Ultrasonic texture characterization of aluminum, zirconium and titanium alloys

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Ultrasonic texture characterization of aluminum, zirconium and titanium alloys

by

Alan Joseph Anderson

A dissertation submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Major: Metallurgy

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Iowa State University

Ames, Iowa

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This is to certify that the Doctoral dissertation of

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For the Graduate College
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CHAPTER 1. GENERAL INTRODUCTION

Texture, which is a measure of the preferred crystallographic orientation of the grains within a material, strongly affects many properties of materials. Studies of texture are important for many reasons. It is important to understand the relation of texture to properties so that one can engineer the texture to optimize those properties. In order to do so, one must select the appropriate materials processing procedure. The measurement of texture is central to those strategies. The most common methods of measuring texture, based on diffraction techniques, provide excellent information but require the removal of samples for testing. They are very useful for texture-property studies, but they can seriously affect process optimization when a plant needs to be stopped or shut down for cutting samples out of the product, or when the texture information is obtained after a significant delay, during which time a significant amount of material has been processed.

Nondestructive techniques can provide information about texture without requiring the removal of samples from the product currently being processed. This can greatly increase the optimization of processing schemes. In addition, nondestructive techniques for texture characterization could be used during processing, thereby providing texture information continuously. This continuous information could ultimately be used for intelligent processing of materials. For example, during the rolling of sheet metal, the texture changes each time the sheet is deformed. Sending metal through a series of rolling mills would produce sheet metal with a particular texture. If one measures the texture after the first mill.
one could use that information to fine tune the remaining mills allowing intimate control of the product's final structure.

This kind of intelligent processing is far from reality, albeit closer than one might imagine, and is outside the scope of this dissertation. Instead, the author intends to study the nondestructive techniques for measuring texture in several materials as they may apply to particular industries. Texture characterization has many important implications in industry. For example, the production of aluminum can bodies depends greatly on the knowledge of formability parameters and texture of the alloys used. Another example discussed in this dissertation is formation of hydride platelets in irradiated zirconium alloys; formation tends to occur in the basal plane of the hexagonal crystallites. In each case, there is a considerable need to understand and control texture.

While nondestructive techniques for predicting texture have been presented thus far as an obvious choice for quantifying texture, they are not without limitations. Nondestructive techniques are based on the use of waves, typically ultrasonic waves, propagating through the material. Texture is sensed because of the anisotropy of the wave speed in individual crystallites, with a preferred orientation producing a macroscopic anisotropy. These waves travel at great speeds thereby providing a quick method for predicting texture. However, since their speeds are controlled by the elastic properties of the material, they are only able to obtain limited information. Diffraction techniques provide more information, but they are generally more expensive, require considerable sample preparation, and are more time consuming. Choice of texture characterization technique, i.e. diffraction or nondestructive,
requires a balance between ease of use, speed, and amount of information desired for a particular application.

1.1 Dissertation Organization

This document discusses in detail some research completed on texture characterization in metal alloys. Chapter 1 presents an introduction to texture characterization and includes some discussion of work previously done in this field. In addition, background material is presented. Chapter 2 contains a document published in *Textures of Materials* and discusses characterization of aluminum alloys, specifically aluminum can body stock alloys. This chapter provides a brief description of the theories behind texture characterization and ultrasonic measurement of texture for alloys with cubic crystallites. Chapter 3 continues this study with work completed after publication. This work attempts to clear up some of the outstanding issues remaining in the published study. Chapter 4 discusses another aspect of research done on aluminum can body stock alloys, describing the possible correlation between earing and a particular texture parameter.

Chapter 5 continues with texture characterization for hexagonal materials, specifically zirconium, zirconium alloys, and titanium. The results of this study have been prepared for publication and submitted to *Metallurgical Transactions*. This chapter includes considerable discussion of the theories behind texture characterization and ultrasonic measurement of texture as it applies to hexagonal materials. Finally, Chapter 6 concludes this dissertation with general conclusions about texture characterization.
1.2 Texture Characterization

Materials, specifically polycrystalline metals and ceramics, are composed of thousands of smaller regions called grains. These grains usually have near-perfect crystalline structure, may or may not have the same chemical composition, and most likely have different orientations with respect to the sample axis. The distribution of orientations is what texture describes. To understand the significance of texture, consider its influence on elastic properties. For some materials a pure single crystal will have nearly isotropic elastic properties. Thus the distribution of orientations will have little affect on the macroscopic elasticity. However, if the crystallites have a significant single crystal anisotropy, then the sample properties will depend on this distribution of orientations. For a sample with completely random distribution of orientations, i.e. no texture, the sample will appear isotropic even though each of the single crystals is not isotropic. When the distribution shows peaks near one or several orientations then this preferred orientation combined with the single crystal anisotropy will give rise to an anisotropy in the sample.

Texture may affect several material properties. The most obvious being elasticity of an anisotropic material, as discussed above. A number of mechanical properties are influenced by texture, primarily because plastic deformation occurs most readily in particular crystallographic directions. Piezoelectricity is another example of a texture dependent property. Hydrides in hexagonal materials tend to form in certain crystallographic directions and thus, depend on the texture. These hydrides then influence the mechanical properties.
Any property which depends on an anisotropic property of a crystallite will be influenced by texture.

1.2.1 Diffraction Techniques for Measuring Texture

Texture has historically been measured with X-ray diffraction. A sample is cut and placed in a goniometer. The source and detector are placed at appropriate angles to detect a certain diffraction peak, for example the \{100\} or \{111\} peaks in a cubic material. The sample is then rotated to a sequence of orientations involving rotations of 360 degrees around \( \eta \) and tilts of 90 degrees about \( \chi \) (see Figure 1b). Intensity is measured at each point, and points of comparable intensities are assigned a contour. All of the contours are then plotted resulting in what is commonly called a pole figure or stereographic projection (see Figure 1c).

Pole figures give a good qualitative description of the texture. However, a single pole figure does not fully define the texture since information is not recovered about rotations of crystallites about the normal to the diffracting planes. A good quantitative measure of the texture requires an in-depth, mathematically taxing study of the individual intensities. This led to the development of the orientation distribution function (ODF). The ODF is the three-dimensional representation of the distribution of orientations described above as quantified by a probability density as a function of three Euler angles describing the crystallite orientation. Typically, the ODF is plotted as rectangular sections of a three-dimensional Euler-angle space. Each section contains contours showing relative weights at each point.

Roe\(^1\) and Bunge\(^2\) concurrently developed this 3-D, quantitative representation of texture using
slightly different conventions. The experimental determination of the ODF involves combining the information in several pole figures to obtain one ODF representing the texture of the sample. This ODF can also be used to predict material properties, such as elastic and plastic properties. This is a useful but time-consuming technique which has recently been improved upon due to the ever increasing speed of modern computers; however, this scheme still requires the measurement of at least three pole figures.

1.2.2 Ultrasonic Techniques for Measuring Texture

Because the construction of the full ODF from diffraction data is time consuming and generally requires special specimens which must be removed from the component of interest, there is a need for simpler measurement techniques for a number of practical applications. The development of such techniques is the subject of this dissertation. In particular, ultrasonic techniques will be considered. Since the majority of previous work in the ultrasonic characterization of texture has been concerned with low carbon steel sheet, motivated by automobile sheet metal forming applications, we will explore applications to different material systems in this dissertation. In aluminum, applications to predict the forming of beverage cans will be considered, and in zircaloy, applications to nuclear cladding will be investigated. Since ultrasonic techniques gain speed at the cost of information, an important issue will be whether the information recovered is adequate for the purpose at hand.
Quantification of texture starts by considering a rotation which brings the sample coordinate system into alignment with the crystallite coordinate system. This rotation, \( g \), can be described by three Euler angles, see Figure 2. Every possible rotation is contained within Euler space. The volume fraction of crystallites with a particular orientation \( g \) is defined as the ODF, \( f(g) \), given by the following:

\[
ODF = \frac{(dV/V)}{(dg)} = f(g) = f(\psi, \Phi, \phi)
\]  

(1)

where \( dV/V \) is the volume fraction of grains with orientation \( g \) within an infinitesimal element of Euler space, \( dg \).

Practical use of the ODF is facilitated by representing it by an expansion in spherical harmonics. The coefficients of this expansion are called the orientation distribution coefficients (ODCs) and are the quantities related to material properties. Throughout this paper we will be using Roe's notation except in Chapter 2 where results were converted to Bunge's notation for consistency with the publication. Any description or calculation will depend slightly on the notation used; familiarity with Roe's notation has led to its use by the author. The ODCs inherently depend on both the sample and crystal symmetries. The ODF can be given by the following equation.

\[
w(\xi, \psi, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \sum_{n=-l}^{l} W_{lmm} Z_{lmm}(\xi) e^{-im\psi} e^{-in\phi}
\]  

(2)
where the $W_{lmn}$ are the ODCs, $Z_{mn}$ are generalized associated Legendre functions, $\theta$, $\psi$, and $\phi$ are Euler angles as given in Figure 2, and $\xi=\cos(\theta)$. The sample symmetry and the crystallite symmetry determine which ODCs will be present. For cubic crystallites the only independent, nonzero ODCs are $W_{000}$, a normalization constant, and $W_{lm0}$ where $l$ is an even number greater than or equal to four and the $m$ are the even numbers between 0 and $l$. Thus the independent ODCs are $W_{000}$, $W_{200}$, $W_{420}$, $W_{440}$, $W_{620}$, $W_{640}$, $W_{820}$, $W_{840}$, $W_{860}$, $W_{880}$ etcetera. For hexagonal crystallites the only nonzero ODCs are $W_{200}$, $W_{220}$, $W_{400}$, $W_{420}$, $W_{440}$, $W_{600}$, $W_{620}$, $W_{640}$, $W_{646}$, $W_{660}$, $W_{666}$ etcetera.

Figure 1. Graphical description of X-ray measurement scheme (a), angles (b) and pole figure (c).
Thompson et al.\textsuperscript{1} realized that, since macroscopic elastic properties could be predicted by the ODF, measurement of elastic properties should provide information on the texture. This turns out to be true but only to a limited extent. Since elasticity is a fourth ranked tensor quantity only, ODCs of fourth order or less can be predicted\textsuperscript{4}. The basic scheme is to measure the anisotropy of the ultrasonic wave speed. This relates to the anisotropy of the elastic moduli which then predicts the ODCs. Equations relating the ODCs to the bulk ultrasonic wave speeds are given in the next section.
1.2.2.1 Relations between ODCs and Ultrasonic Velocities

The key step in relating ultrasonic velocities to texture is relating the elastic stiffnesses to the ODCs because the ultrasonic velocities are directly related to the elastic stiffnesses. The elastic stiffnesses can be thought of as a function involving three quantities: 1) the single crystal elastic constants (SCEC), 2) a measure of the single crystal anisotropy, and 3) a sample average of the orientations (some measure of the texture).

The first step in determining the relationship between ODCs and elastic stiffnesses, and hence, any physical property which depends on the elastic stiffnesses, is to make a transformation from the crystallite world to the sample world. This is done with by multiplying the crystallite elastic constants by directional cosines given by the following using matrix notation:

\[ C'_{ykl} = A_{um} A_{vp} A_{kl} C_{mnop} \]

where,

\[
A_{xy} = \begin{vmatrix}
  l_1 & l_2 & l_3 \\
  m_1 & m_2 & m_3 \\
  n_1 & m_2 & m_3 \\
\end{vmatrix}
\]

where \( A_{xy} \) are directional cosines, \( C'_{ykl} \) are the elastic constants for the sample, and \( C_{ykl} \) are the elastic constants for the crystallite. Rewriting the elastic constants in the reduced notation, substituting in for \( A_{xy} \) and simplifying for each of the equations above, one gets the following set of equations.
where the $\langle \rangle$ imply averaging over all crystallite orientations, and $l$, $m$, $n$ are still the direction cosines written in terms of the Euler angles. Evaluation of $\langle r \rangle$ is done by weighting $r$ with the ODF and integrating over the ranges of all three Euler angles.

$$\langle r \rangle = \int_0^{2\pi} \int_0^1 \int_0^{2\pi} r(\psi, \Phi, \phi) w(\psi, \Phi, \phi) \, d\phi \, d\psi \, d\Phi$$

Then by replacing $r(\psi, \Phi, \phi)$ and $w(\psi, \Phi, \phi)$ with expansion in spherical harmonics, as presented in equation (2), one gets

$$\langle r \rangle = 4\pi^2 \sum_{l=0}^\infty \sum_{m=-l}^l \sum_{n=-l}^l R_{lmm} W_{lmm}$$
Here the $R_{imn}$ are known quantities, and the results for each $<t_i>$ is given below in equation (7):

\[
\begin{align*}
\langle r_1 \rangle &= \frac{1}{5} - \frac{6}{35} \sqrt{2\pi^2} \left\{ W_{400} - \frac{2}{3} \sqrt{10} W_{420} + \frac{1}{3} \sqrt{70} W_{440} \right\} \\
\langle r_2 \rangle &= \frac{1}{5} - \frac{6}{35} \sqrt{2\pi^2} \left\{ W_{400} + \frac{2}{3} \sqrt{10} W_{420} + \frac{1}{3} \sqrt{70} W_{440} \right\} \\
\langle r_3 \rangle &= \frac{1}{5} - \frac{16}{35} \sqrt{2\pi^2} W_{400} \\
\langle r_4 \rangle &= \frac{1}{5} - \frac{16}{35} \sqrt{2\pi^2} \left\{ W_{400} + \frac{5}{2} W_{420} \right\} \\
\langle r_5 \rangle &= \frac{1}{5} - \frac{16}{35} \sqrt{2\pi^2} \left\{ W_{400} - \frac{5}{2} W_{420} \right\} \\
\langle r_6 \rangle &= \frac{1}{5} + \frac{4}{35} \sqrt{2\pi^2} \left\{ W_{400} - \sqrt{70} W_{440} \right\} \\
\langle r_7 \rangle &= \frac{1}{5} + \frac{16}{35} \sqrt{2\pi^2} \left\{ W_{400} + \frac{5}{2} W_{420} \right\} \\
\langle r_8 \rangle &= \frac{1}{5} + \frac{16}{35} \sqrt{2\pi^2} \left\{ W_{400} - \frac{5}{2} W_{420} \right\} \\
\langle r_9 \rangle &= \frac{1}{5} + \frac{4}{35} \sqrt{2\pi^2} \left\{ W_{400} - \sqrt{70} W_{440} \right\}
\end{align*}
\]

This completes the relation between sample elastic stiffness and the ODCs. Now one needs to show the relation between ultrasonic velocities and the elastic stiffnesses. This is presented in the next section.

