9-2012

A test for complementarities among multiple technologies that avoids the curse of dimensionality

Li Yu
Central University of Finance and Economics

Terrance Hurley
University of Minnesota - Twin Cities

James Kliebenstein
Iowa State University, jklieben@iastate.edu

Peter F. Orazem
Iowa State University, pfo@iastate.edu

A Test for Complementarities among Multiple Technologies that Avoids the Curse of Dimensionality

March 2012

Abstract

We propose a strategy to identify the complementarity or substitutability among technology bundles. Differences between the observed distribution of technology choices can be subjected to statistical tests. Combinations of technologies that occur with greater frequency than would occur under independence are complementary technologies. Combinations that occur with less frequency are substitute technologies. We use the strategy to evaluate multiple technology adoptions on U.S. hog farms. As the number of bundled technologies increases, they are increasingly likely to be complementary with one another, even if subsets are substitutes when viewed in isolation.

JEL: O33; L25; C12
Key words: technology bundles; complements, substitutes; independence; hog farms.

This paper has benefited from comments from seminar participants at Iowa State University and at meetings of the Midwest Economics Association and the Agricultural & Applied Economics Association. Funding support from the U.S. Department of Agriculture and the National Pork Board/Primedia Business is gratefully acknowledged.

1. Introduction

Complementarities among new technologies lead to rising returns to innovation and the incentive to adopt multiple technologies. Several studies have examined possible complementarity between technologies using the sign and significance of the effect of technology interactions on productivity (Dorfman, 1996; Stoneman, 2002; Caswell and Zilberman, 1985; Ichniowski, Shaw and Prennushi, 1997; Poppo and Zenger, 2002; and Carree,Lokshin and Belderbos, 2011). All past empirical strategies have suffered the curse of dimensionality which forces the consideration of only a small number of possible technologies. If there are K distinct technologies, there are 2^K possible technology bundles from which to choose. For tractability, researchers have limited their analysis to only two or three of the K technologies, imposing independence between the included and excluded technologies. If independence is inappropriate, these studies will yield biased inference regarding the complementarity or substitutability of the technologies.

Given the potential importance of multiple technology adoption for explanations of growth, agglomeration, and innovation, we need tractable methods for evaluating higher dimensioned technology bundles. We propose a strategy that can accommodate any number of technologies. We illustrate the methodology in an analysis of joint choices of 6 technologies in the hog industry. Complementary bundles are composed of at least 3 technologies. Such highly dimensioned bundles lead to increasing returns to scale that can help explain the sharp increase in market share of large farms.

2. Identifying Whether Technology Bundles Are Complements or Substitutes

It appears intuitively appealing to assume that complementary relationships result in a positive correlation in adoption rates of the two technologies, while substitute relationships result in negative correlation. Nevertheless, the correlation between any two technology adoption rates can mislead if there is even one more technology potentially in the mix.

Our strategy builds from the realization that regardless of the number of technologies in the universe, we can construct an expected probability that any subset of the technologies will be adopted under a maintained hypothesis that all technologies are independent. We can then compare the actual probability that a randomly selected agent picks that technology bundle with the benchmark probability under independence. If the bundle is selected significantly more often than under the null hypothesis of independence, we can view the bundled technologies as mutually complementary. If the bundle is selected significantly less often than predicted under the null hypothesis of independence, we can view the bundled technologies as substitutes.

To formalize this conceptual strategy, suppose that $K > 1$ technologies can be used alone or in combination. Let X_k, $k = 1, 2, ...K$, equal to1 if the k^{th} technology is adopted and 0 otherwise. Define $1 > p_k > 0$, for $k = 1, 2, ...K$ as the probability a specific technology k is adopted.

With K technologies, there are 2^K possible technology bundles. Let the *jth* bundle be $Y_j = \{ X_1^j, X_2^j, ..., X_K^j \}$ which is a series of ones and zeroes corresponding to whether the *kth* technology is adopted in the *jth* bundle Y_j. Let q_j be the frequency the *jth* technology bundle Y_j is adopted such that $1 > q_j > 0$ for $j = 1, 2, ...2^K$. We designate the set of technologies adopted in bundle Y_j as Ω_j^A which is composed of all subscripts k

such that $(k \in \{1,2,...,K\} \wedge X_k = 1)$. A second set of technologies not adopted in Y_j, Ω_j^N ,is composed of all subscripts k such that $(k \in \{1,2,...,K\} \wedge X_k = 0)$.

