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Strongly coupled fluid-particle flows in vertical channels. I. Reynolds-averaged two-phase turbulence statistics

Jesse Capecelatro
University of Illinois at Urbana-Champaign

Olivier Desjardins
Cornell University

Rodney O. Fox
Iowa State University, rofox@iastate.edu

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Abstract
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Keywords
Reynolds stress modeling, tensor methods, channel flows, particle velocity, fluid equations

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Strongly coupled fluid-particle flows in vertical channels. I. Reynolds-averaged two-phase turbulence statistics

Jesse Capecelatro,1,a) Olivier Desjardins,2 and Rodney O. Fox3,4
1Coordinated Science Laboratory, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801-2307, USA
2Sibley School of Mechanical and Aerospace Engineering, Cornell University, Ithaca, New York 14853, USA
3Department of Chemical and Biological Engineering, Center for Multiphase Flow Research, Iowa State University, Ames, Iowa 50011-2230, USA
4Laboratoire EM2C, CNRS, CentraleSupélec, Université Paris-Saclay, Grande Vois des Vignes, 92295 Chatenay Malabry, France

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Simulations of strongly coupled (i.e., high-mass-loading) fluid-particle flows in vertical channels are performed with the purpose of understanding the fundamental physics of wall-bounded multiphase turbulence. The exact Reynolds-averaged (RA) equations for high-mass-loading suspensions are presented, and the unclosed terms that are retained in the context of fully developed channel flow are evaluated in an Eulerian–Lagrangian (EL) framework for the first time. A key distinction between the RA formulation presented in the current work and previous derivations of multiphase turbulence models is the partitioning of the particle velocity fluctuations into spatially correlated and uncorrelated components, used to define the components of the particle-phase turbulent kinetic energy (TKE) and granular temperature, respectively. The adaptive spatial filtering technique developed in our previous work for homogeneous flows [J. Capecelatro, O. Desjardins, and R. O. Fox, “Numerical study of collisional particle dynamics in cluster-induced turbulence,” J. Fluid Mech. 747, R2 (2014)] is shown to accurately partition the particle velocity fluctuations at all distances from the wall. Strong segregation in the components of granular energy is observed, with the largest values of particle-phase TKE associated with clusters falling near the channel wall, while maximum granular temperature is observed at the center of the channel. The anisotropy of the Reynolds stresses both near the wall and far away is found to be a crucial component for understanding the distribution of the particle-phase volume fraction. In Part II of this paper, results from the EL simulations are used to validate a multiphase Reynolds-stress turbulence model that correctly predicts the wall-normal distribution of the two-phase turbulence statistics. © 2016 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4943231]

I. INTRODUCTION

Turbulent disperse two-phase flows with strong momentum coupling exhibit a wide range of important phenomena that directly impact the overall fluid dynamical system. Such mechanisms include the production of fluid-phase turbulent kinetic energy (TKE) via wakes past particles, the accumulation of particles in high-strain regions of the flow, and the spontaneous generation of densely packed particles, i.e., clusters, that have been observed to hinder mixing between the phases and amplify the aforementioned two-way-coupled effects. In wall-bounded flows, non-trivial coupling between the phases leads to additional inhomogeneities in particle concentration that feed back to the underlying turbulence.

a)Electronic mail: jcaps@illinois.edu
To date, most of the work on wall-bounded particle-laden flows found in the literature is focused on moderately dilute suspensions with weak interphase coupling, where the majority of the underlying carrier-phase turbulence manifests from classical mean-shear production.\textsuperscript{16,17,20,21,24,29,30,32} When the mean mass loading \( \varphi \), defined by the ratio of the specific masses of the particle and fluid phases, is order one or larger, the relative motion between the phases leads to additional sources of instabilities as a result of interphase coupling.\textsuperscript{14} Under certain circumstances, drag production becomes the primary source of turbulence generation, giving rise to a separate class of multiphase turbulence referred to as cluster-induced turbulence (CIT).\textsuperscript{6,7} Because the material density ratio \( \rho_p/\rho_f \) is very large in gas-particle flows, CIT is ubiquitous in practical engineering and environmental flows when body forces or inlet conditions generate a mean-velocity difference. Vreman \textit{et al.}\textsuperscript{25} performed simulations of a high-mass-loading turbulent channel flow with a volume fraction of $1.3\%$. It was found that particle-particle interactions have a large influence on the mean and root-mean-square velocities of each phase. Particles were also observed to decrease the thickness of the boundary layer and increase the skin-friction.

Owing to the wide range of length and time scales associated with two-way-coupled fluid-particle flows, the development of predictive reduced-order modeling strategies, e.g., Reynolds-average Navier-Stokes (RANS) and large-eddy simulation (LES), is crucial for the study of practical environmental and industrial applications. To this end, Fox\textsuperscript{13} recently derived the exact Reynolds-averaged (RA) equations for collisional fluid-particle flows. Through phase-space integration, the collisional Boltzmann equation was replaced by a set of macroscale moment equations written in terms of particle-phase volume fraction \( \alpha_p \), particle-phase velocity \( \mathbf{u}_p \), and granular temperature \( \Theta_p \), that are coupled to the carrier phase by including drag interactions. Unlike in most previous derivations of turbulence models for moderately dense granular flows, a clear distinction was made between the granular temperature, which appears in the particle-phase constitutive relations, and the particle TKE \( k_p \), which appears in the turbulent transport coefficients. Capeceletro \textit{et al.}\textsuperscript{2} further developed the RA formulation of Fox\textsuperscript{13} to include transport equations for the volume-fraction variance, drift velocity, and the separate components of the Reynolds stresses of each phase and particle-phase pressure tensor.

In the present study, the RA formulation is extended to fully developed vertical channel flow, and simulations are performed to identify the behavior of the unclosed terms. The results are used to validate a multiphase turbulence model developed in Part II\textsuperscript{14} of this paper. In order for the simulations to provide useful data, multiphase statistics consistent with the RA formulation must be extracted as a function of wall-normal distance. Specifically, when evaluating the particle velocity, it is crucial to accurately separate the spatially correlated (continuous) contribution, and random-uncorrelated component in an Eulerian frame of reference to uniquely obtain the particle-phase TKE and granular temperature. Vance \textit{et al.}\textsuperscript{27} developed an indirect method to estimate these separate components from calculations of two-point velocity correlations in gas-solid turbulent channel flow. It was shown that a discontinuity in the two-point statistics exists at the origin, consistent with the findings by Février \textit{et al.}\textsuperscript{12} for homogeneous isotropic turbulence. As described by Février \textit{et al.},\textsuperscript{12} the distribution of particle velocities remains spatially correlated at large particle-pair separation, while the two-point correlations at small pair separation decreases with increasing particle inertia, and remains smaller than the total velocity variance in the limit of zero pair separation. This behavior illustrates the portion of the particle velocity corresponding to a distribution that is not spatially correlated (referred in the works of Février \textit{et al.}\textsuperscript{12} and Vance \textit{et al.}\textsuperscript{27} as the quasi-Brownian component of the particle velocity). Vance \textit{et al.}\textsuperscript{27} found the partitioning of the particle velocity in dilute gas-solid channel flows to be sensitive to particle inertia, with increases in the Stokes number resulting in an increase of the fraction of the fluctuating energy residing in the random-uncorrelated motion (RUM). Particle-particle collisions were also found to enhance the fraction of the particle kinetic energy residing in the uncorrelated motion.

As described by Vance \textit{et al.},\textsuperscript{27} the two-point velocity correlation is adequate for deducing the partition of particle-phase velocity, but further studies are necessary to provide more precise quantitative measurements. In particular, local instantaneous information is lost in the computation of two-point statistics, and obtaining wall-normal distributions of the velocity partition becomes...
II. REYNOLDS-AVERAGE TRANSPORT EQUATIONS FOR VERTICAL CHANNEL FLOW

Here we extend the RA formulation for coupled gas-solid flows introduced by Fox\cite{footnote13} to account for the presence of walls. The present study considers fully developed vertical channel flow of width $W$, with the span-wise direction denoted by $x$, the wall-normal direction as $y$ ($0 \leq y \leq W$), and the vertical direction as $z$. All statistical quantities depend at most on $y$. This section presents the RA transport equations in terms of RA quantities $\langle \cdot \rangle$, where angled brackets denote an ensemble average in the statistically homogeneous ($x$ and $z$) directions and in time, particle-phase phase average (PA) quantities $\langle \cdot \rangle_p = \langle \alpha_p \cdot \rangle / \langle \alpha_p \rangle$, with $\alpha_p$ the local particle volume fraction, and fluid-phase PA quantities $\langle \cdot \rangle_f = \langle \alpha_f \cdot \rangle / \langle \alpha_f \rangle$, with $\alpha_f = 1 - \alpha_p$ the fluid volume fraction. The unclosed terms that appear in the transport equations are discussed at the close of this section.

