8-2000

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Jinhua Zhao
Iowa State University

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Disciplines

Agricultural and Resource Economics | Agricultural Economics | Economics | Environmental Health and Protection | Environmental Policy | Natural Resource Economics

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Jinhua Zhao

Working Paper 00-WP 252
August 2000

Center for Agricultural and Rural Development
Iowa State University
Ames, Iowa 50011-1070
www.card.iastate.edu

The author is an assistant professor in the Department of Economics at Iowa State University.

The author thanks David Hennessy, Larry Karp, Cathy Kling, Harvey Lapan, and Quinn Weninger for their helpful comments. The usual disclaimer applies.

For questions or comments about the contents of this paper, please contact Jinhua Zhao, 260 Heady Hall, Iowa State University, Ames, IA 50011-1070. Ph: (515) 294-5857; Fax: (515) 294-0221; E-mail: jzhao@iastate.edu.

Abstract

A major concern with TEPs is that stochastic permit prices may reduce firm incentive to invest in abatement capital or technologies relative to other policies such as a fixed emissions charge. However, under efficient permit trading, the price uncertainty is caused by abatement cost uncertainties which affect investment under both permit and charge policies. We develop a rational expectations general equilibrium model of permit trading to show how cost uncertainty affects investment. Differences between the two policies can be decomposed into a general equilibrium effect and a price-vs-quantity effect. Except for the curvature of the payoff functions, uncertainties reduce both effects so that tradable permits in fact help maintain firms' investment incentive under uncertainty. (*JEL:* Q20)

1 Introduction

Tradable emission permits (TEPs) are gaining popularity in environmental regulation as manifested by the successful sulfur trading in the U.S. and the global carbon trading proposed in the Kyoto Protocol. Among the often-cited advantages of TEPs is the argument that it provides more incentive for firms to invest in abatement technologies or capital than the command and control policies (i.e. standards). In the short run it provides as much incentive as an emissions tax. In the long run, a constant emissions tax would provide more incentive than grandfathered permits because the marginal abatement costs go down as firms invest, reducing permit price as well as the benefits of investment. These findings have been discussed in Magat (1978), Milliman and Prince (1989), and Jung, Krutilla and Boyd (1996). However, even in the long run, Parry (1997) showed that the incentive offered by permits would be close to that by a tax for many pollutants.

Despite these findings, there is a serious concern that TEPs may reduce a firm's incentive to invest because permit prices are typically random and the investment is to a great extent irreversible (Xepapadeas (1999) and Chao and Wilson (1993)).[1] In contrast, other policies such as standards or taxes do not introduce this additional uncertainty. Consequently, in a stochastic world, investment incentive under permits may be smaller. These studies typically assume *exogenous* and random permit price processes (Xepapadeas (1999)) or *exogenous* and random demand function for permits (Chao and Wilson (1993)). In Baldursson and von der Fher (1999), uncertainty is due to the entry and exit of polluting firms.

These studies point out an important possibility. However, since permit price is directly determined by firms' abatement costs through (efficient) permit trading, a major force behind price randomness is the cost uncertainties.[2] Such cost uncertainties will affect the investment decisions

[1]That irreversibility and uncertainty (and future learning) reduces investment is a standard conclusion of real option theory (Arrow and Fisher (1974) and Dixit and Pindyck (1994)).

[2]Throughout this paper, we will maintain the efficient permit trading hypothesis. This hypothesis is confirmed in

under other policies as well. TEPs do not create uncertainties in its own right, but rather "transmit" cost uncertainties into permit prices. Thus the relevant question is, compared with other policies, whether cost uncertainties reduce the investment incentive by a larger amount under TEPs when the permit price is endogenously determined by abatement costs through permit trading.

In this paper, we introduce a general equilibrium model of permit trading by price taking firms with stochastic abatement costs and rational expectations about permit prices. In each period, the government grandfathers a fixed number of emission permits. The only exogenous factors in the model are abatement cost shocks. Given the (marginal) costs, efficient permit trading endogenously determines the equilibrium permit price. A firm can invest in capital or technology to reduce its abatement cost. The investment is irreversible. The aggregate investment behavior of the firms (together with the cost shocks) determine the time path of the permit price.

Thus, our model differs from the literature in that price uncertainty is endogenously determined by abatement cost uncertainties in the general equilibrium. In particular, cost shocks change the price *instantaneously* through permit trading and *overtime* through capital or technological investment. Our model captures several salient features of a TEP system. First, (arguably) the most important determinant of permit price is the firms' abatement costs. Firms' input, output and entry/exit decisions do affect permit price, but mainly indirectly through altering the abatement costs. For example, railway deregulation in the U.S. raised the use of low sulfur coal by the utility companies, contributing to the lower-than-expected SO_2 permit price (Burtraw (1996)). Here the regulatory change reduced permit price through lowering the (marginal) abatement costs. We model the cost shocks without restricting them to be from a particular source. Second, a TEP system is in essence similar to a pure exchange economy with fixed endowment of permits. There are no exogenous permit demand or supply functions. Rather, firms *choose* to be permit suppliers

one of the best known TEP systems, the SO_2 trading in the U.S. (Joskow, Schmalensee and Bailey (1998)).

or buyers through investment. Finally, capital or technological investments are difficult to reverse. For example, a utility company will find it costly to get rid of a scrubber it has installed.

We use our model to study how firms' investment incentive responds to industry and firm specific abatement cost uncertainties. There is a sizeable literature on investment decisions under uncertainty and irreversibility, such as Arrow and Fisher (1974), Henry (1974) and Kolstad (1996). In partial equilibrium models with exogenously given price processes, they find that increased uncertainty reduces investment incentive for risk neutral firms. Since the investment is irreversible, firms may find it optimal to hold back their investment (i.e. wait) until the cost shocks are high enough to justify immediate action. Introducing general equilibrium greatly complicates the analysis, mainly because it is difficult to directly search for the equilibrium permit price process. Further, it is not clear whether uncertainty, especially industry-wide uncertainty, will reduce investment. The reason is that if one firm waits, other firms may invest and consequently drive down the permit price, making further investment suboptimal. That is, facing industry shocks, the firms may "compete" for the investment opportunity, reducing the value of waiting and consequently raising the investment incentive. Leahy (1993), Caballero and Pindyck (1996) and Baldursson and Karatzas (1997) showed that this concern does not matter in models of firms making entry and exit decisions facing exogenous demand shocks in competitive equilibrium. The firms may "pretend" that the price will not be affected by other firms' investment, and uncertainty still reduces investment. Our model is different in both the form of uncertainty and the firm decisions. We show that their results, with some modification, still apply to our case.

We then consider firms' investment strategies facing an emissions charge/subsidy that is constant overtime. Following the tradition of Milliman and Prince (1989) and Jung et al. (1996), we choose the charge policy to be "comparable" to the permit policy in that they lead to the same abatement levels *in the current period.* In a deterministic model, future abatement levels will diverge under the tax and permit policies since the policies lead to different investment paths. This policy difference is the general equilibrium effect of permits where equilibrium permit price goes down as firms invest. When abatement costs are stochastic, abatement levels can diverge even without the general equilibrium effect since tax is a price tool and permit is a quantity tool (Weitzman (1974)). We call this policy difference the price-vs-quantity effect. We will separate the two effects in comparing firm investment incentive under the two policies. We find that uncertainty reduces, but does not eliminate, the general equilibrium effect: the investment paths under the two policies converge as uncertainty level increases. Except for the curvature of the payoff functions, uncertainty also reduces the price-vs-quantity effect. Thus TEPs help maintain firms' investment incentive under uncertainty relative to charges.

Like many papers on abatement capital or technological investment, such as Magat (1978), Milliman and Prince (1989), Jung et al. (1996), Farzin, Huisman and Kort (1998) and Farzin and Kort (2000), we only address the positive question of "what happens" under different policies when there is cost uncertainty and investment irreversibility. We do not tackle the normative issue of what constitutes an optimal policy. In fact, we take a rather static view of the policies themselves: the permit and tax levels are fixed throughout time, regardless of firms' investment and cost shocks. These policies are likely to be inefficient, but may resemble the real world better than policies that adjust frequently to investment and cost shocks.

There seems to be a long-standing consensus among (at least) environmental economists that an efficient environmental policy should encourage firms, in the long run, to invest in abatement capital or technology (see, for example, Kneese and Schultze (1975) and Kemp and Soete (1990)). From a purely theoretical standpoint, investment decisions and policy efficiency do not have to be related. After all, it is the environmental externality that the policy is trying to correct. If the policy successfully does so and if there is no distortion in other sectors of the economy, investment decisions should be left to the firms themselves and should be determined by market forces. That is, environmental policy should not even attempt to influence firms' investment incentive.

To the best of our knowledge, there does not exist a formal investigation into why environmental policies should encourage such investment. There are, however, some peripheral evidence that points to possible explanations. If traditionally environmental externalities have been "under-regulated" in the sense that the policies have corrected only part of the externalities, more investment helps reduce the "inefficiency" of these policies by ameliorating the environmental problem and the need for strict regulation.[3] That is, in the long run, (lax) environmental regulation that encourages more investment should be more efficient. Another possibility is that regulators may be subject to "hold-up" by firms who anticipate less strict regulation if they do not invest and thus keep their abatement expensive (Gersbach and Glazer (1999)). In this case, policies that encourage investment help reduce this hold-up problem, and tend to be more efficient. Further, there may be information spillover from adopters of new technologies to potential adopters, so there is less than socially optimal adoption. Empirically, firms have been perceived not to be willing to invest up to the socially optimal level, leading in part to the introduction of "technology-forcing" regulation in certain cases (such as mobile source air pollution). The relevance of our paper for policy analysis should be viewed in this broad context of regulation that targets the environmental externality itself and (indirectly) the long-run investment incentive.

The paper is organized as follows. We construct the general equilibrium model of permit trading in Section 2. We solve for the firms' optimal investment strategies under permits in Section 3, and under an equivalent charge policy in Section 4. We discuss the generality of our model in Section 5, and conclude the paper in Section 6.

[3] While people may disagree about whether we have too much or too little regulation, the fact that many environmental problems are getting worse over time and new regulations are constantly being introduced does point to the possibility of insufficient regulation.

2 Model Setup: Investment Under Permits

Irreversible investment models under uncertainty can quickly become intractable, even without the added difficulty of handling a rational expectations general equilibrium. We will assume special functional forms in order to obtain analytical results. We will discuss the implications of these assumptions in Section 5, showing that they are not likely to change our major conclusions. But at the beginning, we work with more general functions to define and characterize the competitive equilibrium.

