1951

Short-lived synchrotron-induced radioactivities

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SHORT-LIVED SYNCHROTRON-INDUCED RADIOACTIVITIES

by

Forrest I. Boley

A Dissertation Submitted to the Graduate Faculty in Partial Fulfillment of The Requirements for the Degree of DOCTOR OF PHILOSOPHY

Major Subject: Physics

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1951
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The purpose of the present investigation is to use a procedure in which several large numbers of cloud chamber scattering and absorption problems were studied. The second is the first method suffers from the uncertainties of the mechanism. The present author, therefore, the solution to these two problems have been the use of absorption and cloud chamber measurements. The present absorption by these methods would be difficult. Many of the produced radioactive activities are of such short half-life. Also studied are measurable radioactive activities of small solid angle. The x-ray beam intensity of the ion state of the 70-Mev synchrotron is capable of
time, statistical variations may be minimized. Provided with appropriate recording methods, data concerning the energy spectrum and half-life of a radioactive species may be obtained simultaneously.

The various nuclei which were investigated may conveniently be placed in two groups. In the first are those with half-lives of the order of minutes, namely praseodymium\(^{140}\) and iron\(^{53}\). Although certain features of both these isotopes have been studied previously, neither has been observed with the techniques used here. Information concerning endpoints and half-lives of those nuclides in this first group is interesting as a check on previous absorption measurements.

The second group of nuclei consists of those with half-lives of the order of seconds and with one more proton than neutron. That is, \(Z - N = 1\), where \(Z\) is the atomic number and \(N\) is the number of neutrons. These nuclei, when decaying, convert one proton to a neutron and emit a positron. The upper limit of the continuous energy distribution found for the positrons emitted by such nuclei is of special theoretical interest, as will be indicated in the following paragraph.

Theoretical calculations concerning beta-spectra have been made by Fermi (1), with modifications by Gamow and Teller (2). The particular set of nuclei with one more proton than neutron derives interest in that, according to the Wigner (3) theory, when decay occurs between the ground state
of one of these and the ground state of the product nucleus
the total energy difference should just be that attributable
to the Coulomb energy of exchanging the extra proton for a
neutron. That is, the only excess energy involved in p-p
(proton-proton) interactions over n-n (neutron-neutron) ones
is that due to the Coulomb forces acting in the p-p case. If
this is true, due to the high degree of symmetry between these
Z - N = 1 nuclei and the final nuclei, this Coulomb energy may
be calculated quite generally by use of rather simple assump-
tions, such as the uniformity of the charge distribution
within the nuclei and the proportionality of the nuclear
volume to atomic mass number. The result (4) of such a calcu-
lation leads to a value for the Coulomb energy of

$$E_c = G \frac{A - \frac{3}{4}}{A^{1/3}}$$

where \( G = \frac{3}{5} \frac{e^2}{r_0} \). \( e \) is the electronic charge and \( r_0 \) is the
constant in the expression \( R_0 = r_0 A^{1/3} \) for the nuclear radius.
\( A \) is the mass number of the decaying nucleus.

Since the total energy difference between nuclear ground
states is just the maximum energy of the positron plus its
rest mass plus the neutron-proton mass difference, one may
also write

$$E_c = E_{\text{max}} + m_e + (n-p)$$

where \( E_{\text{max}} \) is the kinetic energy endpoint of the positron
spectrum, and where \( m_e, n \) and \( p \) are the masses of the electron,
neutron and proton in energy units. This, of course, is true only if the positron decay occurs between ground states. That is, no gamma ray accompanies the disintegration. No gamma ray has ever been detected for the special class of nuclei considered here.

Using the n-H value of 782 kev given by R. F. Taschek et al (5), \( E_{\text{max}} \) becomes

\[ E_{\text{max}} = C \left[ \frac{(A - 1)}{A^{1/3}} \right] - 1.80 \text{ Mev.} \]

The constant \( C \) may be found by attempting a best fit using experimentally determined values of \( E_{\text{max}} \) for various values of \( A \). This has been done from time to time by other investigators (6, 7, 8, 9) using several values of \( E_{\text{max}} \), but not with all those currently available. These workers have in general found experimental values of \( E_{\text{max}} \) in reasonable agreement with this type of prediction. This work was, however, done with cloud chamber and absorption techniques with endpoints being determined without the assistance of a Fermi plot.

Another interesting relationship should be noted. Bethe and Bacher (10) and Wigner (4) have pointed out that the half-life could be predicted from the Fermi theory of beta-decay. If the nucleus of one of these positron emitters goes to the ground state of the final nucleus, the relationship between the half-life and the maximum total energy is represented fairly accurately by \( T_{1/2} \propto (W_0)^{-5} \) for light nuclei. Thus one should obtain a straight line on plotting the inverse of the
fifth root of the experimentally determined half-life against
the value of $W_0$. $W_0$ is the maximum total positron energy in
units of $mc^2$. This will be given more attention later.

