On the transition between turbulence regimes in particle-laden channel flows

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Abstract
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In the present study, a series of simulations of vertical particle-laden channel flows with increasing mass loading is conducted to analyse the transition from the dilute limit where classical mean-shear production is primarily responsible for generating fluid-phase TKE to high-mass-loading suspensions dominated by drag production. Eulerian–Lagrangian simulations are performed for a wide range of particle loadings at two values of the Stokes number, and the corresponding two-phase energy balances are reported to identify the mechanisms responsible for the observed transition.

Disciplines
Chemical Engineering | Complex Fluids | Membrane Science | Transport Phenomena

Comments
On the transition between turbulence regimes in particle-laden channel flows

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Turbulent wall-bounded flows exhibit a wide range of regimes with significant interaction between scales. The fluid dynamics associated with single-phase channel flows is predominantly characterized by the Reynolds number. Meanwhile, vastly different behaviour exists in particle-laden channel flows, even at a fixed Reynolds number. Vertical turbulent channel flows seeded with a low concentration of inertial particles are known to exhibit segregation in the particle distribution without significant modification to the underlying turbulent kinetic energy (TKE). At moderate (but still low) concentrations, enhancement or attenuation of fluid-phase TKE results from increased dissipation and wakes past individual particles. Recent studies have shown that denser suspensions significantly alter the two-phase dynamics, where the majority of TKE is generated by interphase coupling (i.e. drag) between the carrier gas and clusters of particles that fall near the channel wall. In the present study, a series of simulations of vertical particle-laden channel flows with increasing mass loading is conducted to analyse the transition from the dilute limit where classical mean-shear production is primarily responsible for generating fluid-phase TKE to high-mass-loading suspensions dominated by drag production. Eulerian–Lagrangian simulations are performed for a wide range of particle loadings at two values of the Stokes number, and the corresponding two-phase energy balances are reported to identify the mechanisms responsible for the observed transition.

1. Introduction

Wall-bounded disperse two-phase flows play an important role in many environmental and industrial applications. Some examples include liquid–solid slurry pipelines, sediment deposition in marine flows, foreign debris in gas turbine engines, and fluidized bed reactors. Accurate predictions of such flows are necessary in order to gain a detailed understanding of the fundamental processes taking place, and ultimately improve the design of engineering devices. Meanwhile, non-trivial interactions between the carrier fluid and particulate phase lead to a wide range of two-phase flow regimes that may exist simultaneously within a single flow.

Many studies on wall-bounded particle-laden flows consider moderately dilute suspensions with weak interphase coupling (see e.g. Kulick \textit{et al.} 1994; Wang \& Squires 1996; Rouson \& Eaton 2001; Yamamoto \textit{et al.} 2001; Marchioli \& Soldati 2002; Picciotto \textit{et al.} 2005; Pitton \textit{et al.} 2012; Zhao \textit{et al.} 2013). In this regime, the majority of the underlying carrier-phase turbulence manifests from classical mean-shear production (Zhao \textit{et al.}}
Experiments (e.g. Fessler et al. 1994) and fully-resolved (model-free) calculations (e.g. Garcia-Villalba et al. 2012) have shown that wakes generated by small \((d_p/\eta \ll 1)\) and large \((d_p/\eta \gg 1)\) particles (with \(d_p\) the particle diameter and \(\eta\) the Kolmogorov length scale) could have a direct contribution to the turbulent kinetic energy (TKE) production and dissipation even at low particle seedings. Tanaka (2017) performed fully resolved simulations of particle-laden homogeneous shear turbulence and showed that finite particle inertia generates mean slip velocities between the phases in both the streamwise and shear direction, resulting in a slower increase in fluid-phase TKE. When gravity is directed in the sheared direction, the Reynolds shear stress becomes relatively large between counter-rotating trailing vortices downstream of particles, which was found to contribute to a transient rapid increase in TKE (Tanaka 2017).

When the mean mass loading \(\Phi\), defined by the ratio of the specific masses of the particle and fluid phases, is order one or larger, the relative motion between the phases leads to additional sources of instabilities as a result of interphase coupling (Glasser et al. 1998). Gualtieri et al. (2017) found that sub-Kolmogorov particles with \(\mathcal{O}(1)\) mass loading modify the energy spectrum in homogeneous shear flows, leading to a scaling law \(E(k) \propto k^{-4}\), with \(k\) the wave number, that emerges at small scales where particle forcing balances viscous dissipation. The particles were found to drain energy from the carrier flow at large scales and release it back at small scales. Dritselis (2016) evaluated budgets of the Reynolds stress in a vertical channel flow seeded with sub-Kolmogorov particles at intermediate loading \((\Phi \leq 0.5)\). Each term in the Reynolds stress budget was found to decrease in the presence of particles. The extent to which the budgets were reduced was found to depend on the particle response time, with greater effect when collisions were included. It was observed that large flow scales are generally dragged by small inertial particles, while large inertial particles are capable of interacting with a wider range of moderate flow scales.

At significantly large mass loadings, particles self-organize into dense clusters that greatly impact the overall flow structure (e.g. Agrawal et al. 2001; Capecelatro et al. 2014a). Vreman et al. (2009) performed simulations of a high-mass-loading turbulent channel flow and found that when \(\Phi \gg 1\), particle collisions have a large influence on the mean and root-mean-square velocities of each phase. Particles were also observed to decrease the thickness of the boundary layer and increase the skin friction. A similar modification to the near-wall velocity profile was observed in dense suspensions of neutrally buoyant particles (Picano et al. 2015; Santarelli et al. 2016; Lashgari et al. 2016, 2017). Even at small density ratios, significant turbulence modulation was observed in those studies, with three distinct regimes revealed by the Reynolds stress budget: laminar, turbulent and inertial shear-thickening depending on the relative contribution from viscous and particle stresses on the momentum transfer across the channel. Capecelatro et al. (2016a) recently performed simulations of high-mass-loading \((\Phi = 10)\) channel flows, demonstrating that in this regime the primary source of turbulence generation is a term that is proportional to the product of mass loading, drift velocity and mean slip between the phases, termed drag production. It was also shown that the fluid-phase Reynolds-stress budget differs significantly from what is observed in dilute particle-laden channel flows.