We can now define our baseline adoption rate for each of the 2^K possible technology bundles under the null hypothesis of independence. A larger dimensioned bundle is mutually independent if $q_j^0 = \prod_{k \in \Omega_j^A} p_k \prod_{l \in \Omega_j^N} (1 - p_l)$. These estimated probabilities form the null hypotheses against which complementarity and substitutability can be assessed.

Null hypothesis: independence $H_0 : q_j = q_j^0$
Alternative 1: complementarity $H_C : q_j > q_j^0$
Alternative 2: substitutability $H_S : q_j < q_j^0$

To operationalize the tests, we need estimates of the sampling distributions of the null and alternative hypotheses. Given a random sample of S firms denoted by $i = 1, 2, ...S$, let $X_k^i = 1$ if firm i adopts technology k and 0 otherwise; and $Y_j^i = 1$ if firm i adopts technology bundle j and 0 otherwise.

Under the null hypothesis of independence, the likelihood function for p_k is $L = \prod_{i=1}^{S} \prod_{k=1}^{K} p_k^{X_k^i} (1 - p_k)^{1 - X_k^i}$. Optimizing its log-likelihood equation with respect to p_k yields the estimates $\hat{p}_k = \frac{\sum_{i=1}^{S} X_k^i}{S}$ for $k = 1, 2, ...K$.

The probability of adopting a given technology k can be calculated by the frequency of its occurrence in the random sample under independence assumption:

$$\hat{q}_j^0 = \prod_{k \in \Omega_j^A} \hat{p}_k \prod_{l \in \Omega_j^N} (1 - \hat{p}_l). \qquad (1)$$

Given the sampling data, the log-likelihood of adopting a bundle of technologies *j* is

$\ln L^{(ii)} = \sum_{j=1}^{2^K-1} \ln(q_j) \sum_{i=1}^{S} Y_j^i + \ln\left(1 - \sum_{j=1}^{2^K-1} q_j\right) \sum_{i=1}^{S} Y_{2^K}^i$. Similarly, Maximum Likelihood Estimation yields the estimates

$$\hat{q}_j = \frac{\sum_{i=1}^{S} Y_j^i}{S} \text{ for } j = 1, 2, \ldots 2^K - 1 \text{ and } \hat{q}_{2^K} = 1 - \sum_{j=1}^{2^K-1} \hat{q}_j \ . \qquad (2)$$

Testing is complicated by the fact that the sampling distributions of $\hat{p}_{kl}$ and $\hat{q}_j$ are unknown. Moreover, the test has to incorporate the unknown correlation between $\hat{q}_j$ and $\hat{q}_j^{\,O}$. We use percentile bootstrapping to approximate the sampling distributions and their inter-correlations. The resulting simulated distributions are used to calculate confidence intervals.[1]

3 Application: Multiple Technology Adoption on U.S. Hog Farms

We illustrate the performance of the test for mutual complementarity or substitutability among technologies using data from U.S. hog production. The industry

---

[1]One could also consider applying a multinomial distribution to construct test statistics under the null hypothesis of independent technologies. However, one could only test for independence *between* technology bundles. Once we reject the null hypothesis that bundles are independent, as we did in the expanded version of this paper using a multinomial Log-likelihood ratio test (Yu *et al*, 2011, pp. 10, 16), we cannot go further to test for evidence of substitutability or complementarity between specific technologies *within* each technology bundle. As that is the goal of this paper, the bootstrapping method proves more informative.

has experienced rapid technological innovation over the last twenty-five years in the areas of nutrition, health, breeding and genetics, reproductive management, facilities, and environmental management (McBride and Key, 2003). These technologies have been associated with improved feed efficiency, lower death loss, higher quality meat, more rapid weight gain, and other improved outcomes that raise farmer profits (Rhodes, 1995). Six technologies are included in our application: Artificial Insemination, Split Sex Feeding, Phase Feeding, Multiple Site Production, Early Weaning, and All In/All Out. These technologies were identified by the editors of *National Hog Farmer Magazine* (*NHFM*) and the National Pork Board as the relevant technologies available to farmers over the period. The detailed descriptions of the technologies and their adoption rates are included in the Appendix table A.1.