1.2.2.2 Ultrasonic Velocity Relationships to Elastic Stiffnesses

Bulk ultrasonic velocities are directly related to the sample elastic constants as determined above, and these velocities are defined in terms of a propagation direction and a polarization direction. Hence, a bulk wave propagating in the $i$ direction, in an orthogonal
coordinate system containing three principle directions \((i=1, 2, 3)\), and polarized in the \(j\) direction is labeled, \(V_{ij}\). For example, a shear wave traveling in the \(1\) direction and polarized in the \(2\) direction is labeled \(V_{12}\). The relationship between the bulk velocities and the elastic constants, and hence, the ODCs are given as follows:

\[
\begin{align*}
\rho V_{11}^2 &= c_{11}' = c_{11} - 2c \left[ \frac{1}{5} - \frac{6}{35} \sqrt{2\pi^2} \left( W_{400} - \frac{2}{3} \sqrt{10} W_{420} + \frac{1}{3} \sqrt{70} W_{440} \right) \right] \\
\rho V_{22}^2 &= c_{22}' = c_{11} - 2c \left[ \frac{1}{5} - \frac{6}{35} \sqrt{2\pi^2} \left( W_{400} + \frac{2}{3} \sqrt{10} W_{420} + \frac{1}{3} \sqrt{70} W_{440} \right) \right] \\
\rho V_{33}^2 &= c_{33}' = c_{11} - 2c \left[ \frac{1}{5} - \frac{16}{35} \sqrt{2\pi^2} W_{400} \right] \\
\rho V_{23}^2 &= c_{44}' = c_{44} + c \left[ \frac{1}{5} - \frac{16}{35} \sqrt{2\pi^2} \left( W_{400} + \frac{5}{2} W_{420} \right) \right] \\
\rho V_{13}^2 &= c_{55}' = c_{44} + c \left[ \frac{1}{5} - \frac{16}{35} \sqrt{2\pi^2} \left( W_{400} - \frac{5}{2} W_{420} \right) \right] \\
\rho V_{12}^2 &= c_{66}' = c_{44} + c \left[ \frac{1}{5} + \frac{4}{35} \sqrt{2\pi^2} \left( W_{400} - \sqrt{70} W_{440} \right) \right] \\
c_{23} &= c_{12}' = c \left[ \frac{1}{5} - \frac{16}{35} \sqrt{2\pi^2} \left( W_{400} + \frac{5}{2} W_{420} \right) \right] \\
c_{13}' &= c \left[ \frac{1}{5} - \frac{16}{35} \sqrt{2\pi^2} \left( W_{400} - \frac{5}{2} W_{420} \right) \right] \\
c_{12}' &= c \left[ \frac{1}{5} + \frac{4}{35} \sqrt{2\pi^2} \left( W_{400} - \sqrt{70} W_{440} \right) \right]
\end{align*}
\]

This is a general result for bulk ultrasonic wave propagating in a sample made up of cubic crystallites. For the case of hexagonal crystallites, as in Chapter 5, the relations involving the
ODCs will be different, hence the results will be different. Results for hexagonal materials are presented in Chapter 5.

In addition, the specifics of a wave propagating in a particular sample may also change these results. For example, Chapter 3 presents work done on aluminum sheet materials. The introduction of surfaces which lie close to each other relative to the wavelength causes a perturbation in the wave propagation. The waves propagating in an sheet are referred to as plate waves, and their velocities differ from comparable bulk waves in that the velocity is dependent on frequency. Thus, one must account for the type of wave generated. Details such as these are discussed in more detail in later chapters.

1.2.3 Relationship of Ultrasonic and Diffraction Determination of Texture

Most sheet metal samples with interesting textures contain a rolling texture or some other deformation texture. Deformation mechanisms inherently involve dislocations, and these dislocations may affect the ultrasonic velocity. Other inhomogeneities that may affect the velocity are precipitates and second phases. Reno et al.⁸ and Clark et al.⁶ (in a second paper) discussed the effect of these types of inhomogeneities. Both studies used neutron diffraction to determine the pole figures, and thus, the ODCs. These ODCs are then compared with the ultrasonic results. Since neutron diffraction is independent of elastic properties, these inhomogeneities will not directly affect the neutron prediction of the ODCs. Also, because neutron diffraction and ultrasonic Lamb waves both sample the entire volume, the results should correlate well with any minor differences being attributed to the effect of
these inhomogeneities on the elastic properties and, hence, the ultrasonic ODCs. In both studies a sample of hot rolled aluminum was measured with ultrasonics and neutron diffraction. The ODCs from both techniques gave very similar results leading to the conclusion that dislocations, second phases and precipitates show effects on the ultrasonic technique to second order only. This is confirmed in numerous other studies which are summarized by Thompson et. al.\(^3\). Figures 7, 8, and 9 in reference 3 clearly show nice correlations between X-ray diffraction predicted ODCs and ultrasonically predicted ODCs.

1.3 Current Research

Extensive research related to texture determination is being done. The author is interested in aluminum and several hexagonal materials, zirconium and some zirconium alloys as well as titanium to a lesser degree. Several studies of texture prediction in aluminum alloys have been reported. Clark et al.\(^5\) used several ultrasonic techniques—Lamb waves, shear waves, Rayleigh waves—to predict ODCs. They showed a correlation between \(W_{420}\) and a formability parameter. Neutron diffraction was done to verify their results. Included in this paper by Clark is a detailed discussion of the engineering required to make on-line measurements of texture using ultrasonics. Several other papers report similar results\(^6,7,8\) with seemingly the same final objective: making on-line texture measurement a reality. Although several types of waves are used throughout the literature, the underlying process is essentially the same: relate an anisotropy in the elastic wave speed to the anisotropy due to texture in the sample.
Considerable other work is also being done, but is discussed in more detail in Chapters 2 and 5.

1.4 References


CHAPTER 2. ULTRASONIC CHARACTERIZATION OF ROLLING AND
RECRYSTALLIZATION TEXTURE IN ALUMINUM


A.J. Anderson. R.B. Thompson, R. Bolingbroke, J.H. Root

ABSTRACT

Crystallographic texture, which plays an important role in many forming operations, is developed during sheet metal processing. Improved product quality could be achieved if the texture could be measured, quickly and inexpensively, during different stages of the processing. One potential scheme, involving measurement of the angular dependence of the speed of ultrasonic waves propagating in the plane of the sheet is described, as applied to aluminum. While this technique has been successfully used in steel sheets, application to aluminum is more difficult because of the smaller elastic anisotropy of the crystallites. This paper presents the results of a detailed study of the strengths and weaknesses of the approach. Included are discussions of signal processing factors, the accuracy of the ultrasonically determined $C_l^{\sigma}$, and a comparison between $C_l^{\sigma}$ measured by ultrasonics and by diffraction techniques (both X-ray and neutron). It is found that relative velocity measurements are very good for determining $C_4^{12}$ and $C_4^{13}$. These measurements can easily distinguish between rolling and recrystallization textures, and can also detect more subtle variations in recrystallization textures. A systematic offset in the prediction of $C_4^{11}$ is also reported, which
is believed to be a consequence of unknown alloy dependence of the single crystal elastic
constants and texture free velocities, quantities which play a key role in inferring this quantity
from absolute measurements.

Keywords
Ultrasonics, Aluminum, Nondestructive, Forming, Rolling Texture, Recrystallization Texture

INTRODUCTION

Control of crystallographic texture, which strongly influences various forming
operations, is an important aspect of the sheet metal production process. A first step in such
control is sensing the texture that has been developed at various stages of the process. In the
laboratory, X-ray diffraction is the most familiar approach. Considerable information can be
obtained in a near surface layer. Alternatively, through the oblique sectioning (or composite
sample) technique, a volume average can be obtained. Similar volume average information
can be obtained through neutron diffraction. However, if one desires to nondestructively
obtain information in the field, e.g. at a rolling mill or can manufacturing facility, the choices
are more limited. X-ray diffraction is limited to sensing a near surface layer in the commonly
used reflection mode or requires quite expensive instrumentation and obtains limited
information in the transmission mode. Ultrasonic measurements provide an alternate
approach which, although also limited in the information that can be obtained, has the
advantages of utilizing relatively simple, inexpensive instrumentation and sensing a
volumetric average of the texture. In this paper, the use of ultrasound to characterize the
rolling and recrystallization textures of hot-rolled aluminum sheet is discussed.

The process of producing aluminum sheet for can fabrication involves several steps,
as illustrated in Figure 1. Usually, the process starts with a large aluminum ingot (typically,
\( \sim 2 \text{ m} \times \sim 600 \text{ mm} \times \sim 4.5 \text{ m} \)). This ingot is heated and placed into the hot-rolling mill where
its thickness is reduced to approximately 2.5 millimeters. Important process parameters at
this stage are the temperature and speed of the sheet as it leaves the rolling operation, which
controls the microstructure developed subsequent to rolling. At temperatures sufficiently
above the recrystallization temperature, a well developed recrystallization texture develops
after the deformation. However, if the temperature is too low, natural recrystallization may
be inadequate and the sheet is coiled and annealed. In either case, the sheet is then cold-
rolled to the final thickness and shipped to the can body production facility. During the cold-
rolling, a deformation microstructure appears. The design goal of this processing scheme,
assuming the sheet is strongly recrystallized after hot-rolling/annealing, is to cold-roll the
sheet (i.e. create the deformation texture) in such a way to produce a blend of textures which
will reduce earing during the can body formation (see Figure 1).

Ultrasonic wave propagation within any medium is controlled by the macroscopic
elastic properties of the medium. Since the single crystal elastic constants are anisotropic,
texture of a polycrystal will produce an elastic anisotropy, which will lead to an anisotropy of
the velocity of ultrasonic waves. In sheet samples, measuring the angular dependence of the
velocity in the plane of the sheet allows one to predict quantities that give a measure of the texture (discussed in more detail in section II).

The role of ultrasonic texture monitoring during sheet production is to nondestructively measure textures developed at various stages during the processing to assure that the desired texture is present prior to can body production. This will allow the can body makers to improve their yield. One realistic scheme is to measure the texture after hot-rolling. In times of tighter process control an on-line assessment of texture, and hence degree of recrystallization, at this stage of production would be of great benefit. Another more complex scheme would be to measure the texture on-line during the cold-rolling. This on-line information could be used, in an intelligent processing sense, to control the cold-rolling parameters for optimal texture development, a process that has not yet been realized. This paper will concentrate on evaluating the degree to which ultrasonics can monitor the recrystallization after the hot roll, in support of the former potential application.

Several techniques for the ultrasonic measurement of texture have been developed over the last decade. In steel sheet, several researches have demonstrated the ability to characterize texture and use this information to predict formability parameters such as plastic strain ratios (Sayers, 1982; Cassier, Donadille, Bacroix 1989; Hirao et al., 1989; Clark Jr. et al., 1990; Borsutzki et al., 1993; Kawashima, Hyogushi, Akagi, 1993; Thompson et al., 1993). Much less work has been done in aluminum, in part because the single crystal anisotropy is considerably smaller and hence much more precise velocity measurements are required. Nevertheless, encouraging preliminary results have been obtained. In several
papers by Clark Jr. (1988) and colleagues (Reno, Fields, Clark Jr., 1988) and a paper by Thompson et al (1989), fundamental ultrasonic plate modes were used to predict texture coefficients in an expansion of the orientation distribution function (ODF). These ultrasonically determined coefficients compared favorably with values determined by X-ray diffraction. Thompson et al reported difficulty with one particular texture parameter, $C_{4}^{11}$. but otherwise showed that ultrasonic predictions of texture parameters could be made with reasonable accuracy. Li and Thompson (1989) used higher order plate modes to predict texture parameters with much success. As a part of that study, they suggested that the difficulty with $C_{4}^{11}$ might be due to alloying elements in the aluminum, since predictions made on very high purity aluminum samples showed good correlation to the X-ray diffraction measurements of $C_{4}^{11}$, in contrast to previously reported results on alloys.

Ear formation in the final drawing procedure has been correlated to ultrasonically determined texture parameters by Lu (Lu, Morris and Gu, 1991) and by Thompson et al (1993). Lu defined a texture parameter related to wave speed anisotropy. Thompson et al related Lu's texture parameter to the commonly used ODF coefficient, $C_{4}^{13}$. The combined data sets of both groups showed a strong correlation between $C_{4}^{13}$ and the percent earing. Thompson's conclusion was "the fact that two data sets, obtained on two different alloys made by two different manufacturers and sensed by two different wave types, fits together so nicely on a single plot suggests the existence of an underlying correlation which deserves further study." Man (1994) has provided further theoretical interpretation of this result.
The above discussion suggests that ultrasonics can predict texture parameters and that these measurements are robust and reliable. The application of these techniques to improve production of aluminum sheet is the next step. Anderson and Thompson (1994) and Stiffler, Daly and Wojnar (1994) have shown that degrees of recrystallization can be distinguished using ultrasonic predictions of texture parameters. Here, a more detailed discussion of the former work is provided.

Measurements made on two sets of samples of aluminum sheet will be presented. The first set of three sheets provided were small in physical size and measurements of the texture parameters were made by hand. The samples differed in the type of texture present, two samples were recrystallized to different degrees, and one sample was not recrystallized, i.e., contained the deformation textures. The ultrasonic measurements easily differentiated these textures. The second set of three sheets had more subtle differences, being recrystallized to varying degrees. As the sheets were much larger, it was possible to make the measurements using a more accurate, automated device (Thompson et al., 1993). Subtle variations in texture were sensed in agreement with independent measurements using X-ray and neutron diffraction.

In section II we present the theory underlying the ultrasonic characterization of texture. Section III describes the samples in greater detail and section IV describes the experimental techniques, which include two ultrasonic approaches (for the small and large samples), X-ray diffraction and neutron diffraction. The data obtained on the first set of sheets was not ideal because of their smaller size, and some new regression procedures were
required to accurately extract the texture information, as presented in section V. The texture information inferred from the ultrasonic measurements is discussed and compared to the X-ray and neutron scattering results in section VI. The paper concludes with a summary in section VII.

**THEORY**

**Description of Texture**

Polycrystals exhibit anisotropic macroscopic properties when (a) the individual crystallites are anisotropic and (b) they have a preferred orientation (texture). The latter is typically sensed by diffraction methods and is quantified through the use of an orientation distribution function (ODF). The ODF is the three-dimensional representation of the distribution of orientations quantified by a probability density as a function of three Euler angles describing the crystallite orientation with respect to the sample axis. Typically, the ODF is plotted as rectangular sections of a three-dimensional Euler-angle space. Each section contains contours showing relative densities at each point. Roe (1965) and Bunge (1982) concurrently developed this 3-D, quantitative representation of texture using slightly different conventions. The experimental determination of the ODF involves combining the information in several pole figures to obtain one ODF representing the texture of the sample. This is a useful but time-consuming technique which has recently been improved upon due to the ever increasing speed of modern computers; however, this scheme still requires the
measurement of a number of pole figures. Once the ODF is determined, it can be used to predict material properties, such as elastic and plastic properties.

The ODF can be represented by an expansion in generalized spherical harmonics. The coefficients of this expansion are called the orientation distribution coefficients (ODCs) and are the quantities related to material properties. Throughout this paper the author will be using Bunge's notation. The ODCs inherently depend on both the sample and crystal symmetries. The ODF, \( f(\phi_1, \Phi, \phi_2) \), can be given by the following equation.

\[
\begin{align*}
  f(\phi_1, \Phi, \phi_2) &= \sum_{l=0}^{\infty} \sum_{\mu=-l}^{l} \sum_{\nu=-l}^{l} C_{l}^{\mu \nu} T_{l}^{\mu \nu} \\
  \text{where,} \\
  T_{l}^{\mu \nu} &= e^{-i\mu\phi_2} P_{l}^{\mu \nu}(\cos(\Phi)) e^{-i\nu\phi_1}
\end{align*}
\]

The \( C_{l}^{\mu \nu} \) are the ODCs, \( P_{l}^{\mu \nu} \) are generalizations of the associated Legendre functions; \( M(l) \) and \( N(l) \) represent the number of linearly independent ways the symmetry (crystal and sample, respectively) can be fulfilled, and \( \phi_1, \Phi, \) and \( \phi_2 \) are Euler angles. The sample symmetry and the crystallite symmetry determine which ODCs will be present.

