1985

Transient thermal stress intensity factors of near surface cracks

Wei-Chung Wang
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TRANSIENT THERMAL STRESS INTENSITY FACTORS OF NEAR SURFACE CRACKS

Iowa State University

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Transient thermal stress intensity factors
of near surface cracks

by

Wei-Chung Wang

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Iowa State University
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1985
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I. INTRODUCTION

An important consideration in the design and analysis of engineering components is the possible existence and growth of cracks or crack-like defects. Crack growth caused by cycling loads and brittle or ductile fracture originating from cracks or crack-like defects in structural materials are costly at best and could be catastrophic. Cracks or crack starters are present to some degree in all structures. They may exist as basic defects in the constituent materials or result from welding, load cycling, stress-corrosion cracking and other effects induced in construction or during service life. The urgent need for methods which can quantify the effects of cracks on material performance has led to the evolution and development of the theory of fracture mechanics.

Fracture mechanics is generally used to assure the integrity of a whole structure or individual components. It helps the engineer to quantitatively establish allowable stress levels and inspection requirements for design against fracture. In addition, fracture mechanics is used to analyze the rate at which small cracks grow to critical size. In all these respects, the most significant historical advance in quantifying fracture has been the introduction of the stress intensity factor (SIF) as a single parameter for categorising the onset of crack propagation. The relationship between this quantity and a material's toughness made the SIF a practically useful fracture parameter. Today, its significance dominates efforts for effective
predictions of failure and structural life. However, the greatly improved fracture resistance of modern materials require a closer look at fracture and fracture parameters. Elastic-plastic fracture and stable crack growth phenomena put strain on the traditional linear elastic approaches to fracture. Nonetheless, SIF, in some form or another is likely to dominate design approaches for some time to come and will continue to be widely used in dimensioning, failure analysis, material selection, structural-life prediction, and acceptance tests of structures of various geometries. Much is known about fracture mechanics problems in the isothermal state (both static and dynamic loading). However, even a brief literature survey will reveal that comparatively little work has been done on fracture mechanics for thermal loadings, particularly transient thermal loads. The research reported in this dissertation is concerned with adding to our knowledge of failure and fracture in the field of transient thermal loads.

It is known that when the flow of heat is disturbed by the presence of crack-like imperfections, local intensification of temperature occurs. Large thermal stresses arise in the neighborhood of the crack tips and may, in critical cases, cause crack propagation and/or fracture. The degree of the disturbance of the thermal field becomes even more pronounced when the imperfections are near a boundary that experiences rapid temperature changes. This particular aspect of transient field is the subject of this dissertation.
A practical example of a severe thermal transient is the behavior of a nuclear pressure vessel during a major loss-of-coolant accident (LOCA). For example, when the primary cooling pipe breaks, the heat generated by the core will be carried off by the emergency core coolant injected into the normal coolant path within the vessel. Thus, the vessel wall initially operating at a high temperature, is subjected to a severe thermal shock. If at the time of LOCA there is a defect on or below the inner surface of the vessel, the crack may propagate as a result of the thermal shock. Less dramatic, but also more common, is crack growth under thermal fatigue. This phenomenon exists in almost all engineering components and has received media coverage as a result of wide spread thermal fatigue cracks at nuclear reactor nozzles.

Analytical solutions are generally obtainable through lengthy mathematical manipulations for steady state crack problems. The complexity of the thermal stress field, the steep gradients in temperature and stress and the time dependent characteristics pose great difficulties in obtaining closed-form solution to crack problems under transient thermal loading. Hence, numerical or experimental methods are essential for the solution of this type of problems.

The problem to be studied in this research is to determine the influence on the stress field of near surface line cracks when the surface experiences a rapid temperature change. The problem is formulated as a two dimensional transient problem in a multiply-connected semi-infinite domain and it is assumed that the quasi-static
formulation of thermoelasticity is valid. Then, the coupling terms in the heat conduction equations and the inertia terms in the equations of motion can be neglected. The boundary condition applied in the research was a sudden temperature increase of the cracked surface through heating as opposed to the refrigeration technique used in most of the previous photothermoelastic work.

A new method for whole-field stress analysis based on a symbiosis of two techniques --- classical photoelasticity and modern digital image analysis is effectively employed for acquiring the necessary information from experimentally obtained photographs. The method is essentially an extension of the idea of "half-fringe photoelasticity" (HFP), but in a more general sense.

The stress intensity factors were obtained through a multiparameter-multipoint technique based on the least squares solution of an overdetermined system of nonlinear algebraic equations. Fringe patterns reconstructed through the use of the experimentally determined fracture parameters into the elastic field equations were compared to the experimentally obtained ones to verify the quality of the initial data collection zones near the crack tips.

The variation of the stress intensity factors with time for cracks at five different angular orientations is presented.
II. LITERATURE SURVEY

In 1835, the original energy equation for an isotropic elastic solid was derived by Duhamel. Later, his and Neumann's simultaneous work on the modification of the equations of isothermal elasticity established the analytical foundations of thermoelastic theory. Since then, thermoelastic theory has been well developed, as the books of Melan and Parkus [1], Boley and Weiner [2], Kovalenko [3] and Nowacki [4], show. In particular, thermoelastic problems for both simply and multiply connected bodies with a steady temperature field can be reduced by the use of Muskhelishvili's complex variable method [5] to the corresponding problems of the isotropic theory of elasticity. Bogdanoff [6] also emphasized the utility of complex potentials in solving thermal stress problems.

Since the classical work of Griffith [7], fracture mechanics problems have been of great interest to many researchers. Griffith proposed the first criterion for the initiation of crack propagation based on energy considerations. Irwin [8] in 1957 formulated another criterion based on force considerations and demonstrated the equivalence between the two criteria. Irwin's criterion requires only the stress field in the immediate vicinity of the crack tip to be considered. He showed that the necessary condition for crack growth is that the stress intensity factor (SIF) exceeds some critical value which could be obtained through experiments.
Numerous papers discussed the stress singularities at the crack tips of a Griffith crack\(^1\) for an elastic medium in an isothermal state. The work was summarized in [9, 10, 11].

The applicability of the Griffith-Irwin concept to cracks under thermal loading remained unexplored until the work done by Florence and Goodier [12, 13]. They employed Muskhelishvili's method to solve the linear thermoelastic problem for a uniform heat flow disturbed by various insulated discontinuities (spherical cavity, circular or ovaloid hole, etc.). A crack was studied as the limiting case of an elliptical hole.

Sih [14] considered the singularities of two-dimensional thermal stresses at the tips of a crack in an infinite medium. Employing the complex variable method, he showed that the \(1/\sqrt{F}\) stress singularity is preserved in the thermal stress problems as in the problems with mechanical stresses. Consequently, SIFs depend only upon the crack configuration and the manner in which the temperature (or its gradient) is prescribed on the boundary. Bapu Rao [15] presented an alternative formulation, making use of elliptical coordinates, and reached the same conclusions as Sih.

\(^{1}\) A Griffith crack is a long flat ribbon-shaped cavity in a solid. Two dimensionally speaking, it is represented by a segment of a straight line. It is stressed in such a way that there is no variation in the stress pattern as we pass in a direction parallel to the plane of the crack.
Analytical results of the thermal stress singularities at the tips of an arbitrarily inclined crack in a semi-infinite medium with the thermally insulated edge surface under uniform heat flow was discussed by Sekine [16]. Similar conclusions to Sih's analysis were given. Also, the effects of the ligament size and the angle of inclination of the crack on the SIFs were presented. Atsumi et al. [17] considered a thermal fracture of a circular cylinder with a circumferential edge crack under uniform heat flow.

Sumi and Katayama [18] effectively employed the analytical continuation and the modified mapping collocation methods to calculate the SIFs in a finite rectangular plate with a Griffith crack under a steady state temperature field.

Three dimensional problems for a crack under thermal loading were also solved. These include the expressions for the SIFs in an infinite body with an insulated penny-shaped crack obtained by Goodier and Florence [19]. The linear thermoelastic fracture analysis of an infinite solid having a circumferential edge crack in a spherical cavity subjected to a uniform axial flow was solved by Nakazono et al. [20].

The references cited above [12-20] present some of the analytical solutions to the stress field near a crack tip due to steady state thermal loading. However, the rapid progress of the modern technology calls for the solution of transient thermal problems of fracture mechanics.
The first effort to investigate the effects of a transient temperature distribution on the near crack tip stress field was by Emery [21] in 1966. An approximate superposition method was used to find the SIFs for thermal stresses in thick hollow cylinders. Later in 1969, Emery et al. [22] employed the Green's function method to compute SIFs for edge cracks in a rectangular plate subjected to sudden cooling along one edge of the plate. In both studies, the temperature field was considered to be a function only of the distance from the thermally loaded edge. Shah and Kobayashi [23] used the same scheme and obtained the transient SIF of an elliptical crack embedded in a thick plate, one side of which is subjected to a sudden temperature change. Recently, Nied [24] investigated the transient thermal stress problem for an elastic strip which is insulated on one face and cooled by surface convection on the face containing an edge crack. Numerically obtained SIF was presented as a function of Fourier number and crack length for various values of Biot's number. In his solution, no restriction is placed on the crack length to plate thickness ratio as adopted by Emery et al.

In 1973, Lauriello and Chen [25] applied both the original and modified Griffith theories to study the brittle fracture in rock subjected to transient surface heating over a circular area by a constant flux or constant temperature convective heat source.

Estimating SIFs or propagation and onset of cracks for a nuclear pressure vessel under thermal shock condition has attracted the
attentions of several researchers. Blauel et al. [25] used glass plates and hollow cylinders to investigate the behavior of surface cracks under transient thermal loads. The experimental results were compared to theoretical and numerical calculations based on a linear elastic analysis.

Marston et al. [27] stated that the static crack arrest procedures specified in the ASME Boiler and Pressure Vessel Code should be adequate for assessing the propagation of cracks whose initial depths are less than 20 percent of the vessel's thickness. Cheverton et al. [28-30] numerically and experimentally studied the same problem by using thick-walled steel cylinders. Both finite element method (FEM) and finite difference method (FDM) were used in the analysis of fracture and crack arrest. The FEM and the FDM were used in the static analysis while only the FDM was used in the dynamic analysis. They found that for shallow cracks FEM and FDM results agreed with each other and with the experiments, but for deep cracks the FEM results were closer to the measured values. FEM results were also in good agreement with the measured value in dynamic analysis. Therefore, they concluded that linear elastic fracture mechanics (LEFM) is valid for shallow cracks and the dynamic effects may not be negligible for very deep penetration of the vessel wall.

In the above references [26-30], the effect of cladding on the

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2 Cladding is the material bonded to the inner wall of a nuclear pressure vessel to protect the base metal from chemical corrosion or detrimental radiation damage.
thermal shock resistance was ignored. Nied [31,32] extended the research by performing a parametric study to examine the effect of varying cladding thickness to cylinder wall thickness ratio for various circumferential crack lengths. The effect of yielding of the cladding on the SIF for cracks underneath the cladding was also studied by using a simple plastic strip model.

Using the weight function method, Stamm et al. [33] evaluated the SIFs for semi-elliptical surface cracks and edge cracks for the shakedown situation for a finite-width plate under cyclic thermal loading. Approximate procedures were recommended to calculate stress distributions in the second and following cycles and their influence on the SIFs was discussed.

Due to the analytical difficulties, in all the aforementioned transient studies, the plane of the crack was assumed to produce no effect upon the temperature field, i.e., the crack is fully conducting - heat flow is not interrupted by the crack. In order to carry out the solutions under less restrictive conditions, Ting and Jacobs [34] assumed a fully insulated crack and used the finite element method to obtain the transient temperature distribution and stress field of Griffith cracks positioned parallel to the free surface of a semi-infinite medium. The boundary condition used was a step temperature change at the free surface. They found that for heating at the free boundary both crack opening and sliding modes were present. For cooling
at the free boundary, only the crack sliding mode was found due to the compression in vicinity of the crack tip.

It should also be noted that, in all the transient problems cited here [21-34], the quasi-static theory of thermoelasticity was used, thus, the coupling in the thermoelasticity equations was disregarded.

Refined as they are, the analytical and numerical methods for solving transient thermal stress problems are still unable to provide sufficient accuracy or to be employed within reasonable effort to many real life problems in engineering design and manufacturing. The main reasons for these difficulties are the numerical inaccuracy in the transient temperature determination and difficulties that arise due to complicated shapes and boundary conditions. Hence, experimental methods are excellent alternatives for transient thermal stress problems. One such methods is photothermoelasticity where the birefringent response is generated by the self-equilibrated thermal stresses which, in turn, are generated by the nonlinear temperature distribution in the photoelastic model.

Following the application of photoelasticity to the solution of thermal stress problems by Weibel [35], Gerard and Gilbert [36] first introduced the technique of photothermoelasticity for the measurement and visualization of complete thermal stress fields arising from transient temperature gradients by means of refrigeration. Tramposch and Gerard [37] applied the same technique and introduced a novel sandwich technique with an embedded polariscope to study the three
dimensional thermal stress problems associated with a thick-walled cylinder. Gerard [38], Rothstein and Kirkwood [39] also reported their work using these techniques.