National Hog Farmer Magazine conducted a survey of its subscribers in years 1995, 2000 and 2005. Each year, hog farmers were asked whether they use any of the listed technologies. Each technology is treated as a dichotomous variable taking the value of 1 if the technology is used and 0 otherwise. As shown in table 1, when there are 6 technologies, there are 64 possible technology bundles. At least one-fifth of the possible bundles never occurred in the sample data in 2000 and 2005. Of the selected bundles, most are not statistically distinguishable from the null hypothesis of independence.

The number of bundles that are observed significantly less often than would be predicted under independence declines over time, meaning that evidence of substitute bundles decreases over time. Depending on the year from 7-11% of the technology bundles were chosen significantly more frequently than they would under the independence null, implying the bundled technologies are mutually complementary. As shown in table 2, there are no ‘two-technology’ complementary bundles. Instead, complementary bundles most typically included 4-6 adopted technologies, and the bundles are increasingly likely to be mutually complementary as the number of bundled technologies increases. This interesting finding has implications for the substantial increase in the market share of very large hog farms over the same period. The productivity of the bundled complementary technologies is greater than the productivity of the technologies used in isolation. Because the technologies are expensive, the high dimensioned bundles were adopted by the largest farms. As a result, the largest farms had a productivity advantage compared to smaller farms using a smaller number of technologies. Hence, bundled complementary technologies were a source of increasing returns to scale in hog production at a time when small farms were exiting the market and large farms were becoming more common.

As noted in the introduction, previous methods employed to assess whether technologies are complements or substitutes were plagued by the curse of dimensionality which meant that most studies examined pairwise evaluations, effectively imposing independence between the analyzed technologies and those technologies excluded from consideration. As shown in table 3, pairwise evaluations assuming independence with all other technologies incorrectly implies that many paired technologies are complements when in fact, no pairs are complements when the presence of other technologies are considered. Furthermore, depending on the year, 20% to 47 % of the cases are found to be independent. None of cases are substitutes, in stark contrast to the implications when all technologies are considered jointly. Compared to the testing method that considers all six technologies jointly, the pairwise test yields the correct inference in only 13% of cases in 1995 and 2000 and in only 47% of the cases in 2005. In short, pairwise tests are prone to incorrectly assessing pairs as complements when they are not, and they fail to find substitute relationships in the cases where they occur.

4. Conclusion

This paper proposes a tractable statistical method to test for mutually complementary or substitute technologies. The method exploits the fact that profit maximizing producers will adopt technologies in groups if they are complements with greater frequency than would be predicted if the technologies were mutually independent. This statistical method makes it simple and feasible to check the relationships between technologies which have high dimensional combinations.

Our method solves a series of problems in the current literature on technology adoption. Our method easily accommodates highly dimensioned technology bundles, side-stepping the curse of dimensionality that limited previous applications to only a very few technologies. The method allows for the simultaneous adoption of multiple technologies so that one need not presume some adoption decisions are exogenous or independent. These problems are shown to be particularly problematic in that the correlation between any two technology adoption rates, ignoring the existence of other technologies, may provide misleading inferences on whether the two technologies are complements or substitutes.

Applying the method to a data set that includes six technologies adopted by U.S. hog farmers, we find that as the number of bundled technologies increases, they are increasingly likely to be complementary with one another, even if subsets are substitutes when viewed in isolation. Our findings also suggest that the complementarity among technologies in large bundles is contributing to a form of returns to scale that is leading to increasing growth in average farm size.

Reference

Caswell, M. F.,Zilberman, D., 1985. The Choices of Irrigation Technologies in California. *American Journal of Agricultural Economics.* 67, 224-234.

Dorfman, J., 1996. Modeling Multiple Adoption Decisions in a Joint Framework.*American Journal of Agricultural Economics.*78, 547-557.

Ichniowski, C., Shaw, K., Prennushi, G., 1997. The Effects of Human Resource Management Practices on Productivity: A Study of Steel Finishing Lines. *The American Economic Review.* 87, 291-313.

Carree, M., Lokshin, B., Belderbos, R., 2011. A Note on Testing For Complementarity and Substitutability in Case of Multiple Practices. *Journal of Productivity Analysis.*35 , 263-269.

McBride, W., Key, N., 2003. Economic and Structural Relationship in U.S. Hog Production.*Agricultural Economic Report* No. (AER818) 60, USDA.

Poppo, L., Zenger, T., 2002. Do formal contracts and relational governance function as substitutes or complements?*Strategic Management Journal.* 23, 707-725.

Rhodes, V. J., 1995. The Industrialization of Hog Production. *Review of Agricultural economics.* 17, 107-118.

