Risk-based security assessment for operating electric power systems

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Risk-based security assessment for operating electric power systems

by

Hua Wan

A dissertation submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of

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Major: Electric Engineering (Electric Power)
Major Professors: James D. McCalley and Vijay Vittal

Iowa State University
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1999

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ABSTRACT

The power system is a widespread and complex network whose complete behavior, at present, still remains partially characterized. Power systems have operated in most cases reliably, but conservatively with the help of many deterministic techniques that rely heavily on the modeling of system components and the associated dynamics. Now, with increasing competition and growing demand, the power system, however, has been shifting from a deterministically regulated system to a competitive and uncertain market environment. Power utilities are required to have a comprehensive knowledge of the risks as well as benefits in their transmission operations. Our interest is motivated by this need of the industry to provide a method to quantify the risk of operating a power system with consideration to the probabilistic nature of system behaviors. The objective of this dissertation is to develop a foundation of risk-based bulk power system security assessment that leads to the definition, calculation, and application of the "risk" in operating electric power systems. The work includes three parts of risk assessments: transmission line thermal overload, voltage insecurity and composite risk assessments. Both the probability of insecurity problems and their cost consequences are measured such that an expected monetary impact is given as the measurement of risk. This quantitative measurement of thermal, voltage, and composite risk is helpful for the operator to trade off the benefits and costs in the competitive utility environment. For making this economic tradeoff, several decision criteria, including both deterministic and probabilistic strategies, from conservative to greedy preference, are introduced to aid the operator to make operating decisions. This research establishes a bridge between power system security and economics by the index of risk that is compatible with the economic results of market-based electricity trading. Both the method to quantify the risk and the ways to apply it in decision-making make contributions to the power industry.
1 INTRODUCTION

1.1 Background

The power system is a widespread and complex network whose complete behavior, at present, still remains partially characterized. Since the massive blackout of New York City and most of the Northeast in 1965, power systems have operated in most cases reliably, but conservatively with the help of many deterministic techniques that rely heavily on the modeling of system components and the associated dynamics. These approaches include methods for both steady state and dynamic analysis. The analyses are conducted to obtain system operating limits which satisfy reliability criteria established by the North American Electric Reliability Council (NERC).

Now, with increasing competition and growing demand, power utilities are required to have a comprehensive knowledge of the risks as well as benefits in their transmission operations.

Within the electric network, an individual disturbance with non-zero cost consequence may occur for a number of reasons at any time in any system environment. The disturbance may result in overload, voltage collapse, or transient instability, and draw the prevailing system to an uncontrollable cascading situation leading to widespread power outages. To maintain system reliability under these uncertainties, certain limits are required in operating the power system. The current approach in the industry uses deterministic methods to calculate these limits with significant safety margins to cover "all" the possible unknown uncertainties. In practice, this means that power engineers propose a strong system and then operate it with large security margins. Though investment and operational costs are relatively high, this has resulted in a high degree of reliability in most power systems.

The power system, however, has been shifting from a deterministically regulated system to a competitive and uncertain market environment. A fluctuation of market demand and supply has led to an uncertain market price in the system operation. Although some methods of risk assessment as well as risk management have been introduced into the market-oriented energy trading business, the traditional deterministic reliability criteria are still intact. This has led engineers to face more pressure, from economic imperatives in the marketplace, to operate power systems with lower security margins. In order to be able to operate the system closer to the traditional deterministic limits, or even beyond them, more refined methods for power system security assessment are needed that take into account the probabilistic nature of many uncertain variables in the decision-making environment. Also, from the perspective of long-term operating economics, a quantitative assessment of risk allows us to make decisions that minimize the total cost, cost of security violation and cost of energy supply, over the long run to attain economic efficiency.

Our interest is motivated by this need of the industry to provide a method to quantify the risk of operating a power system with consideration of the probabilistic nature of system behavior. The objective of this dissertation is to develop a foundation of risk-based bulk power system security assessment that leads to the definition, calculation, and application of the "risk" in operating electric power systems.
1.2 Human Ancestor's Philosophy of "Avoiding Risk"

Take a glimpse at the history of the human race. "The entire history of human species is a chronology of exposure to misfortune and adversity and of efforts to deal with risk"[1]. Great dangers in unknown lands, including dreaded beasts, fatal diseases, tidal waves, and a fear of falling over the edge of an imaginary world are testimony to the enormous risk that humans have faced in their exploration of this world. "Many of our primitive ancestor's responses to risk were identical to those of other animals. Without a great deal of thought, people who were threatened by dangerous beasts fled" [1]. As a result they were able to avoid dangerous areas and situations (see Figure 1.)

![Figure 1: Philosophy of Avoiding Risk](image)

1.3 Power System Security

In the course of conquering this world, together with the risks in the natural world, human beings also face numerous risks created through their man-made wonders. One of the wonders, electrical power system, has contributed to the many advances and conveniences in modern human life. The heavy dependency on electrical power systems was fully demonstrated by the blackout of New York City in 1965 and the second major blackout, the PJM blackout of 1967 which caused power to be interrupted in all or portions of four states [2]. The fear of a similar massive outage in the country led the power industry to invest greater effort to increase understanding of electrical power system behavior and to place operations on a secure basis to prevent future problems. This effort produced a reliable, if conservative technique of maintaining system security. "N-1" security, as it often termed, examines the behavior of an N-component system that has lost any one major component.
1.3.1 The Terms

The North American Electric Reliability Council (NERC) uses reliability [3] in a bulk electric system to indicate "the degree to which the performance of the elements of that system results in electricity being delivered to customers within accepted standards and in the amount desired. The degree of reliability may be measured by the frequency, duration, and magnitude of adverse effects on the electric supply." The NERC also suggests that reliability can be addressed by considering two basic and functional aspects of the bulk electric system - adequacy and security. In this research, the discussion is limited to the security aspect of power system reliability, which is defined in [3] as "the system's capacity to withstand disturbances arising from faults and the unscheduled removal of bulk power supply equipment without the further loss of facilities or cascading." Given adequate capability to supply the energy demand, a system is secure, if its components are not stressed beyond their ratings, and bus voltages and system frequency remain within tolerance when any single credible contingency occurs. Unlike the long-term planning where the system conditions are unknown, security analysis measures the system's reliability in the near future, given an operating condition.

1.3.2 The Problems

Security problems most commonly manifest themselves in three different forms. Thermal overload of a circuit occurs when current exceeds the rating of a circuit, and the circuit overheats due to the $I^2R$ losses, resulting in circuit loss of life and subsequent consequences. This is the most universal security problem in the transmission systems. Voltage collapse typically occurs in power systems which are heavily loaded, faulted or have reactive power shortages. It is associated with the reactive power of loads not being met because of limitations on the production and transmission of reactive power, and it may result in system-wide loss of loads and blackout. The third form, rotor angle instability, is concerned with the system dynamic response following a large disturbance in terms of the generators' capability to remain in synchronism. The risk of thermal overload and voltage collapse is the primary topics of interest in this research.¹

1.3.3 The Methods

There are two approaches for reliability assessment: the probabilistic method, which assesses the reliability level of a system and which has largely been used in adequacy assessment, and the deterministic approach, which is predominantly used in both security and adequacy assessment.

As described by the Conference Internationale Des Grande Reseaux Electriques (CIGRE) in its latest position report [5], the security assessment in practice works as follows:

¹ We have partially addressed the transient instability problem already in [4].
1.3.3.1 Identification of Credible Contingency Set

In order to perform security assessment, the normal practice for a utility is to first identify a set of credible contingencies based on a combination of history, experience, obvious relations, worst-case analysis, trial and error, and even hunches of experienced planners, operators, and engineers. In practice, they are usually defined as the loss of any single critical element (i.e., line, transformer, generating unit, etc.) in a power system either spontaneously or preceded by a single-phase fault. Several hundred to a couple of thousand critical component-contingencies are identified for a typical transmission grid. This is usually referred to as the N-1 criterion.

1.3.3.2 Determination of Security Limits

According to the N-1 criterion, the utility plans and operates the power system such that, if any one of the above mentioned credible contingencies occurs, the system will
1. rapidly recover through the use of regular protection systems and automatic control devices, and continue to supply all the load within emergency (voltage and frequency) ratings, and
2. re-enter an adequate state, at most, by means of minor system adjustments, including manual switching.

After obtaining a deterministic boundary by the contingency analysis, a safety margin that works as a buffer for other unknown uncertainties is added to discount the security boundary. All we have done is to make the system survive under any possible, credible contingency. The security limits, therefore, constitute the first line of defense of a power system in an essentially "passive" strategy against the many unforeseen events that can perturb or threaten its operation.

1.3.3.3 Security Criteria

The above mentioned approach to dealing with power system security is fully deterministic. The philosophy behind this approach is similar to our human ancestors fleeing from dangerous lands. If they thought there was one beast in the jungle, they would keep themselves away from its living area. If there were two or more, they would avoid the entire area occupied by any of these beasts.

Similarly, the deterministic criterion, without any probabilistic analysis of whether one could meet a beast and the possibility of being attacked by a beast, simply provides large margins in protecting the system against severe contingencies. In practice, this means that power engineers can propose a strong system and operate it with large security margins. Though investment and operational costs are relatively high, this has resulted in a high degree of reliability in most power systems.

"Some human-like creatures, such as Homo erectus and Homo Sapiens neanderthalensis reacted to threats in these ways but, despite the fact that they were larger and stronger, failed in their efforts to manage the threats to their existence" [1].
1.4 The Gaps

In contrast, "anatomically modern humans (Homo Sapiens Sapiens) not only survived but flourished. Human's continued existence becomes a testimony to the success of our ancestors in managing risk. The difference [between modern humans and other creatures] was the unique human gift of reason. Men and women think, and it is in their ability to think that they deal with risks in ways that are different from those of other creatures... by their ability to anticipate adversity and to prepare for it" [1]. From this sense of evolution, there is a necessity to improve the existing deterministic approach to security assessment.

In the recent changes in regulatory policies towards inter-utility power interchange practices, a major focus of the changing policies is "competition" as a replacement for "regulation" to achieve economic efficiency. A number of changes would be required from operating, planning, and organizational standpoints.

From an operational point of view, transmission corridors are substantial power market influences, i.e., the choice of one limit over another for a transmission corridor can translate into millions of dollars per year for the selling and buying entities. Energy marketers need as much transfer capability as possible to deliver their energy from one place to another for making profits. To maintain system reliability, however, a certain amount of transfer limit has to be maintained regardless of the economic forces behind the marketers.

The blame has been expressed by some engineers that the marketer, who is driven by monetary profits to use economic risk management, is pushing the engineer to continually reduce the current safety margin and to operate systems closer and closer to the deterministic security limits. Today's universal deterministic approach to security assessment, however, presents remarkable gaps between the industry trend and the results it provides. This becomes apparent in the following situations:

- The present deterministic approach to security assessment results in costly operating restrictions.
- The granted security limits, which are mandatory for all the operating decisions, are not justified in their supposed low level of risk.
- The current approach, which does not take into account the probabilistic nature of uncertainties, maintains the security by simply restricting the activities within its conservative margins, leaving the world beyond its boundary uncharted.
- Finally, the incentive for one to explore or chart the world outside does not exist simply because the rule has been set. That is partially why some engineers feel anxious when marketers aggressively push them to operate beyond traditional limits, because these areas have never been very well explored before.

These gaps motivate us to propose an alternative risk-based security assessment to quantify the risk of system operation and to directly connect power system economics and security to benefit both profit-driven marketers and reliability-driven engineers.
1.5 The Objective

The research objective of this dissertation is to develop the foundation for risk-based security assessment of electric power systems. The concept of risk in power system operation will be introduced. The research will develop methods of measuring risk in terms of thermal overload, voltage collapse, and hence provide a composite security index for determining system security level. This work will also propose several decision-making guidelines for operating systems based on the risk under various situations.

1.6 Our Method

The method we propose in this dissertation is a probabilistic method that takes into account the probabilistic nature of a power system’s operating uncertainties. The risk calculated is an expected monetary impact or cost consequence over a short period of time under a given operating condition. Within the scope of security assessment, we use a "forecasted" probability distribution of the uncertainties, such as contingencies and weather under the known information of the current operating condition, multiplied by their associated cost consequences. That is,

Equation 1: Generic Risk Expression

\[ \text{Risk}(X_i) = E(\text{Im}(X_{t+1}) | X_i) \]

where the risk of a current pre-contingency operating condition \( X_i \) is given by the expected value of the future monetary impact \( E(\text{Im}(X_{t+1}) | X_i) \). It is a double integral of the product of probability of uncertainties\(^2\) and the corresponding (post-contingency) impact, along the uncertain event \( E_i \) and uncertain future operating condition \( X_{t+1} \).

The risk calculated has explicit economic meaning and represents the expected cost due to possible insecurity problems. It measures the economic consequence of an uncertainty weighted by its probability of occurrence. This significant property makes risk a direct bridge between the power system economics and security such that the security can be treated in the same way as an ordinary commodity. The security can no longer be a mandatory constraint to ordinary decision-making problems; rather it becomes in itself a decision-making problem, which leads the power industry closer to the ideal of a free market.

From the perspective of risk itself, the following advantages occur:

- Our method is able to quantify the risk of insecurity, which itself has the nature of uncertainty.

\(^2\) The uncertain event set \( \{E_i, \forall i = 0, N\} \) includes the possibility that the current state remains the same and no other event occurs.
• The quantified risk provides an economic consequence of insecurity by taking account of both the probability and severity of uncertain events.

• Managing security becomes a decision problem depending on the preference of the decision-maker to the exposure to the risk, rather than putting mandatory limits by rule.

• The resultant risk charts the system both within the traditional boundary and the world outside by augmenting the traditional deterministic security limits with the inclusion of risk in the assessment.

• The risk is able to function as a lever to adjust the behavior of market participants via economic mechanisms to avoid system security problems rather than the mandatory cutting of transactions by rule.

A detailed method describing the procedure to calculate the individual risks for each security problem, and ultimately arriving at a composite risk index is discussed in Chapter 3, 4, and 5. At this point, it is worthwhile to cite a comparison of both deterministic and probabilistic approaches used in our industry by the latest position report [5] on power system security assessment.

1.7 Deterministic Method versus Probabilistic Method

The weakness of the deterministic approach lies primarily in the arbitrariness of selecting contingencies for the N-1 tests, and of choosing specific values for parameters in studies, which hides the underlying probabilistic nature of many variables. It may lead to ignoring important cases and including unlikely ones. The fear of omitting critical cases usually results in a conservative over-design without any indication that risks are reduced to an acceptable level.

The probabilistic approach, on the other hand, reduces the above mentioned weakness but has been relatively slow in being accepted as a methodology that can be used in practice. One factor that accounts for this is the accuracy of the probabilistic models employed and the precision of the solution methods that are derived. These problems are also present in the deterministic approach, but due to the greater simplicity and transparency of this method, the simplifications and approximations applied are often perceived to be justified. As a consequence, the deterministic approach often unfairly enjoys higher credibility. Another factor is the unavailability of standards for what constitutes acceptable risk.

The author of this dissertation also thinks one reason for the difficulty of probabilistic approaches may be partially due to the primary philosophy of engineering as a discipline whose objective is to make the world more well-defined and precise. Consequently, most engineers are more comfortable accepting a clear deterministic number instead of an uncertain value carrying some amount of variance. However, this world is a world full of uncertainties. Our technology is developed to reduce these uncertainties but never eliminate them. To characterize the existing uncertainties will aid engineers to find better ways to reduce them.
2 REVIEW OF RELATED WORK

2.1 Probabilistic Approaches in Power System Security Assessment

While the deterministic approaches in power system security assessment have become an accepted practice in the industry, an extensive amount of work has also taken place in the area of probabilistic approaches, specifically using probabilistic methods for adequacy assessment in generating reserve capacity evaluations. Most of them can be found in the bibliographic papers [6] and [7]. The sophisticated nature of the security problem, however, has led to relatively, conservative use of probability methods for security assessment. From [5] we note, "The probabilistic nature of power system security (both dynamic and static) has been well recognized since early days of modern power system operation and control. However, probabilistic security assessment has not been much developed. Even if some practical methods have been elaborated, none has been able to impose itself and, as we have seen, in today's everyday practice the problem is still essentially approached deterministically." On the other hand some attention to the probabilistic methods for the evaluation of system security can be found in [8], [9], [10], and [11]. Reference [8] provides a framework integrating both adequacy and security evaluation. An expected load actually interrupted (ELAI) is calculated as the measure of reliability. The author of [8] states that "the development of Security Limits is very important. Clearly, these limits are an input to the reliability evaluation problem as the essence of reliability analysis is the separation of states into success and failure states, and the Security Limits, as well as equipment rating limits, from fundamental criteria for such separation." Reference [9] gives a probabilistic security cost which is similar to the concept of risk. Reference [10] calculates a probability of system remaining secure with consideration of differential equations. The framework of [11] provides a random time for the system shifting from a secure region to an insecure one. Although different measurements and methods are developed, the same "separation" idea, such as "Security Limits" in [8], "Operational Limit Boundary (OLB)" in [9], "Dynamic Security Region" in [10], and "Security Region" in [11], can be found in a variety of probabilistic security assessment techniques. One of the distinctions of the risk-based security assessment in this dissertation is that the ratings and security limits are no longer deterministic.

This work focuses on the risk-based electric power systems' security assessment. There are many publications relating to the topics of risk, probability, statistics, and power system security. The references listed are by no means comprehensive. Our focus in this research endeavor is to review probabilistic techniques that are closely related to the nature of the work we are performing, specifically the security problems of thermal overload and voltage collapse and their corresponding impact on the victims.

2.2 Theoretical Foundations

The transition from a fixed rate of return industry to a participant in a competitive energy market means that the power industry has shifted from its original deterministic, reliability-oriented operation to a profit-
oriented situation. Consequently, the concept of risk, so much a part of the field of economics and finance has to be introduced into the system operation. This is becoming more crucial under the situation of a competitive environment that may bring numerous uncertainties and unknown factors to the originally relatively reliable system operation.

The concept of risk is a well-developed one and has been applied in areas of business, finance, managerial economy, and insurance in the modern business environment. Other industries, such as nuclear power, space systems, chemical process, etc. also develop their risk management due to the characteristics of high hazardous consequences. References [1], [12], and [13] reveal the scope of the ways in which financial risk is managed, in contrast to the typical engineering risk that engineers face. Reference [14] documents a Symposium for the Probabilistic Risk Assessment (PRA) and Management in nuclear power, airline safety, chemical process and other industries. Although the terminology of risk in different area varies, the risk assessment is basically asking three questions [14]:

1. What can go wrong?
2. How likely is that to happen?
3. If it does happen, what are the consequences?

To integrate all of these three questions in our risk assessment, theories on both probability and power systems are needed. References [15], [16], and [17] deal with the theory of probability, statistics, and applied statistical methods. Reference [18] discusses the probability concept used in electric power systems, and [19] and [20] give the models, static and dynamic analysis, and operations and control of modern electric power systems. The risk-based security assessment in this work originates from [4] and [21], which provide both the concept and the method of determining the risk in power system stability analysis.

### 2.3 Topics on Thermal Overload and Voltage Stability

Regarding theory of conductor thermal overload and voltage stability, references [22], [23], and [24] have extensive discussions on these two topics, including basic concepts, mechanisms, models, and analytic methods. Particular topics of transmission line thermal overload and its potential impacts can be found [25] through [48]. A bibliographical publication of voltage stability can be found in [50]. The closely related publications on voltage stability as well as the voltage impact are listed in [51] through [77], where the probabilistic methods and research on maximum load margin in voltage assessment are specially emphasized. The detailed reviews of both overload and voltage stability topics are given in Chapters 3 and 4, respectively.

### 2.4 Topics on Decision-Making

Although the major part of this dissertation is the risk assessment for power system security, a review of some of the literature on the decision-making under uncertainty and applications in power systems is also provided. Basic strategies in [86] and applications in [87] and [88] are introduced in Chapter 6 of this dissertation.
2.5 Differences between the Proposed and Related Work

The proposed work differs from the previous ones in the following respects:

- An obvious distinction is made between the proposed probabilistic approach and the traditional deterministic security assessment, within the theoretical foundations of electric power systems.

- A risk-based security assessment framework is provided. Existing literature discusses only cost-based reliability assessment focusing on the aspect of adequacy without considering performance impact.

- Both probability and impact evaluation is incorporated into the risk assessment. Few existing probabilistic assessment approaches, which compute the probability of insecurity, consider quantification of consequence or further monetary cost of these uncertainties.

- The uncertainties existing in the impact as well as system contingencies and other fluctuations are taken into account in the risk-based security assessment. In this way, the limits, such as on branch flows and on bus voltages are no longer treated as mandatory values as in the existing deterministic and probabilistic approaches to security assessment.

- A bridge between power system security and economics is established by the index of risk. The concept of risk that underlies this work provides an economic measurement of system security which is compatible with the economic results of market-based electricity trading. This is a significant difference from other security indices.
3 TRANSMISSION LINE OVERLOAD RISK ASSESSMENT

3.1 Introduction

3.1.1 Problem Statement

Every conductor used in power system transmission circuits has associated ampacities or current limits, called thermal capacities or thermal ratings. An elevation of the conductor temperature due to flow of more current may bring about both mechanical and electrical damages on the transmission line. Consequently, the line’s thermal ratings are limited by the conductor’s Maximum Design Temperature (MDT).

Normally, power engineers deterministically calculate the thermal ratings of transmission lines by applying the maximum designed temperature and other assumed specific values of input data, such as ambient temperature and wind speed, into the thermal balance equation according to the law of Joule heating ([25], [26]). The Institute of Electrical and Electronics Engineers (IEEE) has published a standard [25] to describe how to calculate these deterministic thermal ratings.

It has long been realized that these ratings are conservative and result in under-utilization of conductors. Besides the firm, conservative and non-uniform current limits, there is neither a measurement nor a motivation provided to assist operators in making more informed decisions.

3.1.2 Previous Work

Past research in this area has been generally devoted to increasing conductor ratings to overcome the under-utilization of existing lines. Three techniques have been applied to overcome this drawback:

- Short-Time and Long-Time Emergency Ratings [27], [28], [29],
- Dynamic Thermal Ratings [27], [28], [30], [31], and
- Probabilistic Methods [32], [33], [34], [35], [36], [37].

