the dimension of $m(x; \gamma)$. However, for the current problem, the response probability π_p includes a kernel estimator; thus, the calibration estimator (7) from using a high-dimensional $m(x; \gamma)$ is computationally heavy. Thus, we suggest two one-dimensional moment conditions by deriving an asymptotic result of the estimator based on (7).

The asymptotic result of the calibration estimator from (7) is given in the following theorem.

Theorem 2 *The asymptotic variance of the estimator $\hat{\gamma}_{p-gmm}$ is $n^{-1}A_m^{-1}B_mA_m^{-1}$:*

$$A_m = E \left(O(X, Y) \pi(X, Y) \{ Y - E_0(Y | X_1) \} [m(X; \gamma) - E_0\{m(X; \gamma) | X_1\}] \right) |_{g^*, \gamma^*},$$

$$B_m = E \left(O(X, Y) [m(X; \gamma) - E_0\{m(X; \gamma) | X_1\}]^2 \right) |_{g^*, \gamma^*}.$$

There are two things to note. First, when $g(x_1)$ is known, the term $E_0[Y | X_1]$ and $E_0[m(X) | X_1]$ will vanish. Second, a consistent estimator for the asymptotic variance, which can be used to construct a confidence

interval, is derived as $\hat{A}_m^{-1}\hat{B}_m\hat{A}_m^{-1}$:

$$\hat{A}_m = \frac{1}{n} \sum_{i=1}^n \frac{1 - \hat{\pi}_i}{\hat{\pi}_i} \delta_i \left\{ y_i - \tilde{E}_0(Y | x_{1i}; \hat{\gamma}) \right\} \left\{ m(x_i) - \tilde{E}_0\{m(X) | x_{1i}; \hat{\gamma}\} \right\},$$

$$\hat{B}_m = \frac{1}{n} \sum_{i=1}^n \frac{1 - \hat{\pi}_i}{\hat{\pi}_i} \left[m(x_i) - \tilde{E}_0\{m(X) | x_{1i}; \hat{\gamma}\} \right]^2,$$

where $\hat{\pi}_i = \pi_p(x_i, y_i; \hat{\gamma})$ denotes an estimated response probability.

Based on the result of Theorem 2, two choices of one-dimensional $m(x; \phi)$ can be suggested. The first choice is the function appearing in Lemma 3.1. This function is derived by maximum likelihood estimation; thus, we can expect a good performance. Specifically, the estimator $\hat{\gamma}_{p-ca1}$ is defined as the solution to (7) using

$$m(x; \gamma) = \frac{\tilde{E}\{\delta \exp(\gamma Y) \pi_p(X_1, Y; \gamma) Y | x\}}{\tilde{E}\{\delta \exp(\gamma Y) | x\}},$$

which is an approximation of the term in Lemma 3.1.

The second choice of $m(x; \gamma)$ is

$$\frac{\tilde{E}\{\delta \exp(\gamma Y) Y | x\}}{\tilde{E}\{\delta \exp(\gamma Y) / \pi_p(X_1, Y; \gamma) | x\}}.$$

We call the estimator from this $m(x; \gamma)$ as $\hat{\gamma}_{p-ca2}$. This is an approximation of $E\{O(X, Y)\pi Y | X\} / E\{O(X, Y) | X\}$. This choice is based on an optimal $m(x)$ if there are no $E_0\{m(X) | X_1\}$ and $E_0(Y | X_1)$, that is, $g(x_1)$ is known. This result is already known in other literature from different perspectives (Rotnitzky and Robins, 1997; Morikawa and Kim, 2017). We derive this result from a more direct approach.

Lemma 3.2 *When $g(x_1)$ is known, the asymptotic variance is minimized when $m^*(X) = E\{O(X, Y)\pi Y | x\} / E\{O(X, Y) | x\}$.*

Although $g(x_1)$ is unknown in practice and the optimality does not hold in general, we can still consider this function as a good candidate of $m(x; \gamma)$.

Finally, we derive asymptotic results of two estimators.

Theorem 3 *The asymptotic variance of the estimator $\hat{\gamma}_{p-ca1}$ is the same as $n^{-1}A_1^{-1}B_1A_1^{-1}$, which is given in Theorem 1, $\hat{\gamma}_{p-score}$. The asymptotic variance of $\hat{\gamma}_{p-ca2}$ is $n^{-1}A_2^{-1}B_2A_2^{-1}$.*

$$A_2 = E \left\{ O(X, Y) \pi \{Y - E_0(Y | X_1)\} \left(\frac{E\{O(X, Y) \pi Y | X\}}{E\{O(X, Y) | X\}} - E_0 \left[\frac{E\{O(X, Y) \pi Y | X\}}{E\{O(X, Y) | X\}} \mid X_1 \right] \right) \right\}$$

$$B_2 = E \left\{ O(X, Y) \left(\frac{E\{O(X, Y) \pi Y | X\}}{E\{O(X, Y) | X\}} - E_0 \left[\frac{E\{O(X, Y) \pi Y | X\}}{E\{O(X, Y) | X\}} \mid X_1 \right] \right)^2 \right\} \Big|_{g^*, \gamma^*}.$$

We can see that the asymptotic variance of $\hat{\gamma}_{p-score}$ and $\hat{\gamma}_{p-ca1}$ are the same. This result is natural because $m(X)$ derived from the asymptotic analysis of $\hat{\gamma}_{p-score}$ is directly used for the construction of the estimator $\hat{\gamma}_{p-ca1}$. As for the comparison between $\hat{\gamma}_{p-ca1}$ and $\hat{\gamma}_{p-ca2}$, it is difficult to say which one is superior theoretically. In §5, we experimentally confirm that the two estimators perform similarly. In addition, the asymptotic variances can be estimated like $\hat{\gamma}_{p-gmm}$. However, in the case of $\hat{\gamma}_{p-ca2}$, it might be difficult to estimate practically because of triply nested expectations.

To avoid using kernels twice, we can use a parametric model for the density of Y given X and $\delta = 1$ to calculate the conditional expectation. As in the previous section, write the estimators using $f(y | x, \delta = 1; \hat{\beta})$ as $\hat{\gamma}_{pw-ca1}$ and $\hat{\gamma}_{pw-ca2}$. For these estimators, the following asymptotic property holds. This result is similar to the property of generalized estimating equation (Liang and Zeger, 1986).

Lemma 3.3 *When the working parametric model $f(y | x, \delta = 1; \beta)$ is well-specified, the forms of the asymptotic variance of $\hat{\gamma}_{pw-ca1}$ and $\hat{\gamma}_{pw-ca2}$ are the same as in Theorem 3. When the working parametric model $f(y | x, \delta = 1; \beta)$ is mis-specified, $\hat{\gamma}_{pw-ca1}$ and $\hat{\gamma}_{pw-ca2}$ are \sqrt{n} -consistent.*

Note that $\hat{\gamma}_{pw-score}$ has a different asymptotic property compared with $\hat{\gamma}_{pw-ca1}$ and $\hat{\gamma}_{pw-ca2}$. When the parametric model is well-specified, the form of the asymptotic variance of $\hat{\gamma}_{p-score}$ will be changed from that of Theorem 1. In addition, when the parametric model is mis-specified, $\hat{\gamma}_{p-score}$ is not consistent anymore. Therefore, $\hat{\gamma}_{pw-ca1}$ is considered to be more robust than $\hat{\gamma}_{pw-score}$ although asymptotic variances of $\hat{\gamma}_{p-ca1}$ and $\hat{\gamma}_{p-score}$ are the same.

Between $\hat{\gamma}_{pw-ca1}$ and $\hat{\gamma}_{pw-ca2}$, it is difficult to say which one is superior theoretically in terms of statistical efficiency. We can state that $\hat{\gamma}_{pw-ca2}$ is computationally superior to $\hat{\gamma}_{pw-ca1}$ because if the parametric working model belongs to an exponential family, the function $m(x; \gamma)$ can be calculated analytically without relying on Monte Carlo integration (Morikawa et al., 2017).