Hovanesian and Kowalski [40] developed a set of similarity relationships, which include temperature-scaling factors, size factors, and time-scaling factors between model and prototype. These relations further substantiate the application of the photoelastic technique for studies of thermal problems.

Burger [41] reviewed the main considerations in model tests of thermal stress problems, with special attention to the photoelastic method. In [42], he proposed a generalized photoelastic technique for the study of transient thermal stresses. He applied it to the study of the stresses in a flat plate subject to varying temperature gradients throughout its thickness. The temperature gradients were established by a combination of heating and cooling on the surfaces of the plate. He found that the stresses maximized only after a considerable time when the temperature change had already been felt by a large portion of the cross section of the plate. In this paper, he did not study thermal shock conditions.

The earliest work on thermal shock by using photoelasticity was reported by Becker [43]. He used dried ice in contact with a notched specimen to show that during the early stages of shock the skin contraction is completely constrained and that \( \alpha E(\Delta T) \) constitutes the upper bound to the stresses. Stress raisers do not play any part in
determining the maximum stresses during the period immediately following shock. Gurtman and Colao [44] also studied the stress concentration around a circular hole in a flat unrestrained plate subjected to a thermal shock which was applied by placing dry ice (-78°C) directly on the top surface of the model. Later, Burger [45] recommended an easy experimental method for subjecting the edges of photoelastic plate models to severe and repeatable thermal shock and results were compared to plane stress and plane strain predictions. In his method, he could photograph the isochromatic fringes earlier than Becker's during the shock. He concluded that if a plane stress condition was assumed to exist on the edge of the plate, the maximum edge stresses would be underestimated. The plane strain prediction based on recorded temperatures predicts maximum stress values within the experimentally determined range.

The main difficulties of the refrigeration technique as used by the aforementioned researchers were that the low temperature caused moisture in the air to condense on the test pieces, and the installation of thermocouples for measuring temperature distribution distorted the transient thermal stress distribution.

In 1979, Tsuji and Oda [46] employed the new technique of photothermoelasticity by means of heating, to study the transient thermal stress concentration in semi-infinite plates with circular and arc-shaped notches. Also, thermal paint instead of thermocouples was used to measure the temperature distributions. Extremely clear photographs of the photoelastic isochromatics were obtained.
Miskioglu and Burger [47-49] used photothermoelasticity by means of heating on polycarbonate materials (PSM-1) to obtain an experimental solution of transient thermal stress concentration factors around an elliptical hole due to a sudden temperature change at the edge of the plate. The solution, which was obtained by the combination of photoelastic data with contemporary statistical computer procedures, was in terms of non-dimensional variables. These consisted of the pertinent geometrical parameters of the ellipse, the material properties and time. They also estimated the maximum stress at the tip of a line crack by considering it as the limiting form of an ellipse of decreasing minor axis. They found that tensile stresses, as opposed to the compressive stresses at the elliptical boundary, were present at the tip for some of the angular orientations tested (0°, 22.5°, 45°). In their work, ligament size (the minimum thickness between the defect and the boundary) was kept constant. Chou [50] extended their work by taking the effects of the change of ligament size into consideration and confirmed the applicability of the prediction equation proposed by them. He also investigated the validity of the equation for the same model made from different material (copper).

Since the application of photoelasticity in determining $K_I$ for plane problems was first demonstrated by Post [51], Wells and Post [52] and Irwin [53], great progress has been made in solving problems with this method in various areas.
Emery et al. [54] adopted photothermoelastic and sandwich techniques to investigate the steady state SIFs of radial cracks at the interior wall of a partially filled annulus. Later, Emery et al. [55] reported on the SIFs for notched plates which were suddenly chilled on the cracked side. The stresses near the tips of the cracks and the notches were singular and observed to agree with isothermal LEFM.

Fessier and Mansell [56], Marloff et al. [57] and Schroedl et al. [58] successfully initiated the stress freezing and slicing techniques for analyzing the stress field near a crack tip. Since then, Smith [59, 60] and Smith and Olaosebikan [61, 62] have contributed greatly to the use of these techniques for three dimensional problems.

In the area of fracture dynamics, Bradley and Kobayashi [63, 64] and Kobayashi and Ramulu [65, 66] used dynamic photoelasticity to measure SIFs during crack propagation.

A number of approaches for extracting SIF from two- or three-dimensional photoelastic stress fields are summarized in references [61, 62, 67, 68]. In general, they can be classified into two categories, i.e., point-matching techniques and global methods.

Examples for the point-matching techniques are Irwin's two-parameter method [53], Smith's extrapolation technique [61], Schroedl and Smith's differencing method [61], Bradley and Kobayashi's shear differentiating method [63], Etheridge and Dally's three-parameter method [69], Gdoutos and Theocaris's extrapolation method [70], Ruiz and Phang's slope method [68] and Smith and Olaosebikan's quadratic method.
All these methods are limited to determining only two ($K_I$ and $\sigma_{0x}$ or $K_I$ and $K_{II}$) of the three quantities which affect the fringe pattern. Also, they take isochromatic data from only one or two points in the fringe field and do not fully utilize the available data.

In order to eliminate uncertainties in the calculation of SIF and take advantage of the full-field characteristics of the fringe pattern, Bradley and Kobayashi [64], Cheng [71], Sanford [72] and Sanford and Dally [73] introduced the global method for making more than one measurements at the stress field near the crack tip and using statistical procedures to accurately evaluate SIF. Also, with the emergence of computers and image-processing techniques (e.g. Voloshin and Burger [74], Muller and Saackel [75], Tsai and Park [76]) becoming a reality in optical stress analysis, the full potential of the global method can be more readily utilized.

It is generally accepted that the near-field equations suggested by Irwin [53] adequately describe the state of stress in the immediate vicinity of a crack tip, excluding a very small region around the crack tip itself. However, near-field measurements can lead to errors because of difficulties inherent to the crack tip zone: fringe clarity, caustic (light scattering from the dimple) at the crack tip, unknown degree of plane-strain constraint, etc. Considerable work has been done in recent years [77-80] to investigate the influence of non-singular (i.e. higher order) terms on the stress field around but not immediately adjacent to the crack tip.
III. THEORY

A. Photoelastic Theory

Photoelasticity is based on a phenomenon that occurs when polarized light passes through models made from certain polymeric plastics. The plastic becomes temporarily birefringent when stressed. The unique stress optical behavior of a plate of such a material causes a ray of polarized light passing through the thickness at any point to be resolved into two components along the two principal stress directions at the point. One of these components is progressively retarded, relative to the other, as they pass through the thickness of the stressed plastic. This relative retardation is proportional to the difference of principal stresses at the point.

For two-dimensional or plane-stress problems and for light at normal incidence to the plane of the model, the stress-optic law can be written as [81]

\[ \Delta = \frac{2\pi hc}{\lambda} (\sigma_1 - \sigma_2) \]  

where

\[ \Delta \] = relative retardation or relative angular shift in radians;
\[ \sigma_1, \sigma_2 \] = in-plane principal stresses;
\[ c \] = relative stress-optic coefficient in brewsters
\[ (1 \text{ brewster} = 10^{-12} \text{ m}^2/\text{N}); \]
\[ h \] = path length of the incident radiation through the photoelastic model or slice in a direction
normal to the principal plane;
\[ \lambda = \text{wave length of the radiation.} \]

For practical use, equation (3-1) is reduced to

\[ (3-2) \quad \sigma_1 - \sigma_2 = \frac{Nf}{h} \]

where

\[ N = \frac{A}{2\pi} = \text{the fringe orders or relative retardation in full cycles;} \]
\[ f = \frac{\lambda}{c}, \text{material fringe value in N/m-fringe.} \]

Since the maximum in-plane shear stress at any point is given by

\[ \tau_{\text{max}} = \frac{1}{2}(\sigma_1 - \sigma_2) \]

Equation (3-2) can also be written as

\[ (3-3) \quad \tau_{\text{max}} = \frac{Nf}{2h} \]

In a dark field circular polariscope setup, the intensity of the transmitted light emerging from the analyzer is

\[ (3-4) \quad I = K\sin^2\frac{\Delta}{2} \]

where \( K \) is a constant.

Hence, extinction occurs when \( I = 0 \), i.e., when

\[ \frac{\Delta}{2} = n\pi \quad \text{for } n = 0, 1, 2, 3, \ldots \]
The resulting dark lines are called isochromatic fringes and the corresponding numbers are called full fringe orders. For these values, \( N = \frac{\Delta}{2\pi} \) is always an integer.

In the light field setup for a circular polariscope, the intensity is

\[
I = K\cos^2\frac{\Delta}{2}
\]

Extinction occurs when \( I = 0 \), i.e., when

\[
\frac{\Delta}{2} = \frac{2n+1}{2}\pi \quad \text{for} \quad n = 0, 1, 2, 3, \ldots
\]

or

\[
N = \frac{\Delta}{2\pi} = \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \ldots
\]

These are often called "half-order" fringes.

The light intensity distribution for equations (3-4) and (3-5) are shown in Figure 3.1 and Figure 3.2, respectively.

In half-fringe-photoelasticity (HFP) [74], the image analysis system, called the "EyeCom System", distinguishes between 256 gray levels in any of the intervals for relative retardation, \( \Delta \), from 0 to \( \pi \), \( \pi \) to 2\( \pi \), etc., i.e., 0 \( \leq \)\( N \leq \)0.5, 0.5 \( \leq \)\( N \leq \)1, etc. The system, of its own, cannot distinguish between the intervals, so the interval being used must be known to the experimenter. Usually, the experimental parameters are chosen in such a way that the range of the fringe order lies between 0 and 0.5, i.e., 0 \( \leq \)\( N \leq \)0.5 [82-84]. However, in a more general sense, the user may allow the system to identify the intervals by inputting the
FIGURE 3.1. Plot of radiation intensity vs. phase retardation for a dark field circular polariscope

\[ I = K \sin^2 \left( \frac{\Delta}{2} \right) \]

FIGURE 3.2. Plot of radiation intensity vs. phase retardation for a light field circular polariscope

\[ I = K \cos^2 \left( \frac{\Delta}{2} \right) \]
fringe order manually or automatically for each interval if more than half fringe order is desired.

From the equations for Tardy compensation [81], we have that the light intensity emerging from a dark field circular polariscope with its analyzer rotated by angle $\mu$ from the dark field position is

$$I_{\mu} = K(1 - \cos2\mu\cos\Delta - \cos2\gamma\sin2\mu\sin\Delta)$$

where

$K = \text{the same constant as in equation (3-4);}$

$\mu = \text{angle of rotation of the analyzer from the dark field position;}$

$\gamma = \text{isoclinic angle;}$

$\Delta = \text{relative retardation.}$

If the analyzer is rotated 90° from this position

$$I_{\mu+90} = K(1 + \cos2\mu\cos\Delta + \cos2\gamma\sin2\mu\sin\Delta)$$

Then

$$I_{\mu} + I_{\mu+90} = 2K$$

$$= 2I_0$$

where $I_0$ is the reference intensity.

The sum of the intensities from two mutually perpendicular settings of the analyzer is a constant, $2I_0$. This value does not depend on whether the model is loaded or not. It is a "reference intensity" for each point in the field of the model and takes account of variations, introduced by the model, light source or the optical elements.
Now equation (3-4) can be rewritten to incorporate the reference intensity from equation (3-6) as

\[ I = I_0 \sin^2 \theta \]

or

(3-7) \[ I = I_0 \sin^2 (N\pi) \]

B. EyeCom System Theory

The EyeCom picture digitizer and display is a product of Loge/Spatial Data Systems, Inc. This is an image processing system that combines three types of man-machine communications; alphanumeric, graphic, and pictorial, into a single unit to provide the tools required for efficient image processing.

The EyeCom picture digitizer and display provides both input and output functions for the processing of pictures. The system is capable of displaying a digitized image of an analog picture either in real time or from a refresh memory.

Figure 3.3 is a schematic diagram of the HFP system [74]. The system consists of a regular polariscope in which the camera or viewing lens is replaced with a video camera. Alternatively, photographic negatives of a stressed model in a polariscope may be placed on a light table and viewed through the EyeCom scanner. For transient problems, this is often the only option and is the one adopted for the research in this dissertation. The system includes the following elements:
FIGURE 3.3. Schematic diagram of the HFP system
• An "EyeCom" scanner which uses a special vidicon television-camera tube to scan the chosen image area. The picture is divided into 480 lines and each line is divided into 640 parts. This division represents 307,200 points or picture elements (called 'pixels'). The brightness (Z value) for each pixel is converted into a video signal digitized with 8 bit resolution and stored in the Z-register. This establishes a scale which divides the range from a preselected darkest to a lightest point into 256 different gray levels.

