SUPPORT MINIMIZED LIMITED VIEW CT USING A PRIORI DATA

R. A. Roberts, O. Ertekin
Center for Nondestructive Evaluation
Iowa State University
Ames, IA 50011

INTRODUCTION

This paper reports progress in work on CT reconstruction of incomplete X-ray (Radon) projection sets in situations where explicit object geometry and composition information is available. Previous work on this problem, reported in [1,2,3], addressed two major issues: 1.) appropriate compensation for missing projection data regarding flaws for which no explicit a priori data is available, and 2.) the scaling and geometric registration of explicit a priori component data. The first of these issues is addressed by restricting interest to the reconstruction of flaws which have high-contrast (high S/N) discontinuous boundaries. This restriction focuses attention on problems such as the inspection of monolithic material structural components for cracks, porosity, inclusions, or dimensional abnormalities. It tends to exclude applications such as the imaging of slight density variations, or the imaging of diffuse boundary structures, such as might be encountered in medical applications. It was noted in the first year of this project that when reconstructing projection data from a compact support discontinuous boundary object, removal of a number of projections invariably increased the "size" of the reconstruction (i.e. increased the number of pixels above noise). This suggested that rather than setting the missing projection values to zero (l_2 norm minimization), it might be desirable to interpolate the missing projections such that the reconstructed object has a minimum size, i.e. minimum support. In a majority cases studied, this approach yields quite reasonable reconstructions of the object geometry, even in extreme cases where half the projection data is missing. When discrepancies between the true object and the reconstruction are significant due to extremely limited data, it was observed that support minimized reconstructions tend to be more intelligible than those of other methods, due to the straight-forward visual interpretation of the support minimized reconstruction. The support minimized reconstruction yields the minimum number of non-zero pixels required for consistency with the measured data. No unnecessary artifacts are present outside the object, and there is no ambiguity as to the necessity of the features which are present. This is not to say that support minimized inversions answer all questions: clearly they do not. Support minimized solutions represent a reasonable answer to the question "how large must a detected flaw be", but does not answer the question "how large could a detected flaw be". Indeed, it may be that this latter question has no satisfactory answer when dealing with limited data. Support minimized reconstruction can be viewed as an extension of the established method of support constraints, in which the allowed support of a reconstruction is specified a priori.[4] Rather than explicitly specifying the allowed support, the support minimized reconstruction dynamically seeks the smallest possible support.
The second issue, regarding the registration of explicit a priori data such as a CAD drawing of a component, was addressed in the second year of the project. The approach taken to this problem is to variationally align the a priori data through the optimization of a set of registration parameters (x-y translation, rotation, size, contrast, and brightness). The measure of alignment which is optimized is the support of the difference between the reconstruction and the a priori data. This approach reconstructs any differences between the a priori data and the reconstruction as compactly as possible.

The work performed in the first two years of the project was based on a conventional back projection algorithm, in which discrete values of the missing projection data were explicitly manipulated as independent variables in the variational reconstruction procedure. This approach was appropriate for experimental algorithm development and algorithm performance experiments. But, because the calculation of each component of the optimization gradient vector requires a separate back projection, this approach was deemed too computationally cumbersome for application on a large scale. It was noted that the reconstruction procedure could be efficiently implemented using a forward projection-based algorithm, and that such an algorithm would be ideally suited for implementation on a SIMD (single instruction, multiple data) parallel computer. Work in the third year of this project, reported here, focused on the implementation of the support minimized limited view reconstruction in a forward projection-based algorithm on a SIMD computer capable of handling common size CT system data sets (say 1Kx1K).

SUPPORT MINIMIZED FORWARD PROJECTION RECONSTRUCTION

The work reported here assumes that the X-ray measurement is adequately modeled by the forward Radon transform

\[ p(\theta, s) = \int_{l(\theta, s)} \mu(x) \, dl(x) \]  

where \( p(\theta, s) \) is the projection of \( \mu(x) \), \( \mu(x) \) is the attenuation coefficient, \( x \) is a two dimensional position vector in the image plane, \( l(\theta, s) \) are line segments connecting source and detectors, where \( \theta \) represents the angular orientation of the segment relative to the \( x \)-coordinates (rotation), and \( s \) represents the perpendicular distance of the segment to the coordinate origin (translation). The discrete version of eqs.(1) is represented by

\[ p_i = \sum_j l_{ij} \mu_j \]  

where the subscripts \( i, j \) denote discrete points in the \( \theta, s \) projection plane and \( x_1, x_2 \) image plane, respectively, and \( l_{ij} \) is the length of the line segment passing through the \( j \)-th image pixel associated the \( i \)-th projection value. The physically measured projection data is denoted \( p_i^m \). In the present work, given a measured projection set, the \( \mu_j \) are determined by minimizing the summed squared error measure

\[ E^m = \sum_i (p_i - p_i^m)^2 \]  

with respect to the image pixels \( \mu_j \) using a conjugate gradient search.