Consider a tradable emissions permit market consisting of N price taking firms with rational expectations about permit prices. We focus on emissions trading and ignore firms' output decisions.[4] Let the total abatement cost (TAC) of firm n be $C(a_n, K_n, n, \epsilon_n, \epsilon_0)$, where a_n is the abatement level, K_n the stock of abatement capital or technology, ϵ_n the firm specific shock, and ϵ_0 the industry shock affecting every firm in the TEP market. By allowing TAC to depend on n, we account for the heterogeneity of the firms, a major advantage of tradable permits. We assume that the cost is increasing and convex in the abatement level: $C_a > 0$ and $C_{aa} > 0$. Capital or technological stock reduces the cost, but at a decreasing rate: $C_K < 0$ and $C_{KK} > 0$. Positive firm and industry shocks increase the cost, but also make capital or technological investment more worthwhile: $C_{\epsilon_n} > 0$, $C_{K_n\epsilon_n} < 0$, $C_{\epsilon_0} > 0$, and $C_{K_n\epsilon_0} < 0$.[5]

We consider firm decisions in continuous time over $[0, \infty)$. We assume that firm specific and industry shocks follow independent generalized Brownian motions:

$$d\epsilon_n = \alpha_n(\epsilon_n, t)dt + \sigma_n(\epsilon_n, t)dz_n(t), \quad n = 0, 1, \ldots, N, \tag{1}$$

where $dz_n(t)$ is the incremental Wiener process, with $E(dz_n(t)) = 0$, $\text{var}(dz_n(t)) = dt$, and

[4]Firms may be in different industries and produce different kinds of outputs. Requate (1998) studies specifically the relationship between output choice and permit trading decisions.

[5]This last assumption is not critical for our general results. Since a random shock can be equally high or low, the effects of cost uncertainty on investment will not change even if we reverse one or more conditions in this assumption.

$\mathrm{cov}(dz_n, dz_m) = 0$ whenever $n \neq m$. Random change in ϵ_0 represents the industry shock and that in ϵ_n represents firm n's specific shock, for $n = 1, \ldots, N$. The term α_n is the trend of ϵ_n, and can be either positive, zero or negative. The term σ_n measures the degree of uncertainty of future ϵ_n values. Firm specific shocks may be caused by the randomness in a firm's internal production process, and industry shocks may be due to the prices of some common inputs used by all the firms. We assume that these shocks are independent of each other.

At any moment t, firm n observes $K_n(t)$, $\epsilon_n(t)$, and $\epsilon_0(t)$ and thus knows its own TAC function. Based on the TAC functions, or the marginal abatement cost (MAC) functions, firms trade permits until the MACs are equalized across all firms. (We assume that the trading is efficient.) The equilibrium permit price depends only on the total number of permits and not on their distribution across firms. Let $\bar{e}$ be the total number of permits distributed by the government, $\bar{e}_n$ be firm n's free permits, and e_n^0 firm n's emission without abatement, all constant overtime. Then

$$\sum_n a_n(t) = \sum_n \left(e_n^0 - \bar{e}_n\right) = \sum_n e_n^0 - \bar{e} \equiv \bar{a} \qquad \forall t \geq 0, \tag{2}$$

where $\bar{a}$ is the total industry abatement. Note that there is a one-to-one correspondence between $\bar{e}$ and $\bar{a}$, so that we can use $\bar{a}$ to represent the government's permit policy. Firm n's total cost (including TAC and permit cost) is given by

$$D(p, K_n, n, \epsilon_n, \epsilon_0) = C(a_n(p, K_n, n, \epsilon_n, \epsilon_0), K_n, n, \epsilon_n, \epsilon_0) + p(e_n^0 - a_n(p, K_n, n, \epsilon_n, \epsilon_0) - \bar{e}_n). \tag{3}$$

The equilibrium permit price equals firms' MACs, and can be written as

$$p = p^*(\{K_n, \epsilon_n\}_{n=1}^N, \epsilon_0, \bar{a}). \tag{4}$$

Firm n can invest in capital or technology to increase its stock K_n. The investment cost function is linear in the investment level, with the unit cost given by κ. Linearity implies that the stock can be non-differentiable (although continuous) in time: if the current stock is too low, firm n can instantaneously adjust the stock to its desired level.

In addition to the instantaneous permit market equilibrium, we need to specify the intertemporal competitive equilibrium of capital or technological investment. Suppose there is a permit price process $\{p(t), t \geq 0\}$, and further $\{p(t), K_n(t), \epsilon_n(t), \epsilon_0(t)\}$ contains all of the information about the future that affects firm n's payoff. A sufficient condition for the latter condition to hold is that $\{p(t), K_n(t), t \geq 0\}$ is Markovian, which we will confirm later. Assuming the firm is risk neutral,[6] its optimal decision on investment is given by

$$V(p(t), K_n(t), \epsilon_n(t), \epsilon_0(t)) \equiv$$
$$\max -E \int_t^{\infty} D(p(\tau), K_n(\tau), n, \epsilon_n(\tau), \epsilon_0(\tau)) e^{-r(\tau-t)} d\tau - \sum_w \kappa(K_n(w^+) - K_n(w^-)) e^{-r(w-t)}, \quad (5)$$

subject to (1), the price process $p(t'), t' \geq t$, and $K_n(w^+) > K_n(w^-)$. The discount rate is r, and w's are the instants when investment occurs, with w^- and w^+ representing the instants just before and after the investment.

Given K_0, the optimization problem generates the optimal investment strategies

$$K_n^*(t) = K_n^*(p(t), K_n(t), \epsilon_n(t), \epsilon_0(t)), \quad n = 1, \ldots, N. \quad (6)$$

It measures the optimal level of stock in period t given the information available. From (4), the rational expectations competitive equilibrium price is given by

$$p(t) = p^*(\{K_n^*(t), \epsilon_n(t)\}_{n=1}^N, \epsilon_0, \bar{a}). \quad (7)$$

Equations (6) and (7) completely characterize the competitive equilibrium. Since $\epsilon_n(t)$ and $\epsilon_0(t)$ are Markovian, we know the resulting $\{p(t), K_n(t)\}$ is also Markovian.

Directly solving the competitive equilibrium proves to be too hard a problem. Instead, we

[6] Our result does not depend on the assumption of risk neutrality. When the firms are risk averse, we can either use the risk adjusted discount rate or use risk neutral probabilities and the riskless discount rate if there are traded assets that can span the risks.

rely on the equivalence between the competitive equilibrium and a "social planner's problem" of maximizing the total firm payoffs subject to the shocks and permit policy (Lucas and Prescott (1971) and Baldursson and Karatzas (1997)). We have to qualify that the social planner is *not* maximizing the social welfare, which would include the pollution damage (or even the choice of an appropriate policy). Rather, we introduce the social planner only as a convenient way of solving the competitive equilibrium, and consequently restrict the planner to maximize the firm payoffs only.

2.1 The Social Planner's Problem

From (2) and (3), we know $\sum_n D(p, K_n, n, \epsilon_n, \epsilon_0) = \sum_n C(a_n, K_n, n, \epsilon_n, \epsilon_0)$. That is, *when all permits $\bar{e}$ are freely distributed by the government*, the social planner can simply minimize the total expected abatement cost:[7]

$$\begin{gathered}\max_{\boldsymbol{K},\boldsymbol{a}} -E\int_0^\infty \sum_n C(a_n(t), K_n(t), n, \epsilon_n(t), \epsilon_0(t))e^{-rt}dt - \sum_w \sum_n \kappa(K_n(w^+) - K_n(w^-))e^{-rw} \\ \text{subject to} \quad \sum_n a_n(t) = \bar{a}, \quad \text{equation (1)}, \quad K_n(w^+) \geq K_n(w^-).\end{gathered} \tag{8}$$

The vector $\boldsymbol{K} = \{K_1, \ldots, K_N\}$ describes the firms' stocks and $\boldsymbol{a} = \{a_1, \ldots, a_N\}$ represent the firms' abatement levels. Time indices w's are the instants at which at least one firm invests.

Again, the optimization involves two steps. First, at each moment t, the planner needs to allocate $\bar{a}$ permits among the N firms, given $\boldsymbol{K}$, $\boldsymbol{\epsilon} = \{\epsilon_1, \ldots, \epsilon_N\}$, and ϵ_0. The resulting minimal social abatement cost is

$$S(\boldsymbol{K}, \boldsymbol{\epsilon}, \epsilon_0, \bar{a}) = \min_{\boldsymbol{a}} \left\{ \sum_n C(a_n, K_n, n, \epsilon_n, \epsilon_0), \quad s.t. \quad \sum_n a_n = \bar{a} \right\}. \tag{9}$$

To facilitate the following dynamic optimization problem, we impose the condition that $S(\cdot)$ is

[7]If some of the permits are auctioned at the market price, the equivalent social planner's objective function must include the cost of purchasing these permits. The analysis becomes more complicated because the marginal abatement cost enters the objective function directly (representing the permit price).

convex in $\boldsymbol{K}$; note that $C(\cdot)$ is convex in K_n. In the second step, we rewrite the problem in (8) by substituting in the optimal permit allocation:

$$
\begin{aligned}
&J(\boldsymbol{K}(t), \boldsymbol{\epsilon}(t), \epsilon_0(t), \bar{a}, \kappa) \equiv \\
&\max_{\boldsymbol{K}} -E\int_t^{\infty} S(\boldsymbol{K}(\tau), \boldsymbol{\epsilon}(\tau), \epsilon_0(\tau), \bar{a}) e^{-r(\tau-t)} d\tau - \sum_w \sum_n \kappa(K_n(w^+) - K_n(w^-)) e^{-r(w-t)}
\end{aligned} \tag{10}
$$

subject to equation (1) and $K_n(w^+) \geq K_n(w^-)$. We solve the problem following Dixit and Pindyck (1994). To reduce clutter, we ignore $\bar{a}$ and κ in $J(\cdot)$ whenever it is convenient. Appendix A shows that

Proposition 1 *The optimal stock $\boldsymbol{K}'$ satisfies the following complementary slackness condition:*

$$
J_{K_n'}(\boldsymbol{K}', \boldsymbol{\epsilon}, \epsilon_0) - \kappa \leq 0, \quad K_n' - K_n \geq 0, \quad \left(J_{K_n'}(\boldsymbol{K}', \boldsymbol{\epsilon}, \epsilon_0) - \kappa\right)\left(K_n' - K_n\right) = 0, \quad \forall n. \tag{11}
$$

The proposition states that whenever $J_{K_n} > \kappa$, more abatement capital is needed (because its marginal value exceeds its marginal cost κ), and firm n should instantaneously invest until the new stock K_n' satisfies $J_{K_n'}(\boldsymbol{K}', \boldsymbol{\epsilon}, \epsilon_0) = \kappa$; note that $J(\cdot)$ is concave in K_n (Appendix A), thus higher K_n reduces J_{K_n}. If $J_{K_n} < \kappa$, irreversibility means that the stock will not be changed. As shocks $\boldsymbol{\epsilon}$ and ϵ_0 change J_{K_n} overtime, $J_{K_n}(\boldsymbol{K}, \boldsymbol{\epsilon}, \epsilon_0) = \kappa$ acts as a barrier to capital adjustment: J_{K_n} can never exceed κ for a positive length of time. Whenever the shocks raise J_{K_n} above κ, instantaneous investments are undertaken to restore the equality. Since $\boldsymbol{\epsilon}(t)$ and $\epsilon_0(t)$ are not differentiable, the resulting $K_n(t)$ is not differentiable whenever firm n invests.