Stoneman, P., 2002. The Economics of Technological Diffusion. Oxford, UK: Blackwell Publishers.

Yu, L., Hurley, T. M., Kliebenstein, J., Orazem, P.F., 2011. Testing for Complementarity and Substitutability Among Multiple Technologies: The Case of U.S. Hog Farms. Iowa State University Working Paper #08013.

Table 1
Number of substitute, complementary and independent technology bundles among the 64 possible bundles.

Bundle Relationships	*1995*	*2000*	*2005*
Not Present	0 [0]	14 [14]	18 [18]
Substitute technologies	33 [36]	12 [15]	4 [7]
Independent technologies	20 [17]	31 [28]	35 [31]
Complementary technologies	11 [11]	7 [7]	7 [8]

Note: The statistics are based on 2000 bootstrapped samples. Numbers are the occurrence of the relationship implied from confidence interval bootstrapped samples at the 5% [10%] significance level.

Table 2
Complementary technology bundles.

Number in Bundle (*n*)	*1995*	*2000*	*2005*
2 technologies	0/15	0/15	0/15
3 technologies	1/20	0/20	0/20
4 technologies	1/15	0/15	0/15
5 technologies	4/6	1/6	2/6
6 technologies	1/1	1/1	1/1

Note: Numerator is the number of complementary relationships found among bundles with *n* technologies using the 5% significance level. The denominator is the number of possible bundles with *n* technologies. We exclude one case in which no technologies are adopted and six cases in which only one technology is adopted as these do not naturally fit into 'substitute', 'complement' or 'independent' categorizations. However, we do include those 7 cases in table 1's summary of the number of cases that occur with greater, equal, or lesser frequency than predicted from independence.

Table 3
Ability of pairwise evaluations to successfully assess complementary or substitute relationships in the presence of more than two technologies in hog production.

		Technology Relationships (Implied/Actual)[b]			
Year	Possible Pairs[a]	Substitutes	Complements	Independent	%Correct
1995	15	0/12	12/0	3/2	13.3%
2000	15	0/2	10/0	5/2	13.3%
2005	15	0/1	8/0	7/11	46.7%

Note: [a] With six technologies, there are 15 possible bilateral relationships.
[b] Top number is the occurrence of the relationship from bilateral correlations, imposing independence with all other technologies. Bottom number is occurrences when 6 technologies are considered jointly.

Table A.1

Description of technologies in the hog production.

Technology	Description	1995	2000	2005
AI	Artificial Insemination focuses on enhancing hog reproductive efficiency and improving the gene pools.	0.323[0.468]	0.571[0.496]	0.647[0.479]
SSF	Split Sex Feeding feeds different rations to males and females. They have different diets for pigs of various weights and separate diets for gilts and barrows for maximum efficiency and carcass quality.	0.418[0.493]	0.498[0.501]	0.383[0.487]
PF	Phase Feeding involves feeding several diets for a relatively short period of time to more accurately and economically meet the pig's nutrient requirements.	0.604[0.489]	0.735[0.442]	0.603[0.49]
MSP	Multiple Site Production produces hogs in separate places in order to curb disease spread.	0.297[0.457]	0.415[0.494]	0.397[0.49]
EW	Early Weaning helps to produce more piglets each year. It may include Segregated early Weaning technology (which gives the piglets a better chance of remaining disease-free when separated from their mother at about three weeks when levels of natural antibodies from the sow's milk are reduced), Medicated Early Weaning (which uses medication of the sow and piglets to produce excellent results in removing most bacterial infections) and Modified Medicated Early Weaning (which is same as MEW but less all-embracing. The range of infectious pathogens to be eliminated is not quite as comprehensive. MMEW can also be used to move pigs from a diseased herd to a healthy herd).	0.166[0.372]	0.327[0.47]	0.261[0.44]
AIAO	All In/All Out allows hog producers to tailor feed mixes to the age of their pigs instead of offering either one mix to all ages or having to offer several different feed mixes at one time. It helps limit the spread of infections to new arrivals by allowing for cleanup of the facility between groups of hogs being raised.	0.574[0.495]	0.695[0.461]	0.627[0.484]

Note: Information is based on the USDA animal and plant health inspection service and ERS; http://www.thepigsite.com/; and National Hog Farmer http://nationalhogfarmer.com/. The number is the average adoption rate and the number in the bracket is the standard deviation.