• A real-time digitizer which digitizes the video signal in 1/30 second. This is too fast for direct transfer to the computer, so a special digitizer data bus transfers the data to the refresh memory where it can be accessed later by the computer.

• A display system or monitor which displays the image from the scanner either in real time or from memory. It also acts as a graphics/numeric terminal for data processing, program development, and graphical data displays.

• A LSI-11/2 computer and peripherals.

In the EyeCom system, the digital output value, Z, is related to the light intensity I by the relationship [85]

\[ Z = K_v I^X \]  

(3-8)

where

\[ Z = \text{the digitized output value for each pixel in the field;} \]
I = light intensity of that pixel;
\( \gamma \) = slope of the vidicon tube sensitivity curve;
\( K_v \) = a proportionality constant for the vidicon camera.

The digitization of the light intensity plot (Figures 3.1 and 3.2) by the EyeCom image analysis system occurs on the I axis such that equal gray level divisions represent neither equal intensity increments nor equal stress (phase retardation) increments. Therefore, the system needs to be calibrated to relate the digitized gray level, called Z-values, to the fringe order.

The calibration procedure for real-time experiment was discussed in [74, 82]. A detailed discussion of the calibration technique for photographic negatives will be included in Chapter IV.

C. Fracture Equations and Photoelasticity

Westergaard [86], in a widely quoted paper, defined an Airy stress function \( \phi(z) \) which is harmonic and can be written as

\[
\phi = \text{Re} Z_I + \gamma \text{Im} Z_I
\]

Where \( \overline{Z}_I \) and \( \overline{Z}'_I \) are the first and second integrals with respect to z of a complex function \( Z_I(z) \) for mode I loading, such that

\[
\overline{Z}_I = \frac{dZ_I}{dz}, Z_I = \frac{dZ_I}{dz} \quad \text{and} \quad \overline{Z}'_I = \frac{dZ'_I}{dz}
\]

where \( Z'_I \) is the first derivative of \( Z_I \) with respect to z, where \( z = x + iy \), a complex variable.
Then

\[ \sigma_x = \frac{\partial^2 \phi}{\partial y^2} = \text{Re}Z_I - y\text{Im}Z_I \]

\[ \sigma_y = \frac{\partial^2 \phi}{\partial x^2} = \text{Re}Z_I + y\text{Im}Z_I \]

\[ \tau_{xy} = -\frac{\partial^2 \phi}{\partial x \partial y} = -y\text{Re}Z_I \]

By appropriate choice of the function \( Z_I(z) \), Westergaard showed solutions for the stress distributions in crack problems where \( \tau_{xy} \) is zero along the \( x \)-axis. However, Sih [87], Evans and Luxmoore [88] and Eftis and Liebowitz [89] have shown that this solution is only valid for equibiaxial stress systems. Earlier than these findings was the work of Irwin [53] who noted that the equation of Westergaard could be modified to include an adjustable non-singular stress term in the \( x \)-direction, \( \sigma_{0x} \).

Thus, the modified Westergaard equations are

\[ \sigma_x = \text{Re}Z_I - y\text{Im}Z_I - \sigma_{0x} \]

\[ \sigma_y = \text{Re}Z_I + y\text{Im}Z_I \]

\[ \tau_{xy} = -y\text{Re}Z_I \]

The inclusion of the non-singular stress, \( \sigma_{0x} \), was necessary to explain the tilt of the isochromatic fringe loops from the normal in the work of Wells and Post [52]. Other photoelastic investigators [90, 91, 92] also used this idea. Nevertheless, the procedure itself is
empirical and fails to make apparent, in any general sense, the fact that the boundary loading has a pronounced influence on the isochromatic patterns close to the crack tip. It was Sih [87] who first recognized the need of relaxing the restriction placed on Westergaard's assumption. He employed Muskhelishvili's complex potentials method of the plane problem, i.e., the stresses may be expressed in terms of two complex functions \( \phi(z) \) and \( \psi(z) \) of the variable \( z = x + iy \). They are

\[
\sigma_x + \sigma_y = 4\text{Re}[\phi'(z)]
\]
\[
\sigma_y - \sigma_x + 2i\tau_{xy} = 2[\phi''(z) + \psi'(z)]
\]

The solution he derived for the opening mode crack problem is

\[
\sigma_x = 2\text{Re}[\phi'(z)] - 2y\text{Im}[\phi''(z)] + A
\]
\[
\sigma_y = 2\text{Re}[\phi'(z)] + 2y\text{Im}[\phi''(z)] - A
\]
\[
\tau_{xy} = -2y\text{Re}[\phi''(z)]
\]

where \( A \) is a real constant depending on the applied load. Hence the problem is reduced to the determination of a single complex function \( \phi(z) \) satisfying the necessary boundary conditions.

Eftis and Liebowitz [89] showed that Sih's equations are equivalent to the modified Westergaard equations if

\[
2\phi'(z) = Z_1(z) + A
\]

where
Sanford [93] pointed out that the modified Westergaard equations proposed by Sih [87] and Eftis and Liebowitz [89] may be inadequate for accurate photoelastic analysis in problems where the boundary or stress gradient ahead of the crack can be expected to play a significant role. He introduced another complex function $\eta(z)$ as

$$\eta(z) = z\psi''(z) + \psi'(z)$$

Then

$$\sigma_x = 2\text{Re}\psi' - 2y\text{Im}\psi'' - \text{Re}\eta$$

$$\sigma_y = 2\text{Re}\psi' + 2y\text{Im}\psi'' + \text{Re}\eta$$

$$\tau_{xy} = -2y\text{Re}\psi'' + \text{Im}\eta$$

This general formulation can be degenerated into its special cases, i.e., Westergaard equations (when $\eta(z) = 0$) and modified Westergaard equations (when $\eta(z) = \sigma_{0x}/2$).

The "generalized" Westergaard equation for opening mode crack problems (superscript 1) can be obtained by setting $2\psi' = Z_1(z) - \eta(z)$ in the above equations. They yield

$$\sigma_x^{(1)} = \text{Re}Z_1 - y\text{Im}Z_1' + y\text{Im}\eta' - 2\text{Re}\eta$$

$$\sigma_y^{(1)} = \text{Re}Z_1 + y\text{Im}Z_1' - y\text{Im}\eta'$$

$$\tau_{xy}^{(1)} = -y\text{Re}Z_1' + y\text{Re}\eta' + \text{Im}\eta$$
Similar arguments can be extended to the shearing mode crack problems (superscript 2) by selecting \( \eta(z) = \text{i}Z_{II}(z) \), the expression of the components of the stress are:

\[
\sigma_{x}^{(2)} = 2\text{Im}Z_{II} + \text{yRe}Z_{II}^{'}
\]

\[
\sigma_{y}^{(2)} = -\text{yRe}Z_{II}^{'}
\]

\[
\tau_{xy}^{(2)} = -\text{yIm}Z_{II}^{'} + \text{Re}Z_{II}
\]

Let the origin of the complex coordinate, \( z \), be located at the crack tip. Then, the Westergaard stress functions, \( Z_{I} \) and \( Z_{II} \), can be expressed as [78,93]:

\[
Z_{I}(z) = \sum_{n=0}^{N} \frac{A_{n}}{(n - \frac{1}{2})} z^{(n - \frac{1}{2})}
\]

and

\[
Z_{II}(z) = \sum_{m=0}^{M} \frac{B_{m}}{(m - \frac{1}{2})} z^{(m - \frac{1}{2})}
\]

Also, the \( \eta(z) \) function is of the form [93]

\[
\eta(z) = \sum_{p=0}^{P} a_{p} z^{p}
\]

Now the general form of the stresses for mixed mode crack problems can be obtained by superimposing the stresses due to opening and shearing modes. With substitution of the assumed stress functions, the components of the stress are:
\[ \sigma_x = \sigma^{(1)}_x + \sigma^{(2)}_x \]
\[ = \sum_{n=0}^{\infty} A_n r^{(n - \frac{1}{2})} \left[ \frac{\cos(n - \frac{1}{2})}{(n - \frac{1}{2})} - \sin\theta\sin(n - \frac{3}{2})\theta \right] \]
\[ + \sum_{m=0}^{\infty} B_m r^{(m - \frac{1}{2})} \left[ \frac{2\sin(m - \frac{1}{2})}{(m - \frac{1}{2})} + \sin\theta\cos(m - \frac{3}{2})\theta \right] \]
\[ + \sum_{p=0}^{P} r^p [\sin\theta\sin(p - 1)\theta - 2\cos p\theta] \]

\[ \sigma_y = \sigma^{(1)}_y + \sigma^{(2)}_y \]
\[ = \sum_{n=0}^{\infty} A_n r^{(n - \frac{1}{2})} \left[ \frac{\cos(n - \frac{1}{2})}{(n - \frac{1}{2})} + \sin\theta\sin(n - \frac{3}{2})\theta \right] \]
\[ + \sum_{m=0}^{\infty} B_m r^{(m - \frac{1}{2})} \left[ - \sin\theta\cos(m - \frac{3}{2})\theta \right] \]
\[ + \sum_{p=0}^{P} r^p [\sin\theta\sin(p - 1)\theta] \]

\[ \tau_{xy} = \tau^{(1)}_{xy} + \tau^{(2)}_{xy} \]
\[ = \sum_{n=0}^{\infty} A_n r^{(n - \frac{1}{2})} \left[ - \sin\theta\sin(n - \frac{3}{2})\theta \right] \]
\[ + \sum_{m=0}^{\infty} B_m r^{(m - \frac{1}{2})} \left[ \frac{\cos(m - \frac{1}{2})}{(m - \frac{1}{2})} - \sin\theta\sin(m - \frac{3}{2})\theta \right] \]
\[ + \sum_{p=0}^{P} r^p [\sin\theta\sin(p - 1)\theta + \sin p\theta] \]

Taking only the first terms of each stress function, we obtain the well-known Irwin's solution:

\[ \sigma_x = \frac{1}{\sqrt{2\pi r}} \left[ K_1 \cos \frac{\theta}{2} \left( 1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) - K_II \sin \frac{\theta}{2} \left( 2 + \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \right) \right] - \sigma_0 \]

\[ \sigma_y = \frac{1}{\sqrt{2\pi r}} \left[ K_1 \cos \frac{\theta}{2} \left( 1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) + K_II \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \right] \]

\[ \tau_{xy} = \frac{1}{\sqrt{2\pi r}} \left[ K_1 \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} + K_II \cos \theta \left( 1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \right] \]
where $K_I = -2\sqrt{2\pi a_0}$ = opening mode stress intensity factor;
$K_{II} = -2\sqrt{2\pi B_0}$ = shearing mode stress intensity factor;
$\sigma_{0x} = 2C_0$.

The maximum in-plane shear stress $\tau_{\text{max}}$ is related to the rectangular components of stress by

$$\tau_{\text{max}} = \left[\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2\right]^\frac{1}{2}$$

The stress optic law in photoelasticity which relates the fringe order $N$ to the maximum in-plane shear stress is

$$2\tau_{\text{max}} = \frac{Nf}{h}$$

Combining equations (3-9) and (3-10) and squaring on both sides

$$\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2 = \left(\frac{Nf}{2h}\right)^2$$

or

$$D^2 + T^2 = \left(\frac{Nf}{2h}\right)^2$$

where

$$D = \frac{1}{2}(\sigma_{xx} - \sigma_{yy})$$
$$T = \tau_{xy}$$

Equations (3-9) and (3-10) when substituted into equation (3-11) represent the general solution for the isochromatic fringe pattern around a crack tip for any size region. The size of the region and the
degree of accuracy desired determines the number of coefficients which must be retained in each series expansion in order to adequately describe the stress state.

It is worth noting that all the equations mentioned above have some inherent limitations, such as absence of body forces and isothermal conditions. They were also developed for stationary cracks. Therefore, care must be taken when applying these equations to different types of problems. The research reported in this dissertation is one example of such a problem. Since the analytical solutions are generally not available for transient thermal problems, a semi-inverse technique was used in the experiments to obtain a representation of the elastic fields surrounding the cracks under transient thermal loading. Coefficients of each power series were postulated as functions of time to conform to transient phenomenon. A similar technique was successfully employed by Bradley and Kobayashi [63] for solving dynamic fracture problems by photoelasticity.

D. Numerical Method of Analysis

To determine SIF's in the HFP technique with the help of the EyeCom system, a multiparameter-multipoint approach based on the least squares solution of an overdetermined system of nonlinear algebraic equations [72, 73] is used. This method utilizes an iterative procedure based on the generalized Newton-Raphson method [94].