4 Estimation of mean functional

We have discussed estimation of γ so far. Here, we discuss estimation for the mean functional $\mu \equiv E(Y)$. We can consider the following three estimators:

$$\hat{\mu}_{ipw} = \frac{1}{n} \sum_{i=1}^n \frac{y_i}{\pi_p(x_{1i}; \hat{\gamma})}, \quad \hat{\mu}_{mp} = \frac{1}{n} \sum_{i=1}^n \left[\delta_i y_i + (1 - \delta_i) \frac{\tilde{E}\{\delta \exp(\hat{\gamma}Y)Y | x_i\}}{\tilde{E}\{\delta \exp(\hat{\gamma}Y) | x_i\}} \right],$$

end

$$\hat{\mu}_{db} = \frac{1}{n} \sum_{i=1}^n \left[\frac{\delta_i y_i}{\pi_p(x_{1i}; \hat{\gamma})} + \left\{ 1 - \frac{\delta_i}{\pi_p(x_{1i}; \hat{\gamma})} \right\} \frac{\tilde{E}\{\delta \exp(\hat{\gamma}Y)Y | x_i\}}{\tilde{E}\{\delta \exp(\hat{\gamma}Y) | x_i\}} \right].$$

The estimator $\hat{\mu}_{ipw}$ was used in Wang et al. (2014). However, if we consider the original motivation of the semiparametric response model first introduced in Kim and Yu (2011), it will be natural to use the estimator $\hat{\mu}_{mp}$. The estimator $\hat{\mu}_{db}$ is introduced using the analogy of doubly robust estimator in the ignorable case (Robins et al., 1994). Note that this estimator is different from other doubly robust form estimators in the nonignorable case (Miao and Tchetgen Tchetgen, 2016; Morikawa and Kim, 2017).

When $\hat{\gamma}$ is a \sqrt{n} -consistent estimator, the following asymptotic result can be established.

Theorem 4 *Under certain regularity conditions, for estimators $\hat{\mu}_{ipw}$, $\hat{\mu}_{mp}$ and $\hat{\mu}_{db}$, we have*

$$\begin{aligned}\hat{\mu}_{ipw} &= C_1 + C_2 + C_3 + o_p(n^{-1/2}), \\ C_1 &= \frac{1}{n} \sum_{i=1}^n E_0(Y | x_{1i}), \quad C_2 = \frac{1}{n} \sum_{i=1}^n \frac{\delta_i}{\pi_i} \{y_i - E_0(Y | x_{1i})\}, \quad C_3 = H_1(\hat{\gamma} - \gamma^*), \\ \hat{\mu}_{db} &= \hat{\mu}_{mp} + o_p(n^{-1/2}) = D_1 + D_2 + D_3 + o_p(n^{-1/2}), \\ D_1 &= \frac{1}{n} \sum_{i=1}^n E_0(Y | x_i), \quad D_2 = \frac{1}{n} \sum_{i=1}^n \frac{\delta_i}{\pi_i} \{y_i - E_0(Y | x_i)\}, \quad D_3 = H_2(\hat{\gamma} - \gamma^*),\end{aligned}$$

where

$$H_1 = E \left[(1 - \pi) \{Y - E_0(Y | X_1)\}^2 \right] |_{g^*, \gamma^*}, \quad H_2 = E \left[(1 - \pi) \{Y - E_0(Y | X)\}^2 \right] |_{g^*, \gamma^*}.$$

There are three things to note. First, we can see that the asymptotic variances of $\hat{\mu}_{mp}$ and $\hat{\mu}_{db}$ are the same. This result makes sense if you consider the ignorable case, which has been well established in literature (Rotnitzky and Vansteelandt, 2014). Second, the asymptotic variance of estimators $\hat{\mu}_{mp}$ and $\hat{\mu}_{db}$ are generally smaller than that of $\hat{\mu}_{ipw}$ because the

estimators use more information in the conditional expectation of $\hat{\mu}_{mp}$ and $\hat{\mu}_{db}$. Third, the asymptotic variance can be estimated by taking the variance in the expression of Theorem 4.

We can also use the working parametric model for calculating the conditional expectation for $E_0(Y | X)$. We write the estimators as $\hat{\mu}_{w-mp}$ and $\hat{\mu}_{w-db}$. We have the following properties.

Lemma 4.1 *When the working parametric model $f(y | x, \delta = 1; \beta)$ is well-specified, the forms of the asymptotic variance of $\hat{\mu}_{w-mp}$ and $\hat{\mu}_{w-db}$ are not changed from that of Theorem 4. Also, when the working parametric model is well-specified, $\hat{\mu}_{w-db}$ is consistent.*

This suggests that when the parametric outcome model is used, $\hat{\mu}_{w-db}$ is considered to be superior to $\hat{\mu}_{w-mp}$ because even if this model is mis-specified, $\hat{\mu}_{w-db}$ is consistent, while $\hat{\mu}_{w-mp}$ is not generally consistent. It is related to a doubly robust form estimators in nonignorable nonresponse cases (Miao et al., 2015; Miao and Tchetgen Tchetgen, 2016) although our estimator $\hat{\mu}_{w-db}$ has a different form. They proposed doubly robust estimators in the sense that it is consistent even if the underlying baseline response model ($g(x_1)$) or outcome model ($f(y | x, \delta = 1)$) is mis-specified under the correct assumption of an odds ratio model ($\exp(\gamma Y)$). In our setup, the response model $g(x_1)$ is nonparametric, and therefore cannot be mis-specified. Our proposed estimator $\hat{\mu}_{w-db}$ is robust in the sense that the estimator is consistent under the correct assumption of the odds ratio model even if the outcome model is mis-specified.

5 Simulation Study

We conducted two simulation studies to compare the performance of the estimators for γ and μ . For the estimators of γ , we compared $\hat{\gamma}_{p-gmm}$, $\hat{\gamma}_{p-score}$, $\hat{\gamma}_{p-ca1}$ and $\hat{\gamma}_{p-ca2}$ or $\hat{\gamma}_{p-gmm}$, $\hat{\gamma}_{pw-score}$, $\hat{\gamma}_{pw-ca1}$ and $\hat{\gamma}_{pw-ca2}$. Note that the estimator $\hat{\gamma}_{p-gmm}$ with $m(x) = x$ corresponds to a baseline estimator proposed in Wang et al. (2014). In the estimator $\hat{\gamma}_{p-gmm}$, we adopt x_1 as $m(X)$. For the estimation of mean functional μ , we compared three estimators: $\hat{\mu}_{ipw}$, $\hat{\mu}_{mp}$ and $\hat{\mu}_{db}$ or $\hat{\mu}_{ipw}$, $\hat{\mu}_{w-mp}$ and $\hat{\mu}_{w-db}$. We consider simulations under three conditions: (1) X and Y are discrete, (2) Y and X_2 are continuous and X_1 is discrete, and (3) X and Y are continuous. For the case of (3), see next section. In all cases, the parameter values under the missing data models were chosen so that the overall missing rate was about 30%.

5.1 Case where X and Y are discrete

Let X_1 be a categorical random variable taking values $\{0, 1, 2, 3\}$ and Y be a binary variable taking values $\{0, 1\}$ and Z be a binary variable taking value $\{0, 1\}$. The random variable X_1 follows a multinomial distribution with probability $(0.25, 0.25, 0.25, 0.25)$ and the random variable X_2 , independent of X_1 , follows a Bernoulli distribution with probability 0.5. The random variable Y follows a Bernoulli distribution with probability $1/\{1 + \exp(1.3 - (X_1 - 1.6)^2 - 1.5X_2)\}$. The random variable δ follows a Bernoulli distribution as follows; M1: $\pi(X_1, Y) = 1/\{1 + \exp(-\phi_0 - \phi_1 X_1 + \gamma Y)\}$, where $(\phi_0, \phi_1, \gamma) = (0.2, 0.8, 0.6)$, M2: $\pi(X_1, Y) = 1/\{1 + \exp(\phi_0 + \phi_1 X_1 + \phi_2 X_1^2 + \gamma Y)\}$, where $(\phi_0, \phi_1, \phi_2, \gamma) = (0.2, -0.4, 0.7, 0.6)$, and M3: $\pi(X_1, Y) = 1/\{1 + \exp(\phi_0 + \phi_1 \sin(X_1) + \gamma Y)\}$, where $(\phi_0, \phi_1, \gamma) = (-1.6, -0.8, 0.6)$. The simulation is

Table 1: Monte Carlo mean square error (MSE) and bias of γ when X and Y are discrete

Model	n		$\hat{\gamma}_{p-gmm}$	$\hat{\gamma}_{p-score}$	$\hat{\gamma}_{p-ca1}$	$\hat{\gamma}_{p-ca2}$
M_1	1000	Bias	0.09	0.01	0.04	0.03
		MSE	0.94	0.61	0.44	0.46
	4000	Bias	0.00	0.01	0.01	0.01
		MSE	0.16	0.10	0.10	0.10
M_2	1000	Bias	0.08	0.02	0.01	0.01
		MSE	0.52	0.39	0.34	0.35
	4000	Bias	0.00	0.00	0.00	0.00
		MSE	0.11	0.086	0.088	0.088
M_3	1000	Bias	0.07	0.02	0.01	0.01
		MSE	0.43	0.35	0.32	0.33
	4000	Bias	0.02	0.00	0.00	0.00
		MSE	0.090	0.072	0.072	0.072

replicated 500 times with two sample sizes; 1000 and 4000. Here, as all the variables are discrete, we use a nonparametric model for y given x and $\delta = 1$.