A. Continuity

The transport of the RA particle-phase volume fraction $\langle \alpha_p \rangle$ reduces to the wall-normal component\cite{footnote12,footnote13}

$$
\frac{d\langle \alpha_p \rangle_p}{dy} = 0,
$$

where the subscript denotes the phase (particle $p$ or fluid $f$) and component ($x$, $y$, or $z$). This notation will be used throughout. Because the wall-normal component of velocity is null at the walls, this expression yields $\langle u_{p,y} \rangle_p(y) = 0$. Similarly, from transport of $\langle \alpha_f \rangle$ we find for the wall-normal fluid velocity $\langle u_{f,y} \rangle_f(y) = 0$. Furthermore, the span-wise PA velocities $\langle u_{p,x} \rangle_p$ and $\langle u_{f,x} \rangle_f$ are null for vertical channel flow, leaving only the vertical components $\langle u_{p,z} \rangle_p(y)$ and $\langle u_{f,z} \rangle_f(y)$ as nonzero quantities.
B. Mean momentum

The nonzero components of the RA particle-phase momentum equation are given by

\[
\frac{1}{\langle \alpha_p \rangle} \frac{d}{dy} \left( \langle u_{p,yy}^2 \rangle_p + \langle P_{p,yy} \rangle_p \right) = \frac{1}{\tau_p} u_{d,y},
\]

\[
\frac{1}{\langle \alpha_p \rangle} \frac{d}{dy} \left( \langle u_{p,zz}'' \rangle_p + \langle P_{p,zz} \rangle_p \right) = \frac{1}{\tau_p} \left( \langle u_{f,z} \rangle_f - \langle u_{p,z} \rangle_p + u_{d,z} \right) - g,
\]

(2)

where \(\tau_p = \rho_p d_p^2/(18 \mu_f)\) is the particle relaxation time, \(g\) is the magnitude of the gravitational acceleration, and \(\mathbf{P}_p = \Theta_p \mathbf{I} - \sigma_p\) is the particle-phase pressure tensor, with \(\Theta_p\) the local granular temperature, \(\sigma_p\) the particle-phase viscous-stress tensor, and \(\mathbf{I}\) the identity tensor. The double-prime notation is used throughout to represent fluctuations about a fluid-phase PA quantity, e.g., \(u_p''(x,t) = u_p(x,t) - \langle u_p \rangle_p\). The first equation in (2) determines \(\langle \alpha_p \rangle(y)\), and the second equation determines \(\langle u_{p,z} \rangle_p(y)\). The fluxes on the left-hand side of (2) involve the total granular energy tensor \(\kappa_p = \frac{1}{2} (u_p'' \otimes u_p'' + \langle \mathbf{P}_p \rangle_p)\), i.e., the sum of the particle-phase Reynolds-stress tensor and the PA pressure tensor. Because the boundary conditions at the walls are different for \(u_p'' \otimes u_p''\) and \(\langle \mathbf{P}_p \rangle_p\) (see Part II of this paper), below we derive a separate transport equation for each contribution.

The drift velocity, defined as \(\mathbf{u}_d = \langle \mathbf{u}_f \rangle_p - \langle \mathbf{u}_f \rangle_f = \langle \mathbf{u}_p'' \rangle_p\), is a very important quantity in high-mass-loading fluid-particle flows, and will be shown in Sec. II C to be directly responsible for generating fluid-phase TKE. In the fully developed channel flows considered in this work, the wall-normal component of the drift velocity, \(u_{d,y}\), is found to be negligible. The first equation in (2) can therefore be integrated to find

\[
\langle \alpha_p \rangle(y) = \frac{C}{\kappa_{yy}(y)},
\]

(3)

where the integration constant \(C\) is found from the integral mass balance

\[
\frac{1}{W} \int_0^W \langle \alpha_p \rangle(y) dy = \overline{\alpha}_p
\]

(4)

and \(\overline{\alpha}_p\) is the average particle-phase volume fraction in the channel. In the remainder of this paper, \(\langle \cdot \rangle = \frac{1}{W} \langle \cdot \rangle dy/W\) will denote a volume-averaged quantity. The distribution of the particle-phase volume fraction across the channel, and hence the momentum coupling with the fluid phase, is thus entirely determined by \(\kappa_{yy}(y)\).

The nonzero components of the RA fluid-phase momentum equation are given by

\[
\frac{1}{\langle \alpha_f \rangle} \frac{d}{dy} \left( \langle \sigma_f \rangle_{f,yy}'' + \langle \mathbf{P}_f \rangle_{f,yy} \right) = \frac{1}{\tau_p} \left( \langle \mathbf{u}_d,\mathbf{u}_d \rangle_f - \langle \mathbf{u}_f,\mathbf{u}_f \rangle_f - \langle \mathbf{P}_f \rangle_{f,yy} \right) - g.
\]

(5)

The triple-prime notation is used throughout to represent fluctuations about a fluid-phase PA quantity, e.g., \(u_f''(x,t) = u_f(x,t) - \langle u_f \rangle_f\). In (5), \(C_f = \frac{1}{\rho_f} \frac{\partial \langle \rho_f \rangle}{\partial z}\) is constant, \(\langle \rho_f \rangle\) is the RA fluid-phase pressure, \(\langle \sigma_f \rangle\) is the RA fluid-phase viscous-stress tensor, and \(\langle \phi \rangle(y) = \rho_p \langle \alpha_p \rangle/(\rho_f(\mathbf{a}_f))\) is the RA mass loading. The fluid pressure can be decomposed as \(\langle \rho_f \rangle_{f,yy}'' = \rho_f^*(y) - \rho_f^* C_f z\). The first equation in (5) determines \(\rho_f^*(y)\), and the second equation determines \(\langle \mathbf{u}_f,\mathbf{u}_f \rangle_f(y)\). Even at large mass loading, the term \(\langle \phi \rangle u_{d,y}\) is negligible for the flows considered herein. As done in (3), the first equation in (5) can therefore be integrated directly to find \(\rho_f^*(y)\); however, this pressure distribution does not appear in any of the other balances.

In the mean momentum balances, the PA Reynolds stresses \(\langle \mathbf{u}_p'' \otimes \mathbf{u}_p'' \rangle_p\) and \(\langle \mathbf{u}_p'' \otimes \mathbf{u}_p'' \rangle_f\), and the drift velocity \(\mathbf{u}_d\) all require closure. In comparison to single-phase turbulence, only the drift velocity is new. The vertical drift velocity \(u_{d,z}(y)\) can be seen as modifying the PA fluid velocity to account for correlations between the fluid and the particles, and it will be shown in Secs. III–V that an accurate prediction of \(u_{d,z}(y)\) is crucial for turbulence modeling when \(\langle \phi \rangle \gtrsim 0.1\). Because
\( \rho_p / \rho_f \gg 1 \) in gas-particle flows, significant mass loading is easily obtained for relatively small particle-phase volume fractions (e.g., \( \bar{\alpha}_p \approx 0.001 \)).

