COST OPTIMIZATION OF ALLOCATING VIRTUAL NETWORK FUNCTIONS WITH PRECEDENCE IN MULTICAST NETWORKS

Ramcharan Chalamalasetty

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COST OPTIMIZATION OF
ALLOCATING VIRTUAL NETWORK
FUNCTIONS WITH PRECEDENCE IN
MULTICAST NETWORKS

Creative Component
Master of Science, Computer Engineering
Iowa State University

By
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Abstract

In the past few years, Network Function Virtualization (NFV) has been widely used to reduce the operational and capital costs of implementing network functions with better performance and easier network management. NFV is a network architecture completely built relying on Virtual Network Functions (VNFs). Quite a few developments have been done for the optimization of resource allocation to implement VNFs in various networks. In this report, an algorithm for reducing the cost of placing the VNFs within a Multicast network along with a choice of selecting the precedence of the VNFs is presented. The proposed approach is formulated as a Mixed Integer Linear Programming (MILP) model based on a main objective of minimizing the cost of resource allocation with Precedence in Multicast Networks. Finally, we evaluate the algorithm through simulations in CPLEX and demonstrate the results.

Index Terms - Network Function Virtualization (NFV), Virtual Network Function (VNF), Multicast Network, Mixed Integer Linear Programming (MILP).
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<td>9</td>
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<td>10</td>
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<td>11</td>
<td>NSF Network Model Results with Precedence between VNFs</td>
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1. Introduction

Multicasting is a process of communicating from a single source to multiple destinations with each destination defined to have its own need. This has become an emerging service mock that is used by many applications because of its advantages across various networks. Let us consider a Mobile Service Provider (MSP) as an example. There are various mobiles that support 4G/LTE cellular networks or some other devices that only support 3G networks. Based on the requirement of the destination device, the MSP has to configure the networks. So technically the MSP has to send higher bandwidth signals from Source which can be split using many types as per the customer’s usage. One more example would be TVs. Some TVs support 4K UHD signals, while others might support SD signals, etc. In order to handle all such requests from the destinations and fulfill them at a faster rate, Multicast communication has been very effective when compared to that of Unicast communication which happens from one source to one destination.

Nowadays, most of these multicast services are implemented using Network Functions (NFs). Examples of NFs can be load balancers, firewalls, intrusion detection systems, routers, etc. Many of the Service Providers (SP) currently use several combinations of NFs to provide the services based on the functionality they wish to achieve. These NFs are generally implemented using physical devices exclusively designed for the NF’s purpose and these devices are connected to each other to satisfy the service requirement. The deployment and management of NFs on the physical devices have become very costly. So, with the main objective of reducing the capital costs and the operational costs \[1\], Network Function Virtualisation (NFV) has been proposed by ETSI (European Telecommunications Standards Institute) \[2\].

NFV is a technique to virtualize the NFs. In other words, it is a process of providing a virtual platform for NFs through Software Virtualisation Methods so
as to be operated with efficient management, better performance and lower costs. These kind of NFs, which can be deployed on NFV infrastructure are known to be as Virtualised Network Functions (VNFs). These VNFs can be cascaded and can also be combined for providing the needed services at reduced operational and investment costs.

The main aim of this report is to design an optimal routing and VNF placement algorithm for the virtual networks that support multicasting topology, where the optimality criterion is to minimize capital and operational costs. In order to meet specific demands for each destination in a multicast topology, the order of VNFs in which they will be deployed is also important. The algorithm that we propose in this report has the ability to place the VNFs as per the destination needs. To evaluate the proposed algorithm, we have used CPLEX as the optimization solver of our MILP model.

Remaining chapters of the report are organized as follows. Chapter 2 provides a review of the literature related to NFV and resource allocation in multicast networks. Chapter 3 describes the problem formulation for a Unicast model. Then in Chapter 4, we show the generalization and development of formulation from a Unicast model to a Multicast model. Chapter 5 presents simulation models and the various implementations using those models along with the obtained results. Finally, a 'Conclusion' Chapter that discusses about what we have achieved from the designed algorithm and possible future developments.
2. Related Work

Multicast services involving NFV components (like VNFs) are being widely used by many applications because of the advantages that we discussed in the previous chapter. So the need of development in this field has grown a lot. There is a huge amount of work that has been currently going on across the improvements of various techniques of to implement NFV for lower network expenditures and better maintenance. In this chapter, we present some of the works that have helped us to understand NFV and multicasting. We also discuss some of the techniques that we included for the solution approach which were already presented across different papers.

The work presented in [4] explains NFV’s relationships with Software Defined Networks (SDNs) and cloud computing, etc. So as to have a better understanding of NFV and the key research areas in the field of NFV, this paper acts as a good reference which also shows the architecture of NFV as well as state-of-the-art survey on NFV. However, this paper do not contribute much towards the implementation of Multicasting using NFV.