The result for the estimation of γ and μ are presented in Table 1 and Table 2 respectively. First, we can see that our proposed estimators perform better than the baseline estimator $\hat{\gamma}_{p-gmm}$ in terms of mean square errors. Next, it is seen that $\hat{\mu}_{mp}$ and $\hat{\mu}_{db}$ are superior to $\hat{\mu}_{ipw}$ in terms of efficiency, which is consistent with our theory in Theorem 4. Finally, it is confirmed that when the sample size is large ($n = 4000$), mean square errors of $\hat{\gamma}_{p-score}$ and $\hat{\gamma}_{p-ca1}$ are almost the same. This matches to theoretical results in § 3. It is also seen that mean square errors of $\hat{\gamma}_{p-ca1}$ and $\hat{\gamma}_{p-ca2}$ are almost the same.

Table 2: Monte Carlo mean square error (MSE) for μ when X and Y are discrete. Multiplied by 1000.

Model	n		$\hat{\gamma}_{p-gmm}$	$\hat{\gamma}_{p-score}$	$\hat{\gamma}_{p-ce1}$	$\hat{\gamma}_{p-ce2}$
M_1	1000	$\hat{\mu}_{ipw}$	0.92	0.73	0.73	0.73
		$\hat{\mu}_{mp}$	0.89	0.73	0.73	0.73
		$\hat{\mu}_{db}$	0.89	0.73	0.73	0.73
	4000	$\hat{\mu}_{ipw}$	0.32	0.24	0.24	0.24
		$\hat{\mu}_{mp}$	0.31	0.24	0.24	0.24
		$\hat{\mu}_{db}$	0.31	0.24	0.24	0.24
M_2	1000	$\hat{\mu}_{ipw}$	1.33	1.04	1.01	1.01
		$\hat{\mu}_{mp}$	1.32	1.01	0.99	1.00
		$\hat{\mu}_{db}$	1.32	1.01	0.99	1.00
	4000	$\hat{\mu}_{ipw}$	0.35	0.28	0.28	0.27
		$\hat{\mu}_{mp}$	0.34	0.27	0.27	0.27
		$\hat{\mu}_{db}$	0.34	0.27	0.37	0.27
M_3	1000	$\hat{\mu}_{ipw}$	129	128	128	129
		$\hat{\mu}_{mp}$	1.32	1.17	1.17	1.17
		$\hat{\mu}_{db}$	1.32	1.17	1.17	117
	4000	$\hat{\mu}_{ipw}$	122	123	122	122
		$\hat{\mu}_{mp}$	0.27	0.24	0.24	0.24
		$\hat{\mu}_{db}$	0.27	0.24	0.24	0.24

5.2 Cases where X_1 is discrete and Y, X_2 are continuous

Let X_1 be a binary random variable, whose distribution is described above, and X_2 be an uniform distribution $U[-1, 1]$. As for the random variable Y , we make an assumption for the conditional distribution of Y given X and $\delta = 1$, and also $\pi(X, Y) = P(\delta = 1|X, Y)$. Note that the distribution of Y is uniquely determined by these two distributions. For the generation of samples in this setting, see Morikawa et al. (2017). First, let the distribution of Y given X and $\delta = 1$ be a Gaussian distribution $N(-1.0 - 0.4X_1 + 0.5X_2^2, 1.0)$. Second, let the response mechanism be the following logistic models; M1: $\pi(X_1, Y) = 1/\{1 + \exp(-\phi_0 - \phi_1X_1 + \gamma Y)\}$, where $(\phi_0, \phi_1, \gamma) = (0.3, 0.4, 0.5)$, M2: $\pi(X_1, Y) = 1/\{1 + \exp(-\phi_0 - \phi_1X_1 - \phi_2X_1^2 + \gamma Y)\}$, where $(\phi_0, \phi_1, \phi_2, \gamma) = (0.3, 0.3, 0.2, 0.5)$, and M3: $\pi(X_1, Y) = 1/\{1 + \exp(-\phi_0 - \phi_1 \sin(X_1) + \gamma Y)\}$, where $(\phi_0, \phi_1, \gamma) = (0.3, 0.3, 0.5)$.

We used a well-specified parametric outcome model to assist with the calculation of the conditional expectation and compared $\hat{\gamma}_{p-gmm}$, $\hat{\gamma}_{pw-score}$, $\hat{\gamma}_{pw-ca1}$ and $\hat{\gamma}_{pw-ca2}$. Note that the estimator for $\exp\{-g(x_1)\}$ is still constructed from an empirical distribution without using a kernel estimator. For the calculation of the condition expectation term based on Monte Carlo integration, which appears in the objective function of $\hat{\gamma}_{pw-score}$, $\hat{\gamma}_{pw-ca1}$ and $\hat{\gamma}_{pw-ca2}$, we obtain 500 samples from the auxiliary distribution $f(y | x, \delta = 1; \hat{\beta})$. However, in the case of $\hat{\gamma}_{pw-ca2}$, actually, we can perform calculation analytically because $f(y | x, \delta = 1; \beta)$ is a normal distribution (Morikawa and Kim, 2017). We write the estimator relying on Monte Carlo integration as $\hat{\gamma}_{pw-ca2-s}$ and the other that does not rely on Monte Carlo integration as $\hat{\gamma}_{pw-ca2-a}$.

Table 3: Monte Carlo mean square error (MSE) and bias of γ when X_1 is discrete, and X_2, Y are continuous

Model	n		$\hat{\gamma}_{p-gmm}$	$\hat{\gamma}_{pw-score}$	$\hat{\gamma}_{pw-ca1}$	$\hat{\gamma}_{pw-ca2-a}$	$\hat{\gamma}_{pw-ca2-s}$
M_1	2000	Bias	0.54	0.05	0.03	0.01	0.01
		MSE	1.30	0.27	0.41	0.36	0.36
	4000	Bias	0.53	0.02	0.01	0.02	0.01
		MSE	1.27	0.15	0.19	0.18	0.21
M_2	2000	Bias	0.61	0.05	0.07	0.08	0.09
		MSE	1.25	0.27	0.45	0.39	0.40
	4000	Bias	0.6	0.03	0.01	0.01	0.01
		MSE	1.3	0.16	0.22	0.16	0.20
M_3	2000	Bias	0.67	0.02	0.03	0.01	0.01
		MSE	1.31	0.24	0.43	0.38	0.38
	4000	Bias	0.62	0.03	0.02	0.01	0.01
		MSE	1.32	0.13	0.18	0.14	0.16

The simulation is replicated 500 times with sample size 2000 and 4000. The result is reported in Table 3 and 4. There are five things to note. First, it is seen that all of the proposed estimators are superior to the baseline estimator $\hat{\gamma}_{p-gmm}$. Secondly, whether $\hat{\gamma}_{p-ca1}$ or $\hat{\gamma}_{p-ca2}$ is better depends on the true distribution. Thirdly, the mean square error of $\hat{\gamma}_{p-score}$ is smaller than that of $\hat{\gamma}_{p-ca1}$ and $\hat{\gamma}_{p-ca2}$. However, as noted earlier, $\hat{\gamma}_{p-score}$ is not a consistent estimator when the working model is mis-specified. Fourth, by comparing two estimator $\hat{\gamma}_{p-ca2-s}$ and $\hat{\gamma}_{p-ca2-a}$, we can see the variance increases negligibly due to Monte Carlo integration. Finally, it is seen that $\hat{\mu}_{mp}$ and $\hat{\mu}_{db}$ are superior to $\hat{\mu}_{ipw}$ in terms of mean square errors.

5.3 Coverage probability

We further examined the confidence intervals of γ and μ under the setting § 5.1 with $n = 4000$. As an estimator for μ , we adopted $\hat{\mu}_{mp}$. We nonpara-

Table 4: Monte Carlo mean square error (MSE) for μ when X_1 is discrete, and X_2, Y are continuous. Multiplied by 100.

Model	n		$\hat{\gamma}_{p-gmm}$	$\hat{\gamma}_{pw-score}$	$\hat{\gamma}_{pw-ca1}$	$\hat{\gamma}_{pw-ca2-a}$
M_1	2000	$\hat{\mu}_{ipw}$	29	9.3	11.4	9.9
		$\hat{\mu}_{w-mp}$	11	2.4	3.8	3.5
		$\hat{\mu}_{w-db}$	10	2.2	3.7	3.4
	4000	$\hat{\mu}_{ipw}$	29	5.4	5.5	5.4
		$\hat{\mu}_{w-mp}$	11	1.4	1.6	1.7
		$\hat{\mu}_{w-db}$	11	1.3	1.6	1.7
M_2	2000	$\hat{\mu}_{ipw}$	27	9.9	10	10
		$\hat{\mu}_{w-mp}$	13	2.2	3.7	3.4
		$\hat{\mu}_{w-db}$	12	2.2	3.7	3.3
	4000	$\hat{\mu}_{ipw}$	26	3.0	6.0	4.8
		$\hat{\mu}_{w-mp}$	10	1.0	1.9	1.3
		$\hat{\mu}_{w-db}$	10	0.9	1.9	1.3
M_3	2000	$\hat{\mu}_{ipw}$	35	8.5	12	12
		$\hat{\mu}_{w-mp}$	12	2.3	4.2	3.6
		$\hat{\mu}_{w-db}$	12	2.1	4.1	3.6
	4000	$\hat{\mu}_{ipw}$	20	6.1	5.9	5.8
		$\hat{\mu}_{w-mp}$	11	1.4	1.8	1.5
		$\hat{\mu}_{w-db}$	11	1.3	1.7	1.5

Table 5: Coverage probability (CI) for γ and μ .