C. Reynolds-stress tensors

In fully developed vertical channel flows, there are four nonzero components for the Reynolds-stress tensors (i.e., \( xx, yy, zz, yz \)). The transport equation for the particle-phase Reynolds stress tensor \( (i, j = x, y, z) \)

\[
\frac{1}{\langle \alpha_p \rangle} \frac{d}{dy} (\langle u''_{p,ij} u''_{p,ij} \rangle_p + \varepsilon_{p,ij}) = \mathcal{P}_{p,ij} + \mathcal{R}_{p,ij} - \epsilon_{p,ij} + \mathcal{D} \mathcal{E}_{p,ij},
\]

\[
\frac{1}{\langle \alpha_f \rangle} \frac{d}{dy} (\langle u''_{f,ij} u''_{f,ij} \rangle_f + \varepsilon_{f,ij}) = \mathcal{P}_{f,ij} + \mathcal{R}_{f,ij} - \epsilon_{f,ij} + \mathcal{D} \mathcal{E}_{f,ij} + \mathcal{D} \mathcal{P}_{ij},
\]

(6)

The terms on the left-hand side and first three terms on the right-hand side of (6) do not involve mixed statistics (i.e., fluid-particle correlations) and have the same form as in single-phase turbulent flow.\(^{22} \) The mean-gradient production terms of each phase,

\[
\mathcal{P}_{p,ij} = -\left(\langle u''_{p,ij} u''_{p,ij} \rangle_p \delta_{jz} + \langle u''_{p,ij} u''_{p,iz} \rangle_p \delta_{ij} \right) \frac{d(u_{p,z})_p}{dy},
\]

\[
\mathcal{P}_{f,ij} = -\left(\langle u''_{f,ij} u''_{f,ij} \rangle_f \delta_{jz} + \langle u''_{f,ij} u''_{f,iz} \rangle_f \delta_{ij} \right) \frac{d(u_{f,z})_f}{dy},
\]

(7)

are closed, where \( \delta_{ij} \) is the Kronecker delta and repeated indices \((kk)\) imply summation.

As in single-phase turbulent channel flow, the terms containing viscous diffusion and pressure redistribution of each phase, \( \mathcal{E}_{p,ij} = \langle P_{p,y} u''_{p,j} \rangle_p + \langle P_{p,y} u''_{p,i} \rangle_p \) and \( \mathcal{E}_{f,ij} = \langle \tau_{f,y} u''_{f,j} \rangle_f + \langle \tau_{f,y} u''_{f,i} \rangle_f \), are unclosed and become important in near-wall regions of the flow. In addition, the triple correlations, \( \langle u''_{p,y} u''_{p,i} u''_{p,j} \rangle_p \) and \( \langle u''_{f,y} u''_{f,i} u''_{f,j} \rangle_f \), and pressure-rate-of-strain/dissipation-rate tensors,

\[
\mathcal{R}_{p,ij} - \epsilon_{p,ij} = \langle P_{p,ik} \nabla u''_{p,j} \rangle_p + \langle P_{p,jk} \nabla u''_{p,i} \rangle_p,
\]

\[
\mathcal{R}_{f,ij} - \epsilon_{f,ij} = \frac{1}{\langle \alpha_f \rangle} \left( \langle \tau_{f,ik} \nabla u''_{f,j} \rangle_f + \langle \tau_{f,jk} \nabla u''_{f,i} \rangle_f \right),
\]

(8)

all require closure. Here, the fluid-phase stress tensor is modeled as \( \tau_f = (p_f / \rho_f) \mathbf{I} - \sigma_f \). The traceless, symmetric pressure-rate-of-strain tensors, given by

\[
\mathcal{R}_{p,ij} = \langle \Theta_p \nabla u''_{p,i} \rangle_p + \langle \Theta_p \nabla u''_{p,j} \rangle_p - \frac{2}{3} \langle \Theta_p \nabla \cdot u''_{p} \rangle_p \delta_{ij},
\]

\[
\mathcal{R}_{f,ij} = \frac{1}{\rho_f \langle \alpha_f \rangle} \left( \langle p_f \nabla u''_{f,i} \rangle_f + \langle p_f \nabla u''_{f,j} \rangle_f - \frac{2}{3} \langle p_f \nabla \cdot u''_{f} \rangle_f \delta_{ij} \right),
\]

(9)

are the most important terms in Reynolds-stress models.\(^{22} \)

The remaining terms in the Reynolds-stress balances contain fluid-particle correlations and become important when interphase coupling is significant. The drag-dissipation-and-exchange tensors of each phase

\[
\mathcal{D} \mathcal{E}_{p,ij} = \frac{1}{\tau_p} \left( \langle u''_{p,f,j} u''_{p,j} \rangle_p + \langle u''_{p,f,i} u''_{p,i} \rangle_p \right),
\]

\[
\mathcal{D} \mathcal{E}_{f,ij} = \frac{\phi}{\tau_p} \left( \langle u''_{f,p,j} u''_{f,j} \rangle_p + \langle u''_{f,p,i} u''_{f,i} \rangle_p \right),
\]

(10)

describe how the Reynolds stresses are both dissipated and exchanged between the phases. In (10), the tensors \( \langle u''_{p,f,i} u''_{p,j} \rangle_p \) and \( \langle u''_{f,p,i} u''_{f,j} \rangle_p \) require closure. The drag-production term

\[
\mathcal{D} \mathcal{P}_{ij} = \frac{2 \phi}{\tau_p} \langle u_{p,z} \rangle_p \delta_{iz} \delta_{jz} \langle u_{f,z} \rangle_f \]

(11)

describes how fluid-phase Reynolds stresses are produced by a mean-velocity difference between the phases. From (6) we observe that drag production in vertical channel flow only produces \( \langle u''_{f,z} \rangle_f \).
When the mass loading is large, drag production can be much larger than mean-gradient production (i.e., \( DP \gg P \)). Note that \( DP_{ij} \) will be closed once the model for \( u_{d,z}(y) \) in (2) has been chosen so that no additional closure is required.

### D. PA particle-phase pressure tensor

The PA particle-phase pressure-tensor is governed by

\[
\frac{1}{\langle \alpha_p \rangle} \frac{d}{dy} \langle \alpha_p \rangle \left( \langle u_{p,y}^2 \rangle_p + \langle Q_p \rangle_p \right) = \epsilon_p - R_p + \mathcal{P}_p - \frac{2}{\tau_p} \langle \mathbf{P}_p \rangle_p + C, \tag{12}
\]

where \( Q_p \) is the wall-normal component of the granular-flux tensor, \( C \) is the collisional-dissipation tensor modeled via

\[
C = \frac{12}{\sqrt{\pi} d_p} \langle \alpha_p \rangle^{1/2} (\Delta^* - \mathcal{P}_p), \tag{13}
\]

where \( \Delta^* = \frac{1}{4} (1 + e)^2 \Theta^* + \frac{1}{4} (1 - e)^2 \mathbf{I}, \) \( \mathbf{I} \) is the identity matrix, \( e \) is the coefficient of restitution for particle-particle collisions. The laminar mean-gradient production term \( P_p = -\langle \mathbf{P}_p \rangle_p \cdot \nabla \langle u_p \rangle_p - \langle \nabla \langle u_p \rangle_p \rangle \cdot \langle \mathbf{P}_p \rangle_p \) is closed. We briefly note that \( C \) is the only term appearing in the RA equations that is not derived explicitly from the corresponding EL equations, but is instead modeled. The validity of this model will become apparent in Sec. IV B 5 when the wall-normal distributions of each term in (12) are presented. It should also be noted that the pressure-rate-of-strain/dissipation-rate tensor in (12) is exactly the same as \( \epsilon_p - R_p \) in (6). In other words, redistribution/dissipation in the particle-phase Reynolds stresses is the principal production term for \( \langle \mathbf{P}_p \rangle_p \). This is analogous to single-phase flow where dissipation of TKE leads to viscous heating. In this case, however, redistribution/dissipation is directly attributed to anisotropy in the particle-phase stress tensor.

In (12), the turbulent flux \( \langle u_{p,y}^2 \rangle_p \), \( \epsilon_p \), \( R_p \), and \( C \) are all unclosed. As in single-phase turbulent channel flows, the turbulent flux is null at the walls, but dominant away from the walls. Similarly, \( \langle Q_p \rangle_p \) is important near the walls in vertical channel flows. A transport equation for the PA granular temperature is obtained by taking one-third of the trace of (12). It is noteworthy that the collision frequency depends on the uncorrelated granular energy \( \Theta_p \) and not on the trace of \( \kappa_p \). For this reason, separate transport equations for \( \langle \mathbf{P}_p \rangle_p \) and the particle-phase Reynolds stresses are needed to account correctly for collisions.