Many algorithms were developed for finding a minimum cost multicast tree within a network. One such such work which also involves NFV on SDN is presented in [5]. It introduces the routing algorithms for building NFV-enabled Multicast topology on Software Defined Networks. The authors in [5] propose different approximation algorithms considering the problem as a Steiner Tree problem. Another similar work in [6], presents a resource allocation algorithm for VNFs by transforming the model into a queuing model in a Cloud Center (a SDNNFV enabled network). However, either of the these contributions do not discuss the precedence relationships between VNFs.

When allocating resources in Multicast networks, there are high chances of al-
locating the same resources at the nodes across the flows to different destinations. As this is a duplicate allocation at the nodes, the costs have to be calculated only once in these cases. This can happen similarly with the flows as well. In order to avoid such duplication in the network, we have to use some constraints which are presented in the chapter 4 as Generalization constraints. Similar constraints are introduced in [7]. Although, the approach in [7] solves the Multicast traffic grooming problem by providing a MILP formulation.

Most of the works that we described here introduce various algorithms that can be used for multicast services involving VNFs. As to our knowledge, we haven’t come across an optimistic resource allocation algorithm that uses the defined precedence between resources. Hence, we propose an optimal algorithm that can minimize the network cost with the allocation of VNFs to VMs (nodes) in accordance to the precedence of VNFs.
3. Problem and its Formulation

We define the problem as “Reducing the cost for the flow and placement of the VNFs used along the flow paths within unicast and multicast networks”. Our model is designed to optimize the flow within the network having VNFs with a minimal cost and to follow the precedence relationship between VNFs. The remaining categories in this chapter will explain how the path is formulated to include the VNFs and the different variables, constraints that we use to provision the flow with minimal cost while observing precedence in a Unicast Network.

The Unicast Formulation

We model the network as a graph consisting of nodes and links that connect THE VNFs which can be placed at any node. Generally, there can be preference within networks to place a specific VNF before another VNF, as each VNF can have its respective functions. So we include the precedence constraints and provide the precedence relationship between VNFs as an input to the program.

Each flow within a network can have its own requirements like a flow from Source to Destination 1 might probably need just a Firewall, but the flow from Source to Destination 2 could need a Firewall and as well as deep packet inspection while the other flow may not be needing any network function at all and so on. To handle all these cases, we have included the choice of selecting the VNFs (from the pre-defined set of VNFs) for each destination also as an input parameter to the program.

We started the development of the algorithm with the assumption to expand a Unicast network into a Multi-cast network. So our initial work is done on a Unicast network (with One Source and one Destination along with 2 VNFs) consisting of 6 nodes. As we progressed we resolved all the problems that we faced and have
constrained the objective function as shown in this section. We tested various cases to check whether the expected results are obtained in every case.

Table 1: Definitions of Parameters and Variables for a Unicast Network

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>Set of Nodes</td>
</tr>
<tr>
<td>VNF</td>
<td>Set of Virtual Network Functions (VNFs)</td>
</tr>
<tr>
<td>i, j, l, m, a, b</td>
<td>Nodes ∈ N</td>
</tr>
<tr>
<td>n, p</td>
<td>VNFs where n, p ∈ VNF</td>
</tr>
<tr>
<td>S</td>
<td>Source Node ∈ N</td>
</tr>
<tr>
<td>D</td>
<td>Destination Node ∈ N</td>
</tr>
<tr>
<td>Wn</td>
<td>Matrix of binary inputs defining the need of VNF ‘n’ across the path to the destination</td>
</tr>
<tr>
<td>Kn^np</td>
<td>Matrix of binary inputs defining the precedence of ‘n’ and ‘p’ where n, p ∈ VNF</td>
</tr>
<tr>
<td>bi_j</td>
<td>Cost of bandwidth on link i, j</td>
</tr>
<tr>
<td>VC_i^n</td>
<td>Cost of placing VNF ‘n’ on Node ‘i’</td>
</tr>
<tr>
<td>M</td>
<td>Large Constant Value</td>
</tr>
<tr>
<td>FSD^ij</td>
<td>Binary Variable representing Flow between link (i, j) from S to D</td>
</tr>
<tr>
<td>Vi^n</td>
<td>Binary Variable representing the allocation of VNF ‘n’ at Node ‘i’</td>
</tr>
<tr>
<td>Hi</td>
<td>Number of Hops between S and i in the Flow from S to D</td>
</tr>
<tr>
<td>Xi^n</td>
<td>An auxiliary variable used for linearization of the product between VNFs and Hops in the Flow</td>
</tr>
<tr>
<td>TC</td>
<td>Total Cost</td>
</tr>
</tbody>
</table>