**Ultrasonic Determination of ODCs**

Several authors have realized that, since elastic properties could be predicted by the ODF, measurement of elastic properties should provide information on the texture. This turns out to be true but only to a limited extent. Since elasticity is a fourth ranked tensor quantity, only ODCs of fourth order or less can be predicted (Sayers, 1982). The basic
scheme is to measure the anisotropy of the ultrasonic wave speed. This relates to the
anisotropy of the elastic moduli which then predicts the ODCs. Equations relating the ODCs
to the speeds of symmetric Lamb waves in plates or sheets are given in equation (1)
(Thompson et al., 1989), where \( \rho \) is the density. \( V_{s0}(\Theta = 0^\circ, 45^\circ, 90^\circ) \) is the velocity of the
fundamental symmetric Lamb wave in the indicated direction. \( P \) and \( L \) are polycrystalline
averages of the single crystal elastic constants corresponding to the Lame elastic constants \( \alpha \)
and \( \alpha + 2\mu \), respectively. \( C \) is a measure of the single crystal anisotropy \((C_{11}-C_{12}-2C_{44})\). The
type of wave used is not crucial as any elastic wave will provide information about elastic
properties: however, the formulae used will differ in detail.

\[
\begin{align*}
C_{4}^{13} &= \frac{3\sqrt{15}\rho}{2C} \left[ V_{s0}^2(0^\circ) - V_{s0}^2(90^\circ) - 2V_{s0}^2(45^\circ) \right] \\
C_{4}^{12} &= \frac{3\sqrt{105}\rho}{2(1 + \frac{2P}{L})C} \left[ V_{s0}^2(90^\circ) - V_{s0}^2(0^\circ) \right] \\
C_{4}^{11} &= \frac{15\sqrt{21}\rho}{23 + 8\left( \frac{P}{L} \right) + 8\left( \frac{P}{L} \right)^2} \left[ V_{s0}^2(0^\circ) + V_{s0}^2(90^\circ) + 2V_{s0}^2(45^\circ) - \frac{4}{\rho}\left( L - \frac{P^2}{L} \right) \right] 
\end{align*}
\] (2)

It is important to note that \( C_{4}^{12} \) and \( C_{4}^{13} \) can be inferred from relative velocity
measurements, whereas \( C_{4}^{11} \) depends on the difference of an average absolute measurement
and theoretical expectations for an isotropic polycrystalline aggregate. Thus \( C_{4}^{11} \) is
influenced by the greater uncertainties in absolute velocity measurements as compared to
relative velocity measurements as well as uncertainties in the effects of alloying elements on the isotropic moduli, \( L \) and \( P \).

**Diffraction Determination of ODCs**

The theory required to obtain ODCs from diffraction data is well-developed, and has been fully described previously (Roe, 1965; Bunge, 1982). The intensity of a diffraction peak is directly proportional to the volume fraction of grains, within the measurement volume, whose \((hkl)\) plane-normals are parallel to the bisector of the incident and diffracted beams. A specimen is oriented systematically to sweep this bisector through all possible directions in the specimen. The diffraction peak intensity, \( P \), is measured as a function of direction, specified as a vector, \( y \), with reference to specimen-fixed axes. Maps of \( P(y) \) are called pole figures and are measured for each of a selection of crystallographic directions, specified by a vector, \( h \). The collection of pole figures can be represented by a series expansion of spherical harmonics, with the expansion coefficients being the ODCs we seek:

\[
P_h(y) = \sum_{l=0}^{\infty} \sum_{\nu=1}^{N(h)} \left[ \frac{4\pi}{2l+1} \sum_{\mu=1}^{M(h)} C_{l\mu}^{\nu} k_i^{\nu}(h) k_i^{\nu}(y) \right]
\]

Here the functions \( k_i^{\nu}(h) \) and \( k_i^{\nu}(y) \) contain the effects of crystallographic and specimen symmetry, respectively. When a complete set of pole figure data is available, the ODCs can be obtained by a least squares analysis, with more complex procedures employed otherwise.
X-ray and neutron diffraction provide the raw pole figure data for the determination of ODCs. The advantages of neutron diffraction for measurements of the volumetric average texture in a bulk specimen, and various neutron techniques have been described previously (Bunge, 1989). X-ray diffraction is, by far the most accessible, and widely-used technique for texture analysis. However, as already explained, when used in a nondestructive mode, it is most sensitive to a thin layer of material near the measurement surface.

SAMPLES

Experiments were conducted on two sets of samples, each consisting of AA3004 can stock which had undergone hot rolling. Experiments on the first set of samples were intended to determine whether the ultrasonic technique could differentiate rolling and recrystallization textures. They were cut from 3 sheets, having respective thicknesses of 2.654 mm (sheet 1), 2.652 mm (sheet 2) and 2.570 mm (sheet 3). Sheets 1 and 3 were in the form of rectangles, with long dimensions on the order of 250 mm and small dimensions on the order of 125 mm. Sheet 2 was somewhat smaller than the others, which posed some difficulty in the velocity measurement. Sheet 2 had been processed through a hot mill and was in an unrecrystallized state. The other two samples (sheets 1 and 3) were at the same stage of processing and were in a fully recrystallized condition, showing a predominantly cube texture. Chemical composition differences led to a stronger cube texture in sheet 3 as compared to sheet 1. Experiments on the second set of sheets were designed to evaluate whether the ultrasonic technique was sensitive to more subtle variations in texture. These
were all in the fully recrystallized condition, but contained different amounts of cube texture, with sheet 4 having the weakest value, sheet 5 an intermediate value and sheet 6 the strongest value. In addition, there were differences from the center to the edge of each sheet. These samples had much larger lateral dimensions, being in excess of 600 mm x 600 mm.

**EXPERIMENTAL APPROACH AND OBSERVATIONS**

Two distinct techniques were used for the ultrasonic measurements. On the second set of larger samples, a fully automated instrument was available for use (Thompson et al., 1993). However, the samples studied initially were too small for use with that instrument, so a second technique had to be employed. In that case, we adopted a modification of a technique previously used by Clark Jr. et al. (1987; 1993).

**Manual Ultrasonic Measurements on Small Sheets**

The classical approach to measuring velocity involves plotting the arrival time of a particular waveform feature as a function of propagation distance. Here, we utilized $S\text{\textsubscript{o}}$, Lamb modes of plates (Auld; 1973), excited and detected at a frequency of 448 kHz by electromagnetic-acoustic transducers (EMATs) (Thompson, 1990). Unfortunately, the spatial length of our EMATs, coupled with a minimum separation required to allow the high gain electronic receivers to recover from the electrical break-through pulse, made it impractical to gather data at a large number of separations for waves traveling in the smaller dimensions of our samples. Hence we modified a procedure originally employed by Clark.
Clark was interested in predicting steel sheet formability based on the angular dependence of the ultrasonic velocity. His strategy was to infer the velocity from the delay of a particular zero crossing of a signal which had propagated between a pair of EMATs, separated by a fixed separation (Clark Jr. et al., 1987; Clark Jr. et al., 1993). To determine the exact path length that should be used to obtain the correct absolute velocity, a calibration experiment was required. This involved measuring an absolute velocity on one sheet. This absolute velocity was then used to determine an effective distance between the transmitting and receiving transducers at the fixed separation, the distance being chosen equal to the product of the measured absolute velocity and the time of the observed zero-crossing. Once this effective distance was known, the angular dependence of the velocity was inferred from the angular dependence of the time of the zero-crossing. Clark checked his procedure by making absolute measurements in three directions on a sheet (θ = 0°, 45°, 90° with respect to the rolling direction), using one of these as a calibration, and comparing the other two absolute velocities to the velocities inferred from the zero-crossing times and effective distance determined by the above procedure. The results agreed to within parts in 10⁴.

In our work, absolute velocity measurements were made along the longest direction of each of the three sheets in the first set of samples by selecting a particular zero-crossing in the received burst and plotting its time of arrival versus the EMAT separation. This separation was varied by inserting aluminum spacers of known lengths, ranging from 10 to 20 cm, between the EMATs. The time was measured with a time averaging counter. The slope of the plot of separations versus time determined the absolute velocity.
In principle, the accuracy of this procedure can be determined from that of the time and distance measurements (Johnson and Thompson, 1993). Each measurement of time was based on the average of 1000 successive waveforms, performed automatically by the counter. The standard deviation of this population was typically 11 ns, implying that the standard deviation of the estimate of the mean was $(11 \text{ ns})/\sqrt{1000} \approx 0.3 \text{ ns}$. The spacers had lengths known to $\pm 0.0127 \text{ mm}$ ($\pm 0.5 \times 10^{-3} \text{ inches}$). Based on these values, the uncertainty in the velocity, estimated from 11 pairs of separation and time data taken over separations ranging from 10 to 20 cm should in principle (Johnson and Thompson, 1993) be $\pm 6 \times 10^{-4} \text{ mm}/\mu\text{s}$. about $0.01\%$. This in principle accuracy is limited by the uncertainty in the spacer length rather than the time measurement.

When standard regression analysis techniques were applied to the data, the results in Table 1 were obtained. The observed standard deviations were about an order of magnitude greater than those expected in principle. We do not have a detailed understanding of the reason, but recognize some contributing factors such as plate curvature, EMAT lift-off effects (Clark Jr. et al., 1987), and lack of perfect contact between EMATs and spacers. Nevertheless, the accuracy of $0.1\%$ is sufficient to obtain the desired information, as will be seen below.

As noted above, the small dimensions of the samples precluded the application of this procedure for other directions. Hence, motivated by the previously discussed work of Clark et al., the transducers were fixed at an unknown distance, $d_1$, and the time delay to a particular
zero-crossing was measured at 0°, 45° and 90° to the rolling direction for each sheet. Figure 2 schematically illustrates this procedure. The above steps were repeated for four d's. The raw times are given in Table 2. These were also measured with a time averaging counter. In some cases, there was a little more noise than in the absolute measurements, but based on the statistical estimates, the precision in the determination of time was at least 1 ns. We have only quoted times to an accuracy of 10 ns in Table 2, based on our experience of greater errors in absolute measurements than would be expected in principle based on the accuracies of individual time and distance measurements.

Table 1. Absolute Velocities on First Set of Sheets.

<table>
<thead>
<tr>
<th>Sheet</th>
<th>Direction</th>
<th>Velocity (mm/µs)</th>
<th>Standard Deviation (mm/µs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>TD</td>
<td>5.360</td>
<td>0.009</td>
</tr>
<tr>
<td>2</td>
<td>TD</td>
<td>5.418</td>
<td>0.005</td>
</tr>
<tr>
<td>3</td>
<td>RD</td>
<td>5.274</td>
<td>0.005</td>
</tr>
</tbody>
</table>

Further processing of this raw data to obtain velocities and texture parameters will be discussed in the following sections. However, it is clear from the raw data that the rolling texture of sheet 2 is clearly differentiated from the recrystallization textures of sheets 1 and 3, since $t(45°) > t(0°)$ and $t(45°) > t(90°)$ in sheet 2 while $t(45°) < t(0°)$ and $t(45°) < t(90°)$ in
sheets 1 and 3.

**Automated Measurements on Large Sheets**

Measurements on the second set of sheets were made using an automated texture measurement instrument called Ultraform (Thompson et al., 1993). Ultraform uses the same basic technique as the hand measurements with some modifications. \( S_0 \) Lamb waves are again excited and detected by EMATs at a frequency of approximately 550 kHz. The instrument uses two transmitters and one receiver in each of the three directions needed, 0°, 45°, 90°. For a given direction, a triggered wave is transmitted simultaneously from both transmitters. It is detected as a pair of signals at the receiver. These waveforms are digitized and then processed in the frequency domain to determine the wave velocities. The central feature is a plot of the relative phase of the two received waveforms versus frequency, the slope of which is simply related to velocity after correction for a slight geometrical dispersion.

**Table 2.** Zero-crossing delays in \( \mu \text{s} \).

<table>
<thead>
<tr>
<th>d</th>
<th>Sheet 1 (recrystallized)</th>
<th>Sheet 2 (rolled)</th>
<th>Sheet 3 (recrystallized)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>t(0°)</td>
<td>t(45°)</td>
<td>t(90°)</td>
</tr>
<tr>
<td>#1</td>
<td>30.32</td>
<td>30.21</td>
<td>30.22</td>
</tr>
<tr>
<td>#2</td>
<td>32.21</td>
<td>32.09</td>
<td>32.09</td>
</tr>
<tr>
<td>#3</td>
<td>34.04</td>
<td>33.90</td>
<td>34.04</td>
</tr>
<tr>
<td>#4</td>
<td>35.92</td>
<td>35.78</td>
<td>35.80</td>
</tr>
</tbody>
</table>
associated with the finite ratio of plate thickness to wavelength. Laboratory evaluation of Ultraform indicates absolute accuracies in velocity of $10^3$ and relative accuracies of $10^4$ (Thompson et al., 1993). Ultraform then calculates the ODCs from these velocities using equation (1) above.

**X-ray diffraction**

The texture was independently characterized on each set of sheets using X-ray diffraction (Mo X-rays), performed on the mid-plane of the sheet. Four incomplete pole figures, (200), (111), (220) and (113), were obtained in each case, from which orientation distribution functions were computed. In the second set of sheets, in which transverse variations in texture were of interest, the texture was characterized at both the sheet center and edge.

**Neutron diffraction**

On sheets 2 and 3, neutron diffraction characterization of the texture was also performed. motivated by the desire to gain further information in a few cases where there were significant differences between the ultrasonic and X-ray predictions. Neutron measurements were considered more appropriate than the mid-plane X-ray measurements for comparing to the ultrasonic results since both the neutron and ultrasonic techniques provide through thickness averages while the mid-plane X-ray measurements, in the reflection mode utilized, sense only a thin surface layer.
Neutron diffraction measurements were performed in Canada at Chalk River Laboratories, a part of Atomic Energy of Canada Limited, with the E3 neutron diffractometer at the NRU reactor. The specimen was mounted on an Eulerian cradle, which was programmed to orient the material systematically, sweeping the bisector of the incident and diffracted beams through a complete hemisphere of specimen directions. Complete pole figures were obtained for the (111), (200) and (220) diffraction peaks from the aluminum alloy. A background pole figure was also measured by placing the neutron detector at a scattering angle free of diffraction peaks and performing the usual orientational scan. The incident and diffracted beams were collimated by Soller slits to have angular divergences of about 0.5° in the scattering plane. The cross sections of these beams were large, 50 mm x 50 mm, so regardless of orientation, the complete specimen volume was always fully bathed in the neutron beam. As a result of these experimental arrangements, there were no significant geometrical corrections to make to the pole figures prior to the analysis that yielded the ODCs.

PROCESSING THE MANUAL ULTRASONIC DATA

The raw data obtained on each of the three initial samples was an absolute velocity in one direction (Table 1) and a matrix of 12 times of arrival (3 directions x 4 transducer separations) of a selected zero-crossing of the received toneburst (see Table 2). As noted previously, the velocities were initially inferred based on a technique applied by Clark Jr. et
al (1987; 1993) to steel sheet, which involved assigning an effective propagation distance to each fixed transducer separation.

A triggered waveform can be thought of as a pulse starting at zero time (see Figure 3). Because of the finite length of the EMAT transducers, and the fact that they are excited by a toneburst, a particular feature in a received signal is delayed from the trigger pulse by more than the transducer separation divided by the wavespeed. In Clark's method, the determination of the transducer separation ignored this fixed delay associated with the tone burst width. In applying his technique to our set of sheets, we had the option of determining the effective distance on one sheet and using it to interpret all data (as was done by Clark in studying a large set of steel sheets) or determining it on each sample, as is possible in our study given the data in Table 1. Choosing the latter approach, and averaging the velocities obtained at the four separations for each angle, we obtained the velocities shown in the first row of (4).