The limited view CT reconstruction problem concerns the case where projection values \( p_i^m \) are unavailable over a range of angle \( \theta \). In this case, the inversion problem does not have a unique solution, and hence additional measures must be applied to select a desired solution from the infinite number of possible solutions. The support minimized reconstruction includes a measure of image support in the error functional, in the form

\[ E^s = \sum_i (1 + e^\eta |\mu_j|^{-\eta})^{-1} \]
where $\varepsilon$ is a noise threshold, and $n$ is a positive number determining transition sharpness. Note that as $n$ becomes large, the argument of the summation in eq.(4) becomes a step function positioned at $\mu_i = \varepsilon$, in which case $E^2$ is a measure of the area of the image above the noise threshold. The total objective function to be optimized is written

$$E = E^m + \gamma E^s$$

where the parameter $\gamma$ balances the contribution of the measurement- and support-derived penalties. The components of the gradient vector are determined as

$$\frac{\partial E}{\partial \mu_k} = \sum_i 2(p_i - p_i^m) l_{ik} + \gamma n \varepsilon \eta \mu_k |\mu_k|^{-\eta-1} (1 + \varepsilon n \mu_k)^{-\gamma-1}$$

Eqs.(5 and 6) are used by a conjugate gradient minimization routine to minimize eq.(5).

The algorithm is implemented on a 16K processor MasPar MP1 computer. The algorithm contains two compute intensive steps, which are optimized for execution on the SIMD machine. The first is the forward projection step, eq.(2). This is performed by distributing the image pixels over the two-dimensional processor array. Horizontal and vertical projections are then performed using horizontal and vertical summation commands. Following each summation step, the image is rotated in angle (requiring appropriate pixel interpolation), then the summation step is repeated. The second compute intensive step is the evaluation of the summation in eq.(6). This is performed by essentially reversing the forward projection step. Indeed, it is noted that this summation represents a non-filtered backprojection of the difference between the simulated and measured projections. At each angular orientation of the pixel array, the projection difference values associated with that angle are appropriately weighted and summed to the pixels. The pixel array is then rotated in angle over the processors, and the step is repeated. It was noted that for each search direction in the gradient search, only one forward projection, eq.(2), need be performed, due to the linearity of the forward mapping. Thus backward and forward projections are required only when the search direction changes.

It was observed that, when the support functional is not being used, the convergence rate of the variational reconstruction is often significantly increased by filtering the projection difference $p_i - p_i^m$ prior to performing the back-projection summation in eq.(6), where the applied filter $h(s)$ has a Fourier transform $\mathcal{F}(k)$ which approximates $|k|$. Indeed, at the initiation of the reconstruction, when $p_i = 0$, this step represents a conventional filtered backprojection reconstruction. The accelerated convergence results from the fact that the filtered backprojection ideally points directly to the minimum of the $(l)^2$ norm objective function. It was noticed, however, that the benefits of this filtering on the algorithm convergence rate is reduced at the latter stages of the optimization, or when the support measure is employed, apparently due to the fact that the topology of the objective function surface in these cases bears little resemblance to the $(l)^2$ norm surface. When eq.(6) is optimized with no support minimization, $\gamma=0$, the reconstruction is a conventional constrained support solution, where the a priori specified non-zero support is restricted to the specified pixel array.

The algorithm incorporates a priori component data by variationally aligning the data with the reconstructed image. Six parameters control the alignment, corresponding to translation $(r_1, r_2)$, rotation $(r_3)$, scale $(r_4)$, contrast $(r_5)$, and brightness $(r_6)$. The a priori data $\mu^a(x)$ is expressed as a function of continuous position variables $x_1, x_2$. Discrete values of $x$ in the image plane corresponding to the index $j$ are denoted $x_{1j}, x_{2j}$. The algorithm operates on the difference between the reconstruction and the registered a priori data, expressed

$$d_j = |\mu_j| - r_5 \mu^a(r_4 (R_{11} x_{1j} + R_{12} x_{2j}) + r_1, r_4 (R_{21} x_{1j} + R_{22} x_{2j}) + r_1 - r_6$$

where $R_{ij}$ is the rotation tensor $R_{11} = \cos(r_3), R_{12} = \sin(r_3)$. The algorithm seeks to minimize the support of $d_j$, thus the measure $E^s$ is determined as
\[ E^x = \sum_i (1 + e^{\eta |d_i|^\eta})^{-1} \]  

The registration variables are treated as six additional independent variables in the optimization procedure, hence the solution is sought to the simultaneous equations

\[ \frac{\partial E}{\partial \mu_n} = 0, \quad \frac{\partial E}{\partial r_k} = 0 \]  

resulting in six additional terms in the gradient vector.