Our design helps in allocating the VNFs with precedence along the flow in these models. So to construct our algorithm while satisfying all the possible scenarios, we considered several variables and parameters that would be needed in a Unicast model are described in the Table 1.
**Objective Function**

The objective of this optimization problem is to minimize the Total Cost in a network while allocating VNFs following the precedence relationship defined and is given by equation (1):

\[
TC = \sum_{ij} (b_{ij} F_{ij}^{SD}) + \sum_i \sum_{n} (VC_i^n.V_i^n) + \sum_i H_i \quad \forall \ i, j, S, D \in N; \ n \in VNF
\]

Here, the first term is for the sum of all the costs of bandwidths that are used in the network which is a product of the flows on links and their cost. The second term is the sum of all the installation costs over the nodes within the network wherever there is a VNF placement at a node. The final term is the sum of the minimum number of hops from the source to each of the nodes, which is used in the constraints to enforce precedence of the allocation of VNFs.

**Constraints**

a) Flow Constraints:

In order to ensure that there is no incoming flow to the Source and there is no Outgoing flow from the Destination, equations (2) - (5) are defined as below:

\[
\sum_{iS} F_{iS}^{SD} = 0 \quad \forall \ i, j, S, D \in N
\]

(2)

\[
\sum_{Si} F_{Si}^{SD} = 1 \quad \forall \ i, j, S, D \in N
\]

(3)

\[
\sum_{Dj} F_{Dj}^{SD} = 0 \quad \forall \ i, j, S, D \in N
\]

(4)

\[
\sum_{iD} F_{iD}^{SD} = 1 \quad \forall \ i, j, S, D \in N
\]

(5)

For the flow conservation within the network, equation (6) is defined.

\[
\sum_j F_{ij}^{SD} = \sum_j F_{jl}^{SD} \quad \forall \ i, j, S, D \in N; \ j \neq S, D
\]

(6)
b) VNF Placement Constraints:

These constraints determine the placement of VNFs along the flow from S to D.

\[ \sum_i V_{ni} = W_n \quad \forall \quad i \in N; n \in VNF \quad (7) \]

\( W_n \) is the binary input to determine whether we need the VNF 'n' within the network. If it is '0', then we do not use the VNF 'n' in the flow from S to D. The equation (7) will ensure it will be placed at node 'i'. The below equation (8) will guarantee that the VNF 'n' will be placed at node 'i' along the flow from S to D.

\[ \sum_i V_{ni} \leq \frac{(\sum_a F_{il}^{SD} + \sum_a F_{ai}^{SD})}{2} \quad \forall \quad a, i, l, S, D \in N; n \in VNF \quad (8) \]

Eq. (9) shows that the number of hops at the Source node are zero, while the eq. (10) guarantees the nodes 'j' is exactly one hop farther from node 'i' along the path from the Source to the Destination, which helps to maintain the precedence relationship between VNFs.

\[ H_S = 0 \quad \forall \quad S \in N \quad (9) \]

\[ 1 - F_{ij}^{SD} - \frac{H_i + 1 - H_j}{M} \geq 0 \quad \forall \quad i, j, S, D \in N \quad (10) \]

The eq. (10) is observed in [8] and it is used here as one of the precedence constraints that helps to determine the number of hops between 2 nodes where the VNF can be allocated. This equation also makes sure that if there is a link 'ij' used in the flow from S to D, then the number of hops from S to node 'j' is exactly one hop greater than the number of hops from S to node 'i'. Equations (11) and (12) helps to determine if the node (where the VNF placement can happen) is in the path to the destination. If the number of hops to node 'i' from S is zero, then the \( V_{ni} \) will be zero which indicates that the VNF 'n' will not be placed at node 'i'. Similarly with node 'j' for VNF 'p'.

\[ V_{ni} \leq H_i \quad \forall \quad i \in N; n \in VNF \quad (11) \]

\[ V_{ji} \leq H_j \quad \forall \quad j \in N; p \in VNF \quad (12) \]
Eq. (13) is the key equation that governs the Precedence in the network.

\[
\sum_i V^n_i . H_i \leq \sum_j V^p_j . H_j \quad \forall \quad i, j \in N; n, p \in VNF
\]  

(13)

The above equation is a product of two variables which is therefore non-linear. This is linearized by introducing auxiliary variables \( X^n_i \) and \( X^p_j \), as shown below.

\[
X^n_i = V^n_i . H_i \quad \forall \quad i \in N; n \in VNF
\]

\[
X^p_j = V^p_j . H_j \quad \forall \quad j \in N; p \in VNF
\]

With the above substitutions, equation (13) can be written as (14):

\[
\sum_i X^n_i \leq \sum_j X^p_j \quad \forall \quad i, j \in N; n, p \in VNF
\]  

(14)

c) Linearization Constraints:

The following constraints are used to evaluate \( X^n_i \) and \( X^p_j \), where \( M \) is a very large number. Although used in [9], these constraints are standard.