The remaining task is to determine the function $J(\cdot)$. Suppose the state $(\boldsymbol{K}, \boldsymbol{\epsilon}, \epsilon_0)$ is such that no investment is needed for any firm (the continuation region). The Bellman equation is

$$
J(\boldsymbol{K}, \boldsymbol{\epsilon}, \epsilon_0) = -S(\boldsymbol{K}, \boldsymbol{\epsilon}, \epsilon_0, \bar{a})dt + e^{-rdt}\left\{E[J(\boldsymbol{K}, \boldsymbol{\epsilon} + d\boldsymbol{\epsilon}, \epsilon_0 + d\epsilon_0)]\right\}.
$$

Applying Ito's lemma and using the fact that the shocks are independent, we obtain the following partial differential equation

$$\sum_{n=0}^{N}\left\{\frac{1}{2}\sigma_n(\epsilon_n,t)^2 J_{\epsilon_n\epsilon_n}(\boldsymbol{K},\boldsymbol{\epsilon},\epsilon_0)+\alpha_n(\epsilon_n,t)J_{\epsilon_n}(\boldsymbol{K},\boldsymbol{\epsilon},\epsilon_0)-rJ(\boldsymbol{K},\boldsymbol{\epsilon},\epsilon_0)\right\}-S(\boldsymbol{K},\boldsymbol{\epsilon},\epsilon_0,\bar{a})=0. \quad (12)$$

The optimality conditions in (11) imply the following boundary conditions:

$$\text{(Value-matching)} \qquad J_{K_n}(\boldsymbol{K},\boldsymbol{\epsilon},\epsilon_0)=\kappa, \quad n=1,\ldots,N, \quad (13)$$

$$\text{(Smooth-pasting)} \qquad J_{K_n\epsilon_m}(\boldsymbol{K},\boldsymbol{\epsilon},\epsilon_0)=0, \quad n=1,\ldots,N, \quad m=0,1,\ldots,N, \quad (14)$$

where $\boldsymbol{K}$ is evaluated at the investment barrier $\boldsymbol{K}^b(\boldsymbol{\epsilon},\epsilon_0)$ to be determined jointly with the function $J(\cdot)$. (In particular, $\boldsymbol{K}^b$ is given by $J_{K_n}(\boldsymbol{K}^b,\boldsymbol{\epsilon},\epsilon_0)=\kappa$, $\forall n$.) The social planner's optimal solution is completely characterized by (12) - (14).

2.2 Special Functional Forms

To solve (12) - (14) analytically, we make specific assumptions about the stochastic processes of $\boldsymbol{\epsilon}$ and ϵ_0 and the cost function $C(\cdot)$. For the balance of the paper, we assume that $\boldsymbol{\epsilon}$ and ϵ_0 follow geometric Brownian motions. That is,

$$\alpha_n(\epsilon_n(t),t)=\alpha_n\epsilon_n(t); \qquad \sigma_n(\epsilon_n(t),t)=\sigma_n\epsilon_n(t). \quad (15)$$

To make the problem interesting, we impose $\alpha_n < r$, $n=0,1,\ldots,N$. Otherwise, the cost of abatement would increase too quickly to allow any capital or technological investment. We assume that firm n's abatement cost is quadratic in the following form:

$$C(a_n,K_n,n,\epsilon_n,\epsilon_0)=\frac{1}{2}c(K_n,n)\epsilon_0 a_n^2+d(K_n,n)\epsilon_n, \quad n=1,\ldots,N, \quad (16)$$

with $c_{K_n}<0$, $c_{K_nK_n}>0$, $d_{K_n}<0$ and $d_{K_nK_n}>0$. $c(K_n,n)\epsilon_0$ is the *unit* marginal abatement cost, and $d(K_n,n)\epsilon_n$ is the fixed cost of abatement. The industry shock affects both the total and marginal costs of abatement, while the firm specific shock only affects the total cost. As we show later on, not allowing ϵ_n to affect the marginal abatement cost enables us to obtain a clean and intuitive solution to the optimization problem.

Substituting (16) into (9), we know cost minimization requires

$$c(K_n, n)a_n = c(K_m, m)a_m = \frac{p}{\epsilon_0}, \quad m, n = 1, \cdots, N; \qquad \sum_n a_n = \bar{a}, \tag{17}$$

where p is the shadow value of total abatement $\bar{a}$, which is also the equilibrium permit price. Further, we can rewrite the social cost as

$$S(\boldsymbol{K}, \boldsymbol{\epsilon}, \epsilon_0, \bar{a}) = L(\boldsymbol{K}, \bar{a})\epsilon_0 + \sum_{n=1}^{N} d(K_n, n)\epsilon_n, \tag{18}$$

where $L(\boldsymbol{K}, \bar{a}) = \sum_n \frac{1}{2} c(K_n, n) a_n(\boldsymbol{K}, \bar{a})^2$. Since $d(K_n, n)$ is convex in K_n, to guarantee that $S(\cdot)$ is convex in $\boldsymbol{K}$, we impose the sufficient condition that $L(\cdot)$ is convex in $\boldsymbol{K}$. Appendix B shows other characteristics of $L(\cdot)$.

3 Optimal Investment Under TEPs

We solve for the social planner's (and then the firms') optimal investment strategies based on the special functional forms. To gradually build up the intuition, we first study the effects of industry shock alone, and then reintroduce the firm specific shocks.

3.1 Industry Shock Alone

In this section, we assume that there are no firm specific shocks, in particular, $\epsilon_n = 1$, $n = 1, \ldots, N$. Appendix C shows that in the socially optimal solution, the investment barrier for firm n is

$$\epsilon_{0,n}^b(\boldsymbol{K}) = O_0^1 (r - \alpha_0) \frac{\kappa + d_{K_n}(K_n, n)/r}{-\partial L(\boldsymbol{K}, \bar{a})/\partial K_n}, \tag{19}$$

where $O_0^1 = \frac{\beta_0^1}{\beta_0^1 - 1}$, with $\beta_0^1 > 1$ being a constant decreasing in σ_0^2. Thus O_0^1 increases in σ_0^2. Further, $O_0^1 = 1$ if $\sigma_0^2 = 0$ and $\lim_{\sigma_0^2 \to \infty} O_0^1 = \infty$. Note that we defined the barrier inversely as the industry shock ϵ_0 being a function of $\boldsymbol{K}$. The barrier has several features. First, $\epsilon_{0,n}^b(\boldsymbol{K}) > 0$

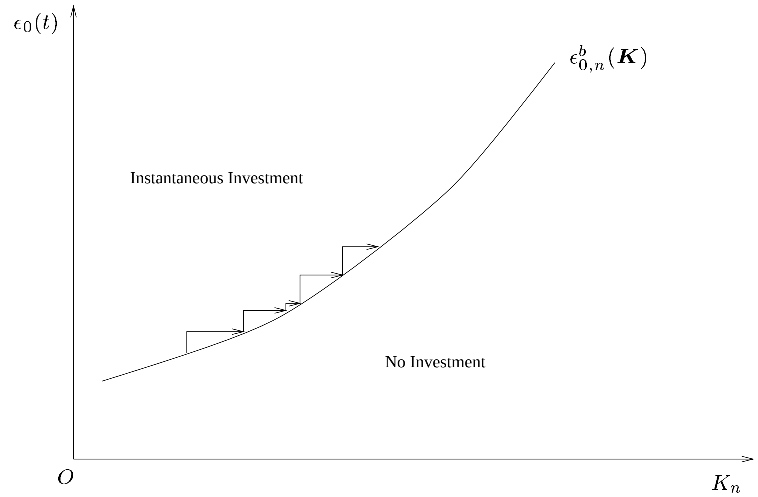


Figure 1: Barrier Control Policy for Firm n

since $\kappa > \frac{-d_{K_n}(K_n,n)}{r}$ (otherwise, the fixed abatement cost alone would justify the investment). We can also show (Appendix C) that $\frac{\partial \epsilon^b_{0,n}(\boldsymbol{K})}{\partial K_m} > 0$ for $m \neq n$. That is, if other firms already have high stocks, the social planner would have less incentive to let firm n invest when positive shock occurs. The reason is that firm n is abating less due to its low stock (or high unit cost). The cost saving from investing would then be lower. Appendix C also shows that $\frac{\partial \epsilon^b_{0,n}(\boldsymbol{K})}{\partial K_n} > 0$, i.e. firm n's investment barrier increases in its own stock.

Thus (19) says that given $\boldsymbol{K}$, firm n should invest to achieve $J_{K_n} = \kappa$ if and only if $\epsilon_0 > \epsilon^b_{0,n}(\boldsymbol{K})$. In other words, if positive shocks occur such that $\epsilon_0(t) > \epsilon^b_{0,n}(\boldsymbol{K})$, instantaneous investment should be undertaken to raise $\epsilon^b_{0,n}(\boldsymbol{K})$ to $\epsilon_0(t)$. Higher shock ϵ_0 calls for more investment because as ϵ_0 increases, the marginal value of investment (the marginal reduction in total abatement cost) also increases. However, the barrier is higher as K_n increases because of the declining returns of the stock: $L_{K_nK_n} > 0$ and $d_{K_nK_n} > 0$. Figure 1 shows the barrier and the barrier control policy for firm n, holding other firms' stocks fixed.

Equation (19) has an intuitive interpretation. If $O^1_0 = 1$, the equation simply says that the marginal cost of investment, κ, should equal the marginal benefit, which is the reduction in all future costs of abatement, equal to the sum of $\frac{-\epsilon_0 \partial L/\partial K_n}{r-\alpha_0}$, the reduction in the variable cost, and $\frac{-d_{K_n}}{r}$, the reduction in the fixed cost. The term $O_0^1 > 1$ measures the option value effect: for firm n to invest, the needed cost shock is higher by the factor O_0^1. Since O_0^1 increases in σ_0, we know that higher uncertainty raises the barrier to invest.