Consider a set of functions of the form:

\[ G_k(A_0, A_1, \ldots, A_N, B_0, B_1, \ldots, B_M, \alpha_0, \alpha_1, \ldots, \alpha_p) = 0 \]
Taking a truncated Taylor's series expansion of equation (3-12) yields:

\[(G_k)_i^{i+1} = (G_k)_i + \left( \frac{\partial G_k}{\partial A_0} \right)_i \Delta A_0 + \left( \frac{\partial G_k}{\partial A_1} \right)_i \Delta A_1 + \ldots + \left( \frac{\partial G_k}{\partial A_N} \right)_i \Delta A_N
\]
\[+ \left( \frac{\partial G_k}{\partial B_0} \right)_i \Delta B_0 + \left( \frac{\partial G_k}{\partial B_1} \right)_i \Delta B_1 + \ldots + \left( \frac{\partial G_k}{\partial B_M} \right)_i \Delta B_M
\]
\[+ \left( \frac{\partial G_k}{\partial \alpha_0} \right)_i \Delta \alpha_0 + \left( \frac{\partial G_k}{\partial \alpha_1} \right)_i \Delta \alpha_1 + \ldots + \left( \frac{\partial G_k}{\partial \alpha_P} \right)_i \Delta \alpha_P \]

where the subscript \(i\) refers to the \(i\)th iteration step, and \(\Delta A_0\), \(\Delta A_1\), \ldots, \(\Delta A_N\), \(\Delta B_0\), \(\Delta B_1\), \ldots, \(\Delta B_M\), \(\Delta \alpha_0\), \(\Delta \alpha_1\), \ldots, \(\Delta \alpha_P\) are corrections to the previous estimates of \(A_0\), \(A_1\), \ldots, \(A_N\), \(B_0\), \(B_1\), \ldots, \(B_M\), \(\alpha_0\), \(\alpha_1\), \ldots, \(\alpha_P\), respectively.

The corrections are determined so that \((G_k)_i^{i+1} = 0\), and thus

\[
(3-13) \quad \left( \frac{\partial G_k}{\partial A_0} \right)_i \Delta A_0 + \left( \frac{\partial G_k}{\partial A_1} \right)_i \Delta A_1 + \ldots + \left( \frac{\partial G_k}{\partial A_N} \right)_i \Delta A_N + \left( \frac{\partial G_k}{\partial B_0} \right)_i \Delta B_0
\]
\[+ \left( \frac{\partial G_k}{\partial B_1} \right)_i \Delta B_1 + \ldots + \left( \frac{\partial G_k}{\partial B_M} \right)_i \Delta B_M + \left( \frac{\partial G_k}{\partial \alpha_0} \right)_i \Delta \alpha_0 + \left( \frac{\partial G_k}{\partial \alpha_1} \right)_i \Delta \alpha_1
\]
\[+ \ldots + \left( \frac{\partial G_k}{\partial \alpha_P} \right)_i \Delta \alpha_P = -(G_k)_i
\]

In matrix notation equation (3-13) becomes

\[
(3-14) \quad \begin{bmatrix} G \end{bmatrix} = \begin{bmatrix} C \end{bmatrix} \begin{bmatrix} A \end{bmatrix}
\]

where

\[
\begin{bmatrix} G \end{bmatrix} = \begin{bmatrix} -G_1 & -G_2 & \ldots & -G_L \end{bmatrix}^T
\]
Since matrix $[C]$ is not square ($L > M + N + P$) equation (3-14) has no unique solution. However, it can be shown that a solution in the least squares sense can be obtained by multiplying both sides from the left by $[C]^T$.

$$[C]^T[G] = [C]^T[C][\Delta]$$

or

$$(3-15) \quad [\Delta] = [E]^{-1}[C]^T[G]$$

where $[E] = [C]^T[C]$.

To apply this solution scheme to the isochromatic equation, equation (3-11) is rewritten in the form:

$$G_k = D_k^2 + T_k^2 - \frac{N_k f}{2h} = 0$$

where $k$ refers to the value of the function evaluated at a point in the field $(r_k, \theta_k)$ at which the fringe order is $N_k$.

Thus, the procedure for determining the best fit values of the coefficients can be described as follows:

(a) from the fringe pattern select a sufficiently large
set of data points \((r'_k, \theta'_k, N'_k)\) over the region to be investigated.

In this research, 50 data points were used;

(b) assume initial estimates for \(A_0, A_1, \ldots, A_N, B_0, B_1, \ldots, B_M, \alpha_0, \ldots, \alpha_p\);

(c) compute the elements of the matrices \([G]\) and \([C]\) for each data point;

(d) compute \([\Delta]\) from equation (3-15);

(e) revise the estimates of the unknowns, i.e.

\[
(A_0)_{i+1} = (A_0)_i + \omega \Delta A_0 \\
(A_1)_{i+1} = (A_1)_i + \omega \Delta A_1 \\
\vdots \\
\vdots \\
\vdots \\
(a_p)_{i+1} = (a_p)_i + \omega \Delta a_p;
\]

(f) repeat steps (c), (d), and (e) until \([\Delta]\) goes below a prescribed value depending upon the accuracy required.

It should be noted that if \(\omega = 1\) in step (e), the common Newton-Raphson method is resumed as suggested in \([72, 73]\) (See Appendix A).

Due to the fact that the convergence rate of Newton-Raphson method is generally fast, it should always be employed first for all the iteration processes. However, with increasing number of coefficients, the convergence of the solution becomes more and more sensitive to errors in the data points and to the values of the initial guesses of the coefficients. To overcome this sensitivity, the coefficients determined
in the 1-coefficient fit, which always converged, were used as the initial guess for the 2-coefficient fit and the coefficients determined in this fit used as the initial guess in the 3-coefficient case and so on in order to obtain an initial guess as close to the final values as possible. Furthermore, the generalized Newton-Raphson method is often used very effectively to find solutions for large sets of algebraic equations by selecting appropriate values of relaxation factor, $\omega$. No attempts were tried in this research to find optimal value of $\omega$ for each iteration process, since it was only used to expedite convergence when necessary.

A computer program TTSIF (Transient Thermal Stress Intensity Factors) using the above techniques was developed. It is summarized in the block diagram in Figure 3.4.
FIGURE 3.4. Block diagram of program TTSIF (Transient Thermal Stress Intensity Factors)
IV. EXPERIMENTAL TECHNIQUE

A. Model Material

As reported in [95], PSM-1, a specially annealed polycarbonate plastic, is the most suitable material for photothermoelastic analysis. It not only has a high but also constant thermoelastic figure of merit and thermal diffusivity in the temperature range of -10°C to 55°C. The material properties of PSM-1 are given in Table 4.1. Aside from its excellent transparency, this material is ductile, rather than brittle; and it is completely free of time-edge effects.

While much easier to machine than most photoelastic plastics, PSM-1 is more sensitive to localized heating from the cutting operation. However, the unique insensitivity of PSM-1 to moisture permits the use of water or other aqueous coolants during machining. When excessive heating is avoided, optically clean boundaries (free of residual birefringence) can be produced. Since the material is relatively ductile, the machining precautions normally required to prevent chipping and cracking of brittle plastics are unnecessary. Coarse, heavy cuts can be made through adequate cooling without danger of fracturing the work piece. But the work piece should never be allowed to remain in stationary contact with a rotating tool so that continuous rubbing occurs, since the local heat generation may induce residual birefringence.
TABLE 4.1. Material properties of PSM-1 over the range -10°C to 55°C

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's Modulus</td>
<td>E</td>
<td>2.39 GN/m²</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.47 x 10⁵ psi</td>
</tr>
<tr>
<td>Poisson's Ratio</td>
<td>ν</td>
<td>0.38</td>
</tr>
<tr>
<td>Photoelastic Fringe Coefficient</td>
<td>f₀</td>
<td>7 kN/ fringe-m</td>
</tr>
<tr>
<td></td>
<td></td>
<td>40 lb/ fringe-in.</td>
</tr>
<tr>
<td>Coefficient of Thermal Expansion</td>
<td>α</td>
<td>1.46 x 10⁻⁴ °C⁻¹</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8.11 x 10⁻⁵ °F⁻¹</td>
</tr>
<tr>
<td>Specific Heat</td>
<td>C_p</td>
<td>0.307 W-hr/kg °C</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.264 Btu/lb °F</td>
</tr>
<tr>
<td>Thermal Conductivity</td>
<td>k</td>
<td>0.365 W/m °C</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.211 Btu/ft-hr °F</td>
</tr>
<tr>
<td>Thermal Diffusivity</td>
<td>αₜ</td>
<td>1.01 x 10⁻³ m²/hr</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.09 x 10⁻² ft²/hr</td>
</tr>
<tr>
<td>Figure of Merit</td>
<td>Qₜ</td>
<td>46.61 fringe/m °C</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.26 fringe/in. °C</td>
</tr>
</tbody>
</table>

B. Test Specimen

The original test piece was a rectangular plate measuring 106.6 mm x 152.4 mm x 6.4 mm (4.0 in. x 6.0 in. x 0.25 in.) as shown in Figure 4.1. A line crack was simulated in the model as a narrow slit with
length $2a = 20.3 \text{ mm (0.8 in.)}$ and width $0.15 \text{ mm (0.006 in.)}$. The slit was machined with a fine-pitched jewlers slitting saw $0.15 \text{ mm (0.006 in.)}$ thick. All the test pieces used in this research were prepared by the machine shop of the Engineering Research Institute at Iowa State University. Special care was taken to ensure that residual stresses after machining were negligible. The geometrical parameters of the test pieces are given in Table 4.2. In this table, $\beta$ is the angle that the line crack makes with the heated edge of the plate and $c$ is the distance of the center of the line crack from the heated edge. The ratio $c/a$ measures the "depth" of the line crack below the surface of the plate (i.e., the heated edge). This depth was adjusted to keep the ligament size; i.e., the minimum thickness of the material between the line crack and the edge, the same for all test pieces. In this research, a nominal ligament size of $3.8 \text{ mm (0.15 in.)}$ was adopted. Figure 4.2 is a 30 times enlargement of the slit for model $\beta = 0^\circ$. It shows the geometric details of the crack tip.

The plate was large enough to simulate a semi-infinite plate with a line crack close to its edge. The vertical sides of the plate were far enough from the crack that the edge effect which they cause did not affect the temperature and stress distribution around the crack. This can be easily confirmed by observing the fringe patterns in Figure 4.3. Moreover, the far-field edge stresses are depicted by the fringes which remain parallel with the plate edge over a sufficiently long distance.
FIGURE 4.1. Test specimen. All dimensions in mm. Crack parameters are nominal values. Exact values for each model are recorded in Table 4.2.
TABLE 4.2. Geometrical parameters of the specimen

Slit length = 2a = 20.3 mm (0.8 in.)
Slit width = w = 0.15 mm (0.006 in.)
Ligament size = l = c - asinβ = 3.8 mm (0.15 in.)

<table>
<thead>
<tr>
<th>β°</th>
<th>0</th>
<th>22.5</th>
<th>45.0</th>
<th>75.0</th>
<th>90.0</th>
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<td>c(mm)</td>
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<td>11.0</td>
<td>13.6</td>
<td>14.6</td>
</tr>
<tr>
<td></td>
<td>0.150</td>
<td>0.303</td>
<td>0.433</td>
<td>0.536</td>
<td>0.550</td>
</tr>
<tr>
<td>2a(mm)</td>
<td>20.4</td>
<td>21.0</td>
<td>20.4</td>
<td>20.7</td>
<td>21.0</td>
</tr>
<tr>
<td></td>
<td>0.804</td>
<td>0.825</td>
<td>0.803</td>
<td>0.816</td>
<td>0.821</td>
</tr>
<tr>
<td>w(mm)</td>
<td>0.20</td>
<td>0.20</td>
<td>0.15</td>
<td>0.20</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>0.009</td>
<td>0.008</td>
<td>0.006</td>
<td>0.008</td>
<td>0.007</td>
</tr>
<tr>
<td>l(mm)</td>
<td>3.76</td>
<td>4.22</td>
<td>4.06</td>
<td>3.83</td>
<td>3.86</td>
</tr>
<tr>
<td></td>
<td>0.148</td>
<td>0.166</td>
<td>0.160</td>
<td>0.151</td>
<td>0.152</td>
</tr>
</tbody>
</table>

C. Experimental Apparatus

The heating apparatus is shown in Figure 4.4. The smallest imperfections or dirt particles in the area of contact disturbed the temperature field sufficiently to cause irregularities in the stress field. The top edge of the heating copper block (weight = 1.32 Kg or 2.90 lb.) was, therefore, machined smooth for good contact with the photo thermoelastic model and all surfaces were kept clean. Also, the top edge of the block was slightly wider than the test model. Then,
FIGURE 4.2. 30 times enlargement of the slit, $\beta = 0^\circ$
FIGURE 4.3. Dark field fringe patterns showing the region away from the crack. AA is the heated edge of the model.
when the test model was lowered onto the block, the whole boundary of the model would be in uniform contact with the edge.