Surprisingly, it remains to be seen how wall-bounded particle-laden flows transition from the dilute limit in which classical mean-shear production is primarily responsible for generating fluid-phase TKE, to the high-mass-loading regime dominated by drag production. In this study, we explore the transition between these two states and identify the key mechanisms responsible for the striking differences. Eulerian–Lagrangian simulations are conducted for \(0 \leq \Phi \leq 20\) in a \(Re_\tau = 300\) turbulent channel flow at two Stokes num-
bers. Details on the mathematical formulation are presented in §2. The vertical channel configuration is then described in §3, and visualisations and energy budgets are reported in §4.

2. Mathematical description

In this section we present the governing equations describing solid spherical particles suspended in a constant-density gas-phase channel. Unlike in single-phase flows where the Navier–Stokes equations can be directly averaged to obtain a set of mean-flow equations (Pope 2000), special care needs to be taken for turbulent multiphase flows. As discussed in Fox (2014), averaging the microscale equations (i.e. a model that resolves the boundary layers around each particle), will fail to retain important fluid–particle coupling terms. For example, in flows where fluctuations in the particle-phase volume fraction \( \alpha_p \) play an important role (see, e.g. Capecelatro et al. 2015), the averaged microscale equations will not contain information about such fluctuations. Deriving the mean-flow equations starting from a mesoscopic description will retain more physics by explicitly accounting for important interphase coupling terms. The mesoscale equations are presented in the following section, followed by the macroscopic mean flow equations.

2.1. Volume-filtered Euler–Lagrange equations

The displacement of an individual particle \( i \) with diameter \( d_p \) is calculated via

\[
v_p^{(i)} = \frac{dx_p^{(i)}}{dt} \quad \text{and} \quad \frac{dv_p^{(i)}}{dt} = \mathbf{A}^{(i)} + \mathbf{F}_c^{(i)} + \mathbf{g}
\]

(2.1)

where \( v_p^{(i)}(t) \) is the instantaneous particle velocity at time \( t \), \( \mathbf{g} = [g, 0, 0]^T \) is the acceleration due to gravity and \( \mathbf{F}_c \) is the collision force modelled using a modified soft-sphere approach (Capecelatro & Desjardins 2013) originally proposed by Cundall & Strack (1979). In this work, we consider inelastic collisions with a coefficient of restitution of 0.9 for both particle–particle and particle–wall collisions. The interphase-exchange term is given by

\[
\mathbf{A}^{(i)} = \frac{1}{\tau_p} \left( u_f^{(i)}[x_p^{(i)}] - v_p^{(i)} \right) - \frac{1}{\rho_f} \nabla \sigma_f[x_p^{(i)}] + \frac{1}{\rho_p} \nabla \cdot \sigma_f[x_p^{(i)}]
\]

(2.2)

where \( \tau_p = \rho_p d_p^2 / (18 \rho_f \nu_f) \) is the particle relaxation time, with \( \rho_p \) and \( \rho_f \) the particle- and fluid-phase material densities, respectively, and \( \nu_f \) is the kinematic viscosity of the fluid. The fluid velocity \( u_f = [u_f, v_f, w_f]^T \), modified pressure gradient \( \nabla p_f^* \) and divergence of the viscous-stress tensor \( \nabla \cdot \sigma_f \) are taken at \( x_p^{(i)} \), the center position of particle \( i \). The term \( \nabla p_f^* \) is a body force that contains the hydrodynamic pressure \( p_f \) and is adjusted dynamically in order to maintain a constant mass flow rate in the channel.

To account for the presence of particles in the fluid phase without requiring to resolve the boundary layers around individual particles, a volume filter is applied to the constant-density Navier–Stokes equations (Anderson & Jackson 1967), thereby replacing the point variables (fluid velocity, pressure, etc.) by smoother, locally filtered fields. The volume-filtered conservation equations for a constant-density fluid are given by

\[
\frac{\partial \alpha_f}{\partial t} + \nabla \cdot \alpha_f u_f = 0
\]

(2.3)

and

\[
\frac{\partial \alpha_f u_f}{\partial t} + \nabla \cdot (\alpha_f u_f \otimes u_f) = - \frac{1}{\rho_f} \nabla p_f^* + \nabla \cdot \sigma_f - \frac{\rho_f}{\rho_p} \alpha_p \mathbf{A} + \alpha_f \mathbf{g}
\]

(2.4)
where $\alpha_f$ is the fluid-phase volume fraction and $\alpha_p = 1 - \alpha_f$. The fluid-phase viscous-stress tensor is defined as

$$\sigma_f = (\nu_f + \nu^*) \left[ \nabla u_f + (\nabla u_f)^T - \frac{2}{3} \nabla \cdot u_f I \right] \tag{2.5}$$

where $I$ is the identity tensor and $\nu^*$ is an effective viscosity that accounts for enhanced dissipation due to unresolved fluid-velocity fluctuations generated at the particle scale (Gibilaro et al. 2007). Details on the interphase-exchange term $\mathcal{A}$ are discussed in §3.2.