**Table 3. Ultrasonic Velocities on First Set of Samples (in cm/μs).**

<table>
<thead>
<tr>
<th>Sheet 1</th>
<th></th>
<th></th>
<th>Sheet 2</th>
<th></th>
<th></th>
<th>Sheet 3</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0°</td>
<td>45°</td>
<td>90°</td>
<td>0°</td>
<td>45°</td>
<td>90°</td>
<td>0°</td>
<td>45°</td>
</tr>
<tr>
<td>Clark's Approach</td>
<td>0.5499</td>
<td>0.5520</td>
<td>0.5512</td>
<td>0.5615</td>
<td>0.5559</td>
<td>0.5572</td>
<td>0.5412</td>
<td>0.5449</td>
</tr>
<tr>
<td>Modified Approach</td>
<td>0.5480</td>
<td>0.5507</td>
<td>0.5497</td>
<td>0.5638</td>
<td>0.5565</td>
<td>0.5581</td>
<td>0.5418</td>
<td>0.5466</td>
</tr>
</tbody>
</table>
In examining the data, however, we noticed that the effective separation, $d$, depended systematically on the sheet, being essentially equal in sheets 1 and 2 but systematically smaller by about 0.6% in sheet 3. Since the transducers were rigidly held at the same distance on all sheets, this raised a concern. It was initially speculated that, since the plates had slightly different thicknesses, the effect was associated with geometrical dispersion (a slight dependence of guided wave speed on the plate thickness). However, theoretical analysis did not support this analysis. It was then observed that the changes in effective distance appeared to be related to the absolute velocity in the calibration direction, as given in Table 1. This led to the following interpretation and modified data processing procedure.

Consider the bottom sketch in Figure 3, which schematically shows that the observed time of the selected zero-crossing can be considered to be composed of a fixed delay time, $\tau_o$, associated with the finite temporal length of the excitation pulse and the spatial length of the EMATs, and the propagation delay, $\Delta t$. To account for these effects, we define a modified effective distance using equation (4). This equation uses $l$ as the separation counter, and $j$ as the sheet counter. The $t_j(\Theta_o)$ are the known measured times for waves propagating in the calibration direction, $\tau_o$ is the unknown fixed delay and $V_j(\Theta_o)$ is the known absolute velocity in a particular direction on sheet $j$. For our set of data, there are twelve independent forms of equation (4), where $l$ ranges over 4 separations and $j$ ranges over three sheets. These are

$$D_l = [t_j(\Theta_o) - \tau_o] \cdot V_j(\Theta_o)$$  (4)
solved, in a least squares sense, for the value of $\tau_0$ and the four $D_i$. Given these values, the velocities in the other directions are determined from the inverse of the slope of a regression line passing through these four points on the $t_i(\theta)$ versus $D_i$ plot and forced through a $y$-intercept of $\tau_0$. We found that this procedure fit the data considerably better than when $\tau_0$ was taken to be zero, equivalent to Clark's procedure. The resulting velocities are presented in the second row of (4). Although the shifts were small, ranging from 0.1% to 0.4%, they were considered to be significant in terms of the high precision desired.

Examination of the raw time data indicates several interesting points. First, the shifts in time between the sheets and directions is quite small, being on the order of tenths of microseconds. Nevertheless, with modern electronics, such shifts can be easily measured. This is confirmed by noting that the zero-crossing delays order in a fashion consistent with the absolute velocities for the directions and materials where those were determined. Second, it is clear that $V^2 > V^1 > V^3$. $[v^{1,3}(0) - v^{1,3}(90)] < 0$, $[v^2(0) - v^2(90)] > 0$, $v^{1,3}(45) > [v^{1,3}(0) + v^{1,3}(90)]/2$, $v^2(45) < [v^2(0) + v^2(90)]/2$, where $V = [v(0) + v(90) + 2v(45)]/4$ and the superscripts define sheets, not exponents. Equation (1) then implies that $C_{41}^{11}(2) < C_{41}^{11}(1) < C_{41}^{11}(3)$, $C_{41}^{12}(2) > 0$, $C_{41}^{12}(1,3) < 0$, $C_{41}^{13}(2) < 0$, $C_{41}^{13}(1,3) > 0$ where the argument here denotes the sheet number. We thus again see that there is a clear differentiation between the rolled (sheet 2) and recrystallized (sheets 1 and 3) samples. Figure 4 presents the predicted angular form of the velocities, obtained by fitting the function $A + B\cos(2\theta) + C(1-\cos(4\theta))$ to the data at $\theta = 0^\circ, 45^\circ, 90^\circ$. The aforementioned distinctions between the three sheets are clearly evident.
It should be noted that the simpler procedure of Clark was quite adequate for his needs. Differences between the conditions encountered in our work and his work are the much greater propagation distances in his experiments, making $\tau_e$ a smaller fraction of the observed times, the much greater elastic anisotropy of steel, making the required precision in velocity determination less stringent, and the fact that Clark's final output was to be empirically correlated with plastic strain ratios in steel whereas we wish to make direct comparisons to values of $C_4''$ determined by X-ray or neutron diffraction.

**QUANTITATIVE CHARACTERIZATION OF TEXTURE**

**Differentiation of Rolling and Recrystallization Textures**

Having convinced ourselves that the data passes all qualitative tests, we calculate the ODCs using equation (1) above based on the velocities inferred from the raw data by the two methods discussed above. The results of these calculations along with neutron diffraction data for Sheets 2 & 3 are plotted in Figure 5 versus the X-ray diffraction data. Results for $C_4''$ are not included in this figure, but will be discussed in a later section. The ultrasonically determined values of $C_4^{12}$ and $C_4^{13}$ for Sheets 1 & 3 compare very favorably with the X-ray and Neutron diffraction values which suggests that recrystallization textures are easily measured using ultrasonics. It should also be noted that, in each case, the new signal processing produced results that were closer to both the X-ray and neutron diffraction data than did Clark's method. The data for sheet 2 does not support as simple an interpretation. For $C_4^{13}$, we notice that the predictions of the modified ultrasonic technique are farther from
the X-ray measurements than are those based on Clark's method. We note, however, that the modified method leads to predictions that are closer to the neutron measurements. It is well known that the rolling texture has gradients through the thickness (Ren, 1996), leading to differences between the mid-plane X-ray and neutron values for $C_4^{13}$. The fact that the modified ultrasonic prediction is closer to the neutron measurements is consistent with the fact that both measure volumetric averages.

The situation with $C_4^{12}$ in sheet 2 is more complex, as both ultrasonic methods yield predictions of comparable magnitude, but opposite sign, to the predictions of either diffraction method. We do not have a definitive interpretation of this result. We note, however, that this sample was smaller than desired and energy reflected from the edges could have interfered with the measurement. It is also possible that there was a mislabelling of the sample axes when the ultrasonic data were taken because an interchange of the RD and TD would account for the difference. Other microstructural features could be playing a role, for example, precipitates or dislocations will also affect the measured velocity. However, since no such sign discrepancies have been reported on the large number of aluminum samples that have been studied previously (Clark Jr. et al., 1988; Reno, Fields, Clark Jr., 1988; Cassier, Donadille, Bacroix, 1989; Borsutski et al., 1993; Thompson et al., 1993), we expect that the experimental difficulties is the likely cause. Unfortunately, we cannot remeasure the ultrasonic velocities because the samples have been subsequently destroyed. Because of these complications, which we believe to be unique to this particular experiment, the results for $C_4^{12}$ on sheet 2 are not presented in Figure 5.
(4) presents the numerical values of the ultrasonic (new algorithm) and X-ray determinations of the ODCs. In addition to $C_4^{12}$ and $C_4^{13}$ that were presented graphically, $C_4^{11}$ is included. The obvious discrepancies between the ultrasonic and X-ray values will be discussed in section VI C.

**Subtle Variations in Recrystallization**

The second set of sheets were recrystallized to varying degrees. Measurements were made to determine how easily ultrasonics could distinguish between degrees of recrystallization. In addition to variation between the sheets, each sheet contained a variation in the texture along the width from the center to the edge of the rolled sheet.

Measurements were taken at two inch intervals across each sheet using Ultraform. The ultrasonic results are plotted in Figure 6 along with X-ray diffraction results. The X-ray results were taken at the center and edge only. One can easily see that the ultrasonic measurements for $C_4^{12}$ and $C_4^{13}$ have the same magnitude and vary in the same sense across the sheet as the X-ray predictions. $C_4^{11}$ values also show the same trends across the sheet, but differ in magnitude. This difference will be discussed in greater detail in the next subsection.

**Offset in $C_4^{11}$**

Predictions of $C_4^{11}$ for every sheet seemed to have a significant negative offset with respect to the X-ray and neutron diffraction results. To explain this offset in the prediction of $C_4^{11}$, one must look into the calculation of this ODC. Equation (1) shows the relation
between $C_4^{11}$ and the measured velocities, as well as that for the other ODCs. There are two important differences between the calculation of $C_4^{11}$ and that for $C_4^{12}$ and $C_4^{13}$. First, $C_4^{11}$ depends on absolute, rather than relative velocity measurements. Second, the average of the absolutely measured velocities must be compared to the value that would be observed in the absence of texture, whose square is described by the last term in the square brackets.

$\frac{4}{\rho} \times \left( L - P^2 / L \right)$. This term would be expected to depend on alloying. In the absence of knowledge of the alloying effect, the isotropic value for pure aluminum has been used in our calculation, based on the single crystal elastic constants (SCECs) and use of the Hill averaging procedure to obtain $L$ and $P$. While $C_4^{12}$ and $C_4^{13}$ also depend on the SCECs through $L$ and $P$, the dependence is in a multiplicative factor instead of a subtractive term. Propagation of any error associated with the SCECs leads to a much larger error in $C_4^{11}$ than in $C_4^{12}$ or $C_4^{13}$.

Clearly, further work is required in the determination of $C_4^{11}$. The accuracy in the ultrasonic determination of $C_4^{11}$ using the scheme described in this paper depends strongly on the selection of SCECs. The proper determination of the SCECs for the particular alloy being studied, a result that has been successfully employed in steel (Forouraghi, 1995; Daniel, Jonas, 1990) would presumably yield improved results and this will be the subject of further study. Other ultrasonic schemes, which infer $C_4^{11}$ from relative rather than absolute velocity measurements should also be considered (Li and Thompson, 1989; Kawashima, 1990; Kawashima, Hyogushi and Akagi, 1993)
DISCUSSION AND CONCLUSIONS

Interpretation of our results is aided by knowledge of the ODCs that would be exhibited by the ideal textures present in rolled and recrystallized sheet, as are also presented in (4). These calculations are based on a Gaussian spread of 10° about the ideal orientation. As is well known, the Cu, S and Brass textures are typical of rolled sheet while the cube and Goss textures are typical of recrystallized sheet.

Examination of (4) suggests that \( C_4^{13} \) should be an excellent parameter to differentiate rolling from recrystallization textures, since it has negative values of about the same magnitude for the three rolling textures and positive values of about the same values for the recrystallization textures. These values were observed in both the ultrasonic and X-ray data. The change of the angular dependence of the ultrasonic velocity shown in Figure 4 from one exhibiting a valley near 45° to one with a peak near 45° is a direct consequence. Figure 7 plots the values of \( C_4^{13} \) determined by ultrasonics and neutron diffraction against the X-ray values for the six samples examined, with two entries each for samples 4-6 corresponding to the center and edge positions. The good agreement suggests that this ultrasonically observable parameter could provide a useful assessment of recrystallization in field environments such as those found in rolling mills or can manufacturing plants.

Stiffler et al (1994) have also studied this possibility, reporting a correlation between degree of recrystallization and \( C_4^{13} \) in 3XXX beverage can stock. In contrast to the present work, which utilized \( S_o \) guided modes, Stiffler made measurements with \( S_{H_6} \) modes. The two sets of results seem fully consistent. Stiffler's material had values of \( C_4^{13} \) equal to -2.260
in the rolled condition and 1.569 in the recrystallized condition. A series of sheets whose degree of partial recrystallization was inferred from micrographs exhibited intermediate values of $C_4^{13}$ which were roughly proportional to the degree of recrystallization. The sheets studied in the present work fall in a similar range of $C_4^{13}$.

In summary, ultrasonics is a quick and precise method of characterizing texture in aluminum and aluminum alloys. Not only are strong textures differentiated, but subtle changes can be sensed as well. $C_4^{12}$ and $C_4^{13}$ can be determined precisely, and our results compare very well with other methods of determining ODCs. An important potential application of these techniques appears to be monitoring the degree of recrystallization.

Acknowledgment

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Roe, R., Description of Crystallite Orientation in Polycrystalline Materials. III. General Solution to Pole Figure Inversion. In *Journal of Applied Physics*. 1965, 36, 2024


Table 4. Comparison of ultrasonic, X-ray, neutron and ideal ODCs for all samples. Results for sheets 1, 2 and 3 are computed using the modified method discussed above.

### Ultrasonically measured ODCs

<table>
<thead>
<tr>
<th></th>
<th>(C_{41}^{11})</th>
<th>(C_{42}^{12})</th>
<th>(C_{43}^{13})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sheet 1</td>
<td>-4.06</td>
<td>-0.36</td>
<td>0.6</td>
</tr>
<tr>
<td>Sheet 2</td>
<td>-7.64</td>
<td>1.19</td>
<td>-1.46</td>
</tr>
<tr>
<td>Sheet 3</td>
<td>-2.06</td>
<td>-0.46</td>
<td>1.16</td>
</tr>
<tr>
<td>Sheet 4 (Center. Edge)</td>
<td>-1.80</td>
<td>-1.70</td>
<td>-0.53</td>
</tr>
<tr>
<td>Sheet 5 (Center. Edge)</td>
<td>-1.92</td>
<td>-1.79</td>
<td>-0.40</td>
</tr>
<tr>
<td>Sheet 6 (Center. Edge)</td>
<td>-2.54</td>
<td>-2.25</td>
<td>-0.45</td>
</tr>
</tbody>
</table>

### X-ray ODCs (neutron ODCs)

<table>
<thead>
<tr>
<th></th>
<th>(C_{41}^{11})</th>
<th>(C_{42}^{12})</th>
<th>(C_{43}^{13})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sheet 1</td>
<td>1.1 (n/a)</td>
<td>-0.428 (n/a)</td>
<td>0.712 (n/a)</td>
</tr>
<tr>
<td>Sheet 2</td>
<td>-0.004 (-0.21)</td>
<td>-0.393 (-0.67)</td>
<td>-1.202 (-1.60)</td>
</tr>
<tr>
<td>Sheet 3</td>
<td>2.07 (2.39)</td>
<td>-0.776 (-0.49)</td>
<td>1.698 (1.84)</td>
</tr>
<tr>
<td>Sheet 4 (Center. Edge)</td>
<td>1.36</td>
<td>1.72</td>
<td>-0.47</td>
</tr>
<tr>
<td>Sheet 5 (Center. Edge)</td>
<td>1.30</td>
<td>1.56</td>
<td>-0.42</td>
</tr>
<tr>
<td>Sheet 6 (Center. Edge)</td>
<td>1.51</td>
<td>1.58</td>
<td>-0.58</td>
</tr>
</tbody>
</table>

### Ideal textures

<table>
<thead>
<tr>
<th></th>
<th>(C_{41}^{11})</th>
<th>(C_{42}^{12})</th>
<th>(C_{43}^{13})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Goss</td>
<td>-1.69</td>
<td>-7.57</td>
<td>4.16</td>
</tr>
<tr>
<td>Cube</td>
<td>6.78</td>
<td>0</td>
<td>5.74</td>
</tr>
<tr>
<td>Cu</td>
<td>-1.69</td>
<td>2.51</td>
<td>-3.35</td>
</tr>
<tr>
<td>S</td>
<td>-1.69</td>
<td>0.09</td>
<td>-2.76</td>
</tr>
<tr>
<td>Brass</td>
<td>-1.69</td>
<td>-2.51</td>
<td>-3.35</td>
</tr>
</tbody>
</table>
Figure 1. Flow diagram representing typical production sequence for aluminum can-body sheet. Final product shows ears formed during production of the can.
Figure 2. Representation of measurement scheme.

