

2020

## Extracting resilience metrics from distribution utility data using outage and restore process statistics

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## Extracting resilience metrics from distribution utility data using outage and restore process statistics

### Abstract

Resilience curves track the accumulation and restoration of outages during an event on an electric distribution grid. We show that a resilience curve generated from utility data can always be decomposed into an outage process and a restore process and that these processes generally overlap in time. We use many events in real utility data to characterize the statistics of these processes, and derive formulas based on these statistics for resilience metrics such as restore duration, customer hours not served, and outage and restore rates. Estimating the variability of restore duration allows us to predict a maximum restore duration with 95% confidence.

### Disciplines

Controls and Control Theory | Systems and Communications

### Comments

This is a pre-print of the article Carrington, Nichelle'Le K., Ian Dobson, and Zhaoyu Wang. "Extracting resilience metrics from distribution utility data using outage and restore process statistics." *arXiv preprint arXiv:2011.00693* (2020). Posted with permission.

# Extracting resilience metrics from distribution utility data using outage and restore process statistics

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**Abstract**—Resilience curves track the accumulation and restoration of outages during an event on an electric distribution grid. We show that a resilience curve generated from utility data can always be decomposed into an outage process and a restore process and that these processes generally overlap in time. We use many events in real utility data to characterize the statistics of these processes, and derive formulas based on these statistics for resilience metrics such as restore duration, customer hours not served, and outage and restore rates. Estimating the variability of restore duration allows us to predict a maximum restore duration with 95% confidence.

## I. INTRODUCTION

Under normal conditions, component outages in electric power distribution systems occur at a low rate and are restored as they occur. However, when there is severe weather or other extreme stresses, the component outages occur at a high rate, and the outaged components accumulate until they are gradually restored. This paper studies these resilience events in which outaged components accumulate. We process 5 years of outage data recorded by a distribution utility to obtain many such real events for statistical analysis. Each event has a resilience curve that tracks the cumulative number of components or customers outaged as they accumulate and then are restored.

It is customary to divide resilience curves into successive, non-overlapping phases. For example, Nan [1] describes a disruptive outaging phase followed by a recovery phase, while Panteli [2] describes a resilience trapezoid with the three phases of progressive disturbances, then a degraded or assessment phase, then recovery in a resilience triangle. Similarly, Yodo [3] describes resilience curves in terms of successive unreliability, disrupted, and recovery phases with triangles, trapezoids, and other curves. Carrington [4] uses the nadir of resilience curves from utility data to separate an outage phase from a recovery phase. Many other papers have similar accounts of resilience phases.

These distinct phases of resilience are conceptually compelling. Moreover, the dimensions, slopes, and areas of the resilience triangles and trapezoids define standard resilience metrics of event duration, average rates of outage or recovery, and overall impact.

However, working with our utility data suggests a different point of view in which outage and restore processes routinely

overlap in time<sup>1</sup>. That is, for practical processing of real distribution system outage data we decompose the resilience curve not into successive phases but into outage and restore processes that occur together for much of the time. Moreover, we will show that standard resilience metrics can still be obtained with this approach. Our point of view is general: outage and restore processes can always be combined into a resilience curve, and any resilience curve can always be decomposed into an outage process and a restore process.

We start by extracting from the utility data many resilience events and their resilience curves and the outage and restore processes. These track the number of components out during each event. Then we obtain the statistics of the outage and restore processes and derive the resilience metrics. The outage process is statistically characterized by the times between successive outages or the outage rate. The restore process starts after a delay and is statistically characterized by the times between successive restores or the restore rate. The duration of the restore process and of the entire event are then easily obtained standard metrics.

There is also a conventional customer resilience curve tracking the number of customers out during the event that we also obtain from the utility data. This customer resilience curve can also be decomposed into a customer outage process and a customer restore process. We can measure the impact of an event by the customer hours lost, which is the area under the customer resilience curve and a well-known resilience metric [1]–[3]. We compute the mean customer hours lost from the statistics of the customer outage and restore processes.

Previous pioneering work on queueing models of reliability and resilience has used outage and restore processes. Zapata [5] models distribution system reliability with outages as a point process arriving at a queue that is serviced by a repair process with multiple crews to produce an output that is a restore process. Wei and Ji [6] analyze distribution system resilience to particular severe hurricanes with an outage process arriving at a queue with a repair process to produce a restore process. In [6], these processes vary in both time and space as the hurricane progresses. Both [5] and [6] statistically model the outage process and the repair process of components, and then calculate the restore process. With our focus on studying the overall system resilience with real data, we can model the restore process directly from the data, and avoid the

<sup>1</sup>The average fraction of event duration for which outage and restore processes overlap is 0.61 for events with 10 to 20 outages, 0.89 for events with 100 to 200 outages, and 0.95 for events with 1000 to 2000 outages.

complexities of explicitly modeling the repair of components and assuming an order in which they are repaired.

Our methods require a sufficient number of events for good statistics, so that this paper addresses the more common, less extreme events. Therefore there is little overlap of this paper with [6], which addresses individual instances of the most extreme events (direct hits by a hurricane), for which there are few events for a given utility.

There are several methods of estimating the number of outages in an anticipated storm [7]–[13], including practical utility application in [13]. Since some of our statistics show a dependence on the number of outages, this capability to predict the number of outages will be useful in applying the results in this paper to anticipated storms.

The utility data also yields estimates of the variability of the outage and restore processes, enabling estimates of the variability of the restore duration. Then, given the estimate of the number of outages, we can use our restore time statistics to predict upper bounds of the restore duration of an anticipated storm, such as its 95th percentile. The upper bound is intended to help the utility predict when the restore process will be completed with more confidence.