2.2. Phase-averaged flow equations

Analogous to Favre averaging in variable density flows, the phase average (PA) denoted by $\langle (\cdot) \rangle_f = \langle \alpha_f (\cdot) \rangle / \langle \alpha_f \rangle$ is useful in multiphase modelling. Here, angled brackets denote an average in the homogeneous $x$- and $z$-directions, i.e. $\langle \cdot \rangle(y) = \int f(\cdot) \, dx \, dz$, with $y$ the wall-normal direction. Fluctuations about the PA fluid velocity are expressed as $u''_f(x,t) = u_f(x,t) - \langle u_f \rangle_f$, with $\langle u''_f \rangle_f = 0$. It is important to note that because the fluid velocity is correlated to volume-fraction fluctuations, $\langle u''_f \rangle \neq 0$ in general. With this, the PA fluid TKE is

$$k_f = \frac{1}{2} \langle u''_f \cdot u''_f \rangle_f. \tag{2.6}$$

Similarly, the PA operator with respect to the particle phase can be defined via $\langle (\cdot) \rangle_p = \langle \alpha_p (\cdot) \rangle / \langle \alpha_p \rangle$, with fluctuations about the PA particle velocity given by $u''_p(x,t) = u_p(x,t) - \langle u_p \rangle_p$. Here, $u_p = [u_p, v_p, w_p]^T$ is the spatially correlated component of the particle velocity (Février et al. 2005; Capecelatro et al. 2014b) used in constructing the particle-phase TKE, given by

$$k_p = \frac{1}{2} \langle u''_p \cdot u''_p \rangle_p. \tag{2.7}$$

As described in Capecelatro et al. (2015), special care needs to be taken when decomposing the local particle velocity $u''_p$ into its spatially correlated and uncorrelated components, i.e. $u''_p = u_p[x_i] + \delta u''_p$, where $\delta u''_p$ represents the random uncorrelated motion used in defining the granular temperature (Février et al. 2005)

$$\Theta = \frac{1}{3} \delta v^{(i)} \cdot \delta v^{(i)}_p. \tag{2.8}$$

With these definitions, the total granular energy is

$$\kappa = k_p + \frac{3}{2} \langle \Theta \rangle_p. \tag{2.9}$$

In fully developed two-phase vertical channel flows, the evolution of $k_f$ is given by (Capecelatro et al. 2016a)

$$\frac{1}{\langle \alpha_f \rangle} \frac{d}{dy} \left( \frac{1}{2} \langle \alpha_f \rangle \langle u''_f \cdot u''_f \rangle_f + \mathcal{E} \right) = \mathcal{P}_{\text{shear}} - \varepsilon + \mathcal{D}\mathcal{E} + \mathcal{DP}. \tag{2.10}$$

The terms on the left-hand side and first two terms on the right-hand side of (2.10) do not involve mixed statistics (i.e. fluid–particle correlations) and have the same form as in single-phase turbulent flow (Pope 2000). The triple correlation $\frac{1}{2} \langle u''_f \cdot u''_f \rangle_f$ will be referred to hereinafter as turbulent convection. The term containing pressure redistribution and viscous diffusion,

$$\mathcal{E} = (pu''_f) / \rho_f - \langle \sigma_{f,y} \cdot u''_f \rangle, \tag{2.11}$$

typically becomes important in near-wall regions of the flow. The dissipation rate is given
Turbulence transition in particle-laden channel flow

by

\[ \varepsilon = \frac{1}{\langle \alpha_f \rangle} \langle \sigma_f : \nabla u'' \rangle, \]  
(2.12)

and in the absence of a disperse phase, production due to mean shear, given by

\[ P_{\text{shear}} = -\langle u'' v''' \rangle_f \frac{d\langle u_f \rangle_f}{dy}, \]  
(2.13)

is the primary mechanism by which \( k_f \) is generated.

The remaining terms in the Reynolds-stress balance contain fluid–particle correlations that become important when interphase coupling is significant. The drag-dissipation-and-exchange rate

\[ \mathcal{D} = \frac{\phi}{\tau_p} \left( \langle u''^p \rangle_p - \langle u'' \rangle_f ight) \]  
(2.14)

describes how the turbulent kinetic energies are both dissipated and exchanged between the phases, where \( \phi(y) = \rho_p (\alpha_p) / (\rho_f (\alpha_f)) \) is the average mass loading. Finally, the drag-production term

\[ \mathcal{D} = \frac{\phi}{\tau_p} u_d \left( \langle u_p \rangle_p - \langle u_f \rangle_f \right) \]  
(2.15)

describes how fluid-phase turbulent kinetic energy is produced by a mean-velocity difference between the phases. The drift velocity, defined as \( u_d = \langle u_f \rangle_p - \langle u_f \rangle_f = \langle u''^p \rangle_p \) is directly responsible for generating fluid-phase TKE. In the limit \( \phi \gg 1 \), drag production can be much larger than mean-gradient production (i.e. \( \mathcal{D} \gg P_{\text{shear}} \)), except very near the walls where \( u_d = 0 \) due to the no-slip boundary condition for the fluid (Capecelatro et al. 2016a). It should be noted that (2.14) and (2.15) were formulated assuming Stokes drag. Accounting for drag coefficients with non-linear dependencies on Reynolds number and volume fraction (e.g. Tenneti et al. 2010) will result in higher-order terms that need to be accounted for (similarly to what appears in filtered two-fluid models, e.g. Igci et al. 2008), but are neglected in this work. A discussion on the effect of using a non-linear drag correlation can be found in appendix A.

