4-2014

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Keywords
Strain Gradient, Taylor Factor, Intrinsic Material Length, Entire Strain Range, Conventional Plasticity Theory

Disciplines
Materials Science and Engineering | Mechanics of Materials | Metallurgy

Comments
This is a manuscript of an article published as Tian, Liang, Alan Russell, and Iver Anderson. "A dislocation-based, strain–gradient–plasticity strengthening model for deformation processed metal–metal composites." *Journal of Materials Science* 49, no. 7 (2014): 2787-2794. The final publication is available at Springer via DOI: 10.1007/s10853-013-7982-5. Posted with permission.

A dislocation-based, strain-gradient-plasticity strengthening model for deformation processed metal-metal composites

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Abstract

Deformation processed metal-metal composites (DMMCs) are high strength, high electrical conductivity composites developed by severe plastic deformation of two ductile metal phases. The extraordinarily high strength of DMMCs is underestimated by the rule of mixture (or volumetric weighted average) of conventionally work-hardened metals. In this paper, a dislocation-density-based, strain-gradient-plasticity model is proposed to relate the strain gradient effect with the geometrically necessary dislocations emanating from the interface to better predict the strength of DMMCs. The model prediction was compared with the experimental findings of Cu-Nb, Cu-Ta, Al-Ti DMMC systems to verify the applicability of the new model. The results show that this model predicts the strength of DMMCs better than the rule-of-mixture model. The strain gradient effect, responsible for the exceptionally high strength of heavily cold worked DMMCs, is dominant at large deformation strain since its characteristic microstructure length is comparable with the intrinsic material length.

Keywords: Composites; Interfaces; Strain gradients; Dislocation density; Hardening

1. Introduction

Deformation processed metal-metal composites (DMMCs) are composites with an excellent combination of high strength and high conductivity. This combination of characteristics makes them candidates for numerous engineering applications, including high-voltage power transmission conductors, high-field pulsed magnets, fly-by-wire flight control system, electric motor armature windings, etc. [1]. The production of DMMCs begins with powder metallurgy or solidification of a liquid solution of two metal elements that are mutually insoluble in the solid state [1]. The mixture of two ductile metal phases is then subjected to plastic deformation processing (i.e., extruding/swaging/wire drawing or rolling) to deform the minor phase into elongated nano-scale (cross-section) filaments or lamellae with nanometer scale spacings [1]. The strength of DMMCs increases greatly with the deformation true strain [2-8], deviating substantially from the conventional rule of mixture model [2]. Bevk et al. [2] observed the exceptionally high strength of Cu-Nb with relatively small amounts of filamentary Nb (with moderate strength) and confirmed that the strength and volume fraction of the filament phase was insignificant to achieve the high strength of DMMCs. However, they also found that the strength of Cu-Nb composites could be predicted well by the rule-of-mixtures model at small deformation true strains (~5 or less). Various strengthening mechanisms or models have been proposed to explain the extraordinarily high strength of DMMCs subjected to a large deformation true strain [9-13]. Bevk et al. [2], Spitzig et al. [3] and other investigators [4-8] have confirmed that the high strength of DMMCs was underestimated by a simple rule-of-mixtures (ROM) model for several different DMMC compositions. Spitzig et al. [9] suggested a Hall-Petch barrier model, which attributed the strengthening effect to the role of the interface as a barrier to dislocation motion. This model described the dependence of the strength of a composite on the filamentary spacing by a Hall-Petch relationship. Funkenbusch et al. [10] proposed a work hardening model, which explained the incremental strength above the ROM prediction as a consequence of geometrically necessary dislocations emitted from the interface to accommodate the strain incompatibility across the interphase boundary. However, these two mechanisms are not completely exclusive. One mechanism will dominate over the other depending on the deformation processing and crystal structure of the two metal phases. Interphase boundaries appear to be the major strengthening mechanism at large deformation strain (above 10) because they act as sinks for dislocations and inhibit the development of Frank-Read dislocation sources due to the fine interphase spacing [11]. The geometrically necessary dislocations (GNDs) are mainly responsible for the incremental strength when two metal phases with different crystal structure are deformed to a moderate strain so that strain accommodation by GNDs between two phases is necessary. These two models are not perfect. The Hall-Petch barrier model failed to explain the high strength of cold-worked two phase materials relative to single phase material with similar filament (i.e. grain) spacing [10]. The work hardening model was assessed only as a semi-quantitative model because it contains many adjustable parameters, making it applicable to some specific DMMC systems [12]. Raabe et al. [13] brought up a modified linear rule of mixture model to include a Hall-Petch contribution from phase boundaries for simulating the strength of fcc/bcc type DMMCs. However, this model requires mathematical assumptions to explain the origin of the Hall-Petch contribution from the interface, which casts some doubt over the universality of the model. Therefore, opportunities exist to further refine models addressing the physical origin of the anomalously high strength of DMMCs.