3.1.2.1 Short- and Long-Time Emergency Ratings

It is generally believed that conductors can sustain over-the-limit operation conditions for a short period of time without significant damage or impact. For example, D.G. Pavin uses higher maximum temperatures to determine temporary overload limits for a 15 minute or 4 hour emergency period [29]. Based on these higher allowable temperatures, one can calculate the higher Short and Long Time Emergency Ratings (STE, LTE) using the same method as provided by the IEEE standard [25]. A similar method can be found in [39].

3.1.2.2 Dynamic Thermal Ratings

Dynamic Thermal Ratings (DTR) are computed from real time meteorological measurements rather than fixed values. They vary as a function of time, depending on variation in ambient conditions. By installing
additional sensors to monitor either ambient data or the conductor heating conditions, this technique releases the ratings from the highly conservative weather assumption of the deterministic method. However, a significant problem with a dynamic line-rating (DTR) system is equipment cost [38]. Another disadvantage of DTR is that it acts well only as a thermal “monitor.” In other words, it prevents the or current thermal damages. DTR, however, cannot prevent future damages. To provide future ratings for the scheduled system condition, it still needs similar conservative assumptions as made by all the deterministic methods to avoid damage in the future.

3.1.2.3 Probabilistic Method

Ambient weather conditions are the major determinants of conductor temperature under a given line flow. Uncertain variables of air temperature, wind speed, solar radiation, and other factors affect the temperature of a conductor. Typically, a designer uses conservative assumptions, for example, 40°C air temperature and 2 ft/s wind speed, to guarantee the extremely low probability of the actual temperature exceeding its limit. This results in a secure but fairly conservative line thermal rating.

Due to the uncertain nature of transmission line overload, there is also a growing interest in using probabilistic methods [32], [33], [34], [35], [36], [37], which account for the variability and stochastic nature of the ambient conditions, without requiring real time meteorological measurements. The probabilistic method calculates the probability that the conductor temperature exceeds its maximum designed temperature under a given current level, such that line ratings can be decided by a permissible overload probability.

3.1.3 Our Approach

In this dissertation, we develop a new method that combines probability and impact calculation to provide the risk of transmission line overload. The risk, which indicates the expectation of the cost consequence associated with thermal overload, may be effectively used to make decisions regarding the line loading and system operation. The assessment, hence, consists of two issues: probability evaluation and impact evaluation. The “component” assessment, which is the thermal risk of a single transmission line under a given line current, and the “system” assessment, which is the thermal risk of a desired control region under a given operating condition, are also discussed and illustrated in the following sections.

3.2 Method Overview

The objective of the approach presented is to assess the risk associated with the transmission line overload given an operating condition. There are two tiers of assessment: “component” assessment and “system” assessment. The purpose of the component assessment is to evaluate the thermal risk of a single transmission line. The term “overload” here; it is not simply that the line flow goes beyond a magic number. Rather, it is that the line temperature exceeds its maximum design temperature. The reason is that a given over-limit line flow does not necessarily result in a harmful thermal overload when the ambient condition is not...
line with a given line flow. The purpose of the system assessment is based on both the probability distribution of line flow under a system-wide environment and the result of component assessment, to calculate the regional or system-wide thermal risk associated with an operating condition.

3.2.1 Component Thermal Risk Assessment [37]

The component thermal risk is the expected impact of thermal overload on a transmission line given a specified current flowing through the line. An illustrative diagram in Figure 2 shows the general components and method to compute the risk.

![Diagram of Component Thermal Risk](image)

Figure 2: Diagram of Component Thermal Risk

The risk is calculated as the expected value of thermal impact including loss of conductor life due to annealing and loss of line clearance due to sag. These two types of impact are all determined by the conductor temperature that is the result of line flow and ambient conditions. Based on the probabilistic presentation of ambient conditions, the probability function of conductor temperature can be obtained and then multiplied with its impact to compute the expected impact value, i.e., the risk.

known. The impact(s) associated with the thermal problem is, therefore, uncertain and depends on the uncertain ambient environment.
3.2.2 System Thermal Risk Assessment

Component thermal risk assessment provides us the detailed, individual-dependent thermal risk study of each transmission line. It also provides a line's thermal risk-current curve under different current levels.\(^4\)

When pursuing system assessment, the line flow on each transmission line is determined by the system operating condition. Different lines, therefore, have different loading levels, ambient environment and capabilities to sustain the thermal burdens, and result in various levels of risk. The system thermal risk assessment is to obtain the regional or system-wide aggregate thermal risk based on the distribution of line flows and ambient conditions. It is computed as the sum of each individual line's thermal risk where each individual risk is obtained by calculating the expected value of impact based on the distribution of line current and the component risk-current curves. Figure 3 is an illustrative diagram for the system thermal risk assessment.

![Diagram of System Analysis](image)

Figure 3: Diagram of System Analysis

3.3 Analytical Development

In this section, we concentrate on the analytic details of the risk assessment of transmission line thermal overload. We first develop the risk assessment for a single conductor and then describe the entire details of risk assessment for an operating condition.

---

\(^4\) For the short time overload, this risk curve depends on both current level and the duration time. We can define the time frame we are interested in studying. One-hour basis study is generally assumed in this report for the security assessment except particularly mentioned and the chapter for planning.
3.3.1 Conductor Heating Model

Electrical current flowing through a conductor experiences resistance and dissipates part of the energy into the conductor. This loss of energy associated with the current flow is shown as the heat gain of the conductor due to resistance. Besides gaining heat through resistance, it is assumed that exposure to the sun is associated with solar radiation gain.

A bare overhead conductor is also exposed to its surrounding ambient circumstance. Heat gain resulting from electric current in a conductor and solar radiation is dissipated in the form of heat to the surrounding atmosphere. This heat dissipation can take any of three forms: conduction, convection, and radiation. Owing to the minimal contact of the conductor with the suspension insulators at each support tower, losses due to heat conduction are negligible in comparison with losses due to convection and radiation.

These losses are primarily driven by the temperature difference between the conductor and its ambient surroundings, while they are significantly enhanced by the cooling effects of wind in the form of convection. A chart of heat flow is shown in Figure 4.

![Figure 4: Conductor Heat Flow](image)

3.3.1.1 Steady State Heat Balance Equation

The steady state conductor temperature is determined from the thermal balance between the heat gain and losses in the conductor. This behavior, in ANSI/IEEE Standard [25], is expressed by the heat balance equation, given in Equation 2. Other thermal models, which include more ambient factors, can be found in [22]. What is needed here is an effective thermal model to compute the conductor temperature under given line flow and various ambient conditions. As shown in Figure 2, the IEEE model used can be substituted for other models depending on the user's preference.
Equation 2: Steady State Heating Model

\[ I^2 R(\theta) + Q_s = Q_r(\theta, \theta_a) + Q_c(\theta, \theta_a, \mu) \]

Here, \( I^2 R \), \( Q_s \) are the Joule (resistance) and solar heat gain, respectively, \( Q_r \) and \( Q_c \) are the heat loss by radiation and convection, \( \theta \), \( \theta_a \) and \( \mu \) are conductor temperature, air temperature and wind velocity, respectively.

At this equilibrium point, the dissipation of heat due to conductor radiation and air convection is equal to the total heat gain of the conductor. The conductor temperature remains at this steady state temperature unless its heat balance is disturbed.

In the following sections, we discuss each heat gain and heat loss term in detail.

3.3.1.2 Convected Heat Loss

The formula of convected heat loss is classified into forced convected heat loss and natural convection heat loss by the IEEE standard [25] because the convection is significantly different depending on whether the conductor is exposed to strong wind or a still atmosphere.

- **Forced Convected Heat Loss**

  \[
  Q_{c1} = \left[ 1.01 + 0.37\left( \frac{DP_f}{\mu_f} \right)^{0.52} \right] k_f (\theta - \theta_a) \\
  Q_{c2} = 0.1695 \left( \frac{DP_f}{\mu_f} \right)^{0.52} k_f (\theta - \theta_a) \\
  Q_c = \max(Q_{c1}, Q_{c2})
  \]

  The convected heat loss \( Q_c \) is the largest value of \( Q_{c1} \) and \( Q_{c2} \), \( D \) is the conductor diameter in inches, \( D \) is the density of air in \( \text{lb/ft}^3 \), \( \mu_f \) is absolute viscosity of air in \( \text{lb/(hr)(ft)} \), and \( k_f \) is thermal conductivity of air in \( \text{W/(\text{hr})(\theta\circ\text{C})} \).

- **Natural Convection Heat Loss at Sea Level**

  With the wind speed being zero, the natural convection heat loss at sea level is expressed as,

  \[
  Q_c = 0.072 D^{0.75} (\theta - \theta_a)^{1.25}
  \]

- **Natural Convection Heat Loss at Altitudes Above Sea Level**

  \[
  Q_c = 0.283 \rho_f^{0.5} D^{0.75} (\theta - \theta_a)^{1.25}
  \]
The data of air density $\rho_f$, air viscosity $\mu_f$, and the coefficient of air thermal conductivity $k_f$ against different altitudes are listed in the tables given in [25] and [26].

3.3.1.3 Radiated Heat Loss

The radiated heat loss of a conductor is given by the expression

$$Q_r = 0.138 D \varepsilon \left[ \left( \frac{\theta - 273}{100} \right)^4 + \left( \frac{\theta + 273}{100} \right)^4 \right]$$

where $\varepsilon$ is the constant of thermal emissivity which is 0.23 for a new conductor and 0.91 for a flat-black well-weathered conductor.

3.3.1.4 Solar Heat Gain

The amount of heat received from the sun is expressed as,

$$Q_s = a Q_D A' \sin(\alpha)$$

where $Q_D$ is the direct solar radiation on the earth surface in $W/m^2$, $A'$ is the projected area of the conductor, and $a$ is the solar-absorption coefficient. This is 0.23 for the new conductor and 0.97 for the black conductor. In the case of a round, horizontally placed conductor, the effective angle of incidence of the sun's rays, $\alpha$, is given by,

$$\alpha = \cos^{-1} \left[ \cos H_c \cos(Z_c - Z_t) \right]$$

where $H_c$ is the altitude of the sun above the horizon, $Z_c$ is the azimuth of the sun, and $Z_t$ is the azimuth of the conductor.\(^5\)

3.3.1.5 Dynamic Heat Balance Equation

When an increase in current occurs suddenly in a conductor, the conductor temperature does not rise instantaneously because of the heat capacity of the conductor. The required time at each current level for the conductor temperature reaching the steady state level is approximately 60 minutes [29]. This time delay depends on the specific heat capacity of the conductor, the weight, and also the ambient weather conditions. The time-temperature characteristics of ACSR is expressed [27] as a first-order differential equation according to

**Equation 3: Conductor Heating**

\[
\text{Heat stored in conductor} = \text{Heat gained from } I^2 R \text{ and solar radiation} - \text{Heat lost via radiation and convection}
\]

or

\[ Z_t = 180^\circ \text{ for a north-south line.} \]
Equation 4: Dynamic Heating Model

\[
P \frac{\partial \theta}{\partial t} = I^2 R(\theta) + Q_f - Q_s(\theta, \theta_s) - Q_c(\theta, \theta_s, u)
\]

where \( P = 4.186(453.6)(C_1 W_1 + C_2 W_2) \) is the heat capacity of the conductor, \( C_1 \) is the specific heat capacity of aluminum, \( W_1 \) is the weight per unit length of aluminum, \( C_2 \) is the specific heat capacity of steel, \( W_2 \) is the weight per unit length of steel, and \( \theta \) is the instantaneous conductor temperature.

3.3.2 Impacts of Thermal Overload

An elevation of the conductor temperature may bring about both mechanical and electrical effects. Three primary factors must be considered when defining the thermal limit of a power line [39]: sag, loss of strength, and limitations of the conductor fittings. We assume that properly designed, selected, and installed fittings are not limiting factors for the thermal limit [39]. Hence, only the significance of sag and loss of strength are considered in this dissertation.\(^6\)

3.3.2.1 Loss of Clearance due to Sag

The thermal expansion caused by the increase of temperature can result in the line dropping beneath its safety clearance (see Figure 5.) Under certain conditions, this may cause flashover to the ground, resulting in a ground fault, outage of the circuit, and weakening of the system with the possibility of cascading events or a severe underlying human injury. This was the case in the well-known July 2, 1996 WSCC outage where a 345-KV line sagged and touched a 15 ft tree [38]. Hence, the impact, which depends on whether flashover occurs, is a function of line temperature. The limiting condition for sag is that line temperature should not exceed a limiting temperature for which the line sags through all of the designed safety margins, as specified by the National Electrical Safety Code (NESC) [40].

We express the temperature corresponding to this limiting condition as \( \theta_L = \theta_{MDT} + M_g / K_s \),\(^7\) where \( \theta_{MDT} \) is the maximum design temperature at which a designed safety margin \( M_g \) is maintained, and \( K_s \) is the sagging coefficient which represents the increasing sag due to the temperature rising by 1°C. We then express the impact of sag as,

Equation 5: Impact of Loss of Clearance

\[
\text{Im}_{\text{sag}}(\theta) = \begin{cases} 
\text{Im[Fault]} & \text{if } \theta > \theta_L \\
0 & \text{otherwise}
\end{cases}
\]

\(^6\) Other impacts, such as fitting failures, can be analyzed and included in a similar way as the impact we consider in this dissertation.

\(^7\) Other sag-temperature model can be found in [41], [42], [43], [44], and [45], where both the elastic and inelastic conductor elongation such as creep are considered.
where \( \text{Im}[\text{Fault}] \) is the impact (or financial cost) corresponding to an outage of the overloaded circuit. This impact is dependent upon operating conditions, and its quantification requires analysis by power flow and stability simulation. Generally, however, if a circuit is so heavily loaded that it sags and flashes over, it is likely that the impact of its outage will be substantial and highly undesirable. Therefore, we set \( \text{Im}[\text{Fault}] \) to be very large.

![Figure 5: Conductor Sag](image)

The safety clearance by the NESC [40] is specified large enough so that under most conditions, the probability of occurrence for a flashover is extremely small. However, when weather conditions are extreme (high ambient temperature and little wind) or during an emergency short time overload when current is very high, this probability increases and the risk of sag can be substantial.

### 3.3.2.2 Loss of Strength due to Annealing

Annealing, the recrystallization of metal, is a gradual and irreversible process when the grain matrix established by cold working is consumed, causing loss of tensile strength [46]. When fully recrystallized, the metal would be in the softened state as it was before cold working. Whenever the line's remaining strength decreases to the tensile load, it indicates the end of service life, and thus replacement of the line is required. The conductor's expected total life, therefore, is the amount of time for the conductor strength to reach the tensile load, under the condition that the conductor operating temperature is always maintained at its maximum allowable temperature. For instance, the projected or anticipated total life of the conductors in New York state is 25-40 years [28]. But a higher annealing rate caused by higher operating temperatures will accelerate this process and therefore reduce the lifetime of the conductor.
Conductor strength reduction curves as a function of both conductor temperature and time is illustrated in Figure 6. The rate of strength loss varies with conductor operating temperatures, as illustrated by these curves. The expressions to describe this phenomenon have been presented in references [42], [46], [47], and [48].

When the conductor operating temperature is higher than the maximum allowable value $\theta_{MDT}$, the annealing rate is greater than that for which it was designed, and therefore the conductor's expected life decreases. The annealing impact of thermal overload is proportional to this decrease of expected life. We compute the decrease in expected life as the difference between the overload time interval $\tau$ during which time the conductor operates at a temperature $\theta > \theta_{MDT}$, when conductor strength reduction is $\Delta S(\theta, \tau)$, and the expected time $t$ required to lose this same amount of strength at the design temperature $\theta_{MDT}$. We denote this decrease in expected life as $\Delta t = t - \tau$.

![Figure 6: Illustration of Loss of Life by Strength Reduction Curve](image)

When computing risk for continuous operation at an elevated temperature $\theta > \theta_{MDT}$, we have that $t = t_c(\theta_{MDT})$ and $\tau = r_c(\theta)$ so that $\Delta t$ is given by

$$\Delta t = t_c(\theta_{MDT}) - r_c(\theta)$$

where $r_c(\theta)$ is the remaining conductor life for conductor continuous operation at $\theta$, during which time the conductor strength reduces from the present strength to its tensile load, and $t_c(\theta_{MDT})$ is the expected remaining

---

8 The tensile load shown in Figure 6 may decrease with an increase of line sag based on a hyperbolic function of line sag.
conductor life for continuous operation at $\theta_{MDT}$, when the same amount of strength (denoted as $\Delta S_c = \Delta S(\theta_{MDT}, t_c) = \Delta S(\theta, t_c)$) is lost.

When computing risk for temporary operation at an elevated temperature $\theta > \theta_{MDT}$, $\Delta t$ is given by $^9$

$$\Delta t = \begin{cases} 
    t_x(\theta_{MDT}) - t_c(\theta) & \Delta S < \Delta S_c \\
    t_c(\theta_{MDT}) - t_c(\theta) & \text{otherwise}
\end{cases}$$

where $t_x$ is the temporary overload time interval at the operating temperature $\theta > \theta_{MDT}$, during which time the tensile strength reduces by $\Delta S_x(\theta, t_x)$, $t_c(\theta_{MDT})$ is the time required for the same amount of strength reduction when the conductor operates at the design temperature $\theta_{MDT}$.

In both cases, the annealing impact of thermal overload is then given by

**Equation 6: Impact of Loss of Life**

$$Im_{\text{anneal}}(\theta) = \begin{cases} 
    \frac{\Delta t \times C_t}{t_0} & \theta > \theta_{MDT} \\
    0 & \text{otherwise}
\end{cases}$$

where $C_t$ is the cost of re-conductoring the circuit, and $t_0 = t_c(\theta_{MDT})$ is the expected remaining conductor life.

### 3.3.3 Probabilistic Model

#### 3.3.3.1 Probability Distribution of Conductor Temperature

Because of the uncertainty of ambient conditions, the probability density function of conductor temperature $\theta$ is governed by the joint probability function of ambient conditions.

- **Continuous Loading**

  For the continuous loading case, given the current, the probability of conductor temperature being $\theta$ is the summation of the joint probability of all ambient conditions$^{10}$ ($z$) such that the temperature determined by them under the given current $I$, by the steady state thermal balance Equation 2, is $\theta$, i.e.,

---

$^9$ If the temperature $\theta$ or temporary overload time $t_x$ is so high that the conductor's strength "quickly" decreases to its tensile load within the time $t$, the problem degenerates to the continuous case.

$^{10}$ There are many combinations of wind speed and ambient temperature that result in a particular conductor temperature under the current $I$. 

Equation 7: Probability Function of Conductor Temperature under Continuous Loading

\[ \Pr(\theta|I) = \sum_{\xi} \Pr(\xi) \quad \forall \xi \in \{ \theta: \theta(\xi, I) = \theta \} \]

- **Short-Time Emergency Loading**

  When an increase in current occurs suddenly in a conductor, the conductor temperature rises according to its time-temperature characteristics of Equation 4. When a conductor eventually reaches its steady state temperature, i.e., \( \frac{d\theta}{dt} = 0 \), this equation degenerates into Equation 2 as shown in Section 3.3.1.1.

  Given a short-time emergency (STE) overload, which could be much higher than the continuous rating, the conductor temperature does not necessarily reach a level determined by the steady state thermal balance Equation 2 during the short time interval \( r \). Therefore, to determine conductor temperature after a short-time overload, one needs to consider the line temperature prior to the overload, and the temperature increase during the time interval \( r \). Equation 8 provides the probability of the conductor temperature, given a short-time overload current \( I \) during the time interval \( r \).

Equation 8: Probability Function of Conductor Temperature under Short-Time Loading

\[ \Pr(\theta|I) = \sum_{\xi} \Pr(\xi) \quad \forall \xi \in \{ \theta: \theta(\xi, I) = \theta \} \]

Here \( \theta_0 \) is the conductor temperature just prior to the short-time overload.

- **Long-Time Emergency Loading**

  It may be of interest to sustain an overload for time periods greater than 1 hour, e.g., for 3 to 4 hours. In this case, the dynamic behavior of conductor temperature is negligible, since this time frame is longer than that of the time-temperature transient period. The probability expression is then the same as that of the continuous loading.

3.3.3.2 Probabilistic Model of Ambient Conditions

The temperature of the line is influenced by the ambient conditions, but detailed analysis of thermal mechanics is beyond the scope of this dissertation. In the analysis presented, three major factors: ambient temperature, wind speed, and solar radiation are considered. The random behavior of air temperature \( \theta_a \) and wind speed \( u \) are modeled as Normal and Weibull distributions [34] and [35] respectively, as in Equation 9, where the optimal parameters of distribution functions can be obtained using point estimation [14] from historical data.
Equation 9: Probability Function of Ambient Conditions

\[
\begin{align*}
Pr(\theta | \mu, \sigma) &= \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(\theta - \mu)^2}{2\sigma^2}} \\
Pr(u | \gamma, \beta) &= \frac{\gamma}{\beta} u^{\gamma-1} e^{-u^\gamma/\beta}
\end{align*}
\]

Here, \( \mu, \sigma, \gamma \) and \( \beta \) are the scale and shape parameters for both distributions. They can be estimated by the method shown in the following section.

The solar radiation is assumed constant, since its influence is relatively small. Therefore, the vector \( z \) used in Equation 7 and Equation 8 consists of the air temperature \( \theta \) and wind velocity \( u \).\(^\dagger\) The joint distribution of \( z \) is the product of the distribution functions of \( \theta \) and \( u \), which is \(^\ddagger\)

\[
Pr(z) = Pr(\theta | \mu) \times Pr(u)
\]

We would emphasize that the assumption of these probability functions is only a prior distribution that will be discussed in the chapter on decision-making. The probability density function (PDF) of such a distribution may vary significantly from the near-normal distribution due to the non-normal PDF's of the individual meteorological distributions. To obtain a "true" or "near-true" probability distribution is a continuous task for both statisticians and engineers. A better estimation of the distributions provides better information for the decision-maker. However, one typically has to make the decision before all facts are completely known. This means one is unable to obtain all the knowledge (moments) of the true distribution function but rather only a limited set of moments or even only a mean value. Even if we obtained a "perfect" function to describe the historical statistics of weather completely, whether it would fit the future condition is still questionable. For our risk analysis, a particular form of probability function does not influence the method used in computing the risk, which is the expected value of impact. Using computers, the expected value can be computed from any distribution function, even if there is no analytical form for the function. Meanwhile, we expect the prior distribution function will be continuously updated based on any posterior information and knowledge, as explained in Chapter 6.