Now we move from the social planner's problem to those of individual firms. The investment barrier for firm n in (19) still applies in the competitive equilibrium, but it is expressed as a function of the stocks of *all firms*. This is natural for a social planner with information on all firms. But an individual firm typically only observes its own stock, its own abatement level (i.e. its trading of the permit) and the market price of permits. The investment barrier in the competitive equilibrium should reflect this information constraint. Based on (19), Appendix C derives the following investment barrier for firm n in terms of the permit price in the competitive equilibrium:

$$p_0^b(K_n, a_n) = O_0^1(r - \alpha_0)\frac{2\left[\kappa K_n - \eta_{K_n}^d d(K_n, n)/r\right]}{\eta_{K_n}^c a_n}, \tag{20}$$

where $\eta_{K_n}^c = -c_{K_n} K_n/c > 0$ is the elasticity of the abatement cost coefficient with respect to the stock, and $\eta_{K_n}^d = -d_{K_n} K_n/d > 0$ is that of the fixed abatement cost. Appendix C shows that under rather general (and appealing) conditions, p_0^b is increasing in K_n (after accounting for K_n's effect on a_n).

At any moment, the permit price is determined in (17) through efficient permit trading. When a new industry shock occurs, and before the firms invest, permit price p changes in proportion to the change in ϵ_0 (cf. (33) in Appendix B). The investment rule in (20) says that if there is a positive shock in ϵ_0 such that p rises above $p_0^b(K_n, a_n)$, firm n will invest immediately until $p_0^b(K_n, a_n)$ equals the permit price.[8] Intuitively, investment allows a firm to abate more and sell more (or buying fewer) permits, thus the firm is more willing to invest if the permit price is high. It is clear from

[8] Of course, if many firms invest, the industry marginal abatement cost decreases, lowering p. This general equilibrium effect reduces the investment needed of the firms.

(20) that

Proposition 2 *Facing only the industry-wide shock, an individual firm is less likely to invest the higher the cost of capital κ, the current stock K_n, and the level of uncertainty about the shock. Investment is more likely the higher the permit price, the firm's abatement quantity, the elasticities of marginal and fixed cost reduction from investment, and the firm's fixed abatement cost.*

It is obvious that a firm has more incentive to invest if the investment is cheaper, if it is more effective in reducing the abatement cost, or if the firm's abatement cost is already high. A firm's abatement cost is increasing in its abatement level (cf. (16)). Investment is thus more effective in cost reduction as abatement level is higher. Consequently, firms which are undertaking more abatement have higher incentive to invest.

The industry-wide uncertainty reduces a firm's incentive to invest. There are three forces underlying the option value coefficient O_0^1. Investment irreversibility and evolution of ϵ_0 provide the firm with incentive to wait for sufficiently high cost shock to actually invest. As we discussed earlier, firms do not want to delay investment for too long because other firms may grab the investment opportunity and drive down the permit price. "Competition" for investment raises the firm's investment incentive. The third factor is the general equilibrium effect: Given a large positive shock to ϵ_0, many firms will invest and the permit price will decrease. Anticipating the price reduction, each individual firm's incentive to invest goes down. It turns out that the second and third factors cancel each other out. As we will show in Section 4, the barrier in (20) is equivalent to one where the firm "pretends" that the price is *exogenously* given and is proportional to ϵ_0 (equals a constant times ϵ_0, cf. (17)). That is, in determining its investment strategy, the firm can simply ignore the competition for investment opportunity and the general equilibrium effect. This observation is consistent with the findings of Leahy (1993), Caballero and Pindyck (1996) and

Baldursson and Karatzas (1997) except that the "exogenous price" is itself random.[9]

3.2 Firm Specific and Industry Shocks

Suppose now that there is no industry shock, with $\epsilon_0 = 1$, while firm specific shocks are given in (1) and (15). Appendix D shows that the social planner's optimal decision is given by the following investment barrier for firm n, $n = 1, \cdots, N$:

$$\epsilon_{s,n}^b(\boldsymbol{K}) = O_n^1 (r - \alpha_n) \frac{(\kappa + L_{K_n}/r)}{-d_{K_n}}, \tag{21}$$

where $O_n^1 = \frac{\beta_n^1}{\beta_n^1 - 1}$ and is increasing in σ_n^2. Firm n should invest whenever its specific shock ϵ_n exceeds $\epsilon_{s,n}^b(\boldsymbol{K})$. Without uncertainty, $O_n^1 = 1$. Equation (21) can then be rewritten as $-\frac{\epsilon_{s,n}^b(\boldsymbol{K}) d_{K_n}}{r - \alpha_n} - \frac{L_{K_n}}{r} = \kappa$, which simply says that the expected marginal reduction in the present value of abatement cost from investment should equal the marginal investment cost.

Repeating the same procedure of going from (19) to (20), we obtain firm n's optimal investment barrier in the competitive equilibrium:

$$\epsilon_n^b(K_n, p, a_n) = O_n^1 (r - \alpha_n) \frac{\kappa K_n - \frac{1}{2} \eta_{K_n}^c p a_n / r}{\eta_{K_n}^d d}. \tag{22}$$

In this equation, we have effectively "separated" the investment barriers of different individual firms: even though the firms interact with each other in the competitive equilibrium, the critical value of ϵ_n for firm n to invest is independent of the shocks of other firms. This simplifying result is due to the independence among the firm specific shocks and the assumption that these shocks only affect fixed abatement costs (see Appendix D for more discussion).

Comparing (20) and (22), we see that under the industry or firm specific shocks, a firm's investment barrier responds to the same influencing factors in the same direction. The only difference

[9] We choose not to decompose the three effects analytically in this paper. These effects are important, but are not our focus. They have been dealt with in the literature under various situations, and most of the intuition applies in our model. We thus refer readers to the cited literature for the analytical decomposition.

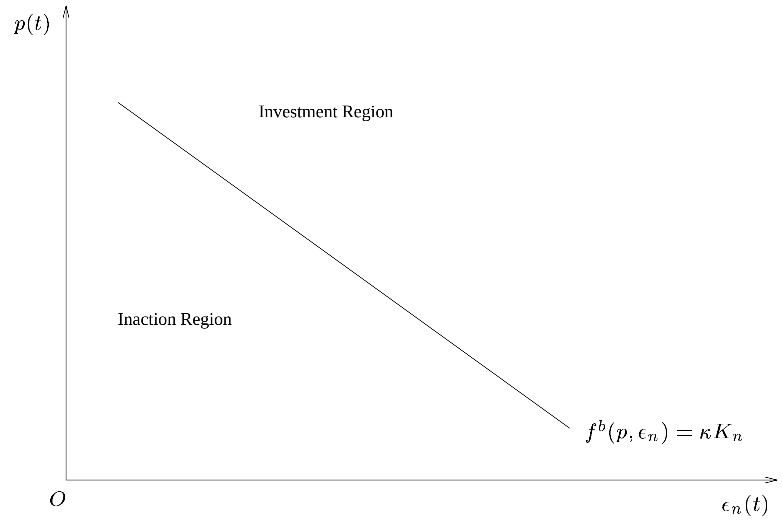


Figure 2: Firm Investment Barrier Facing Both Shocks

lies in the functional forms and the magnitude of the responses. Further, the industry and firm shocks affect the investment barriers in similar fashions. The barrier is raised by O_n^1, for $n = 0$ or $n = 1, \ldots, N$. In particular, if the firm and industry shocks follow identical and independent processes, i.e. $\alpha_0 = \alpha_n$ and $\epsilon_0 = \epsilon_n$, $n = 1, \ldots, N$, then the shocks raise the investment barriers by the same proportion.

Now we reintroduce the industry shock. Given its stock and abatement level, a firm makes its investment decision based on the observed values of both the permit price (incorporating the industry shock) and its own firm specific shock. Investment may be necessary when one of the shocks is sufficiently high, even if the other is relatively low. Through similar procedures to those in deriving $p_0^b(\cdot)$ and $\epsilon_n^b(\cdot)$, we obtain the following barrier function:

$$
\begin{aligned}
& f^b(p,\epsilon_n) = \kappa K_n, \\
\text{where} \quad & f^b(p,\epsilon_n) = \frac{1}{O_0^1}\frac{1}{2(r-\alpha_0)}\eta_{K_n}^c a_n p + \frac{1}{O_n^1}\frac{1}{r-\alpha_n}\eta_{K_n}^d d(K_n,n)\epsilon_n.
\end{aligned} \tag{23}
$$

That is, whenever positive industry and/or firm shocks occur so that $f^b(p,\epsilon_n) > \kappa K_n$, instantaneous investment is undertaken to restore the equality. No investment occurs when $f^b(p,\epsilon_n) < \kappa K_n$. Figure 2 depicts the investment barrier.

We can verify that the investment barriers $p_0^b(\cdot)$ in (20) and $\epsilon_n^b(\cdot)$ in (22) are special cases of (23). In particular, if there is only the industry shock, with $\epsilon_n = 1$, $\alpha_n = 0$ and $O_n^1 = 1$, we obtain $p_0^b(\cdot)$ from (23). If there are only firm specific shocks, with $\epsilon_0 = 1$, $\alpha_0 = 0$ and $O_0^1 = 1$, (23) reduces to (22). We noted that under either the industry or the firm specific shocks, the investment barrier for a firm is influenced by the same factors in similar fashions. The qualitative effects of these factors are preserved when both shocks are present:

Proposition 3 *When there are both industry and firm specific shocks, an individual firm's incentive to invest is decreasing in the cost of capital κ, the current stock K_n, and the uncertainty levels of both the industry and its own shocks. It is increasing in the permit price level, the firm specific shock, the firm's current abatement level, the elasticity of cost reduction to investment, and the firm's fixed abatement cost.*

4 Optimal Investment Under Emission Charges

In this section, we turn to the policy of an emission tax (or equivalently an abatement subsidy) that remains constant over time and compare firms' investment incentive under the tax and permit policies. Under the tax/subsidy system, each firm's abatement and investment decisions are independent of those of other firms, since the payoff from abatement (through reduced charges) is determined by the fixed tax rate. The model is simpler without the general equilibrium requirement: we only need to study how a representative firm n responds to the shocks ϵ_0 and ϵ_n.

Let τ be the rate of emission tax or abatement subsidy. In each period, given its capital or technological stock K_n, firm n's decision on abatement level is

$$\max_{a_n} -\frac{1}{2}c(K_n, n)\epsilon_0 a_n^2 - d(K_n, n)\epsilon_n + \tau a_n, \tag{24}$$

which implies that $a_n^* = \frac{\tau}{c(K_n, n)\epsilon_0}$. To make the tax comparable to the permit policy, we set

$\tau = p^*(\boldsymbol{K}(0), \epsilon_0(0), \bar{a})$: given the *current* stocks and shocks, the two policies lead to the same abatement level.

Since τ is fixed, a shock in ϵ_0 would change the abatement level even without affecting the stock K_n. However, we noted in (17) that under the permit policy, an industry shock leads to a proportional permit price change and does not affect the abatement level prior to the firm's investment. The difference arises because tax is a price tool and permit is a quantity tool, and the case has been analyzed in a more general setting in Weitzman (1974). To facilitate our analysis, we decompose the differences in firm investment strategies into two parts: the *general equilibrium effect* and the *price-vs-quantity effect*. In particular, we consider a tax policy where the tax rate would fluctuate directly with the industry shock: $s = b\epsilon_0$ with $b = \tau/\epsilon_0(0)$. The constant b represents the "real" tax (or subsidy) the firms face: it fixes the "real" marginal cost of each firm, regardless of the industry shock. We will show that the difference between policies s and $\bar{a}$ captures the general equilibrium effect and that between s and τ captures the price-vs-quantity effect.