### III. APPROACH

#### A. Channel configuration

To evaluate the unclosed terms appearing in the RA equations presented in Sec. II, fully developed particle-laden flows between plane, parallel walls are computed in an EL framework. Gravity acts in the negative \( z \)-direction in a channel of dimensions \( 10W \) in the streamwise \( (z) \), \( 1.5W \) in the spanwise \( (x) \), and \( W \) in the wall-normal \( (y) \) directions. Periodic conditions are imposed in the streamwise and spanwise directions. To maintain a statistical stationary state, the constant \( C_f \) in (5) is chosen such that the average fluid velocity in the vertical direction, defined by (where \( \bar{u}_f = 1 - \bar{u}_p \))

\[
\bar{u}_{f,z} = \frac{1}{W \bar{u}_f} \int_0^W \langle \alpha_f \rangle(y) \langle u_{f,z} \rangle_f(y) dy, \tag{14}
\]

is constant. Three cases are considered, corresponding to \( \bar{u}_{f,z} / \mathcal{V}_f = 1 \) (riser configuration), 0, and \(-1 \) (downer configuration), where \( \mathcal{V}_f = \left| \tau_p \bar{u}_f \right| \) is the magnitude of the terminal velocity of an isolated particle in a corresponding quiescent flow. Each case consists of 4476233 monodisperse particles of diameter \( d_p = W/250 \) and density \( \rho_p = 2000 \rho_f \), corresponding to a volume-average particle-phase volume fraction of \( \bar{\varphi}_p = 0.01 \), and average mass loading \( \bar{\varphi} = 20.2 \). The particle Reynolds number, \( Re_p = \rho_f \mathcal{V}_f d_p / \mu_f = 10 \) for all cases considered in the present study.
FIG. 1. Near-wall grid resolution relative to the particle diameter. Wall-normal grid spacing (—); center of particle position (+).

Unlike in one-way coupled flows with prescribed carrier-phase turbulence, grid resolution requirements for CIT are unknown a priori. It is assumed that the majority of the fluid-phase velocity fluctuations are generated by clusters with length scales $L \gg d_p$, and thus grid spacing is chosen to be on the order of the particle diameter. Uniform spacing is imposed in the $x$ and $z$ directions of size $\Delta x = \Delta z = 3.125 d_p$, with a total grid size of $800 \times 138 \times 120$. Because clusters tend to fall in the near-wall region in vertical wall-bounded flows, entrainment of the carrier-phase leads to high gradients of fluid quantities that must be adequately captured. To this end, grid stretching is applied in the wall-normal direction varying from $0.025 d_p \leq \Delta y \leq 2 d_p$, as depicted in Fig. 1. The grid stretching was designed such that the center of each particle remains greater than $d_p/2$, thus restricting the collision detection search to the nearest neighboring cells. Note that a uniform auxiliary grid for handling collision detection could be introduced to alleviate this constraint, but was not found necessary for the present study. The volume-filtered EL formulation described in Sec. III B decouples the grid-size-to-particle-diameter ratio during interphase-exchange processes, allowing for the grid spacing to be smaller than the particle diameter without compromising the use of microscale models (e.g., drag). The numerical approach has been extensively validated for vertical wall-bounded flows in our previous work.5,8

B. Volume-filtered Euler-Lagrange formulation

The displacement of an individual particle $i$ is calculated using Newton’s second law of motion,

$$\frac{dv_p^{(i)}}{dt} = \mathcal{A}^{(i)} + F_c^{(i)} + g, \quad (15)$$

where $v_p^{(i)}(t)$ is the instantaneous particle velocity at time $t$, $F_c$ is the collision force modeled using a modified soft-sphere approach originally proposed by Cundall and Strack.10 In this work, we consider inelastic collisions with a coefficient of restitution $e = 0.9$ for both particle-particle and particle-wall collisions. The interphase-exchange term is given by

$$\mathcal{A}^{(i)} = \frac{1}{\tau_p} \left( u_f[x_p^{(i)}] - v_p^{(i)} \right) - \frac{1}{\rho_p} \nabla p_f^* [x_p^{(i)}] + \frac{1}{\rho_p} \nabla \cdot \sigma_f [x_p^{(i)}], \quad (16)$$

where the modified pressure gradient $\nabla p_f^*$ and divergence of the viscous-stress tensor $\nabla \cdot \sigma_f$ are taken at $x_p^{(i)}$, the center position of particle $i$. The term $\nabla p_f^*$ is a body force that contains the hydrodynamic pressure $p_f$ and is adjusted dynamically in order to maintain a constant mass flow rate in the channel.
In real systems with moderate Reynolds numbers and particle volume fractions, particles will experience drag with a nonlinear dependence on volume fraction and velocity (see, e.g., Tenneti et al.\textsuperscript{26}), but to simplify the RANS analysis, the higher-order terms are neglected here. To account for the presence of the particle phase in the fluid without requiring to resolve the boundary layers around individual particles, a volume filter is applied to the constant-density Navier-Stokes equations, thereby replacing the point variables (fluid velocity, pressure, etc.) by smoother, locally filtered fields. The resulting fluid-phase momentum equation is given by

\[
\frac{\partial \alpha_f u_f}{\partial t} + \nabla \cdot (\alpha_f u_f \otimes u_f) = -\frac{1}{\rho_f} \nabla p_f^\ast + \frac{1}{\rho_f} \nabla \cdot \sigma_f - \frac{\rho_p}{\rho_f} \alpha_p \mathbf{A} + \alpha_f \mathbf{g}.
\]  

(17)

To transfer the fluid variables to the particle location, second-order tri-linear interpolation is used. To extrapolate the particle data back to the Eulerian mesh, we apply the volume-filtering approach used in deriving the fluid-phase equations of motion (17). We begin by defining a filtering kernel \( G \) with a characteristic length \( \delta_f \), such that \( G(r) > 0 \) decreases monotonically with increasing \( r \), and is normalized such that it integrates to unity. Given a quantity \( \mathcal{A}^{(i)}(t) \) located at the center of particle \( i \) at time \( t \), and assuming \( G \) does not vary significantly over the volume of the particle (i.e., \( \delta_f \gg d_p \)), its Eulerian projection is given by

\[
\alpha_p \mathbf{A}(\mathbf{x}, t) \approx \sum_{i=1}^{N_p} \mathcal{A}^{(i)}(t)G(|\mathbf{x} - \mathbf{x}_p^{(i)}|)V_p,
\]

(18)

where \( N_p \) is the total number of particles in a single realization of the flow and \( V_p = \pi d_p^3/6 \) is the particle volume. This expression replaces the discontinuous Lagrangian data with an Eulerian field that is a smooth function of the spatial coordinate \( x \). Using (18) with \( \mathcal{A}^{(i)} = 1 \), we obtain the particle volume fraction \( \alpha_p \), and \( \mathcal{A}^{(i)} = \mathbf{A}^{(i)} \) gives the momentum exchange term \( \mathbf{A} \) seen by the fluid in (17). Further details on the numerical implementation can be found in Capecelatro and Desjardins.\textsuperscript{4}

C. Decomposition of particle fluctuating energy

As shown in Sec. II, separate transport equations are solved for the particle-phase Reynolds stresses \( \langle u''_p \otimes u''_p \rangle_p \) and particle pressure tensor \( P_p \). Taking the trace of each yields the particle-phase TKE, \( k_p \), and PA granular temperature, \( \langle \Theta_p \rangle_p \), respectively, which are used to define the total particle-phase fluctuating energy by

\[
\kappa_p = \frac{1}{2} \langle v'_p \cdot v'_p \rangle = k_p + \frac{3}{2} \langle \Theta_p \rangle_p,
\]

(19)

where the single-prime notation represents a fluctuation about a RA quantity, e.g., \( v'_p = v_p - \langle v_p \rangle \). Note that when angled brackets are used on a Lagrangian quantity it represents a particle average. Evaluating the separate contributions of \( \kappa_p \) requires introducing a separation of length scales into the averaging procedure in order to properly decompose these separate contributions. To accomplish this, a similar filtering approach used during the interphase exchange is employed, as given by (18). In our previous work on homogeneous CIT,\textsuperscript{6,7} it was demonstrated that \( k_p \) and \( \langle \Theta_p \rangle_p \) depend strongly on the choice of the filter size. It was found that an averaging volume that adapts to the local particle field is capable of separating the spatially correlated and uncorrelated components of granular energy with less sensitivity compared to a constant filter size. In such an approach, the adaptive filter allows for a sufficient number of particles to be sampled in dilute regions of the flow, while remaining optimally compact in dense clusters. Given an ensemble of identical (i.e., monodisperse) particles, and assuming there are no sharp gradients in volume fraction, an averaging volume will sample \( N_p \) particles with a filter size

\[
\delta_f(\alpha_p) = \left( \frac{N_p d_p^3}{\alpha_p} \right)^{1/3}.
\]

(20)

It should be noted that in the present study, it was not known \textit{a priori} whether a single value of \( N_p \) is capable of accurately separating the spatially correlated and uncorrelated contributions from the
particle-phase velocity as a function of distance from the wall. Verification of the adaptive filter is presented in Sec. IV, along with one-point and two-point statistics obtained from the filter.