3.3.3.2.1 Parametric Estimator of Normal Distribution

The mean and variance of a Normal distribution can be estimated by the sample mean and sample variance [14].

\(^\dagger\) More variables may be included in the vector \( z \) if desired for use in more detailed models.

\(^\ddagger\) If the correlation between each other is negligible, then we have \( Pr(z) = Pr(\theta | \mu) \times Pr(u) \).
where $X_i$ is one of the $n$ samples from a Normal distribution.

### 3.3.3.2.2 Parametric Estimator of Weibull Distribution [18]

The PDF of a Weibull distribution is given by

$$f_X(x|\gamma, \beta) = \frac{\gamma}{\beta} x^{\gamma-1} e^{-x/\beta}$$

The Cumulative Distribution Function (CDF) can be obtained by integrating this PDF function with respect to the random variable $x$.

$$F_X(x|\gamma, \beta) = \frac{\gamma}{\beta} x^{\gamma-1} e^{-x/\beta} dx$$

$$= 1 - e^{-\left(\frac{x}{\beta}\right)^\gamma}$$

This leads to

$$\ln\left(\ln\left(\frac{1}{1 - F_X(x)}\right)\right) = \gamma \ln(x) - \ln(\beta)$$

Then we have a linear transformed function:

$$y = \gamma z - \ln(\beta)$$

where

$$y = \ln\left(\ln\left(\frac{1}{1 - F_X(x)}\right)\right)$$

$$z = \ln(x)$$

The slope ($\gamma$) and the interception ($-\ln(\beta)$) can be obtained by a linear fit shown in Figure 7. The parameters of the Weibull distribution are then estimated.

![Figure 7: Parametric Estimation of Weibull Distribution](image-url)
3.3.4 Risk of Thermal Overload

3.3.4.1 Thermal Risk Incurred by a Single Conductor with a Given Flow

Risk, which is the expected impact of an event, is the product of the event's probability and its impact. For our case, the event is thermal overload, i.e., temperature of conductor being greater than the permissible value, and the risk is the expectation of costs that may result. Given the current $I$, we may compute thermal overload risk as the probability of the temperature being greater than $\theta_{MDT}$, times its related impacts, i.e.,

$$
Risk(I) = E(Im \mid I) = \int_{\theta > \theta_{MDT}} \Pr(\theta \mid I) \times (Im_{seg}(\theta \mid I) + Im_{convec}(\theta \mid I))d\theta
$$

Here, $\theta$ is the conductor temperature which is influenced by the line current $I$ together with the ambient conditions, $Pr(\theta \mid I)$ is the conditional probability density function (PDF) of the conductor temperature $\theta$ given $I$, and $Risk(I)$ is the risk regarding this line loading.

3.3.4.2 Thermal Risk Incurred by a Transmission Network with a Given Operating Condition

Given an operating condition of the transmission network $X$, the thermal risk associated with this condition is calculated as the expected thermal impact, i.e., the conditional expected impact under line flow times the probability of line flows, based on the Bayes' Theorem [14].

$$
Risk(X) = E(Im \mid X) = \sum_{i=1}^{n_{line}} E(I_{im} \mid I_i) \times Pr(I_i \mid X) = \sum_{i=1}^{n_{line}} Risk(I_i) \times Pr(I_i \mid X)
$$

where $E(Im \mid X)$ is the total expected thermal impact of the operating condition $X$. It is the sum of all the expected thermal impacts from each transmission line; $n_{line}$ is the number of lines in the concerned
transmission area, the $Risk(I)$ is the expected thermal impact on an individual line which is calculated from Equation 10; $Pr(I | X)$ is the probability of line current given a particular operating condition.

To compute the thermal risk for a system operation condition, we need to answer the following questions:

1. How much is the probability of a line current given a system operating condition?
2. How much is the thermal risk on each line when the line current is given?

We have answered the second question in Section 3.3.4.1. The probability of a line flow is dependent on the uncertainties of system operation. There are contingencies, load fluctuations, load correlation on buses, and other deviations of system parameters.

### 3.3.4.2.1 Risk of Thermal Overload with Uncertainty of Contingencies

In this case, the probability function of line flow depends on the probability function of contingencies given that the post-contingency line flow can be uniquely determined by the contingency. Then,

$$Risk(X) = \sum_{i=1}^{n_{\text{contingency}}} \sum_{k=0}^{n_{\text{contingency}}} Risk(I_{ik} | E_k) \times Pr(E_k | X)$$

where $n_{\text{contingency}}$ is the number of possible contingencies, $E_k$ is the k-th contingency, $k = 0$ represents the no-contingency case whose probability is $(1.0 - \sum_{k=1}^{n_{\text{contingency}}} E_k)$, $I_{ik}$ is the post-contingency flow on the line $i$.

### 3.3.4.2.2 Risk of Thermal Overload with Uncertainty of Loads and Other Parameters

Besides the uncertainty of contingencies, the loads and other system parameters may be uncertain. The method of estimating the risk of thermal overload under these uncertainties is similar to that used in estimating the risk of voltage insecurity in Chapter 6, where both bus voltages and line flows can be obtained through a power flow solution.

### 3.4 Illustrations

#### 3.4.1 Component Analysis

##### 3.4.1.1 Deterministic Data

The example consists of a 1000ft. "Drake" conductor 795 kcmil 26/7 ACSR, and for every 1°C temperature increase, the sag of the line increases by 0.6". The conditions in Table 1 are used to compute the
Table 1: Deterministic Conditions for Computing Line Ratings

<table>
<thead>
<tr>
<th>Condition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ambient Air Temperature ($\theta_a$)</td>
<td>40 °C</td>
</tr>
<tr>
<td>Wind Speed ($u$)</td>
<td>2 ft/s</td>
</tr>
<tr>
<td>Maximum Design Temperature ($\theta_{MDT}$)</td>
<td>100 °C</td>
</tr>
</tbody>
</table>

Table 2: Example of Conductor Ratings by Deterministic Method

<table>
<thead>
<tr>
<th>Condition</th>
<th>Maximum Temperature</th>
<th>Ratings (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>100°C, 9 hrs</td>
<td>992</td>
</tr>
<tr>
<td>LTE</td>
<td>115°C, 3 hrs</td>
<td>1140</td>
</tr>
<tr>
<td>STE</td>
<td>125°C, 15 min</td>
<td>1310</td>
</tr>
</tbody>
</table>

deterministic thermal ratings. The line ratings, by the deterministic method of IEEE standard [25], are listed in Table 2.

3.4.1.2 Probability Distribution of Ambient Conditions

Ambient conditions are not always as severe as assumed in calculating the deterministic ratings. Suppose that the mean and standard deviation of air temperature and wind speed around this conductor, according to historical data, are as listed in Table 3. Their probability density functions using Equation 9 are shown in Figure 8. We see that air temperature of 40 °C is beyond four standard deviations of its mean, so that its probability is less than 10^{-4}.

For the long transmission line, the ambient conditions tend to vary along the line. If, however, we assume that the statistical description of this variation is uniform along the line length, then the probability that a particular ambient condition happens at one location is the same as the probability that this condition happens at another location (not necessarily at the same time). We can still use the statistics at one location to describe the weather distribution along the entire line. Otherwise, we should use the data collected at the location where the ambient conditions are statistically most critical.

Table 3: Statistics of Ambient Conditions

<table>
<thead>
<tr>
<th>Condition</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air Temperature, °C</td>
<td>15</td>
<td>6.3</td>
</tr>
<tr>
<td>Wind Speed, ft/s</td>
<td>3.5</td>
<td>1.3</td>
</tr>
</tbody>
</table>
3.4.1.3 Probability Distribution of Conductor Temperature

Conductor temperature is a function of current as well as ambient conditions (see Equation 2 and Equation 4). Even when the line carries its deterministic limiting current, its temperature is typically below its maximum design value $\theta_{MDT}$. Figure 9 shows that conductor temperature in most cases varies from $11^\circ C$ to $138^\circ C$ with different ambient conditions when continuous limiting current ($992\,A$) flows through the line. We see that only for very high temperature and very low wind speed, does the temperature exceed $\theta_{MDT}$ which is $100^\circ C$.

The distribution of conductor temperature when the line carries its continuous limiting current is shown in Figure 10. It shows that the average conductor temperature is $61.8^\circ C$, which is much lower than the maximum design value ($100^\circ C$). The probability of the temperature being greater than $100^\circ C$ is only about 1 percent (0.011). This is why the deterministic rating is thought to be very conservative, and it provides motivation to consider increasing this rating beyond $992\,A$. One way to determine the rating increase is to compare the additional risk to the additional benefit. In what follows, we illustrate how to compute additional risk. However, benefit assessment is dependent on line location and the economics of power transmission, and it must be done on a case-by-case basis. Another way to determine the rating increase is by applying the equal risk criterion. This is the approach we will illustrate in Section 6.4.1.
Figure 9: Conductor Temperature under Various Ambient Conditions

Figure 10: Probability Density Function of Conductor Temperature, given $I = 992(A)$
3.4.1.4 Thermal Risk of a Given Loading Level

Combining the distribution of temperature with its potential impacts, the risk for various current levels is calculated using Equation 10. The results are shown in Figure 11, Figure 12, and Figure 13 where the sagging impact is assumed much higher than that of the conductor's annealing.\textsuperscript{13}

3.4.1.4.1 Thermal Risk of Continuous Loading

For the continuous loading, the risk of thermal overload incurred by the conductor is shown in Figure 11. It shows the risk of thermal overload incurred by the conductor running at different levels of continuous current. This thermal risk increases with the increase of line loading. The figure also shows that the risk of the deterministic continuous rating is not zero, since the ambient condition could be more severe than that of the deterministic 40°C air temperature. Because this small amount of risk has been accepted implicitly in the industry, we will use it to identify ratings associated with temporary overload.

3.4.1.4.2 Thermal Risk of Temporary Overload

Figure 12 shows the iso-risk contours for the long time emergency (LTE) overload that lasts more than one hour.\textsuperscript{14} With a higher risk level, the equal risk contour for the conductor loading expands to the right of the plot,

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure11.png}
\caption{Risk of Carrying Continuous Over-Current}
\end{figure}

\textsuperscript{13} We choose 100 times the single line re-conducting cost as the flashover impact, which depends on the importance of the conductor in the whole system.

\textsuperscript{14} The overload time can be arbitrarily defined. In general, it could be any time appropriate to the situation.
which means the conductor can tolerate more current under a temporary overload without increase of risks. It can also be observed from Figure 12 that the time-inverse effect of iso-risk curves become inelastic because heavy loading makes the sag become the dominant hurt, which does not depend on overload duration for long run to the conductor.

![Figure 12: Risk of Long-Time Emergency Overload](image)

When the duration of overload is less than 60 minutes, the conductor temperature will not reach a steady state value due to the influence of temperature transient behavior. Therefore, the probability of the conductor temperature exceeding the allowable temperature is reduced, and current levels are higher for the same level of risk.

If the conductor initially operates at the continuous loading at 992A with the mean conductor temperature of 61.8°C, the iso-risk contours for this short-time overload are shown in Figure 13, where the curve for 0.005 risk\textsuperscript{15} varies from 1225A to 1700A according to duration of short-time overload. The figure shows the conductor can tolerate more overload current and longer time if a higher risk is accepted. When the dynamic of conductor temperature calms down, the risk curves converge to those corresponding curves in the LTE risk plot (Figure 12).

\textsuperscript{15} The value of risk is normalized by the line's re-conducting cost.
3.4.2 System Analysis

The system analysis is a modified IEEE RTS-96 system [49]. The local transmission area that we are particularly interested in is the area that contains line 130-120, 230-120, and 230-130. The line flows of each line under different line outage conditions are shown in Figure 14, Figure 15, and Figure 16, where the system load is proportionally allocated throughout the entire system, and the change in load is followed by the generation at 230. The probabilities of these outages are listed in [49]. We also assume those 230 KV lines have the same thermal characteristics as assumed in Section 3.4.1, and they have the same re-conductoring cost of $108,000 per mile. Under these assumptions, the deterministic continuous, 3-hour and 15-minute temporary ratings are 395 MVA, 454 MVA, and 521 MVA, respectively, according to Table 2. We can compare these ratings and the line flows in Figure 14, Figure 15, and Figure 16 to get an intuitive feeling of overload risk.

Figure 17 shows the quantitative risk of transmission line thermal overload against various system load levels. This is illustrated in a semi-log plot. As expected intuitively, the line 120-230, in this case, suffers much more risk of overload when load level is more than 3400 MW. The line 130-120 has little risk of overload. The line 230-130 has negligible overload risk that cannot even be shown in the same risk plot of the other two lines. Figure 18 shows the total transmission lines overload risk in our specified local area under various system conditions. With the current load level of 3200 MW, the area's transmission line overload risk is almost nothing (less than $10^{-3}$ per hour) which indicates the local area is not a transmission line constrained area.
Figure 14: Flows on Line 130-120

Figure 15: Flows on Line 230-120
Figure 16: Flows on Line 230-130

Figure 17: Risk of Thermal Overload on Lines
3.5 Summary

A systematic approach to measuring the risk associated with the thermal overload is presented in this chapter. The approach and its components are illustrated in Figure 2 and Figure 3. The probabilistic ambient models, the evaluation of probability distribution of conduction temperature, and the corresponding impacts are addressed. Finally, the method to evaluate the risk and how to apply the risk assessment into transmission line thermal ratings are also discussed.

The quantitative measurement of thermal risk is helpful for the operator in trading off the benefits and costs in a competitive utility environment. The risk function can also be included, along with a benefit term, in the optimization functions to reach an optimal decision for power system operation.
4 VOLTAGE RISK ASSESSMENT

4.1 Introduction

4.1.1 Problem Statement

The voltages in a power system are required to be maintained within such a range that they satisfy the requirements of electric customers and are stable under any disturbances. However, a voltage collapse may occur on power systems that are heavily loaded, weakened by transmission outages, or subjected to reactive power shortages. This association with reactive power deficiencies may result in uncontrollable system-wide voltage collapse, loss of loads, and blackout.

4.1.2 Previous Work

The prevailing practice in industry of providing energy to customers and avoiding voltage insecurity is to maintain a deterministic reliability margin on bus voltages, reactive power requirements, transfer capabilities, or system loading levels such that the system can survive the collapse under any single component failure. Figure 19 is a typical plot of several P-V curves used in analyzing a voltage-constrained network.

The solid line is a P-V curve for a case without any contingencies. It shows the maximum loading capability as 4070 MW if no contingency occurs. With the possibility of contingencies, the P-V curve typically becomes more restrictive. The most constraining contingency in this case gives a maximum loadability at 3689 MW. At this operating point, the system is not safe enough because a small deviation in system conditions together with the outage of the line 230-120 will result in a collapse of the entire system. In setting the operating guidelines, a safety margin, for example, 3%, is selected, and the total capability is established. The available capability is then the distance between the current operating point and the security boundary discounted by the safety margin (Figure 19). This procedure effectively avoids the collapse by creating a firm boundary against all the "dangerous" possibilities. However, it does not provide answers to the following questions:

- Risk Quantification: How safe or how risky is the current system's operating condition?
- Trend: How does the risk change as the operating conditions are relieved or stressed?

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16 Some reliability councils may have other requirements. However, maintaining a deterministic security boundary is a widely accepted de facto reliability criterion.

17 The system for which these curves are obtained is described in Section 4.4.2.

18 The available capability, that is the distance between the security boundary and the current operating point, is usually intuitively thought to be a measure of a system's safeness. However, one still does not know what the real difference is between 10 MW and 100 MW of available capability. The question is what will happen, and what will the consequences be in these situations?
• Security-Economy Tradeoff: How is increased risk associated with heavier use of facilities offset by the corresponding increase in benefit?

We wish to develop a risk index for voltage insecurity that provides a quantitative justification of system reliability in terms of the system economics. There have been numerous techniques and indices in measuring the voltage stability problem. We generally classify them into deterministic and probabilistic methods.

![Figure 19: P-V Curves at a Load Bus](image)

4.1.2.1 Deterministic Method

Most of the previous work on voltage stability study is by the deterministic method where there is no uncertainty considered in the analysis. Performance indices of voltage ([19] and [51]) are used often with satisfactory results in many on-line or off-line security assessment tools. Other indices, such as Sensitivity Factors, Singular Values and Eigenvalues, Loading Margin and Closest Loadability, Tangent Vector Index, etc., have been proposed, as summarized by Dr. Canizares in [52]. Recently, some researchers have presented their results by using first and second order approximation to the loading margin to estimate the loadability, for example [53], and Available Transfer Capability (ATC). We would like to mention briefly the loading margin and the margin sensitivity techniques because they closely relate to our approach where these deterministic techniques are used probabilistically to compute the probability of voltage collapse.
4.1.2.1 Loading Margin and Margin Sensitivity

"For a particular operating point, the amount of additional load in a specific pattern of load increase that would cause a voltage collapse is called the loading margin to voltage collapse. Loading margin is the most basic and widely accepted index of voltage collapse. Every paper on voltage collapse indices uses loading margin as the horizontal scale when the performance of the index is graphed" [52]. As shown in Figure 19, the loading margin to voltage collapse is the change in loading between the operating point and the nose of the P-V curve. There are basically two methods to obtain the loading margin: Continuation Power Flow ([54], [55], and [56]) and Direct Methods [54], [56] and [57]). The advantages, listed in [52], of the loading margin as a voltage collapse index are:

- The loading margin is straightforward, well accepted, and easily understood.
- The loading margin is not based on a particular system model, but can be used with either static power system models or dynamic system models.
- The loading margin is an accurate index that takes full account of the power system nonlinearity and limits, such as reactive power control limits encountered as the loading increases. Limits are not directly reflected as sudden changes on the loading margin.
- Once the loading margin is computed, it is easy and quick to compute the margin sensitivity with respect to any power system parameters or controls ([60], [61], and [62]).
- The loading margin accounts for the pattern of load increase. [20]

The disadvantages of the loading margin as a voltage collapse index are [52]:

- The loading margin requires the assumption of a direction of load increase.
- The loading margin requires computation at points away from the current operating point and hence, are computationally more expensive than indices only using information at the operating point.

As mentioned earlier, the margin sensitivity with respect to any power system parameters or controls developed in [60], [61], and [62] can be computed easily as long as a loading margin is obtained. It can also alleviate the dependence of the loading margin on an assumed pattern of load increase and can be used in the probabilistic assessment as described in our approach.

4.1.2.2 Probabilistic Method

As mentioned in [64], "there are comparatively few applications of probabilistic analysis to voltage stability problems." References [9], and [63] through [70] measure the probability of voltage instability and the corresponding indices. For the voltage collapse problem, [9] measures the probability of the system demand

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19 The voltage collapse problem can be stated as an optimization problem ([58] and [59]) that allows for the use of the well-known optimization techniques such as Interior Point Methods, Lagrangian Methods, etc.

20 This can be viewed as a disadvantage as well.
moving beyond the system collapse boundary. The economic significance of security using the Expected Demand Not Served (EDNS), Expected Unserved Energy (EUE), and Expected Outage Cost (EOC) are also discussed in [9]. Reference [63] calculates the probability and frequency of voltage instability and the expected voltage stability margin by examining its voltage stability indicator at any load bus exceeding a threshold value during outages. On the other hand, a probabilistic load flow method based on linearization of AC power flow equations is used to assess the voltage instability reflecting the random variation of loads, generation uncertainties, dispatching effects and outages in [64]. The voltage standard deviations and the voltage/reactive injection sensitivities \( \frac{AV}{dQ} \) are proposed as effective voltage collapse indices. Other indices include, a probabilistic evaluation of the load margin based on Tangent Vector and Monte Carlo simulations given in [65], and an expected voltage instability proximity using \( \frac{AV}{dQ} \) sensitivities discussed in [68]. References [66], [67], and [69] apply Interior Point and Lagrangian optimization techniques probabilistically in solving for voltage stability problems. Such indices as the Probability of Unsolvable Cases (PUC), Loss of Load Expectation (LOLE), and Instability Risk Factor\(^{21}\) at PQ buses are proposed. The previous work on probabilistic power flow (see [6] and [7]) can also be treated as a probabilistic method on voltage insecurity. They provide the probabilistic description of system voltages when system does not have voltage collapse problem.

4.1.3 Our Approach

In this dissertation, a method that combines both probability and impact calculations is developed. The technique provides the risk of voltage insecurity where the deterministic loading margin and margin sensitivity method is used in the probability calculation. The risk, which indicates the expectation of the cost consequence associated with two voltage insecurity problems of voltage collapse or voltage-out-of-limit\(^{22}\) may be effectively used to make decisions regarding the system operation.

Our risk assessment generally consists of two issues: probability and impact evaluations. This method is decomposed into two tiers of assessments: "component" and "system" assessments. The component assessment gives a detailed study of the impact at each bus under given bus voltages. The system assessment, however, with consideration of system uncertainties, gives the voltage risk of a given operating condition for a desired region. Both assessments are discussed and illustrated in the following sections.

4.2 Method Overview

The objective of our approach is to assess the risk associated with the voltage insecurity given an operating condition. The purpose of the component assessment is to evaluate the voltage risk at a single load bus with a given bus voltage. Its result will be used in the system assessment that measures the possibility of both voltage

\(^{21}\) It is defined as the probability of maximum active power demand less than actual load demand at a PQ bus [69].

\(^{22}\) It is not a deterministic voltage limit as treated by other reliability assessment.
collapse and voltage-out-of-limit under a system-wide environment. Both assessments relate to the evaluation of probability and its impact.

### 4.2.1 Component Voltage Risk Assessment

The component voltage risk is the expected impact of load interruption at a load bus given a specified bus voltage where the cost consequence of voltage insecurity is evaluated in detail. It indicates the amount of money that one can expect to lose due to load interruption under a given bus voltage. Two measurements are needed to evaluate this bus-oriented risk under given voltage levels: probability of service interruption and expected interruption cost. The risk at the bus is the product of these two measurements.