Strain-gradient-plasticity theories interpret size-dependent deformation behavior of metals at the micrometer scale [14, 15]. Previous experiments on micro-torsion [15], micro-bending [16] and micro-indentation [17] confirmed the size effect present in the deformation response of metals. For example, Fleck et al. [15] observed that the shear strength of twisted thin copper wires increased by a factor of three as the wire diameter decreases from 170 to 12 µm. A similar bending hardening effect was observed by Stolken and Evans [16] as the Ni foil thickness decreased from 50 to 12.5 µm. De

Guzmana et al. [17] conducted micro-indentation tests on Ni and Cu samples and showed that the measured hardness increased with decreasing penetration depth from 2000 to 200nm. Lloyd et al. observed a substantial strength increase for SiC-particle-reinforced composite when the particle diameter decreased from 16μm to 7.5μm with a fixed particle volume fraction of 15% [18]. The conventional plasticity theories cannot explain this size-dependent material behavior at the micro-scale, because no internal material length scale was implemented in their constitutive models [19, 20, 21]. In strain gradient plasticity theories, a material length scale was introduced to compare with the characteristic microstructure dimension to determine the strain-gradient effect [14, 19, 20]. The strain gradient effect can also be used to interpret the grain-size-dependent flow stress of polycrystalline materials—the Hall-Petch relation [19]. How the plasticity behavior of metals is affected by the strain gradient is determined by dislocation motion, which has long been known to be the most important mechanism of plastic deformation [14, 19, 20]. Strain gradients are accommodated by the geometrically necessary dislocations (GNDs) to distort the crystalline lattice structure [22, 23]. An increased density of GNDs will work harden crystals like that of statistically stored dislocations (SSDs) by Taylor's hardening law [24].

Various strain gradient plasticity continuum models have been put forward to describe the collective behavior of material defects and to predict the size-dependent mechanical behavior of materials due to the difficulty to perform atomistic simulation and discrete dislocation dynamics calculations [19, 20]. Fleck and Hutchinson developed a higher order couple stress theory to relate strain gradients with the effective measure of a curvature tensor [15] and later with both rotation and stretch gradients [25]. A higher order stress was defined as the work conjugate of strain gradient to satisfy the Clausius-Duhem thermodynamic inequality. It requires additional boundary conditions for finite element implementation. The phenomenological nature of Fleck-Hutchison theories requires many fitting parameters, which are difficult to test fully by limited strain-gradient dominated experiments. A mechanism-based strain gradient theory proposed by Gao et al. distinguished the micro-scale at which dislocation interaction follows the Taylor's hardening law from the meso-scale where high-order strain-gradient plasticity is established. An effective strain gradient measure was derived from three deformation modes to be linked with geometrically necessary dislocations [20]. This model used Taylor's hardening law as the foundation so that it can predict the linear dependence of the square of plastic flow stress on strain gradient that was observed experimentally by Nix and Gao [26]. It has fewer adjustable parameters than Fleck-Hutchison theory, and it interpreted the experiments of size-dependent plasticity well. Gao et al. [27] presented another way to link Taylor's hardening law with continuum theories without resorting to the high-order stresses at the meso-scale cell in mechanism-based strain-gradient theory. The essential idea is to calculate the density of GNDs as nonlocal variables expressed in terms of plastic strain by representing strain gradients as a non-local integral of strains. The constitutive equations of this theory resemble those of conventional plasticity theory and are much simpler than high-order theory of Fleck-Hutchinson and mechanism-based strain-gradient theory.