Figure 3. Schematic of waves generated during measurements.
Figure 4. Angular variation of ultrasonic velocity in the plane of the sheet. Values at 0°, 45°, and 90° are the measured ultrasonic velocities and the remainder of the curve is plotted based on the expected theoretical variation of A + Bcos(2θ) + C(1-cos(4θ)).
Figure 5. Comparison of ODCs predicted by two slightly different ultrasonic methods and neutron diffraction to X-ray diffraction:
(a). Sheets 1 and 3 (recrystallization textures),
(b). Sheet 2 (rolling texture).
Figure 6. Spatial variations in ODCs from center of sheet to edge of sheet.
Figure 7. Comparison of $C_4^{13}$ determined by X-ray diffraction, ultrasonics and neutron diffraction for all six sheets.
CHAPTER 3. CONTINUATION OF ALCAN STUDY

To answer the question of how the single crystal elastic constants affect the predictions of $W_{400}$, one needs to independently determine these constants. Two samples of aluminum were provided; one produced from 99.9% pure aluminum and the second produced from an alloy similar to the can body stock alloy used in previous studies—3XXX. Al-1.2Mn-1.0Mg. The samples were rectangular in shape and were large enough to allow measurement of bulk waves propagating in each of the three principle directions.

3.1 Background

By reviewing the equations presented in Chapter 1 for bulk waves propagating in cubic materials, one sees that there are, technically, six unknowns: $W_{400}$, $W_{420}$, $W_{440}$, $c_{11}$, $c_{12}$, and $c_{44}$. For most work, one assumes that the SCECs are known; however, the SCECs can be strongly affected by changes in composition of the alloy. This effect is seen in predictions of $W_{400}$ presented in Chapter 2. One might imagine that since we can measure nine different bulk velocities, we could independently determine the six unknowns. Unfortunately, this is not true because the nine equations are not independent. In reality only four of the equations are truly independent; therefore other assumptions must be made to allow predictions of the ODCs and SCECs.

To illustrate the dependence of the equations (8) in Chapter 1, one can look at combinations of these equations. It should be noted that equations (8)(g), (8)(h), and (8)(i)
are identical to the previous three equations, (8)(d), (8)(e), and (8)(f), with the exception that they depend on \( c_{12} \) instead of \( C_{44} \). These three equations, (8)(g), (8)(h), and (8)(i), are not generally used for determining ultrasonic velocities. Therefore, we have six equations with which to find the six unknowns. For example, one might want to measure the quantity \( \rho \left( \nu_{11}^2 - \nu_{22}^2 \right) \) to predict \( W_{420} \). This result is

\[
\rho \left( \nu_{11}^2 - \nu_{22}^2 \right) = -\frac{32}{35} \sqrt{5} \pi c^2 W_{420}
\]

Unfortunately, the combination \( \rho \left( \nu_{44}^2 - \nu_{55}^2 \right) \) gives the exact same result. Therefore, at least one of our six equations is dependent on the other equations. Continuing this same line of thought, it can be shown that the combinations \( \rho \left( \nu_{11}^2 + \nu_{22}^2 - 2 \nu_{33} \right) \) and \( \rho \left( \nu_{44}^2 + \nu_{55}^2 - 2 \nu_{66} \right) \) give exactly the same result as well. Thus, there are really only four equations which can be used to find the six unknowns, and more information is required. By making some further assumptions, i.e. providing us with more information, we can determine our six unknowns. First, one can probably assume that the single crystal anisotropy, \( c \), will not change much as the elastic constants change. Thus only one of the elastic moduli, \( c_{11} \) or \( c_{44} \), needs to be known to have enough information to predict the other quantities. Again, assume that the relation between \( c_{11} \) and \( c_{44} \) will not change appreciably even though each quantity may change; this assumption is that Poisson's ratio is constant, thereby providing a relation between \( c_{11} \) and \( c_{44} \).

Another technique is to look at a combination of elastic constants rather than looking at specific elastic constants. For example, it has been shown that the sums of the three
velocities in each direction are free of texture and equal to a linear combination of the bulk elastic moduli as given by the following:

\[ \rho (V_{11}^2 + V_{12}^2 + V_{13}^2) = \rho (V_{21}^2 + V_{22}^2 + V_{23}^2) = \rho (V_{31}^2 + V_{32}^2 + V_{33}^2) = \lambda + 4\mu \]

This can be shown to be true regardless of whether the pure-mode polarization directions are aligned with the sample directions (see Appendix A). This issue is important for the samples studied and will be discussed in more detail below.

Similarly, one can measure a combination of elastic constants using only longitudinal waves. Paralleling the development of equations for sheets, one can equate the bulk velocity travelling at 45° to a combination of ODCs. Then by combining equations for velocities travelling at 0° (a principal direction), 45°, and 90° (another principal direction) with the velocity of a wave travelling in the normal direction, one can eliminate the ODCs leaving a combination of elastic moduli as given by the following equation.

\[ \lambda + 2\mu = \rho \frac{2}{5} \left( V(0)^2 + V(90)^2 + 2V(45)^2 - \frac{3}{2} V_{33}^2 \right) \]

This procedure has the advantage of eliminating the need for shear wave measurements, which, as discussed in the next sections, did not produce good results for these particular samples.
3.2 Measurements

Bulk longitudinal velocities were measured in a pulse-echo mode with the samples submersed in water using a quarter-inch, 20 MHz transducer. A pulse-generator triggers the pulse which travels from the transducer, through the water, through the sample, reflects off the bottom face of the sample and travels back through the sample and water to the transducer. The received waveform is then displayed on a LeCroy 9410 oscilloscope connected to a computer with a GPIB board. The waveform is captured on the computer and saved. Consecutive echoes are recorded for determining the velocity. Longitudinal measurements were made in each of the three sample directions.

Shear wave measurements were made in contact mode using Panametric shear wave couplant and a quarter-inch, 5 MHz transducer. Again the wave is triggered using a pulse-generator, whereby the transducer transmits the wave through the sample and receives the reflected echo. The transducer was placed in such a way to measure the pure shear mode propagating through the sample. Once consecutive echoes were recorded with the computer, the transducer was rotated 90° to measure the shear wave with the opposite polarization. Raw data is presented in Figure 1 through Figure 6. These figures show the raw waveform and magnitude spectrum for each echo recorded by each measurement type in each of the three directions for both samples. An attempt was made to record the first three echoes in each case; unfortunately, for several cases the third echo was not strong enough to record accurately. However, the calculation of the velocity only requires two echoes, thus the first
Figure 1. Raw data for direction 1 in the pure aluminum sample.
Figure 2. Raw data for direction 2 in the pure aluminum sample.
Figure 3. Raw data for direction 3 in the pure aluminum sample.
Figure 4. Raw data for direction 1 in the alloyed aluminum sample.
Figure 5. Raw data for direction 2 in the alloyed aluminum sample.
Figure 6. Raw data for direction 3 in the alloyed aluminum sample.
and second echo were used for each case and the third echo, when recorded, was disregarded at this stage.

In addition, velocity measurements in directions 45° to the principle directions were made by machining faces on the samples which were 45° to the original sample faces.

Velocity measurements were computed with software and are presented in Table 1. The software computes the velocities using a phase-slope algorithm and a cross-correlation algorithm. The average of these two results were used throughout this study. Prediction of the ODCs and SCECs is complicated because the shear wave pure-mode polarization directions did not coincide with the sample directions. Theories presented thus far assume that the pure-mode polarization directions do coincide with the sample directions. However, as discussed in the previous section, one can compare a texture-free combination of the elastic moduli, and this combination of elastic moduli does not depend on propagation direction.

Therefore, one can determine an estimation of the change in the elastic constants due to alloying by comparing the combination of bulk elastic moduli to the same combination for a pure sample. In Table 2 the combination of bulk elastic moduli is computed for each of the three directions in each of the two aluminum samples. The last row shows the percent change for each direction between the samples, and the rightmost column shows the largest percent change within a particular sample.
Table 1. Velocities computed for two aluminum samples.

<table>
<thead>
<tr>
<th></th>
<th>Pure aluminum sample</th>
<th></th>
<th>Typical can-body stock aluminum alloy sample</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Direction</td>
<td>Longitudinal</td>
<td>Shear Measurement 1</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>6.414E+03</td>
<td>3.006E+03</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6.328E+03</td>
<td>3.095E+03</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>6.361E+03</td>
<td>3.016E+03</td>
</tr>
</tbody>
</table>

Table 3 gives longitudinal velocities measured in each of the three principle directions plus the 45° direction in the 1-2 plane, the quantity $\lambda + 2\mu$, and the percent difference between the quantity $\lambda + 2\mu$ for the pure sample and the alloyed sample.

3.3 Conclusions

It is clear by looking at Table 2 that the variation in $\lambda + 4\mu$ within each sample is greater than the variation between samples. This suggests that either the elastic moduli are not changing appreciably or that the precision of our velocity measurements is not sufficient for detecting the changes in the moduli.
Table 2. Texture-free comparison of elastic moduli.

<table>
<thead>
<tr>
<th></th>
<th>( \lambda + 4\mu )</th>
<th>( p(V_{11}^2 + V_{12}^2 + V_{13}^2) )</th>
<th>( p(V_{21}^2 + V_{22}^2 + V_{23}^2) )</th>
<th>( p(V_{31}^2 + V_{32}^2 + V_{33}^2) )</th>
<th>%change within sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alloy</td>
<td>158.1512</td>
<td>162.1738</td>
<td>157.8273</td>
<td></td>
<td>2.6801</td>
</tr>
<tr>
<td>Pure Al</td>
<td>161.2489</td>
<td>155.5602</td>
<td>159.0041</td>
<td></td>
<td>3.5278</td>
</tr>
<tr>
<td>%change between samples</td>
<td>1.9210</td>
<td>4.2514</td>
<td>0.7400</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3. Texture-free comparison of elastic moduli using longitudinal waves only.

<table>
<thead>
<tr>
<th>Direction</th>
<th>1</th>
<th>45° (1-2)</th>
<th>2</th>
<th>3</th>
<th>( \lambda + 2\mu )</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pure Al</td>
<td>6369</td>
<td>5922</td>
<td>6329</td>
<td>6394</td>
<td>109.33E+9</td>
<td>4.54</td>
</tr>
<tr>
<td>Alloved Al</td>
<td>6486</td>
<td>6457</td>
<td>6341</td>
<td>6325</td>
<td>114.53E+9</td>
<td></td>
</tr>
</tbody>
</table>

It is possible that the precision of the shear velocities is adversely affected by the considerable shape change of the waveform during propagation. This shape change is clearly evident in Figure 1 through Figure 6. This change can create problems with the algorithms, phase-slope and cross-correlation, used to compute the velocities. Therefore, one should verify the velocities with another technique. Table 4 shows the elastic moduli computed using shear velocities determined from a "first-feature" technique. This technique employs selecting the very first observable feature of successive waveforms for computing the time delay. Unfortunately, it is very difficult to determine the first feature of the shear waveforms in Figure 1 through Figure 6. Thus, it is again difficult to say whether the changes in moduli
are due to the differences between the samples or the imprecision of the velocity measurements.

Table 3, however, shows a much clearer distinction between the two samples. Even when compared to the reference values for pure aluminum, the percent difference for $\lambda+2\mu$ for the pure and alloyed samples are 1.2% and 5.7%, respectively. These are certainly reasonable values. In addition, these results show the expected trends for changes in elastic moduli due to alloying. Because the longitudinal velocities are much easier to reproduce and can be measured with greater precision, the results produced using only longitudinal waves were expected to provide better accuracy.

Table 4. Texture-free comparison of elastic moduli computed using first feature method.

<table>
<thead>
<tr>
<th></th>
<th>$\lambda + 4\mu = \rho (V^2_{11} + V^2_{12} + V^2_{13})$</th>
<th>$\rho (V^2_{21} + V^2_{22} + V^2_{23})$</th>
<th>$\rho (V^2_{31} + V^2_{32} + V^2_{33})$</th>
<th>%change within sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alloy</td>
<td>162.8400</td>
<td>162.9716</td>
<td>152.3156</td>
<td>6.5386</td>
</tr>
<tr>
<td>Pure Al</td>
<td>163.3892</td>
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<td>160.7892</td>
<td>3.1741</td>
</tr>
<tr>
<td>%change between samples</td>
<td>0.3361</td>
<td>3.0141</td>
<td>5.2700</td>
<td></td>
</tr>
</tbody>
</table>

3.4 References

CHAPTER 4. EARING STUDY

Deep-drawing aluminum cans produces ears which are caused by the elastic and plastic anisotropy of the metal. The purpose of this study was to tie together some empirical work relating ODCs to earing\(^1\) with other studies showing effects of texture on earing\(^2,3\). The basis of this work stems from a theory proposed by Papadakis and Thompson\(^4\) which relates earing profile heights, expressed as a Fourier series, to ODCs.

4.1 Background

Earing profiles are typically measured using an LVDT or similar device while the part (e.g. aluminum can) is rotated 360° starting at a line which corresponds to the location of the rolling direction in the original blank. This produces, on a strip chart, a curve showing peaks and valleys through a range of 0° to 360°, as demonstrated in Figure 1. One can express this curve as a Fourier series,

\[
F = \frac{B_0}{2} + \sum_{k=1}^{\infty} B_k \cos(k\theta) + \sum_{k=1}^{\infty} A_k \sin(k\theta)
\]

where \(B_k\) and \(A_k\) are the Fourier coefficients, and \(\theta\) is the angle. Since we have assumed that our earing was produced on a cup drawn from rolled sheet, we would expect the coefficients of the sine terms to be negligible because rolling textures show only bilateral symmetry\(^4\). In addition, one expects \(B_1\) and \(B_2\) to be related to the two-fold and four-fold earing.
Figure 1. Schematic showing process for collecting earing profiles from drawn cans.
respectively, with $B_0$ describing some average peak height. Therefore, given a set of data corresponding to a curve describing the earing, one could find the coefficients of the Fourier series and seek to determine relationships to the ODCs.

The relation to the ODCs is not, however, completely known. The ODCs, $W_{400}$, $W_{420}$, and $W_{440}$ can be qualitative linked to the texture through the following. $W_{400}$ measures the planar anisotropy; $W_{420}$ measures the two-fold changes in the texture; $W_{440}$ measure four-fold changes in the texture. Thus, one would expect $W_{420}$ to be related to the second order Fourier coefficient, $B_2$, and $W_{440}$ to be related to $B_4$. Therefore, $W_{420}$ and $W_{440}$ should be related to two-fold and four-fold earing, respectively. Previous results also suggest a correlation between $W_{440}$ and four-fold earing. Figure 2 shows a plot of results clearly showing this correlation. The data set labeled “Reynolds data” is from reference 1, and the data set labeled “Lu’s data” is from reference 2. However, the third data set, labeled “Our data (new model),” is given in Table 1. A description of the samples and the ODCs are published in reference 3 without the earing data. Percent earing presented in this plot is defined as:

$$\text{%Earing} = \left[ \frac{h_p - h_v}{h_v} \right] \times 100$$  \hspace{1cm} (2)

where $h_p$ is the average height of the peaks near 45°, 135°, 225°, and 315°, and $h_v$ is the average height of the valleys near 0°, 90°, 180°, 270°.
Figure 2. Correlation between percent earing and $W_{440}$ as shown by three different data sets.
Table 1. Earing data for "Our Data (new model)" in Figure 2.