Given a bus voltage, the amount of load being interrupted at a bus depends on whether this voltage exceeds the tolerance of the load connected to the bus or not. With the statistically distributed load characteristics for various load classes in an aggregated bus load, the tolerance, or the interruption voltages of an aggregated load becomes random. The probability that this random load tolerance covers the given load voltage, leads to the probability of the load being served or not being interrupted. Multiplying the probability that a load does lead to interruption and the expected interruption cost, it gives an expectation impact on the load of this bus due to the possible insecurity risk of the given voltage. The illustrative diagram of component voltage risk study is shown in Figure 20.

For each bus, the component study gives a "risk-voltage" plot which encapsulates the detailed study of expected voltage impact at this bus. It will be used in the system study where the probabilistic bus voltages, and hence the risk, are obtained via the uncertain system conditions.

![Diagram of Component Voltage Risk Study](image-url)
4.2.2 System Voltage Risk Assessment

The result of component assessment provides a detailed, bus-oriented expected impact if a particular voltage profile occurs. However, the bus voltages are determined by the operating condition and other system-oriented factors, such as contingency, system load deviation, load distribution factors on each bus, and so on.

The purpose of system voltage risk assessment is to estimate the voltage risk for a given operating condition under these uncertainties. In contrast to the component voltage risk assessment where the major task is to evaluate the expected impact of a voltage profile at a bus, the major task of the system voltage risk assessment is to obtain the probability description of the voltage profiles throughout buses. The final result of system assessment, i.e., the risk, is obtained by combining both the system-induced probability assessment and the component-induced impact assessment.

From the viewpoint of the entire system, we assume that there are two distinct outcomes for the future performance of system voltages, i.e., whether they collapse or not. The bifurcation point shown on the P-V curve provides the boundary between these two outcomes. With the system operating without suffering a voltage collapse, the voltage may still go beyond the tolerance of loads, resulting in service interruption. Our risk includes both these outcomes.

Equation 12: General Expression for Voltage Risk

\[
Risk = Pr(Collapse) \times Im(Collapse) + Pr(Collapse) \times Im(NoCollapse)
\]

By Equation 12, the risk assessment includes two issues: measuring the probability of collapse and the expected impact of both collapse and out-of-limit.

According to the operating condition and future uncertainties, different buses have different voltage levels with the associated probabilities. Applying the bus-oriented risk-voltage curves from the component study, we can then determine the total system risk by aggregating the associated bus risks weighted by the occurrence probability of bus voltages. The illustrative diagram is shown in Figure 21.

4.3 Analytical Development

In this section, we give the details of the voltage risk assessment. We first develop the component voltage risk assessment for a single bus, and then the system-wide voltage risk assessment.

4.3.1 Definition

We define the "risk," as a condition under which there is a possibility of an adverse deviation from a desired outcome that is expected or hoped for [1]. There are two primitives included within this definition:
future uncertainties and impact of outcomes. Furthermore, we define the degree of risk\textsuperscript{23} as the expectation\textsuperscript{24} of the dollar-based impacts of these outcomes, which would be the amount of impact multiplied by the corresponding probability of outcome.

4.3.2 Component Voltage Risk Assessment

The objective of component voltage risk assessment is to find the voltage risk at a bus under a given bus voltage. With a given voltage, the risk evaluated measures the probability of the service being interrupted and the cost consequence associated with this interruption. The result is provided by a bus-oriented risk-voltage curve such that different buses have different risk-voltage curves according to the associated load mix and service interruption costs.

4.3.2.1 Load Interruption Voltages

In situations where under- or over-voltage protection is widely used in both power system's distribution networks and within the load itself, the distribution components and the loads are protected from over-current or other potential damage due to unacceptable load voltage. Additionally, cascading voltage collapse is avoided by employing load-shedding schemes which will automatically trip the individual load or load groups. Under these conditions the voltage violates its set threshold leading to service interruption of users whose services have been

\textsuperscript{23} We will use the term "risk" to mean the degree of risk in the later sections.
\textsuperscript{24} Expected value is a measure of risk in the theory of "risk management," although other measures, such as probability and variance, can be used to quantify the degree of risk [1].
tripped ([71] and [23]). Also, some loads may drop off by themselves without any action of protective relays when voltage is unsustainable.

Table 4 [72], summarizes the examples of interruption voltages due to distribution protections.

Because the load interruption voltages are equipment dependent, they should be modeled individually. However, an entire distribution system is usually modeled as a single aggregated load at a bus in typical power flow and stability studies. This aggregated load at a bus is the composition of many individual loads with a number of load characteristics.

Table 4: Electric Service Deviation Tolerances for Load-And-Control Equipment [72]

<table>
<thead>
<tr>
<th>Device</th>
<th>Voltage Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Communication Equipment</td>
<td>± 5 %</td>
</tr>
<tr>
<td>Computers, Data Processing Equipment</td>
<td>± 10 %</td>
</tr>
<tr>
<td>Contactors, Motor Starts</td>
<td>− 15 % to + 10 %</td>
</tr>
<tr>
<td>Lighting</td>
<td>− 10 %, − 25 %</td>
</tr>
<tr>
<td>Fluorescent</td>
<td>+ 18 %</td>
</tr>
<tr>
<td>Incandescent</td>
<td>± 10 %</td>
</tr>
<tr>
<td>Motors, Standard Induction</td>
<td>Variable</td>
</tr>
<tr>
<td>Resistance Loads, Furnaces, Heaters</td>
<td></td>
</tr>
<tr>
<td>Others</td>
<td></td>
</tr>
</tbody>
</table>

Reference [73], [74], and [75] present component-based methods to specify an aggregated load model, where the load characteristic of interest is the sensitivity of real and reactive power to voltage, rather than load interruption voltage. In these approaches, the load mix is specified for each bus in terms of residential, commercial, and industrial classes. This data could be derived from customer billing information or other statistics and simulations.

The component-based approach is attractive because it provides for an upward aggregation of the available information, i.e., the load class mix data, into the model to be used in the study. It also avoids the tedious work of modeling each individual load by grouping similar loads into class or subclasses. The same approach can be used to model load interruption voltage.

We assume that the load mix model at a bus is specified as in [73]. Additionally, we further assume the interruption voltages within a load class \( c \) are normally distributed. That is,

Equation 13: Probability Distribution of Interruption Voltage

\[
V_{L,c} \sim N(\mu_{L,c}, \sigma_{L,c}^2)
\]

\[
V_{U,c} \sim N(\mu_{U,c}, \sigma_{U,c}^2)
\]

25 If detailed information of load mix is obtained, the load can also be classified on a subclass level, like resistance heating, room air conditioner, lighting, water heating, etc.
where both the lower and upper tolerance limits ($V_{L,c}$ and $V_{U,c}$) within load class "$c"$ are randomly distributed with mean ($\mu_{L,c}$ and $\mu_{U,c}$) and standard deviations ($\sigma_{L,c}$ and $\sigma_{U,c}$). These parameters can be estimated through load statistics as in Table 4.

### 4.3.2.2 Probability of Service Interruption

The service interruption at a bus occurs when the bus voltage is beyond the sustainable limits of the individual loads. The probability of the load class "$c"$ being interrupted under a given voltage level $V_{bus}$ is computed as:

$$\Pr(V_{L,c} > V_{bus} \text{ or } V_{U,c} < V_{bus} \mid V_{bus})$$

where both $V_{L,c}$ and $V_{U,c}$ are randomly distributed as in Equation 13.

As shown in Figure 22, the shaded area satisfies the condition $V_{L,c} < V_{bus} < V_{U,c}$. An integral of the joint probability of the random $V_{L,c}$ and $V_{U,c}$ on this area represents the probability that the load class "$c"$ is served. Therefore, the probability of service interrupted is 1.0 minus this probability of load being served.

![Figure 22: Probability of Load Being Served](image)

### 4.3.2.3 Expected Impact of a Given Voltage

With $K_{bus,c}$ as the percentage share of a load class "$c"$ at a particular bus, the expected service interruption impact is its expected service interruption cost multiplied by its expected interruption amount. If we sum up all the load classes at a bus, we have
Equation 14: Expected Impact at a Bus with a Given Voltage\textsuperscript{26}

\[
E(\text{Im}_{\text{bus}} | V_{\text{bus}}) = E(P_{\text{bus}}) \times \sum_c E(C_{\text{bus},c}) \times E(K_{\text{bus},c}) \times \Pr(V_{L,c} > V_{\text{bus}} \text{ or } V_{U,c} < V_{\text{bus}} | V_{\text{bus}})
\]

where \( P_{\text{bus}} \) is the (forecasted) total amount of load at a particular bus, \( K_{\text{bus},c} \) is the percentage share of a load class \( c \) in \( P_{\text{bus}} \), \( E(K_{\text{bus},c}) \times \Pr(V_{L,c} > V_{\text{bus}} \text{ or } V_{U,c} < V_{\text{bus}} | V_{\text{bus}}) \) represents the expected amount of service interruption in load class \( c \); \( C_{\text{bus},c} \) is the service interruption cost associated with the load class \( c \) at this bus. The independence of \( P_{\text{bus}}, C_{\text{bus},c}, V_{L,c} \) and \( V_{U,c} \) is used in Equation 14. The expectation of service interruption cost for each load class \( E(C_{\text{bus},c}) \) is obtained by regression methods \textsuperscript{[17]} based on customer survey or historical data. References \textsuperscript{[3]} and \textsuperscript{[76]} give a summary and survey on these cost evaluations. Other publications on evaluating load interruption cost of residential, commercial, or industrial segments can be found in \textsuperscript{[6]} and \textsuperscript{[7]}. The probability term, \( \Pr(V_{L,c} > V_{\text{bus}} \text{ or } V_{U,c} < V_{\text{bus}} | V_{\text{bus}}) \), has been given in Section 4.3.2.2.

4.3.2.4 "Risk – Voltage" curve

With various levels of bus voltage, Equation 14 can give a risk-voltage curve that shows the voltage risk against different voltage levels at a bus. This procedure can be done bus by bus without knowing the operating condition of the entire power system.

4.3.3 System Voltage Risk Assessment

The objective of system voltage risk assessment is to find the risk associated with voltage insecurity for a local region or an entire system under a given operating condition. With a given operating condition, the risk measures both the probability of voltage insecurity (voltage collapse and voltage-out-of-limit) and the associated cost consequence.

4.3.3.1 Assumptions

- We invoke the assumption usually made for security assessment, i.e., a short-term operating condition is given. The objective of this chapter is to determine the "risk" of voltage insecurity under this operating condition.
- The given operating condition has a strong correlation with the condition in the near future. We can predict the expectation of the future condition very well so that the variation of the future condition is small and some linear approximations are valid.

\textsuperscript{26} We use the notation \( \Pr(A | B) \) to represent the conditional probability of event \( A \) under the given condition \( B \). Similarly, \( E(A) \) will be the expected value of \( A \), and \( E(A | B) \) is the conditional expectation of \( A \) given \( B \).
• The variation of the future condition away from its expectation, except for the contingencies, is due to small parametric deviations. They may include deviation of load distribution factors, load power factors, line parameters, and so on.

• The steady state model of the power system is assumed. We are interested in the post-contingency performance after an uncertain disturbance occurs.

• Some assumptions on the probability distribution are made. They include a Poisson distribution of contingencies, Multi-Variate Normal (MVN) distribution of parametric deviations and Normal distribution of load interruption voltages. The description of these distribution functions can be found in References [15] and [16].

• The occurrence of contingencies is independent of each other, and it is also independent of other system parametric deviations and the operating condition. The individual parametric deviations, e.g., deviations of the real or reactive load distribution factors at each bus are considered correlated depending on the statistical data of these deviations. Other uncertainties outside the power system are assumed independently distributed.

• The impact of contingency is assumed to include the influence of voltage-out-of-limit and its direct effect in terms of customer load interruption. We do not include any sympathetic effects that might lead to loss of additional components.

4.3.3.2 Framework for Risk-based Voltage Assessment

We assume that there are two distinct outcomes for the future performance of system voltages, collapse or no collapse. The bifurcation point shown on the P-V curve provides the boundary between these two outcomes. When the system is operating without suffering a voltage collapse, the voltage may go below the tolerance of loads, resulting in load interruption. Our general risk expression includes both of these outcomes.

Equation 15: Voltage Risk of a Given Operating Condition

\[
Risk(X_0) = E(Im \mid X_0) \\
= \Pr(Collapse \mid X_0) \times E(Im(Collapse)) + \\
[1.0 - \Pr(Collapse \mid X_0)] \times E(Im(NoCollapse))
\]

where \( X_0 \) stands for the current operating condition. The risk, \( Risk(X_0) \), depends on the probability of voltage collapse \( \Pr(Collapse \mid X_0) \) under the condition \( X_0 \), the expected impact of collapse \( E(Im(Collapse)) \), and the expected impact of no collapse (or impact of voltage-out-of-limit), \( E(Im(NoCollapse)) \).
We will address each of these terms in the following sections. We drop the notation of the given operating condition $X_0$ in the following derivations for simplicity. The reader should be aware that all the derivations are based on $X_0$, i.e., all of the expressions are functions of $X_0$.

### 4.3.3.3 Probability of Voltage Collapse

There are several uncertainties associated with the voltage collapse under the scope of short-term operating time frame. They are (1) contingencies, (2) short-term system load, (3) short-term parametric deviations, e.g., deviations of load distribution factors, generation dispatch, and other parametric uncertainties if desired.

#### 4.3.3.3.1 Contingency

The occurrence of a contingency, by assumption, follows a Poisson distribution, i.e.,

\[
E_i \sim \text{Poisson}(\lambda_i t)
\]

where the $\lambda_i$ is the occurrence rate of the contingency $E_i$, the time frame $t$ is the time used to estimate our future risk, i.e., we are assessing the operating risk within the next $t$ hours.

#### 4.3.3.3.2 Short-term Load Fluctuation

Besides contingencies, the expectation of load drift and variation may be another uncertainty in the near future. A short-term load forecast provides an expectation of system load level $\mu_L$ and its standard deviation $\sigma_L$. By our assumption, it is normally distributed, as illustrated by

\[
L \sim N(\mu_L, \sigma_L^2)
\]

#### 4.3.3.3.3 Short-term Parametric Deviation

In reality, the load distribution factors among load buses, load power factors, generation participation factors, and other system parameters will not be certain in the future, even though we may forecast or estimate them very well. We assume that

---

27 Most regression and load forecasting models implicitly assume the normality in their statistical models.
• the parameters are random in the future, and they follow a Multi-Variate Normal (MVN) distribution around their expected values, and

• the deviations of parameters, although random, are small such that linear approximation of maximum loadability with respect to these random parameters is valid.

Let us denote the expectation of these parameters as \( E(K_p) \), where the parametric column vector \( K_p \) virtually may include all the possible system parameters, such as load distribution factors, generation participation factors, and so on.

Based on the given expectation of \( E(K_p) \), i.e., a given parametric pattern, a Continuation Power Flow (CPF) or other techniques ([53], [55], [56], and [57]) will provide an expectation of maximum loadability \( E(L_{mi}) \) and the margin sensitivities \( S_p \) with respect to these parameters ([60], [61], and [62]). Then, according to the second assumption in this section,

\[
L_{mi} = E(L_{mi}) + S_p^T (K_p - E(K_p))
\]

where \( L_{mi} \) is the system's maximum loadability, this is now random due to the random parameters \( K_p \).

By the normality assumption of the parametric deviations, \( K_p \) follows a Multi-Variate Normal distribution with mean vector \( E(K_p) \) and variance-covariance matrix \( V_p \).

Equation 18: Probability Distribution of Parameters

\[
K_p \sim MVN(E(K_p), V_p)
\]

where \( E(K_p) \) is the vector of expected system parametric scenario, and \( V_p \) is the variance-covariance matrix of these parameters. The elements of the variance-covariance matrix represent both the variances of each parameter and the correlation with respect to deviation of other parameters. This matrix can be estimated from the sample statistics of historical data [14].

It can be proven that \( L_{mi} \), a linear function of the MVN distributed \( K_p \) also follows a Normal distribution [16]. Its expected value is \( E(L_{mi}) \), and the variance is \( S_p^T \times V_p \times S_p \).^29

The probability distribution of maximum loadability is therefore,

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^28 This has been discussed in Section 4.1.2.1.1.

^29 Reference [16] has a thorough proof of the theory of linear models in statistics.
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Equation 19: Probability Distribution of Maximum Loadability

\[ L_{\text{mi}} \sim N(\mu_{L_{\text{mi}}}, \sigma_{L_{\text{mi}}}^2) \]

and it depends on the value of the parameters, their variability, and how they correlate with each other.

4.3.3.3.4 Probability of Collapse

Under a given topology determined by a contingency, when both load level \( L \) and maximum loadability \( L_{\text{mi}} \) are random, the probability of voltage collapse is the probability that the loading margin \( M_i = L_{\text{mi}} - L \) is negative. The probability distributions of \( L \) and \( L_{\text{mi}} \) are obtained through Equation 17 and Equation 19, respectively. Since both are Normal distributions, the resultant loading margin \( M_i \) will be also Normally distributed, with a mean of \( \mu_{M_i} \) and a variance of \( \sigma_{M_i}^2 \). That is,

Equation 20: Probability of Collapse under a Contingency

\[ \text{Pr}(\text{Collapse} \mid E_i) = \text{Pr}(M_i < 0 \mid E_i) \]

where the random loading margin \( M_i = L_{\text{mi}} - L \) has a Normal distribution, and

Equation 21: Probability Distribution of Loading Margin

\[ M_i \sim N(\mu_{M_i}, \sigma_{M_i}^2) \]

\[ \mu_{M_i} = E(L_{\text{mi}}) - \mu_L \]

\[ \sigma_{M_i}^2 = \sigma_{L_{\text{mi}}}^2 \times \sigma_L^2 + \sigma_P^2 \]

Through the use of the Total Probability Theorem [14], the total probability of voltage collapse under the system exposed to uncertain contingencies is

Equation 22: Probability of Collapse

\[ \text{Pr}(\text{Collapse}) = \sum_{E_i} \text{Pr}(\text{Collapse} \mid E_i) \times \text{Pr}(E_i) \]

where the conditional probability \( \text{Pr}(\text{Collapse} \mid E_i) \) and the probability of contingency, \( \text{Pr}(E_i) \), are given in Equation 20 and Equation 16, respectively.
4.3.3.4 Expected System Impact With and Without Voltage Collapse

4.3.3.4.1 Expected Impact Without Voltage Collapse

Section 4.3.2.3 gives the expected impact under a given bus voltage. The bus voltage, however, also depends on (1) contingencies, (2) short-term system load level, and (3) short-term parametric deviations. Although a particular load model is not assumed in the voltage risk analysis, a component-based load model like in [73], [74], and [75] can be used since the information of load segments have been obtained.

With small deviations of system parameters, a linear approximation of voltage around its expectation is assumed such that a Multi-Variate Normal distribution of bus voltages is obtained.

\[ V = E(V | L, E_i) + \left( \frac{\partial V}{\partial E_p} \right) \times (K_p - E(K_p)) \]

\[ E(V | L, E_i) \sim MVN\left( E(V | L, E_i), \left( \frac{\partial V}{\partial E_p} \right) V_p \left( \frac{\partial V}{\partial E_p} \right)^T \right) \]

Equation 23: Multi-Variate Normal Probability Function of Bus Voltages

where \( \frac{\partial V}{\partial E_p} \) is the sensitivity matrix of bus voltages with respect to the variation of system parameters. If the voltage does not collapse, the expectation of bus voltages \( E(V | L, E_i) \) is obtained by solving the power flow based on the expected system condition, and the contingency \( E_i \), \( V_p \) is again the variance-covariance matrix of parametric deviations as defined in Section 4.3.3.3.3.

With the above Normal distribution of bus voltages, the expected voltage impact for the study system at a given load level and given contingency is

\[ E(\text{Im} | L, E_i) = \sum_{\text{bus}} \int_{\text{bus}} E(\text{Im}_{\text{bus}} | V_{\text{bus}}) \text{Pr}(V_{\text{bus}}) dV_{\text{bus}} \]

where \( E(\text{Im}_{\text{bus}} | V_{\text{bus}}) \) is given in Equation 14, and \( \text{Pr}(V_{\text{bus}}) \) is the Normal probability density function provided in Equation 23.

Under exposure to the uncertain load level and contingencies, the expected impact of voltage-out-of-limit when voltage does not collapse is
Equation 24: Expected System Impact without Voltage Collapse

\[ E(\text{Im}(\text{NoCollapse})) = \sum_{E_i} \left( \int_{L} E(\text{Im} | L, E_i) \times \text{Pr}(L) dL \times \text{Pr}(E_i) \right) \]

which lumps all the possible contingencies\(^{30}\) and the possible load drifting. The probability function of a load level, \( \text{Pr}(L) \), and a contingency, \( \text{Pr}(E_i) \), are given in Equation 16 and Equation 17.

4.3.3.4.2 Expected Impact with Voltage Collapse

It is possible to mitigate the impact of voltage collapse via corrective or restorative operating actions. It is also possible that partial interruption can mitigate the voltage collapse and prevent full interruption. However, the effectiveness of these actions is very uncertain. Therefore, we assume here that voltage collapse results in total system blackout.

The expected impact is then the interruption cost of the entire system's load, i.e.,

Equation 25: Expected System Impact with Voltage Collapse

\[ E(\text{Im(Collapse)}) = \sum_{\text{bus}} \left( P_{\text{bus}} \sum_c C_{\text{bus,c}} \times K_{\text{bus,c}} \right) \]

where all the loads in the system and all the load components at a bus are interrupted.

4.3.3.5 Risk of Voltage Insecurity

Both the probability and impact terms in Equation 15 have been given in Equation 22, Equation 24, and Equation 25. The result, which is the voltage risk associated with the given operating condition \( X_0 \), can be used for:

- a quantitative measurement of voltage insecurity for any operating point. It is useful as a decision-making aid in determining operating limits associated with voltage problems.

- a risk curve with respect to various system conditions to find the trend of voltage risks.

- a marginal risk with respect to each transaction or injection which may be used to price the congestion.

Since the risk represents the expected cost associated with the insecurity problem, it can give a market

\(^{30}\) Theoretically, one must include all contingencies here, but practically, one only includes the "credible" contingencies.
incentive to mitigate the congestion. The collective revenue based on these price incentives can be allocated to the real congestion victims, for example, the interrupted customers.