4.1 The General Equilibrium Effect

Substituting $a_n^* = \frac{b}{c(K_n,n)}$ into (24), we obtain the firm's per period payoff as

$$S_n(K_n, \epsilon_n, \epsilon_0, b) = \frac{1}{2}\frac{b^2}{c(K_n, n)}\epsilon_0 - d(K_n, n)\epsilon_n. \tag{25}$$

The payoff increases in ϵ_0: higher industry shock raises the subsidy rate s the firms receive. Adopting the same approach as the social planner's problem in the last section, we get the firm's investment barrier

$$f^b(s, \epsilon_n) = \kappa K_n, \tag{26}$$

where $f^b(\cdot)$ is given in (23), and is increasing in both of its arguments. Thus, if either an industry or a firm specific shock occurs so that $f^b(b\epsilon_0, \epsilon_n^b) > \kappa K_n$, firm n will invest to restore the equality.

The firm's investment *strategy* under fluctuating tax is the same as that under permits. This observation confirms our earlier discussion that under permits, the firm can "pretend" that the permit price is exogenously set at a level proportional to ϵ_0 and ignore the general equilibrium effect. However, identical investment *strategies* do not necessarily lead to the same investment *levels* under the two policies. Under TEPs, when some or all firms invest, permit price p decreases, reducing $f^b(p, \epsilon_n)$ and the required investment. Under the fluctuating tax policy, the tax rate $s = b\epsilon_0$ remains fixed. This general equilibrium effect under permits is the only source of difference between the investment paths under the two policies. If abatement costs are constant over time (i.e. no uncertainty), the coefficients $O_i^1 = 1$ and $\alpha_i = 0$, for $i = 0, n$. Then the difference between (23) and (26) corresponds precisely to the deterministic analysis in Milliman and Prince (1989) and Jung et al. (1996). Our interest is to investigate how this difference depends on the uncertainty levels of ϵ_0 and ϵ_n.

It is informative to start with special cases. Suppose there is no industry shock with $\epsilon_0 = 1$ so that $s = b$. Then (26) is reduced to (22) with p replaced by s. That is, given $\boldsymbol{K}$, the minimum shock to ϵ_n required for firm n to invest is the same under equivalent fluctuating tax and permit policies (i.e. when $s = p$). Since the firm specific shocks are independent, at each instant there is a strictly positive probability that some other firms will invest (as long as $\boldsymbol{K} < \infty$). Strictly speaking, the probability of investment by any other firm is

$$\Pr\left\{\epsilon_i(t) > \epsilon_i^b, \quad \text{for some} \quad i \neq n\right\} = 1 - \prod_{i \neq n} \Pr\left\{\epsilon_i(t) \leq \epsilon_i^b\right\} > 0. \tag{27}$$

That is, if ϵ_n changes such that firm n decides to invest, it is possible that other firms also invest, reducing the permit price p. Then the investment level of firm n will be smaller under permits than under the fluctuating tax policy with strictly positive probability. The difference of course is due to the general equilibrium effect under permits.

Suppose the uncertainty level σ_i^2 increases for all $i = 1, \ldots, N$. Then ϵ_i^b increases, but as Sarkar (2000) and Dixit and Pindyck (1994) showed, $\Pr\{\epsilon_i(t) > \epsilon_i^b\}$ may actually increase in some cases. That is, if ϵ_i becomes more volatile, the barrier may be "hit" more frequently even though the barrier itself is higher, increasing the expected investment. Whether or not this scenario arises in abatement investment is an empirical issue. Our paper is motivated by the concern that uncertainty reduces investment, and we therefore assume that this probability is decreasing in σ_i^2. That is, as firm specific shocks become more volatile, it is less likely that other firms will invest or permit price p will decrease. Then firm n's (expected) investment level under permits will be closer to that under fluctuating tax. In the extreme, if $\sigma_i^2 \to \infty$, no firm will invest and the investment paths are identical under the two policies. Uncertainty reduces, but does not eliminate, the general equilibrium effect discussed in Milliman and Prince (1989) and Jung et al. (1996).

Now we consider the special case of industry-wide shock alone. With $\epsilon_n = 1$, (26) is simplified to (20). When an industry shock occurs so that $p = s > p^b(K_n, a_n)$, firm n invests under both policies. If there are other firms which also want to invest, i.e. if $p = s > p^b(K_i, a_i)$ for some $i \neq n$, permit price p decreases, reducing the magnitude of firm n's investment under permits. Again, as uncertainty σ_0^2 increases, the investment barrier $p^b(K_i, a_i)$ increases and we only consider the case where the probability that firm i invests decreases. As the industry shock becomes more volatile, it is less likely that other firms also invest or price p decreases. Then firm n's investment increases and is closer to that under the fluctuating tax policy. Uncertainty in ϵ_0 again reduces, but does not eliminate, the difference between investment levels under the permit and fluctuating tax policies. In summary,

Proposition 4 *Investment levels tend to be higher under the fluctuating tax s than under the permits $\bar{a}$. In the case where uncertainty reduces the probability of investment, the difference in investment levels is reduced, but not eliminated, by both the industry and firm-specific cost uncer-*

tainties.

4.2 The Price-vs-Quantity Effect

Given firm n's stock K_n, the tax rate τ fixes the marginal abatement cost, i.e. a price tool, and the fluctuating rate s fixes the abatement level, i.e. a quantity tool. Firms do not interact with each other, thus the only difference between τ and s is due to the price-vs-quantity effect.

Substituting $a_n^* = \frac{\tau}{c(K_n,n)\epsilon_0}$ into (24), we know firm n's instantaneous payoff rate is

$$T_n(K_n, \epsilon_n, \epsilon_0, \tau) = \frac{1}{2}\frac{\tau^2}{c(K_n,n)\epsilon_0} - d(K_n,n)\epsilon_n. \tag{28}$$

In contrast to policy s, the payoff is decreasing and convex in the industry shock. Further, higher industry shock ϵ_0 *reduces* the effectiveness of investment in increasing the payoff. Appendix E shows that firm n's investment barrier is

$$\begin{aligned} &g^b(\epsilon_0, \epsilon_n) = \kappa K_n, \\ \text{with} \quad &g^b(p, \epsilon_n) = \frac{O_0^2}{2(r - \sigma_0^2 + \alpha_0)}\frac{\tau^2}{c(K_n,n)}\frac{1}{\epsilon_0}\eta_{K_n}^c + \frac{1}{O_n^1}\frac{1}{r-\alpha_n}\eta_{K_n}^d d(K_n,n)\epsilon_n, \end{aligned} \tag{29}$$

where $O_0^2 = \frac{\beta_0^2+1}{\beta_0^2} > 0$ is the option value coefficient. That is, whenever *negative* industry and/or positive firm shocks occur so that $g^b(\epsilon_0, \epsilon_n) > \kappa K_n$, instantaneous investment is undertaken to restore the equality.

For the problem to be interesting (in particular for investment to be finite), we impose the condition that $r > \sigma_0^2 - \alpha_0$ (Appendix E). Then we can show that $\beta_0^2 < -1$ and is increasing in σ_0^2. That is, $0 < O_0^2 < 1$ and is decreasing in σ_0^2. From (29), we know

Proposition 5 *Under the constant emissions charge τ, a firm is more likely to invest when the industry shock ϵ_0 is low and/or the firm shock ϵ_n is high. Its investment incentive is decreasing in the cost of capital κ, the current stock K_n, and the uncertainties in both the industry and firm shocks. The incentive is increasing in the tax level τ, and the effectiveness of investment in reducing*

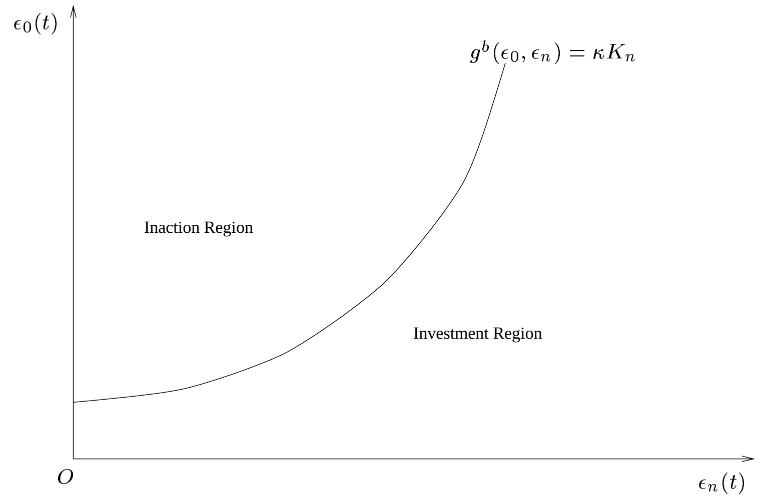


Figure 3: Investment Barrier Under Tax τ

the costs.

Comparing Propositions 3 and 5 and equations (23) and (29) indicates that the investment incentive under permits $\bar{a}$ and charges τ is subject to similar exogenous influencing factors in similar fashions. The only difference is that under charges, it is the negative, instead of the positive, industry shock that causes more investment. The reason is that higher ϵ_0 actually reduces the marginal benefit of stock K_n (cf. (28)). Figure 3 graphs the investment barrier under τ: investment occurs when ϵ_0 is low or ϵ_n is high.

The price-vs-quantity effect is fully reflected by the difference in the investment barriers under s and t, i.e. the difference between (26) and (29). Since firm shock ϵ_n does not affect the abatement level, its impact on the investment incentive is the same in (26) and (29), and the price-vs-quantity effect does not exist for ϵ_n. To streamline our analysis, we focus on the industry shock and assume $\epsilon_n = 1$. Let $E_n = 2(\kappa K_n - \eta^d_{K_n} d(K_n, n)/r)c(K_n, n)/\eta^c_{K_n}$, which is independent of σ_0^2. From (26) and (29), we can rewrite the barriers under s and τ as

$$\epsilon_0^s = E_n O_0^1 \frac{r-\alpha_0}{b^2} \tag{30}$$

$$\epsilon_0^\tau = \frac{\tau^2}{E_n} \frac{O_0^2}{r-\sigma_0^2+\alpha_0}. \tag{31}$$

We know ϵ_0^s increases and ϵ_0^τ decreases in σ_0^2.