<table>
<thead>
<tr>
<th>Sheet</th>
<th>% W440</th>
<th>% Earing</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>First Set of Sheets</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sheet 1</td>
<td>1.949</td>
<td>2.2</td>
</tr>
<tr>
<td>Sheet 2</td>
<td>-4.716</td>
<td>-6</td>
</tr>
<tr>
<td>Sheet 3</td>
<td>3.744</td>
<td>6.2</td>
</tr>
<tr>
<td><strong>Second Set of Sheets</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Center</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sheet 1</td>
<td>3.552</td>
<td>2.9</td>
</tr>
<tr>
<td>Sheet 2</td>
<td>4.072</td>
<td>3.1</td>
</tr>
<tr>
<td>Sheet 3</td>
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<td>3.2</td>
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<tr>
<td><strong>Second Set of Sheets</strong></td>
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<tr>
<td>Edge</td>
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<tr>
<td>Sheet 1</td>
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<tr>
<td>Sheet 2</td>
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<td>6.9</td>
</tr>
<tr>
<td>Sheet 3</td>
<td>5.206</td>
<td>6.6</td>
</tr>
</tbody>
</table>

4.2 Experimental Procedure

Earing profiles for this study were obtained previously during a field test of an automated texture measuring device\(^1\). Five blanks were cut from fifteen samples and were drawn into cups. LVDT was used to produce the earing profiles on strip charts. These profiles were scanned on a computer and digitized using software which reads changes in pixel colors in video memory.

Computation of the Fourier coefficients can be done using the following equations\(^6\).

\[
B_n = \frac{1}{\pi} \int_{0}^{2\pi} F(\theta) \cos(n\theta) \, d\theta, \quad n=1,2,3,\ldots
\]

\[
A_n = \frac{1}{\pi} \int_{0}^{2\pi} F(\theta) \sin(n\theta) \, d\theta, \quad n=1,2,3,\ldots
\]
Because the data, \( F(\theta) \), was not at equispaced abcissa values, a linear interpolation routine was used to generate values for \( F(\theta) \) at appropriate values during the integration routine. In addition some averaging was done on the data prior to integration to help reduce the effects from the non-symmetrical nature of the raw earing profile. This averaging consisted of taking the mean of the data at every abcissa from each of the five plates within a sample. In addition, bilateral symmetry was forced on the data by averaging points that should be equal, based on the assumption that the following should be true.

\[
F(\theta) = F\left(\theta + \frac{\pi}{2}\right) = F(\theta + \pi) = F\left(\theta + \frac{3\pi}{2}\right) = F(\theta + 2\pi)
\]

Because there may be some misalignment between the rolling direction on the blank and the starting location for our data caused by either a misplacement of the blank in the drawing apparatus or a misplacement of the sample in the LDVT device, a routine was written to attempt to realign the earing profile. This was accomplished by maximizing the four-fold symmetry of the profile. The routine to do this computes \( B_4 \) for a profile then shifts the profile in \( \theta \), recomputes \( B_4 \), and repeats until it finds a maximum value for \( B_4 \). Once the four-fold symmetry is maximized, the averaging mentioned above is done, and the Fourier coefficients are predicted.
4.3 Results and Discussion

Once the Fourier coefficients are computed, as discussed above, they can be plotted against the ODCs predicted from ultrasonic velocities. Three plots are created based on our assumed relations between the Fourier coefficients and the ODCs. These relations are plotted with Figure 3 containing $B_0$ versus $W_{400}$, Figure 4 containing $B_2$ versus $W_{420}$, and Figure 5 containing $B_4$ versus $W_{440}$. A fairly nice, linear relationship is shown between $B_0$ and $W_{400}$ and $B_4$ and $W_{440}$; however, there does not seem to be much correlation between $B_2$ and $W_{420}$.

In addition to viewing the direct comparison between the Fourier coefficients and the ODCs, one could compare the percent earing measured by the manufacturer to a percent earing predicted from the Fourier coefficients. Since the coefficients simply describe the earing profile, one should be able to relate the percent earing, as given in equation (2), to the Fourier coefficients. This is easily done by substituting equation (1) evaluated at the appropriate angles into equation (2) and simplifying. The result can be given by the following.

\[
\text{%Earing} = \frac{\sum_{n=1}^{\infty} \left(-1\right)^n B_{4n} - B_{4n}}{\frac{B_0}{2} + \sum_{n=1}^{\infty} \left|B_{4n}\right|}
\]  

(5)

For the purposes of this study, we will only need to compute for $n$ up to four, so the above equation simplifies to the next equation.
Figure 3. Attempted correlation between $W_{400}$ and $B_q$, a Fourier coefficient fitted to earring profiles.
Figure 4. Attempted correlation between $W_{420}$ and $B_2$.
Figure 5. Attempted correlation between $W_{440}$ and $B_4$. 

$y = 303.74x - 0.9388$

$R^2 = 0.7807$
\[ \%Earring = \frac{-2B_4}{\frac{B_0}{2} + B_4} \] (6)

It is apparent that the form of this equation depends on the definition of the percent earing. Since we have defined percent earing in terms of the differences in height of the peaks and valleys of the four-fold ears, we would expect this equation to depend largely on the fourth order Fourier coefficients.

Once the percent earing is computed from the Fourier coefficients, it can be plotted against \( W_{440} \) as done in Figure 2. This is shown in Figure 6 along with the data previously shown in Figure 2.

While it is clear that some relation must exist between earing and \( W_{440} \), one does not see as nicely defined correlation between the Fourier coefficients of an earing profile and \( W_{440} \). However, closer inspection of Figure 6 shows that the percent earing computed from the Fourier coefficients correlates slightly better with \( W_{440} \) than the usual definition of percent earing. This suggests that the usual definition of percent earing is subject to greater scatter due to variations from symmetries greater than 40, for example, 60, 80, 100, etc. Some of the scatter in Figure 2 may also be due to these higher order variations. Thus, a better quantification of percent earing could be made using Fourier coefficients computed from the
Figure 6. Representation of correlation between percent earing and $W_{440}$ including percent earing computed from Fourier coefficients.
earing profile, as done in equation (5). because the inclusion of higher order Fourier coefficients will account for some of the variations due to symmetries greater than 40.

4.4 References


CHAPTER 5. ULTRASONIC MEASUREMENT OF THE KEARNS TEXTURE FACTORS IN ZIRCALOY, ZIRCONIUM AND TITANIUM

A paper prepared for publication in Metallurgical Transactions A.

A.J. Anderson, R.B. Thompson, C. S. Cook

Abstract

Texture, developed during the processing of zirconium and zirconium alloys, e.g. Zircaloy, plays an important role in determining the properties and performance of the resultant structural components. Extending the work done on cubic metals, ultrasonic velocity measurements were used to nondestructively characterize the texture in hexagonal sheet metals. A theory is developed relating texture coefficients $W_{LMN}$ ($L \leq 4$, found in an expansion of the crystallite orientation distribution function in terms of generalized spherical harmonics) to industrially measured Keams factors, which predict the fraction of basal poles aligned in a particular sample reference direction. Ultrasonic characterization of texture has been performed on three sheets of zirconium (one pure, two Zircaloy) and one sheet of titanium. These results are compared to the Keams factors measured by X-ray diffraction (for the two Zircaloy sheets) and to the $W_{LMN}$ measured by neutron diffraction (for the pure zirconium and the titanium). Results show that ultrasonics predicts both the ODCs and the Keams factors very well.
1. Introduction

The preferred crystallographic orientation, i.e. texture, developed in Zircaloy during processing has important effects on the mechanical and physical properties of the material in the resultant components, especially their anisotropies. For example, nodular corrosion in Zircaloy has been shown to be dependent on the texture.\(^1\)\(^2\) Studies show that propagation of stress corrosion cracks depend greatly on the orientation of the basal poles.\(^3\) Also important to the nuclear industry is the effect of the texture on the directions of growth of hydride platelets and irradiation strains during service. The texture of sheet and tubing products is greatly influenced by processing variables such as reduction schedule and annealing.\(^4\)

Measurement of texture has, to date, been done with X-ray and neutron diffraction. While these diffraction techniques provide good quantification of texture, the methods are generally destructive and have limitations. In their usual form, X-ray measurements sense a thin layer exposed to the surface either by virtue of its initial position or machining. Neutron techniques provide volume average information but require measurements at special facilities. While diffraction techniques employing measurements of multiple pole figures can provide a complete description of texture, industry has found it convenient to use a small subset of this information contained in the Kears factors.\(^5\) This elementary quantification of texture is based on a weighted average of the information contained in one pole figure to determine the effective fraction of basal poles aligned along the reference direction.

Ultrasonic waves have been used to characterize texture in a variety of material systems including the cubic metals, steel\(^6\)\(^9\) and aluminum\(^10\)\(^12\), and the hexagonal metals, titanium.
zirconium, and Zircaloy.\textsuperscript{13-16} Although ultrasonic measurements provide more limited information than diffraction measurements, there is considerable motivation for their use. In addition to being nondestructive, these ultrasonic techniques are fast, relatively inexpensive, and measure a through-thickness average of the texture making them attractive alternatives to the currently used X-ray techniques. This paper investigates the possibility of using these techniques to predict Kearns factors. The qualitative feasibility of such an approach has been demonstrated by Konishi and Honji\textsuperscript{17}, who showed a correlation between the velocity of a longitudinal ultrasonic wave propagating in the normal direction of sheets and tubes and the Kearns factor in the normal direction. However, their explanation used the simplified model of Rosenbaum and Lewis\textsuperscript{18} for the effects of texture on velocity. They note that a more complete description is possible but was not necessary for their needs. Development of this more complete description is one of the objectives of the present paper. Motivations are not only the desire to strengthen the theoretical foundation of their work, but also to provide a basis for determining Kearns factors in additional directions, e.g. transverse and rolling directions, on plate and tubing material.

II. Mathematical Background

A. Description of Kearns factor

The Kearns factors (sometimes referred to as basal pole factors) are industrially important measures of texture in hexagonal materials like zirconium, its alloys (Zircaloy), and titanium.\textsuperscript{19,20} They describe the effective number of crystallites with the basal pole
aligned along particular sample axes, for example, the rolling, transverse or normal direction in a rolled sheet. These are denoted by $f_i$ where $i$ refers to one of the sample directions, e.g. for sheet products: 1 = rolling direction (RD), 2 = transverse direction (TD), and 3 = normal direction (ND). The $f_i$ are given by:

\[
f_1 = f_{RD} = \frac{1}{N} \int_0^{\pi/2} \int_0^{2\pi} l(\chi, \eta) \sin^3(\chi) \cos^2(\eta) \, d\eta \, d\chi
\]

\[
f_2 = f_{TD} = \frac{1}{N} \int_0^{\pi/2} \int_0^{2\pi} l(\chi, \eta) \sin^3(\chi) \sin^2(\eta) \, d\eta \, d\chi
\]

\[
f_3 = f_{ND} = \frac{1}{N} \int_0^{\pi/2} \int_0^{2\pi} l(\chi, \eta) \sin(\chi) \cos^2(\chi) \, d\eta \, d\chi
\]

where $l(\chi, \eta)$ is the x-ray intensity for a basal pole figure having $\chi$ as the polar angle and $\eta$ as the azimuthal angle, and

\[
N = \int_0^{\pi/2} \int_0^{2\pi} l(\chi, \eta) \sin(\chi) \, d\eta \, d\chi
\]

a normalization constant.
B. Orientation Distribution Function and Coefficients

Methods for describing texture have been developed concurrently by Roe\textsuperscript{21} and Bunge\textsuperscript{22}. Their methods are similar but differ in notation and in how they deal with symmetries in the sample axes and crystal axes. This paper will use Roe's notation. His theory is based on use of an orientation distribution function (ODF) which quantifies the probability that a crystallite's orientation with respect to the sample will be described by the Euler angles $(\theta, \psi, \phi)$. The ODF can be written as an expansion in generalized spherical harmonics:

$$w(\xi, \psi, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \sum_{n=-l}^{l} W_{lmn} Z_{nmn}^{(l)}(\xi) e^{-jm\psi} e^{-jm\phi}$$

(3)

where $\xi = \cos(\theta)$, the $W_{lmn}$ are the orientation distribution coefficients (ODCs), $Z_{nmn}$ are generalizations of the associated Legendre functions as defined by Roe\textsuperscript{21} and $j = \sqrt{-1}$. Similarly, a normalized pole figure can be represented by the expansion:

$$q_{i}(\zeta, \eta) = \frac{1}{2\pi} \int_{0}^{1} \int_{-1}^{1} I_{i}(\zeta, \eta) d\zeta d\eta = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} Q_{lm}^{i} P_{l}^{(m)}(\zeta, \eta) e^{-jm\eta}$$

(4)

where $\zeta = \cos(\chi_{i})$ and the $P_{l}^{(m)}$ are normalized associated Legendre functions as defined by Roe\textsuperscript{21}. The subscript $i$ refers to a particular pole figure. Since we wish to relate ODCs to the
Kearns factors which are simply related to the basal pole figure, the i can be ignored. It can be shown that the coefficients for the pole figure expansion are related to the ODCs:

\[ Q_{lm} = 2\pi \sqrt{\frac{2}{2l+1}} \sum_{n=-l}^{l} W_{lnm} P_l^{n}(\Xi) e^{im\Phi} \]  \hspace{1cm} (5)

where \( \Xi = \cos(\Theta) \), and \( \Theta \) and \( \Phi \) are angles describing the orientation of the selected pole with respect to the crystallite. For a basal pole figure \( \Theta = 0 \) so \( \Xi = 1 \). Therefore, only the ODCs with \( n = 0 \) will have a nonzero value. Then by utilizing Roe’s normalization,

\[ P_l^0(1) = \sqrt{\frac{2l+1}{2}} \]  \hspace{1cm} (6)

equation (5) will reduce to

\[ Q_{lm} = \pi W_{lm0} \]  \hspace{1cm} (7)

Equation (7) gives us a way to relate the ODCs to the pole figure expansion coefficients, thereby allowing us to use the definition of the Kearns factors to relate them to the ODCs. This is done in the following section.
III. Theory

A. Relationship Between Keams factor and ODCs

By substituting equation (4) into equation (1), the Keams factors can be written in terms of normalized associated Legendre functions:

\[
\begin{align*}
    f_1 &= f_{RD} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} Q_{lm} \int_0^{2\pi} e^{-i m \theta} d\theta \int_0^1 P_l^m(\zeta)(1-\zeta^2) d\zeta \\
    f_2 &= f_{TD} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} Q_{lm} \int_0^{2\pi} e^{-i m \theta} d\theta \int_0^1 P_l^m(\zeta)(1-\zeta^2) d\zeta \\
    f_3 &= f_{ND} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} Q_{lm} \int_0^{2\pi} e^{-i m \theta} d\theta \int_0^1 P_l^m(\zeta) \zeta^2 d\zeta
\end{align*}
\]

It is clear that the integral over \( \eta \) in each of equations (8) will restrict the values of \( m \). For the case of \( f_3 \), the integral over \( \eta \) is zero unless \( m = 0 \). For \( m = 0 \) the integral gives \( 2\pi \).

Likewise, for the cases of \( f_1 \) and \( f_2 \), the integral over \( \eta \) has values of \( \pi/2 \) for \( m = \pm 2 \). \( \pi \) for \( m = 0 \) and zero for all other values of \( m \). By rewriting the \( \zeta \) terms in equations (8) in terms of normalized associated Legendre functions of order \( k = 0 \) or \( k = 2 \), we can take advantage of the following orthogonality relation

\[
\int_{-1}^{1} P_l^m(\zeta) P_{l'}^m(\zeta) d\zeta = \delta_{l,l'}
\]

Thus \( l \) must also be either 0 or 2 for the integral to have a nonzero result. In addition, one must realize that the ODCs are influenced by symmetry. For an aggregate of hexagonal...
crystallites with orthotropic symmetry; these symmetries imply \( W_{220} = W_{220}^{-1} \), where the overbar implies a negative value of the index. Finally, by using equation (7) to eliminate \( Q_m \), we obtain three equations relating the Kearns factors to the ODCs:

\[
\begin{align*}
    f_1 &= f_{RD} = \frac{4\pi^2\sqrt{2}}{3} W_{000} - \frac{4\pi^2\sqrt{10}}{15} W_{200} + \frac{8\pi^2\sqrt{15}}{15} W_{220} \\
    f_2 &= f_{TD} = \frac{4\pi^2\sqrt{2}}{3} W_{000} - \frac{4\pi^2\sqrt{10}}{15} W_{200} - \frac{8\pi^2\sqrt{15}}{15} W_{220} \\
    f_3 &= f_{ND} = \frac{4\pi^2\sqrt{2}}{3} W_{000} + \frac{8\pi^2\sqrt{10}}{15} W_{200}
\end{align*}
\]

Since \( W_{000} \) is a normalization constant and is equal to \( 1/(4\pi^2\sqrt{2}) \), it can be seen that \( f_1 + f_2 + f_3 = 1 \) as is required by the initial definitions.