Model		$\hat{\gamma}_{p-gmm}$	$\hat{\gamma}_{p-ca1}$	$\hat{\gamma}_{p-ca2}$
M_1	γ	0.94	0.95	0.95
	μ	0.94	0.95	0.94
M_2	γ	0.93	0.94	0.95
	μ	0.94	0.95	0.94
M_3	γ	0.95	0.95	0.96
	μ	0.94	0.95	0.95

metrically estimated the asymptotic variances based on the forms in the §3 and §4. Note that our method for constructing confidence intervals does not rely on bootstrapping unlike Shao and Wang (2016) because there is no guarantee that bootstrapping would work. The result is reported in Table 5 with the appropriate coverage rate.

6 Numerical illustration

We apply our method to the Korean Labor and Income Panel Survey data, which was used in several papers (Kim, 2011; Shao and Wang, 2016; Morikawa et al., 2017). These data include $n = 2506$ Korean residents and four variables: the response variable y is income (10^6 Korean Won) in the year, x_1 : income in the previous year, x_2 : gender, x_3 : age, x_4 : education level, where x_1 and y are continuous variables, and x_2 has two categories, x_3 has three categories, and x_4 has two categories.

We made an artificial incomplete dataset by assuming the two response models; M1: $\pi = 1/\{1 + \exp(-1.3 - 0.3\sqrt{x_1} - 0.2x_1 + 0.6y)\}$ and M2: $\pi = 1/\{1 + \exp(-1.2 - 0.5x_1 + 0.6y)\}$. Thus, under the assumption of $\pi = 1/\{1 + \exp(-g(x_1) + \gamma y)\}$, we estimate γ and μ and their confidence interval

Table 6: Estimated value and confidence interval (CI) of $\hat{\mu}_{w-db}$ with significant level $\alpha = 0.05$. The true mean is 1.85.

Model		$\hat{\gamma}_{p-gmm}$	$\hat{\gamma}_{pw-ca1}$	$\hat{\gamma}_{pw-ca2}$
M_1	Value	1.80	1.86	1.86
	CI(0.05)	± 0.12	± 0.06	± 0.07
M_2	Value	1.77	1.84	1.85
	CI(0.05)	± 0.19	± 0.08	± 0.09

according to $\hat{\gamma}_{p-gmm}$, $\hat{\gamma}_{pw-ca1}$ and $\hat{\gamma}_{pw-ca2}$. As for $\hat{\gamma}_{p-gmm}$, we used x_2 , x_3 , x_4 as nonresponse instrumental variables. For the estimation of μ , we used $\hat{\mu}_{w-db}$. We used the bandwidth when regressing δ on X_1 based on cross validation using the np R package (Hayfield and Racine, 2008).

Note that the true mean income was 1.85, which is calculated using the complete data. We reported an estimated value and confidence interval of μ for each case in Table 6. We can see that $\hat{\gamma}_{p-ca1}$, $\hat{\gamma}_{p-ca2}$ are superior to $\hat{\gamma}_{p-gmm}$.

7 Discussion

We have developed several methods doing parameter estimation for the semi-parametric response model. To summarize, we have the following practical suggestions. When the instrumental variables are discrete, we recommend using calibration estimators $\hat{\gamma}_{p-ca1}$ or $\hat{\gamma}_{p-ca2}$ for the estimation of γ by plugging in a nonparametric estimator in the conditional expectation term and then, estimating μ using the doubly robust form estimator $\hat{\mu}_{db}$. When the instrumental variables are continuous, we recommend using $\hat{\gamma}_{pw-ca2}$ for the estimation of γ and using $\hat{\mu}_{w-db}$ for the estimation of μ with the aid of parametric working outcome model.

For future research, the following directions will be considered. The first is how to choose instrumental variables practically from available covariates. One can consider the following three step procedure: (a) making several models as in our work assuming each covariate is an instrumental variable, (b) performing estimation and calculating estimated likelihood, (c) select the model which maximizes the estimated likelihood. The second direction is exploring more efficient estimators under the semiparametric response model or under the semiparametric response model and parametric outcome model.

A Proof of results

To derive asymptotic results, we assume the following conditions to use Lemma 8.11 in Newey and Mcfadden (1994) and Theorem 6.18. in van der Vaart (2002). Essentially, it requires three types of conditions: (1) stochastic equicontinuity, (2) the objective function can be linearized with respect to nonparametric components (this also includes a convergence rate condition of nonparametric estimators), and (3) uniform convergence of the differentiation of estimating equation with respect to a parameter of interests. For simplicity, we write down required conditions for Theorem 1 when the sample space of X is continuous. For other theorems and results where X is discrete, we can add conditions similarly.

We define $\left\{ \frac{\delta}{\pi(x,y;g,\gamma)} - 1 \right\} h(x; \gamma)$ as $\phi(w; g, \gamma)$. We denote the true $g(x_1)$ as $g^*(x_1)$ and the true γ as γ^* . For the norm of functional space including $g(x_1)$, we introduce a norm $\| \cdot \|$ in a Hölder space.

(C1): The response probability is uniformly bounded below from zero.

(C2): $W_i = (X_i, Y_i, R_i)$ are independent and identically distributed.

(C3): The parameter space Γ for γ is compact.

(C4): The kernel $K(x)$ has bounded derivatives of order k , satisfies $\int K(x)dx = 1$, and has zero moments of order up to $m - 1$ and nonzero k -th order moment.

(C5): For all y , $\pi(\cdot, y)$ and $\exp\{g_\gamma(\cdot)\}$ are differentiable to order k and are bounded on an open set containing support of x .

(C6): There exists $v \geq 4$ such that $E(|\delta \exp(\gamma Y)|^v)$ and $E(|1 - \delta|^v)$, and $E(|\delta \exp(\gamma Y)|^v | x)f(x)$ and $E(|1 - \delta|^v | x)f(x)$ are bounded for all supports of x .

(C7): As $h \rightarrow 0$, $n^{1-2/v}h/\log n \rightarrow \infty$, $n^{1/2}h^{1+2k}/\log n \rightarrow \infty$ and $n^{1/2}h^{2k} \rightarrow 0$.

(C8): $0 = E\{\phi(w; g_\gamma, \gamma)\}$ has a unique solution with respect to γ in the interior of Γ . (This condition is needed for consistency. In that sense, it overlaps the condition below.)

(C9): Consistency: $(\hat{\gamma}_{p-gmm}, \hat{g}_{\hat{\gamma}}) \xrightarrow{p} (\gamma^*, g^*)$. This is discussed in Section C.

(C10): The class of functions $\{\phi(w; g, \gamma) : |\gamma - \gamma^*| < \delta, \|g - g^*\| < \delta\}$ is a Donsker class for some $\delta > 0$.

(C11): The map $\gamma \rightarrow E\{\phi(w; \gamma, g_\gamma)\}$ is differentiable at γ^* , uniformly in γ in a neighborhood of γ^* with nonsingular derivative matrices.

(C12): The map $(\gamma, g) \rightarrow \phi(w; \gamma, g)$ is continuous in L^2 space induced from a true distribution.