In this paper we present a dislocation-density-based strain-gradient model for the strengthening effect of deformation processed metal-metal composites (DMMCs). The motivation to incorporate the strain gradient effect into the deformation behavior of DMMCs comes from the fact that the sub-micron size (as low as 20nm [2]) filamentary microstructure in DMMCs is obviously comparable with a common intrinsic material length scale from a fraction of a micron to tens of microns. In addition, the interfaces between fiber and matrix are provided as a realistic source to generate the geometrically necessary dislocations to accommodate a strain gradient. Instead of a continuum formulation, an effective strain gradient is proposed to be directly proportional with the deformation true strain during severe plastic deformation of DMMCs and inversely proportional to the characteristic microstructure length [15, 19, 20]. This assumption should be plausible for DMMCs due to the uniaxial deformation mode during deformation processing and taken by Gao et al. [20] as an argument to discuss the strain gradient effect. A material-dependent parameter is introduced in this assumption to average the effect of different orientations of the slip systems, which is hard to implement in continuum models [14]. An analytic expression of the yield strength of DMMCs was derived in the following section to compare with the experimental results to test the effectiveness of this model.

2. Model formulation

In the section, a physically based strain-gradient plasticity strengthening model for DMMCs is presented based on a modified rule of mixtures. The yield strength of each metal phase in DMMCs obeys Taylor's dislocation hardening law with an additional GNDs density term from the interface. An explicit relation between SSDs (or GNDs) and deformation true strain is established to show the predominant strain gradient effect at large deformation true strain and hence predict the extraordinarily high strength of heavily deformed DMMCs. The internal material length scale for each phase can be totally derived from physical parameters of the material as in the mechanism-based strain-gradient theory [20]. This model eliminates the mathematical complexity of previously developed continuum strain gradient models due to the uniaxial deformation mode of DMMCs, but captures all the essential features of strain-gradient plasticity theory to make it physically plausible.

Plastic deformation in monolithic metallic materials comes from the collective motion of numerous dislocations. A dislocation density concept was found to be useful as the average treatment of dislocation processes to be linked with the macroscopic stress and strain. The plastic hardening originates from the interaction of mobile dislocations with immobile dislocations or other crystal microstructures (e.g. precipitates, solute atoms, grain boundaries) that act as obstacles to dislocation motion. A critical shear stress, known as shear strength, is required to untangle the interaction between dislocations and obstacles and hence to initiate plastic deformation. Taylor's hardening law relates the shear strength to the dislocation density by

$$\tau = \tau_0 + \alpha G b \sqrt{\rho_t} \text{¿} \tau_0 + \alpha G b \sqrt{\rho_S + \rho_G} \tag{1}$$

Where τ_0 is the extrapolated shear strength at zero dislocation density, α is a material constant ranging from 0.1 to 0.5 [23], G is the shear modulus, b is the Burgers vector length, ρ_t is the total dislocation density, ρ_S is the density of statistically stored dislocations, ρ_G is the density of geometrically necessary dislocations. By taking a direct sum of SSDs and GNDs into the strain hardening relation, we make no distinction between SSDs and GNDs in the effectiveness of blocking the motion of mobile dislocations [27]. For crystalline materials, the tensile yield strength is related to the shear strength by a Taylor factor m as

$$\sigma = \sigma_0 + m \alpha G b \sqrt{\rho_S + \rho_G} \tag{2}$$

where $\sigma_0 = m \tau_0$ is the extrapolated yield strength at zero dislocation density. The Taylor factor m reflects the degree of crystalline anisotropy at the continuum level [19]. For a perfectly isotropic solid, $m = \sqrt{3}$. For face-centered cubic (fcc) polycrystalline metals, $m = 3.08$ [19].