**B. Relationship between ODCs and Elastic Constants**

To infer the Kearns factors from ultrasonic measurements, we must first relate these Kearns factors to anisotropic elastic stiffnesses, which in turn control the ultrasonic wavespeeds. It has been shown that, for aggregates of hexagonal crystallites, the elastic constants, and hence the ultrasonic velocities, are related to the ODCs by the following:

...
where $A_1$, $A_2$, $A_3$, and $B$ are measures of the elastic anisotropy and the $C_{ij}$ are the isotropic values of the polycrystal elastic constants, as defined in reference 13.

C. Ultrasonic Measurement Strategy

The ultrasonic velocities are directly related to the above elastic constants. Reviewing this set of equations, one realizes that all of the elastic constants are determined by the five ODCs of order $l=2$ and $l=4$: $W_{200}$, $W_{220}$, $W_{400}$, $W_{420}$, and $W_{440}$. To determine the Kearns factors, only $W_{200}$ and $W_{220}$ need be known. However, because of the structure of the equations, all five ODCs must be determined and hence five independent measurements are...
needed to determine all five ODCs. Ideally, one would like to infer the ODCs and hence the f, by making relative velocity measurements. This improves accuracy and eliminates uncertainties associated with such parameters as the C°,j and the dependence of measured velocities on propagation path lengths. Relative measurements can be made by either rotating the polarization of a wave type for a fixed direction and path length or by changing directions of propagation in a sheet or plate for a fixed wave type and path length. The former technique leads to birefringence, and the latter makes use of Lamb wave velocity anisotropy as discussed below.

In birefringence measurements, one determines the normalized difference between the velocities of shear waves propagating in the ND (through the thickness) with polarizations in the RD and the TD. A shear wave transducer is placed on the sample and the velocity of a wave propagating in the ND, polarized in the RD, is measured. The shear wave transducer is then rotated 90° and the velocity is measured again; the polarization is now in the TD. Figure 1 shows a schematic of the measurement procedure. The difference of the squares of these velocities is one measure of the birefringence, which is related to the ODCs by equation (12).

\[
\rho \left[ V_{31}^2 - V_{32}^2 \right] = \frac{16\pi^2 A_3 \sqrt{15}}{210} W_{220} + \frac{32\pi^2 B\sqrt{2}}{105} W_{420} = 2\rho\bar{V}^2 \left( \frac{\Delta\bar{V}}{\bar{V}} \right) \quad (12)
\]

Here \( V_{ij} \) denotes the velocity of a bulk wave propagating in the i-direction with polarization in the j-direction, and we have taken advantage of the facts that \( \rho V_{31}^2 = C_{55} \) and \( \rho V_{32}^2 = C_{44} \).
\( \bar{V} \) is the average of \( V_{31} \) and \( V_{32} \). \( \left( \Delta V / \bar{V} \right) \) is the traditional measure of birefringence, given by

\[ \left( \bar{t} \right)^2 \] where \( \bar{t} \) is the average arrival time of the two waves and \( \Delta t \) is their difference

\( (t_{TD} - t_{RD}) \). It is sometimes convenient to express this in terms of the sheet thickness, \( h \), and number of reverberations, \( n \), in which case the relation is,

\[
\frac{\Delta V}{\bar{V}} = \sqrt{\frac{C_{44} \rho}{2hn}} \tag{13}
\]

For sheet samples, other measurements can be made using Lamb waves, which propagate in the plane of the sheet. Lamb waves are similar to bulk ultrasonic waves but differ due to the presence of the top and bottom surfaces. In particular, the requirement that stresses vanish at the surfaces causes the dynamic displacement to vary through the thickness of the sheets and the velocities to differ from those of plane waves such as those used in the birefringence technique. General solutions for Lamb wave velocities in anisotropic sheet materials have been obtained previously and that derivation will not be repeated here. In general, the velocities of Lamb waves propagating in certain directions within the sheet sample, for example, \( 0^\circ \), \( 45^\circ \), and \( 90^\circ \), can be determined as functions of the ODCs based on the elastic constants presented in equation (11). These relations are then inverted to predict ODCs from velocities measured in these propagation directions. The final solutions for \( S_0 \) waves in hexagonal polycrystals are presented in equation (14) and a schematic of the displacements occurring in this wave type is shown in Figure 2. Likewise, \( SH_0 \) wave solutions for hexagonal polycrystals are presented in equation (15) with a schematic of the displacements.
shown in Figure 2. Also shown in Figure 2 is a schematic of the measurement procedure for both of these wave types.

It should be noted that the $S_0$ mode velocities used in equation (14) are the velocities in the long wavelength limit. Lamb waves are dispersive which means that the velocity depends on the frequency and thickness of the sheet.\textsuperscript{23} To account for this dispersive nature, one can correct for dispersion by calculating the long wavelength limit of the velocity.\textsuperscript{27} It is also important to note that equations (14) and (15) imply that $W_{200}$ and $W_{400}$ are determined from absolute velocity measurements while $W_{220}$, $W_{420}$, and $W_{440}$ are determined by relative velocity measurements.

Variables described in equations (14) and (15) were defined following equation (11). Note that $C_{11}$, $C_{12}$, and $C_{44}$ are equivalent to the Lamé elastic constants $\lambda + 2\mu$, $\lambda$, and $\mu$, respectively. In this work, they were computed from single crystal elastic constants using the Hill\textsuperscript{28} approximation. Equations (12), (14) and (15) can be used to explicitly find the ODCs from measurements of the angular dependence of the plate wave velocity and measurements of the birefringence. The first formula in equation (14) can be used in conjunction with the first formula in equation (15) to independently determine $W_{200}$ and $W_{400}$ by measuring the angular dependence of the $S_0$ mode velocities and the $SH_0$ mode velocities. Similarly, $W_{220}$ & $W_{420}$ can be determined from the second formula in equation (14) and the birefringence, equation (12). We are then left with two independent ways to determine $W_{440}$, from either $S_0$ mode velocities (last formula in equation (14)) or $SH_0$ mode velocities (last formula in equation (15)).
IV. Experimental Results

A. Prediction of ODCs

Four sheets were examined in this study. Two were Zircaloy sheets, one being 29.718 mm thick (referred to as the "thick" sheet), and the other being 5.588 mm (referred to as the "thin" sheet). Additionally, commercially pure titanium and zirconium sheets, used in a previous study, were examined. The $S_0$ velocity, $SH_0$ velocity, and birefringence measurements used in our analysis were made during the course of this study on the two
Zircaloy sheets. For the titanium and zirconium sheet, the S\textsubscript{0} and SH\textsubscript{0} mode velocities reported for these samples by Li et al.\textsuperscript{26} were used complemented by birefringence measurements made during the course of this study.

Velocity measurements were made using electromagnetic acoustic transducers (EMATs)\textsuperscript{19} at a frequency of approximately 330 kHz. S\textsubscript{0} velocities and SH\textsubscript{0} velocities were measured using a procedure developed by Clark.\textsuperscript{12} This procedure measures an absolute velocity in one of the principle directions (0°, 45°, 90° to the RD) on each sheet by measuring time delay to a specific wave feature (e.g. zero-crossing) at several known, fixed distances. The inverse of the slope of this time versus distance plot gives the absolute velocity in the selected direction. The transmitting and receiving EMATs are then fixed at a separation. However, the exact path length to be used in the velocity calculation is unknown due to the finite length of the meander coil producing the wave and of the tone burst exciting the transmitter. Hence, the time delays to a specific wave feature are measured in each of the three directions, 0°, 45°, 90° to the RD. The velocities in these three directions are calculated by determining an effective distance between the transducers for the fixed separation from the time delay measurements and the absolute velocity measurement in that direction. In addition, the measurements on the thick sheet of Zircaloy required correction for dispersion in order to determine the long wavelength limit required by the theory. This was accomplished by a technique presented by Thompson et al.\textsuperscript{5,27} Table 1 gives the velocities measured on each of the sheets. For each velocity, the time measurement was determined by averaging 2000 successive waveforms, done automatically by a time averaging counter. The
precision of these velocity measurements is approximately ±0.003 mm/μs for the \( S_0 \) mode velocities and approximately ±0.005 mm/μs for the \( \text{SH}_q \) mode velocities.\(^{26}\)

To calculate the ODCs, one needs to determine the polycrystalline elastic constants to be used in the equations. These average polycrystalline elastic constants are computed from single crystal elastic constants using the Hill averaging scheme and are listed in Table 2.\(^{26}\) From these polycrystalline elastic constants and the velocities in Table 1, ODCs were inferred and are listed in Table 3 along with the neutron diffraction measurements made on the titanium and zirconium sheets.\(^{26}\) Included in this table are calculated uncertainties which will be discussed in more detail in Section V. \( W_{\text{iso}} \), which can be calculated from either \( S_0 \) or \( \text{SH}_q \) velocity measurements, is tabulated using both methods.

Table 1. Experimentally determined velocities (mm/μs) for all four samples.

<table>
<thead>
<tr>
<th></th>
<th>Zircaloy (Thick)</th>
<th>Zircaloy (Thin)</th>
<th>Zirconium</th>
<th>Titanium</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{S_0}(0) )</td>
<td>4.2835</td>
<td>4.1512</td>
<td>4.0941</td>
<td>5.3236</td>
</tr>
<tr>
<td>( V_{S_0}(45) )</td>
<td>4.2634</td>
<td>4.1376</td>
<td>4.0805</td>
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<tr>
<td>( V_{S_0}(90) )</td>
<td>4.2585</td>
<td>4.1584</td>
<td>4.0987</td>
<td>5.4506</td>
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<td>( V_{\text{SH}_0}(0) )</td>
<td>2.3228</td>
<td>2.3328</td>
<td>2.3309</td>
<td>2.9540</td>
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<tr>
<td>( V_{\text{SH}_0}(45) )</td>
<td>2.3308</td>
<td>2.3343</td>
<td>2.3594</td>
<td>2.9582</td>
</tr>
<tr>
<td>( \frac{\Delta V'}{V'} )</td>
<td>-0.0303</td>
<td>0.0084</td>
<td>-0.0324</td>
<td>-0.0448</td>
</tr>
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Table 2. Polycrystalline elastic constants computed using Hill averaging scheme.

<table>
<thead>
<tr>
<th>Material</th>
<th>$C^{11}_{11}$</th>
<th>$C^{12}_{12}$</th>
<th>$C^{44}_{44}$</th>
<th>$A_{1}$</th>
<th>$A_{2}$</th>
<th>$A_{3}$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zr/ Zircaloy</td>
<td>145.18</td>
<td>72.28</td>
<td>36.45</td>
<td>-49.05</td>
<td>-5.22</td>
<td>54.27</td>
<td>38.99</td>
</tr>
<tr>
<td>Ti</td>
<td>162.86</td>
<td>76.07</td>
<td>43.40</td>
<td>-61.80</td>
<td>-141.81</td>
<td>203.61</td>
<td>16.69</td>
</tr>
</tbody>
</table>

Table 3. Prediction of ODCs for all four sheets with a comparison to neutron diffraction predictions for the zirconium and titanium sheets.

<table>
<thead>
<tr>
<th></th>
<th>Zircaloy (Thick)</th>
<th>Zircaloy (Thin)</th>
<th>Zirconium</th>
<th>Titanium</th>
<th>Ultrasonic</th>
<th>Neutron</th>
<th>Ultrasonic</th>
<th>Neutron</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{200}$</td>
<td>0.0239</td>
<td>0.0225</td>
<td>0.0106</td>
<td>0.0130</td>
<td>0.0175</td>
<td>0.0171</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>±0.0011</td>
<td>±0.0011</td>
<td>±0.0011</td>
<td>±0.0004</td>
<td>±0.0002</td>
<td>±0.0006</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$W_{220}$</td>
<td>-0.0032</td>
<td>0.0009</td>
<td>-0.0054</td>
<td>-0.0058</td>
<td>-0.0052</td>
<td>-0.0055</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>±0.0004</td>
<td>±0.0004</td>
<td>±0.0004</td>
<td>±0.0003</td>
<td>±0.0001</td>
<td>±0.0002</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$W_{400}$</td>
<td>0.0150</td>
<td>0.0206</td>
<td>0.0005</td>
<td>0.0008</td>
<td>0.0270</td>
<td>0.0058</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>±0.0011</td>
<td>±0.0011</td>
<td>±0.0011</td>
<td>±0.0002</td>
<td>±0.0019</td>
<td>±0.0002</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$W_{420}$</td>
<td>-0.0065</td>
<td>0.0018</td>
<td>-0.0058</td>
<td>-0.0050</td>
<td>-0.0073</td>
<td>-0.0069</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>±0.0003</td>
<td>±0.0002</td>
<td>±0.0002</td>
<td>±0.0001</td>
<td>±0.0007</td>
<td>±0.0002</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$W_{440}$</td>
<td>0.0007</td>
<td>0.0001</td>
<td>0.0025</td>
<td>0.0024</td>
<td>0.0008</td>
<td>0.0022</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(SH0)</td>
<td>±0.0004</td>
<td>±0.0004</td>
<td>±0.0004</td>
<td>±0.0002</td>
<td>±0.0008</td>
<td>±0.0001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$W_{440}$</td>
<td>0.0012</td>
<td>0.0027</td>
<td>0.0024</td>
<td>0.0024</td>
<td>0.0032</td>
<td>0.0022</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(S0)</td>
<td>±0.0010</td>
<td>±0.0010</td>
<td>±0.0010</td>
<td>±0.0002</td>
<td>±0.0020</td>
<td>±0.0001</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
B. Prediction of Kearns factors

The ODCs predicted above were then used to predict the Kearns factors, based on equation (10). Table 4 shows a comparison of the predictions of the Kearns factors based on ultrasonics and diffraction. X-ray diffraction was used on the two Zircaloy sheets, as conducted by Westinghouse, while neutron diffraction was used for the titanium and zirconium sheets. A graphical comparison is given in Figure 3.

Prediction of Kearns factors for all four samples compared to diffraction predictions ("UT" stands for ultrasonic prediction and "Diffr" stands for diffraction prediction).

<table>
<thead>
<tr>
<th></th>
<th>Zircaloy (Thick)</th>
<th>Zircaloy (Thin)</th>
<th>Zirconium</th>
<th>Titanium</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>UT</td>
<td>Diffr</td>
<td>UT</td>
<td>Diffr</td>
</tr>
<tr>
<td>$f_3$</td>
<td>0.731</td>
<td>0.730</td>
<td>0.708</td>
<td>0.758</td>
</tr>
<tr>
<td></td>
<td>±0.018</td>
<td>±0.018</td>
<td>±0.018</td>
<td>±0.018</td>
</tr>
<tr>
<td>$f_2$</td>
<td>0.069</td>
<td>0.074</td>
<td>0.164</td>
<td>0.132</td>
</tr>
<tr>
<td></td>
<td>±0.013</td>
<td>±0.012</td>
<td>±0.012</td>
<td>±0.012</td>
</tr>
<tr>
<td>$f_1$</td>
<td>0.200</td>
<td>0.198</td>
<td>0.127</td>
<td>0.126</td>
</tr>
<tr>
<td></td>
<td>±0.013</td>
<td>±0.012</td>
<td>±0.012</td>
<td>±0.012</td>
</tr>
</tbody>
</table>
V. Discussion

A. Error Analysis

Table 3 includes predicted uncertainties associated with the ultrasonic estimates of the ODCs as well as uncertainties in neutron diffraction measured ODCs as reported by Li et al.\textsuperscript{26} Ultrasonic ODC uncertainties were determined by standard propagation of uncertainty techniques, i.e.

\[
(\Delta W)^2 = \sum_{i=1}^{n} \left( \frac{\partial W}{\partial P_i} \Delta P_i \right)^2
\]

where \( n \) is the number of parameters, \( P_i \) are the parameters, and \( \Delta P_i \) are the uncertainties associated with the parameter. For example, the velocities measured in each direction are parameters. By taking the derivative of each formula listed in equations (12), (14) and (15) with respect to each of the velocities, multiplying by the uncertainty associated with each velocity, squaring and summing, one gets the square of the uncertainty associated with each formula.