3. Vertical particle-laden channel flow

3.1. System description

In this study we consider a turbulent channel flow with friction Reynolds number \( Re_\tau = 300 \) based on the channel half-width \( \delta \) and \( u_\tau \), the friction velocity of the corresponding unladen channel. The domain size is \( 20\delta \times 2\delta \times 3\delta \) in the streamwise \( (x) \), wall-normal \( (y) \), and spanwise \( (z) \) direction, respectively. Periodic boundary conditions are imposed in the homogeneous \( x \)- and \( z \)-directions with gravity \( g = -9.81 \text{ m s}^{-2} \) acting in \( x \). Uniform mesh spacing is imposed in the \( x \)- and \( z \)-directions of size \( \Delta x^+ = \Delta z^+ = 4.6 \), with a total grid size of \( 1250 \times 138 \times 188 \). Here, the superscript + denotes normalization with the viscous scales for length \( \nu/u_\tau \) and time \( \nu/u_\tau^2 \). The mesh spacing is continuously stretched in the wall-normal direction as described in Kim et al. (1987), with \( y_j = \cos \theta_j \), for \( \theta_j = (j-1)\pi/(N_y - 1), j = 1, 2, \ldots, N_y \), where \( N_y \) is the number of grid points in \( y \). This corresponds to a maximum wall-normal spacing of \( \Delta y^+ = 6.78 \) at the channel center, and \( \Delta y^+ = 0.08 \) at the channel wall.

Once the flow reaches a statistically stationary state, the channel is seeded with a random distribution of solid spherical particles. The number of particles used in each
In addition, setting \( A \) et al. velocity in clustered gas–solid flows (Capecelatro constant value of \( \delta \) and drag-production terms (2.14)–(2.15). It was found in our previous work that using a dynamically based on \( \alpha \) the spatial coordinate \( \alpha \) of the discontinuous Lagrangian data with an Eulerian field that is a smooth function of where \( G \) at particle \( \i \) is used. To send the particle data back to the Eulerian mesh, a quantity \( A \) transfer the fluid variables to the particle location, second-order tri-linear interpolation conservative low-Mach number solver with second-order accuracy in space and time. To The Eulerian–Lagrangian equations are solved in NGA (Desjardins et al. 2008), a fully conservative low-Mach number solver with second-order accuracy in space and time. To allow for fully developed clusters; while for the larger Stokes number case cluster formation will be hindered by the channel walls. As the spatially correlated energy \( k \) is largely associated with clusters at high mass loading (Capecelatro et al. 2015), it can be expected that the larger Stokes number case will have higher levels of uncorrelated granular motion as compared to fully developed cluster-induced turbulence.

### 3.2. Discretization

The Eulerian–Lagrangian equations are solved in NGA (Desjardins et al. 2008), a fully conservative low-Mach number solver with second-order accuracy in space and time. To transfer the fluid variables to the particle location, second-order tri-linear interpolation is used. To send the particle data back to the Eulerian mesh, a quantity \( A^{(i)}(t) \) located at particle \( i \) at time \( t \) is projected to the grid via

\[
\alpha_p A(x, t) = \sum_{i=1}^{N_p} A^{(i)}(t) \mathcal{G}(|x - x^{(i)}_p|) V_p
\]

where \( \mathcal{G} \) is a filtering kernel with characteristic size \( \delta_f = 8d_p \). This expression replaces the discontinuous Lagrangian data with an Eulerian field that is a smooth function of the spatial coordinate \( x \). Using (3.2) with \( A^{(i)} = 1 \) yields the particle volume fraction \( \alpha_p \), and \( A^{(i)} = \mathcal{A}^{(i)} \) gives the momentum exchange term \( \mathcal{A} \) seen by the fluid in (2.4). In addition, setting \( A^{(i)} = v_p^{(i)} \) in (3.2) will yield \( u_p \) that appears in the drag-exchange and drag-production terms (2.14)–(2.15). It was found in our previous work that using a constant value of \( \delta_f \) will provide a poor representation of the spatially correlated particle velocity in clustered gas–solid flows (Capecelatro et al. 2015). Instead, \( \delta_f \) is adjusted dynamically based on \( \alpha_p \) when computing \( u_p \) such that a constant number of particles, \( N_p \), are sampled, i.e.

\[
\delta_f(\alpha_p) = \left( \frac{N_p d_p^2}{\alpha_p} \right)^{1/3}. \tag{3.3}
\]
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Physical parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_b$</td>
<td>5.02 m s$^{-1}$</td>
</tr>
<tr>
<td>$2\delta$</td>
<td>3.6 cm</td>
</tr>
<tr>
<td>$g$</td>
<td>9.81 m s$^{-2}$</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>2000 kg m$^{-3}$</td>
</tr>
<tr>
<td>$\rho_f$</td>
<td>1 kg m$^{-3}$</td>
</tr>
<tr>
<td>$\nu_f$</td>
<td>1.8 x 10$^{-5}$ m$^2$ s$^{-1}$</td>
</tr>
<tr>
<td>$e$</td>
<td>0.9</td>
</tr>
<tr>
<td>$d_p$</td>
<td>45.5 µm</td>
</tr>
<tr>
<td>$\mathcal{L}$</td>
<td>0.16, 16.1 cm</td>
</tr>
</tbody>
</table>

Table 1: Fluid–particle parameters used in the channel simulations. Reynolds numbers are defined as $Re_\delta = 2U_b\delta/\nu$, $Re_\tau = u_\tau\delta/\nu$ and $Re_p = \tau_p g d_p/\nu$. The Stokes numbers are defined as $St_\delta = \tau_p U_b/(2\delta)$ and $St_\tau = \tau_p u_\tau/\delta$. $\eta_c$ and $\eta_w$ are the Kolmogorov length scales at the centre and wall of the single-phase channel flow, respectively.