GNDs are used to accommodate the strain gradient and the curvature of a crystal lattice [22]. The effective density of GNDs is given as [20]

$$\rho_G = \frac{\eta}{b} \tag{3}$$

where η is the effective strain gradient. Without the strain-gradient effect, the strain hardening relation (2) can be rewritten as

$$\sigma = \sigma_0 + m\alpha Gb\sqrt{\rho_S} = \sigma_0 + (\sigma_Y - \sigma_0) f(\varepsilon) \tag{4}$$

Where $f(\varepsilon)$ is a function satisfying $f(0)=1$ that can be obtained from uniaxial tension testing, σ_Y is the yield strength without any strain hardening ($\varepsilon = 0$). For ductile metals, the yield strain ε_Y is taken as 0.2% so that the elastic strain is negligible during severe plastic deformation processing of metal phases. A common power hardening law is chosen as $f(\varepsilon) = 1 + \varepsilon^n$, where n is the work hardening exponent for the metal [21]. The density of SSDs can be derived from eq. (4) as

$$\frac{(\sigma_Y - \sigma_0) f(\varepsilon)}{m\alpha Gb}$$
¿
¿
$\rho_S =$ ¿ (5)

With ρ_S and ρ_G given by eq. (5) and (3), respectively, we can rewrite eq. (2) as

$$\sigma = \sigma_0 + (\sigma_Y - \sigma_0)\sqrt{f(\varepsilon)^2 + l\eta} \tag{6}$$

where $l = m^2 \alpha^2 \left(\frac{G}{\sigma_Y - \sigma_0}\right)^2 b$ is identified as the intrinsic material length scale defined by Gao et al. [20]. The intrinsic material length totally hinges on the structure and properties of the monolithic metal phase.

The strength of DMMCs follows a modified rule of mixture relation [13] as

$$\sigma_c = \sigma_A V_A + \sigma_B V_B \tag{7}$$

where σ_c is the yield strength of metal-metal composites, V_A and V_B are the volume fraction of metal phase A and B, respectively. Equation (7) is based on the assumption that the yield strain is roughly the same for two ductile metal phases A and B. σ_A, σ_B are the yield strength of metal phase A and B considering the work hardening effect from both SSDs and GNDs, given by eq.(2) and (6) as

$$\sigma_A = \sigma_{A0} + m_A \alpha_A G_A b_A \sqrt{\rho_{SA} + \rho_{GA}} ¿ \sigma_{A0} + (\sigma_{AY} - \sigma_{A0}) \sqrt{f(\varepsilon)^2 + l_A \eta_A}$$

$$\sigma_B = \sigma_{B0} + m_B \alpha_B G_B b_B \sqrt{\rho_{SB} + \rho_{GB}} ¿ \sigma_{B0} + (\sigma_{BY} - \sigma_{B0}) \sqrt{f(\varepsilon)^2 + l_B \eta_B} \tag{8}$$

where ρ_S and ρ_G for metal phases A and B are given by eq (5) and (3), respectively. The effective strain gradient η can be taken to be proportional to the deformation true strain ε and inversely proportional to the characteristic length of the microstructure [20]. When the characteristic microstructure length is much larger than the intrinsic material length, the strain gradient hardening effect by GNDs become negligible in comparison with the strain hardening by SSDs, and the strain gradient plasticity model eq. (6) will degenerate to the conventional work-hardening model eq. (4). In

DMMCs, the cross-section thicknesses of fiber and matrix are chosen as the characteristic length (see Fig. 1.). So we get $\eta_A = k_A \frac{\varepsilon}{t_A}$, $\eta_B = k_B \frac{\varepsilon}{t_B}$, where k_A, k_B are the material-dependent parameters introduced for metal phases A and B, respectively. The k parameter is an arbitrary fitting parameter used non-mechanistically in this model. The thickness of metal phase A and B should satisfy

$\frac{t_A}{t_B} = \frac{V_A}{V_B}$ from previous work on Al/Ti DMMCs [6]. The deformation true strain ε is calculated as

$\varepsilon = 2 \ln \frac{t_0}{t_B}$, where t_0 is the initial thickness of fiber phase B [1]. So we have $t_B = t_0 \exp\left(\frac{-\varepsilon}{2}\right)$.