With a similar overall aim, previous work estimates individual component restoration times from utility data in different ways. For example, Jaech [14] predicts a gamma distribution of individual component outage restoration times and customer hours lost with a neural network that processes utility records and wind speed, outage time and date, and hence obtains upper bounds of individual component restoration times. Chow [15] analyzes the contributions of timing, faults, protection, outage types and weather to individual component restoration times. Liu [16] fit generalized additive accelerated failure time models to hurricane and ice storm utility data. The individual outages were then combined to give system restoration curves at the county level.

We summarize the innovations in the paper:

(1) We extract from 5 years of utility data many events in which multiple outages accumulate before being restored and perform a novel statistical analysis of these resilience events.

(2) We separate the events into overlapping outage and restore processes for both components and customers, and obtain statistics of these processes. The restore process is directly characterized from the data without modeling component repair.

(3) We derive formulas for the mean and standard deviations of restore and event durations from the statistics of the outage and restore processes, and illustrate their application to estimating an upper bound for the restore duration.

These innovations allow us to extract and compute standard metrics for resilience events from practical utility data.

## II. OUTAGE AND RESTORE PROCESSES

The resilience events of interest occur when outaged components accumulate before being restored. Each event has a conventional resilience curve  $C(t)$  for component outages.  $C(t)$  is the negative of the cumulative number of component outages as a function of time  $t$ . For example, the orange curve

at the bottom portion of Figure 1 shows  $C(t)$  for an event with 10 component outages. We now explain the outage and restore processes and how they relate to the restoration curve.

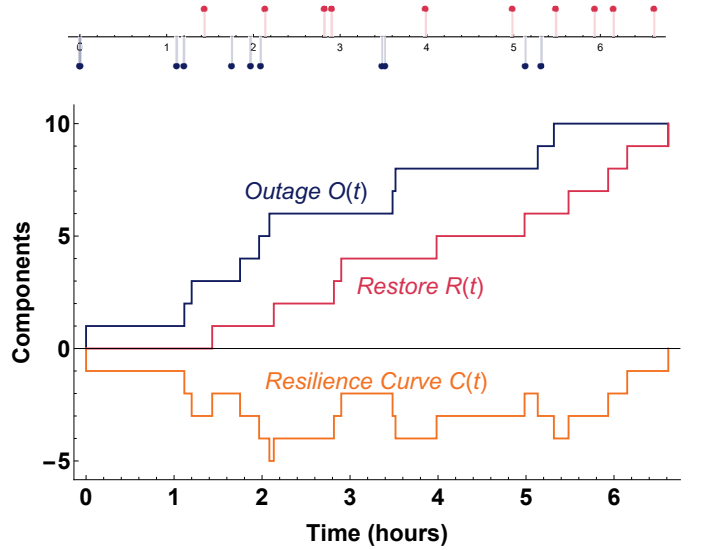


Fig. 1. A component resilience curve and its associated outage and restore processes.

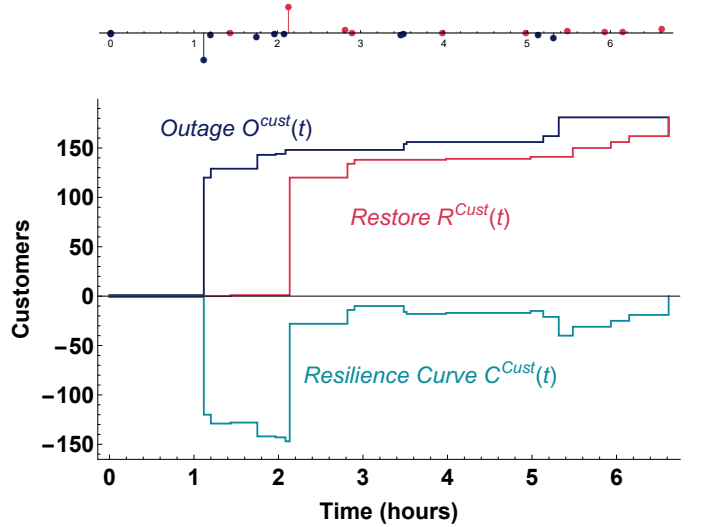


Fig. 2. A customer resilience curve and its associated customer outage and customer restore processes for the same event as Figure 1.

### A. Examples of component outage and restore processes

We start with no components outaged. Then 10 components outage at times  $o_1 \leq o_2 \leq \dots \leq o_{10}$  as shown by the tick marks below the top time line of Figure 1. The restore times are  $r_1 \leq r_2 \leq \dots \leq r_{10}$  as shown by the tick marks above the top time line of Figure 1. The restore times are numbered in the time order that they occur. The cumulative number of outages  $O(t)$  at time  $t$  and the cumulative number of restores

$R(t)$  at time  $t$  are defined by counting 1 for each outage or restore before time  $t$ :

$$O(t) = \sum_{k \text{ with } o_k \leq t} 1 \quad (1)$$

$$R(t) = \sum_{k \text{ with } r_k \leq t} 1 \quad (2)$$

Figure 1 shows the cumulative number of outages  $O(t)$  and the cumulative number of restores  $R(t)$ . In this case,  $O(t)$  and  $R(t)$  increase from zero to the total number of outages 10.

The cumulative number of component outages at time  $t$  is  $O(t) - R(t)$ . The component resilience curve  $C(t)$  is defined as the negative of the cumulative number of component outages at time  $t$ :

$$C(t) = R(t) - O(t) \quad (3)$$

Figure 1 shows how the resilience curve  $C(t)$  can be decomposed into the restore process minus the outage process.