The block was wrapped with a resistance heating tape (Cole-Parmer Instrument Company, Model C-3111-20), and all but the topmost part was insulated. The temperature of the heating surface was maintained at 55°C±0.5°C with a proportional temperature controller (Omega Engineering, Inc., Model 4001). The temperature sensor for the controller was placed between the heating tape and the block. The temperature of the heating block was monitored with two copper-constantan thermocouples embedded in the block 50.8 mm (2 in.) apart and 3 mm (0.12 in.) below the contacting edge. The temperature from this pair of thermocouples was read and printed each time that a picture was taken. These temperature readings were then averaged over the test period to calculate the mean temperature, $T_m$, of the heater. Copper is a good thermal conductor ($k_c = 386 \text{ W/m°C}$) and has a moderate specific heat ($C_{pc} = 0.0787 \text{ W-hr/Kg°C}$). It is therefore a good material for a heat source which is required to maintain a near constant edge temperature when the "cold" model is lowered onto the block. The photoelastic material is a poor conductor ($k = 0.365 \text{ W/m°C}$) when compared with copper. Used in combination with copper as in this study, the temperature of the contact surface of the block dropped only very slightly (<0.5°C) when the cold test specimen was placed on top of it. The original temperature was recovered within 2 minutes.
FIGURE 4.4. Schematic of the heating apparatus. All dimensions in mm.
D. Experimental Procedure

The tests for each model were conducted as follows. After the heating plate had reached an equilibrium temperature, the test piece which was at room temperature was placed on the heating block. The laboratory was air conditioned so that the room temperature for all of the tests performed never averaged beyond 23°C to 25°C. Photographs of the dark field photoelastic isochromatics were taken with a 35 mm camera loaded with Kodak Tri-X Pan Film (speed : ASA 400) and equipped with a close-up lens. For all test models, photographs were taken at 15 sec., 30 sec., 1 min., at 1 min. intervals up to 5 min., and at 7 min., 10 min., 15 min., 20 min., and 30 min. after the specimen was placed on the heating block. The temperature readings of the two thermocouples on the heating block were recorded simultaneously at the same time as each photograph was taken. The two thermocouple readings were kept to within 0.2°C to ensure uniform temperature along the top edge of the block.

For the calibration purpose of the HFP system, photographs were taken of the photoelastic isochromatics of a beam made from the same material as the test models. The beam was loaded in pure bending and photographs were taken on the same roll of film to maintain the same film characteristics, after processing, as the photos of the thermal isochromatics. To ensure the gradual change of gray levels over the whole field, experience showed that two f-stop's less than the exposure indicated by the camera's light meter should be used. The film was developed in D76 for 4.5 minutes (i.e., half of the manufacturer's
recommended developing time) at 65°F. This exposure and processing procedure produced the best images for the EyeCom system. Finally, the negatives were placed on a light table under the video camera for semi-automatic analysis with the EyeCom system.

E. Calibration of the EyeCom System

As mentioned earlier, the EyeCom system needs to be calibrated before it can be used for a photoelastic analysis, i.e., the relationship between digitized light intensities, $Z$, and fringe orders, $N$, must be established first. The calibration procedure can be divided into two steps: internal calibration and external calibration [96, 97]. Internal calibration is performed by the built-in system programs. Its purpose is to setup the proper video signal for the digitizers. External calibration is carried out by the programs developed by the user for different situations. The calibration technique for real-time experiment was discussed in [74, 82]. Here, the calibration procedure for photographic negatives is briefly described.

1. Internal Calibration

Internal calibration of the system is performed by the built-in program SETUP. It provides interactive adjustment of the EyeCom digitization parameters by determining proper values for the setup, zero and range registers. These registers control the video amplifier chain that provide the proper video signal for the digitizers as follows [85]:
• Setup --- The digital value, 0 to 255, in this register sets the proper black level of the scanner video signal to produce linear operation of the amplifier chain in the linear mode and correct logarithmic operation in the log mode.

• Zero --- The value in this register, 0 to 255, determines the video level corresponding to a digitized Z value of zero.

• Range --- The value in this register, 0 to 63, determines the range of video levels covering the span from a Z-value of zero to Z-value of 255.

The setup register is adjusted by first providing a black picture from the scanner which is achieved by capping the lens. The computer program SETUP is then used to initialize the zero register and to setup the appropriate zero value and range. The brightest white level of the picture is adjusted by the f-stop of the lens on the scanner. Then the zero and range registers are set to digitize only that portion of the image gray scale of interest to the user. Since a pure bending specimen was used as a calibration medium in this research, darkest and brightest points in the isochromatic fringe pattern of the beam recorded on the negative were manually chosen and pointed out by using the joystick cursor.

The zero and range registers are then set with the correct values to place the darkest area at a Z-value of zero, and the brightest area at a Z-value of 255. Refresh memory picture and graphics are also initialized and enabled for later use. The whole procedure is summarized in Figure 4.5.
Close the camera lens to get the setup number which represents the dark current of the camera

Show darkest and brightest points in the optical field to set in exposure

Enable refresh picture and graphics

FIGURE 4.5. Block diagram of internal calibration
2. External Calibration

There are two kinds of distortion, linear and nonlinear, that always exist in any signal processing system. The EyeCom system is no exception. Linear distortion can be modelled by a linear function. For example, if $M$ is the measured value from the system, then it can be related to the actual value of the parameter, $m$, as follows:

$$(4-1) \quad M = C_1 m + C_2$$

where $C_1$ and $C_2$ are unknown constants related to various process. The amplitude and the location of the baseline of the signal depend on $C_1$ and $C_2$, respectively. Typical examples of linear distortion are amplitude (or magnitude) distortion and phase (or positional) distortion.

On the other hand, nonlinear distortion is most easily treated parametrically.

In general, $M = f(m)$, where $f$ is an unknown function. Parameters which may be affected by distortions and possible remedies for these distortions are briefly summarized as follows:

(a) Spatial factor

Due to the positional distortion of the video signal, the image appearing on the screen, may be uniformly stretched or moved. For example, the image may be skewed, shifted, rotated, or suffered unknown changes in scale. Skewing can be eliminated by adjusting the camera. Shifting and rotation can be cured by establishing a landmark on the
image, e.g., specifying the crack tip and crack orientation for fracture problems. Change in scale is corrected by using a grid paper of known dimension to set up the "length scaling factor" (see "Data Collection Method").

The effect of nonlinear distortion may cause the screen image to be twisted or bent (so called "Barrel distortion"). This can be solved by tuning the camera.

(b) Noise

Noise in the signal processing system causes from unwanted electrical signals that accompany the message signals. These unwanted signals arise from a variety of sources and can be classified as man-made (e.g., radio signal, 60 cycle hum due to inadequate power supply filtering, and so forth) and naturally occurring (e.g. internal circuit noise and extra-terrestrial radiation). The effects of many noise sources can be reduced or eliminated completely by careful engineering design. However, there are always some unavoidable causes of electrical noise. One such unavoidable cause is the thermal motion of electrons in conducting media -- wires, resistors, and so forth. Thermal noise corrupts the desired signal in an additive fashion and its effects can be minimized by appropriate signal processing techniques.

A simple built-in hardware function in EyeCom system can reduce the electrical noise by a factor of 4. It is performed by scanning the object 16 consecutive times and averaging the results.
Optical noise includes imperfection of the light source, polariscope element, lens and the model. Its harmful effects can usually be reduced by averaging the intensities over several pixels around each point. This effectively puts the image slightly out of focus and smears out the imperfections. All images in this dissertation were "filtered" in this way by scanning an area of 5 pixels x 5 pixels around each point and recording the average value as the Z-value for the central point. This sounds like severe averaging but because the viewing area around the tips were greatly enlarged on the EyeCom screen, 5 pixels represent only about 0.13 mm on the model. For comparison, the slit width typically appeared on the screen is 10 pixels wide.

(c) Light intensities

There are three steps that must be dealt with before we can determine fringe orders from the digitized light intensities, Z. They are:

1. Nonlinearities in the vidicon camera and the film.

2. Smearing of the fringe patterns close to the crack tip and variation of the light intensities caused by the experimental set up. Two such effects are the uneven illumination of the diffuse light radiation source and aperture effects.

3. Deriving the fringe order by using the "corrected" digitized light intensity, Z', obtained from the previous two steps.

Since the reference intensity, I₀, in equation (3-7) is an unknown quantity, and the degree of nonlinearities existing in the system and
the negatives are unknown, it is difficult to find the exact
relationship between Z and I. We adopted a parametric method to solve
this problem.

Since EyeCom system's Z is a distorted measure of I, Z can be
simply expressed as a function of I, i.e.

\[ Z = g(I) \]

If we normalize the equation (3-7) with respect to \( I_0 \), we have

\[ I' = \frac{I}{I_0} = \sin^2(N\pi) \quad \text{and} \quad 0 \leq I' \leq 1 \]

For a beam in pure bending, the fringe order \( N \) varies linearly
across the beam. This can be written as

\[ N = mX + b \]

or

\[ I' = \frac{I}{I_0} = \sin^2[(mX + b)\pi] \]

where \( m \) = the slope of the line = \( \frac{\Delta N}{\Delta X} \);

\( b \) = the intercept

= 0, if the origin is taken at the neutral axis;

\( X \) = distance from the neutral axis.

The relationships of \( N \), \( Z \), and \( I' \) with \( X \) are shown in Figure 4.6.

For the darkest point in any half fringe interval, the minimum
theoretical intensity is \( I = 0 \). If \( I = 0 \) occurs at \( X = X_0 \), then

\[ \pi(mX_0 + b) = n\pi, \quad n = 0, 1, 2, \ldots \]
Known theoretical distribution of fringe order, N, across a beam in pure bending.

Digitized light intensity, Z, is read from EyeCom system

\[ Z = K_v I' \]

Typical half-fringe interval

\[ I' = \frac{I}{I_o} = \sin^2(mX + b) \]

FIGURE 4.6. Relationships of N, Z and I' with X
or

\[ mX_0 + b = n \]

Likewise, the brightest point in the half fringe interval is \( I = I_0 \). If \( I = I_0 \) occurs at \( X = X_1 \), then

\[ \pi (mX_1 + b) = (n + \frac{1}{2})\pi, \quad n = 0, 1, 2, \ldots \]

or

\[ mX_1 + b = n + \frac{1}{2} \]

Choose \( n = 0 \) and solve for \( m \) and \( b \).

\[
m = \frac{1}{2(X_1 - X_0)} \quad \text{and} \quad b = -\frac{X_0}{2(X_1 - X_0)}
\]

and equation (4-2) becomes

\[
(4-3) \quad I' = \sin^2 \left[ \frac{\pi(X - X_0)}{2(X_1 - X_0)} \right]
\]

In any experiment, locations for \( X_0 \) and \( X_1 \) can be selected on a digitized image of the picture. For a beam in bending where the fringe order varies horizontally across the EyeCom screen the two values \( X_0 \) and \( X_1 \) can be found by pointing to two points, one to the left of the regions of maximum \( Z \); one to the right of the adjacent minimum \( Z \). This is shown by the two crosses on the middle of Figure 4.6. The computer program PUBEC (Pure Bending Calibration) determines the exact positions of \( X_0 \) and \( X_1 \). At the same time, the digitized light intensities, \( Z \), from this picture of the beam in bending is placed in a one to one
relationship with the theoretical value $I'$ for the same position. In this way, a "substitution table" between $Z$ and $I'$ is produced.

A beam in pure bending produces the cyclical variations of constant amplitude if the radiation source is highly monochromatic. Recall that we took the pictures for the crack specimens in the transient thermal field and the beam in bending on the same roll of film to ensure the same film characteristics, developed in the same tank and read into EyeCom system with the same setup. Therefore, both maximum and minimum densities on the negatives of beam and specimen were the same. Through use of the substitution table, the nonlinear distortions in the system and negatives are taken care of.

However, with the presence of the effects as described in step 2, the amplitude of the light intensity varies outward from the crack tip as shown in Figure 4.7. Since all these effects are linearly related to the resultant intensity and are spatially dependent, they obey the rule as expressed by equation (4-1). Note that the plot of the light intensity distribution to the right of the crack tip as shown was shifted to the left by 50 pixels so that the whole distribution can be within the size of the screen. Also, the vertical stripes shown in the plots are used to indicate the selected half fringe intervals for data collection which will be discussed in the next section.

Therefore, after the substitution table is applied to the input data from the model, these other effects must be taken care of before determining the fringe order. Let $Z'$ be the "corrected" digitized light
FIGURE 4.7. Light intensity distribution of the test specimen, $\beta = 75^\circ$
intensity (for each Z, the value which has been obtained from substitution table and yet contains these other effects) to distinguish it from I' (for each Z, the value which is obtained from substitution table and contains none of these other effects). Then

\[ Z' = C_1^{I'} + C_2 \]
\[ = C_2 \sin^2(N\pi) + C_2 \]

Also note that there are certain points, in particular, peaks in the light intensity distribution where received light intensity (Z') is only the function of envelopes wrapping it (Figure 4.8). For example, when

\[ N = 0, 1, 2, \ldots; Z' = C_2 \]

and when

\[ N = \frac{1}{2}, \frac{1}{2}, 2\frac{1}{2}, \ldots; Z' = C_1 + C_2 \]

That is, the birefringent term (N) falls out from the equation and Z' can be determined totally by C₂ and/or C₁.