\[
X^n_i \geq [M.V^n_i] - M + H_i \quad \forall \quad i \in N; n \in VNF
\]  

(15)

\[
X^p_j \geq [M.V^p_j] - M + H_j \quad \forall \quad j \in N; p \in VNF
\]  

(16)

\[
X^n_i \leq H_i \quad \forall \quad i \in N, n \in VNF
\]  

(17)

\[
X^p_j \leq H_j \quad \forall \quad j \in N, p \in VNF
\]  

(18)

\[
X^n_i \geq 0 \quad \forall \quad i, D_k \in N
\]  

(19)

\[
X^p_j \geq 0 \quad \forall \quad j, D_k \in N
\]  

(20)

\[
X^n_i \leq M.V^n_i \quad \forall \quad i \in N; n \in VNF
\]  

(21)

\[
X^p_j \leq M.V^p_j \quad \forall \quad j \in N; p \in VNF
\]  

(22)
4. Multicast Algorithm

The formulation from Chapter 3 for the Unicast model is improvised and generalized, so that it can be used for developing a formulation for Multicast Networks. The parameters and variables needed for this model are shown in Table 2. The model doesn’t quite have to be changed except some additional generalization constraints shown towards the end of this chapter which are used to avoid the costs of duplicate links and the costs of placing VNFs at same nodes across several flows to multiple destinations.

We considered a 10 Node network (with one Source, three Destinations and 3 VNFs) to implement all our test cases while designing the algorithm for a Multicast Network.

Objective Function

The objective of this optimization model is to minimize the Total Cost in a network while allocating VNFs for each destination by following the precedence relationship defined between VNFs. It is given by equation (23):

\[ TC = \sum_{ij} (b_{ij}G_{ij}) + \sum_i \sum_n (V C_i^nU_i^n) + \sum_i \sum_{D_k} H_i(D_k) \quad \forall \quad i, j, D_k \in N; n \in VNF \]

(23)

However, this is similar to eq. (1) defined in the previous Chapter. But here we sum up the cost of the links and the VNFs that are used in the flows for each destination and the final term is similarly used for the sum of minimum number of hops from the Source to each of the nodes used across flows to each Destination from Source, which is used in the constraints to enforce precedence of the allocation of VNFs.
Table 2: Definitions of Parameters and Variables for a Multicast Network

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>Set of Nodes</td>
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<td>VNF</td>
<td>Set of Virtual Network Functions (VNFs)</td>
</tr>
<tr>
<td>i, j, l, m, a, b</td>
<td>Nodes $\in N$</td>
</tr>
<tr>
<td>n, p</td>
<td>VNFs where $n, p \in \text{VNF}$</td>
</tr>
<tr>
<td>$D_k$</td>
<td>Set of Destination Nodes</td>
</tr>
<tr>
<td>$b_{ij}$</td>
<td>Cost of bandwidth on link $i, j$</td>
</tr>
<tr>
<td>$VC^n_i$</td>
<td>Cost of placing VNF ’n’ on Node ’i’</td>
</tr>
<tr>
<td>$W_n(D_k)$</td>
<td>Matrix of binary inputs defining the need of VNF ’n’ across the path to each destination</td>
</tr>
<tr>
<td>$K^{np}$</td>
<td>Matrix of binary inputs defining the precedence of ’n’ and ’p’ where $n, p \in \text{VNF}$</td>
</tr>
<tr>
<td>M</td>
<td>Large Constant Value</td>
</tr>
<tr>
<td>$F_{ij}^{SD_k}$</td>
<td>Binary Variable representing Flow between link $(i, j)$ from S to $D_k$</td>
</tr>
<tr>
<td>$V^n_i(D_k)$</td>
<td>Binary Variable representing the allocation of VNF ’n’ at Node ’i’ used in $D_k$</td>
</tr>
<tr>
<td>$H_i(D_k)$</td>
<td>Number of Hops between S and i in the Flow to each $D_k$</td>
</tr>
<tr>
<td>$X^n_i(D_k)$</td>
<td>An auxiliary variable used for linearization of the product between VNFs and Hops in the Flow to each $D_k$</td>
</tr>
<tr>
<td>$G_{ij}$</td>
<td>An auxiliary variable used for generalization of Link Costs</td>
</tr>
<tr>
<td>$U_i(D_k)$</td>
<td>An auxiliary variable used for generalization of VNF Costs</td>
</tr>
<tr>
<td>TC</td>
<td>Total Cost</td>
</tr>
</tbody>
</table>
Constraints

a) Flow Constraints

Moreover, equations (24) - (27) are alike to eq. (2) - (5). But these are defined for each destination as below:

$$\sum_{i} F_{iS}^{SD_k} = 0 \quad \forall \ i, j, S, D_k \in N$$  \hspace{1cm} (24)

$$\sum_{S_i} F_{Si}^{SD_k} = 1 \quad \forall \ i, j, S, D_k \in N$$  \hspace{1cm} (25)

$$\sum_{D_{kj}} F_{D_{kj}}^{SD_k} = 0 \quad \forall \ i, j, S, D_k \in N$$  \hspace{1cm} (26)

$$\sum_{iD_k} F_{iD_k}^{SD_k} = 1 \quad \forall \ i, j, S, D_k \in N$$  \hspace{1cm} (27)