From the above relations, the effective strain gradient can be directly related to the deformation true strain as

$$\eta_A = k_A \frac{V_B \varepsilon \exp\left(\frac{\varepsilon}{2}\right)}{V_A t_0} \quad , \quad \eta_B = k_B \frac{\varepsilon \exp\left(\frac{\varepsilon}{2}\right)}{t_0} \qquad (9)$$

Fig. 1 A schematic of metal-metal composite with A being the matrix metal phase, B being the fiber metal phase

Combining eqs. (7), (8) and (9), the yield strength of DMMCs will be given by

$$\sigma_c=\sigma_{A0}V_A+\sigma_{B0}V_B+V_A(\sigma_{AY}-\sigma_{A0})\sqrt{f(\varepsilon)^2+l_A k_A\frac{V_B\varepsilon\exp\left(\frac{\varepsilon}{2}\right)}{V_A t_0}}+V_B(\sigma_{BY}-\sigma_{B0})\sqrt{f(\varepsilon)^2+l_B k_B\frac{\varepsilon\exp\left(\frac{\varepsilon}{2}\right)}{t_0}} \tag{10}$$

Eq. (10) gives the yield strength of DMMCs, σ_c, as a function of the deformation true strain ε and the volume fraction, the initial dimension, the physical properties and the strain hardening behavior of two metal phases. It clearly explains why strain gradient effects are predominating at large deformation true strain and heavily cold-worked DMMCs have extraordinarily high strengths. Both the strain gradient effects and the strength of DMMCs increase exponentially with deformation true strain. The volume fraction and strength of each phase have less impact on strength of DMMCs than deformation true strain, especially at large deformation true strain, which was observed experimentally by Bevk et al. [2]. This strengthening model for DMMCs captures the essential feature of strain gradient effects by establishing a link between the density of GNDs and the effective strain gradient while maintaining a simple mathematical structure due to the uniaxial deformation processing model of DMMCs. This model can be easily modified to more accurately predict the strength of DMMCs by representing the intrinsic material length as a function of strain, strain rate, grain size and temperature, etc. [19]. The purpose of this paper is to show a physically plausible method to incorporate the strain gradient effects into the strengthening behavior of DMMCs, which has not been done in previous studies [9, 12, 13].

3. Model predictions versus experimental results

In this section, the model prediction of the strength of Cu/Nb, Cu/Ta and Al/Ti DMMCs will be compared with the corresponding experimental results. The upper bound of rule of mixtures prediction is also given to confirm the strain gradient effects that are responsible for the extraordinarily high strength at large deformation true strain. The physically plausible link between the effective strain gradient and the density of GNDs makes the model predict the experimental results quite well.

3.1. Cu/Nb and Cu/Ta DMMCs

The strength of Cu/Nb DMMCs has been investigated extensively in the past [2, 3, 28, 29]. The two phase Cu/Nb composites were produced by rapid cooling of the homogeneous liquid of two elements into two-phase solid eutectics to minimize the crucible contamination and to ensure clean interphase boundaries [2]. The Cu/20 vol% Nb composites with two different initial Nb dendrite sizes due to different melting procedures in ref. [3] are taken as the experimental results for us to compare with the strain-gradient-based model. The input parameters for the model are set up as follows: the power hardening law for the strength of Cu is given by 103+(210-103)(1+ $\varepsilon^{0.54}$) MPa [30]. For Nb, it is 240+(300-240)(1+ $\varepsilon^{0.29}$) MPa [31]. The heavily deformed fcc-Cu and bcc-Nb exhibited crystallographic <111> and <110> texture parallel to the wire axis, respectively. The Taylor factors for heavily deformed fcc-phase and bcc-phase are taken as 3.16 and 2.15, respectively from ref. [13]. The constant α is estimated as 0.3. The shear modulus for Cu is 46 GPa and for Nb, 37.5 GPa [32]. The Burgers vectors for Cu and Nb can be estimated by the interatomic spacing, which is twice the atomic radius in the close-packed slip systems. So we get $b_{Cu}=0.256$ nm and $b_{Nb}=0.292$ nm. Therefore, the intrinsic material lengths for Cu and Nb can be calculated to be l_{Cu} = 42.52 μm and l_{Nb} = 47.45 μm , which are on the order of microns. The two initial Nb dendritic sizes are $t_0=6.2\,um$ and $t_0=3.8\,um$ in ref.