In general, envelopes wrapping the light intensity distribution should be determined before partial fringe orders of each half fringe interval can be found. But, fringe orders corresponding to these peaks are the only available information for us to find the envelope. Experience showed us that C₂ varies very little for any given interval. Also, C₁ varies monotonically within a given interval and the shape of
FIGURE 4.8. Piecewise constant approximation to the envelopes wrapping the light intensity distribution
the light intensity distribution is known to be a sinusoid of varying amplitude. Hence, a simple approximation to describe the envelopes wrapping the light intensity distribution is to assume a piecewise constant model as shown by the dotted lines in Figure 4.8. The deviations between the real envelopes and the model become large when approaching the crack tip. But, we don't collect data very near the crack tip anyway. So, these points of known fringe order and the piecewise constant are adequate to characterize the light intensity distribution.

For any half fringe interval, if the maximum and minimum digitized light intensities are $Z'_{\text{max}}$ and $Z'_{\text{min}}$, then the relationship between any $Z'$ and $I'$ is expressed as

$$Z' = C_1 I' + Z'_{\text{min}}$$

when

$$Z' = Z'_{\text{max}}, \ I' = 1$$

so

$$Z'_{\text{max}} - Z'_{\text{min}} = C_1$$

Therefore,

$$I' = \frac{Z' - Z'_{\text{min}}}{Z'_{\text{max}} - Z'_{\text{min}}}$$

(4-4)

Incorporating equation (4-2) with (4-4), we have

$$\frac{Z' - Z'_{\text{min}}}{Z'_{\text{max}} - Z'_{\text{min}}} = \sin^2(N\pi)$$
Therefore, with the use of substitution table and equation (4-5), partial fringe orders within any selected half fringe interval can be determined.

The substitution table is generated in the program PUBEC as shown in the block diagram in Figure 4.9. The procedure described for equation (4-5) is included in subroutine FEF (see "Data Collection Method").

F. Data Collection Method

The raw data at any point in a thermally stressed model are expressed as the fringe value (N) at a coordinate point (r, θ). To find SIFs from the fringe data, it is necessary to solve for the coefficients of the power series of equation (3-11) by the overdetermined least squares method. The number of data points collected must be more than the sum of the number of coefficients used. In the analysis used in this research, 50 data points (N, r, and θ) were taken in a distributed fashion over a selected region of the fringe pattern for each crack tip. These data were then used as input to the least squares algorithm to obtain a best fit set of coefficients.

Since the line cracks studied in this research are internal cracks, there are two sets of fringe patterns per crack, one is for the "far-end" tip (i.e., the crack tip farthest away from the boundary) and the other for the "near-end" tip (i.e., the crack tip close to the boundary).
Store the image of fringe pattern caused by the pure bending of a beam

Light intensities across the depth of the beam are collected and plotted

One half fringe interval is selected for calibration

Substitution table between Z values and normalized stress-optical I value is constructed

FIGURE 4.9. Block diagram of program PUBEC (Pure Bending Calibration)
Fifty distributed data points were chosen from the fringe pattern for each crack tip on each of the negative states at the pre-selected times during each transient. In the far-end case, 5 data points were collected on each of 5 radial lines on either side of a crack. Three regions were used for the near-end case. One region for the boundary fringe loop ahead of the crack tip with 5 data points on each of the 2 radial lines. Two regions, one for each of the fringe loops on either side of the crack. Each of these data sets have 4 data points on each of 5 radial lines. Figure 4.10 shows how these two data sets would typically be located.

Collecting data points in close vicinity of the crack tip make the results sensitive to factors, such as the geometric and stress field effects of the finite radius of the root at the crack tip, optical effects caused by high stress gradient at the crack tip, and nonlinear stress-strain behavior of the material due to high stress levels at the crack tip. Recognizing these difficulties, measurements in an enlarged region around the crack tip would be more advantageous.

1. Data Acquisition by Program CODATS

The computer program CODATS (Collect Data Systematically, Figure 4.11) is used to collect fringe data from the photographic negatives of thermally stressed models. The negative of the stress field for one time event is positioned properly under the vidicon camera such that the crack tip and its vicinity show up on the EyeCom screen. Then the fringe pattern around the crack tip, i.e., the image on the screen is
FIGURE 4.10. Example of the distribution of data points, \( \beta = 75^\circ \) at \( t = 4 \) min. Point T indicates the crack tip and line DT shows the crack direction.
stored in the computer memory for data collection. This image is an average of the sum of 16 continuous frames to eliminate electrical noise. A substitution table generated by program PUBEC is taken from a file as described under "Calibration of the System".

Major subroutine COLMS (Collect Multiple Fringe Systematically, Figure 4.12) is then called. The first assignment of COLMS is to establish a "length scaling factor" that correctly relates the distances on the video screen with actual length on the model. This is performed by subroutine GCF (Get Conversion Factor) as follows. A transparent sheet of millimeter graph paper is placed into the field of view aligned with the camera and projected onto the screen. Two points on the grid, 1 cm apart, are chosen. The coordinates of these two points are entered by using the joystick cursor. From these two points, the length on the screen image is found corresponding to 1 cm of the actual length on the model. Hence, the length scaling factor is established. In practice, we mount the calibration grid onto the specimen before the first photographic time sequence during a test. This image of the grid on the original negative is used for the calibration procedure described above.

After the length scale is calculated, the crack information is obtained through subroutine GCI (Get Crack Information). The crack tip is indicated with the joystick, the crack length and crack orientations relative to actual model are entered. GCI then sets the crack tip as the origin for the polar coordinate system \( r \) and \( \theta \). A special arrangement was made in this routine to relate the EyeCom coordinate
FIGURE 4.11. Block diagram of program CODATS (Collect Data Systematically)
Obtain conversion factor by using grid paper (subroutine GCF)

Obtain crack information, i.e., tip coordinates, crack length and orientation (subroutine GCI)

Estimate polar coordinates of point of interest to determine the region of data collection from fringe pattern (subroutine EAR)

Specify no. of regions (NRE)

Specify no. of angles (NRG) and no. of data points along each angle (NDP) for one region

Plot the light intensity along a line connecting crack tip and the point of interest (subroutine LIP)

Determine the partial fringe order for the selected half fringe interval (subroutine FEF)

FIGURE 4.12. Block diagram of subroutine COLMS (Collect Multiple Fringe Systematically)
After the length scale is calculated, the crack information is obtained through subroutine GCI (Get Crack Information). The crack tip is indicated with the joystick, the crack length and crack orientations relative to actual model are entered. GCI then sets the crack tip as the origin for the polar coordinate system $r$ and $\theta$. A special arrangement was made in this routine to relate the EyeCom coordinate system and the commonly used polar coordinate system in fracture mechanics.

Subroutine EAR (Estimate Angle and Radial Distance) is then used to determine a preferred region for data collection. As the user moves the cursor and presses a special key <BS> on EyeCom keyboard, polar angle, radial distance and $r/a$ ratio are shown both graphically and numerically on the EyeCom screen. The number of regions is subsequently entered into the system by the user and the data collection process started. For each region of interest, the number of radial lines and number of points along each line are specified by the user. For each radial line, light intensity distribution is plotted by calling subroutine LIP (Light Intensity Plot). Typical light intensity distributions for a 75° line crack is shown in Figure 4.7. The light intensity corresponding to each point along the line is the average value of light intensities of a 5 pixel x 5 pixel area around the point to further reduce the system noise. Special considerations were included in the routine to ensure that the light intensity plot always fits on the EyeCom screen for the convenience of data collection.
After all the half fringe intervals along one radial line are treated, the relationship between fringe order and radial distance for every point along the radial line is established.

Since the upper and lower values of \( r/a \) for each half-fringe interval are calculated and printed on the screen, the user can specify the upper and lower values of \( r/a \) that should be used for each radial line and enter them into the system. A doubly-clamped cubic spline interpolation routine SPLINE is used to find the fringe orders corresponding to the radial positions of the points specified earlier and stored in a file for the data analysis.

The procedure is repeated for the other radial lines in the same region and/or other regions until the data collection is finished. The final data file consists of fringe orders, polar angles (in radians) and radial distances (in inches) for the 50 data points.

The EyeCom coordinates corresponding to the upper and lower limits of \( r/a \) for all radial lines generated are recorded during the period of data collection. From these, a rectangular region that includes all data points can be drawn on the screen by using the maximum and minimum EyeCom coordinates obtained from the recorded values. A typical data collection region and points collected are shown in Figure 4.13.

G. Pattern Recognition of Fringe Field

In Chapter III-D, the numerical method of analysis by which the experimentally observed fringe pattern is matched to Equation (3-11) is
FIGURE 4.13. Distribution of the data points for the 75° model at time $= 4 \text{ min.}$, $\Delta T = 30.3^\circ \text{C}$, Line AA is the heated edge of the model. Data points were enhanced.
described. The coefficients inherent in the stress terms of Equation (3-11) are found through an overdeterministic least squares procedure that finds the best fit to the isochromatic data from 50 independent points. As a final step, these best fit coefficients were used to reconstruct an isochromatic fringe pattern which was compared to the experimental fringe pattern in a visual check for the adequacy of the assumed model for reconstructing the fringe pattern from the experimentally determined coefficients $A_n$, $B_m$, and $\alpha_p$. Two analytical models were used. For the crack tip nearest to the edge, the model was the generalized Westergaard. For the far-tip, the model was the modified Westergaard. The choice of the model will be discussed in Chapter V.

Locally supported pattern recognition software was developed for the HFP system. In order to back plot the calculated fringe values from equation (3-11), the rectangular region of data collection specified by program CODATS is divided into small rectangular cells of uniform size. Program RC (Rectangular Coordinates) calculates the EyeCom coordinates for the four nodes of each cell and stores them in a file. Then, the fringe order corresponding to each node is calculated from equation (3-11) in program CAFO (Calculate Fringe Orders) by using the coefficient values obtained from program TTSIF. These fringe values are also stored in a file for the use in reconstructing the fringe patterns.

Each rectangular cell is further divided into four triangles where one vertex of all the triangles shares a common point, i.e., the
centroid of the cell. Contouring is then performed for each triangle by
inverse interpolation through an existing program SIMCON [98].

Finally, the reconstructed fringe pattern (in graphic mode) is
superimposed on the image of the fringes on the negatives, that is on
the experimentally obtained pattern that resides in the refresh
memory. A visual check for differences between the two fringe patterns
provided information on the quality of the two main stages in the
determination of SIFs. They are:

- The accuracy of reading the initial fringe values that were
  used in establishing the stress field governed by Westergaard
  relations.
- The adequacy of the number of coefficients used in the
  Westergaard relations for describing the stress field in the
  area of data collection.

If the visual match was good then the results were accepted. If
the match was poor, both stages were checked and improved.

There is another useful feature available to the user of EyeCom. A
built-in hardware procedure can be used to identify the least
significant binary bit\(^3\) for each pixel and to assign "black" to a zero
and "white" to a one. The resulting picture generated by this procedure
is called a "bit map" of the image stored in the refresh memory. When
displayed on the monitor screen, it appears as fringe-like lines

---------------------

\(^3\) The least significant bit of a binary number is the modulo 2
remainder of the number. It is the right digit in the 8 bit binary
representation of the light intensities.
connecting the pixels with the same least significant bits. This image does not convey any direct stress information but is a useful and rapid technique for checking symmetry of loading, stress gradients, and stress concentration [74]. Figure 4.14 shows an example of a "bit map".

The whole procedure and programs used in the data analysis are summarized in Figure 4.15. All the programs reside in the Experimental Stress Analysis Program Bank, Department of Engineering Science and Mechanics, Iowa State University.
FIGURE 4.14. Example of a bit map, $\beta = 75^\circ$ at $t = 7$ min.
FIGURE 4.15. Block diagram of the data analysis programs
V. RESULTS AND DISCUSSION

Isochromatic fringe patterns that give the full stress field near the crack tips are used to obtain the transient thermoelastic stress intensity factors. Typical isochromatic fringe patterns for both near- and far-end crack tips and the related fringe counts are shown in Figure 5.1. Figure 5.1 presents the data (fringe field) the crack tip regions as they appear on the negatives. These are the actual low level fringe orders that were used in all subsequent analyses. They show one of the main advantages of the HFP approach, namely that the actual thermal "step", $\Delta T$, can be so small that all material properties can be considered to remain linear throughout the tests discussed in this dissertation. There is no need for large $\Delta T$, or alternatively, to fringe multiply by optically increasing the effective length of the light path through the model with mirrors and multiple passes. The latter procedures multiples the effect of the dimple that occurs at the crack tip and also the gradient effect whereby the path of the two orthogonally polarized beams through the model becomes progressively more separated.