4.4 Illustrations

4.4.1 Component Analysis

For a simple illustration, we assume an aggregated load at a bus has 100% residential load\(^{31}\) with mean interruption voltages at 0.85 (lower mean), 1.15 (upper mean), and a 0.02 of standard deviation. Based on the lower mean of the interruption voltage of this residential load class, one can expect at least half of the load to be shut off when the voltage goes below 0.85. On the other hand, more than half of the load can be expected to be lost if the voltage is too high, for example in this case, higher than 1.15.

The expected cost consequence or impact of load interruption at this bus under various voltage levels is computed by Equation 14 and shown in Figure 23, where an expected cost of $50 per MWhour for an average 6 hour\(^{32}\) service interruption is assumed for this residential load.

This bus oriented\(^{33}\) component analysis can be done off-line based on the statistical data of load mix and their interruption costs. It is independent of the following system risk assessment and more detailed information and assessment on service interruption impact can be studied here. The result is given by a risk-voltage plot at each bus and then used for system voltage risk assessment.

4.4.2 System Analysis

We provide an illustration of the proposed risk analysis on a modified IEEE Reliability Test System (IEEE-RTS 96) [49]. We have chosen a scenario where three contingencies, each one being an outage of a transmission line, provoke voltage insecurity\(^{34}\) on the load buses. An illustration of the system diagram is shown in Figure 24.

---

\(^{31}\) The load mix assumption is purely for simplicity. Adding more load classes according to a real load statistics does not change the general shape of Figure 23. Summation of \(c=1\) and the summation from \(c=1\) to \(n\) in Equation 10 and 11 do not have much difference except for data required and computational result.

\(^{32}\) To obtain the \(E(C_{bus,c})\) in Equation 14, a simple linear regression on interruption duration has been assumed here such that \(E(C_{bus,c}) = \beta_{bus,c} \times t = 50 \times 6\). Generally, it may be any nonlinear function, such as exponential function of the duration \(t\) where other regression methods on \(t\) could be used.

\(^{33}\) The component analysis in Chapter 3, "Transmission Line Thermal Overload Risk Assessment", is line-oriented instead of bus-oriented here.

\(^{34}\) Here, we only consider the voltage problem. However, these contingencies may also cause thermal overload and transient instability in the system. Our generalized approach provides uniform measurement in assessing the composite risk associated with all three types of security problems ([21], [37], and [77]). This attractive feature of the approach will be illustrated in Chapter 5.
Figure 23: Risk-Voltage Plot at a Bus

Figure 24: Local Illustration of IEEE RTS 96 System
The time frame of interest is one hour. Under this time frame, we assume the forecasted expectation\(^{35}\) of the future system will be the same as the current operating condition. The standard deviation of this future load level is assumed to be 2%. We further assume the deviation of load sharing factors on each bus to be the parametric variation that has 5% standard deviations around the expected values. The occurrence of contingencies is estimated from annual outage rates for the corresponding transmission line. Both the probability of each contingency, including that of no-contingency condition which is \(1.0 - \sum E_i \Pr(E_i)\), and the corresponding maximum loadability based on expected system parameters are listed in Table 5.

<table>
<thead>
<tr>
<th>Contingency</th>
<th>Occurrence Probability</th>
<th>Loadability (x100 MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No outages</td>
<td>0.9999</td>
<td>40.70</td>
</tr>
<tr>
<td>Outage 130-120</td>
<td>4.58e-5</td>
<td>39.14</td>
</tr>
<tr>
<td>Outage 230-130</td>
<td>4.58e-5</td>
<td>37.32</td>
</tr>
<tr>
<td>Outage 230-120</td>
<td>4.58e-5</td>
<td>36.89</td>
</tr>
</tbody>
</table>

The value of outage rates we assumed here is for simplicity and for illustrating the different effects even when the outage rates are the same. One should improve the accuracy of these values by relating them to line length or actual historical or real time data of each line.

4.4.2.1 Probability of Collapse

Suppose that the current load level is at 3600 MW, and the load sharing factors on each load bus are as listed in [49]. We wish to compute the probability of voltage collapse under the current operating condition.

With 2% standard deviation, the true load has 95% probability of fluctuating within an interval of \(3600 \pm 1.96 \times 72\) MW.\(^{36}\) (see Table 6)

<table>
<thead>
<tr>
<th>Expected Load Level</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>36.00 x 100 MW</td>
<td>0.72 x 100 MW</td>
</tr>
</tbody>
</table>

Furthermore, the system load may be distributed among load buses with different load sharing factors, the 5% standard deviations of load sharing factors cause the P-V curve or the maximum loadability to be uncertain. The randomness of the loadability under different contingencies is listed in Table 7. They are obtained from

\(^{35}\) We emphasize that this is only an expectation because the future system will almost always deviate from this forecasted system.
Equation 19. The randomness of load margin, the distance between the random maximum loadability, and the random load level for each contingency in Equation 21, are given in Table 8. Based on the Normal distribution of load margin, we can obtain the probability of collapse under each contingency which shows indeed the probability of the random load margin as being less than zero. The results are listed in the last column of Table 8.

Table 7: Randomness of Loadability due to Uncertain Load Sharing Factors

<table>
<thead>
<tr>
<th>Contingency</th>
<th>Occurrence Probability</th>
<th>Expected Loadability (x 100 MW)</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Outage</td>
<td>0.999</td>
<td>40.70</td>
<td>0.3839</td>
</tr>
<tr>
<td>Outage 130-120</td>
<td>4.58e-5</td>
<td>39.14</td>
<td>0.4179</td>
</tr>
<tr>
<td>Outage 230-130</td>
<td>4.58e-5</td>
<td>37.32</td>
<td>0.3970</td>
</tr>
<tr>
<td>Outage 230-120</td>
<td>4.58e-5</td>
<td>36.89</td>
<td>0.3353</td>
</tr>
</tbody>
</table>

Table 8: Randomness of Load Margin

<table>
<thead>
<tr>
<th>Contingency</th>
<th>Expected Margin (x 100 MW)</th>
<th>Standard Deviation</th>
<th>Probability of Collapse under Contingency</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Outage</td>
<td>4.70</td>
<td>0.8160</td>
<td>4.3e-9</td>
</tr>
<tr>
<td>Outage 130-120</td>
<td>3.14</td>
<td>0.8325</td>
<td>8.0e-5</td>
</tr>
<tr>
<td>Outage 230-130</td>
<td>1.32</td>
<td>0.8225</td>
<td>0.0547</td>
</tr>
<tr>
<td>Outage 230-120</td>
<td>0.89</td>
<td>0.7942</td>
<td>0.1306</td>
</tr>
</tbody>
</table>

The probability of collapse over the next hour, for the load being 3600 MW, is therefore only $8.5 \times 10^{-6}$. It is calculated by summing up all the products of collapse probability under contingency and the probability of the corresponding contingency, i.e.,

$$8.5 \times 10^{-6} = 0.9999 \times 4.3 \times 10^{-9} + 4.58 \times 10^{-5} \times 8.0 \times 10^{-5} + 4.58 \times 10^{-5} \times 0.0547 + 4.58 \times 10^{-5} \times 0.1306$$

Figure 25 provides plots of collapse probabilities against different loading levels under each contingency (including "no outage" cases). We also show the total probability is the sum of the collapse probabilities weighted by the contingency probabilities.

---

36 1.96 is in the 95th percentile of a standard Normal distribution, and 72 MW is the standard deviation of load that is obtained by 3600x2% MW according to our assumption.
4.4.2.2 Expected Impact

4.4.2.2.1 Expected Impact of No-Collapse (Voltage-Out-of-Limit)

It is possible that the voltage may decline below the load's tolerable range when it is still stable. As a simple illustration, we assume all the load buses have identical load class mix, say 100% residential load with mean interruption voltage at 0.85 (lower mean) and 1.15 (upper mean), and a 0.02 of standard deviation, as assumed in the example of component assessment. The expected cost consequence or impact due to voltage-out-of-limit under various load levels is shown in Figure 26. This result is obtained by combining both the expected P-V curves as in Figure 19 and the expected interruption curve as in Figure 23.

The impact curve in Figure 26 suggests the potential cost of load interruption due to voltage-out-of-limit under the condition that the system does not suffer the voltage collapse problem. It is represented by the term $E(\text{Im(NoCollapse)})$ in Equation 15. It shows that the risk of voltage out-of-limit is increasing with the system is stressed more demand for electricity, although the voltage is still stable at this time. This is because the system voltage level decreases with increase of load level as shown in the P-V curve in Figure 19.

![Figure 25: Probability of Voltage Collapse](image)

---

37 Given a load level, we will have a voltage level as illustrated in Figure 19. This voltage, by looking up Figure 23, will lead to some amount of expected impact of load interruption at a bus. Therefore, we arrive at the plotted graph in Figure 26.
4.4.2.2 Expected Impact of Collapse

For the impact of collapse, we assume the outcome will be an entire system blackout as mentioned in Section 4.3.3.4.2. The rate of cost consequence is also uniformly assumed as $50 per MWhour for a flat expected duration of 6 hours. When load level is at 3600 MW, this impact is expected to raise the cost to be $1.08 million per hour.

4.4.2.3 Risk of Voltage Insecurity

Through Equation 15, a semi-log plot of risk associated with voltage problems is depicted in Figure 27. It is the sum of two parts: risk of collapse and risk of voltage-out-of-limit with no-collapse.

The left boundary of the shaded area in Figure 27 is the worst-case single contingency security boundary. This is the traditional firm security limit of this system. The right boundary of the shaded area is the no-contingency limit. The solid curve of total risk, which indicates the expected impact of voltage insecurity problems, including both voltage collapse and out-of-limit, varies with the different loading levels of this system. The risk of losing voltage stability and voltage limits gets higher when more load is demanded in the system.
4.4.2.4 Marginal Risk of Voltage Insecurity

With the result of risk, one may compute the marginal risk with respect to any desired parameters to obtain risk sensitivities. One application of this marginal risk is for the system control to find an optimal preventive scheme to mitigate the risk of voltage insecurity.

It is also of interest to use the marginal risk with respect to system participants or transactions in order to price or allocate the congestion cost to each participant. In this case, we interpret the risk as an expected cost due to possible transmission congestion problems, in contrast to the real production cost resulting from the transmission loss, because reliability has a price. The risk is an additional implicit cost to the real cost of energy delivered with the possibility of security problems. Then the marginal risk represents an incremental congestion cost due to incremental transmission participation, specifically in the delivered energy. It can be used as

- a price signal to price the transmission congestion, and
- an allocation factor to allocate the revenue collected by congestion cost to congestion victims.

Figure 28 plots a marginal risk against various loading positions. For example, it suggests an expected $4.25 per MWhour of additional implicit cost charged for the possibility of losing voltage security when the system is running at the level of 3600 MW. It also shows the market participants should be priced more congestion cost when the system is stressed by more load demand.
4.5 Summary

A probabilistic method to compute the operating risk of voltage collapse and voltage-out-of-limits is presented in this chapter. The resulting risk represents the expected future cost of voltage insecurity based on information from the current operating condition. It gives a quantitative measure of voltage insecurity both within and outside the traditional security boundary. It is promising in its potential to

- quantify a composite risk in hybrid security problems,
- act as a leading indicator for reliability, and
- facilitate the pricing of power system security.
5 COMPOSITE RISK ASSESSMENT

5.1 Introduction

5.1.1 Problem Statement

As expressed in Chapter 1, within the electric network, an individual disturbance with non-zero cost consequence may occur for any number of reasons or at any time in any system environment. The disturbance may result in power system security problems: thermal overload, voltage instability, or transient instability, and draw the prevailing system to an uncontrollable cascading situation leading to widespread power outages. To maintain system security or reliability, certain transfer limits are required regardless of the economic forces behind the markets.

From an operational viewpoint, however, transmission corridors are substantial power market influences, i.e., the choice of one limit over another for a transmission corridor can translate into millions of dollars for the selling and buying entities. By seeking profit in the market environment, energy marketers need as much transfer capability as possible to deliver their energy from one place to another for profit.

Our interest is motivated by this contradiction induced by the recent changes in regulatory policies towards inter-utility power interchange practices with a major focus on "competition" as a replacement of "regulation" to achieve economic efficiency. Instead of "regulating" the market by traditional firm, non-economic, obligatory security rules, the new environment is seeking an economic signal or lever that reflects the system security such that it can adjust the behavior of market participants to allow the "free" market to satisfy the security requirement by itself.

5.1.2 Previous Work

Reliability indices have been defined to monitor the frequency and duration of outages. As stated in [78], "There are two basic categories of indices: customer based indices and load based indices. Customer based indices record the frequency and duration of outages for individual customers and are most informative in mainly residential areas. Load based indices monitor information on the duration and frequency of interruption of load and are relevant for circuits that are mostly industrial or commercial." Many indices have been developed mainly for the adequacy assessment in [6] and [7]. Again, from [78], "The most commonly used indices are the System Average Interruption Frequency Index (SAIFI), the System Average Interruption Duration Index (SAIDI), the Customer Average Interruption Duration Index (CAIDI), and the Average Service Availability Index (ASAI)." Also, there have been numerous works conducted in power system security assessment that provide various security indices for a power system. The performance index ([19] and [51]), as mentioned in Chapter 4, measures the security level of a system condition based on the deviation of branch flows and bus voltages from their normal values. In addition to the review of probabilistic approaches to power
system security assessment in Section 2.1, [79] proposes a probabilistic security index for the purpose of identify operating limits. Reference [80] uses a method called "generalized tetrachoric series" to calculate the statistics of eigenvalue locations based on the sensitivities of the eigenvalues. It may be used for a probabilistic assessment of small-signal or oscillatory stability. References [81] and [82] discuss the probabilistic transient stability analysis.

However, all of these indices, both performance and probability indices, treat the power system security as the deterministic obligation to the system operation. Most of them do not have, or do not need to be concerned with the significance of economics or the cost related to the security.

### 5.1.3 Our Approach

In this chapter, we use the results in Chapters 3, and 4 and other related work ([83] and [84]) to build a composite risk index for power system security. This index reflects the expected economic loss due to possible system security problems that manifest themselves in three different forms: thermal overload, voltage insecurity, and transient instability. Because the risk indices developed for each security problem have a uniform economic meaning, i.e., the expected monetary impact or cost consequence of the insecurity, they are additive and hence, become a composite risk index that reflects the overall security level for a given operating condition.

### 5.2 Method Overview

We develop a two-tiered approach to conduct the composite risk assessment for a given operating condition. The two tiers are the "component" assessment and the "system" assessment. In component assessment, we evaluate the expected impact or cost consequence on an individual power system component, for instance, a transmission line, a transformer, or an aggregated bus load, under various given electrical conditions of the component. In system assessment, we focus on evaluating the power system uncertainties that have influence on the electrical conditions of the system components, given a system operating condition. The result is given as a composite expected monetary impact of a given operating condition over the next period of time considered. The following sections illustrate how this approach works.

### 5.2.1 Component Assessment

In component assessment, the objective is to assess the consequence, as well as monetary cost associated with each power system component, given an electrical condition for operation. A component assessment measures the expected impact incurred by each victim under possible insecure conditions. Such a study assesses lines, transformers, loads and generation units.
5.2.1.1 "Component" Risk on Transmission Line under Given Line Flows

As described in Chapter 3, a given flow through a transmission line may result in thermal overload of the conductor, and hence, result in related physical damages and even human injuries. The cost consequence or the impact of this thermal overload has been presented in detail in Chapter 3. Moreover, it has been also discussed that the possible overload depends not only on the given line flow, but also on the ambient weather around the transmission line. An expected value of the overload impact, by the probabilistic description of ambient weather conditions, is calculated as the "component" overload risk for the given line under the given load flow. This procedure can be repeated under various line flows such that a "risk-flow" curve is created for a transmission line.

The component assessment encapsulates the detailed impact calculation and thermal model of a transmission line into its final risk-flow curve such that one can determine the expected monetary impact on a transmission line from its line flow without knowing its intrinsic physical properties and financial cost. It is also convenient for the component assessment itself that it does not need to know what happens on the power system. What does matter for the component study is the line flow, in other words, "give me the line flow, and I'll tell you how much money you are going to lose based on the weather condition."

The risk-flow curve is created on a line by line basis. Each transmission line has its own risk-flow based on its local weather condition and physical properties.

5.2.1.2 "Component" Risk on Transformer under Given Flows

As described in [83], a given flow through a transformer may result in elevation of the temperature of winding and insulation, and hence, bring about possible loss of life and equipment damage on the transformer. The elevation of temperature is dependent on the uncertain ambient weather conditions. Thus, an expected monetary cost consequence of transformer overload is calculated as the component overload risk by the probabilistic description of ambient weather conditions. This procedure is repeated under various flows such that a risk-flow curve is created for a transformer.

Similar to the component assessment of a transmission line, a transformer's risk-flow curve encapsulates the detailed internal thermal model and probabilistic impact calculation. In this way, on the system side, one can determine the monetary loss only by the transformer loading level without any knowledge of the intrinsic properties of the transformer.

The risk-flow curve is created for each transformer based on its local weather condition and physical properties.

5.2.1.3 "Component" Risk on Load under Given Bus Voltages

In Chapter 4, it has been demonstrated that the end users of electricity may be interrupted under out-of-limit voltage. Different load classes have different distributions of voltage tolerance and interruption cost. Under a given bus voltage, an expected monetary impact on customers due to service interruption is calculated
as the component voltage risk at a bus based on its aggregated probabilistic description of load interruption voltages for the load mix at the bus. This procedure is repeated under various bus voltages such that a risk-voltage curve is provided for a load bus.

This component voltage risk study for a bus encapsulates the detailed evaluation of expected impact and load mix into its final risk-voltage curve such that one, on the system side, is able to determine the expected monetary impact on a bus by only providing the information of bus voltages.

The risk-voltage curve is created for each bus in a transmission network according to its local load mix.

5.2.1.4 Component Risk on a Generator under Given System Conditions

In [84], neural networks have been developed for the power system stability study. Given a system condition, the synchronism of a generation unit depends on the general occurrence of a fault, fault type, and fault location.

This component stability risk study encapsulates the time consuming stability simulations into the resultant neural networks such that one, on the system side, is able to quickly determine the stability of a generator through feeding the system condition and contingency into the trained neural networks without performing tedious stability studies.

The component stability study and neural networks training are conducted for each unit such that trained neural nets are available for each generation unit for the ongoing system risk assessment of transient stability.

5.2.2 System Assessment

A higher tier above the "component" assessment is the "system" assessment whose objective is to provide a composite security risk over the next period of time under a given system condition. An illustrative diagram is shown in Figure 29.

5.2.2.1 Risk of Overload and Voltage Insecurity

A contingency analysis is used in the system assessment to provide the branch flows and bus voltages under a set of credible contingencies. For each contingency, the resultant flows and voltages are fed into the risk-flow and risk-voltage curves, which are created by the component assessment, to obtain the risks of overload and voltage insecurity under the contingency. The risk of each insecurity problem is finally calculated by summing up all the contingency risks with weights of contingency occurrence probability.

5.2.2.2 Risk of Transient Instability

The operating condition is also fed into a stability risk assessment, as described in [84], to obtain the risk of transient stability. A sum of all the risks for each insecurity problem is ultimately calculated as the composite risk of the given operating condition.
5.3 Analytical Development

5.3.1 Component Assessment

The detailed analytic developments of component studies for the risk of transmission line thermal overload, voltage insecurity, and transformer overload are in Chapter 3 and 4 and References [83] and [84], respectively. They are briefly stated in Section 5.2.1. The reader can refer to the related chapters for a detailed description.

5.3.2 System Assessment

5.3.2.1 Definition and Goal

We have defined the “risk,” as a condition under which there is a possibility of an adverse deviation from a desired outcome that is expected or hoped for [1]. There are two primitives included within this definition: future uncertainties and impact of outcomes. We also defined the “degree of risk” as the expectation of the
dollar-based monetary impacts or cost consequences of those outcomes. It would be the amount of impact multiplied by the corresponding probability of outcome.

Our goal is to evaluate the degree of risk of losing power system security under a given operating condition. The degree of risk is measured by the expected cost consequence of insecurity over a short period of time considered, say over the next hour.

5.3.2.2 Assumptions

The composite risk assessment is developed for providing a risk-based power system security assessment. It represents the so called "insecurity cost" of an operating condition. The assumptions are the same as those listed in Section 4.3.3.1 where voltage risk is assessed.

The assumed probability distributions are treated as "prior" distributions whose distribution parameters, for example the mean and variance in a Normal distribution are estimated from the historical data, or even subjectively. Therefore, the emphasis is on the method to evaluate the risk based on a probabilistic description of uncertainties rather than what the probabilistic description really is. It is believed that any "prior" probability distribution can be improved by the "posterior" distribution when additional information is provided [85].

5.3.2.3 Risk Expression

As addressed in Chapter 4, we assume there are two distinct outcomes for the future performance of system voltages, collapse or no-collapse. The model of steady state power system determines whether the system has stable steady state equilibrium points under various uncertainties such as contingencies or not. In the case when the voltage collapses, or the system does not have a "post-contingency" equilibrium, it also has no basis for the transient stability, which is considered a large signal stability problem. Our general risk expression is then as follows:

Equation 26 : Risk Expression

\[
Risk(X_0) = E(\text{Im}|X_0) = \text{Pr}(\text{Collapse}|X_0) \times E(\text{Im}(\text{Collapse})) + [1 - \text{Pr}(\text{Collapse}|X_0)] \times E(\text{Im}(\text{NoCollapse}))
\]

where \(X_0\) stands for the current operating condition. The risk, \(Risk(X_0)\), depends on the probability of voltage collapse \(\text{Pr}(\text{Collapse}|X_0)\) under the condition \(X_0\), the expected impact of collapse \(E(\text{Im}(\text{Collapse}))\) and the expected impact of no-collapse \(E(\text{Im}(\text{NoCollapse}))\).

\[\text{We do not consider the individual's utility function of the monetary impacts.}\]
We will address each of these terms in the following sections. We drop the notation of the given operating condition $X_0$ in the following derivations for simplicity. The reader should be aware that all the derivations are based on $X_0$, i.e., all of the expressions are functions of $X_0$.