It is clear from (3) that any outage and restore processes  $O(t)$  and  $R(t)$  define a resilience curve  $C(t)$ . Moreover, any resilience curve  $C(t)$  can be uniquely decomposed as (3) into outage and restoration processes  $O(t)$  and  $R(t)$  that increase from zero to the number of outaged components  $n$ . In mathematics this decomposition is known as the Jordan decomposition [17] of functions of bounded variation<sup>2</sup>.

There is noticeable variety in the forms of the component resilience curves in our utility data. The last 5 examples in Figure 3 show these curves and their decompositions into outage and restore processes. For comparison, the first example in Figure 3 shows a conventional, idealized case of a trapezoidal resilience curve.

Considering the outage and restore processes separately is useful because they correspond to different aspects of system resilience: the outage process results from individual component strengths under bad weather stress and the restore process results from the number and performance of the restoration crews.

### B. Extracting events from utility data

The historical outage data was recorded by one distribution utility from 2011 to 2016 over a territory including rural areas and small cities. 32 291 outages were recorded during this time. The start and end time of the outages were recorded by fuse cards based on the loss of power at that location. The number of customers out for each outage was also recorded.

We use our method from [4] to define events. The start of an event is defined by an initial outage that occurs when all components are functional, and the end of the same event is defined by the first subsequent time when all the components

are restored. That is, the event starts when the cumulative number of failures  $C(t)$  first changes from zero and ends when  $C(t)$  returns to zero.

In particular, we sort the combined component outage and restore times by their order of occurrence and then calculate the cumulative number of outages  $C(t)$  at all the outage and restore times. Each restore at time  $r_n$  for which  $C(r_n) = 0$  is the end of an event and the immediately following outage is the start of the next event. Note that if the event has  $n$  outages, then it must have  $n$  restores to allow the cumulative number of outage  $C(t)$  to return to zero at time  $t = r_n$ .

Applying this event processing to the historical data yields 2618 events. The component and customer resilience curves for each event are decomposed into outage and restore processes as explained in more detail in the next subsection.

### C. Component outage and restore processes

This subsection explains the outage and restore processes in more detail and shows how their statistics are extracted from the events in the utility data.

Suppose that  $o_1 \leq o_2 \leq \dots \leq o_n$  are the component outage times in an event in order of occurrence and that  $\Delta o_k = o_{k+1} - o_k$ ,  $k = 1, \dots, n-1$  are the times between successive component outages. The outage time differences  $\Delta o_k$ ,  $k = 1, \dots, n-1$  can be regarded as independent samples from a probability distribution of outage time differences  $\Delta o$ . We want to find the mean and standard deviation of  $\Delta o$  as a function of  $n$ .

To do this, we combine the outage time differences  $\Delta o_k$  for all the events with  $n$  outages and calculate their mean and standard deviation. Then we fit the empirical mean and the standard deviation as functions of  $n$  with a linear combination of a constant and 2 exponential functions to smooth and interpolate the data as shown in Figure 4. The functional fits are

$$\overline{\Delta o} = 7.45 + 23.3e^{-0.0388n} + 32.2e^{-0.00391n} \quad \text{min} \quad (4)$$

$$\sigma(\Delta o) = 25.6 + 19.5e^{-0.0375n} + 30.9e^{-0.00153n} \quad \text{min} \quad (5)$$

Suppose that  $r_1 \leq r_2 \leq \dots \leq r_n$  are the component restore times in order of occurrence. Note that the component outaged in the  $k$ th outage can be different from the component restored in the  $k$ th restore. (We call  $r_1, r_2, \dots, r_n$  component restore times to minimize any confusion with the restoration or repair times of particular components.) In effect, we disregard *which* component is restored and only track that *some* component is restored.

Let  $\Delta r_k = r_{k+1} - r_k$ ,  $k = 1, \dots, n-1$  be the times between successive component restores. The restore time differences  $\Delta r_k$ ,  $k = 1, \dots, n-1$  can be regarded as independent samples from a probability distribution of restore time differences  $\Delta r$ . We extract the statistics of  $\Delta r$  from the utility data as a function of the number of outages  $n$  similarly as  $\Delta o$ . Figure 5 plots the mean restore time difference  $\overline{\Delta r}$  and the standard deviation of the restore time difference  $\sigma(\Delta r)$  and the functions fitted. The functional fits are

$$\overline{\Delta r} = 7.64 + 30.8e^{-0.0514n} + 33.8e^{-0.00391n} \quad \text{min} \quad (6)$$

$$\sigma(\Delta r) = 35.3 + 43.7e^{-0.0224n} \quad \text{min} \quad (7)$$

<sup>2</sup>The total variation of  $C(t)$  is  $2n$ , which is bounded. In our case the Jordan decomposition (3) is minimal and unique since we require that  $O(t) = R(t) = 0$  for  $t < o_1$  and  $O(t) + R(t) = 2n$  for  $t > r_n$  [18, defn. 2.4(2), thm. 2.5(2)], [19, sec. 9-4]. Since  $C(t)$  is minus the cumulative number of outages, if there are simultaneous restores and outages, for example,  $m_r$  restores and  $m_o$  outages all occurring precisely at time  $t$ , then only their difference  $m_r - m_o$  contributes to  $C(t)$ , and we assume that only  $|m_r - m_o|$  contributes to the total number  $n$  of outages or restores. Note that  $O(t), R(t), C(t)$  are right continuous.