A.1 Proof of Theorem 2

We have:

$$\begin{aligned}
0 &= \frac{1}{n} \sum_{i=1}^n \left\{ \frac{\delta_i}{\pi(x_{1i}, y_i; \hat{g}_\gamma, \gamma)} - 1 \right\} m(x_i) \\
&= \frac{1}{n} \sum_{i=1}^n [\delta_i \{1 + \exp(-\hat{g}_\gamma + \gamma y_i)\} - 1] m(x_i) \\
&= \frac{1}{n} \sum_{i=1}^n [\delta_i \{1 + \exp(-g_\gamma + \gamma y_i)\} - 1] m(x_i) + \text{Rm}(\mathbf{w}) \\
&= \frac{1}{n} \sum_{i=1}^n \left\{ \frac{\delta_i}{\pi(x_{1i}, y_i; g_\gamma, \gamma)} - 1 \right\} m(x_i) + \text{Rm}(\mathbf{w}), \tag{8}
\end{aligned}$$

where the residual term $\text{Rm}(\mathbf{w})$ is equal to

$$\frac{1}{n} \sum_{i=1}^n \delta_i [\exp\{-\hat{g}_\gamma(x_{1i})\} - \exp\{-g_\gamma(x_{1i})\}] m(x_i).$$

From the assumptions (C1)-(C8), we have

$$\begin{aligned} \|E\{\delta \exp(\gamma y) \mid x\} f_1(x) - \tilde{E}\{\delta \exp(\gamma y) \mid x\} f_1(x)\| &= o_p(n^{-1/4}), \\ \|E(1 - \delta|x) f_1(x) - \tilde{E}(1 - \delta|x) f_1(x)\| &= o_p(n^{-1/4}), \\ \left\| \frac{\tilde{E}\{\delta \exp(\gamma y) \mid x\}}{\tilde{E}\{1 - \delta|x\}} - \exp\{-g_\gamma(x)\} - \text{Rm}_2(\mathbf{w}) \right\| &= o_p(n^{-1/2}), \end{aligned}$$

where

$$\text{Rm}_2(\mathbf{w}) = \frac{1}{n^2} \sum_{i=1}^n \sum_{j=1}^n \delta_i m(x_i) \exp(\gamma y_i) K_h(x_{1i}, x_{1j}) \frac{1 - \delta_j - \exp\{-g_\gamma(x_{1i})\} \delta_j \exp(\gamma y_j)}{E\{\delta \exp(\gamma Y) \mid x_{1i}\}} + o_p(n^{-1/2}).$$

Thus,

$$\begin{aligned} \text{Rm}(\mathbf{w}) &= \text{Rm}_2(\mathbf{w}) + o_p(n^{-1/2}) \\ &= \binom{n}{2}^{-1} \sum_{i \neq j} 2^{-1} (\zeta_{ij} + \zeta_{ji}) + o_p(n^{-1/2}), \end{aligned}$$

where

$$\zeta_{ij} = \delta_i m(x_i) \exp(\gamma y_i) K_h(x_{1i}, x_{1j}) \frac{1 - \delta_j - \exp\{-g_\gamma(x_{1i})\} \delta_j \exp(\gamma y_j)}{E\{\delta \exp(\gamma Y) \mid x_{1i}\}}.$$

From the theory of U-statistics (van der Vaart, 1998), we have

$$\begin{aligned} \text{Rm}(\mathbf{w}) &= \frac{1}{n} \sum_{i=1}^n \{E(\zeta_{ij}|w_i) + E(\zeta_{ji}|w_i)\} + o_p(n^{-1/2}) \\ &= \frac{1}{n} \sum_{i=1}^n E\{\delta_i m(x_i) \exp(\gamma y_i) \mid x_{1i}\} \frac{1 - \delta_i - \exp\{-g_\gamma(x_{1i})\} \delta_i \exp(\gamma y_i)}{E\{\delta \exp(\gamma Y) \mid x_{1i}\}} + o_p(n^{-1/2}) \\ &= \frac{1}{n} \sum_{i=1}^n \frac{E\{\delta m(X) \exp(\gamma Y) \mid x_{1i}\}}{E\{\delta \exp(\gamma Y) \mid x_{1i}\}} \left\{ 1 - \frac{\delta_i}{\pi(x_{1i}, y_i; g_\gamma, \gamma)} \right\} + o_p(n^{-1/2}). \end{aligned}$$

From the first line to the second line, we use the fact $E(\zeta_{ij}|w_i) = 0$ and the order of the bias term of the nonparametric estimator is $o_p(n^{-1/2})$. By plugging the above $\text{Rm}(\mathbf{w})$ in (8), we have

$$\begin{aligned} 0 &= \frac{1}{n} \sum_{i=1}^n \left\{ \frac{\delta_i}{\pi_i(x_{1i}, y_i; \hat{g}_\gamma, \gamma)} - 1 \right\} m(x_i) \\ &= \frac{1}{n} \sum_{i=1}^n \left\{ \frac{\delta_i}{\pi_i(x_{1i}, y_i; g_\gamma, \gamma)} - 1 \right\} \left[m(x_i) - \frac{E\{\delta \exp(\gamma Y) m(X) \mid x_{1i}\}}{E\{\delta \exp(\gamma Y) \mid x_{1i}\}} \right] + o_p(n^{-1/2}). \end{aligned}$$

Therefore, the asymptotic variance is calculated as $A^{-1}BA^{-1}$:

$$\begin{aligned} A &= \nabla_\gamma E \left(\frac{1}{n} \sum_{i=1}^n \left\{ \frac{\delta}{\pi(X_1, Y; g_\gamma, \gamma)} - 1 \right\} \left[m(X) - \frac{E\{\delta \exp(\gamma Y) m(X) \mid X_1\}}{E\{\delta \exp(\gamma Y) \mid X_1\}} \right] \right) \Big|_{\gamma^*} \\ &= E \left(\frac{1}{n} \sum_{i=1}^n \nabla_\gamma \left\{ \frac{\delta}{\pi(X_1, Y; g_\gamma, \gamma)} \right\} \left[m(X) - \frac{E\{\delta \exp(\gamma Y) m(X) \mid X_1\}}{E\{\delta \exp(\gamma Y) \mid X_1\}} \right] \right) \Big|_{\gamma^*} \\ &= E \left(\frac{1-\pi}{\pi} \pi \{Y - E(Y \mid X_1, \delta = 0)\} [m(X) - E\{m(X) \mid X_1, \delta = 0\}] \right) \Big|_{g^*, \gamma^*}, \\ B &= E \left(\left\{ \frac{\delta}{\pi(X_1, Y; g_\gamma, \gamma)} - 1 \right\}^2 \left[m(X) - \frac{E\{\delta \exp(\gamma Y) m(X) \mid X_1\}}{E\{\delta \exp(\gamma Y) \mid X_1\}} \right]^2 \right) \Big|_{\gamma^*} \\ &= E \left(E \left[\left\{ \frac{\delta}{\pi(X_1, Y; g_\gamma, \gamma)} - 1 \right\}^2 \mid X \right] \left[m(X) - \frac{E\{\delta \exp(\gamma Y) m(X) \mid X_1\}}{E\{\delta \exp(\gamma Y) \mid X_1\}} \right]^2 \right) \Big|_{\gamma^*} \\ &= E \left(\frac{1-\pi}{\pi} [m(X) - E\{m(X) \mid X_1, \delta = 0\}]^2 \right) \Big|_{g^*, \gamma^*}. \end{aligned}$$

Note that from the first line to the second line, the calibration condition is used. From the second line to the third line, we used

$$\begin{aligned} &\nabla_\gamma \left\{ \frac{1}{\pi(x_1, y; g_\gamma, \gamma)} \right\} \Big|_{\gamma^*} \\ &= \left[\exp(-g_\gamma + \gamma y) y - \exp(-g_\gamma + \gamma y) \frac{E\{\delta Y \exp(\gamma Y) \mid x_1\}}{E\{\delta \exp(\gamma Y) \mid x_1\}} \right] \Big|_{\gamma^*} \\ &= \left[\frac{1-\pi}{\pi} \{y - E(Y \mid x_1, \delta = 0)\} \right] \Big|_{g^*, \gamma^*} \end{aligned}$$

A.2 Proof of Theorem 3

We can replace the conditional expectation including the nonparametric component with the expectation as follows because of the calibration condition. This type of reasoning is already known in the literature. For example, see Chapter 10.2 in Tsiatis (2006) and page 408 in van der Vaart (2002). Thus, we obtain

$$\begin{aligned} & \frac{1}{n} \sum_{i=1}^n \left\{ \frac{\delta_i}{\pi_i(x_{1i}, y_i; \hat{g}_\gamma, \gamma)} - 1 \right\} \frac{\tilde{E}\{\delta \exp(\gamma Y) \pi(X_1, Y; \hat{g}_\gamma, \gamma) Y | x_i\}}{\tilde{E}\{\delta \exp(\gamma Y) | x_i\}} \\ &= \frac{1}{n} \sum_{i=1}^n \left\{ \frac{\delta_i}{\pi_i(x_{1i}, y_i; \hat{g}_\gamma, \gamma)} - 1 \right\} \frac{E\{\delta \exp(\gamma Y) \pi(X_1, Y; \hat{g}_\gamma, \gamma) Y | x_i\}}{E\{\delta \exp(\gamma Y) | x_i\}} + o_p(n^{-1/2}). \end{aligned} \tag{9}$$