IV. RESULTS

The simulations are initialized with a random distribution of particles subject to gravity in a quiescent flow. Due to the non-trivial coupling between the phases, the transients persist for approximately 10τp before reaching a fully developed statistically stationary state. After this initial transient, results are measured at each computational time step (Δt = 4 × 10^{-5}τp), over a duration of approximately 25τp. To properly capture particle collisions, the value of Δt was chosen to restrict the fastest particles in the domain from moving more than one-tenth of their diameter per time step.

A. Two-point statistics

To assess the accuracy of the filtering procedure in separating the components of granular energy, two-point velocity correlations computed using the filtered particle-phase velocity are compared against the corresponding velocity correlations from the Lagrangian data (i.e., the exact interparticle velocity correlation). The two-point correlations are computed in planes parallel to the channel walls and for a given separation in x or z. The correlations are averaged over the statistically homogeneous x–z planes and over time. The normalized Lagrangian two-particle velocity correlation is defined as

\[ R_{ij}(r, y) = \frac{1}{2\kappa_{ij}} \left( \sum_{m=1}^{N_p} \sum_{n=m}^{N_p} \delta (x - x^{(m)}(t)) \delta (x + r - x^{(n)}(t)) \frac{\langle \alpha_p(x,t) \alpha_p(x + r, t) u''_{p,i}(x, t) u''_{p,j}(x + r, t) \rangle}{\langle \alpha_p(x,t) \alpha_p(x + r, t) \rangle} \right), \]  

where δ is the Dirac delta function. Similarly, the normalized filtered two-point velocity correlation is given by

\[ \tilde{R}_{ij}(r, y) = \frac{1}{2\kappa_{ij}} \left( \sum_{m=1}^{N_p} \sum_{n=m}^{N_p} \delta (x - x^{(m)}(t)) \delta (x + r - x^{(n)}(t)) \frac{\langle \alpha_p(x,t) \alpha_p(x + r, t) u''_{p,i}(x, t) u''_{p,j}(x + r, t) \rangle}{\langle \alpha_p(x,t) \alpha_p(x + r, t) \rangle} \right). \]  

Due to the presence of gravity, the statistics may exhibit strong anisotropy and therefore depend strongly on the directionality of particle pair separation r. Because the particles have finite size, only pair separations larger than the particle diameter are considered. Consequently, only the correlated contribution of the particle-phase velocity is retained at small pair separations after averaging, and thus only contributions from \( k_p \) are captured. It should be noted that in the limit of zero pair separation, the filtered particle velocity correlation reduces to

\[ \tilde{R}_{ij}(0, y) = \frac{\langle \alpha_p^{2} u''_{p,i} u''_{p,j} \rangle}{\langle \alpha_p^{2} \rangle}. \]  

In dilute systems with \( \alpha_p \ll 1 \), \( \tilde{R}_{ij}(0, y) \approx 2k_p(y) \). In high-volume-fraction suspensions, as considered herein, these expressions are not equivalent, and thus \( \tilde{R}_{ij}(0, y) \) does not provide a quantitative measure of \( k_p \) (or \( \Theta_p \)), but instead captures qualitative trends. Thus, the two-point statistics are used for verification purposes, and quantitative measures of the spatially correlated and uncorrelated components will be provided in Secs. IV B 1–IV B 6.

Comparisons between Lagrangian particle velocity correlations and particle velocity correlations obtained using the filter with \( N_p = 0.1 \) are provided in Figs. 2 and 3. Overall excellent agreement can be observed. It can be seen that a single value of \( N_p \) is sufficient for separating the spatially correlated and uncorrelated contributions of the particle velocity at all distances from the wall and for each case under consideration. We note that it was found that varying \( N_p \) by two orders of magnitude changed the resulting velocity correlations by less than a couple percent.

Due to the normalization of the velocity correlations with the velocity variance from the individual particles, \( 2k_{p,z,z} \), values less than 1 at zero pair separation are indicative of finite granular temperature. From Figs. 2 and 3, a general trend can be observed that granular temperature
FIG. 2. Streamwise correlation functions of the streamwise particle velocity normalized by $2k_{p,z}$ in the corresponding plane. Two-particle Lagrangian correlation (symbols), filtered correlation with $N_p = 0.1$ (lines), $u_f, z/V_t = -1$ (black solid line, black open circle), $u_f, z/V_t = 0$ (blue dashed line, blue square), $u_f, z/V_t = 1$ (red dashed-dotted line, red lozenge), $y/W = 0.05$ (a), $y/W = 0.2$ (b), $y/W = 0.35$ (c), $y/W = 0.5$ (d).

increases away from the wall. The coherent motion of clusters in the vicinity of the wall gives rise to large values of $k_p$, while large values of $\langle \Theta_p \rangle$ in the central region of the channel arise from an increased particle-cluster collision rate due to high-shear and cluster-breakup events. Meanwhile, no significant differences can be observed between the three mass flow rates.

B. One-point statistics

1. Volume fraction distribution

Figure 4 shows the instantaneous particle concentration in the vicinity of a cluster for $u_f, z/V_t = -1$ at $y/W = 0.03$. Clusters are made up of many particles that entrain the fluid phase as they fall, and thus experience locally reduced drag. As a result, particles within clusters tend to exhibit large values of $k_p$, which is preserved in the wake of the cluster as they are shed off its trailing edge. Unlike $k_p$, $\Theta_p$ tends to be very small within clusters, with maximum values located in front of falling clusters where the particle-phase velocity is strongly compressed in the vertical direction. It will be shown in Secs. IV B 2–IV B 6 that this compressive heating of the particle phase appears as a production term for $\langle \Theta_p \rangle_p$ and dissipation of $k_p$.

From Fig. 5, clustering is observed to be most pronounced in the near-wall region, regardless of the directionality of the flow. The case with the downward flow is seen to have the most distinct clustering at the wall. It is hypothesized that the downward flow provides the least resistance to falling particles in the vicinity of the channel wall, allowing the clusters to accumulate more particles and grow larger between breakup events. Volume-fraction fluctuations are also seen to be greatest in the near-wall region of the channel, with larger values associated with the downer configuration,
FIG. 3. Spanwise correlation functions of the streamwise particle velocity normalized by $2\kappa_{p,zz}$ in the corresponding plane. Two-particle Lagrangian correlation (symbols), filtered correlation with $N_p = 0.1$ (lines), $\bar{\nu}_{f,z}/V_t = -1$ (black solid line, black open circle), $\bar{\nu}_{f,z}/V_t = 0$ (blue dashed line, blue square), $\bar{\nu}_{f,z}/V_t = 1$ (red dashed-dotted line, red lozenge). $y/W = 0.05$ (a), $y/W = 0.2$ (b), $y/W = 0.35$ (c), $y/W = 0.5$ (d).

and smallest fluctuations associated with the riser configuration. However, the volume-fraction fluctuations are smaller than those observed in homogeneous CIT.\textsuperscript{7} As depicted in Fig. 6, fluid-velocity fluctuations are highly correlated to the local volume fraction distribution. Fluid is strongly entrained downward in clusters, while high-speed jets pass through regions devoid of clusters, a phenomenon referred to as jet bypassing.\textsuperscript{25} Such strong coupling between the phases gives rise to a finite drift velocity that contributes to drag production $D\Phi$ of fluid-phase TKE in (6).