APPLICATION TO LIMITED VIEW DATA

The utility of support minimization in reconstructing compact, discontinuous boundary flaws is first demonstrated. Fig. (1) compares the reconstruction of simulated objects intended to represent cracks and voids in a component. This simulation assumes the component background has been subtracted, as in the experimental example which follows. The true object is compared with reconstructions using half the required 180 degrees of projection data (angles 90 through 180 degrees are removed), using b) filtered back projection re-

![Fig. 1 Comparison of simulated flaw reconstruction. a) true object, b) filtered back projection, c) constrained support, d) minimized support.](image-url)
construction, c) constrained support reconstruction (minimization of eq.6 with \( \gamma=0 \)), and d) support minimized reconstruction (\( \gamma>0, \eta=4 \) and \( \varepsilon=0.03 \) object maximum). Most of the crack is undetected in b), and there is no evidence of the longest members of the crack. In c), the geometry of the crack is detected, but the amplitudes of the longest members of the crack are only a fraction of their true amplitude, and there are significant artifacts exterior to the objects. In d), the crack is completely recovered. Indeed, the only discrepancy between d) and a) is a slight inhomogeneity in amplitude of the larger diameter object. This is expected, since the support functional is essentially a measure of object geometry, rather than composition. If there is a large number of pixels above the support threshold \( \varepsilon \), then there is a large number of degrees of freedom in constructing an object interior whose projection agrees with the available projection data. Additional measures can be applied to aid in the reconstruction of the object interiors, based on additional \textit{a priori} information, such as a preference for object homogeneity, as demonstrated in [2].

The algorithm is applied to an experimental phantom CT projection set, collected on a second-generation scanner having 32 detectors set over a 10 degree arc. The projections have 374 translational positions and 1152 angles over 360 degrees. Details of the 4 in. dia. aluminum phantom are depicted in fig.(2). A complete data reconstruction is compared in fig.(3) with a limited data reconstruction using half the projections (projection angles 90 through 180 degrees are removed). The constrained support algorithm (minimization of eq.6 with \( \gamma=0 \)) was used to obtain figs.(3a,b). A 256x256 pixel array was used with total width equal to the 374 point projection, so as to keep (pixel size) > 1.414 x (translation step), and thus avoid numerical instabilities which arise from oversampling. The \textit{a priori} data employed in the reconstruction is shown in fig.(4). To simulate the detection of flaws in a component, two holes, the smallest (.039 in., neighboring the large central hole) and one moderately large (.151 in.), were removed from the \textit{a priori} image of the actual phantom (compare fig.4 to fig.2 and fig.3a). The reconstruction proceeds by minimizing eq.(5) using the measures of eqs.(3 and 8). The support weighting \( \gamma \) was specified to allow a disagreement between simulated and measured projections, eq.(3), comparable to the measurement noise level. This significantly reduces noise in the reconstruction. The result of the reconstruction using \textit{a priori} data is shown in fig.(4). Both the large and small holes not contained in the \textit{a priori} data are reconstructed. These "flaws" are compactly reconstructed, rather than spatially smeared, as discussed in reference to fig.(1). Quantitative comparisons of vertical line profiles through these flaws, taken from figs.(3a) and (5), are shown in fig.(6) and fig.(7).

![Fig. 2 Drawing of experimental CT phantom.](image)
SUMMARY AND DISCUSSION

This paper reported on progress in the implementation of previously-developed tech­niques for the variational reconstruction of limited view CT data in a parallel SIMD com­puter algorithm capable of handling real-world size data sets. The algorithm is based on a forward projection model, which compensates for the missing projection data through the optimization of functional measures of solution attributes. Past work has shown that the minimization of object support can be an effective aid in reconstructing high-contrast, discontinuous boundary objects, particularly when used in conjunction with functional measures of other properties such as object homogeneity. Past work has also demonstrated effective means for incorporating explicit a priori object information into the reconstruction using a variational alignment procedure. These past developments were implemented in an algorithm which runs on a 16K MasPar MP1 parallel computer, and results demonstrating the operation of this algorithm were presented here.

A potential significant advantage of using a forward projection-based algorithm is the simplicity with which non-ideal (non-linear) measurement effects can be incorporated into the reconstruction. Development of this feature will be the subject of upcoming work.

Fig. 3 Comparison of experimental phantom constrained support reconstructions using a) complete data, b) projection angles 0 through 90.

Fig. 4 Image used as a priori data. Note missing holes (compare with fig. 2 and 3a).
Fig. 5 Support minimized reconstruction using *a priori* data.

Fig. 6 Vertical line profiles through large "flaw" hole (center), a) from fig.3a, b) from fig.5.
Fig. 7 Vertical line profiles through small "flaw" hole (center), a) from fig.3a, b) from fig.5.

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REFERENCES