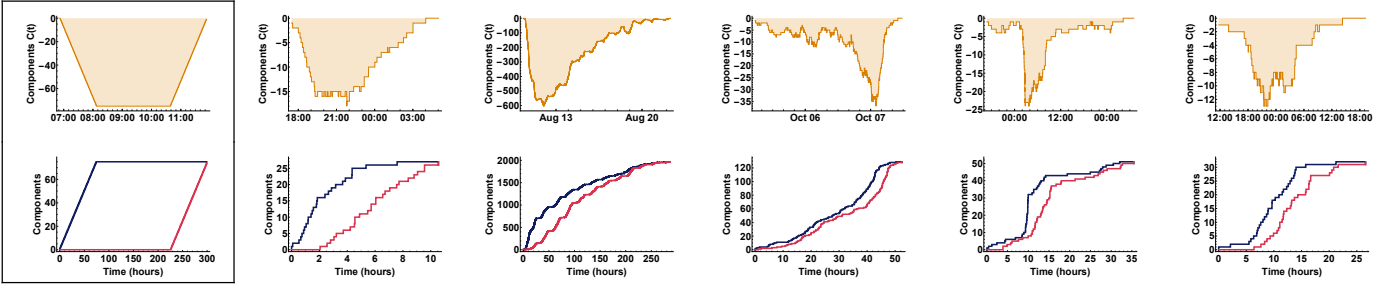


Fig. 3. Component resilience curves (top row) and their corresponding decompositions into outage and restore processes (bottom row). The first example is an idealized case with trapezoidal resilience curve and the rest are examples from utility data.

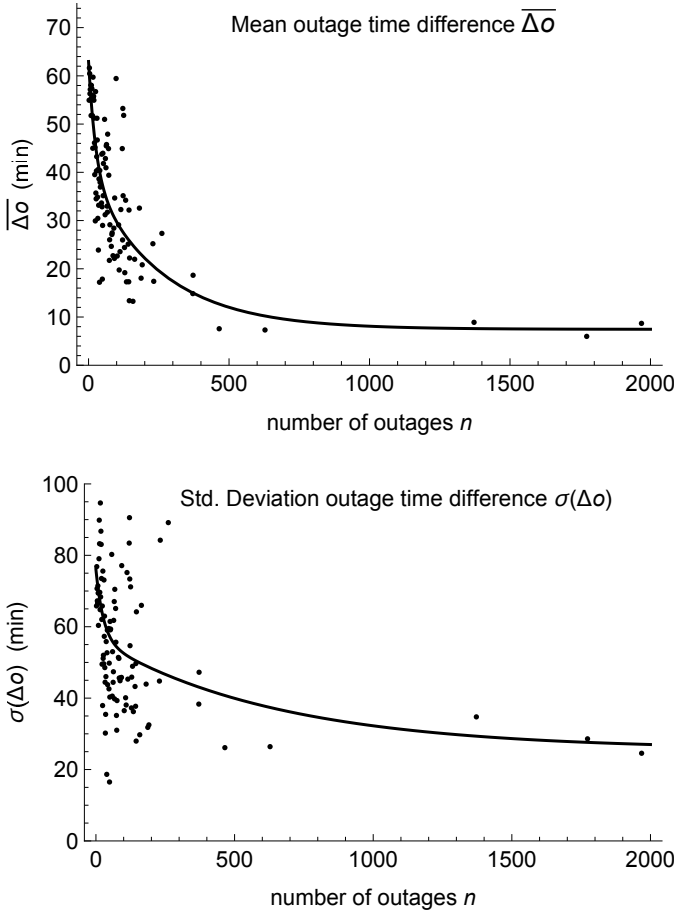


Fig. 4. Mean and standard deviation of outage time difference empirical data (dots) and fitted curve as a function of number of outages  $n$ .

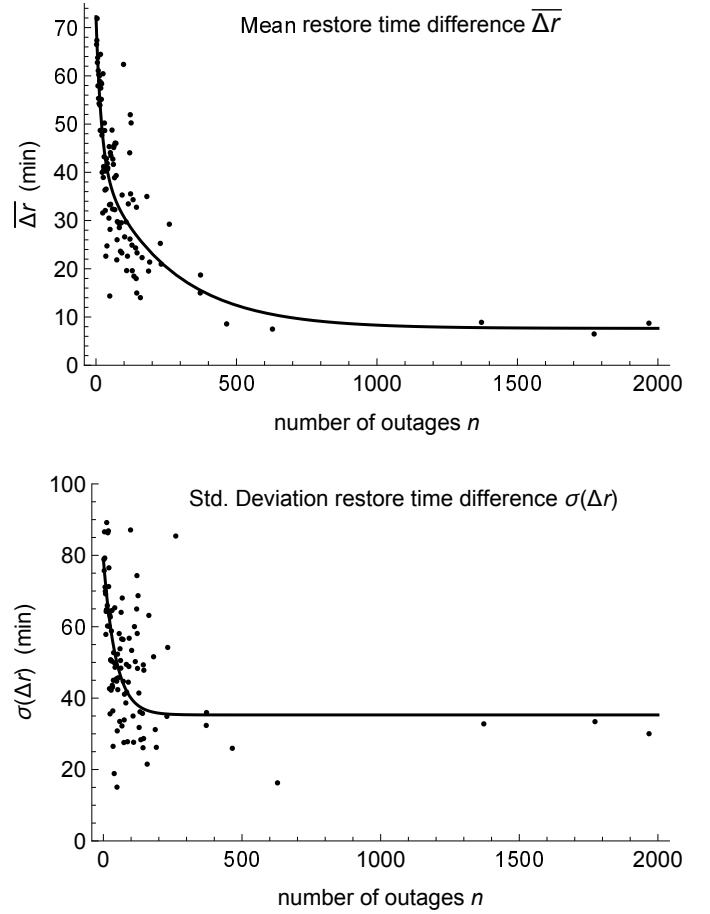


Fig. 5. Mean and standard deviation of restore time difference empirical data (dots) and fitted curve as a function of number of outages  $n$ .