Wall-normal profiles of drift velocity are shown in Fig. 7. Regions of high particle-phase volume fraction are seen to be associated with strongly negative values of the vertical component $u_{d,z}$, while the wall-normal component $u_{d,y}$ is relatively small. In the literature,\textsuperscript{13,33} the latter is usually modeled as a turbulent flux proportional to $d\ln(\langle \alpha_p \rangle)/dy$. From the EL data in Fig. 7(b) (see also Fig. 10(b)), we observe that $u_{d,y}$ is negligible for the vertical channel flows investigated in this work. As discussed in Sec. II B, the volume fraction distribution can be determined from the wall-normal RA particle-phase momentum equation (2) via $\langle \alpha_p \rangle(y) = C/\kappa_{yy}(y)$ in (3) when $u_{d,y}$ is negligible. Comparisons between $\langle \alpha_p \rangle(y)$ and $C/\kappa_{yy}(y)$ extracted from the EL simulations with $C = 6.4 \times 10^{-5}$, $6.0 \times 10^{-5}$, and $5.3 \times 10^{-5}$ for $\bar{\nu}_{f,z}/V_t = -1$, 0, and 1, respectively, are provided in Fig. 8. For clarity, the curves have been moved up (down) by the multiplicative factor given in the caption. At the channel wall, the values obtained from (3) are maximum while the $\langle \alpha_p \rangle_p$ maximum is located slightly inside the channel. Overall, it can be observed that values obtained from (3) match relatively well with the volume fraction distribution.

2. Mean-velocity profiles

Shown in Fig. 9 are profiles of the mean stream-wise velocity of the particle and fluid phases for the three mass flow rates. Both phases exhibit the strongest downward flow in the near-wall region of the channel as a result of falling clusters. The no-slip condition imposed on the fluid at the
wall results in very strong near-wall gradients in the streamwise fluid velocity. Meanwhile, particle collisions at the wall do not hinder their downward motion, and thus as shown in Fig. 7, this is the location of maximum streamwise drift velocity (albeit with $u_d$ equal to zero at the wall owing to the no-slip boundary condition for the fluid velocity).

### 3. Mean-momentum budgets

It was found in the present study that the three cases under consideration exhibit similar trends in many of the statistics. Although the magnitudes are different, only results for $\overline{u_{f,z}}/\mathcal{V}_t = 0$ will be reported in the following analysis, but results from all three cases are used to validate a turbulence model in Part II. The individual terms in mean-momentum equations (2) and (5) are shown in

---

**FIG. 4.** Instantaneous snapshot of correlated ($k_p$) and uncorrelated ($\Theta_p$) granular energy in the vicinity of a cluster for $\overline{u_{f,z}}/\mathcal{V}_t = -1$ at $y/W = 0.03$. Left clip plane is colored by normalized particle volume fraction. Lines show iso-contours of $\alpha_p = 3\alpha_p$.

**FIG. 5.** RA particle-phase volume-fraction profiles. $\overline{u_{f,z}}/\mathcal{V}_t = -1$ (black solid line), $\overline{u_{f,z}}/\mathcal{V}_t = 0$ (blue dashed line), $\overline{u_{f,z}}/\mathcal{V}_t = 1$ (red dashed-dotted line). (a) Mean and (b) variance.
FIG. 6. Instantaneous snapshots of \(x-z\) plane colored by streamwise fluid-phase velocity fluctuations for \(\overline{u}_{f,z}/V_t = -1\). Lines show iso-contours of \(\alpha_p \equiv 2\langle \alpha_p \rangle (y)\). \(y/W = 0.03\) (a), \(y/W = 0.25\) (b), and \(y/W = 0.5\) (c).

Figs. 10 and 11, respectively. The vertical momentum budgets are dominated by the drag and gravity (particles) or drag and pressure gradient (fluid). For the particle phase, the laminar and turbulent transport terms are about the same order of magnitude, but are relatively small. For the fluid phase, the transport terms are two orders of magnitude smaller than the drag/pressure-gradient terms in the vertical momentum budgets. Indeed, except in the laminar boundary layer near the wall, there is very little momentum transport in the wall-normal direction in either phase. This situation is quite different from the behavior observed in single-phase turbulent channel flows.\(^{22}\)

From Fig. 11(a), we observe that

\[
\frac{1}{V_t} (\langle u_{f,z} \rangle_f - \langle u_{p,z} \rangle_p + u_d,z) \approx \frac{\overline{\varphi}}{\langle \varphi \rangle} = \frac{\overline{\alpha_p \langle \alpha_f \rangle}}{\overline{\alpha_f \langle \alpha_p \rangle}},
\]

\(24\)

FIG. 7. RA drift-velocity profiles. \(\overline{u}_{f,z}/V_t = -1\) (black solid line), \(\overline{u}_{f,z}/V_t = 0\) (blue dashed line), and \(\overline{u}_{f,z}/V_t = 1\) (red dashed-dotted line). (a) Streamwise and (b) wall-normal components.
FIG. 8. RA volume fraction (—) compared to the model given by (3) (○). $\pi_{f,z}/V_t = -1$ is moved down by a factor of 1/2. $\pi_{f,z}/V_t = 1$ is moved up by a factor of 3/2.

which generalizes the result found in homogeneous CIT\textsuperscript{7} where $\varphi = \langle \varphi \rangle$. The left-hand side of (24) is shown in Fig. 10(a). Given the relation in (3), we can conclude that $\langle u_{f,z} \rangle_f - \langle u_{p,z} \rangle_p + u_{d,z} \approx V_t \kappa_{yy} / \kappa_{yy}$ in fully developed vertical channel flows. If the drift velocity is modeled by $u_{d,z} = C_g(y)(\langle u_{p,z} \rangle_p - \langle u_{f,z} \rangle_f)$, then the mean slip velocity between the phase can be written as

$$\langle u_{f,z} \rangle_f - \langle u_{p,z} \rangle_p = \frac{V_t \kappa_{yy}}{(1 - C_g) \kappa_{yy}}.$$ \hfill (25)

For very dilute flows, $C_g \approx 0$ and $\kappa_{yy}$ is determined by coupling with the fluid-phase turbulence generated by mean-velocity gradients. In contrast, for strongly coupled fluid-particle flows the values of $C_g$ and $\kappa_{yy}$ are mainly influenced by turbulence generated by clusters. From (25), we observe that for channel flow the profile of $\kappa_{yy}(y)$ affects the mean slip velocity, but the profile of $C_g(y)$ is also important. These ideas are revisited in Part II\textsuperscript{34} of the present study.

4. Second-order statistics

The spatially correlated, uncorrelated, and total kinetic energies are shown in Fig. 12. As in homogeneous CIT, the fluid-phase TKE exceeds the particle-phase TKE, except very near the wall where, by definition, $k_f = 0$. For the particle phase, the correlated component $k_p$ varies between 0.6 and 0.85 of the total granular energy $k_p$, which is smaller than in homogeneous CIT\textsuperscript{7} where
FIG. 10. RA particle-phase momentum balance (2) normalized by gravity for \( \tau_{f,z}/W_t = 0 \). (a) Vertical momentum balance \((i = z)\) and (b) wall-normal momentum balance \((i = y)\). \(-\frac{1}{\langle \alpha_p \rangle} \frac{d}{dy} \langle \alpha_p \rangle (P_{p,y})_p \) (black solid line), \(-\frac{1}{\langle \alpha_p \rangle} \frac{d}{dy} \langle \alpha_p \rangle (u_p''',u_p''',i)_p \) (blue dashed line), \(1 \tau_p \left[ u_{d,i} + \delta_{i,z} \left( \langle u_f,z \rangle_f - \langle u_p,z \rangle_p \right) \right] \) (red dashed-dotted line), \(-g\) (black dotted line).