For the flow conservation within the network for each flow from Source to each Destination, eq. (28) is defined.

$$\sum_{j} F_{ij}^{SD_k} = \sum_{j} F_{jl}^{SD_k} \quad \forall \ i, j, S, D_k \in N; j \neq S, D_k$$  \hspace{1cm} (28)

b) VNF Placement and Precedence Constraints

Eq. (29) will be the important one which inputs the selection of VNFs for each destination. $W_n(D_k)$ is the matrix of binary inputs to determine whether we need the VNF ‘n’ within the network for destination $D_k$. Equation (29) will ensure it will be placed at node ‘i’ and equation (30) guarantees that the VNF ‘n’ will be placed at node ‘i’ along the flows from $S$ to each Destination.

$$\sum_{i} V_i^n(D_k) = W_n(D_k) \quad \forall \ i, D_k \in N; n \in VNF$$  \hspace{1cm} (29)

$$\sum_{i} V_i^n(D_k) \leq \frac{(\sum_{il} F_{il}^{SD_k} + \sum_{ai} F_{ai}^{SD_k})}{2} \quad \forall \ a, i, l, S, D_k \in N; n \in VNF$$  \hspace{1cm} (30)

Eq. (31) shows that the number of hops at the Source node are zero and eq. (32) is similar to eq. (10) which guarantees the node ‘j’ is exactly one hop far from
Source when compared to the number of hops to node ‘i’ from Source along the paths from Source to each of the Destinations.

\[ H_{S}(D_k) = 0 \quad \forall \quad S, D_k \in N \] (31)

\[ 1 - F_{ij}^{SD_k} - \frac{H_i(D_k) + 1 - H_j(D_k)}{M} \geq 0 \quad \forall \quad i, j, S, D_k \in N \] (32)

Equations (33) and (34) help to determine if the node (where the VNF placement can happen) is used in the path to each destination. If the number of hops to node 'i' from S is zero, then \( V_{i}^{n}(D_k) \) will be zero which indicates that the VNF 'n' will not be placed at node 'i' in the flow from Source to Destination \( D_k \). Similarly with node 'j' for VNF 'p' is shown in (34).

\[ V_{i}^{n}(D_k) \leq H_{i}(D_k) \quad \forall \quad i, D_k \in N; n \in VNF \] (33)

\[ V_{j}^{p}(D_k) \leq H_{j}(D_k) \quad \forall \quad j, D_k \in N; p \in VNF \] (34)

Eq. (35) plays a significant role which in fact governs the Precedence in the network for each destination.

\[ \sum_{i} V_{i}^{n}(D_k).H_{i}(D_k) \leq \sum_{j} V_{j}^{p}(D_k).H_{j}(D_k) + [1 - W_{p}(D_k)].M \]
\[ \forall \quad i, j, D_k \in N; n, p \in VNF \] (35)

The eq. (35) consists of product of two variables which is Non-Linear. This is linearized by introducing auxiliary variables \( X_{i}(D_k) \) and \( X_{j}(D_k) \) as shown below.

\[ X_{i}(D_k) = V_{i}^{n}(D_k).H_{i}(D_k) \quad \forall \quad i, D_k \in N; n \in VNF \]

\[ X_{j}(D_k) = V_{j}^{p}(D_k).H_{j}(D_k) \quad \forall \quad j, D_k \in N; p \in VNF \]

After the above substitutions, the equation (35) can be written as (36):

\[ \sum_{i} X_{i}(D_k) \leq \sum_{j} X_{j}(D_k) + [1 - W_{p}(D_k)].M \quad \forall \quad i, j, D_k \in N; p \in VNF \] (36)
c) Linearization Constraints:

These constraints are used in evaluating $X_i(D_k)$ and $X_j(D_k)$, where M is a very large number. However, as discussed in the previous chapter these constraints are standard which are used for flow to each destination.