Let  $\Delta r_0 = r_1 - o_1$  be the delay in the start of the restoration process relative to the start of the event at time  $o_1$ . One factor contributing to  $\Delta r_0$  is utility inspection crews and clean-up crews working to ensure the safety of the area and assess the damage needing repair. There is no clear trend in variation of  $\Delta r_0$  with  $n$ , so we combine the data for  $\Delta r_0$  for all events with 2 or more outages and compute

$$\overline{\Delta r_0} = 132 \text{ min} \quad (8)$$

$$\sigma(\Delta r_0) = 92.4 \text{ min} \quad (9)$$

#### D. Customer outage and restore processes

The utility data records the number of customers outaged by each component outage, allowing us to similarly analyze resilience curves tracking the number of customers out and outage and restore processes for the number of customers. Generalizing (1) and (2), the cumulative numbers of customers

out  $O^{\text{cust}}(t)$  and customers restored  $R^{\text{cust}}(t)$  at time  $t$  are

$$O^{\text{cust}}(t) = \sum_{k \text{ with } o_k \leq t} c_k^{\text{out}} \quad (10)$$

$$R^{\text{cust}}(t) = \sum_{k \text{ with } r_k \leq t} c_k^{\text{res}} \quad (11)$$

The customer resilience curve  $C^{\text{cust}}(t)$  is now obtained similarly to the component resilience curve (3) as

$$C^{\text{cust}}(t) = R^{\text{cust}}(t) - O^{\text{cust}}(t). \quad (12)$$

Figure 2 shows an example of customer processes  $O^{\text{cust}}(t)$  and  $R^{\text{cust}}(t)$  and the resilience curve  $C^{\text{cust}}(t)$ . The processes  $O^{\text{cust}}(t)$  and  $R^{\text{cust}}(t)$  increase from zero to the total number of customers out, which is 181 in this example.

When an outage that disconnected customers is restored, the same number of customers are restored. However, outages are not necessarily restored in the order that the outages occurred. Therefore the numbers customers restored  $c_1^{\text{res}}, c_2^{\text{res}}, \dots, c_n^{\text{res}}$  are a permutation of the customers out  $c_1^{\text{out}}, c_2^{\text{out}}, \dots, c_n^{\text{out}}$ .

The customers out  $c_1^{\text{out}}, c_2^{\text{out}}, \dots, c_n^{\text{out}}$  can be regarded as  $n$  independent samples from a distribution of number of customers out  $c$ . The customers restored  $c_1^{\text{res}}, c_2^{\text{res}}, \dots, c_n^{\text{res}}$  can also be regarded as  $n$  independent samples from  $c$ . We combine the data for customers out for all events<sup>3</sup> and compute the mean and standard deviation of the number of customers:

$$\bar{c} = 54.0 \quad (13)$$

$$\sigma(c) = 180. \quad (14)$$

The customers out for each outage are determined by the location of the outage in the network, and the distribution of the number of customers out is determined by the overall network design and its vulnerabilities.

### III. RESILIENCE METRICS

We express event durations in terms of the time differences of the restore process and then derive formulas for the mean and standard deviations of the restore duration and the event duration. We also combine the time differences with the customers outage statistics to derive a formula for the mean customer hours lost. The average outage and restore rates are easily obtained.

The restore process starts at time  $r_1$  and ends at time  $r_n$ , so the restore duration is

$$D_R = r_n - r_1 \quad (15)$$

$$\begin{aligned} &= (r_2 - r_1) + (r_3 - r_2) + \dots + (r_n - r_{n-1}) \\ &= \Delta r_1 + \Delta r_2 + \dots + \Delta r_{n-1} \end{aligned} \quad (16)$$

The mean restore duration is then

$$\overline{D_R} = (n-1)\overline{\Delta r} \quad (17)$$

Assuming that  $\Delta r_1, \Delta r_2, \dots, \Delta r_{n-1}$  are independent, we obtain  $\sigma^2(D_R) = (n-1)\sigma^2(\Delta r)$  and

$$\sigma(D_R) = \sqrt{n-1}\sigma(\Delta r) \quad (18)$$

<sup>3</sup>2% of the customer data are blank entries that we replaced with 0.

In (16), the restore duration  $D_R$  is measured until the last restore of the event. If it is preferred to measure the restore duration until, say, 95% of the outages are restored, then this can easily be done by replacing  $n-1$  in (17) and (18) by  $\lceil 0.95n-1 \rceil$ , where the ceiling function  $\lceil \cdot \rceil$  rounds up to the nearest integer.

Each event starts at the first outage time  $o_1$  and ends at the last restore time  $r_n$ . Then the event duration is

$$D_E = r_n - o_1 = (r_1 - o_1) + (r_n - r_1) = \Delta r_0 + D_R \quad (19)$$

Using (17), the mean event duration is

$$\overline{D_E} = \overline{\Delta r_0} + (n-1)\overline{\Delta r} \quad (20)$$

and, since  $\Delta r_0$  and  $D_R$  are independent, we use (18) to obtain  $\sigma^2(D_E) = \sigma^2(\Delta r_0) + (n-1)\sigma^2(\Delta r)$  and

$$\sigma(D_E) = \sqrt{\sigma^2(\Delta r_0) + (n-1)\sigma^2(\Delta r)} \quad (21)$$

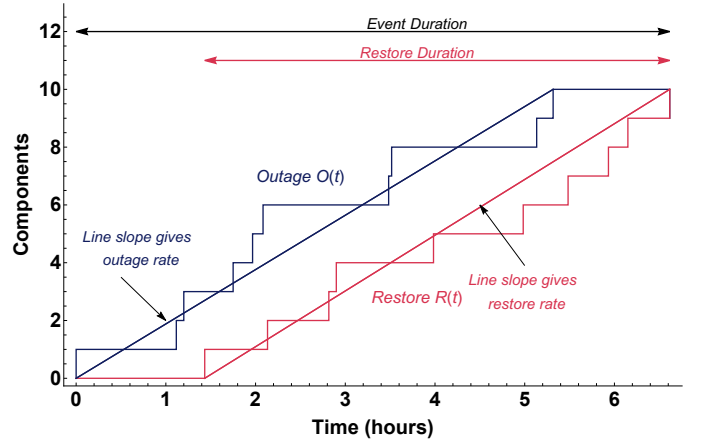


Fig. 6. Resilience metrics (durations and rates) for the component outage and restore processes.