Non-dimensional parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Re_\delta$</td>
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</tr>
<tr>
<td>$Re_\tau$</td>
<td>300</td>
</tr>
<tr>
<td>$Re_p$</td>
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</tr>
<tr>
<td>$St_\delta$</td>
<td>1.79, 17.9</td>
</tr>
<tr>
<td>$St_\tau$</td>
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</tr>
<tr>
<td>$d_p/\eta_c$</td>
<td>0.16, 0.49</td>
</tr>
<tr>
<td>$d_p/\eta_w$</td>
<td>0.78, 2.46</td>
</tr>
<tr>
<td>$d_p^+ \delta$</td>
<td>0.74, 2.35</td>
</tr>
<tr>
<td>$\mathcal{L}/(2\delta)$</td>
<td>0.05, 4.46</td>
</tr>
</tbody>
</table>

Applying the adaptive filter in an a priori manner to the particle data was found to yield a partition of $\kappa$ into $k_p$ and $\langle \Theta \rangle_p$ insensitive to the choice of parameters. Further details can be found in Capecelatro et al. (2015, 2016a).

Special care must be taken in the near-wall region where particles are larger than the mesh spacing due to the grid stretching employed. Because the projection used to transfer particle data to the mesh is tied to a filter size $\delta_f$, and not the grid spacing (as is typically done when considering traditional interpolation), the grid-size-to-particle-diameter ratio is decoupled during the interphase-exchange process. To ensure the solution remains unconditionally stable, and the cost remains low when the grid spacing is significantly smaller than the filter size, (3.2) is solved using a two-step implicit filtering procedure (Capecelatro & Desjardins 2013). However, special care needs to be taken to properly account for inter-particle collisions in regions where the particle diameter is larger than the mesh spacing. To this end, a uniform auxiliary grid with spacing $2d_p$ is used to handle efficient collision detection throughout the domain. The numerical approach has been extensively validated for vertical wall-bounded flows in our previous work (Capecelatro et al. 2014b; Capecelatro & Desjardins 2015).

4. Results and discussion

4.1. Visualisation

Visualisations of the instantaneous fluid-phase velocity and particle positions for $St_\tau = 2.1$ are shown in Fig. 1. The velocity field can be seen to change dramatically with increasing particle loading. At $\Phi = 0.2$ the flow closely resembles a single-phase channel
with particles accumulating near the channel centreline. The turbulence intensity decreases at Φ = 1 and particles appear to be more homogeneously distributed. At Φ = 4 the velocity magnitude is significantly decreased and the channel is observed to have relaminarized (i.e. the spatial variations in fluid velocity are significantly reduced). Above Φ = 4, the velocity magnitude increases due to strong interphase coupling as particles spontaneously cluster. Jet bypassing can be observed at Φ = 20 as clusters entrain the gas near the channel wall, causing high-speed jets to manifest in regions of low α_p. It is important to note that with increasing mass loading the corresponding pressure drop increases in order to maintain equal mass flow rates for each case. Thus, relaminarization does not imply drag reduction, rather a switch from drag at the channel walls to drag at the particle surfaces. The specific dissipation mechanisms responsible for the observed behaviour will be quantified in later sections.

4.2. Mean two-phase statistics

For each case presented in this work, the initial transient persists for approximately 10τ_p before reaching a fully developed statistically stationary state. After this initial transient, results are measured at each computational timestep (∆t = 2 × 10^{-5}τ_p), over a duration of approximately 10τ_p. The total TKE, defined as \( (\rho_f \alpha_f k_f + \rho_p \alpha_p k_p) / \rho_m \) based on the mixture density \( \rho_m = \rho_f \alpha_f + \rho_p \alpha_p \) is shown in Fig. 2(a). The total TKE is observed to have non-monotonic behaviour. Because particles more closely follow fluid streamlines in the lower Stokes number flow, they exhibit higher correlated energy \( k_p \) compared to the flows seeded with larger particles, giving rise to larger overall TKE. In the low-mass-loading regime, the level of fluid-phase TKE is reduced as particles are added to the flow (as shown by the energy budgets in later sections), and the additional energy by the
particles are not sufficient to balance this loss. With $\Phi > 2$ interphase coupling gives rise to drag production, resulting in an overall increase in TKE.

The skin-friction coefficients of the channel flows based on the total force $F_w$ exerted by the flow on the channel walls and the bulk velocity is given by

$$c_f = \frac{F_w}{\rho_f \alpha_f S U_b^2},$$

where $S$ is the total wall surface area. As shown in Fig. 2(b), at low mass loading ($\Phi \leq 2$) the skin-friction coefficients are less than the unladen flow. Vreman (2007) showed similar behaviour in turbulent particle-laden pipe flow. However, above $\Phi = 2$ the skin-friction coefficients drastically increase with increasing loading due to enhancement in particle concentration near the wall, as seen in Fig. 3.

A transition in the particle distribution can be seen from the profiles of normalized volume fraction in Figs. 3(a) and 3(c). At low Stokes number and low mass loading, the channel flow exhibits similar phenomenon of turbophoresis reported in the literature (e.g. Marchioli & Soldati 2002), resulting in particle accumulating near the channel wall. At higher mass loading, fluid-phase TKE near the wall is reduced enough such that the vortex strength is not sufficient to sustain turbophoresis and particles accumulate away from the wall (Capecelatro & Desjardins 2015). At high Stokes numbers, particles accumulate near the channel centreline at low mass loading and cluster near the wall at the highest loading. As noted earlier, at high Stokes numbers the cluster size in the wall-normal direction is affected by the channel walls. Hence, particle distribution across the channel is likely to be strongly dependent on the normalized cluster length for this case.