$$X_i(D_k) \geq [M.V^n_i(D_k)] - M + H_i(D_k) \quad \forall \ i, D_k \in N; n \in VNF$$  \hspace{1cm} (37)

$$X_j(D_k) \geq [M.V^n_j(D_k)] - M + H_j(D_k) \quad \forall \ j, D_k \in N; p \in VNF$$  \hspace{1cm} (38)

$$X_i(D_k) \leq H_i(D_k) \quad \forall \ i, D_k \in N$$  \hspace{1cm} (39)

$$X_j(D_k) \leq H_j(D_k) \quad \forall \ j, D_k \in N$$  \hspace{1cm} (40)

$$X_i(D_k) \geq 0 \quad \forall \ i, D_k \in N$$  \hspace{1cm} (41)

$$X_j(D_k) \geq 0 \quad \forall \ j, D_k \in N$$  \hspace{1cm} (42)

$$X_i(D_k) \leq M.V^n_i(D_k) \quad \forall \ i, D_k \in N; n \in VNF$$  \hspace{1cm} (43)

$$X_j(D_k) \leq M.V^n_j(D_k) \quad \forall \ j, D_k \in N; p \in VNF$$  \hspace{1cm} (44)

As described at the start of this Chapter, below are the generalization constraints for Flow and placement of VNFs which minimize the Total Cost. These constraints not needed in a Unicast model as the model itself will find a single and an optimal path from Source to Destination.

d) For Generalization of the Flow over each Destination:

Here we use another auxiliary variable $G_{ij}$ in order to generalize the flow and minimize the cost for the flow from Source to all destinations.

$$G_{ij} \geq \sum_{D_k} \frac{F^{SD_k}_{ij}}{M} \quad \forall \ i, j, S, D_k \in N$$  \hspace{1cm} (45)
\[ G_{ij} \leq \sum_{D_k} F_{ij}^{SD_k} \quad \forall \ i, j, S, D_k \in N \] (46)

e) For Generalization of the Placement over each Destination:

In this section, we introduce one more auxiliary variable \( U^n_i \) in order to generalize the placement of VNFs across the flows to each destination and minimize the cost for the placement of VNFs by not including the costs of duplicated allocation of VNFs at the nodes.

\[ U^n_i \geq \sum_{D_k} \frac{V^n_i(D_k)}{M} \quad \forall \ i, D_k \in N; n \in VNF \] (47)

\[ U^p_j \geq \sum_{D_k} \frac{V^p_j(D_k)}{M} \quad \forall \ j, D_k \in N; p \in VNF \] (48)

\[ U^n_i \leq \sum_{D_k} V^n_i(D_k) \quad \forall \ i, D_k \in N; n \in VNF \] (49)

\[ U^p_j \leq \sum_{D_k} V^p_j(D_k) \quad \forall \ j, D_k \in N; p \in VNF \] (50)
5. Implementation and Results

The implementation of the algorithm is in CPLEX. While the code is written in OPL (Optimization Programming Language) and it is stored as a .mod file. The data is given in a separate file (.dat format). We also generated a .lp file, which provides the list of constraints and their corresponding values while the program ran. This file helped us to resolve all the issues that we faced while designing the algorithm.

The key components to be observed in the test cases are:

1) Flow from Source to Destination
2) VNF Placement
3) Precedence between VNFs is satisfied or not

The expected result would be the lowest cost flows from Source to Destinations with the VNF placed along the flows and following the precedence conditions that will be defined as an input.

Unicast Model

The designed MILP model is tested on several cases in different networks. Starting with the Complete Unicast Network (a set of 6 Nodes inclusive of a Source node and a Destination Node with 3 VNFs to be placed among these nodes) as shown in Figure 1, the following scenarios are tested. The links are shown below:

\[
\text{links} = \{<1,2>, <1,3>, <1,4>, <1,5>, <1,6>, <1,1>, \\
<2,1>, <2,3>, <2,4>, <2,5>, <2,6>, <2,2>, \\
<3,1>, <3,2>, <3,4>, <3,5>, <3,6>, <3,3>, \\
<4,1>, <4,2>, <4,3>, <4,5>, <4,6>, <4,4>, \\
<5,1>, <5,2>, <5,3>, <5,4>, <5,5>, <5,6>, <5,5>, \\
\}.
\]
with their respective link costs as:

\[
\text{linkcost} = [50, 2, 1, 2, 16, 0, \\
50, 50, 10, 3, 1, 0, \\
2, 50, 15, 24, 50, 0, \\
1, 10, 15, 19, 7, 0, \\
2, 3, 24, 19, 1, 0, \\
16, 1, 50, 7, 1, 0];
\]

and the cost of placing the VNF at each Node:

\[
\text{vnfcost} = [[3, 3, 3], \\
[10, 18, 10], \\
[10, 10, 10], \\
[18, 10, 40], \\
[18, 20, 10], \\
[16, 25, 30]];
\]

**Case 1:** No Precedence between VNFs and only VNF 1 and VNF 3 should be placed along the flow from S to D.

In this case, we observed that the Objective is 32, the placement of VNF 1 is at node 2 and the placement of VNF 3 at node 5. While the path from S to D is 1− > 5− > 2− > 6.