The restore rate is

$$\lambda_R = (\Delta r)^{-1} \quad (22)$$

The outage rate during events<sup>4</sup> is

$$\lambda_O = (\overline{\Delta o})^{-1} \quad (23)$$

In (22) we obtain the restore rate  $\lambda_R$  from  $\overline{\Delta r}$ , which is a quantity averaged over events. This restore rate  $\lambda_R$  should be distinguished from the instantaneous restore rate  $\lambda_R^{\text{inst}}(t)$ , which has been observed in [6] to vary with time for the largest events. The restore rate  $\lambda_R$  averaged over events can usefully apply even as the instantaneous restore rate varies.<sup>5</sup>

The restore process depends largely on the restoration capability available to the utility. The restore process metrics

<sup>4</sup>The outage rate measured over a year is much lower than the outage rate during events because it accounts for the time between events.

<sup>5</sup>Consider the idealized case of a non homogeneous Poisson recovery process. Suppose there are 3 restoring processes on the time intervals  $D^{(1)}, D^{(2)}, D^{(3)}$ , each of duration  $T$  and with  $n$  restores. Then the expected value of  $\int_{t \in D^{(i)}} \lambda_R^{\text{inst}}(t) dt$  is  $n-1$  and the duration  $T = \sum_{k=1}^{n-1} \Delta r_k^{(i)} = (n-1)\overline{\Delta r}^{(i)}$  for  $i = 1, 2, 3$ . Then we estimate  $\lambda_R = \frac{1}{3T} \sum_{i=1}^3 \int_{t \in D^{(i)}} \lambda_R^{\text{inst}}(t) dt$  as  $\frac{3(n-1)}{(n-1)(\overline{\Delta r}^{(1)} + \overline{\Delta r}^{(2)} + \overline{\Delta r}^{(3)})} = (\overline{\Delta r})^{-1}$ .

are  $\overline{\Delta r_0}$  and  $\overline{D_R}$  or  $\lambda_R$ . Increasing the number of utility crews or their effectiveness would decrease  $\overline{\Delta r_0}$  and  $\overline{D_R}$ , and increase  $\lambda_R$ .

The outage process depends on a combination of the weather impact and the condition and strength of the grid components. The number of outages  $n$  will vary with the weather and the condition of the grid components, increasing if the weather is more extreme or more prolonged, or if the grid components are weaker.

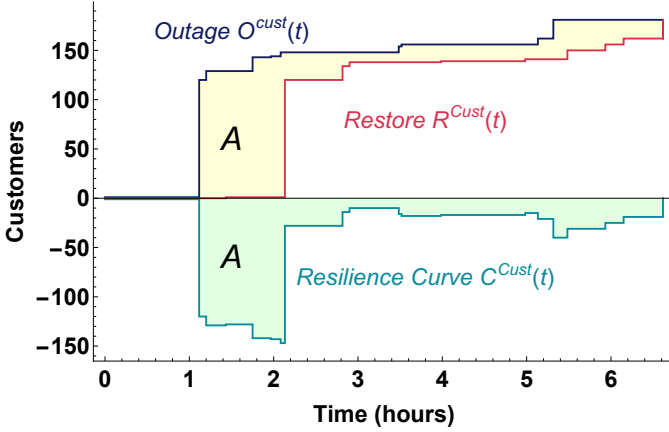


Fig. 7. Area  $A$  under resilience curve is the customer hours metric and is equal to the area  $A$  between the outage and restore processes.

We use the customer hours lost  $A$  to quantify the customer impact of an event.  $A$  is the area under the customer resilience curve:

$$A = - \int_{o_1}^{r_n} C^{cust}(t) dt \quad (24)$$

The minus sign in (24) makes  $A$  a positive area. Using (12),  $A$  is also the area between the customer outage and restore curves:

$$A = \int_{o_1}^{r_n} [O^{cust}(t) - R^{cust}(t)] dt \quad (25)$$

The two interpretations of area  $A$  are illustrated in Figure 7.

Consider the rectangle formed by the event duration and the total number of customers out. Let  $A_R$  be the area in this rectangle above the restore curve and  $A_O$  be the area in this rectangle above the outage curve. Then  $A = A_R - A_O$  where

$$A_R = \left( \sum_{j=1}^n c_j^{res} \right) \Delta r_0 + \sum_{i=2}^n \sum_{j=i}^n c_j^{res} \Delta r_{i-1} \quad (26)$$

$$A_O = \sum_{k=2}^n \sum_{\ell=k}^n c_\ell^{out} \Delta o_{k-1} \quad (27)$$

Using the independence of the terms in (26), (27), the independence of the time differences and the customers out, and  $\sum_{i=2}^n \sum_{j=i}^n 1 = \frac{1}{2}n(n-1)$  gives

$$\begin{aligned} \overline{A} &= \overline{A_R} - \overline{A_O} \\ &= n\overline{c} \overline{\Delta r_0} + \frac{1}{2}n(n-1)\overline{c}(\overline{\Delta r} - \overline{\Delta o}) \end{aligned} \quad (28)$$