In the low-mass-loading limit, the streamwise drift velocity that contributes to both $D P$ and $D E$ is relatively low. With increasing loading, the magnitude of the drift velocity increases as seen in Fig. 3(d), giving rise to drag production. The resulting profiles in normalized fluid/particle energies are shown in Figs. 4 and 5. For the channel flows loaded with large particles, the magnitude of $k_f$ decreases with increasing $\Phi$ until interphase coupling becomes strong enough at $\Phi = 4$ to produce significant TKE. Mass loading is
also seen to affect the relative contribution to the granular energy. At low mass loading, \( \kappa \) is entirely comprised of uncorrelated motion due to the high inertia of the particles and the large normalized cluster length, such that \( \kappa \approx \langle \Theta \rangle_p \). At higher mass loading, particles begin to accumulate and fall as clusters formed near the channel walls, resulting in high values of \( k_p \). Due to the nature of the particle–wall collisions, neither \( k_p \) nor \( \langle \Theta \rangle_p \) are null at the wall (Capecelatro et al. 2016a, b). Their relative contributions to \( \kappa \) is expected to depend on the Stokes number for decaying turbulence (Février et al. 2005) (i.e. \( k_p/\kappa \) increases with decreasing \( St \)) and in turbulent flows generated by mean shear. For fully developed cluster-induced turbulence where the fluid-phase turbulence is produced by clusters, the Stokes number based on the fluid integral scales is constant (Capecelatro et al. 2015) so that relative contributions to \( \kappa \) are nearly constant. As expected, at the channel centreline \( k_p < k_f \) while \( \langle \Theta \rangle_p \) is produced at the channel wall due to mean slip and transported to the centreline by wall-normal fluctuations in \( u_p \) (Capecelatro et al. 2016b).

### 4.3. TKE spectra

The spectral distributions of streamwise fluid-phase Reynolds stresses at the channel centerline \( (y^+ = 290) \) and near the wall \( (y^+ = 50) \) are shown in Fig. 6. In general, two-way coupling is seen to increase energy at high wave numbers (small scales) at low mass
loading. A broadband reduction in the streamwise fluid-phase TKE is observed at high mass loading, with greater effect in the high Stokes number cases. Gualtieri et al. (2013, 2017) reported similar trends, albeit at lower mass loading, for homogeneous shear flows. In those studies, the range of scales that contribute to the Reynolds stress was found to progressively broaden with increasing mass loading. At $\Phi \approx 0.8$, momentum coupling by the particles was found to excite the Reynolds shear stress throughout the entire range of scales, similar to what is seen here. For the low Stokes number flows considered here, turbulence enhancement at small scales occurs on length scales smaller than approximately $14d_p$, and for larger particles at scales $< 4d_p$. At low Stokes numbers, the inertial subrange is unaffected at the channel centerline for all values of mass loading. At $y^+ = 50$ a broadband reduction of TKE occurs for $\Phi \geq 1$. For the higher Stokes number case, broadband reduction occurs both near the wall and at the channel centerline, but only for $\Phi \geq 4$.

4.4. Fluid-phase energy budgets

The fluid energy budgets are shown for the $St_\tau = 0.21$ cases in Fig. 7 and the $St_\tau = 2.1$ cases in Fig. 8. Three distinct regimes are identified for both particle classes. At sufficiently low mass loading (e.g. $\Phi = 0.2$), the fluid-phase TKE energy budget closely resembles that of a single-phase turbulent channel, in which mean shear production is...
Figure 5: Two-phase energy profiles normalized by $U_b^2$ for $St_{f} = 2.1$ showing $k_f$ (−), $k_p$ (−−−) and $\langle \Theta \rangle_p$ (−·).

the primary source of TKE, balanced by viscous dissipation. With increasing $\Phi$, the magnitudes of mean-shear production and viscous dissipation decrease, yet remain the dominant terms. At $\Phi = 2$, mean gradients in fluid velocity have greatly diminished and the flow relaminarizes. Above $\Phi = 2$, the relative contributions of drag production and exchange increase as a result of increasing drift velocity between the phases (due to the
formation of clusters) as is seen in Fig. 3(d). At $\Phi = 20$, drag production is balanced by drag exchange, with viscous dissipation only contributing very near the wall. Unlike in the low-mass-loading regime, energy is being produced throughout the channel for $\Phi > 2$.

The transition between turbulence regimes is seen to occur faster in the lower Stokes number case. At low mass loading, the lower Stokes number flows are more effective at dissipating fluid-phase TKE. Recall that the drag-dissipation-and-exchange rate (2.14) is proportional to the spatially correlated particle-phase velocity fluctuations $u''_p$. Because the uncorrelated granular motion arises when particles deviate from fluid streamlines (e.g. due to particle trajectory crossings), the lower Stokes number particles exhibit higher correlated energy and thus larger values of $D\mathcal{E}$. While all the terms appearing in the fluid-phase energy budget are approximately null at $\Phi = 2$ for both particle sizes, the interphase coupling terms ($D\mathcal{P}$ and $D\mathcal{E}$) are non-negligible at $\Phi = 4$ for the channel seeded with smaller particles, while all the terms remain relatively small for the channel seeded with larger particles.
Figure 7: Fluid-phase TKE budget normalized by viscous scales $u_t^2/\nu$ for $St_\tau = 0.21$. Turbulent convection (---), $-\varepsilon$ (---), $d\mathcal{E}/dy$ (---), $\mathcal{P}_{\text{shear}}$ (---), $\mathcal{D}\mathcal{P}$ (---) and $\mathcal{D}\mathcal{E}$ (***).