So here, the VNF 3 is placed first and the VNF 1 is placed next. The Figure 2 show the placement, path and the Objective.

**Case 2:** Precedence between VNFs is chosen as: \( VNF1− > VNF3− > VNF2 \)

Here the Objective is increased to 50, with the same path as the previous case while the placement of VNFs are changed. Now, we have all the 3 VNFs placed at node 2 which led to the cheapest cost path. The Figure 2 shows the CPLEX results.
Case 3: This case is with no VNFs, so no Precedence in order to find the cheapest and the best path from S to D. The resulted objective is very less and is 6 according to the link costs that are shown above. The path from S to D is $1 \rightarrow 5 \rightarrow 6$. The VNFs are not placed anywhere which satisfy our condition and the outputs are shown in Figure 4.

As we were able to get the expected outcomes from the Unicast model, we expanded our interest to Multicast networks and the results from the Multicast model are shown in the next section.

Multicast Model

For testing of the Multicast algorithm, we examined a complete 10 Node network with one Source Node and 3 Destination Nodes along with 3 VNFs as shown in Figure 5. The links, link costs and vnf costs considered are as below:

\[
\text{links} = \\
\{<1,2>,<1,3>,<1,4>,<1,5>,<1,6>,<1,7>,<1,8>,<1,9>,<1,10>,<1,1>, \]
\n\{<2,1>,<2,3>,<2,4>,<2,5>,<2,6>,<2,7>,<2,8>,<2,9>,<2,10>,<2,2>, \}
\]
Figure 2: Unicast Results with No Precedence between VNFs

\[
<3,1>, <3,2>, <3,4>, <3,5>, <3,6>, <3,7>, <3,8>, <3,9>, <3,10>, <3,3>, <4,1>, <4,2>, <4,3>, <4,5>, <4,6>, <4,7>, <4,8>, <4,9>, <4,10>, <4,4>, <5,1>, <5,2>, <5,3>, <5,4>, <5,6>, <5,7>, <5,8>, <5,9>, <5,10>, <5,5>, <6,1>, <6,2>, <6,3>, <6,4>, <6,5>, <6,7>, <6,8>, <6,9>, <6,10>, <6,6>, <7,1>, <7,2>, <7,3>, <7,4>, <7,5>, <7,6>, <7,8>, <7,9>, <7,10>, <7,7>, <8,1>, <8,2>, <8,3>, <8,4>, <8,5>, <8,6>, <8,7>, <8,9>, <8,10>, <8,8>, <9,1>, <9,2>, <9,3>, <9,4>, <9,5>, <9,6>, <9,7>, <9,8>, <9,10>, <9,9>, <10,1>, <10,2>, <10,3>, <10,4>, <10,5>, <10,6>, <10,7>, <10,8>, <10,9>, <10,10>;
\]

linkcost = [50, 2, 1, 2, 1, 7, 8, 9, 10, 0,
Figure 3: Unicast Results with VNF Precedence

50,50,10,3,1,7,8,9,10,0,
2,50,5,6,50,7,8,9,10,0,
1,10,5,1,7,7,8,9,10,0,
2,3,6,1,1,7,8,9,10,0,
1,1,50,7,1,7,8,9,10,0,
7,7,7,7,7,7,8,9,10,0,
8,8,8,8,8,8,8,9,10,0,
9,9,9,9,9,9,9,9,10,0,
10,10,10,10,10,10,10,10,10,0];

\text{vnfcost} = [[1000,1000,1000],
Figure 4: Unicast Results without VNFs

\[
\begin{align*}
[1, 9, 7], \\
[1, 8, 20], \\
[3, 5, 25], \\
[4, 24, 4], \\
[10, 10, 15], \\
[8, 10, 35], \\
[4, 15, 19], \\
[9, 90, 5], \\
[16, 1, 30]
\end{align*}
\]

The similar scenarios that we considered while testing the Unicast model are implemented here as well.

**Case 1:** No Precedence between VNFs and the VNF requirement for each destination is:
a) VNF 1 and VNF 2 for Destination 1 (D1)
b) Only VNF 3 for Destination 2 (D2)
c) VNF 1, VNF 2 and VNF 3 for Destination 3 (D3)

The CPLEX results showed that the placement of VNFs are according to the requirement with an objective of 55 and the paths for each destination are:
D1: 1→4→8 with both VNF 1 and VNF 2 placed at node 4
D2: 1→4→9 with VNF 3 placed at node 5
D3: 1→4→5→10 with VNF 1 and VNF 2 placed at node 4 and the VNF 3 placed at node 5

The results for this case from CPLEX are shown in Figure 6.

**Case 2:** With Precedence between VNFs as \( VNF_3 \rightarrow VNF_1 \rightarrow VNF_2 \) and the VNF requirement for each destination is same as previous case.