An alternative expression for (28) can be obtained using (20):

$$\overline{A} = n\overline{c} \overline{D_E} - \frac{1}{2}n(n-1)\overline{c}(\overline{\Delta r} + \overline{\Delta o}) \quad (29)$$

The terms in (28) and (29) can be understood by examining the corresponding areas in Figure 8. For example, the area  $\overline{A_R}$  above the average restore curve is the area of the rectangle with sides  $\overline{\Delta r_0}$  and  $n\overline{c}$  plus the area of the triangle with sides  $(n-1)\overline{\Delta r}$  and  $n\overline{c}$ . And the area  $\overline{A_O}$  above the average outage curve is the area of the triangle with sides  $(n-1)\overline{\Delta o}$  and  $n\overline{c}$ .

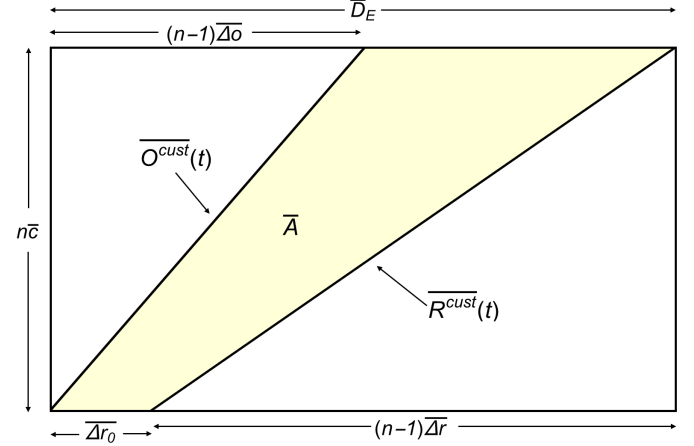


Fig. 8. Averaged dimensions and customer outage and restoring processes shown to calculate the customer hours  $\overline{A}$  in (28) and (29)

It is useful to describe the outage and restore processes with separate parameters and separate metrics because they respond to different resilience investments. For example, a program of renewing or strengthening components would affect the outage process whereas an increased number of repair crews would affect the restoration process. In more detail, (20) shows the effects on the average event duration of reducing the number of outages  $n$  (by hardening the infrastructure) reducing  $\overline{\Delta r_0}$  (by deploying more inspection crews) and reducing the average time between restores  $\overline{\Delta r}$  (by deploying more repair crews). For a larger event,  $n$  is larger and deploying more repair crews will have a larger effect because  $\overline{\Delta r}$  is multiplied by  $n-1$ . Formula (28) shows the corresponding effects on the customer hours  $\overline{A}$ . Hardening the upstream system or installing more reclosers can reduce the average customers disconnected per outage  $\overline{c}$  and proportionally reduce  $\overline{A}$ . Reducing the number of outages  $n$  or the  $\Delta r$  has an even greater effect for larger events because the second term of (28) grows like  $n^2$ .

#### IV. RESULTS

This section gives numerical results illustrating the application of the formulas for the statistics of the metrics.

We can evaluate restore duration mean  $\overline{D_R}$  and standard deviation  $\sigma(D_R)$  for a given number of outages from (6) and (7). For example, if there are  $n = 10$  outages then the restore duration has mean 527 min and standard deviation 211 min. If there are  $n = 100$  outages, then the restore duration has mean 3038 min and standard deviation 397 min. If these formulas are to be used for predicting restoration duration for an incoming storm, then the number of outages  $n$  can be predicted by a number of methods as reviewed in the introduction. As well



as estimating the mean, it is useful in applying the restore duration to compute its variability with its standard deviation.

The event duration  $D_E$  is the restore duration  $D_R$  plus the delay until the first restore  $\Delta r_0$ . From (8) and (9),  $\Delta r_0$  has mean 132 min and standard deviation 92.4 min. Then we can evaluate event duration mean and standard deviation from (20) and (21). For example, if there are  $n = 10$  outages, then the event duration has mean 660 min and a standard deviation of 230 min. If there are  $n = 100$  outages, then the event duration has mean 3171 min and a standard deviation of 408 min.

When an outage event occurs, the primary question the customers want an answer for is ‘‘How long will the power be out?’’. To help determine what should be announced to the public to answer this question, it is useful to estimate an upper bound on the restore duration that will be satisfied with a specified confidence level.

For each given value of number of outages  $n$ , the restore duration  $D_R$  approximately follows a gamma distribution. We can estimate from (6) and (7) the mean and standard deviation of  $D_R$  and then calculate the gamma distribution with that mean and standard deviation. That is, we estimate the gamma distribution with the method of moments. Then we can easily evaluate the 95th percentile of the gamma distribution. This estimates an upper bound on the restore duration that exceeds the actual restore duration with probability 0.95. The curves in Figure 9 show an increasing and initially decelerating increase of mean restore duration as the number of outages increase, and a similar increase in the 95th percentile of restore duration. The dots in Figure 9 are the restore durations for the events in the data; they show how the mean and 95th percentile of restore duration calculated from the estimated gamma distribution summarize the empirical data. The event data becomes sparser as the number of outages increase.