[3]. The prediction of the strain-gradient strengthening model is compared with the experimental results and the rule of mixtures prediction in Fig. 2(a). The strain-gradient strengthening model matches well with the experimental data over the entire strain range, while rule of mixtures predicts the strength accurately only at low deformation strain. The fitting material parameters k for Cu is 0.01 and for Nb is 0.201 for $t_0 = 6.2\,um$. For $t_0 = 3.8\,um$, k_{Cu} and k_{Nb} are 0.01 and 0.213, respectively. The above two values of fitted material parameter k for either Cu or Nb are quite close, but k_{Cu} is very different from k_{Nb}, which indicates that k may be a material-related parameter.

Fig. 2 (a) Comparison of the tensile strength of Cu-20 vol% Nb DMMC between experimental results [3] and our strain gradient hardening model. Two different initial Nb dendritic sizes were analyzed to show the size effect. (b) Comparison of the tensile strength of Cu-20 vol% Ta DMMC between experimental results [3] and our strain gradient hardening model. Two different initial Ta dendritic sizes were analyzed to show the size effect. An upper bound of rule of mixtures prediction is given for comparison purposes

Similar comparison between our model and experimental results is made for Cu/20 vol % Ta DMMC. The Cu/20 vol% Ta with two different initial Ta dendritic sizes (7.1µm and 3.5µm) were used to produce DMMCs for the strength test [3]. The parameters for Ta are as follows: the power hardening law for Ta is given by 275+(450-275)(1+ $\varepsilon^{0.21}$) MPa [33]. The Taylor factor for heavily deformed bcc Ta is estimated to be 2.15 [3]. The constant α is 0.3. The shear modulus for Ta is 69 GPa [32]. The Burgers vector of Ta is estimated by 0.292nm. Therefore, the intrinsic material length of Ta can be calculated to be 18.89μm. The comparison between the prediction of the strain gradient strengthening model and the experimental results is given in Fig 2(b). The fitted material parameters for Cu and Ta are $k_{Cu}=0.01$, $k_{Ta}=0.1465$ for $t_0=7.1um$. For $t_0=3.5um$, $k_{Cu}=0.01$ and $k_{Ta}=0.1926$. The two k values for Ta are close, which suggests that k is material dependent.

The rule of mixtures is obviously unable to predict the high strength at large deformation true strain (greater than 5) because it neglects the contribution of GNDs as an immobile dislocation forest blocking dislocation motion. The number of GNDs increased substantially to accommodate the strain gradient due to non-uniform deformation of two phases at the interface, work-hardening the composite to diverge from the rule of mixtures prediction. At low deformation true strain, the strain gradient is negligible mainly due to the relatively large characteristic microstructure length, so that the strain gradient hardening effect is weak, and our model coincides with the rule of mixtures.

The initial dendritic size effect on the strength of DMMCs is explained by different physical mechanisms in our strain gradient hardening model and Hall-Petch barrier model, although they both give an inverse-square-root relation. The Hall-Petch barrier model attributed this size effect to the increased interface area that acts as a barrier to dislocation motion for fine filaments. In contrast, our strain gradient hardening model proposed that at the same deformation true strain, the initial finer dendritic size will lead to finer filament thickness, which will increase the strain gradient. The increased strain gradient has to be accommodated by an increased density of GNDs that work-harden the composite. The increased dislocation density is experimentally observed by Trybus et al. [34]. Therefore, the strain gradient hardening model should be more physically plausible than the Hall-Petch barrier model that considers only the hardening from the impeded dislocation motion.