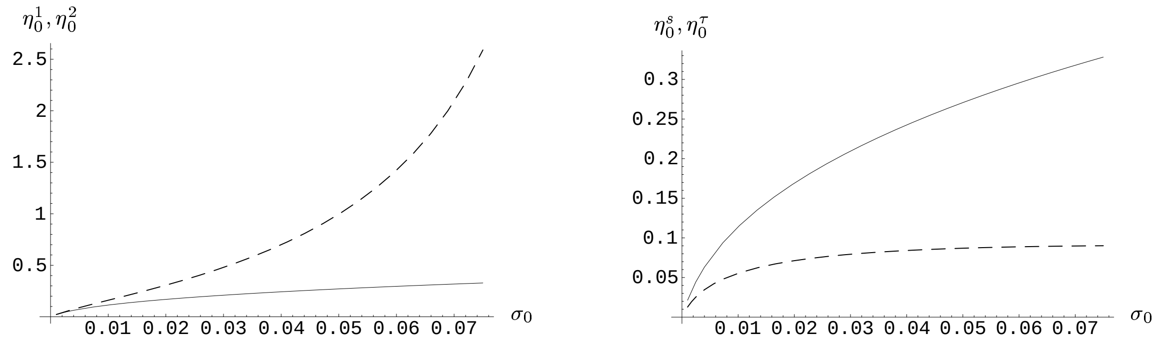
To investigate how uncertainty changes the (expected) investment level under the two policies, we need to find out how uncertainty affects the barriers ϵ_0^s and ϵ_0^τ, as well as the probabilities that these barriers are exceeded by ϵ_0 as in (27). Similar to the arguments leading to Proposition 4, we only consider cases where uncertainty reduces the expected investment, and impose the condition that the expected investment is reduced whenever the barrier is raised. Then we study only the effects of σ_0^2 on the barriers, and consider one policy to be *more sensitive* to uncertainty if its associated investment barrier tightens more when σ_0^2 increases.

Define the elasticities of the two barriers to σ_0^2 as $\eta_0^s = \frac{\partial \epsilon_0^s}{\partial \sigma_0^2}\frac{\sigma_0^2}{\epsilon_0^s}$ and $\eta_0^\tau = -\frac{\partial \epsilon_0^\tau}{\partial \sigma_0^2}\frac{\sigma_0^2}{\epsilon_0^\tau}$. Similarly define the elasticities of the two option value terms O_0^1 and O_0^2 as $\eta_0^1 = \frac{\partial O_0^1}{\partial \sigma_0^2}\frac{\sigma_0^2}{O_0^1}$ and $\eta_0^2 = -\frac{\partial O_0^2}{\partial \sigma_0^2}\frac{\sigma_0^2}{O_0^2}$. From (30) and (31), we know

$$\eta_0^s = \eta_0^1; \qquad \eta_0^\tau = \eta_0^2 - \frac{\sigma_0^2}{r-\sigma_0^2+\alpha_0}. \tag{32}$$

Thus the *sensitivity* of the investment barrier under the variable charge s depends entirely on the sensitivity of its option value coefficient O_0^1, independent of the current shock, the capital or technology stock, or the abatement cost. This result is natural: the only reason that a risk neutral firm cares about the cost uncertainty under s is the existence of the option value of delaying the investment. For policy τ, there is an added effect due to the "curvature" of the payoff function: it is convex in ϵ_0.[10] Thus higher uncertainty raises a firm's investment payoff through this curvature effect, offsetting (partially) the option value effect.

[10]In particular, the objective function is increasing in $\frac{1}{\epsilon_0}$, which is rising at the expected rate $\sigma_0^2 - \alpha_0$ (Appendix E).



(a) Option Value Coefficient

(b) Overall Investment Barrier

Figure 4: Elasticities: "—" under s; "- - -" under τ

Firms are reluctant to invest under the two policies for exactly opposite reasons: fearing that future values of ϵ_0 may be too low under s and too high under τ. As a result, the pure option value effects η_0^1 and η_0^2 are different under the two policies. It is difficult to compare η_0^1 and η_0^2 analytically, even though we know their functional forms. Numerical examples indicate that $\eta_0^1 < \eta_0^2$, especially when uncertainty level is high. Figure 4 shows the four elasticity measures responding to uncertainty for the case of $r = .085$ and $\alpha_0 = .02$. Panel (a) shows the comparison of η_0^1 and η_0^2. Thus, based solely on option values, uncertainty reduces the investment incentive *proportionally* more under fixed tax τ than under variable tax s.

Under τ, the curvature factor encourages investment, and reduces the effects of uncertainty in retarding investment. This factor is decreasing in r and α_0 and increasing in σ_0^2. Since we imposed a limit on the uncertainty level (i.e. $r > \sigma_0^2 - \alpha_0$), the curvature factor cannot fully offset the option value effect. But as r and α_0 decreases and uncertainty increases, the curvature factor becomes more important. In summary, we know

Proposition 6 *The price-vs-quantity effect exists only for the industry shock. The sensitivity of the investment barrier under variable tax s depends only on the option value coefficient, while that under fixed tax τ depends also on the curvature effect. Based on the option value effect,* *increased uncertainty reduces the investment incentive proportionally more under τ than under s. The curvature effect becomes more significant as the uncertainty level increases and r or α_0 decreases.*

5 Generality of the Model

There are a number of assumptions that helped us obtain the analytical results but also made our model somewhat special. In this section, we show that these assumptions do not change our major conclusions. One may argue that we did not explicitly model the decisions and shocks on the output side. However, we can interpret the abatement cost function $C(a_n, K_n, n, \epsilon_n, \epsilon_0)$ as a reduced form that already incorporated the optimal output decisions and shocks. For example, given output price and production function, the optimal output level is uniquely determined by the arguments of $C(\cdot)$. Then $C(\cdot)$ is the "net" cost that includes the cost of production, net of the revenue. If all firms face the same random output price, this random process is included in ϵ_0, and if the random output price affects individual firms, its process is incorporated in ϵ_n. Similarly, any other factors directly or indirectly affecting firms' abatement decisions (such as certain policy shocks) can be incorporated in the cost function one way or another. In this sense, our model is rather general.

Another special feature of our model concerns how the shocks affect the variable and fixed parts of the abatement cost, shown in (16). We can easily extend the model to let the industry shock affect the fixed cost as well. We apply the same method of deriving the effects of ϵ_n and obtain a similar investment barrier to (23). In fact, if there is perfect correlation among ϵ_0 and ϵ_n, $n = 1, \ldots, N$, (23) describes the barrier for firm n facing the industry shock alone that affects both its variable and fixed cost. We assumed away the fixed cost effect of the industry shock mainly to reduce clutter.

The model becomes much more complicated if we let the firm specific shock to affect the variable and marginal abatement costs. The social planner's problem becomes impossible to solve. We can apply the findings of Leahy (1993) and Caballero and Pindyck (1996), and solve the firm's investment strategy pretending that the price is exogenous. Then we obtain an investment barrier similar to (23), except that now the uncertainty's effect on the investment level becomes ambiguous. In addition to the option value effect captured by the option value coefficient, there is also the price-vs-quantity effect because each firm takes the permit price as a constant independent of the firm specific shock. If the option value effect dominates the price-vs-quantity effect, our major results still hold. By assuming away the firm shocks from the variable cost, we are able to eliminate the price-vs-quantity effect, and highlight the interaction of the option value and the general equilibrium effects.

The variable and marginal abatement costs are assumed to be linear in the industry uncertainty. This assumption influences the price-vs-quantity effect in comparing the fixed and variable tax policies, since an important part of the effect is driven by the "curvature" of the payoff function. For example, if the payoff function under variable charge s is convex in the industry shock ϵ_0, investment will decrease less as uncertainty rises. Therefore, the curvature factor in the price-vs-quantity effect is *not* a general result, even though the option value factor can be extended to other functional forms.

We assumed linear investment cost and no capital or technological depreciation. Introducing depreciation complicates the derivation, since even with independent shocks, the optimal strategy will be characterized by a partial differential equation with free boundaries, which is notoriously difficult to solve analytically. It will not change our major results, since depreciation will not remove the existence of option values (Abel and Eberly (1997)). Linear investment cost is responsible for the barrier control strategy, and the investment path would be differentiable in time if a convex investment cost function is assumed. Our chief result, however, is not the barrier control strategy itself. Our interest is in the impacts of uncertainty on investment level under different policies. These results are not likely to change even if we assume more general cost functions. For example, Abel and Eberly (1994) showed in a partial equilibrium model with a general adjustment cost function that uncertainty reduces investment.

6 Conclusion

A major concern with tradable emission permits is whether uncertainties in permit prices retard firms' incentive to invest in abatement capital or technology. But when the permit market works efficiently, permit price uncertainty can only be caused by stochastic abatement costs. We developed a rational expectations general equilibrium model where price taking firms undertake irreversible capital or technological investments in response to the cost shocks and the consequent price uncertainties. Cost uncertainties determine price uncertainties both through instantaneous permit trading and by affecting investment. We showed that both industry and firm specific cost uncertainties reduce the investment incentive in the equilibrium.

However, these uncertainties also reduce the investment incentive under an emissions charge policy. The relative magnitude of investment decrease under the two policies can be decomposed into two effects: the general equilibrium effect as identified in Magat (1978), Milliman and Prince (1989), and Jung et al. (1996), and the price-vs-quantity effect similar to Weitzman (1974), which in turn is decomposed into the option value and curvature effects. Higher uncertainty reduces both the general equilibrium effect and the option value effect, implying that the investment incentive is reduced less by uncertainty under permits than under charges. In this sense, tradable permits in fact helps maintain firms' investment incentive under uncertainty. The curvature effect implies that uncertainty helps investment incentive under fixed charges, since in our model the payoff function is convex in the industry shock under charges while linear under permits. This particular effect will change if the functional forms are altered, and as such, does not represent a general conclusion.

Following the tradition of the real options literature, we have represented the firms' investment incentive by investment barriers: investment is undertaken only when a barrier is exceeded. We did not translate the barriers into expected investment, but instead drew conclusions based on the barriers only. For this reason, whenever possible, we have used the term "investment incentive" instead of "investment level." More research is needed to formally extend our results to those based on the expected investment.

If the permit trading itself is imperfect and is subject to significant random shocks, investment incentive *will* be adversely affected under tradable permits. This effect is over and above that of abatement cost uncertainty that we have identified in this paper. It is an interesting and important empirical question to determine, for particular emissions and permit markets, the relative magnitude of the various sources of shocks.

We have ignored the normative issue of optimal policy design, taking the (most likely inefficient) fixed permits or fixed charge policies as given. Therefore, a policy that encourages investment incentive is not necessarily the more efficient policy. Of course, if there is no distortion in the capital and R&D sectors, the permit policy is efficient if the damage function of the emissions increases from sufficiently low levels to sufficiently high levels at the permit amount $\bar{e}$. The charge policy is efficient if the marginal damage is constant at the charge level τ. An interesting extension of our model is to investigate the optimal policies when the damage function is of a more general form.