\( k_p/\kappa_p \approx 0.9 \). This difference can be attributed to the fact that the channel walls reduce the turbulence integral length scale compared to CIT, such that the particle Stokes number in the channels is significantly higher than observed in CIT. Also, the TKE levels seen in Fig. 12 are significantly smaller than those seen in CIT for the same flow conditions. This difference is due to the fact that \( u_{d,z} \) which appears in the fluid-phase TKE drag-production term, is smaller than in CIT and depends on the distance from the wall (see Fig. 7). In fact, the shape of the TKE profiles is largely determined by the shape of \( u_{d,z}(y) \) (as opposed, for example, to wall-normal variations in the drag terms). The mixed statistics involving fluid-particle correlations, i.e., \( k_{f@p} = \frac{1}{2} \langle u''_f \cdot u'''_p \rangle_p \) and \( k_{fp} = \frac{1}{2} \langle u''_f \cdot u''_p \rangle_p \), are also shown in Fig. 12. These terms are responsible for redistributing kinetic energy between the phases, and become important in the presence of clusters. From a modeling perspective, it is interesting to note that \( k_{f@p}/k_f > 1 \) appears to be nearly independent of \( y \), and thus does not require solving additional transport equations. Additionally, \( k_{fp} < \kappa_p \) indicates that \( k_f \) and \( k_p \) are correlated, as opposed to \( k_f \) and \( \kappa_p \) as is often assumed in turbulence models for particle-laden flows.\(^{13}\) Models for the fluid-particle velocity correlations are presented in Part II.\(^{34}\)

The normalized components of the second-order statistics are shown in Fig. 13. It was found that the component profiles are very similar for the three cases conducted in the present study. Thus, changing the mean fluid velocity over the range used in this work does not significantly modify the balance between cluster-induced and mean-shear-induced turbulence. Because the production terms for the latter (see (7)) are proportional to the \( yz \) components of the Reynolds stresses, and

FIG. 11. RA fluid-phase momentum balance (5) normalized by gravity for \( \tau_{f,z}/W_t = 0 \). (a) Vertical momentum balance \((i = z)\) and (b) wall-normal momentum balance \((i = y)\). \(-\frac{1}{\rho_f} \frac{d}{dy} \langle \rho_f \rangle \nabla_i \langle p_f \rangle \) (red dashed-dotted line), \(-\frac{1}{\rho_f} \frac{d}{dy} \langle \rho_f \rangle (\sigma_f,g)_{ij} \) (blue dashed line), \(-\frac{1}{\rho_f} \frac{d}{dy} \langle \rho_f \rangle (\sigma_f,g)_{ij} \) (black solid line), \(-\frac{1}{\rho_f} \frac{d}{dy} \langle \rho_f \rangle (\sigma_f,g)_{ij} \) (black solid line), \(1 \tau_p \left[ u_{d,i} + \delta_{i,z} \left( \langle u_f,z \rangle_f - \langle u_p,z \rangle_p \right) \right] \) (magenta dotted line), \(-g\) (black dotted line).
these components are very small in Fig. 13, our channel flows are dominated by cluster-induced turbulence production. For this reason, the $zz$ components of the Reynolds stresses contain more than 90% of the total TKE for the fluid phase. In the particle phase, this percentage is slightly smaller than in homogeneous CIT, but still more than 80%.

Next, we can observe from the components of the particle-phase pressure tensor in Fig. 13 that significant anisotropy exists in vertical channel flows, just as was observed in homogeneous
CIT. However, the $yz$ component of $\langle \mathbf{P} \mathbf{p} \rangle_p$ differs significantly from zero across the channel. This behavior is due to the laminar mean-gradient production term $\mathcal{P}_p$ in (12). Because $\langle u''_{p,i} u''_{p,j} \rangle_p$ and $\langle P_{p,ij} \rangle_p$ are, respectively, the turbulent and laminar wall-normal fluxes of RA particle-phase vertical momentum (see (2)), we can conclude that for the particle phase the laminar mean-momentum transport is not negligible compared to turbulent transport across the entire channel. Again, this situation is very different from single-phase turbulent channel flows where the laminar part is only important very near the walls.\textsuperscript{22} Also, it is noteworthy that the components of $\langle \mathbf{P} \mathbf{p} \rangle_p$ are approximately isotropic at the wall. This observation would suggest that particle-wall collisions in the presence of clusters have a strong isotropization effect even in fairly dilute flows where particle-particle collisions are not dominant.

Finally, we note that the components of total granular energy $\langle \mathbf{v}'_p \mathbf{v}'_p \rangle$ exhibit anisotropy that is intermediate between the correlated and uncorrelated components. This, however, is to be expected from their definitions: $\langle \mathbf{v}'_p \mathbf{v}'_p \rangle = \langle u''_{p,i} u''_{p,j} \rangle_p + \langle P_{p,ij} \rangle_p$. From a modeling perspective, it is important to recall that separate transport equations are needed for $\langle u''_{p,i} u''_{p,j} \rangle_p$ and $\langle P_{p,ij} \rangle_p$, not only because they appear separately in the equations, but also because the boundary conditions are different. For example, $\langle P_{p,ij} \rangle_p$ is nearly isotropic (and nonzero) at the wall, but $\langle u''_{p,i} u''_{p,j} \rangle_p$ has its largest anisotropy at the wall. Moreover, although $k_p > 0$ at the wall due to the mean velocity slip, the no-penetration condition leads to $\langle u'^2_p \rangle_p = 0$. Thus, to implement the boundary conditions correctly, the spatially correlated and uncorrelated components must be solved for separately.

In summary, the normalized second-order statistics at the channel centerline are in reasonably good agreement with values found in fully developed, homogeneous CIT,\textsuperscript{7} and the differences are due to the fact that the channel width used in this work is too small to allow for the cluster-induced turbulence to become fully developed. Near the wall, the different boundary conditions on the

FIG. 14. Fluid-phase Reynolds-stress balance (6) normalized by $\tau_p k^2$ for $\pi_{f,zz}/V_f = 0$, $-\frac{1}{\alpha_f} \frac{d}{d\alpha_f}(\alpha_f)\mathcal{E}_{f,ij}$ (blue dashed line), $\mathcal{R}_{f,ij}$ (magenta dotted line), $-\varepsilon_{f,ij}$ (red dashed-dotted line), $\mathcal{D}\mathcal{E}_{f,ij}$ (black dashed filled circle line), $\mathcal{P}_{f,ij}$ (black dashed filled triangle line), and $\mathcal{D}\mathcal{P}_{ij}$ (black dashed filled square line). (a) $zz$, (b) $yy$, (c) $xx$, and (d) $yz$. 

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components of the second-order statistics affect the anisotropy but, because mean-gradient turbulence production is negligible, the wall-boundary-layer statistics are very different than those observed in single-phase turbulent channel flows. Presumably, this situation would change if the fluid-phase mean velocity were increased enough to make the mean-gradient turbulence production sufficiently larger.

5. Energy budgets

The individual terms in Reynolds-stress balances (6) for the fluid and particle phases are shown in Figs. 14 and 15, respectively. The fluid-phase Reynolds-stress budget differs significantly from what is observed in low-mass-loading particle-laden channel flows, where the mean-shear production, dissipation, transport, and drag terms all contribute to the overall fluid-phase TKE. In contrast, the channels considered herein have negligible contributions from mean-shear production and the turbulent flux terms. As was observed in homogeneous CIT, fluid-phase TKE is produced by particles falling under the influence of gravity, which is dissipated to heat in the fluid directly by viscous dissipation while a large fraction is transferred to the particle phase through drag exchange. In Fig. 14(a), it is seen that the vertical $zz$ component primarily involves a balance between drag production $\mathcal{D}P_{zz}$ and drag exchange $\mathcal{DE}_{f,zz}$. Very near the wall, viscous and pressure diffusion/dissipation contributions ($\epsilon_{f,zz}$ and $\mathcal{E}_{f,zz}$) are comparable to $\mathcal{DE}_{f,zz}$, but are negligible away from the wall. As shown in Fig. 14(b), the wall-normal component $yy$ mainly involves contributions from $\mathcal{R}_{f,yy}$, $\mathcal{E}_{f,yy}$, and $\mathcal{DE}_{f,yy}$ throughout the majority of the channel. At the wall, energy is mostly produced by pressure-rate-of-strain term ($\mathcal{R}_{f,yy}$) with a small contribution from viscous dissipation ($\epsilon_{f,yy}$), and dissipated due to pressure/viscous diffusion and redistribution ($\mathcal{E}_{f,yy}$). At the channel center, $\mathcal{DE}_{f,yy}$ acts as a sink of TKE that is balanced by $\mathcal{R}_{f,yy}$ and $\mathcal{E}_{f,yy}$. From Fig. 14(c), it can be seen that the $xx$ component of fluid TKE mainly involves a balance between

![Figure 15. Particle-phase Reynolds-stress balance (6) normalized by $\tau_p g^2$ for $V_i = 0$.](image-url)
Note that the terms appearing in the $yy$ and $xx$ components remain less than 4% of the $zz$ component, and thus pressure redistribution is unable to significantly reduce the anisotropy in the fluid-phase Reynolds stresses. Finally, the off-diagonal $yz$ component is mostly generated by pressure strain, and dissipated by $\mathcal{D}E_{f,zy}$ and $E_{f,yz}$ throughout the majority of the channel. It can be seen that $R_{f,yz}$ and $E_{f,yz}$ increase significantly at the wall owing to the high shear in the vicinity of the boundary layer.