If we substitute $\hat{g}_\gamma(x_1)$ in the conditional expectation with a linearized estimator as in the proof of Theorem 2.1, the expression (9) is equal to

$$\frac{1}{n} \sum_{i=1}^n \left\{ \frac{\delta_i}{\pi_i(x_{1i}, y_i; \hat{g}_\gamma, \gamma)} - 1 \right\} \frac{E\{\delta \exp(\gamma Y) \pi(X_1, Y; g_\gamma, \gamma) Y | x_i\}}{E\{\delta \exp(\gamma Y) | x_i\}} + o_p(n^{-1/2}).$$

Then, as in the proof of Theorem 1, the above term is equal to

$$\frac{1}{n} \sum_{i=1}^n \left\{ \frac{\delta_i}{\pi_i(x_{1i}, y_i; g(x_1), \gamma)} - 1 \right\} \left[q(x_i) - \frac{E\{\delta \exp(\gamma Y) q(X) | x_{1i}\}}{E\{\delta \exp(\gamma Y) | x_{1i}\}} \right] + o_p(n^{-1/2}),$$

where

$$q(x) = \frac{E\{\delta \exp(\gamma Y) \pi(X_1, Y; g_\gamma, \gamma) Y | x\}}{E\{\delta \exp(\gamma Y) | x\}}.$$

Then, the asymptotic variance is obtained as $A^{-1}BA^{-1}$:

$$\begin{aligned}
A &= \nabla_{\gamma} E \left(\frac{1}{n} \sum_{i=1}^n \left\{ \frac{\delta_i}{\pi_i(X_{1i}, y_i; g_{\gamma}, \gamma)} - 1 \right\} \left[q(X_i) - \frac{E\{\delta \exp(\gamma y) q(x_i) \mid x_{1i}\}}{E\{\delta \exp(\gamma y) \mid X_{1i}\}} \right] \right) \Big|_{\gamma^*} \\
&= E \left(\frac{1}{n} \sum_{i=1}^n \nabla_{\gamma} \left\{ \frac{\delta_i}{\pi_i(X_{1i}, Y_i; g_{\gamma}, \gamma)} \right\} \left\{ q(X_i) - \frac{E\{\delta \exp(\gamma y) q(X_i) \mid X_{1i}\}}{E\{\delta \exp(\gamma Y) \mid X_{1i}\}} \right\} \right) \Big|_{\gamma^*} \\
&= E \left(O(X, Y) \pi \{Y - E_0(Y \mid X)\} [E_0\{\pi Y \mid X\} - E_0\{\pi(X, Y)Y \mid X_1\}] \right) \Big|_{g^*, \gamma^*}, \\
B &= E \left(O(X, Y) [E_0\{\pi Y \mid X\} - E_0\{\pi(X, Y; g, \gamma)Y \mid X_1\}]^2 \right) \Big|_{g^*, \gamma^*}.
\end{aligned}$$

From the second line to the third line, we use

$$E\{E(\pi Y \mid X, \delta = 0) \mid X_1, \delta = 0\} = E(\pi Y \mid X_1, \delta = 0).$$

A.3 Proof of Lemma 3.1 and Theorem 1

We have

$$\begin{aligned}
0 &= \sum_{i=1}^n \delta_i \{1 - \pi(x_{1i}, y_i; \hat{g}_{\gamma}, \gamma)\} y_i + (1 - \delta_i) \frac{\tilde{E}\{-\delta \exp(\gamma Y) \pi(X_1, Y; \hat{g}_{\gamma}, \gamma) Y \mid x_i\}}{\tilde{E}\{\delta \exp(\gamma Y) \mid x_i\}} \\
&= \frac{1}{n} \sum_{i=1}^n \left\{ \frac{\delta_i}{\pi_i(x_{1i}, y_i; \hat{g}_{\gamma}, \gamma)} - 1 \right\} \frac{E\{\delta \exp(\gamma Y) \pi(X_1, Y; \hat{g}_{\gamma}, \gamma) Y \mid x_i\}}{E\{\delta \exp(\gamma Y) \mid x_i\}} + o_p(n^{-1/2}).
\end{aligned}$$

From the first line to the second line, we use a result in the proof of Morikawa et al. (2017). This concludes the proof of Lemma 3.1. For the rest of the proof, we leave it to the proof of Theorem 3.

A.4 Proof of Lemma 3.2

When $g(x_1)$ is known, the asymptotic variance is written as $n^{-1}A^{-1}BA^{-1}$:

$$\begin{aligned}
A &= E \{O(X, Y) \pi Y m(X)\} \Big|_{\gamma^*} = E [E \{O(X, Y) \pi Y \mid X\} m(X)] \Big|_{\gamma^*}, \\
B &= E [E \{O(X, Y) \mid X\} m^2(X)] \Big|_{\gamma^*}.
\end{aligned}$$

We can use the Cauchy Schwartz inequality for the expectation:

$$A^{-1}BA^{-1} \geq E [E \{O(X, Y) | X\}^{-1} E \{O(X, Y)\pi Y | X\}^2] |_{\gamma^*}$$

The equality holds when $m(X; \gamma) |_{\gamma^*}$ is a proportional to

$$E \{O(X, Y) | X\}^{-1} E \{O(X, Y)\pi Y | X\} |_{\gamma^*}.$$

This concludes the proof.

A.5 Proof of Lemma 3.3

For the former statement, we can prove similarly as in Theorem 3. For the latter statement, it is proved because estimators are reduced to the form of $\hat{\gamma}_{p-gmm}$ when the outcome model is misspecified.

A.6 Proof of Theorem 4

First, we show the asymptotic form of $\hat{\mu}_{ipw}$. Although a similar result is obtained in Wang et al. (2014), our result is slightly different. The difference is that $E(y | x_1, \delta = 0)$, which appear in the terms of B_1 and B_2 , is $E(y | x_1)$ in their result.

The estimator can be expanded as:

$$\begin{aligned} \hat{\mu}_{ipw} &= \frac{1}{n} \sum_{i=1}^n \delta_i y_i [1 + \exp\{-\hat{g}_{\hat{\gamma}}(x_{1i})\} + \hat{\gamma} y_i] \\ &= J_1 + J_2 + J_3 + o_p(n^{-1/2}), \end{aligned}$$

where

$$\begin{aligned}
J_1 &= \frac{1}{n} \sum_{i=1}^n \frac{\delta_i y_i}{\pi(x_i, y_i; g^*, \gamma^*)} \\
J_2 &= \frac{1}{n} \sum_{i=1}^n \delta_i y_i \exp(\hat{\gamma} y_i) [\exp\{-\hat{g}_\gamma(x_{1i})\} - \exp\{-g_\gamma(x_i)\}] \\
J_3 &= \frac{1}{n} \sum_{i=1}^n \delta_i y_i [\exp\{-g_\gamma(x_i) + \hat{\gamma} y_i\} - \exp\{-g_{\gamma^*}(x_i) + \gamma^* y_i\}].
\end{aligned}$$

As in the proof of Theorem 2.1, the term J_2 is

$$\begin{aligned}
J_2 &= \frac{1}{n} \sum_{i=1}^n \frac{E\{\delta \exp(\hat{\gamma} Y) Y \mid x_{1i}\}}{E\{\delta \exp(\hat{\gamma} Y) \mid x_{1i}\}} \left\{ 1 - \frac{\delta_i}{\pi(x_{1i}, y_i; g_\gamma(x_{1i}), \hat{\gamma})} \right\} + o_p(n^{-1/2}) \\
&= \frac{1}{n} \sum_{i=1}^n \frac{E\{\delta \exp(\gamma Y) Y \mid x_{1i}\}}{E\{\delta \exp(\gamma Y) \mid x_{1i}\}} \Big|_{\gamma^*} \left\{ 1 - \frac{\delta_i}{\pi(x_{1i}, y_i; g_\gamma(x_{1i}), \hat{\gamma})} \right\} + o_p(n^{-1/2}) \\
&= \frac{1}{n} \sum_{i=1}^n E(Y \mid x_{1i}, \delta = 0) \left\{ 1 - \frac{\delta_i}{\pi(x_{1i}, y_i; g_\gamma(x_{1i}), \hat{\gamma})} \right\} + o_p(n^{-1/2}) \\
&= \frac{1}{n} \sum_{i=1}^n E(Y \mid x_{1i}, \delta = 0) \left\{ 1 - \frac{\delta_i}{\pi(x_{1i}, y_i; g^*(x_{1i}), \gamma^*)} \right\} + o_p(n^{-1/2}) \\
&\quad - \frac{1}{n} \sum_{i=1}^n \delta_i E(Y \mid x_{1i}, \delta = 0) \exp\{-g_{\gamma^*}(x_i)\} \exp(\gamma^* y_i) \{y_i - E(Y \mid x_{1i}, \delta = 0)\} (\hat{\gamma} - \gamma^*)
\end{aligned}$$

From the first line to the second line, we have used a calibration condition.