Appendix: Model Details

A Proof of Proposition 1

Suppose at state $\{\boldsymbol{K}, \boldsymbol{\epsilon}, \epsilon_0\}$, at least one firm needs to increase its stock. Applying Bellman's Principle of Optimality to (10), we get

$$J(\boldsymbol{K}, \boldsymbol{\epsilon}, \epsilon_0) = \max_{\boldsymbol{K}'} -S(\boldsymbol{K}, \boldsymbol{\epsilon}, \epsilon_0, \bar{a})dt + e^{-rdt}\left\{E[J(\boldsymbol{K}', \boldsymbol{\epsilon} + d\boldsymbol{\epsilon}, \epsilon_0 + d\epsilon_0)] - \kappa \sum_n (K_n' - K_n)\right\},$$

where the expectation E is conditional on $\boldsymbol{\epsilon}$ and ϵ_0. Since $S(\cdot)$ is convex in $\boldsymbol{K}$ (cf. equation (9)), or $-S(\cdot)$ is concave in $\boldsymbol{K}$, we can show that $J(\cdot)$ is concave in $\boldsymbol{K}$.[11] Thus the necessary and sufficient condition for the maximization problem on the right hand side is given by the following Kuhn-Tucker conditions:

$$E[J_{K_n'}(\boldsymbol{K}', \boldsymbol{\epsilon} + d\boldsymbol{\epsilon}, \epsilon_0 + d\epsilon_0)] - \kappa \leq 0, \quad K_n' - K_n \geq 0,$$

$$\left(E[J_{K_n'}(\boldsymbol{K}', \boldsymbol{\epsilon} + d\boldsymbol{\epsilon}, \epsilon_0 + d\epsilon_0)] - \kappa\right)\left(K_n' - K_n\right) = 0,$$

$$n = 1, \ldots, N$$

As $dt \to 0$, $d\boldsymbol{\epsilon} \to \mathbf{0}$ and $d\epsilon_0 \to 0$ with probability one. Thus we can remove the expectation operation and obtain (11).

B Characteristics of Function $L(\boldsymbol{K}, \bar{a})$

Applying the envelope theorem to the minimization problem in (9), we know

$$p(\boldsymbol{K}, \epsilon_0, \bar{a}) = \frac{\partial S(\boldsymbol{K}, \boldsymbol{\epsilon}, \epsilon_0, \bar{a})}{\partial \bar{a}} = \frac{\partial L(\boldsymbol{K}, \bar{a})}{\partial \bar{a}} \epsilon_0. \tag{33}$$

[11] Chapter 11 of Dixit and Pindyck (1994) showed this point for the case of $N = 1$. Their approach can be directly generalized to $N > 1$. Theorem 9.8 of Stokey and Lucas (1989) strictly proved a case of $N = 1$ for discrete time optimization. Again, their proof can be generalized to $N > 1$ and continuous time.

Thus industry shocks affect the permit price directly: without any capital adjustment, price p is affine in ϵ_0. Similarly, from the envelope theorem, $\frac{\partial S}{\partial K_n} = C_{K_n} = \frac{1}{2} c_{K_n} a_n^2 \epsilon_0 + d_{K_n} \epsilon_n$. But from (18), $\frac{\partial S}{\partial K_n} = L_{K_n} \epsilon_0 + d_{K_n} \epsilon_n$. Thus

$$\frac{\partial L(\boldsymbol{K}, \bar{a})}{\partial K_n} = \frac{1}{2} c_{K_n}(K_n, n) a_n(\boldsymbol{K}, \bar{a})^2. \tag{34}$$

C Investment Barrier Facing Industry Shock Alone

Based on (15), we can verify that the homogeneous part of the differential equation (12) has the following solution:

$$J^h(\boldsymbol{K}, \boldsymbol{\epsilon}, \epsilon_0) = \sum_{n=0}^{N} \left[B_n^1(\boldsymbol{K}) {\epsilon_n}^{\beta_n^1} + B_n^2(\boldsymbol{K}) {\epsilon_n}^{\beta_n^2} \right], \tag{35}$$

where $B_n^i(\boldsymbol{K})$, $i = 1, 2$, $n = 0, \ldots, N$, are constants of integration to be determined by the boundary conditions, and $\beta_n^1 > 1$ and $\beta_n^2 < 0$ are the roots of the fundamental quadratic

$$\frac{1}{2} \sigma_n^2 \beta(\beta - 1) + \alpha_n \beta - r = 0. \tag{36}$$

We can show that $\partial \beta_n^1 / \partial \sigma_n < 0$.

When $\epsilon_n = 1$, $n = 1, \ldots, N$, the only random variable is ϵ_0, the industry shock. Given the function form in (18), and using (35), we can verify that the general solution to (12) is

$$J(\boldsymbol{K}, \epsilon_0) = B_0^1(\boldsymbol{K}) \epsilon_0^{\beta_0^1} + B_0^2(\boldsymbol{K}) \epsilon_0^{\beta_0^2} - \frac{L(\boldsymbol{K}, \bar{a}) \epsilon_0}{r - \alpha_0} - \frac{\sum_n d(K_n, n)}{r}. \tag{37}$$

If $\epsilon_0 = 0$, the variable abatement cost is zero (cf. (16)). The benefit of investment in reducing the fixed abatement cost is deterministic. All abatement investment occurs at time zero. Afterwards, no investment is needed and we are in the continuation region. Thus (37) applies when $\epsilon_0 = 0$. Further, the total abatement cost is simply the present value of the total fixed cost. That is, $J(\boldsymbol{K}, 0) = -\frac{\sum_n d(K_n, n)}{r}$. Since $\beta_0^2 < 0$, $\lim_{\epsilon_0 \to 0} \epsilon_0^{\beta_0^2} = \infty$. Thus $B_0^2(\boldsymbol{K}) = 0$. Then (37) is simplified as

$$J(\boldsymbol{K},\epsilon_0) = B_0^1(\boldsymbol{K})\epsilon_0^{\beta_0^1} - \frac{L(\boldsymbol{K},\bar{a})\epsilon_0}{r-\alpha_0} - \frac{\sum_n d(K_n,n)}{r}. \tag{38}$$

The second and third terms on the right hand side measure the present value of expected total cost of abatement *given the current stock.* The first term then measures the value of having the flexibility to adjust the stocks as the shocks occur.

Parameter $B_0^1(\boldsymbol{K})$ is still unknown. We determine it jointly with the investment barrier $\boldsymbol{K}^b(\epsilon_0)$, using the two barrier equations (13) and (14). Substituting (38) into (13) and (14), we get

$$J_{K_n} = \frac{\partial B_0^1(\boldsymbol{K})}{\partial K_n}\epsilon_0^{\beta_0^1} - \frac{\epsilon_0}{r-\alpha_0}\frac{\partial L(\boldsymbol{K},\bar{a})}{\partial K_n} - \frac{1}{r}d_{K_n}(K_n,n) = \kappa$$
$$J_{K_n\epsilon_0} = \beta_0^1\frac{\partial B_0^1(\boldsymbol{K})}{\partial K_n}\epsilon_0^{\beta_0^1-1} - \frac{1}{r-\alpha_0}\frac{\partial L(\boldsymbol{K},\bar{a})}{\partial K_n} = 0,$$

where $\boldsymbol{K}$ is evaluated at the barrier $\boldsymbol{K}^b$. Solving the two equations for B_0^1 and ϵ_0, we obtain equation (19).

Now we study how $\epsilon_{0,n}^b$ depends on K_m, $m \neq n$. Only the denominator $-\partial L/\partial K_n$ is affected by K_m, and from (34), we know $\frac{\partial}{\partial K_m}\left(-\frac{\partial L}{\partial K_n}\right) = -c_{K_n}a_n\frac{\partial a_n}{\partial K_m}$. Efficient permit trading means that $\frac{\partial a_n}{\partial K_m} < 0$, since as K_m increases, firm m's marginal abatement coefficient $c(K_m,m)$ decreases. Thus firm m will abate more, and consequently firm n will abate less. Thus $\frac{\partial}{\partial K_m}\left(-\frac{\partial L}{\partial K_n}\right) < 0$ and $\frac{\partial \epsilon_{0,n}^b(\boldsymbol{K})}{\partial K_m} > 0$.

Since $d_{K_nK_n} > 0$, the numerator on the right hand side of (19) is increasing in K_n. For the denominator, since $L(\cdot)$ is convex in K_n, we know $\frac{\partial}{\partial K_n}\left(-\frac{\partial L}{\partial K_n}\right) < 0$. Thus, $\frac{\partial \epsilon_{0,n}^b(\boldsymbol{K})}{\partial K_n} > 0$.

Next we derive (20). From (17) and (33), we know $a_n(\boldsymbol{K},\bar{a}) = \frac{p(\boldsymbol{K},\epsilon_0,\bar{a})}{c(K_n,n)\epsilon_0}$. Substituting a_n to (34), we get

$$\frac{\partial L(\boldsymbol{K},\bar{a})}{\partial K_n} = \frac{1}{2}\frac{c_{K_n}(K_n,n)}{c(K_n,n)^2}\frac{p(\boldsymbol{K},\epsilon_0,\bar{a})^2}{\epsilon_0^2}. \tag{39}$$

Substituting this expression to (19) and using the two elasticity definitions, we know on the investment barrier,

$$\epsilon_0 = \frac{2(r-\alpha_0)\left[\kappa K_n - \eta^d_{K_n} d/r\right] O^1_0}{\eta^c_{K_n} a_n p/\epsilon_0}.$$

Multiplying by p/ϵ_0 on both sides, we get (20).

Now we show how $p^b_0(K_n, a_n)$ depends on K_n. Note that

$$\frac{\kappa K_n - \eta^d_{K_n} d/r}{\eta^c_{K_n} a_n} = \frac{\kappa + d_{K_n}/r}{-a_n c_{K_n}/c}. \tag{40}$$

Since $d_{K_n K_n} > 0$, we know the numerator on the right hand side is increasing in K_n. Firm n's optimal abatement decision is $a_n = \frac{p}{c\epsilon_0}$. Thus the denominator in (40) is $-\frac{p}{\epsilon_0}\frac{c_{K_n}}{c^2}$. Under perfect competition, there are many firms and a change in K_n is not likely to affect p. That is, we can regard p as a constant. Since $L(\cdot)$ is convex in K_n and from (39), we know $\frac{c_{K_n}}{c^2}$ is increasing in K_n. Thus the denominator in (40) decreases in K_n, which leads to $\frac{\partial p^b_0}{\partial K_n} > 0$.