Components of the particle-phase Reynolds stress balance are shown in Fig. 15. It can immediately be seen that the magnitude of the individual terms is significantly smaller than the terms appearing in the fluid-phase TKE balance. While the $zz$ component of the fluid-phase TKE is primarily generated by drag production, it can be seen in Fig. 15(a) that the particle-phase TKE is generated by contributions from drag exchange and mean-shear production. At the channel center, $\mathcal{D}E_{p,zz}$, $\mathcal{E}_{p,zz}$, and the turbulent flux term increase the fluctuating energy of the particle phase, which is dissipated by $\epsilon_{p,zz}$. Meanwhile, at the channel wall, $\mathcal{D}E_{p,zz}$ removes TKE, which is mostly produced by $\mathcal{E}_{p,zz}$. The remaining components of the particle-phase Reynolds-stress balances are approximately an order of magnitude smaller than the those for the $zz$ component, and thus are unable to significantly reduce the anisotropy in the particle-phase Reynolds stresses.

The individual terms appearing in the particle-phase pressure tensor balance (12) are shown in Fig. 16. Here, the collision term is found from a kinetic theory closure (see Part II for details) instead of computing it directly from the EL simulations. Thus, the non-zero balance in Fig. 16 is likely due to discrepancies in the modeled collisional dissipation tensor $C$ with the soft-sphere model used in the EL simulations. In other words, a better approximation of the true contributions from $C$ could be found by forcing the balances to be exactly zero for each component $\langle P_{p,ij}\rangle_p$. In any case, it can be observed that the primary role of collisions is to redistribute energy from $\langle P_{p,zz}\rangle_p$ to the other diagonal components.

![Figure 16](image-url)
As was observed with the previous energy balances, the terms appearing in the vertical $zz$ direction have the largest magnitudes. However, the anisotropy among the components of the PA particle-phase pressure tensor is not as great owing to the reorientation by particle-particle collisions. We can note that except near the walls, the particle-particle collisions are too infrequent to make the stress tensor nearly isotropic. The principal production term of the $zz$ component is $\epsilon_{p,zz}$, which acts as the primary source of dissipation of the particle-phase TKE. This is analogous to single-phase flow where dissipation of TKE leads to viscous heating, except in this case, dissipation is directly attributed to anisotropy in the particle-phase stress tensor. Laminar mean-gradient production $P_{p,zz}$ is also seen to increase the fluctuating energy in the $zz$ component, whereas dissipation is due to collisions and drag exchange. In contrast, $C$ appears as a source of production in the $yy$ and $xx$ components, acting to reorient the energy from the vertical direction and enhance the isotropy. We note that the sum of the terms in the $zz$ component is very near zero, and thus $C$ adequately measures the level of dissipation of the uncorrelated granular energy. The model, however, is less effective in the remaining components. Because $C$ depends on the uncorrelated granular energy $\Theta_p$ and not on the total particle fluctuating energy $\kappa_p$, separate transport equations for $\langle P_p \rangle_p$ and the particle-phase Reynolds stresses are needed to account correctly for collisions, as is further demonstrated in Part II.

Finally, as noted earlier for the other balances, the spatial transport terms (laminar and turbulent) are relatively small compared to the source terms in the balances for the PA particle-phase pressure tensor.

6. Fluid-particle correlations

Fluid-particle velocity correlations appear as unclosed terms in the drag-dissipation-and-exchange tensors $\mathcal{DE}_p$ and $\mathcal{DE}_f$ of each phase (refer to (10)). As shown in Figs. 14 and 15, these terms play a significant role in the overall TKE balances throughout the channel width. Note that the exact (but unclosed) transport equations for the fluid-particle velocity correlations can be derived by starting directly from the mesoscale model equations, but are not included in the present work. However, the interested reader is referred to Ref. 7 for further details. Components of the fluid-phase velocity correlations seen by the particles $\langle u_f''' \otimes u_f''' \rangle_p$ and fluid-particle cross-correlations $\langle u_f''' \otimes u_p''' \rangle_p$ are shown in Fig. 17. For both terms, the majority of the fluctuating energy resides in the $zz$ component. This is a direct result of particle clustering and the fact that clusters entrain the fluid in their vicinity due to momentum coupling. The anisotropy of the fluid-phase TKE seen by the particles is nearly the same as $k_f$ (see Fig. 13(a)), and thus can be adequately modeled by introducing correlation coefficients, as is discussed in more detail in Part II.

V. SUMMARY

In the present work, Eulerian–Lagrangian simulations of high-mass-loading vertical channel flows were studied with the purpose of exploring the fundamental physics of wall-bounded
multiphase turbulence. The exact RA equations for high-mass-loading suspensions were presented, and wall-normal distributions of the unclosed terms retained in the context of fully developed channel flow were evaluated. It was shown that the decomposition of the particle-phase fluctuating energy into spatially correlated and uncorrelated components is crucial for understanding the relative importance of the terms in the balance equations, which is needed to develop a predictive multiphase turbulence model. To this end, an adaptive spatial filter was applied to the Lagrangian data with an averaging volume that adjusts to the local particle concentration. Two-point velocity correlations computed from the filtered particle-phase velocity were compared against the corresponding velocity correlations from the Lagrangian data (i.e., the exact interparticle velocity correlation). Overall excellent agreement was observed for each case under consideration at all distances from the wall, verifying the capability of the filter to accurately partition the particle-phase velocity into its spatially correlated and uncorrelated components, and express them as instantaneous local Eulerian fields consistent with the RA formulation.

Clusters were observed to fall at the walls of the vertical channel, leading to locally reduced drag and large values of the correlated particle-phase TKE \( k_p \). Meanwhile, the PA granular temperature was observed to increase weakly away from the wall where clustering is less distinct. Results obtained using the adaptive spatial filter revealed strong anisotropy of the Reynolds stresses both near the wall and far away. It was shown that this anisotropy is a crucial component for predicting the distribution of the RA particle-phase volume fraction. It was shown that the decomposition of the particle-phase fluctuating energy into its spatially correlated and uncorrelated components is necessary to account for the boundary conditions at the wall. For example, the wall-normal component of the correlated particle velocity (i.e., both for the mean and in the Reynolds stresses) must be zero due to wall damping (as in single-phase turbulent flows). However, the spatially uncorrelated granular temperature (and the components of the particle-phase pressure tensor) need not be zero at the wall. Furthermore, the spatially correlated particle velocity can have a mean slip at the wall due to finite-Knudsen-number effects, and thus the particle-phase Reynolds stresses tangential to the wall are nonzero at the wall. A set of boundary conditions, consistent with these observations, for the particle-phase TKE \( \langle \mathbf{u}_p'' \otimes \mathbf{u}_p'' \rangle_p \) and particle-phase pressure tensor \( \langle P_p \rangle_p \) in dilute flows with inelastic collisions is derived in Part II of this paper from the well-known Johnson and Jackson boundary conditions for granular flows.

Simulation results from the present paper are used to validate a multiphase turbulence model in Part II. Owing to the strong anisotropy of the Reynolds-stress tensors, a Reynolds-stress model is a natural choice for strongly coupled fluid-particle flows. In comparison to single-phase channel flows where the turbulence is produced by the mean fluid velocity gradients, for the particle-laden flows considered in this work the nonzero shear stresses \( \langle \mathbf{u}_f'''_y \mathbf{u}_f'''_z \rangle_f \) and \( \langle \mathbf{u}_p'''_y \mathbf{u}_p'''_z \rangle_p \) are very small. Thus, the wall-normal turbulent transport of momentum is very weak, and the mean-gradient TKE production term is negligible compared to drag production. From the perspective of turbulence modeling, it is unlikely that two-equation models (e.g., \( k-\varepsilon \)) developed for single-phase turbulent channel flows and based on a turbulent viscosity closure will be successful for the more complex particle-laden flows investigated in this work.

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