From the third line to the fourth line, we have used a delta method. Similarly,

from the delta method, the term J_3 is

$$J_3 = \frac{1}{n} \sum_{i=1}^n \delta_i y_i \exp\{-g_{\gamma^*}(x_i)\} \exp(\gamma^* y_i) \{y_i - E(Y \mid x_{1i}, \delta = 0)\} (\hat{\gamma} - \gamma^*) + o_p(n^{-1/2}).$$

Organizing the above result, we obtain

$$\begin{aligned}\hat{\mu}_{ipw} &= C_1 + C_2 + C_3 + o_p(n^{-1/2}), \\ C_1 &= \frac{1}{n} \sum_{i=1}^n E(Y | x_{1i}, \delta = 0), \\ C_2 &= \frac{1}{n} \sum_{i=1}^n \frac{\delta_i}{\pi(x_i, y_i; g_{\gamma^*}, \gamma^*)} \{y_i - E(Y | x_{1i}, \delta = 0)\}, \\ C_3' &= H_1'(\hat{\gamma} - \gamma^*),\end{aligned}$$

where

$$\begin{aligned}H_1' &= \frac{1}{n} \sum_{i=1}^n \delta_i \exp\{-g^*(x_i) + \gamma^* y_i\} \{y_i - E(y | x_{1i}, \delta = 0)\}^2 \\ &= \frac{1}{n} \sum_{i=1}^n \delta_i \left(\frac{1}{\pi_i} - 1\right) |_{g^*, \gamma^*} \{y_i - E(y | x_{1i}, \delta = 0)\}^2.\end{aligned}$$

From Slutsky's theorem, we have

$$C_3' = E[(1 - \pi)\{Y - E[Y | x_{1i}, \delta = 0]\}^2] |_{g^*, \gamma^*} (\hat{\gamma} - \gamma^*) + o_p(n^{-1/2}).$$

This concludes the proof.

Next, we show the asymptotic form of $\hat{\mu}_{mp}$. From Theorem 1 in Kim (2011), the following is yielded:

$$\begin{aligned}\hat{\mu}_{mp} &= \frac{1}{n} \sum_{i=1}^n \frac{\delta_i y_i}{\pi(x_i; g_{\hat{\gamma}}, \hat{\gamma})} + \left\{1 - \frac{\delta_i}{\pi(x_i; g_{\hat{\gamma}}, \hat{\gamma})}\right\} \frac{E(\delta \exp(\gamma^*) Y | x_i)}{E(\delta \exp(\gamma^*) Y | x_i)} + o_p(n^{-1/2}) \\ &= \frac{1}{n} \sum_{i=1}^n \frac{\delta_i}{\pi(x_i; g_{\hat{\gamma}}, \hat{\gamma})} \{y_i - E(Y | x_i, \delta = 0)\} + E(Y | x_i, \delta = 0) + o_p(n^{-1/2}) \\ &= \frac{1}{n} \sum_{i=1}^n \frac{\delta_i}{\pi(x_i; g_{\gamma^*}, \gamma^*)} \{y_i - E[Y | x_i, \delta = 0]\} + E(y | x_i, \delta = 0) + o_p(n^{-1/2}) \\ &\quad + H_2'(\hat{\gamma} - \gamma^*),\end{aligned}\tag{10}$$

where

$$H'_2 = \frac{1}{n} \sum_{i=1}^n \frac{\delta_i \{1 - \pi(x_i)\}}{\pi(x_i)} \Big|_{g^*, \gamma^*} \{y_i - E(Y | x_i, \delta = 0)\} \{y_i - E(Y | x_{1i}, \delta = 0)\}.$$

From the second line to the third line, we use the delta method. In addition, we have

$$\begin{aligned} & E \left[\frac{\delta_i \{1 - \pi(x_i)\}}{\pi(x_i)} \Big|_{g^*, \gamma^*} (y_i - E(Y | X_i, \delta = 0)) \{y_i - E(Y | X_{1i}, \delta = 0)\} \right] \\ &= E [(1 - \delta) \{Y - E(Y | X, \delta = 0)\} \{Y - E(Y | X, \delta = 0) + E(Y | X, \delta = 0) - E(Y | X_1, \delta = 0)\}] \\ &= E [\{Y - E(Y | X, \delta = 0)\} \{Y - E(Y | X, \delta = 0) + E(Y | X, \delta = 0) - E(Y | X_1, \delta = 0)\} | \delta = 0] P(\delta = 0) \\ &= E [\{Y - E(Y | X, \delta = 0)\}^2 | \delta = 0] P(\delta = 0) \\ &= E [(1 - \delta) \{Y - E(Y | X, \delta = 0)\}^2] = H_2. \end{aligned}$$

Slutsky's theorem concludes the proof.

Next, we show the result of $\hat{\mu}_{db}$. The estimator $\hat{\mu}_{db}$ is expanded as $\hat{\mu}_{mp}$:

$$\begin{aligned} \hat{\mu}_{db} &= \frac{1}{n} \sum_{i=1}^n \frac{\delta_i y_i}{\pi(x_i; \hat{g}_{\hat{\gamma}}, \hat{\gamma})} + \left\{ 1 - \frac{\delta_i}{\pi(x_i; \hat{g}_{\hat{\gamma}}, \hat{\gamma})} \right\} \frac{\tilde{E}\{\delta \exp(\hat{\gamma} Y) Y | x_i\}}{\tilde{E}[\delta \exp(\hat{\gamma} Y) | x_i]} \\ &= \frac{1}{n} \sum_{i=1}^n \frac{\delta_i y_i}{\pi(x_i; \hat{g}_{\hat{\gamma}}, \hat{\gamma})} + \left\{ 1 - \frac{\delta_i}{\pi(x_i; \hat{g}_{\hat{\gamma}}, \hat{\gamma})} \right\} \frac{E\{\delta \exp(\gamma^* Y) Y | x_i\}}{E\{\delta \exp(\gamma^* Y) | x_i\}} + o_p(n^{-1/2}) \\ &= \frac{1}{n} \sum_{i=1}^n \frac{\delta_i}{\pi(x_i; \hat{g}_{\hat{\gamma}}, \hat{\gamma})} \{y_i - E(Y | x_i, \delta = 0)\} + E(Y | x_i, \delta = 0) + o_p(n^{-1/2}) \\ &= \frac{1}{n} \sum_{i=1}^n \frac{\delta_i}{\pi(x_i; \hat{g}_{\hat{\gamma}}, \hat{\gamma})} \{y_i - E(Y | x_i, \delta = 0) - E(Y | x_{1i}, \delta = 0) + E(E(Y | x_i, \delta = 0) | x_{1i}, \delta = 0)\} \\ &\quad + E(Y | x_i, \delta = 0) + o_p(n^{-1/2}) \\ &= \frac{1}{n} \sum_{i=1}^n \frac{\delta_i}{\pi(x_i; g_{\hat{\gamma}}, \hat{\gamma})} \{y_i - E(Y | x_i, \delta = 0)\} + o_p(n^{-1/2}). \end{aligned}$$

From the first line to the second line, the calibration condition is used to replace the nonparametric estimator with a true distribution. From the third line to the fourth line, we use the same argument as in Theorem 1 to replace $\hat{g}_{\hat{\gamma}}$ with $g_{\hat{\gamma}}$. The final line is the same expression as (10). This concludes the proof.

A.7 Proof of Lemma 4.1

For the former statement, we can use the same argument as in the proof of Theorem 4. For the latter statement, when the working model is misspecified, there exists $s(x)$ satisfying

$$\frac{\tilde{E}\{\delta \exp(\hat{\gamma}Y)Y \mid x; \hat{\beta}\}}{\tilde{E}\{\delta \exp(\hat{\gamma}Y) \mid x; \hat{\beta}\}} = s(x; \hat{\gamma}) + o_p(1).$$

Then, we have

$$\begin{aligned} \hat{\mu}_{db} &= \frac{1}{n} \sum_{i=1}^n \frac{\delta_i y_i}{\pi(x_i; \hat{g}_\gamma, \hat{\gamma})} + \left\{ 1 - \frac{\delta_i}{\pi(x_i; \hat{g}_\gamma, \hat{\gamma})} \right\} \frac{\tilde{E}\{\delta \exp(\hat{\gamma}Y)Y \mid x_i\}}{\tilde{E}\{\delta \exp(\hat{\gamma}Y) \mid x_i\}} \\ &= \frac{1}{n} \sum_{i=1}^n \frac{\delta_i y_i}{\pi(x_i; \hat{g}_\gamma, \hat{\gamma})} + \left\{ 1 - \frac{\delta_i}{\pi(x_i; \hat{g}_\gamma, \hat{\gamma})} \right\} s(x_i; \hat{\gamma}) + o_p(1) \\ &= \frac{1}{n} \sum_{i=1}^n \frac{\delta_i y_i}{\pi(x_i; \hat{g}_\gamma, \hat{\gamma})} + o_p(1) = \hat{\mu}_{ipw} + o_p(1). \end{aligned}$$

This concludes the proof.