5.3.2.4 Probability of Voltage Collapse (Losing Equilibrium)

The detailed derivation of probability of voltage collapse is in Chapter 4. We summarize it as follows:

**Equation 27: Probability of Voltage Collapse**

$$\Pr(Collapse) = \sum_{E_i} \Pr(Collapse \mid E_i) \times \Pr(E_i)$$

where the conditional probability $\Pr(Collapse \mid E_i)$ is the probability of collapse under a given contingency $E_i$, $\Pr(E_i)$ is occurrence probability of the contingency. Both are given in Chapter 4, where any other system-wide uncertainties, for instance, system load level, load distribution factors, branch parameters, and so on, could be included in the term $\Pr(Collapse \mid E_i)$.

5.3.2.5 Expected Impact of Collapse

The consequence of voltage collapse is not only the interruption of loads but also the shutdown of generation units or even an entire plant. This is because the system is also transient unstable if it does not have a stable equilibrium point. Hence, in the case when system load level moves outside the bifurcation point of a PV curve, there are non-zero unit tripping costs associated with it.

As explained in Chapter 4, it is possible to mitigate the impact of voltage collapse via corrective or restorative operating actions. It is also possible, and perhaps inevitable, that partial interruption can mitigate the voltage collapse and prevent full interruption. However, the effectiveness of these actions is very uncertain and usually results in a series of cascading events. Therefore, we assume here that voltage collapse results in a total system blackout which includes blackout of both loads and loss of all generation.

The expected impact is then the interruption cost of the entire system loads and generation units.

**Equation 28: Expected System Impact with Voltage Collapse**

$$E(\text{Im}(Collapse)) = \sum_{\text{bus}} E(\text{Im}_{\text{bus,Load}}) + \sum_{\text{bus}} E(\text{Im}_{\text{bus,Gen}})$$

---

For a simplest case where only the uncertainty of contingency is considered, the conditional probability term $\Pr(Collapse \mid E_i)$ is deterministically either "0" or "1" according to the operating condition. Most previous works on probabilistic voltage stability are based on this simplification. We, however, propose a method to evaluate the probability of collapse under any arbitrary random system parameters (see Chapter 4).
where $\sum_{\text{bus}} E(\text{Im}_{\text{bus, Load}})$ is the expected cost consequence of all the loads in the system when interrupted (as derived in Chapter 4), $\sum_{\text{bus}} E(\text{Im}_{\text{bus, Gen}})$ is the expected cost consequence on generation units when all the units in the system are lost. The cost consequence, including replacement, repair, and startup costs, for each unit is given in [84].

5.3.2.6 Expected Impact of No-Collapse

Under the condition that the voltage does not collapse, there are several possible insecure outcomes when the system is exposed to uncertainties such as contingencies, load, and etc. The outcomes include the thermal overload of transmission lines, overload in transformers, voltage-out-of-limit on loads, and transient instability on generation units. Each insecurity problem has its own uncertainty dependency and impact characteristics.

Equation 29: Expected System Impact without Voltage Collapse

$$E(\text{Im(NoCollapse)}) = E(\text{Im(Line)}) + E(\text{Im(Transformer)}) + E(\text{Im(Load)}) + E(\text{Im(Generator)})$$

where $E(\text{Im(NoCollapse)})$ is the monetary expected impact when voltage is steady state stable, it is the sum of expected impact of transmission line overload $E(\text{Im(Line)})$, transformer overload $E(\text{Im(Transformer)})$, load interruption due to voltage-out-of-limit $E(\text{Im(Load)})$, and loss of generator due to transient instability $E(\text{Im(Generator)})$.

5.3.2.6.1 Expected Monetary Impact on Transmission Line due to Thermal Overload

A detailed derivation of this can be found in Chapter 3 and 4. We summarize its result as follows. The expected monetary impact on a line due to thermal overload is given in Equation 30 where the random effect of contingency, load drifting and overload impact are explicitly considered.

Equation 30: Expected Impact on Line

$$E(\text{Im(Line)}) = \sum_{L, E_i} \left( L E(\text{Im(line)} | L, E_i) \times Pr(L) dL \right) \times Pr(E_i)$$

which lumps all the possible contingencies and the possible load drifting. The probability function of a contingency, $Pr(E_i)$, and a load level, $Pr(L)$, are given by Equation 31 and Equation 32, respectively, according to the assumption in Section 5.3.2.2.

---

40 Theoretically, one must include all contingencies here, but practically, one only includes the "credible" contingencies.

41 If only the uncertainty of contingency is considered, one may set all the other probability distributions, such as probability of load level and system parameters, as 1.0. This will give a simplified case of Equation 30 as described in Chapter 3.
Equation 31: Probability Distribution of Contingency

\[ E_i \sim \text{Poisson}(\lambda_i) \]

Equation 32: Probability Distribution of Load Level

\[ L \sim N(\mu_L, \sigma^2_L) \]

In Equation 30, the term \( E(\text{Im(line)} \mid L, E_i) \) is the expected overload impact on a line under a given contingency and a given load level. The randomness of system parameters is represented as their influences on the line flows. Therefore, the term \( E(\text{Im(line)} \mid L, E_i) \) is calculated as,

\[
E(\text{Im(line)} \mid L, E_i) = \sum_{\text{lines}} \int_{\text{line}} E(\text{Im(line)} \mid I_{\text{line}}) \Pr(I_{\text{line}}) dI_{\text{line}}
\]

where \( E(\text{Im(line)} \mid I_{\text{line}}) \) is given by the risk-flow curve from the component risk assessment of transmission line thermal overload. It gives the expected impact on a line under various given flows. The probability of line flows \( \Pr(I_{\text{line}}) \), depending on random system parameters under a given contingency and a given load level, follows a Normal probability density function of line flow which is provided in

\[
I \sim \text{MVN}
\left(
E(I \mid L, E_i) + \left( \frac{\partial I}{\partial \mathbf{K}_p} \right) \mathbf{V}_p \left( \frac{\partial I}{\partial \mathbf{K}_p} \right)^T
\right)
\]

where \( \frac{\partial I}{\partial \mathbf{K}_p} \) is the sensitivity matrix of line flows with respect to the variation of system parameters. Under the condition that the voltage does not collapse, the expectation of line flows \( E(I \mid L, E_i) \) is obtained by solving the power flow based on the expected system condition. The contingency \( E_i \), \( \mathbf{V}_p \) is the variance-covariance matrix of parametric deviations as defined in Chapter 4. This Multi-Variate Normal distribution of line flows is derived from the assumption that the system parameters \( \mathbf{K}_p \) follow a Multi-Variate Normal distribution and only have small deviations from the current operating condition. That is,

Equation 33: Probability Distribution of Parameters

\[
\mathbf{K}_p \sim \text{MVN}(E(\mathbf{K}_p), \mathbf{V}_p)
\]
5.3.2.6.2 Expected Monetary Impact on Transformer due to Thermal Overload

Similar to our dealing with the transmission line, the expected monetary impact on a transformer is given in Equation 34.

**Equation 34: Expected Impact on Transformer**

\[
E(\text{Im}(\text{Transformer})) = \sum_{E_i} \left( \int_{L} E(\text{Im}(\text{Transformer}) \mid L, E_i) \times \text{Pr}(L) dL \times \text{Pr}(E_i) \right)
\]

which lumps all the possible contingencies and the possible load drifting. The probability function of a contingency, \( \text{Pr}(E_i) \), and a load level, \( \text{Pr}(L) \), are given by Equation 31 and Equation 32, respectively.

In Equation 34, the term \( E(\text{Im}(\text{Transformer}) \mid L, E_i) \) is the expected transformer overload impact under a given contingency and a given load level. This term is calculated from:

\[
E(\text{Im}(\text{Transformer}) \mid L, E_i) = \sum_{I_{\text{sflr}}} \int_{I_{\text{sflr}}} E(\text{Im}_{\text{sflr}} \mid I_{\text{sflr}}) \times \text{Pr}(I_{\text{sflr}}) dI_{\text{sflr}}
\]

where the randomness of system parameters is represented as their influences on the flows of transformers. The term \( E(\text{Im}_{\text{sflr}} \mid I_{\text{sflr}}) \) is provided by the risk-flow curve from the component risk assessment of transformer thermal overload risk; it gives the expected impact on transformer under various given flows. \( \text{Pr}(I_{\text{sflr}}) \) is the Normal probability density function of flow on a transformer which is obtained in the same manner as is described in Section 5.3.2.6.1.

5.3.2.6.3 Expected Monetary Impact on Load due to Voltage-Out-of-Limit

A detailed description of this is given in Chapter 4. We summarize its result as follows. The expected monetary impact on load due to voltage-out-of-limit is given in Equation 35 where the random effect of contingency, load drifting, and load interruption are explicitly considered.

**Equation 35: Expected Impact on Load**

\[
E(\text{Im}(\text{Load})) = \sum_{E_i} \left( \int_{L} E(\text{Im}(\text{Load}) \mid L, E_i) \times \text{Pr}(L) dL \times \text{Pr}(E_i) \right)
\]

which lumps all the possible contingencies and the possible load drifting. The probability function of a load level, \( \text{Pr}(E_i) \), and a contingency, \( \text{Pr}(L) \), are given by Equation 31 and Equation 32, respectively, based on the assumption in Section 5.3.2.2.
In Equation 35, the term \( E(\text{Im}(\text{Load}) \mid L, E_1) \) is the expected load interruption impact under a given contingency and a load level. This term is calculated from

\[
E(\text{Im}(\text{Load}) \mid L, E_1) = \sum_{\text{bas}} \int_{V_{\text{bus}}} E(\text{Im}_{\text{bus}} \mid V_{\text{bus}}) \Pr(V_{\text{bus}}) dV_{\text{bus}}
\]

where \( E(\text{Im}_{\text{bus}} \mid V_{\text{bus}}) \) is given by the risk-voltage curve from the component voltage risk assessment. It gives the expected impact on load under various given voltages. \( \Pr(V_{\text{bus}}) \) is the Normal probability density function of bus voltage which is provided in

\[
V \sim \text{MVN}
\left( E(V \mid L, E_1), \left( \frac{\partial V}{\partial E_p} \right) V_p \left( \frac{\partial V}{\partial E_p} \right)^T \right)
\]

This Multi-Variate Normal distribution of bus voltages is derived from the assumption that the system parameters \( K_p \) follow a Multi-Variate Normal distribution and only have small deviations from the current operating condition by our assumption.

### 5.3.2.6.4 Expected Monetary Impact on Generator due to Transient Instability

As described in earlier sections, the risk of transient instability in [84] is actually a conditional expectation of monetary impact (cost consequence) under the condition that the system does have steady state equilibrium; in other words, this occurs under the condition that voltage does not collapse. Consequently, the risk calculated in [84] is the expected impact of transient instability in Equation 29.

As summarized in [84], the expected monetary impact on a generator due to transient instability is

\[
\text{Equation 36: Expected Impact on Generator}
\]

\[
E(\text{Im}(\text{Generator})) = \Pr(K \mid X_0) \times \text{Im}(K \mid X_0)
\]

The first term represents the conditional probability \( \Pr(K \mid X_0) \) of occurrence for an event \( K \) defined as a transient instability of a generator over the next period of time considered, given the operating condition \( X_0 \). It depends on the occurrence probability of a fault, fault type, and fault location. The consequences of a transient instability are evaluated by assessing the direct and indirect financial costs (or impacts) incurred due to tripping of units; they depend on generation level, which is one of the parameters in the operation condition \( X_0 \). The evaluation of the assessment is made from the perspective of a generation company that owns the units that are at risk to go out of step. The detailed description of calculating each of terms is in [84].
5.4 Illustrations

5.4.1 System Description

We provide an illustration of assessing composite risk on a modified IEEE Reliability Test System (IEEE-RTS 96) [49]. The modification we made includes adding two dummy buses, Bus #250 and Bus #270, on the line 130-130 and 130-120, to simulate various locations of fault for transient instability study. Other modifications on system topology are as the same as that made in [84]. The area 3 which has one generation plant connects to Bus #13, three transmission lines, 230-130, 230-120, and 130-120, two transformers at Bus #120, which connect to Area 2 through Bus #90 and Bus #100, and a self-supplied area load at Bus #130. This area is a net exporting area that transfers its excess energy through transmission line 130-120 to the boundary bus #120, and then to the southern low voltage area #2 via two transformers. We want to find a composite risk incurred by this local area for a system operating condition over the next hour. An illustration of the system diagram is shown in Figure 30.

We have chosen a scenario where three contingencies, each one being an outage of a transmission line, provoke possible transmission line overload, transformer overload, voltage insecurity and transient instability in the area considered. The contingencies and their occurrence probabilities are listed in Table 9.

As in the illustration of voltage stability, from Chapter 4, we choose the system load level as the variable representing the various operating conditions. The result is given by a plot of composite risk levels for each of the system load level to show how the risk evolves along different operating conditions. The successive evolution of area load is balanced by a proportional increase of generations throughout the entire system, not necessarily within the local area, according to the expected participation factors of generation units. This method of identifying the dispatch is an approximation of the full Economic Dispatch Calculation (EDC) which would normally be used. In our example, we use the original shares of generation within the total generation level as the expected participation factors. For simplicity, we also assume the system has been provided adequate generation reserves for the increase of load such that the real limits of generators are ignored. The load sharing and generation participation factors could be either random or even correlated depending on whether we expect the randomness and independence of their effects. We select these pro rata increase of load and generation as the expected sharing factors (Equation 37). If the randomness of these sharing parameters needs to be considered, the variance-covariance matrix of these parameters also needs to be estimated for our risk assessment, (see Section 4.3.3.3.3). In our case, a 5% independent deviation of real load sharing factors is assumed. The reactive part of load is, however, assumed to be dependent on the real part of a load. That means the power factor is included as fixed values.

---

42 The modifications made on the original RTS-96 system are used for the purpose of creating a transient unstable case such that all of these security problems can be illustrated simultaneously.
43 One may choose other variables, by replacing the load level with other desired variable for the horizontal variable of the risk plot, to see its impact on the risk level.
Figure 30: Illustration of IEEE RTS-96 System

Table 9: Expected Maximum Loadability under Contingencies

<table>
<thead>
<tr>
<th>Contingency</th>
<th>Occurrence Probability</th>
<th>Loadability (x100 MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No outages</td>
<td>0.9999</td>
<td>66.86</td>
</tr>
<tr>
<td>Outage 230-120</td>
<td>4.58e-5</td>
<td>58.05</td>
</tr>
<tr>
<td>Outage 130-120</td>
<td>4.58e-5</td>
<td>54.51</td>
</tr>
<tr>
<td>Outage 230-130</td>
<td>4.58e-5</td>
<td>65.11</td>
</tr>
</tbody>
</table>

One may also use load forecasting and economic dispatch to obtain these factors.
Equation 37: Pro Rata Load Sharing and Generation Participation Factors

\[
LD_{\text{share}_i} = \frac{\text{Load}_i}{\sum \text{Load}_i} \]
\[
GEN_{\text{share}_i} = \frac{\text{Gen}_i}{\sum \text{Gen}_i} \]

According to the above scenario, the expected maximum loadability where the system does exist, equilibrium for voltage stability are obtained through successive simulation of Continuation Power Flow under each contingency. The results are listed in the last column in Table 9.

5.4.2 Component Assessment

In component assessment, the objective is to assess its impact and expected monetary cost associated with each power system component, under a given electrical condition for operation. It measures the expected impact incurred by each victim under possible insecure conditions. The results are given by the risk-flow and risk-voltage curves for each individual branch and load bus.

5.4.2.1 Transmission Line Thermal Overload

There are three transmission lines in the considered area, line 130-120, 230-130, and 230-120. For simplicity, we assume all of three have the same thermal, weather conditions, and designed service life that are specified in Section 3.4. According to the deterministic method, the normal (continuous), 3 hour, and 15 min short-time ratings are 400 MVA, 450 MVA, and 520 MVA, respectively [25]. Figure 31 shows the per unit expected impact on line under various flows. For calculating real dollar-based risk, one need to multiply the per unit risk by the line's replacement cost per mile and its length. The re-conductoring cost is assumed as $108,000 per mile for all three lines. So the only difference on the real dollar-based risks for different lines is by the line length in this example. For instance, the expected thermal impact on line 230-120 under various line flows, through component risk assessment, is shown in Figure 31. One may continuously run this line at its 100% continuous rating level, 400 MVA. This will lead to an expected $10 loss per hour, or $87,600 per year. This relatively small amount of monetary loss is due to low occurrence rate of these weather conditions that result in line temperature exceeding its maximum design value.

Figure 31 is obtained regardless of the system conditions, like system load level, contingency, etc. It is calculated from the expected uncertain weather conditions and the line's thermal characteristics under the given line flows, regardless of how these line flows are obtained. These risk-flow curves, encapsulating the detailed thermal information of individual lines, will be used in the upper tier of system risk assessment.
5.4.2.2 Transformer Thermal Overload

There are two transformers in the considered area, transformer 120-90 and 120-100. Both connect as the interface between the considered area 3 and the southern area 2. Both are 400 MVA transformers. The weather conditions and thermal characteristics of both are assumed as the same, as illustrated in [83]. For a given load level, the real dollar-based expected overload impact on each of the transformer is therefore, the same as shown in Figure 32.

In a fashion similar to the component study for a transmission line, Figure 32 is obtained regardless of the system conditions. It is calculated from the expected uncertain weather conditions and the transformers' thermal characteristics under the given operating flows, regardless of how these flows are obtained from the network condition. These risk-flow curves, encapsulating the detailed thermal information of individual transformers, will be used in the upper tier of system risk assessment.

5.4.2.3 Load Interruption due to Voltage-Out-of-Limit

There is an aggregated bus load at Bus #130 in the considered area. For a simple illustration, we assume the aggregated load at bus #130 has 100% residential load\(^{45}\) with mean interruption voltages at 0.85 (lower mean), 1.15 (upper mean), and a 0.02 of standard deviation. Based on the lower mean of the interruption voltage of this residential load class, one can expect at least half of the load being shut off when the voltage goes below 0.85. On the other hand, more than half of the load lost can be expected if the voltage is too high, e.g., higher than 1.15 in this case.

The expected cost consequence or impact of load interruption at the bus #130 under various voltage levels is computed as described in Section 4.3.2, and shown in Figure 33, where an expected cost of $50 per MWhour for an average 6 hours service interruption is assumed for this residential load. For instance, one can expect a $3 per MWhr loss when the voltage drops to 0.9. If there is a 300MW aggregated load connected to bus #130, $900 per hour, or $7.8 million averaged per year, is expected to be lost when voltage drops to 0.9. However, in this case, less than $0.3 per hour, which is calculated by multiplying the $0.001 per MWhr by 300 MW, is expected to be lost when the voltage is maintained above 0.95.

Figure 33 is calculated from the expected load interruption cost under the given operating voltages, regardless of how those voltages are obtained from the network condition. This risk-voltage curve, encapsulating the detailed service interruption information of individual loads, will be used in the upper tier of system risk assessment.

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\(^{45}\) See Section 4.4.
Figure 31: Expected Impact on Lines under Various Line Flows

Figure 32: Expected Impact on Transformer under Various Flows
5.4.3 System Assessment

The time frame of interest is one hour. Under this time frame, we assume the forecasted expectation of the future system, characterized by the system load level as assumed in Section 5.4.1, will be the same as the current operating condition. The standard deviation of this future load level is assumed to be 5%. We further assume the deviation of load sharing or distribution factors on each bus to be the parametric variation that has 5% standard deviations around the expected values. The occurrence of contingencies is estimated from annual outage rates for the corresponding transmission line. Both the probability of each contingency, including that of no-contingency condition which is \(1.0 - \sum \Pr(C_i)\), and the corresponding maximum loadability based on expected system parameters are listed in Table 9.

5.4.3.1 Probability of Collapse

Suppose that the system load level would be expected at 5000 MW, and be expected distributed to each load bus according to the pro rata factors. We wish to compute its probability of voltage collapse.

With 5% standard deviation, the true load is expected to fluctuate within an interval of \(5,000 \pm 1.96 \times 250\) MW\(^{48}\) with a probability of 95%. This randomness of system load is listed in Table 10.

---

\(^{46}\) We emphasize that this is only an expectation because the future system will almost always deviate from this forecasted system.

\(^{47}\) Any other system parameter could be used if desired.

\(^{48}\) 1.96 is in the 95\(^{th}\) percentile of a standard Normal distribution, and 250 MW is the standard deviation of load that is obtained by 5,000x5% MW according to our assumption.
Furthermore, the system load is expected to distribute among load buses with random load sharing factors, the 5% standard deviations of load distributing factors cause the P-V curve or the maximum loadability to be uncertain. In the example, we assume the fluctuation of real load sharing factors is independent with each other, and the reactive load has perfect correlation with the corresponding real load fluctuation, i.e., the power factor remains constant. The resultant randomness of the loadability under different contingencies is calculated in Table 11. The randomness of loading margin, which is the distance between the random maximum loadability and the random load level for each contingency, are then given in the middle columns in Table 12.

Based on the Normal distribution of load margin, we can obtain the probability of collapse under each contingency. It is indeed the probability that the random load margin will be less than zero. The results are listed in the last column of Table 12. For instance, the probability of collapse over the next hour, for the load being 5,000 MW, is therefore only $2.16 \times 10^{-6}$. It is calculated by summing up all the products of collapse probability under contingency and the probability of the corresponding contingency, i.e.,

$$2.16 \times 10^{-6} = 0.9999 \times 1.2 \times 10^{-10} + 4.58 \times 10^{-5} \times 1.34 \times 10^{-3} + 4.58 \times 10^{-5} \times 4.58 \times 10^{-2} + 4.58 \times 10^{-5} \times 1.61 \times 10^{-8}$$

Figure 34 provides plots of collapse probabilities against different expected system loading levels under each contingency (including "no outages" case). We also show the total probability of collapse that is the sum of the collapse probabilities weighted by the contingency probabilities.