In Figure 5.2, the image in EyeCom memory is displayed with the intensities multiplied 4 times. This is an EyeCom system capability called arithmetic shift. The images in Figure 5.2 were not used in any of the data analysis for this dissertation. They are presented to demonstrate the facility of EyeCom and give a better visual indication of the stress and fringe field.
FIGURE 5.1. Screen photographs of near- and far-end bright field isochromatic fringes for $\beta = 75^\circ$, $t = 5$ min., $\Delta T = 31^\circ$C. AA is the heated edge of the model.
FIGURE 5.2. Light intensities of the same screen photographs of Figure 5.1, magnified four times
Typical time sequences of photographs of the fringe patterns are shown in Figures 5.3 to 5.5. In Figure 5.3, the fringe sequence for both crack tips of the 90° crack are shown. The two upper rows show the far-end tip region. The two lower rows show the near-end tip region. The lines that coincide with AA are the edge of the model that is in contact with the hot-block which was at a temperature = 55°C. $T_1$ = hot-block temperature at 3 mm below contact surface and $T_0$ = room temperature. According to Sanford and Dally's method for classifying mode I, mode II or mixed mode stress intensity factors from isochromatic fringe patterns [99], the isochromatic fringes of the far-end tips are classic mode I (opening mode) patterns where $K_{II} = 0$ and $\sigma_{0x} > 0$. The near-end fringe patterns are not represented in Sanford's paper. They are similar to the far-end tip field except that there are fringe loops ahead of the crack tips. This loop indicates a boundary effect.

Similar examples of the fringe patterns for the 45° and 22.5° cracks are respectively shown in Figure 5.4 and Figure 5.5. The fields in these two sets of photos are mixed mode. Figure 5.6 shows the variation of fringe patterns with angular orientation $\beta$ at a common time $t = 7$ min.

An overall review of the fringe patterns for different cracks shows that, in some cases, the disturbance of the thermal field due to the presence of the crack is so small that the SIFs cannot be determined by the present technique. The technique developed in Chapter IV requires that at least one half or one full fringe order must be present in the
FIGURE 5.3. Photographic prints of dark field fringe patterns at different times for $\beta = 90^\circ$, (mode I). $t$ = time after test model was placed onto the heater. $T_0$ = room temperature = 25$^\circ$C, $T_i$ = surface temperature
FIGURE 5.4. Dark field fringe patterns at different times for $\beta = 45^\circ$ (mixed mode). $t =$ time after test model was placed onto the heater. $T_0 = 25^\circ C$
FIGURE 5.5. Dark field fringe patterns at different times for $\beta = 22.5^\circ$ (mixed mode). $t$ = time after test model was placed onto the heater. $T_0 = 25^\circ$C
For $\beta = 0^\circ$ the 2 tips are similar.

$T_i/T_o = 54.7^\circ C/24.5^\circ C$

$\beta = 0^\circ$

$T_i/T_o = 55.0^\circ C/25^\circ C$

$\beta = 22.5^\circ$

$T_i/T_o = 55.0^\circ C/25^\circ C$

$\beta = 45^\circ$

$T_i/T_o = 55.5^\circ C/24.5^\circ C$

$\beta = 75^\circ$

$T_i/T_o = 55.0^\circ C/25^\circ C$

$\beta = 90^\circ$

FIGURE 5.6. Dark field fringe patterns for different angles of orientation at $t = 7$ min.
image field. In the other words, dark and light intensity references for a selected half fringe interval are indispensable to the application of the piecewise constant model for approximating the envelopes that wrap the light intensity distribution as described in Chapter IV-E. Therefore, no SIFs will be reported for the 0° (Figure 5.7), or the 22.5° crack for the far-end tip at 15 seconds and after 2 minutes (Figure 5.5), or the 45° crack again for the far-end tip at 15 seconds and after 7 minutes (Figure 5.4), or all cracks for the near-end tip before 3 minutes (Figures 5.3 to 5.5).

The different stages in the analysis are described below.

A. Determination of Stress Intensity Factors

In Chapters III and IV, the numerical procedures for matching experimental fringe patterns to the power series representation of the crack tip stress fields were developed. These procedures were used to interpret the fringe patterns for near- and far-end crack tips.

Figures 5.3 to 5.5 show that boundary effects play a significant role on the fringe patterns. Without boundary effects, the fringe order ahead of the crack along the line 8 is constant [93] and the modified Westergaard equations must be used. When a boundary is close enough to influence the SIFs, the isochromatic fringe order varies ahead of the crack and the generalized Westergaard equations must be used to describe the stress state for the region which the 1/√F singularity zone does not dominate any more.
FIGURE 5.7. Dark field fringe patterns at different times for $\beta = 0^\circ$. $T_0 = 24.5°C$. $t = $ time after test model was placed onto the heater. $T_i = $ temperature at time $t$. 

- $T_i = 54.8°C$  
  $t = 1$ min. 

- $T_i = 55.3°C$  
  $t = 3$ min. 

- $T_i = 54.9°C$  
  $t = 5$ min. 

- $T_i = 54.7°C$  
  $t = 7$ min. 

- $T_i = 55.0°C$  
  $t = 15$ min. 

- $T_i = 55.0°C$  
  $t = 30$ min.
The size of the region over which fringe data are acquired determines the number of coefficients needed in each power series expansion to adequately describe the stress field over the region. The stress intensity factors $K_I$ and $K_{II}$ were computed by using the method described in Chapter III. The material properties used were from [95] as given in Table 4.2. The material used for the models in this dissertation was not independently calibrated.

1. Near-End Crack Tips

The computed values of $K_I$ for different times are shown in Figure 5.8 as a function of the number of coefficients $n$ in $A_n$, $m$ in $B_m$ and $p$ in $C_p$ that was used in each of the three power series expansions of equation (3-11). In general, the values for $K_I$ change as the number of coefficients increase from 1 to 3. This change is sharp for the $75^\circ$ and $90^\circ$ cracks and more gentle for the $22.5^\circ$ and $45^\circ$ cracks. For 3 or more coefficients in each of expansions, $K_I$ stabilizes in all test models except the $90^\circ$ case. The odd behavior of the $90^\circ$ crack is also observed in the reconstructed fringe patterns for different numbers of coefficients. This is shown in Figure 5.9. The fringe orders shown in these reconstructed fringe patterns are in an increment of 0.25 fringe order. The corresponding bit map is also included. One of the features of the HFP system is that the graphics display on the monitor screen can be superimposed on the bit map as shown for the 4 coefficient model of the $75^\circ$ crack in Figure 5.9. The comparison between the reconstructed fringes and the related bit map is, in most cases, the cleanest visual check for the adequacy of the back plot.
FIGURE 5.8. Changes in $K_I$ for different times when the order of the coefficients in the analytical model of equation (3-11) is increased from 1 to 5. Arrows indicate the correct values for $K_I$. 
FIGURE 5.8 (continued)
FIGURE 5.8 (continued)
FIGURE 5.8 (continued)

Note that in Figure 5.9, no results are shown for 1 and 2 coefficients. This is because the one coefficient model is essentially the modified Westergaard model which is not suitable for the near-end case and the 2 coefficient model did not provide good back plots at any time. It is clearly inadequate for describing the fringe field. The superimposed fringe patterns of Figure 5.9 show that the 3 and 4 coefficient models both match the salient features of the experimental patterns over the sampled regions around the crack tips. No further improvement is obtained when 5 coefficients are used. The results for the 90° model is exceptional in that the correlation between the back
FIGURE 5.9. Reconstructed fringe patterns for 3, 4 and 5 coefficients in the expansion and the bit maps for the same time for different cracks.
3 Coefficients

4 Coefficients

15.8 mm on the model

5 Coefficients

Bit Map
(4 Coefficients Superimposed)

$\beta = 75^\circ$; $t = 4$ min.

FIGURE 5.9 (continued)
\[ \beta = 45^\circ; \quad t = 10 \text{ min.} \]

FIGURE 5.9 (continued)
\( \beta = 22.5^\circ; t = 3 \text{ min.} \)

FIGURE 5.9 (continued)
plots becomes worse when the number of coefficients in the expansion is increased from 4 to 5. Neither the far-field fringe pattern nor the fringe loop ahead of the crack match well. This poor match corresponds to the abrupt increase in the derived value for $K_I$ from a relatively stabilized value with 3 and 4 coefficients to a new instability when 5 coefficients are used. For the 22.5° and 45° cracks, only the far-field fringe patterns do not show good match. No significant difference can be found in the 75° crack when the number of coefficients increases from 3 to 5.

Similarly, Figure 5.10 shows the variation of $K_{II}$ values versus the number of coefficients in the power series expansion. Relative stability in the values for the SIF shows up in all cracks except the 22.5° crack where mode II (shearing mode) is predominant. $K_{II}$ values increase from stabilized 4 coefficient values to 5 coefficient values.

Note that the $K_{II}$ values for all the 1 coefficient models are positive as shown by the solid symbols in Figure 5.10. All others were negative values. Physically speaking, the sign of $K_{II}$ is immaterial. However, for the purpose of reporting exact results as obtained from the iterative procedure, $K_{II}$ values for 1 and 2 coefficient model are not connected by lines.

The argument and evidence given above led to the decision to use 4 coefficients in each of the three (two for 90° crack) power series expansion of the generalized Westergaard equations. This meant that for each SIF a total of $4 \times 3 = 12$ ($4 \times 2 = 8$ for 90° crack) parameters had to be found by iteration.
FIGURE 5.10. Changes in $K_{II}$ for different times when the order of the analytical model is increased (open symbol indicates negative value, solid symbol indicates positive value)
FIGURE 5.10 (continued)
FIGURE 5.10 (continued)
2. Far-End Crack Tips

The values of $K_I$ for the far-end crack tips at different times were also computed as the number of coefficients $A_n$, $B_m$ and $a_p$ that was used in each of the three (2 for $90^\circ$ crack) power series expansions was increased. The results are plotted in Figure 5.11 except that, instead of plotting $K_I$ versus the number of coefficients in each of the series $A_n$, $B_m$ and $a_p$ of equation (3-11), the total number of coefficients in the three (two for $90^\circ$ crack) expansions are used.

Since there are no fringe loops formed ahead of the far-end crack tips, only one coefficient of the far-field stress function need to be used, however, same number of coefficients will be used in mode I and
FIGURE 5.11. Changes in $K_I$ when the order of the analytical model is increased from 3 to 7 parameters for all crack inclinations except 90° when the order was from 2 to 4. Arrows indicate the correct values for $K_I$. No. of Parameters No. of Parameters
Time: 4 minutes
(Far-End Crack Tip)

Time: 5 minutes
(Far-End Crack Tip)

Time: 7 minutes
(Far-End Crack Tip)

Time: 10 minutes
(Far-End Crack Tip)

FIGURE 5.11 (continued)
mode II stress functions, i.e., uneven number of coefficients will be used in the three power series. Therefore, it would be more convenient to use parameters than coefficients for the presentation of results. Here, the parameter is defined as the sum of the coefficients used in mode I, mode II and far-field stress functions. For example, in the mixed mode case, 5 parameters means 2 coefficients used in both mode I and mode II stress functions with 1 coefficient used in the far-field stress function. In the mode I case, only one coefficient of the far-field stress function is used. Therefore, for a 3 parameter model, 2 coefficients of the mode I stress function shall be used.
Unlike the near-end crack tips, the stability characteristics of the $K_I$ values with increasing number of coefficients does not exist. $K_I$ generally decrease as the number of parameters increase except for the 22.5° case. Like the near-end case, 90° crack is the one most predominant by mode I. $K_I$ values become lower when the crack becomes less inclined to the heated edge.

The reconstructed fringe patterns obtained by using 3, 5 and 7 parameters (2, 3 and 4 parameters for 90° crack) in the power series expansion are shown in Figure 5.12.

For the 90° crack, very slight differences can be seen between 2 and 3 parameter models. Distinct differences can be found for 75°, 45° and 22.5° when parameters used increased from 3 to 5. In all test models, the 7 parameter model (4 parameters for 90°) is worse than the 5 parameter model (3 parameters for 90°) except the 22.5° case for which only slight differences can be observed. Since the fringe orders in the region of the far-end tips are lower than those of the near-end tips, the degree of agreement between the observed fringes and the reconstructed fringes can be seen better in the corresponding bit maps. These are illustrated in the last frame of each of the sets of photographs of Figure 5.12.

The 5 parameter (3 parameters for 90°) model provided the best fit solution to the stress fields. For pure mode I problems, a similar conclusion was drawn by Etheridge and Dally [69], Rossmanith and Irwin [77], Rossmanith and Chona [100] and Chona et al. [101]. This
FIGURE 5.12. Reconstructed fringe patterns plus the bit maps with reconstructed fringe patterns

$\beta = 90^\circ; t = 5\text{ min.}$
Figure 5.12 (continued)

$\beta = 75^\circ; t = 4 \text{ min.}$
3 Parameters

5 Parameters

11.5 mm on the model

7 Parameters

Bit Map
(5 Parameters Superimposed)

$\beta = 45^\circ$; $t = 3$ min.

FIGURE 5.12 (continued)
β = 22.5°; t = 1 min.