D Derivation of Equation (21)

The derivation is similar to the case of industry shock alone in Appendix C, although the existence of multiple shocks complicates things a bit. With $\epsilon_0 = 1$, we know $S(\boldsymbol{K}, \boldsymbol{\epsilon}, \bar{a}) = L(\boldsymbol{K}, \bar{a}) + \sum_n d(K_n, n)\epsilon_n$. Then the general solution to (12) is

$$J(\boldsymbol{K}, \boldsymbol{\epsilon}) = \sum_{n=1}^{N}\left(B^1_n(\boldsymbol{K}){\epsilon_n}^{\beta^1_n} + B^2_n(\boldsymbol{K}){\epsilon_n}^{\beta^2_n} - \frac{d(K_n, n)\epsilon_n}{r-\alpha_n}\right) - \frac{L(\boldsymbol{K}, \bar{a})}{r}. \tag{41}$$

Again, if $\epsilon_n = 0$ for all n, the fixed abatement cost is zero and the benefit of investment is deterministic. All investment should be undertaken at time zero, so that we are in the continuation region, i.e. (41) applies. Further, the total cost is $J(\boldsymbol{K}, \boldsymbol{0}) = -L/r$. Thus $B^2_n(\boldsymbol{K}) = 0$ for all n, and (41) is simplified as

$$J(\boldsymbol{K}, \boldsymbol{\epsilon}) = \sum_{n=1}^{N}\left(B^1_n(\boldsymbol{K}){\epsilon_n}^{\beta^1_n} - \frac{d(K_n, n)\epsilon_n}{r-\alpha_n}\right) - \frac{L(\boldsymbol{K}, \bar{a})}{r}. \tag{42}$$

To figure out the investment barrier, we apply the two barrier equations (13) and (14) and get

$$J_{K_n} = \sum_{m=1}^{N} \frac{\partial B_m^1(\boldsymbol{K})}{\partial K_n} {\epsilon_m}^{\beta_m^1} - \frac{d_{K_n}(K_n, n)\epsilon_n}{r-\alpha_n} - \frac{L_{K_n}(\boldsymbol{K}, \bar{a})}{r} = \kappa \tag{43}$$

$$J_{K_n \epsilon_n} = \beta_n^1 \frac{\partial B_n^1(\boldsymbol{K})}{\partial K_n} {\epsilon_n}^{\beta_n^1 - 1} - \frac{d_{K_n}(K_n, n)}{r-\alpha_n} = 0 \tag{44}$$

$$J_{K_n \epsilon_j} = \beta_j^1 \frac{\partial B_j^1(\boldsymbol{K})}{\partial K_n} {\epsilon_j}^{\beta_j^1 - 1} = 0, \quad j \neq n. \tag{45}$$

for $n = 1, \ldots, N$, where $\boldsymbol{K}$ is evaluated at the barrier $\boldsymbol{K}^b$.

Equation (45) indicates that $\frac{\partial B_j^1(\boldsymbol{K})}{\partial K_n} = 0$ whenever $j \neq n$. That is, the parameter B_j^1 depends only on firm j's *own* stock. This result is due to the assumptions that the firm specific shocks are independent of each other, and that the shocks only affect the fixed abatement costs. (If ϵ_n enters firm n's variable cost part, the function $L(\cdot)$ would depend on $\boldsymbol{\epsilon}$, and $B_n^1(\cdot)$ would be a function of $\boldsymbol{K}$, rather than K_n only.) Thus we can replace $B_n^1(\boldsymbol{K})$ by $B_n^1(K_n)$ in (43) and (44), and solving the two equations for ϵ_n, we obtain the investment barrier for firm n in (21).

E Investment Barrier Under Tax τ

Parallel to the derivation of (12), we obtain the following differential equation for firm n's net payoff function J^n:

$$\frac{1}{2}\sigma_0^2 \epsilon_0^2 J^n_{\epsilon_0 \epsilon_0} + \frac{1}{2}\sigma_n^2 {\epsilon_n}^2 J^n_{\epsilon_n \epsilon_n} + \alpha_0 \epsilon_0 J^n_{\epsilon_0} + \alpha_n \epsilon_n J^n_{\epsilon_n} - rJ^n + T_n = 0.$$

Using (28), we know the solution to this differential equation is

$$\begin{aligned} J^n =& B_0^1(K_n)\epsilon_0^{\beta_0^1} + B_0^2(K_n)\epsilon_0^{\beta_0^2} + \frac{1}{2}\frac{\tau^2}{c(K_n, n)} \frac{1}{r-(\sigma_0^2 - \alpha_0)} \frac{1}{\epsilon_0} \\ &+ B_n^1(K_n){\epsilon_n}^{\beta_n^1} + B_n^2(K_n){\epsilon_n}^{\beta_n^2} - \frac{d(K_n, n)\epsilon_n}{r - \alpha_n}, \end{aligned} \tag{46}$$

where β's are again the roots of the fundamental quadratic (36). We can show that $\beta_i^1 > 1$ and $\beta_i^2 < -1$, for $i = 0, n$, as long as $r - (\sigma_0^2 - \alpha_0) > 0$ and $r - \alpha_n > 0$.

As $\epsilon_0 \to 0$ and $\epsilon_n \to 0$, the firm faces zero fixed abatement cost but infinite marginal cost. Then it undertakes no abatement and receives no subsidy. Thus its net payoff is zero: $J^n \to 0$. Applying this result to (46), we know $B_0^2 = 0$ and $B_n^1 = 0$.

The boundary conditions for J^n is given by $J^n_{K_n} = \kappa$, $J^n_{K_n \epsilon_0} = 0$, and $J^n_{K_n \epsilon_n} = 0$. Applying (46) to these boundary conditions, we obtain (29).

Now we show the reason for imposing the condition $r > \sigma_0^2 - \alpha_0$. Let $y = \frac{1}{\epsilon_0}$. Applying Ito's lemma, we know the stochastic process for y is

$$dy = (\sigma_0^2 - \alpha_0)ydt - \sigma_0 ydz_0.$$

If $r \leq \sigma_0^2 - \alpha_0$, the expected payoff to the firm (cf. (28)) would be infinite since part of the objective function is increasing at a faster rate than the discount rate. Firms would have incentive to invest without bounds. Thus we need to impose $r > \sigma_0^2 - \alpha_0$ in our model.

References

Abel, Andrew B. and Janice C. Eberly, "A Unified Model of Investment Under Uncertainty," *American Economic Review*, 1994, *84* (5), 1369–1384.

—— **and** ——, "An Exact Solution for the Investment and Value of a Firm Facing Uncertainty, Adjustment Costs, and Irreversibility," *Journal of Economic Dynamics and Control*, 1997, *21*, 831–852.

Arrow, Kenneth J. and Anthony C. Fisher, "Environmental Preservation, Uncertainty, and Irreversibility," *Quarterly Journal of Economics*, 1974, *88*, 312–319.

Baldursson, Fridrik M. and Ioannis Karatzas, "Irreversible Investment and Industry Equilibrium," *Finance and Stochastics*, 1997, *1*, 69–89.

—— **and Nils-Henrik M. von der Fher**, "Prices vs. Quantities: the Irrelevance of Irreversibility," mimeo, University of Olso 1999.

Burtraw, Dallas, "The SO2 Emissions Trading Program: Cost Savings Without Allowance Trades," *Contemporary Economic Policy*, 1996, *14*, 79–94.

Caballero, Ricardo J. and Robert S. Pindyck, "Uncertainty, Investment, and Industry Evolution," *International Economic Review*, 1996, *37(3)*, 641–662.

Chao, Hung-Po and Robert Wilson, "Option Value of Emission Allowances," *Journal of Regulatory Economics*, 1993, *5*, 233–249.

Dixit, Avinash K. and Robert S. Pindyck, *Investment Under Uncertainty*, Princeton, N.J.: Princeton University Press, 1994.

Farzin, Y. Hossein and P. M. Kort, "Pollution Abatement Investment when Environmental Regulation is Uncertain," *Journal of Public Economic Theory*, 2000, *2*, 183–212.

——, **K. J. M. Huisman, and P. M. Kort**, "Optimal Timing of Technology Adoption," *Journal of Economic Dynamics and Control*, 1998, *22*, 779–799.

Gersbach, Hans and Amihai Glazer, "Markets and Regulatory Hold-Up Problems," *Journal of Environmental Economics and Management*, 1999, *37*, 151–164.

Henry, Claude, "Investment Decisions Under Uncertainty: The Irreversibility Effect," *American Economic Review*, 1974, *64*, 1006–1012.

Joskow, Paul L, Richard Schmalensee, and Elizabeth M. Bailey, "The Market for Sulfur Dioxide Emissions," *American Economic Review*, 1998, *88*, 669–685.

Jung, Chulho, Kerry Krutilla, and Roy Boyd, "Incentives for Advanced Pollution Abatement Technology at the Industry Level: An Evaluation of Policy Alternatives," *Journal of Environmental Economics and Management*, 1996, *30*, 95–111.

Kemp, R. and L. Soete, "Inside the 'Green Box': On the Economics of Technological Change and the Environment," in C. Freeman and L. Soete, eds., *New Explorations in the Economics of Technological Change*, London: Pinter, 1990.

Kneese, A. and C. Schultze, *Pollution, Prices, and Public Policy*, Washington, DC: Brookings Institution, 1975.

Kolstad, Charles D., "Fundamental Irreversibilities in Stock Externalities," *Journal of Public Economics*, 1996, *60*, 221–23.

Leahy, John V., "Investment in Competitive Equilibrium: the Optimality of Myopic Behavior," *Quarterly Journal of Economics*, 1993, *108(4)*, 1105–1133.

Lucas, Robert E. Jr. and Edward C. Prescott, "Investment Under Uncertainty," *Econometrica*, 1971, *39(5)*, 659–681.

Magat, Wesley, "Pollution Control and Technological Advance - a Dynamic Model of the Firm," *Journal of Environmental Economics and Management*, 1978, *5*, 1–25.

Milliman, Scott and Raymond Prince, "Firm Incentives to Promote Technological Change in Pollution Control," *Journal of Environmental Economics and Management*, 1989, *17*, 247–265.

Parry, Ian W. H., "Pollution Regulation and the Efficiency Gains from Technological Innovation," Discussion Paper 98-04, Resources for the Future 1997.

Requate, Till, "Incentives to Innovate Under Emission Taxes and Tradable Permits," *European Journal of Political Economy*, 1998, *14*, 139–165.

Sarkar, Sudipto, "On the Investment Uncertainty Relationship in a Real Options Model," *Journal of Economic Dynamics and Control*, 2000, *24*, 219–225.

Stokey, Nancy L. and Robert E. Lucas Jr. with Edward C. Prescott, *Recursive Methods in Economic Dynamics*, Cambridge, MA: Harvard University Press, 1989.

Weitzman, Martin L., "Price Versus Quantities," *Review of Economic Studies*, 1974, *41*, 477–491.

Xepapadeas, Anastasios, "Environmental Policy and Firm Behavior: Abatement Investment and Location Decisions Under Uncertainty and Irreversibility," Working Paper T0243, NBER 1999.