4.5. Where does the energy go?

We have seen up to this point that increasing particle loading in an initially unladen channel flow will reduce fluid-phase TKE until the mass loading is high enough for drag production to contribute. As was seen in Figs. 7 and 8, the level of reduction is tied to both the mean mass loading and Stokes number. Yet it remains to be seen how the energy is transferred between phases and ultimately dissipated to heat the fluid. In our previous work (Capecelatro et al. 2015) we showed that in homogeneous (unbounded) gas–solid flows in which mean gradients are null, mean kinetic energy produced by particles settling under gravity is balanced by mean drag, which generates viscous heating of the fluid. In the presence of clusters, drag production produces fluid-phase TKE that is eventually dissipated to heat the fluid. The route to fluid heating occurs (i) directly by viscous dissipation resulting from resolved small-scale velocity fluctuations in the fluid phase ($\varepsilon$) and (ii) indirectly by viscous dissipation of unresolved fluid velocity fluctuations, e.g. in the viscous boundary layer around individual particles. The remaining fraction of the fluid-phase TKE is transferred to the particle phase by drag exchange ($\mathcal{D}\mathcal{E}$) and ultimately dissipated to heat the fluid through fluctuating drag.

In order to analyse the mechanisms responsible for fluid-phase dissipation in the channel flows considered here, we must introduce the averaged particle-phase equations of
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Figure 8: Fluid-phase TKE budget normalized by viscous scales $u_4^4/\nu$ for $St_\tau = 2.1$. Turbulent convection (---), $-\varepsilon$ (---), $dE/dy$ (---), $P_{\text{shear}}$ (---), $DP$ (---) and $DE$ (---).

The transport of particle-phase TKE is given by

$$\frac{1}{(\alpha_p)} \frac{d}{dy} \left( \frac{1}{2} \langle v''_p \cdot u''_p \rangle + \varepsilon_p \right) = P_p - \varepsilon_p + DE_p,$$

(4.2)

where $\varepsilon_p = \langle P_p \cdot u''_p \rangle$, with $P_p$ the particle-phase pressure tensor that contains the
uncorrelated granular energy and whose trace is the granular temperature \( \langle \Theta \rangle_p \). The particle-phase dissipation rate is given by

\[
\varepsilon_p = \langle P_p : \nabla u''_p \rangle_p - \langle \Theta \nabla \cdot u''_p \rangle_p
\]  

and

\[
P_p = -\langle u''_p v''_p \rangle_p \frac{d\langle u_p \rangle_p}{dy}
\]

is the only mechanism by which \( k_p \) is generated. Finally, the particle-phase drag-dissipation-and-exchange rate is

\[
\mathcal{D}\varepsilon_p = \frac{1}{\tau_p} \left( \langle u''_f \cdot u''_p \rangle_p - \langle u''_p \cdot u''_p \rangle_p \right),
\]

and is the only source of \( k_p \) in the absence of mean shear. To close the set of equations, a transport equation for the averaged particle-phase pressure tensor is needed, which is given by

\[
\frac{1}{\langle \alpha_p \rangle} \frac{d}{dy} \left( \frac{1}{2} \langle \alpha_p \rangle \langle u''_p \rangle_p + \langle Q_y \rangle_p \right) = P_p +\varepsilon_p - \frac{2}{\tau_p} \langle P_p \rangle_p + \mathcal{C},
\]

where \( Q_y \) is the wall-normal component of the granular-flux tensor (Jenkins & Savage 1983), \( \mathcal{C} \) is the collisional dissipation rate proportional to the coefficient of restitution (Passalacqua et al. 2011), and \( P_p \) represents mean-shear production of granular pressure (Capecelatro et al. 2016a). In summary, the particle phase exchanges energy with the fluid via \( \mathcal{D}\varepsilon_p \), and dissipation of particle-phase Reynolds stresses, \( \varepsilon_p \), appears as the principal production term for \( \langle P_p \rangle_p \). The main role of \( \mathcal{C} \) in (4.5) is to drive \( \langle P_p \rangle_p \) towards isotropy.

The channel-averaged dissipation terms are shown in Fig. 9 as a function of mass loading for both particle sizes. Here, the overbar denotes a volume-average quantity weighted on \( \alpha_f \) and \( \alpha_p \) for terms appearing in the fluid-phase and particle-phase equations, respectively, such that the sum contributes to the total dissipation based on the mixture density \( \rho_m \). In the limit \( \Phi \) approaches zero (i.e. approaching a single-phase channel), dissipation is entirely due to \( \varepsilon \). At \( \Phi = 0.2 \), dissipation in the channel loaded with large particles (\( St_\tau = 2.1 \)) is almost entirely due to \( \varepsilon \). Due to the larger correlated motion attributed to the smaller particles (as seen by the finite contribution of \( k_p \) in Fig. 4), \( \mathcal{D}\varepsilon \) plays a role in dissipating TKE for \( St_\tau = 0.21 \) as low as \( \Phi = 0.2 \). As a result, the TKE budget for the smaller particles appears to transition faster from a single-phase flow dominated by
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mean shear production to CIT, as was seen in Figs. 7 and 8. In the high mass loading limit, $\varepsilon$ is null and the drag exchange terms in each phase, $DE$ and $DE_p$, dominate. The collisional dissipation is seen to only contribute above $\Phi \geq 10$, corresponding to a mean volume fraction $\bar{\alpha}_p \geq 5 \times 10^{-3}$.