3.2 Al/Ti DMMC

Several Al matrix DMMCs have been investigated to produce lighter and stronger composite materials [6-8, 35]. Al/20 vol% Ti DMMC, produced by powder metallurgy and deformation processing, is taken as our test material to compare the strain gradient hardening model with experimental results. The initial Ti powder size was roughly around 60 µm, close to that of Al powders [6]. The high ductility of two metal phases allows severe plastic deformation to proceed without any intermediate annealing. A Ti filament thickness of 50 nm can be obtained after a deformation true strain of 12.1. The strain gradient effect is intensive for such fine characteristic microstructure length. The parameters for the strain gradient hardening model are set up as follows: the power hardening law for Al is

$\sigma_{Al}=6.81+(45-6.81)(1+\varepsilon^{0.297})$ fitted by the experimental data in ref [36]. For Ti, the power law is

$\sigma_{Ti}=87+(215-87)(1+\varepsilon^{0.13})$ [37]. The Taylor factors for heavily deformed fcc-Al and hcp-Ti are estimated to be 3.16 and 2.16, respectively because Ti exhibited a curled filament morphology similar to that of Nb. The shear moduli for Al and Ti are 25 GPa and 43 GPa, respectively. Constant α is still taken as 0.3. The Burgers vectors for Al and Ti are 0.286 nm and 0.294 nm. The intrinsic material length can be calculated as $l_{Al}=110.14\,\mu m$ and $l_{Ti}=13.93\,\mu m$. The comparison between our strain gradient hardening model and the experimental results is given in Fig. 3. The fitted k values for Al and Ti are 0.053 and 0.04. The strain gradient hardening model shows a good fit with experimental results during the entire strain range while rule of mixtures can predict the strength of Al-Ti composite only at low true strain levels.

Fig. 3 The comparison of the strength of Al-20 vol % Ti DMMCs between our strain gradient hardening model prediction and the experimental results. The upper bound of rule of mixtures is given to illustrate the intense strain gradient effect at large deformation true strain

For these three test cases, the strain-gradient hardening model predicts the strength of DMMCs regardless of the crystallographic structures of the two metal phases and the production methods. The microstructure of DMMC has a crucial effect on its strength. A fine microstructure requires a large strain gradient to accommodate at the interface, generating a large amount of GNDs to strengthen the composite. The model can be easily extended to include other effects like grain size, strain rate, temperature by modifying the corresponding material length scale.

4. Conclusions

In this paper, a dislocation-density-based strain gradient hardening model is presented to predict the strength of DMMCs. This model is based on the modified rule of mixtures in which the strength of each phase obeys the Taylor's dislocation hardening law with additional contribution from GNDs emitted from the interface to accommodate the strain gradient. A physical link between the density of GNDs and the effective strain gradient has been established to incorporate the strain gradient hardening effect. The strain gradient effect can be strongly affected by the deformation true strain, the type of composite constituent phases , and the characteristic microstructure length scale. An intrinsic material length scale was introduced to compare with the characteristic microstructure length scale to evaluate the extent of the strain gradient effect. Large deformation true strain leads to a small characteristic microstructure length, causing a strong strain-gradient effect when it is comparable with relatively large intrinsic material length. Our strain-gradient hardening model can predict the experimental strengths of Cu/Nb, Cu/Ta and Al/Ti DMMCs well over the entire deformation strain range. The dominating strain gradient effect at large deformation true strain is responsible for the anomalous high strength of heavily cold-worked DMMCs, which deviates from the rule of mixtures prediction. The rule of mixtures can fit with experimental results only at low strains. The strain gradient model can also account for the initial dendritic size effect on the strength of DMMCs. This hardening model is more physically plausible than the modified rule of mixtures model by Raabe et al. [13] by providing the GNDs as the physical origin of strain gradient. It requires fewer fitting parameters than the work-hardening model proposed by Funkenbusch et al. [10]. The experimentally observed increased dislocation density [34] also makes our model superior to the Hall-Petch barrier model, which assumes that the interfaces only block dislocation motion and have no effect on increasing dislocation density. In addition, the strain gradient hardening model has the flexibility to incorporate various physical conditions (e.g. temperature, grain size, strain rate, etc.) to be easily extended to match various experimental conditions, an advantage compared with previous models.

Acknowledgement

The authors appreciate the financial support of the Iowa State University Research Foundation, the Electric Power Research Center of Iowa State University, and the Department of Energy through Ames Laboratory contract no. DE-AC02-07CH11358.

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