### Table 10: Randomness of System Load

<table>
<thead>
<tr>
<th>Expected Load Level</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>50.00 x 100 MW</td>
<td>250 MW</td>
</tr>
</tbody>
</table>

### Table 11: Randomness of Loadability due to Uncertain Load Sharing Factors

<table>
<thead>
<tr>
<th>Contingency</th>
<th>Occurrence Probability</th>
<th>Expected Loadability (x100 MW)</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Outage</td>
<td>0.999</td>
<td>66.86</td>
<td>1.114</td>
</tr>
<tr>
<td>Outage 130-120</td>
<td>4.58e-5</td>
<td>58.05</td>
<td>0.980</td>
</tr>
<tr>
<td>Outage 230-130</td>
<td>4.58e-5</td>
<td>54.51</td>
<td>0.982</td>
</tr>
<tr>
<td>Outage 230-120</td>
<td>4.58e-5</td>
<td>65.11</td>
<td>1.115</td>
</tr>
</tbody>
</table>

### Table 12: Randomness of Load Margin

<table>
<thead>
<tr>
<th>Contingency</th>
<th>Expected Margin (x100 MW)</th>
<th>Standard Deviation</th>
<th>Probability of Collapse Under Contingency</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Outage</td>
<td>16.86</td>
<td>2.737</td>
<td>1.2e-10</td>
</tr>
<tr>
<td>Outage 130-120</td>
<td>8.05</td>
<td>2.685</td>
<td>1.34e-3</td>
</tr>
<tr>
<td>Outage 230-130</td>
<td>4.51</td>
<td>2.686</td>
<td>4.58e-2</td>
</tr>
<tr>
<td>Outage 230-120</td>
<td>15.11</td>
<td>2.737</td>
<td>1.61e-8</td>
</tr>
</tbody>
</table>
In some cases, one may only consider the uncertainty of contingencies with deterministically known load levels and other parameters. The probability of collapse, shown in Figure 35 for our example, is then only determined by the cumulative probability of contingencies. It has a similar shape with the total probability of collapse in Figure 34, except for the abrupt steps due to discrete probability of contingencies.
5.4.3.2 Expected Impact of Collapse

For the impact of collapse, we assume the outcome will be an *entire system blackout*, as mentioned in Section 5.3.2.5, where both all the loads and generation units are lost. For the considered area, when the system load level is at 5,000 MW or 465 MW in the local area, the expected impact on load would be $139,500, where the rate of cost consequence is uniformly assumed as $50 per MWhour for a flat expected duration of 6 hours. In terms of the cost associated with the generation units, a fixed startup and repair cost is estimated at $156,000 per generation unit. There are three units in the plant of the area considered. The expected impact associated with generators is $468,000. The total expected impact of collapse in the local area considered would be around $0.6 million when system load would reach 5,000 MW (see Figure 36).

![Figure 36: Expected Impact of Collapse](image)

5.4.3.3 Expected Impact of No-Collapse

Under the condition that the voltage does not collapse, there are several possible insecure outcomes when the system is exposed to uncertainties such as contingencies, load, etc. The outcomes include the thermal overload of transmission lines, overload in transformers, voltage-out-of-limit on loads, and transient instability on generation units. Each insecurity problem has its own uncertainty dependency and impact characteristics. The expected composite impact of no-collapse is the sum of all of them.

---

49 It is hard to know just what would happen if we exceeded to system loadability. All we know for sure is that it is a very undesirable outcome, perhaps entirely unacceptable under any circumstances. We simply assign such outcomes a "large" consequence, so that we are assured of large risk if the outcome becomes likely.
5.4.3.3.1 Expected Impact on Lines due to Thermal Overload

We obtained the risk-flow curves for each transmission line in the desired area. To determine the risk on line according to the system operating condition, a power flow is needed to solve for how much MVA flow is transferring on the line. As described in Section 5.4.1, we use the expected system load level as the representative of system operating condition, and successively increase it to see how the risk evolves along various operating conditions. This procedure can be more easily performed by a Continuation Power Flow.\(^5\) According to this scenario, the line flows from bus #130 to bus #120 are shown in Figure 37 against system load levels. The flows on the other two lines in this area are in Figure 38 and Figure 39. It can be seen, in most cases, that the line 130-120, which exports most energy from the local area to the southern area, is heavily loaded, compared to the other two lines.

As addressed in the component line overload risk study, all of the three lines have a deterministic continuous rating around 400 MVA. Their component risk-flow curves are shown in Figure 31. By the system line overload risk assessment, where the system-side uncertainties are considered, the expected overload impact on the considered lines are calculated and shown in Figure 40, where the total overload impact in the area is represented by the high-lighted curve.

In Figure 40, the total thermal risk on transmission lines is almost dominated by the risk of line 130-120, which makes the 130-120 risk curve is not very visible. This is consistent with the fact that line 130-120 transfers most of the energy from the northern to the southern part of the system.

![Figure 37: Flows on Line 130-120 versus Area Load Levels](image)

\(^5\) One may also perform this overload risk study by solving for ordinary power flows in a successive way. The Continuation Power Flow, however, shows the turning point and is more helpful for the voltage risk assessment.
Figure 38: Flows on Line 230-120 versus Area Load Levels

Figure 39: Flows on Line 230-130 versus Area Load Levels
5.4.3.3.2 Expected Impact on Transformers due to Thermal Overload

The flows on transformers are obtained in the same manner as used for line flows. Under the occurrence of contingencies, the flows on transformer 120-90 and 120-100 versus area load levels are shown in Figure 41 and Figure 42, respectively.

The component risk study on these transformers is given in Figure 32 by the transformer's risk-flow curve. All of the three lines have a deterministic rating at 400 MVA. By the system overload risk assessment, where the system-side uncertainties are considered, the expected overload impact on the considered transformers are calculated and shown in Figure 43, where the total overload impact in the area is represented by the high-lighted curve.

5.4.3.3.3 Expected Impact on Load due to Voltage-Out-of-Limit

There is an aggregated load bus, bus #130, in the area considered. The voltage levels at this bus according to various operating conditions under contingencies are obtained from the Continuation Power Flows in the same manner, and at the same time, as the branch flows are obtained (See Figure 44).

The risk-voltage curve at this bus, obtained from the component risk study on bus loads, is depicted in Figure 33. Using the system overload risk assessment, where the system-side uncertainties are considered, the expected overload impact on the considered load is calculated by combining both Figure 44 and Figure 33, as shown in Figure 45. The total voltage impact in the area is represented by the high-lighted curve in Figure 45. The risk of voltage-out-of-limit is not significant in this case because the local area is a net export area where the voltage does not dip too much around generation area (see Figure 44).
Figure 41: Flows on Transformer 120-90 versus Area Load Levels

Figure 42: Flows on Transformer 120-100 versus Area Load Levels
Figure 43: Expected Impact on Transformers

Figure 44: Voltage at bus #130 versus Area Load Levels
5.4.3.3.4 *Expected Impact on Generation Units due to Transient Instability*

One generation plant with three units is in the local area we considered. The possibility of fault, such as one-phase, two-phase, three-phase and two-phase-to-ground fault occurs at various locations in the area, may cause the units at bus #13 to lose their synchronism, and result in tripping units. The expected impact on units due to possibility of transient instability is shown in Figure 46.
5.4.3.4 Composite Risk

The composite risk is the sum of

1) risk of collapse, the probability of collapse times expected impact of collapse,
2) risk of overload on lines, the probability of no-collapse times expected impact on lines,
3) risk of overload on transformers, the probability of no-collapse times expected impact on transformers,
4) risk of interruption on loads, the probability of no-collapse times expected impact on loads, and
5) risk of transient instability, the probability of no-collapse times the expected impact on units.

The composite risk, together with all of the above individual risks associated with each power system insecurity problems, is shown in Figure 47.\textsuperscript{51} It provides quantitative measurements about how much money is going to be lost due to possible power system insecurity, given various operating conditions. For example, when the system is operating at the level of 3,000 MW that is almost close to the total system capability, there is a composite risk level of $13.75 associated with this operating condition. Among the overall $13.75 of risk, $12.69 is because of possible transient instability, around $1.05 is due to transformer overload, only $0.004 is associated with transmission line overload, and almost zero (less than $1 \times 10^{-4}$) risk is due to voltage collapse and out-of-limit at this circumstance. This is consistent with the fact we purposely modified the original unstressed system to be transient stability stressed system.

The resultant composite risk has many advantages in its use as a new security index. The risk in Figure 47 gives a quantitative measurement of insecurity for the operating positions. This measurement is based on the fundamental factors that determine the security level, specifically event likelihood, consequence, and their related uncertainties. Computation of this risk measurement does not require a preliminary specification of a specific boundary. As a result, it eliminates the need for the traditional presuppositions necessary for a deterministic environment where hard boundaries are determined by worst-case "credible" events. It can measure risk both within the traditional security boundary and outside the boundary. It is useful as a decision-making aid in determining operating limits associated with security problems. For example, one might compare the risk of the deterministic security limit, given in Figure 47, which is approximately $102 over the next hour, with the benefit associated with the loading position, to decide whether it is worthwhile to operate at that level.

Security has a price. Instead of limiting the operating condition with a significant security margin inside a deterministic boundary, the risk suggests a price of insecurity. The risk implies an expectation of future cost due to possible insecurity problems. It adds an additional implicit cost to the cost of energy delivered. Figure 48 plots a marginal risk in the local area with respect to various system's loading positions. For example, it

\textsuperscript{51} Figure 47 is a semi-log plot on risk measurements. The transient instability risk stops at its left end because of the property of logarithm where the risk is zero. Another thing we need to be aware of is that the small and flat risk close to the bottom of the semi-log plot, represents a very small amount of risk where the truncate error of software makes this risk show up on the plot.
suggests an expected $0.024 per MWhour of additional implicit cost charged for the possibility of losing security when the system is running at the level of 3,000 MW. The idea of "Bus Incremental Risk"\footnote{This comes from the pricing method of "Bus Incremental Cost (BIC)".} can be introduced to price the cost of security at each bus.

The risk, or the expectation of monetary impact, discussed in this dissertation, however, only provides an expectation of future insecurity costs. It does not guarantee that the future outcome will be exactly the same as this statistical expectation. More information, such as variance of this risk, may be included together with risk to make better operating decisions.

5.5 Summary

A probabilistic method to compute the operating composite risk is presented in this chapter. The resulting risk represents the expected future cost of hybrid power system security problems: transmission line overload, transformer overload, voltage collapse, voltage-out-of-limit, and transient instability, based on the information from the current operating condition. The risk gives a quantitative measure of security both within and outside the traditional security boundary. It is promising in many areas such as power system decision-making for operation, security monitoring, and pricing.
Figure 47: Composite Risk of Insecurity
Figure 48: Area Marginal Risk
6 DECISION-MAKING UNDER RISK

6.1 Introduction

6.1.1 Problem Statement

The electric power industry is shifting from a regulated vertically integrated business environment with a captive market to a de-regulated competitive market environment. This change will result in a profound impact on power system operation. It will also require operating criteria to include uncertainty in arriving at operating limits. We have seen the uncertainty of market demand and supply, and hence, the price uncertainties in the system operation. The methods of risk management that have been used emphasize market trading that focuses on the economic aspects. However, the traditional deterministic reliability criteria are still intact, even though the essence of reliability is also a decision-making problem under uncertainty.

To maintain system reliability under these uncertainties, certain limits are required regardless of the economic forces behind the markets. The current practice in the electric power industry is to use deterministic methods to calculate these limits and keep the system reliable.

The blame has been expressed by some engineers that the marketer, who is driven by monetary profits to use economic risk management, is pushing the engineer to continually reduce the current safety margin and to operate systems closer and closer to the deterministic security limits. Today's universal deterministic approach to security assessment, however, presents remarkable gaps between the industry trend and the results it provides.

The contradiction between economics and reliability has motivated us to propose an alternative risk-based security assessment to chart the system operation and connect power system economics and reliability together. With the quantified risk, we are able to control or manage the system with more informed decisions.

6.1.2 Previous Work

There are many decision strategies used by modern financial management. They are generally classified into deterministic and probabilistic criteria. We will describe them in Section 6.2.

The dominant strategy used in power system operations is the deterministic criterion as iterated in the previous chapters. It belongs to the general strategy of Maximin benefit or Minimax cost criteria. The best example of this strategy is the well-known topic of security constrained optimal power flow [85], where the benefit is maximized under the constraints that all the N-1 deterministic security limits are satisfied. This strategy aims to maximize the minimum benefit induced by the possible maximum cost of insecurity.

Other strategies can be found in [87] and [88] where the "Minimax Regret" criterion is proposed for power system reliability management.
6.1.3 Our Approach

We will introduce several basic strategies mainly used in financial management into power system operations. For different decision-makers under various situations, different strategies may be used. However, the ultimate goal is to make more informed decisions. Although all the strategies, both deterministic and probabilistic criteria, are useful, we believe the probabilistic decision-making strategies based on quantitative risk assessment provide a distinct advantage over deterministic methods in that they provide a framework for using available information and measure the uncertainty of the available information. These advantages result in more informed decisions.

In Section 6.2, we describe a number of general decision-making strategies. From this description, one may observe how a deterministic strategy relates to other strategies that make use of probabilistic characterization of information. This description offers a framework under which risk-based security assessment may be used in practice, which is the primary goal of this chapter. In addition, Section 6.3 outlines how one of these strategies, the decision-making method using expected monetary values, may be employed using risk-based security assessment and Section 6.4 provides some illustrations.

6.2 General Decision Strategies

6.2.1 Decision-Making with No Prior Distribution (Deterministic Criteria)

The first type of decision-making is characterized by completely ignoring any probabilistic characteristic of the problem. Among them, the maximin or minimax criterion is the most common criteria.

6.2.1.1 Maximin or Minimax Criterion

The Maximin Criterion suggests to "examine the minimum gain associated with each action and then take the action that maximizes the minimum gain. This is a pessimistic criterion that directs attention to the worst outcome and then makes the worst outcome as desirable as possible." It is also called the Minimax Criterion, "if the outcomes of the action are stated in terms of loss or disutility then one minimizes the maximum loss"[86].

A simple example as in Table 13 where S1 and S2 are two possible states of nature, A1 through A3 are three actions, the numbers listed in the table represent the gain of each action for the outcome when a state of nature occurs. By this example, we decide to take the action A2 because it has the largest minimum gain (least maximum loss) compared to other actions.

<table>
<thead>
<tr>
<th>Table 13: Maximin Criterion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>----</td>
</tr>
<tr>
<td>S1</td>
</tr>
<tr>
<td>S2</td>
</tr>
<tr>
<td>Minimum gain</td>
</tr>
<tr>
<td>Maximin</td>
</tr>
</tbody>
</table>
6.2.1.2 Minimax Regret Criterion

A second strategy, Minimax Regret\(^{53}\) suggests the application of the minimax criterion, which seeks to avoid "hurt", to a "regret" table. The regret table is created by the following rules. "If the decision maker takes an action and the state of nature occurs for which the gain is largest for this action, then he/she will have no regret. However, if he/she takes an action for which the gain is not the largest, and that same state of nature occurs, then he/she will have a regret of the difference between the largest gain and that which he/she receives"[86]. This strategy is illustrated in Table 14 and Table 15, where the largest gain is obtained from Table 14, and the regret table is computed in Table 15.

Table 14: Maximum Gains

<table>
<thead>
<tr>
<th></th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>0</td>
<td>8</td>
<td>20</td>
</tr>
<tr>
<td>S2</td>
<td>30</td>
<td>18</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 15: Regret Table and Minimax Regret Criterion

<table>
<thead>
<tr>
<th></th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>20-0=20</td>
<td>20-8=12</td>
<td>20-20=0</td>
</tr>
<tr>
<td>S2</td>
<td>30-30=0</td>
<td>30-18=12</td>
<td>30-0=30</td>
</tr>
<tr>
<td>max. regret</td>
<td>20</td>
<td>12</td>
<td>30</td>
</tr>
<tr>
<td>min. regret</td>
<td>20</td>
<td>12</td>
<td>30</td>
</tr>
</tbody>
</table>

6.2.1.3 Hurwicz \(\alpha\) Index Criterion

Hurwicz suggests an application to "examine some weighted combination of the maximum and minimum gain and then take the action which has the most desirable weighted value. The weights \(\alpha\) and \(1-\alpha\) are numbers between zero and one" [86]. An example with \(\alpha=\frac{3}{4}\) is illustrated in Table 16.

Table 16: Hurwicz \(\alpha\) Index Criterion

<table>
<thead>
<tr>
<th></th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>0</td>
<td>8</td>
<td>20</td>
</tr>
<tr>
<td>S2</td>
<td>30</td>
<td>18</td>
<td>0</td>
</tr>
<tr>
<td>(\alpha) index</td>
<td>(\frac{3}{4}*0+\frac{1}{4}*30=7.5)</td>
<td>(\frac{3}{4}*8+\frac{1}{4}*18=10.5)</td>
<td>(\frac{3}{4}*0+\frac{1}{4}*20=5.0)</td>
</tr>
<tr>
<td>max.</td>
<td>(10.5)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

\(^{53}\) This concept was also discussed in [87] and [88] for applying to power system planning.
However, no one has suggested empirically obtaining the \( \alpha \) with decision makers; "but if someone feels that the Hurwicz index characterizes his criterion, the burden of proof is upon him" [86].

6.2.1.4 Laplace Criterion

This criterion is based on the assumption that all possible states are equally likely, that is, it calculates the flat average gain for each action and takes the action with the largest average gain. An example is shown in Table 17.

<table>
<thead>
<tr>
<th>( S_1 )</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>0</td>
<td>8</td>
<td>20</td>
</tr>
<tr>
<td>S2</td>
<td>30</td>
<td>18</td>
<td>0</td>
</tr>
</tbody>
</table>

Average gain: (0+30)/2=15 \( \quad \) (8+18)/2=13 \( \quad \) (20+0)/2=10

6.2.1.5 Selecting a Criterion

Each of the above criteria ignores the probabilistic nature of states. However, there is an implicitly fixed prior distribution. So a criterion and its resulting action are optimal only if they are the best against this prior distribution. This implies that "our first step in solving the decision-making problem is to search a suitable prior distribution which depends upon the information that we possess concerning the states of nature" [86]. This results in a further class of decision-making criterion, i.e., decision-making with prior distribution.

6.2.2 Decision-Making with Prior Distribution

One might argue that the decision-making ignoring the probability of the states of nature is not suitable. An alternative is the decision making with a prior probability distribution which is "characterized by the decision maker having either partial or complete knowledge of the probability distribution on the state of nature" [86]. This type of decision-making under uncertainty can be viewed as including a probability distribution to obtain the maximum expected value of gain.

6.2.2.1 Probability Functions

Probability functions are vital in every decision problem under uncertainty. There are three ways to determine the prior probability functions. They are logical, empirical, and subjective [86].

- The logical approach determines the probability of an event by considering the logical possibilities. Take an event that an ideal coin when flipped will turn up heads as the example; its probability is

\[ \text{(It will be explained in Section 6.2.2.3.)} \]
Pr(Heads) = 1/2 since there are only two outcomes and the result will either be, heads or tails, and the coin is ideal.

- The empirical approach to developing a probability function consists of considering the frequency ratio from a large number of trials, i.e., number of observations divided by the number of trials. However, only after repeated observations can we speak of the empirical probability functions.

- The subjective approach is to assign subjective values to the probability of events. It takes account of a certain kind of numerical measure of somebody's opinion.

"The difference between logical, empirical, and subjective probabilities comes down to a mere difference of interpretation as to the source of the probability statement"[86]. It is believed that this distinction is an important philosophical point. But when it comes to solving decision makers' problems, it is believed the recognition of the use of the calculus of probability is more important than the origin of the probability statement, because any prior probability can be revised in the face of new evidence and experience.

6.2.2.2 Maximizing Expected Value

It can be argued that choosing the action or strategy with maximum expected utility value is a reasonable criterion of choice. Given the utility function, "a decision maker's preferences among risky prospects"[86], we can solve the decision problem by maximizing expected utility. We assume $util(X)$ as a utility function. It can be expressed by the sum of functions in powers of $(X - C)$, using Taylor's Series expansion, where the constant $C$ is the expected value $E(X)$, which is the expected value of gain given an action $A$. We obtain

Equation 38: Taylor's Expansion of Utility Function

$$util(X | A) = util(E(X)) + (X - E(X)) \frac{\partial util(E(X))}{\partial X}$$

$$+ \frac{1}{2} (X - E(X))^2 \frac{\partial^2 util(E(X))}{\partial X^2}$$

$$+ \frac{1}{3} (X - E(X))^3 \frac{\partial^3 util(E(X))}{\partial X^3} + ...$$

Taking the expectation of the above equation, since $E(X - E(X)) = 0$, we obtain the expected utility of the action $A$, which is
Equation 39: Decomposition of Expected Utility under Action

\[ eu(A) = E(\text{util}(X | A)) \]

\[ = \text{util}(E(X)) + \frac{1}{2} \sigma^2 \frac{\partial^2 \text{util}(E(X))}{\partial x^2} + \frac{1}{3!} g_1 \frac{\partial^3 \text{util}(E(X))}{\partial x^3} + \frac{1}{4!} g_2 \frac{\partial^4 \text{util}(E(X))}{\partial x^4} + \ldots \]

where \( \sigma^2 = E(X - E(X))^2 \) is the variance of the distribution of \( X \), \( g_1 = E(X - E(X))^3 \) is the skewness of the distribution, and \( g_2 = E(X - E(X))^4 \) is the kurtosis.

Equation 39 gives the expected value of utility in terms of the moments of the distribution and the derivatives of the utility function. For example, if the outcomes of an action are distributed normally, only the first two terms are significant, since the Normal distribution has only two moments, i.e., mean and variance. This assumption leads to a Mean-Variance analysis [89] of the decision choices.

While maximizing expected utility is the general choice criterion, it was however, argued in [86] and [90] that maximizing expected monetary value is equivalent to maximizing expected utility with continuous repeated decisions in the long run.

To illustrate this point, the example from Section 6.2.1 is used, and a probability distribution is attached to each possible state of nature under each action. The result is shown in Table 18. The gain for each action under various uncertainties is given as a monetary gain.