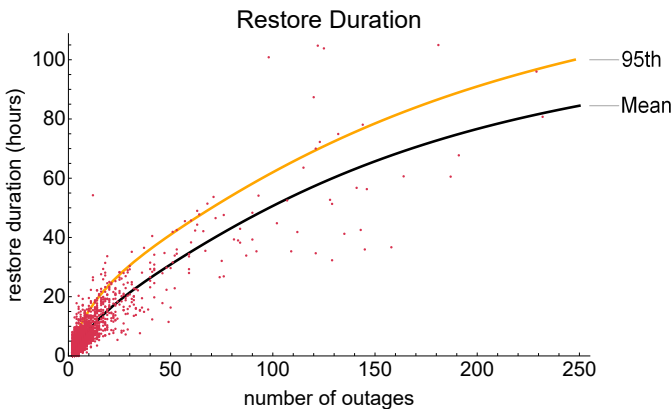


Fig. 9. Curves show mean and 95th percentile of restore duration  $D_R$  versus number of outages. Dots show the restore durations of events in the data.

Figure 10 shows the outage rate  $\lambda_O$  and the restore rate  $\lambda_R$  obtained from (23) and (22) as the number of outages varies. Both rates increase significantly as the number of outages increase. The outage rate results from the interaction of the weather with the grid, and depends on the design margin, age, and maintenance of the grid components. For up to 250 outages, the restore rate is quite close to the outage rate. For more than 50 outages, the restore rate slightly lags the outage

rate, showing the extent to which the utility restoring process succeeds in keeping up with the outage process.

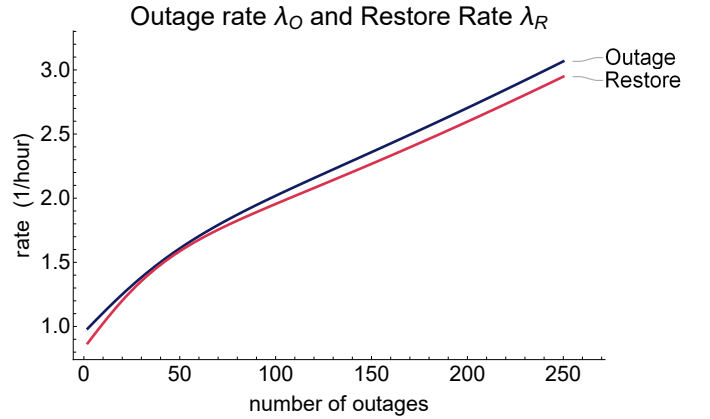


Fig. 10. Outage rate  $\lambda_O$  and restore rate  $\lambda_R$  versus number of outages.

The curve in Figure 11 shows the mean customer hours  $\bar{A}$  calculated from (28) increasing as a function of the number of outages. The dots in Figure 11 show the customer hours  $A$  for each event in the data. There is considerable variability in the customer hours in the data for more than 100 outages. Future work could aim to analyze and quantify this variability.

Although sections II-C and II-D fit the data for the full range of our data up to 2000 outages, the data for the events with more than 250 outages becomes sparse and more variable. In order to be cautious in our conclusions, we limit all the presented results in this section to events with up to 250 outages. Future work with more data or with more elaborate statistical methods might well extend the range of prediction.



Fig. 11. Curve shows mean customer hours  $\bar{A}$  calculated from (28) versus number of outages. Dots show customer hours  $A$  of the events in the data.

## V. CONCLUSIONS

We process 5 years of distribution system outage data to extract and study many resilience events in which outages accumulate and are restored. As appropriate for quantifying resilience, we focus only on the resilience events, and do not analyze the frequency of these events or the times between events that are of interest in other kinds of reliability analysis.

It is usual to separate resilience curves for events into successive non-overlapping phases in time such as outage and recovery. However, our distribution utility data shows that outage and restore processes typically occur together for most of the event. Therefore, instead of using successive phases, we show how resilience curves tracking the number of outaged components or customers can be easily decomposed into outage and restore processes that can occur at the same time. These outage and restore processes describe the same information as the resilience curve, but usefully correspond to different aspects of resilience: the outages are caused by weather interacting with the strength of the components, whereas the restores are done by utility crews. The decomposition of the resilience curve into outage and restore processes is known as the Jordan decomposition in mathematics.

We compute some basic statistics of the outage and restore processes. In particular, we estimate fits for the mean and standard deviations of the times between outages and the times between restores as functions of the number of the outages. The function fitting has the effect of smoothing and interpolating the noisy data. We also estimate the means and standard deviations of the number of customers outaged and of the delay until the restore process starts.

Then, given the predicted number of outages, which is estimated in several ways by previous work [7]–[13], we obtain formulas for the means of standard resilience metrics, such as restore and event durations, restore and outage rates, and the customer hours lost. These formulas for standard resilience metrics usefully quantify the resilience processes and show how the metrics depend on the number of outages, the delay before restoration starts, the average time between restores, and the average number of customers disconnected per outage. The outage rate quantifies the overall grid fragility under weather stress and the restore rate quantifies the overall performance of utility crews. This quantification can help inform investments that improve these metrics. We also estimate the standard deviations of the restore duration and the event duration. This leads to estimates of probable upper bounds of restore durations. These credible upper bounds based on past performance should be useful to utilities for informing customers about their outage duration with confidence.

Our approach models the outage and restore processes directly from utility data and avoids the complexities of modeling individual component restoration or repair times and their order of completion. That is, since the utility data itself incorporates the detailed complexities of resilience, we can give a high-level description and quantification of resilience. This high-level data-driven approach is very much a useful complement to the detailed modeling of the restoration processes by other authors.

In summary, we extract and separate the outage and restore processes from distribution utility outage data in a new way, estimate the statistics of times between successive restores or outages, and then show how standard resilience metrics can be derived from these statistics. The overall effect is to compute some useful resilience metrics from practical utility data.

We gratefully acknowledge support in part from NSF grants 1609080, 1735354, and 1549883.

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