In summary, at relatively low mass loading ($\Phi < 2$) viscous dissipation resulting from small-scale velocity fluctuations is the primary mechanism responsible for energy loss. Within this regime, higher levels of correlated energy ($k_p$) in the low Stokes number flows results in finite (but still small) energy transfer from the particle phase to the fluid via $DE_p$. Because the channel wall separation $2\delta < \mathcal{L}$ for the larger particle cases, higher levels of uncorrelated granular energy $\langle \Theta \rangle_p$ prevent energy transfer from the particle phase to the fluid until the mass loading is sufficiently high and the spontaneous generation of clusters generates $k_p$. At high mass loading ($\Phi = 10$), dissipation is entirely due to energy transfer between the phases and collisions.

5. Concluding remarks

In this work, we present results of vertical turbulent particle-laden channel flows using fully coupled Eulerian–Lagrangian simulations. A transition in turbulence-generation mechanisms is observed going from low to high particle loading. Three distinct regimes are identified based on the mean mass loading: (i) weak interphase coupling at low mass loading ($\Phi \leq 1$) wherein mean-shear production is the dominant mechanism for generating fluid-phase TKE; (ii) moderate coupling at intermediate mass loadings ($2 \leq \Phi \leq 4$) in which the flow relaminarizes; and (iii) strong interphase coupling at high mass loading ($\Phi \geq 10$) where the fluid-phase TKE is entirely generated by mean interphase slip velocities, termed drag production, and balanced by a drag exchange term. Remarkably, the different turbulence production mechanisms in regimes (i) and (iii) exhibit essentially no overlap, resulting from the near absence of fluid-phase turbulence in regime (ii). In addition, for a large Stokes number ($St_\delta = 17.86$), we show that the components of particle-phase granular energy contribute differently in each regime. At low particle loading, granular energy is entirely comprised of spatially uncorrelated motion due to the finite inertia of the particles, resulting in high granular temperature. At high mass loading, clusters spontaneously form and fall near the channel wall, resulting in large contributions of spatially correlated particle-phase TKE. Despite the seeming simplicity of a turbulent channel flow, the results reported here demonstrate the complex behaviour of disperse two-phase flows across granular regimes and point to two key components that must be accounted for in future modelling efforts. First, predictive models should correctly decompose the spatially correlated and uncorrelated components of granular energy as they play fundamentally different roles in the regimes identified. Second, accurate models of the drift velocity are needed to correctly predict drag production that plays a key role in the transition in turbulence regimes.

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Appendix A. The role of non-linear drag

In real systems with moderate Reynolds numbers and particle volume fractions, particles will experience drag with a non-linear dependence on volume fraction and velocity (see, e.g., Tenneti et al. (2011)). In the present manuscript linear Stokes drag was considered to simplify the Reynolds-average analysis. Here we assess the importance of the higher-order terms. Figure 10 shows fluid-phase TKE energy budgets for \( \text{St} = 2.1 \) with \( 1 \leq \Phi \leq 10 \) using the non-linear drag correlation of Tenneti et al. (2011), given by

\[
F_{\text{drag}}^{(i)} = \left( 1 + 0.15 \frac{Re_0^{0.687}}{\alpha_f^2} \right) \frac{\alpha_p}{\alpha_f} F_1(\alpha_f) + \alpha_f F_2(\alpha_f, Re_p),
\]

where \( F_s^{(i)} = (\langle u_f(x_p^{(i)} - v_p^{(i)})/\tau_p \rangle \) is the Stokes drag contribution for particle ‘\( i \)’ used in (2.2), \( F_1(\alpha_f) = 5.81\alpha_p/\alpha_f^2 + 0.48\alpha_p^{1/3}/\alpha_f^4 \), and \( F_2(\alpha_f, Re_p) = \alpha_p^2 Re_p(0.95 + 0.61\alpha_p^3/\alpha_f^2) \).

Due to the non-linear contributions from \( \alpha_f \) and \( Re_p \), phase-averaging the drag contribution becomes cumbersome. Instead, the total drag can be decomposed into mean and fluctuating components via

\[
F_{\text{drag}}^{(i)} = \langle F_{\text{drag}} \rangle + F_{\text{drag}}',
\]

where \( \langle F_{\text{drag}} \rangle \) is defined according to (A 1) using averaged quantities (e.g. \( \langle \alpha_f \rangle, \langle u_f \rangle_f \), etc.), and \( F_{\text{drag}}' = F_{\text{drag}} - \langle F_{\text{drag}} \rangle \) represents the difference.

Although the non-linear drag is seen to impact the magnitude of the terms appearing in the TKE balance, overall the same trend is observed. At low mass loading, mean-shear production is dominated by viscous dissipation, and drag coupling terms have minimal effect. At intermediate values of mass loading (\( \Phi = 4 \)) similar relaminarization is observed, and at high mass loading (\( \Phi = 10 \)) drag production is entirely balanced by drag exchange. We note that non-linear contributions to drag do indeed have meaningful impact on the wall-normal profiles of mean volume fraction interphase slip velocity. Nonetheless, we expect real systems to undergo a similar transition from the dilute limit where classical mean-shear production is primarily responsible for generating fluid-phase TKE to high-mass-loading suspensions dominated by drag production. It is likely that Reynolds number effects on the drag law are masked at high mass loading by other near-field interactions, and thus in general these higher order terms should be accounted for.

REFERENCES


Figure 10: Fluid-phase TKE budget normalized by viscous scales $u^4_\tau/\nu$ for $St_\tau = 2.1$ with linear Stokes drag (left) and the non-linear drag correlation given by Tenneti et al. (2011) (right). Turbulent convection $(-\cdot-)$, $-\varepsilon (-\cdot-)$, $d\mathcal{E}/dy (-\cdot\cdot\cdot)$, $P_{\text{shear}} (-)$, $DP (-\times-)$ and $D\mathcal{E} (-\ast\ast)$.


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