The results are similar. The objective is 55, the VNF placements are same but the paths are:
(a) VNF Placement  
(b) Objective

Figure 6: Multicast Results with No Precedence between VNFs

D1: 1 → 4 → 8 with both VNF 1 and VNF 2 placed at node 4
D2: 1 → 4 → 9 with VNF 3 placed at node 5
D3: 1 → 5 → 4 → 10 with VNF 1 and VNF 2 placed at node 4 and the VNF 3 placed at node 5

showing that the precedence between VNFs is followed and the CPLEX outputs are shown in Figure 7.

Case 3: In this case, no VNFs and hence no precedence relationship.

The outcome showed the lowest cost for the defined Multicast session with VNFs are not placed at all as expected. The objective is 30 and the paths are:
D1: 1 → 8
D2: 1 → 9
D3: 1 → 10

The CPLEX results for this case are shown in Figure 8.

![Multicast Results with No VNFs](image)

(a) VNF Placement  (b) Objective

**Figure 8: Multicast Results with No VNFs**

**NSF Network Model**

In addition to these network topology, we also considered NSF network topology for our testing purposes. This is a 14 node and 21 bidirectional link model with links and their costs are shown in Figure 9. This is extracted from reference.

We considered 3 VNFs and 3 destination nodes (11, 12, and 13) with the Source node as 0 while vnf costs at each node are assumed randomly like any other model and they are shown below:

\[
\text{vnfccost} = \begin{bmatrix}
1000, 1000, 1000, \\
1, 9, 7, \\
1, 8, 20, \\
3, 5, 25, \\
4, 24, 14, \\
10, 10, 15, \\
8, 10, 35, \\
9, 15, 20,
\end{bmatrix}
\]
Figure 9: NSF Network

\[[5, 14, 34], \\
[7, 19, 17], \\
[8, 16, 13], \\
[4, 15, 19], \\
[9, 90, 5], \\
[16, 1, 30]];

We have implemented this model in the similar cases as the previous model and the different test scenarios are:

**Case 1:** No Precedence between VNFs and the VNF requirement for each Destination are as follows:

a) VNF 1 and VNF 2 for Destination 1 (D1)

b) Only VNF 3 for Destination 2 (D2)

c) VNF 1, VNF 2 and VNF 3 for Destination 3 (D3)

The results are similar and as expected. They are shown in Figure 10.

The output paths are:

D1: 0− > 1− > 3− > 10− > 11 with VNF 1 and VNF 2 placed at nodes 1 and 3 respectively

D2: 0− > 1− > 3− > 10− > 12 with VNF 3 placed at node 1
Case 2: With Precedence between VNFs as $VNF_2 > VNF_1 > VNF_3$ and the VNF requirement for each Destination remains same.

The CPLEX outputs show that the paths are changed and the VNF locations are also changed with a slight increase in the cost. These are shown in the Figure 11.

While the paths are:
D1: 0→ 2→ 5→ 13→ 11 with both VNF 1 and VNF 2 placed at node 2
D2: 0→ 2→ 5→ 13→ 12 with VNF 3 placed at node 5
D3: 0→ 2→ 5→ 13 with VNF 1 and VNF 2 placed at node 2 and the VNF 3 placed at node 5

**Case 3:** Here, we consider no VNFs so no precedence also.
As discussed before, this case gives the cheapest path to destinations from Source.

![Figure 12: NSF Network Model Results with No VNFs](image)

The paths from Source to each destination are below:

D1: 0→ 7→ 8→ 11
D2: 0→ 7→ 8→ 12
D3: 0→ 7→ 8→ 12→ 13

We can see in every case, the cost of the duplicate links like 0→ 7→ 8→ in this case are considered only once. As typically, the path is common from node 0 to node 8 and it is split from node 8 to the destinations.
6. Conclusion

In this report, we developed an algorithm to place VNFs along the path to the destinations in Multicast Networks. We introduced the allocation of VNFs at nodes with precedence in the Unicast network. We then advanced our algorithm to Multicast networks. This algorithm is developed as a MILP model and the optimal solutions that are obtained for each of the network topology that we considered are shown in the previous section. The formulation is generalized and ensures an optimal solution also by eliminating the calculation of the duplicate entries for paths or VNF placements.

Moreover, observing all the cases that we considered for testing with different network models, we can say that the defined Precedence relationships are followed between VNFs and these VNFs are placed in the flows to each destinations with the minimal cost.

With this, we can conclude that the algorithm that we designed for a Multicast network model is capable of placing VNFs at the Nodes in the Flow by following the Precedence relationship leading the lowest cost path to destinations.

As a part of our development in this algorithm, we have not included/observed the performance of VNFs as it is out of the scope of this report. But as a future improvement to this work, the performance optimization of VNFs can definitely add a great value.
Bibliography