### Table 18: Maximizing Expected Value

|       | \( \Pr(S_1 | A_1) \) | \( A_1 \) | \( \Pr(S_2 | A_1) \) | \( A_2 \) | \( \Pr(S_1 | A_4) \) | \( A_3 \) |
|-------|-----------------|-------|-----------------|-------|-----------------|-------|
| \( S_1 \) | 0.5             | 0     | 0.8             | 8     | 0.8             | 20    |
| \( S_2 \) | 0.5             | 30    | 0.2             | 18    | 0.2             | 0     |
| expected gain maximum | 15 | 10 | 16 |

#### 6.2.2.3 Relationship between Decision-Making with Prior and without Prior Probabilities

Each of the deterministic criteria in Section 6.2.1 is equivalent to the decision-making with a subjective prior distribution [86]. For example, the maximin criterion expresses the belief that the probability of the possible states depends on which action is chosen, that is, for any action the worst possible state will occur with probability one. This claim is illustrated in Table 19.

A similar conclusion can be drawn for the Minimax Regret, Hurwicz \( \alpha \), and Laplace Criteria. A decision maker using deterministic decision strategy is actually subjectively betting the occurrence of a particular situation.
Table 19: Maximin Criterion vs. Maximum Expected Value

| S_1 | A_1 | Pr(S_1 | A_1) | A_2 | Pr(S_1 | A_2) | A_3 | Pr(S_1 | A_3) |
|-----|-----|-----------|-----|-----------|-----|-----------|
| 1   | 0   | 1         | 8   | 0         | 20  |
| 0   | 30  | 0         | 18  | 1         | 0   |

| minimum gain maximin | 0 | 8 | 0 |
| expected gain maximum | 0 | 8 | 0 |

6.2.3 Decision-Making with Posterior Distribution

"This type of decision making problem is characterized by the possibility of obtaining additional information or data before a decision is rendered"[86]. The decision is then made between the available actions by finding the maximum expected value for each action, with the posterior probabilities which are the revised prior probabilities.

6.2.3.1 Obtaining Posterior Probability

The information we get is not perfect or is not possible to perfect in regard to predicting which state of nature will occur and what the probability function is. The prior distribution used in computing expected value may not be perfect even though we try to make it perfect according to the information and experience we possess.

With the historical experience of states of nature, we have a collection of data, for instance, the wind speeds Z_i's under various weather conditions. According to these data, we can have a conditional probability of obtaining an information when a state occurs, i.e., Pr(Z | S_i). In this case, we can get the probability of wind speeds when a thunderstorm occurs and when there is no thunderstorm by the empirical approach described in Section 6.2.2.1. An example is listed in Table 20.

Table 20: Conditional Probability of Wind Speed under Various Weather Conditions

<table>
<thead>
<tr>
<th></th>
<th>Low</th>
<th>Medium</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>S_1</td>
<td>0.7</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>S_2</td>
<td>0.5</td>
<td>0.3</td>
<td>0.2</td>
</tr>
</tbody>
</table>

We assume the uncertain states of nature, e.g., thunderstorm, have a prior distribution Pr(S_i), say Pr(S_1) = 0.7, Pr(S_2) = 0.3. It indicates the probability that a thunderstorm will occur is 0.7. These probabilities could be obtained by an empirical approach based on previous experience, or even by subjective judgement.
Our objective is to obtain a refined probability function for the uncertain states of nature given additional observed information. To compute the posterior distribution with additional observed information, Bayes’ formula [14] is used to revise the prior probability distribution.

Equation 40: Bayes’ Formula in Computing Posterior Probability

$$\Pr(S_i | Z_j) = \frac{\Pr(S_i) \times \Pr(Z_j | S_i)}{\Pr(S_i) \times \Pr(Z_j | S_i) + \Pr(S_2) \times \Pr(Z_j | S_2) + \ldots}$$

Thus, if a medium wind speed ($Z_2$) happens to be read at a time, the new distribution or posterior probability distribution would be:

Equation 41: Example of Posterior Probability

$$\Pr(S_1 | Z_2) = \frac{0.7 \times 0.2}{0.7 \times 0.2 + 0.3 \times 0.3} = 0.609$$

$$\Pr(S_2 | Z_2) = \frac{0.3 \times 0.3}{0.7 \times 0.2 + 0.3 \times 0.3} = 0.391$$

Note that the observed data ($Z_2$) modifies the prior probabilities, $\Pr(S_1)$ from 0.7 to 0.609, and $\Pr(S_2)$ from 0.3 to 0.391. In this case, we update our prediction of thunderstorm with probability 0.7 to a lower probability 0.6, when we observe the wind speed is not high. This procedure can be continuously repeated to refine the prior distribution, as long as more data is observed. In this manner we continue to build on our experience and make better decisions under exposure to the uncertain world.

6.2.3.2 Decision-Making under Posterior Distribution

To maximize the expected value is the decision strategy under the posterior distributions. In this process, the prior probability in Section 6.2.2.2 is continually refined by observing additional information.

6.3 Power System Decision-Making under Risk

There are several ways to deal with the power system security under risk. We have seen a fruitful history and experience in applying deterministic approaches to avoid risk. We would not discuss the deterministic decision-making strategy used in power systems. Rather, we propose several probabilistic decision-making schemes based on the risk assessment developed in the previous chapters.

In accordance with the definition of the degree of risk provided earlier, the strategy here is based on expected monetary value under either prior or posterior probability functions. Generally, as explained in
Section 6.2.2.2, maximization of expected monetary value may not be sufficient in managing risk under different personal utility functions. More variables, such as variance and even skewness, may be added to the decision-making schemes. Based on Equation 39, one should not have difficulty in adding more terms and constructing more strategies\(^5\) using the generalized expected utility value.

6.3.1 Operation with Risk Restrictions

A brute force approach using the quantitative risk assessment is to determine operating limits by a pre-defined acceptable risk level. In other words, we limit our operating conditions such that the risk, the *expected value* of possible cost consequence, is bounded within a limit. For example, it can be used to determine a thermal rating based on component risk analysis of a transmission line. For the system operation, there are several other ways to apply this criterion.

6.3.1.1 Security Constrained Optimal Power Flow

The strategy is the same as the ordinary security constrained optimal power flow (OPF) [85], i.e.,

**Equation 42 : Security Constrained Optimal Power Flow**

\[
\begin{align*}
\max & \quad \text{Revenue}(X | A) - \text{Cost}(X | A) \\
\text{s.t.} & \quad \text{Postcontingency Flow} < \text{FlowLimit} \\
& \quad \text{Postcontingency Voltage} \in \text{VoltageRange} \\
& \quad \text{other pre-determined limits}
\end{align*}
\]

The key point here is that all the constraints to this optimization problem are pre-determined or deterministic as we see the existing security limits. However, those limits can be determined either deterministically or probabilistically. As we develop the two-tiered risk assessment in previous chapters, these limits can be determined by the "component" risk assessment applied to a pre-defined risk level. Each individual supplier, transmission owner or distribution company, may define their own acceptable risk level for determining the post-contingency flow and voltage limits. The manner in which this optimization problem is solved is identical to the conventional OPF.

This scheme is suitable for each individual to determine their own decision criterion, greedy or conservative, according to their own utility functions and preference to the risk. Meanwhile, the methods in coordinating the system-wide security level are the same as those presently used in the industry. The system security can be maintained by the transaction curtailment through existing congestion management approaches.

\(^5\) One may seek to maximize the expected monetary value with minimum or pre-defined variance.
6.3.1.2 Risk Constrained Optimal Power Flow

Unlike the ordinary security constrained OPF, if each individual does not provide the operating limits according to their individual preference, the system security coordinator could have the opportunities to dispatch the system such that the "system" risk is within a pre-defined level. The operation strategy is then,

Equation 43: Risk Constrained Optimal Power Flow

\[
\max \ Revenue(X | A) - Cost(X | A) \\
\text{s.t.} \\
\text{Risk}(X | A) < \text{RiskLimit}
\]

where \( \text{Risk}(X | A) \) is the risk of an operating condition given a control action \( A \), that is the expected monetary cost consequence associated with the operating condition.

In this case, each individual transfers the decision-making problem to the system security coordinator not by defining their operating limits according to their individual risk level. Rather, they provide their component risk assessment to the central security coordinator such that the coordinator can maneuver the system on behalf of limiting the entire system within a risk-based security boundary. The transferred information consists of risk versus loading level for each component, as illustrated in previous chapters of this dissertation.

This scheme is suitable for the system where all the individuals have the same preference to the risk such that the system security coordinator can apply a uniform scheme limiting the system risk, and identifying the risk allocated to each individual. However, the allocation of system risk to each individual may not be uniform. This may be used in the system curtailment when the system security is the predominant objective.

6.3.2 Operation with Expected Monetary Value Maximization

The more greedy strategy is to release all the security constraints and to maximize expected monetary value under either prior or posterior probability functions.

Equation 44: Maximizing Expected Monetary Value

\[
\max \ Profit(X | A) - Risk(X | A)
\]

where \( Profit(X | A) = Revenue - Cost \) is the (expected) profit of the operating condition by the action \( A \).

\[^{56}\text{We generally believe the benefit is uncertain according to some exogenous variables, such as market prices.}\]
The first order condition of this maximization problem is simply "marginal profit equals to marginal risk", which is similar to the common sense of economics, "marginal revenue equals to the marginal cost", when maximum profit is reached.

**Equation 45: First Order Condition of Decision-Making under Risk**

\[
\frac{\partial \text{Profit}(X | A)}{\partial A} = \frac{\partial \text{Risk}(X | A)}{\partial A}
\]

To reach the expected optimum of an uncertain situation, Equation 45 suggests the selection of a position (or an action) where its marginal risk is equal to the marginal profit. For example, a transmission owner may choose a limiting flow where the marginal risk of overload with respect to an additional flow is the same as the expected marginal profit of the additional flow.

To solve for this optimization problem, both mathematical and market-based approaches could be used. For the market-based solution, the security coordinator calculates the marginal risks against additional market actions, for example, marginal risk against an additional transaction amount. These marginal risks can be used as a pricing signal to adjust, rather than restrict, the behavior of market players such that the free market will reach its optimal equilibrium by the participation of individuals. Since the real commodity trading price includes a security price obtained from an expectation, the extra revenue collected through the security price is transferred to the risk takers, or used as an insurance to cover the possible cost in case the real impact of insecurity would occur.

### 6.4 Illustrations

It is reasonable that each decision maker may apply different decision criteria to make his/her individual-dependent decisions. Even though in a case where the criteria used are the same, the tolerance level of risk may still be different. In this section, three examples using risk assessment are given to illustrate the possible application of risk in power system decision-making problems. The applications by no means are exhaustive, instead they are more focused to "fit" the risk into the present deterministic tradition in order to act as a possible transition to the future.

An application of component risk assessment is given for a transmission line to determine its line ratings. These ratings can be directly used to determine the existing deterministic security analysis. The same approach can be applied by the distribution company to determine its voltage limits for each load bus by the component voltage risk assessment approach. An application of composite "system" risk assessment is given for a security coordinator to determine maximum transaction amount or its curtailment in case the composite risk level would be violated. A third example is to use marginal risk to price the system congestion under the competitive environment.
6.4.1 Determining Ratings Based on Risk Analysis

Three ratings commonly used for ACSR overhead conductors are normal, long-time emergency and short-time emergency ratings [28]. In this section, we describe how these ratings are determined under the risk-based approach.

6.4.1.1 Normal (Continuous) Rating

Thermal rating of a transmission line specifies a maximum amount of current that ensures the risk will remain within a prescribed level. Based on the risk calculation for continuous line loading, normal rating is the continuous current that has the risk level, one is willing to accept.

We assume the chosen deterministic normal rating to be acceptably safe. This guideline is used in this dissertation to give a reference risk level to determine the thermal ratings. One may choose a higher level of risk if the expected benefit is recognized to significantly exceed the additional risk.

As the example in Section 3.4 shows, the continuous rating, obtained directly from the component thermal risk study in Figure 11, depends on the prescribed risk level. If one would accept a risk of 0.01 (which means 1 percent of the cost to re-conductor this circuit, or equivalently, 1 percent of the life loss compared with the designed one), then the continuous limiting current would be 1028.4. This risk is about 2 times as high as that incurred when the deterministic limit of 992.4 is used.

6.4.1.2 Long-Time Emergency Rating

We determine the long-time emergency (LTE) rating as the current level that will incur the same risk under the shorter duration as the normal rating under continuous operation. We call this the Equal Risk Criterion [37]. The LTE rating is higher than the normal rating, not because of a higher maximum allowable temperature, as in the usual way to determine the line ratings ([29], [39]), but because the limited overload time used in the LTE rating calculation, relative to the continuous overload time used in the normal rating calculation, results in a much reduced overload impact. This fact allows that the LTE rating will be computed for any duration less than the remaining conductor life (days, weeks, months), and the resulting rating will be higher than the normal rating, while the incurred risk will be the same as that of the normal rating.

Since the risk of deterministic continuous rating is implicitly accepted, the same amount of the risk associated with the temporary overload should also be acceptable. This is the so called the “equal risk criterion” which is used to decide the long-time and short-time ratings. Based on this criterion, one may guarantee that the temporary ratings are as safe as the deterministic continuous loading in the sense of expected monetary cost.

The long-time ratings can be determined by the iso-risk contours as shown in Figure 49. The contour represents a locus of combination of loading level and its duration where the risk level is the same. The LTE rating is 1225.4 for the same risk level as the continuous one. The LTE ratings are useful during system recovery or for short-time energy exchange.
6.4.1.3 Short-Time Emergency Rating

We also use the equal risk criterion in determining the Short-Time Emergency (STE) rating. Here, the STE rating is also higher than the normal rating because of the much-reduced impact of overload. Furthermore, it is higher than the LTE rating because the dynamics of conductor temperature are effective in this time frame and, for the same current level, the one-hour temperature level is always lower than the steady state level that is used for LTE and normal ratings. STE rating (within 60 minutes), also based on the equal risk criterion, ranges between $1225A$ and over, as indicated in Figure 50. This would be useful during emergency periods and short-time security assessment. From Figure 50, the deterministic 15-minute STE rating goes beyond the risk-based rating. That means operating on this deterministic STE rating for 15 minutes does not ensure the same safety as the normal loading. It will result in a higher expected cost.

6.4.2 The Risk Restriction Used in System Operation

Another application of risk may be for the transaction or load curtailment under a congestion management. One practice of current congestion management is to cut the proposed load when the deterministic security criteria are violated. As an example, we take a simple one-dimensional illustration of the load curtailment to illustrate use of the criteria rather than how to optimize the allocation of curtailment. A security coordinator could pre-define an acceptable composite risk level according to the requirement of a standard.\textsuperscript{37} We propose that this level of risk should be maintained throughout the system operation.

\textsuperscript{37} Such a standard has not been available so far.
As in the current congestion management, system transactions have to be curtailed in case the security is violated. The difference between risk-based congestion management and the current deterministic curtailment is that the system's operating boundary is determined by a limit on the composite risk rather than by limits on flows and voltages. As in the example in Section 5.4, we first illustrate how the traditional security boundary for the system load level is determined by the deterministic criteria:

- The pre-contingency flows in the desired area should be less than their normal ratings. The limit of system load is determined as 3600 MW according to Figure 37, 33, 34, 36, and 37.
- The post-contingency flows in the desired area should be less than their short-time emergency ratings. The limit is determined as 3800 MW for the system load level according to the same figures as above.
- The bus voltages should be maintained within 5% of their normal level. The system load limit is determined as 5600 MW according to Figure 44.
- The system is voltage stable. The limit is determined as 5400 MW.
- The system is transient stable. The load level limit is 2050 MW.

Following the traditional decision-making practice, then, the deterministic security boundary for the system load level is obtained as 2050 MW without any discount of safety margin. If 10% safety margin is required, the limit is then only 1845 MW. If the proposed schedule would exceed this level, the security coordinator has to curtail some amount of load such that the maximum of 1845 MW is maintained. However, the coordinator will not find it easy to justify the reduction of risk of insecurity caused by the curtailment of loads, since there is no quantification of risk.

The risk assessment gives the security coordinator a chance to review the reduction of risk and further aid in deciding how much the load is worthwhile to curtail. If, for example, the considered area would like to accept...
average $1 per hour risk of insecurity as its limit of risk, then the system load could be lifted to 2200 MW according to the composite risk plot in Figure 47. Depending on a standard or an agreement on the acceptable risk level that the system can tolerate, the operating limit can then be determined.

6.4.3 Pricing Congestion by Marginal Risk

A further extension of the idea of risk cap could be an application of marginal risk. In contrast to the traditional security limit, the area faces an increase of risk of $1.0-0.2=$0.8 per hour for the 355 MW increase of transactions in the example of Section 6.4.2. If this 19% (355/1846) increase of transaction does increase the profit of the area by $0.8 per hour, it may be of interest to this area in lifting the load by 19% since the additional profit is expected to cover the additional risk. We have proposed to use the marginal risk as the price of congestion in Section 5.4.3.4. The risk, calculated in this dissertation, represents the expected cost consequence because of possible insecurity of power system given the current operating condition. As we did in the example in Section 5.4.3.4, the marginal risk, which is the first derivative of the regional risk against the system load level,\(^58\) represents the incremental monetary risk if an increment of load is demanded at the current load level. In other words, to trade one additional unit of electricity at the current system condition, the local region is facing a risk to lose extra amount of money due to possible insecurity problems within this region. To cover this amount of "expected" loss, the local area needs to charge all the market players by the corresponding incremental risk. Because the marginal risk varies along the operating condition, this charge of congestion will be different under various system conditions depending on the potential of losing security. The plot in Figure 48 indicates that this amount of charge increases when the system is more stressed by the demand. This pricing signal provides an incentive to market players that they will be charged more due to possible insecurity problems if they utilize facilities in a critical region or corridor. The revenue collected from the congestion price will either go to the risk takers, such as transmission lines, distribution loads, and generation units, depending on allocation of the composite risk, or it will fund an insurance to cover the "real" impact instead of "expected" impact in case a contingency occurs.

6.5 Summary

Both deterministic and probabilistic criteria used in the decision theory are introduced in this chapter. The quantitative probabilistic criteria provide informative strategies other than deterministic criteria. Instead of applying deterministic strategies in the power system operation, probabilistic criteria based on risk assessment may be used in various power system decision problems.

\(^{58}\) A cross marginal risk, which is the first derivative of the local risk against the external load level, can be also calculated to represented the security impact of the external players to the local area. Furthermore, a marginal risk of a specific security problem, for instance, overload risk on a line, with respect to individual buses (a "bus incremental risk") can be treated as a "spot" congestion price on that line for each system injection.
7 CONCLUSION AND SUGGESTIONS FOR FUTURE WORK

7.1 Contribution of This Work

An integrated method has been developed that allows risk-based security assessment in an operating environment considering any type of security problem. The method explicitly calculates the risk of an operating point and permits more informed operating decisions.

A two-tiered infrastructure has been developed to assess the risk of system operation: "component" and "system" risk assessment. Each approach encapsulates its own detailed statistical data and models within its analysis such that the entire risk assessment can be achieved modularly. The component assessment of risk of overload provides a risk-flow curve for each individual line or other serial equipment, the system assessment, however, quantifies the system-wide overload risk by considering the system-wide disturbances. The component assessment of voltage risk provides a risk-voltage curve for each bus which enters the system voltage risk assessment.

Two most important and universal security problems: transmission line thermal overload and voltage insecurity, are quantified by their expected monetary cost consequences. Both are decomposed into "component" and "system" assessment.

A method to compute risk of transmission line thermal overload is developed to quantify the monetary impact of loss of life and loss of clearance incurred by transmission lines.

A method to compute probability of voltage collapse by loading margins is developed. It is used to assess the risk of voltage collapse. In addition, the risk of voltage-out-of-limit without collapse is also quantified for any operating point. Both characterize the risk incurred by end-users under possible voltage problems.

A composite risk index for power system security is provided. The system operator can be informed by this composite risk including risks of transmission line overload, transformer overload, voltage collapse, voltage out-of-limit, and transient instability.

The risk assessment developed in this work leads more informed decision-making in operating power systems. Several decision schemes are given in this work by relating both component and system risk to various steps of system operations, from equipment rating determination to pricing in the marketplace. An example of applying component risk assessment is given by determining line ratings by "equal risk criterion" and an example of applying system risk is given by pricing power system security using "marginal risks".

Finally, the risk charts both traditional and intact regions of power system operation. It acts as a bridge to enable system operators to balance system security with economics to meet the needs of the competitive marketplace. The index developed captures the notion of reliability and associates with it an economic cost in terms of dollars.
7.2 Suggestions for Future Work

In this work, a method to perform risk-based security assessment in the operating environment is developed. To make this research accepted by industry, and eventually an industry standard, more work needs to be done.

Work needs to be done in both probability and impact evaluation. We need a better estimation of probability functions for all the uncertainties. The posterior probability approach may be an effective way to refine the existing probability functions. Also, we need a more comprehensive and detailed estimation of impact of each insecurity problem.

An industry standard for applying probabilistic approaches in power system operation is needed.

Work from the risk assessment of this research can be extended as follows:

- security constrained optimal power flow (OPF)
  As illustrated in Section 6.3.1.1, one can use detailed risk-based component rating analysis to determine the limits for conventional optimal power flow method.

- risk constrained OPF
  In contrast to security constrained OPF, the limitation of optimization problem becomes the system-wide composite risk.

- congestion management
  The risk assessment is helpful for alleviating the network congestion and allocating the corresponding responsibilities.

- risk-based security boundary
  Given a risk restriction, an operating boundary can be determined for guiding the system operation.

- risk-based preventive and corrective control
  Under the risk-based environment, the preventive control becomes a way to mitigate the risk, while, the corrective control is to restore the system to an acceptable risk level. The control schemes may be optimized by obtaining the sensitivity of risk against desired control variables.

- decision-making under risk,
  As indicated in Chapter 6, there are many general strategies for decision-making. Under various situations and preferences, different criteria and the way to use them may be applied. For example, mean-variance
analysis makes a tradeoff between expected value and variance,\textsuperscript{59} a confidence interval gives a range where the random true cost is located.

- Risk-based market pricing
Pricing the network security is getting more and more attention from the market environment. The marginal risk method may be used to provide both discriminative and non-discriminative price signals.

- Value of information
The expected monetary value based on posterior probability provides an additional value to the one based on the prior probability. This additional value becomes the value of the additional information for obtaining the posterior probability functions.

- Other aspects of security problems
They include such as oscillatory instability and effects of protection actions.

\textsuperscript{59} The variance can be obtained by methods used for expected value except for different calculation formula as listed in [15].
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