FIGURE 5.12 (continued)
dissertation is the first to report the behavior of the model versus the method of analysis for a mixed mode problem.

Similarly, Figure 5.13 depicts the variation of $K_{II}$ values versus the number of coefficients in the power series expansion. In general, $K_{II}$ values remain constant after a 3 parameter model was used except the latter events of the 75° crack. It is interesting to see that $K_{II}$ values of 45° crack are larger than those of 22.5° crack. This is opposite to the results obtained from the near-end tips and is a good example of showing the complexity of the thermal stress field in comparison with the mechanical stress field.

B. Effects of Crack Inclination on the Stress Intensity Factors

The magnitude of the theoretical maximum axial stress for a bar under full axial restraint and subjected to a temperature change $\Delta T$ is $\alpha E\Delta T$. Also, the SIF for an infinite plate subjected to uniform tensile stress $\sigma$, and that contains a through-thickness crack of length $2a$ is $\sigma \sqrt{a}$. For the models and the investigation reported in this research, the nondimensional SIFs can then be defined as

\begin{equation}
(5-1) \quad \bar{K}_I = \frac{K_I}{\alpha E(T_m - T_0)\sqrt{a}} \quad \text{and} \quad \bar{K}_{II} = \frac{K_{II}}{\alpha E(T_m - T_0)\sqrt{a}}
\end{equation}

where $T_m$ = temperature of the copper heating block as measured by the thermocouples;

$T_0$ = ambient temperature, i.e., the initial temperature of the test piece;
FIGURE 5.13. Changes in $K_{II}$ for different times when the order of the analytical model is increased from 3 to 7 parameters (all values are negative). Arrows indicate the correct values for $K_{II}$.
FIGURE 5.13 (continued)
The variation of the $K_I$ and $K_{II}$ values with crack inclination at various times is shown in Figure 5.14 and Figure 5.15, respectively. The straight lines in the plots simply connect the data points to show the general trend.

For both far- and near-end crack tips, there is a general decrease in $K_I$ with decreasing crack inclination. A linear relationship exists between $K_I$ values and crack inclination from $22.5^\circ$ to $75^\circ$ for near-end case. No general conclusion can be made for the far-end case due to the
FIGURE 5.14. Variation of $K_I$ with crack inclination for different times. Four coefficient model used for near-end and 5 parameter (3 parameters for 90°) model used for far-end
FIGURE 5.14 (continued)
FIGURE 5.15. Variation of $K_{II}$ with crack inclination for different times. Four coefficient model used for near-end and 5 parameter (3 parameters for 90°) model used for far-end
FIGURE 5.15 (continued)
lack of data of 22.5° crack. From 75° to 90°, obvious deviation from the line connecting data points between 22.5° and 75° can be seen.

The same tendency of $K_{II}$ shows up in all time events for near-end tips. There is a general increase in $K_{II}$ with decreasing crack inclination. With only two data points at 22.5°, it is not possible to draw a firm conclusion but Figure 5.14 suggests that for the far-end tips, $K_{II}$ values peak around 45°.

C. Variation of Stress Intensity Factors with Time

Figures 5.16 and 5.17 respectively depict the variation of $K_I$ and $K_{II}$ with time for each of the cracks considered. For all the cracks tested, except 90°, a common behavior of $K_I$ exists for the near-end crack tip as shown in Figure 5.16. With increasing time, the $K_I$ value increased rapidly to a peak value after which $K_I$ decreased slowly to a steady state after 16 to 20 minutes. This feature can also be observed in Figures 5.2 to 5.5. The time when the peak values of $K_I$ occurred was earlier for cracks with small angles of inclination, $\beta$. This is expected because the thermal obstruction is larger for the less inclined crack.

For the 90° crack the $K_I$ values increased with time and approached constant value. This is because the 90° crack is the one that provides the least obstruction to thermal flow.

The $K_I$ values for the crack tip remote from the boundary follow the same trend as for $K_I$ at near boundary tip. The peaks are more sharply
FIGURE 5.16. Variation of $K_I$ with time for different angles
FIGURE 5.16 (continued)
FIGURE 5.17. Variation of $K_{II}$ with time for different angles

- Crack inclination: $22.5^\circ$
- Crack inclination: $45^\circ$
- Crack inclination: $75^\circ$
FIGURE 5.17 (continued)
defined and the 90° crack behaves like the others. The peak occurred at about 3 min. compared to 5 to 16 minutes for the near-end tips.

It is clear that the extent of the unperturbed temperature and stress fields are smaller than or comparable to the crack length. Even the 0° crack which provides the largest thermal obstruction shows this clearly (Figure 5.7). This explains why the variation of $\overline{R}_I$ versus time is progressively asymptotic to the steady state value.

In Figure 5.17, the $\overline{R}_{II}$ values of the near-end tip of the 75° crack increased gradually until it attained a constant value. However, for the far-end tip, $\overline{R}_{II}$ reached a peak first and then dropped to a constant value. The $\overline{R}_{II}$ values of the 22.5° and 45° cracks of the near-end and the 45° crack of the far-end indicated similar behavior. They all reached a peak rapidly and then dropped to a long time steady value.

Hovanesian and Kowalski [40] worked out a set of similarity relations which included temperature-scaling factors, size factors and time-scaling factors between prototype and model. With subscripts $m$ for model and $p$ for prototype, the time ratio $t_r$ which relates the time for a certain temperature profile to develop in a real structure to that in a model is

$$t_r = \frac{t_p}{t_m}$$

or

$$t_r = \frac{\alpha_{tm}}{\alpha_{tp}} (\lambda)^2 = \frac{\alpha_{tm}}{\alpha_{tp}} \left(\frac{L_p}{L_m}\right)^2$$

where $\alpha_t =$ thermal diffusivity;
L = size of specimen;
λ = size factor between the model and prototype.

Typical values for $\alpha_c$ are: steel = $5 \times 10^{-2}$ m$^2$/hr (0.54 ft$^2$/hr),
PSM-1 = $1.01 \times 10^{-3}$ m$^2$/hr (1.09 $\times 10^{-2}$ ft$^2$/hr). Thus, the time for an
event to occur in the model material may be several hundred times longer
than in a metal prototype of the same size. Such a slow down of events
in the model is an obvious advantage when studying thermal shock and
other rapidly occurring events.

Similarly, these similarity relations predict that for plane-stress
conditions, the stress ratio, $\sigma_r$, from prototype to model is

$$\sigma_r = \frac{\sigma_p}{\sigma_m} = \frac{E_p}{E_m} \frac{\alpha_{tp}}{\alpha_{tm}} \frac{\Delta T_p}{\Delta T_m}$$

where $E$ is the elastic modulus and $\Delta T$ the temperature difference.

Typical values for $E$ are: steel = $21 \times 10^{10}$ N/m$^2$ (30 $\times 10^6$ psi), PSM-1
= $2.39 \times 10^9$ N/m$^2$ (3.47 $\times 10^5$ psi).

In a nuclear power plant, the temperature difference between the
hot steel pressure vessel wall and the emergency core coolant in the
event of a loss-of-coolant accident (LOCA) is about 250°C. Based on the
model experiment in this study, we have time ratio for a 1:2 scale as

$$t_r = \frac{0.00101}{0.05} (2)^2 = 0.08$$

and stress ratio as

$$\sigma_r = \frac{21 \times 10^{10}}{2.39 \times 10^9} \frac{0.05}{0.00101} \frac{250}{30} = 36250$$
With the nondimensional values of $K_I$ and $K_{II}$ presented in Figures 5.16 and 5.17 as well as the time and stress ratios mentioned above, one can estimate the SIFs for a structure which contains internal defect near the surface under transient thermal loading.
VI. CONCLUSION

In this study, combination of photothermoelastic and modern digital image analysis techniques were used for acquiring the needed information from experimentally obtained photographs. The genuine idea of "half-fringe photoelasticity" (HFP) [74] was generalized.

A set of computer programs were developed for calibration of the HFP system, data collection, calculation of the transient thermal SIFs and reconstruction of the fringe patterns.

Different equations were respectively used to obtain the SIFs for far- and near-end crack tips, utilizing a multiparameter-multipoint technique based on the least squares overdeterministic approach developed by Sanford and Dally [73]. Specifically speaking, the modified Westergaard equations were used for the far-end crack tips and the generalized Westergaard equations were adopted for modelling the isochromatic fringe patterns around the near-end crack tips.

Furthermore, a special improvement on the convergence of the iterative procedure used by Sanford and Dally, i.e., the Newton-Raphson method, was attempted. The new method is termed as "generalized Newton-Raphson method" and was found to be very effective for improving the convergence rate of a large set of nonlinear algebraic equations.

The accuracy of the data collected was checked by comparing the reconstructed fringe patterns with the experimentally obtained ones. In addition, the built-in hardware procedure, bit map, was also used.
It was found out from this study that 12 parameter (8 parameters for 90° crack) model provides the best fit solution to the isochromatic fringe patterns around the near-end crack tips. For far-end crack tips, 5 parameter (3 parameters for 90° crack) model proved to be adequate one.

The variation of the stress intensity factors with time for cracks tested is presented. From the results obtained, it is seen that crack under transient thermal loading can be dangerous since the transient thermal SIFs go through a peak value larger than the steady state. This behavior was observed both in the variation of $K_I$ and $K_{II}$ values with time except $K_I$ values of 90° crack and $K_{II}$ values of 75° crack of near-end. For those two cracks, the SIFs increased with time and approached constant values (see Figures 5.16 and 5.17).

Based on the results of the present study, several recommendations can be made for future investigations in this field. They are as follows:

- Full parametric study, i.e., the effects of the ligament size and the crack length on the stress intensity factors, should be carried out. Once the stress intensity factors are known as a function of time, it becomes possible in brittle materials to determine whether catastrophic failure will occur due to unstable crack propagation. Also, the amount of crack growth which occurs after repeated thermal shocks can be estimated using experimental crack growth rate expressions which utilizes
the stress intensity factor information given as a function of crack length.

• Further work to determine the effects of a time dependent boundary condition is suggested. Such repetitive changes of the temperature along the straight boundary of the semi-infinite plate would be of interest when investigating thermal fatigue problems.

• Due to the characteristics of transient problem and the limitation of one memory capacity of EyeCom II system, using of photographic negatives to record the transient events are necessary. Recognizing the problems when using negatives in this study, however, live images, i.e., on-line work may be an alternative way to collect data. During the period of development of this work, another more advanced image analysis system, EyeCom III, was acquired. It has up to 8 memories and more advanced features. The new approach is currently under development.
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APPENDIX A - GENERALIZED NEWTON-RAPHSON METHOD

The solution to an equation \( f(x) = 0 \) may often be found by a simple procedure known as the Newton-Raphson method. Note that \( f \) should be continuously differentiable, but not necessarily linear. This method consists of drawing the tangent to the curve at the point \( A \), as shown in Figure A.1. The \( x \)-intercept of the tangent, or \( x_2 \), is then used as the first approximation. Figure A.1 shows the use of the Newton-Raphson method to solve \( f(x) = 0 \) to the root \( x_T \). From the figure, we have

\[
f'(x_1) = \tan \theta = \frac{f(x_1)}{x_1 - x_2}
\]

where \( f'(x_1) \) denotes the derivative of \( f(x) \) (or slope of the tangent line \( L_1 \) evaluated at \( x = x_1 \)).

Hence

\[ x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}, \quad f'(x_1) \neq 0 \]

An iterative sequence can now be set up as follows:

\[ (A-1) \quad x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad f'(x_n) \neq 0 \]

This formula can be repeatedly used to find improved approximation to the real root \( x_T \).

Of course, it would be of value to have a method which yields a real root in fewer iterations than the Newton-Raphson method. For this reason, instead of constructing the line \( L_1 \) shown in Figure A.1, a line through \((x_1, y_1)\) which intersects the \( x \)-axis closer to \( x_T \) than \( x_1 \) is constructed. Such a line would have an equation of the form
FIGURE A.1. Newton-Raphson Method
This line has a different slope than \( L_1 \). Setting \( y = 0 \) and \( x = x_2 \) in \((A-2)\) yields
\[
x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}, \quad \text{if } f'(x_1) \neq 0
\]
and just as Newton-Raphson method, a recursion formula would be as follows:

\[
(A-3) \quad x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad \text{if } f'(x_n) \neq 0
\]

Set \( \omega = \frac{1}{\tau} \), equation \((A-3)\) becomes

\[
(A-4) \quad x_{n+1} = x_n - \omega \frac{f(x_n)}{f'(x_n)}, \quad \text{if } f'(x_n) \neq 0
\]

which is called the generalized Newton-Raphson formula.

In equation \((A-4)\) the constant \( \omega \) is called an relaxation factor, and the modified Newton-Raphson method which uses equation \((A-4)\) in place of equation \((A-1)\) is called the generalized Newton-Raphson method. A good choice for \( \omega \) can, in general, only be made after experimentation in the range \( 0 < \omega < 2 \). A choice of \( \omega \) different from unity often can increase the convergence rate appreciably.