B Interpretation of $\hat{\gamma}_{p-score}$ from the perspective of maximizing observed likelihood

By maximizing $E\{l(W; \hat{g}_\gamma, \gamma) \mid W_{obs}\}$, where $l(W; \hat{g}_\gamma, \gamma)$ denotes a full-likelihood, we can derive the following EM-type iterative estimator.

- E-step: calculate an approximation of an expected log-likelihood $E\{\log l(W) \mid W_{obs}\}$ as

$$L(\gamma) = \sum_{i=1}^n \delta_i \log \pi(x_{1i}, y_i; \hat{g}_\gamma, \gamma) + (1 - \delta_i) h_{\hat{\gamma}_t}(x_i; \gamma),$$

where

$$h_{\hat{\gamma}_t}(x; \gamma) = \frac{\tilde{E}\{\delta \exp(\hat{\gamma}_t Y) \log(1 - \pi(X, Y; \hat{g}_\gamma, \gamma)) \mid x\}}{\tilde{E}\{\delta \exp(\hat{\gamma}_t Y) \mid x\}}.$$

- M-step: maximize the above function with respect to γ . Update the value as $\hat{\gamma}_{t+1}$.
- By iterating E-step and M-step, we define the converge point as an estimator $\hat{\gamma}_{p-mle}$.

The estimator $\hat{\gamma}_{p-mle}$ is essentially equal to $\hat{\gamma}_{p-score}$ because of the following reason. The estimator $\hat{\gamma}_{p-score}$ solves the equation $E\{\nabla_{\gamma} \log l(W; \hat{g}_{\gamma}, \gamma) | W_{obs}\} = 0$. It can be converted into an EM-style method. From this viewpoint, when Y_{mis} is imputed by $W|W_{obs}$ in the E-step, solving the score equation $\nabla_{\gamma} \log l(W; \hat{g}_{\gamma}, \gamma) = 0$ in $\hat{\gamma}_{p-score}$ is equivalent to maximizing the complete-data profile log-likelihood $\log l(W; \hat{g}_{\gamma}, \gamma)$ with respect to γ in $\hat{\gamma}_{p-mle}$.

C Consistency of estimators

For simplicity, we discuss the consistency of the estimator $\hat{\gamma}_{p-gmm}$ based on the expression (7). For the consistency of other estimators, we can prove similarly. We define

$$\hat{m}(w; \gamma) = \left\{ \frac{\delta}{\pi_p(x, y; \gamma)} - 1 \right\} h(x; \gamma), \quad m(w; \gamma) = \left\{ \frac{\delta}{\pi(x, y; g_{\gamma}, \gamma)} - 1 \right\} h(x; \gamma).$$

For consistency of estimators based on profile semiparametric estimators, three conditions are needed (van der Vaart, 2002); (1) there exists a Glivenko-Cantelli class F of functions with integrable envelope such that $P(\{\hat{m}(w; \gamma)\} \in F) \rightarrow 1$, (2) $\sup_{\gamma \in \Gamma} |\hat{m}(w; \gamma) - m(w; \gamma)| \xrightarrow{P} 0$ for all x , and (3) there exists a point $\gamma^* \in \Gamma$ such that $\inf_{|\gamma - \gamma^*| > \delta} E\{m(w; \gamma)\} > E\{m(w; \gamma)\}_{|\gamma^*} = 0$ for every $\delta > 0$.

The first and second condition are technical conditions. These conditions are reduced to more primitive conditions (Newey and Mcfadden, 1994), which are immaterial for our work. The third condition is a common condition for the consistency of M-estimators without plug-in nonparametric estimators (van der Vaart, 1998).

REFERENCES

References

- Chang, T. and P. S. Kott (2008). Using calibration weighting to adjust for nonresponse under a plausible model. *Biometrika* 95, 555–571.
- Hansen, L. P. (1982). Large sample properties of generalized method of moments estimators. *Econometrica* 50, 1029–1054.
- Hayfield, T. and J. S. Racine (2008). Nonparametric econometrics: The np package. *Journal of Statistical Software* 27(5).
- Hirano, K., G. Imbens, and G. Ridder (2003). Efficient estimation of average treatment effects using the estimated propensity score. *Econometrica* 71, 1161–1189.
- Kenward, M. G. (1998). Selection models for repeated measurements with non-random dropout: An illustration of sensitivity. *Statistics in Medicine* 17, 2723–2732.
- Kim, J. K. (2011). Parametric fractional imputation for missing data analysis. *Biometrika* 98, 119–132.

- Kim, J. K. and J. Shao (2013). *Statistical Methods for Handling Incomplete Data* (1 ed.). CRC Press.
- Kim, J. K. and C. L. Yu (2011). A semi-parametric estimation of mean functionals with non-ignorable missing data. *Journal of the American Statistical Association* 106, 157–165.
- Liang, K.-Y. and S. L. Zeger (1986). Longitudinal data analysis using generalized linear models. *Biometrika* 73(1), 13–22.
- Miao, W., E. T. Tchetgen, and Z. Geng (2015). Identification and doubly robust estimation of data missing not at random with a shadow variable. *arXiv preprint arXiv:1509.02556*.
- Miao, W. and E. J. Tchetgen Tchetgen (2016). On varieties of doubly robust estimators under missingness not at random with a shadow variable. *Biometrika* 103(2), 475–482.
- Morikawa, K. and J. K. Kim (2017). Semiparametric adaptive estimation with nonignorable nonresponse data. *arXiv preprint arXiv:1612.09207*.
- Morikawa, K. and J. K. Kim (2018). A note on the equivalence of two semi-parametric estimation methods for nonignorable nonresponse. *Statistics and Probability Letters* 140, 1–6.
- Morikawa, K., J. K. Kim, and Y. Kano (2017). Semiparametric maximum likelihood estimation with data missing not at random. *Canadian Journal of Statistics* 45(4), 393–409.

- Murphy, S. A. and A. W. Van Der Vaart (2000). On profile likelihood. *Journal of the American Statistical Association* 95(450), 449–465.
- Newey, W. K. and D. L. Mcfadden (1994). Large sample estimation and hypothesis testing. *Handbook of Econometrics IV*, 2113–2245.
- Qin, J. (2017). *Biased Sampling, Over-identified Parameter Problems and Beyond*. ICSA Book Series in Statistics. Singapore: Springer Singapore.
- Qin, J., D. Leung, and J. Shao (2002). Estimation with survey data under non-ignorable nonresponse or informative sampling. *Journal of the American Statistical Association* 97, 193–200.
- Riddles, M. K., J. K. Kim, and J. Im (2016). A propensity-score-adjustment method for nonignorable nonresponse. *J. Surv. Stat. Methodol.* 4(2), 215–245.
- Robins, J. M., A. Rotnitzky, and L. P. Zhao (1994). Estimation of regression coefficients when some regressors are not always observed. *Journal of the American Statistical Association* 89, 846–866.
- Rotnitzky, A. and J. Robins (1997). Analysis of semiparametric regression models with nonignorable nonresponse. *Statistics in Medicine* 16(1), 81–102.
- Rotnitzky, A. and S. Vansteelandt (2014). Double-robust methods. In *Handbook of missing data methodology*. In *Handbooks of Modern Statistical Methods*, pp. 185–212. Chapman and Hall/CRC.
- Rubin, D. B. (1976). Inference and missing data. *Biometrika* 63, 581–590.

- Shao, J. and L. Wang (2016). Semiparametric inverse propensity weighting for nonignorable missing data. *Biometrika* 103(1), 175–187.
- Tang, G., R. J. A. Little, and T. E. Raghunathan (2003). Analysis of multivariate missing data with nonignorable nonresponse. *Biometrika* 90, 747–764.
- Tang, N., P. Zhao, and H. Zhu (2014). Empirical likelihood for estimating equations with nonignorably missing data. *Statistica Sinica* 24(2), 723–747.
- Tsiatis, A. (2006). *Semiparametric theory and missing data*. Springer series in statistics. New York: Springer.
- van der Vaart, A. W. (1998). *Asymptotic Statistics*. Cambridge: Cambridge University Press.
- van der Vaart, A. W. (2002). *Semiparametric Statistics*. Lecture Notes in Mathematics ; 1781. Berlin, Heidelberg: Springer Berlin Heidelberg.
- Wang, S., J. Shao, and J. K. Kim (2014). Identifiability and estimation in problems with nonignorable nonresponse. *Statistica Sinica* 24, 1097 – 1116.
- Zhao, J. and J. Shao (2015). Semiparametric pseudo-likelihoods in generalized linear models with nonignorable missing data. *Journal of the American Statistical Association* 110(512), 1577–1590.