Technically, one also needs to determine additional contributions to the uncertainties associated with each of the averaged elastic constants used in equations (14) and (15). If one uses the averaged elastic constants computed using the Hill approximation with the upper and lower bounds predicted by the Voigt\textsuperscript{30} and Reuss\textsuperscript{31} averaging techniques to define the uncertainty in the averaged elastic constants, then the uncertainty in the ODCs are much larger than the values determined by the precision of the velocity measurement. We have neglected the uncertainty associated with the average elastic constants for two reasons. First,
appropriate values can always be determined by calibration experiments. Second, an estimate based on Voigt and Reuss bounds would neglect possible alloying effects, which can be significant but are generally unknown. Thus, the uncertainties presented in the above tables represent precision with which one can measure these quantities based on the precision of the velocity measurements. We could expect them to be good estimates for the uncertainties in the ODCs inferred from relative measurements but underestimates of the uncertainties in the ODCs inferred from absolute velocity measurements in the absence of calibration experiments.

The uncertainties associated with the Kearns factors are calculated in a similar manner and presented in Table 4. One can see that based on the precision of the measured velocities, ultrasonic techniques provide an accurate way to predict Kearns factors.

B. Comparison of Ultrasonic and Diffraction Results.

The agreement between the ultrasonic and diffraction predictions of the ODCs in zirconium and titanium are generally good, with the exception of \( W_{400} \) in titanium. In all but that case, the differences are small and on the order of the estimated uncertainties. We have not fully determined the source of the disagreement for \( W_{400} \) in titanium, but note that the predictions of this quantity depends on absolute measurements, which are known to be less precise, and on the average elastic constants as predicted in the Hill approximation from tabulated values of single crystal elastic constants. It is interesting to note that a similar discrepancy in the prediction of \( W_{400} \) has been reported in another study.\(^{16}\) A reasonable
speculation is based on the dependence of the elastic constants on alloy content. The tested sample of titanium was “commercially pure” titanium which typically contains approximately 0.2 weight percent oxygen. This small amount of oxygen alloy will change the texture free longitudinal and shear velocities by approximately 0.6% and 1.0%, respectively\(^2\). A change which will significantly affect the predictions of \(W_{400}\) because the absolute velocity measurement is compared to an absolute texture free velocity to predict \(W_{400}\). As noted above, we believe that this problem can be overcome by appropriate calibration experiments. It is interesting that we do not see a similar disagreement in the particular zirconium sample studied. This suggests that the elastic constants are not strongly influenced by the impurities in that sample.

It is also interesting to compare the predictions of \(W_{440}\) made from the \(S_0\) and \(S H_0\) measurements. For the “thick Zircaloy” and zirconium sheets, the agreement is excellent, with the difference in predictions being 0.0005 and 0.0001 respectively. For the titanium sheet, the difference of 0.0024 is on the order of the uncertainty. For the “thin Zircaloy”, the difference in \(W_{440}\) prediction is 0.0026, or about twice the uncertainty or two sigma \((2\sigma)\). This is more than expected but still quite possibly a result of statistical uncertainty.

As would be expected based on the relationship of the \(f_i\) and \(W_{2m0}\), there is also good prediction between the ultrasonic and neutron diffraction predictions of the Kearns factors in the zirconium and titanium sheets studied. For the Zircaloy sheets, the individual ODCs were not determined. However, Table 4 shows that the agreement between the ultrasonic and diffraction predictions of the Kearns factors extends to these materials as well.
Comparison to prior work of Konishi and Honji.

One of the motivations for our research was the significant work of Konishi and Honji\(^7\) who studied the relation between bulk longitudinal ultrasonic velocity and Kearns factors in Zircaloy plates and tubes. Their results are shown in Figure 4. To compare these results to our work, we must first develop a basis for their relation in terms of our theory. This can be done by solving the third relation in equation (10) for \(W_{200}\). Then by neglecting ODCs of order higher than two in equation (11) and realizing that for bulk waves \(C_{11} = \rho V_{11}^2\), one can solve for the bulk velocity in terms of Kearns factors. Repeating this for each of the principle directions, one can show that the longitudinal velocity along a sample reference direction can be related to the Kearns factors in the same reference direction by the following relation:

\[
V_i = \sqrt{\frac{C_{11}'}{\rho} - \frac{2A_i}{7\rho} \left(f_i - \frac{1}{3}\right)}
\]

where the subscript \(i\) refers to the reference direction, for example the RD, TD, or ND in sheet samples. It is important to note that ODCs of orders higher than two are neglected because there is no direct relation between the Kearns factors and ODCs other than \(W_{200}\) and \(W_{220}\). The neglect of contributions of the higher order ODCs should be a good approximation when \(BW_{4m0} / A_i W_{2m0}\) is small. Results from Li et al\(^{\text{26}}\) suggest that \(W_{200}\) is approximately three times larger than the other ODCs, at least for the samples which they studied. This, along with the fact that \(|B / A_i| < 1\) (see Table 2) suggests that the above assumption can be
made for those samples. The velocity predicted by equation (17) is also plotted in Figure 4, along with Konishi's plate and tube data. It can be seen that this first principles theory, with no adjustable parameters, does an excellent job of predicting the trends in the data for larger Kearns factors. For smaller values, it is qualitatively correct but does not reproduce the shape of the data. It should be noted that the predicted velocity is not linear, but for the small range of velocities shown in Figure 4 it appears to be a straight line. The differences between this theory and experiment could be caused by the contributions of the other ODCs, since the smallness of $BW_{4n0} / A_1W_{2m0}$ may not be valid for small Kearns factors. Further work is required to verify this speculation.

VI. Conclusions

It is clear that ultrasonics can be used to predict ODCs in hexagonal materials. The theory presented here provides us with a method to also measure Kearns parameters, which find extensive industrial use in such matters as material purchase specifications. These Kearns factors play an important role in the characterization of Zircaloy sheet. Use of ODF analysis facilitates a more complete understanding of how the Kearns factors affect the ultrasonic velocities. The authors have developed relations between the Kearns factors and ultrasonic velocity which will allow determination of all three Kearns factors from ultrasonic measurements. Analysis of both new and existing data support the theory well. Work needed in the future includes the extension of this analysis to tube geometries.
VII. Acknowledgment

This work was sponsored by the Division of Materials Sciences of the U. S. Department of Energy and was performed at the Ames Laboratory in cooperation with the Center for NDE. Ames Laboratory is operated by Iowa State University for the USDOE under contract W-7405-ENG-82.

VIII. References


6. K. Fourouraghi: Ph.D. Dissertation, Department of Mechanical Engineering, Iowa State University, 1995


Shear Wave Transducer

Figure  Schematic of birefringence measurements.
Figure  Schematic representation of the displacement profile, surface displacements in a cross-sectional plane, and measurement schemes for both the $S_0$ and $SH_0$ mode velocity measurements.
Figure  Graphical comparison of Kearns factors measured by diffraction and Kearns factors measured by ultrasonics.
Figure 4  Velocity in reference direction versus Kearns factors in the same reference direction. Konishi's plate and tube data are superimposed to show agreement.
CHAPTER 6. GENERAL CONCLUSIONS

While on-line texture characterization is not yet a reality, fast, accurate, quantitative, nondestructive techniques exist for characterization of texture. Industries which did not consider the effects of texture previously are starting to take notice because now they can quickly obtain this information.

This dissertation shows that ultrasonic techniques, which are well-founded for materials with strong single crystal anisotropy like iron alloys, can also be used effectively for materials with small single crystal anisotropy like aluminum alloys and for alloys made of hexagonal crystallites. However, for materials with small single crystal anisotropy several more factors need to be considered. Alloy composition can greatly affect SCECs thereby changing predictions of ODCs based on absolute velocity measurements. This can be corrected by determining the SCECs prior to working with a particular alloy. Small errors in signal processing also need to be considered. The approximately one-half bandwidth delay inherent in triggered pulses has a much greater effect on weakly anisotropic materials. This effect is fairly easily corrected.

Texture characterization in hexagonal alloys is straight-forward and similar to the techniques used with cubic alloys. However, there is more texture information available to ultrasonic techniques because of the symmetries of hexagonal crystallites. This dissertation has shown a direct relationship between ODCs and traditionally used industrial texture parameters, thereby providing a means for these industries to obtain considerably more information, more quickly.
6.1 Future Directions

As mentioned previously, the overall goal for the texture characterization field is the development of on-line, in-situ techniques for predicting texture. The ability to measure texture during production would pave the way for intelligent processing of materials, the process of using the on-line information to optimize the processing as it happens. The core techniques seem to be well-developed, but subtleties in these techniques will continue to need attention. For example, a more detailed understanding of the effects of alloying elements on the SCECs is necessary to provide increased precision for predictions of $W_{400}$ in cubic alloys. In addition, there will be many unforeseen hurdles during the development of these on-line techniques.

Another future direction for texture characterization will be the application of these ideas to fields which previously have not considered the effects of texture on their processes and products. The speed and precision with which texture can now be predicted will benefit many of these industries.
It is generally accepted that the sum of squares of the three velocities for the three different pure modes traveling in the same direction are equal to a combination of bulk elastic moduli, i.e.,

\[
\rho (\nu_{11}^2 + \nu_{12}^2 + \nu_{13}^2) = \rho (\nu_{21}^2 + \nu_{22}^2 + \nu_{23}^2) = \rho (\nu_{31}^2 + \nu_{32}^2 + \nu_{33}^2) = \lambda + 4\mu
\]

This sum is, therefore, free of texture. For the case of pure modes traveling in the principle sample directions, this is easy to show by substituting the texture dependent velocities given in equations (8) in Chapter I. However, one can easily show that this is also true when the pure modes do not travel in the principle sample directions.

To show that the above statement is true, one must start with a general equation for the velocities traveling in any direction within a sample. The velocities are found from the solution of the Christoffel equation:

\[
\begin{bmatrix}
\Gamma_{11} - \rho \nu^2 & \Gamma_{12} & \Gamma_{13} \\
\Gamma_{21} & \Gamma_{22} - \rho \nu^2 & \Gamma_{23} \\
\Gamma_{31} & \Gamma_{32} & \Gamma_{33} - \rho \nu^2
\end{bmatrix}
\begin{bmatrix}
d_1 \\
d_2 \\
d_3
\end{bmatrix} = 0
\]

where.
\[ \Gamma_{11} = \overline{C}_{11} n_1^2 + \overline{C}_{66} n_2^2 + \overline{C}_{55} n_3^2 \]
\[ \Gamma_{22} = \overline{C}_{66} n_1^2 + \overline{C}_{22} n_2^2 + \overline{C}_{44} n_3^2 \]
\[ \Gamma_{33} = \overline{C}_{55} n_1^2 + \overline{C}_{44} n_2^2 + \overline{C}_{33} n_3^2 \]
\[ \Gamma_{12} = \Gamma_{21} = \overline{C}_{12} n_1 n_2 \]
\[ \Gamma_{13} = \Gamma_{31} = \overline{C}_{13} n_1 n_3 \]
\[ \Gamma_{23} = \Gamma_{32} = \overline{C}_{23} n_2 n_3 \]

and \( \overline{C}_y \) are given by \( c_y \) in equations (8) in Chapter I. To solve the Christoffel equation, one sets the determinant of the matrix equal to zero. This step gives the following.

\[-(pV^2)^3 + A(pV^2)^2 + B(pV^2) + C = 0\]

where
\[ A = \Gamma_{11} + \Gamma_{22} + \Gamma_{33} \]
\[ B = \Gamma_{12}^2 - \Gamma_{11} \Gamma_{22} - \Gamma_{13} \Gamma_{33} - \Gamma_{22} \Gamma_{33} + \Gamma_{13}^2 + \Gamma_{23}^2 \]
\[ C = \Gamma_{11} \Gamma_{22} \Gamma_{33} - \Gamma_{13}^2 \Gamma_{22} - \Gamma_{12}^2 \Gamma_{33} - \Gamma_{11} \Gamma_{23}^2 + 2 \Gamma_{12} \Gamma_{13} \Gamma_{23} \]

Thus a cubic equation in \( pV^2 \) is obtained which can be solved using standard schemes. Hence, the solutions for \( pV^2 \) are given by.
\[ \rho v^2 = \begin{cases} \left( -\frac{1}{3}B - \frac{1}{9}A^2 \right) \frac{T}{3} + \frac{1}{3}A \\ -\frac{1}{2}T + \frac{1}{2} \left( -\frac{1}{3}B - \frac{1}{9}A^2 \right) \frac{T}{3} + \frac{1}{3}A + \frac{1}{2} \sqrt{3} \\ -\frac{1}{2}T + \frac{1}{2} \left( -\frac{1}{3}B - \frac{1}{9}A^2 \right) \frac{T}{3} + \frac{1}{3}A - \frac{1}{2} \sqrt{3} \end{cases} \]

where,

\[ T = \left( \frac{1}{6BA} + \frac{1}{27}A^3 + \frac{1}{18} \sqrt{3(-4B^3 - B^2A^2 + 18BA + 2C^2 + 4C^3)} \right)^{\frac{1}{3}} \]

The above solution gives each of the three possible wave velocities in a particular direction given by the direction cosines embedded in \( \Gamma_i \). If one takes the sum of the three velocities the following is obtained where the subscript on the velocity refers to one of the three roots.

\[ \left( \rho V_1^2 + \rho V_2^2 + \rho V_3^2 \right) = \frac{1}{3}A + \frac{1}{3}A + \frac{1}{3}A = A = \Gamma_{11} + \Gamma_{22} + \Gamma_{33} \]

\[ = (C_{11} + C_{66} + C_{55})n_1^2 + (C_{66} + C_{22} + C_{44})n_2^2 + (C_{55} + C_{44} + C_{33})n_3^2 \]

Next by substituting the texture dependence for each of the polycrystal elastic constants given in equations (8), and realizing that the sum of squares of unit vectors is unity, it can be shown that the sum of the three velocities is simply \( \lambda + 4\mu \).
\[
\left( \rho \nu_1^2 + \rho \nu_2^2 + \rho \nu_3^2 \right) = \left( C_{11} + C_{66} + C_{55} \right) n_1^2 + \left( C_{66} + C_{22} + C_{44} \right) n_2^2 + \left( C_{55} + C_{44} + C_{33} \right) n_3^2
\]
\[
= (\lambda + 4\mu) n_1^2 + (\lambda + 4\mu) n_2^2 + (\lambda + 4\mu) n_3^2
\]
\[
= (\lambda + 4\mu) \left( n_1^2 + n_2^2 + n_3^2 \right)
\]
\[
= (\lambda + 4\mu) (1)
\]
\[
= (\lambda + 4\mu)
\]

The above discussion tells us that for three velocities propagating in the same direction the sum of the squares of velocities multiplied by the density should be constant regardless of the propagation direction. This allows an independent measure of a combination of the bulk elastic moduli. A measure like this could be useful for determining the approximate change in elastic moduli due to some other effect, e.g. alloying.

References

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