Beta-decay theory involves a function $F(Z,p)$ where $p$ is
the total momentum in $mc$ units of the emitted particle and $Z$ is
the atomic number of the resultant nucleus. $F(Z,p)$ involves
not only a statistical factor of the square of the momentum,
but also a Coulomb correction factor to account for the effect
of the nuclear attraction upon the emitted beta-distribution.
$F(Z,p)$ may be calculated by various approximation methods.
The integral of this function times $(W_0 - W)^2 dp$ taken from zero
momentum to the maximum momentum and multiplied by the half-
life of the activity gives what is called the ft value of the
transition.

According to the Fermi theory this ft value should be
constant for the $2 - N = 1$ nuclei. Comparison ft values will
be calculated for the nuclei investigated here. The ratio of
electron $K$-capture to positron emission probability is less
than $10^{-3}$ for these nuclei from the work of Feenberg and
Trigg (35).

It may be seen from this discussion then that any new
information obtainable concerning this particular type of
nucleus and confirmation of previous absorption and cloud
chamber work by the scintillation spectrometer technique would
be worthwhile.

The scintillation spectrometer appears to be a ready
means of examining these short-lived, high energy beta-activities in order to obtain their half-lives and energy endpoints. That this type of spectrometer lends itself well to this work is seen by the facts that the resolution goes inversely as the square root of the energy (11) becoming better than 10 per cent at 3 Mev, that at these higher energies anthracene provides a crystal of good proportionality (11), and that the effective dead time (12) of the counter is negligible compared to the relatively high initial counting rates encountered in such activities.

In summary this investigation was carried out in an attempt to obtain half-lives and endpoints of several of the short-lived beta-emitters produced by high energy x-rays. Rather interesting theoretical predictions have been made concerning the group to which belong many of those activities studied. Some of these decays have been observed previously by other techniques. Results obtained will be compared with the theory and with other experimental values.
II. INVESTIGATION

A. Objectives

In achieving the primary objective of the investigation, that is the use of a scintillation spectrometer for determining the half-life and energy endpoint of various short-lived synchrotron induced activities, the following general course of action was followed.

It was desirable to look first at something with a long half-life whose endpoint and spectral shape were well known. For this purpose phosphorus$^{32}$ and yttrium$^{90}$ were observed. A check on the operation of the electrical circuitry is thus provided, free of the complications which would result from the use of a rapidly decaying source. In this way one may find the accuracy with which known endpoints can be determined and how closely the known spectral shape can be reproduced.

When the experimental technique proved satisfactory for these activities, the extension was made to others with half-lives of the order of minutes. For these studies praseodymium$^{140}$ and iron$^{53}$ were chosen. Since these observations involved all the features of the equipment except the rapid transfer of the source from the x-ray beam to the spectrometer box, reasonable checks could be made of nearly all the operational features of
the equipment.

Next, of course, activities with half-lives of the order of seconds could be observed. Here it was essential to transfer the sample from the irradiation position to the counting position in the shortest possible time. If the data thus accumulated in a period of several seconds are recorded photographically, to permit later examination at a slower speed, one can obtain not only the spectral distribution but also the half-life. It is with just such a recording "time-stretcher" device that these studies were made. A detailed description of the equipment used for this work is given in the next section.

B. Experimental Apparatus

For the reasons discussed earlier, a scintillation spectrometer was chosen to study short-lived activities. The scintillation spectrometer may be described as follows.

On intercepting particles which are given off by a radioactive source, a scintillation crystal converts the energy of the particle into a flash of light. This flash of light is converted into an electrical signal by means of a photomultiplier tube and the electrical signal after being suitably shaped is recorded.

One of the properties of crystals such as anthracene, stilbene or naphthalene is that when traversed by a particle
such as an electron a flash of light is produced. In some, notably anthracene and stilbene, the intensity of light is very nearly proportional to the energy lost by the particle. If an anthracene crystal of thickness sufficient to stop the particle, that is, to cause it to lose all of its kinetic energy inside, is placed before a beta emitter, the crystal should ideally produce flashes of light the intensity distribution of which would represent the beta spectrum.

A photomultiplier tube is a device which will produce an electrical signal when excited by light. It consists of a photocathode which emits electrons by the photoelectric effect. Following this is a set of dynodes between each of which is a potential difference. The electrons from the photocathode are focused on the first dynode surface, where they produce many secondary electrons. These are in turn focused on the second dynode where more secondary electrons are produced. This multiplying process is repeated at each dynode and the final electron burst is collected by the anode. Ideally the magnitude of charge in the final burst should be proportional to the intensity of the incident light pulse. If such a photomultiplier tube receives the light flashes from the crystal, the output signals from the tube would be proportional to the energy of the original beta particles, assuming the photomultiplier tube to be a proportional device. These output pulses could then be recorded in any desirable manner.
Several effects prevent the foregoing from being more than just an idealized description. First of all, some of the electrons on entering the crystal follow a tortuous enough path that they find themselves once again outside with some unexpended energy remaining. This occurs preferentially to electrons which have lost the least energy in the crystal, since they are perforce nearer the surface. This leads to an apparent preponderance of lower energy electrons, thus making it difficult to attach much significance to the lower half of the energy spectrum.

Second, the crystal does not produce flashes of light of exactly equal intensity for monoenergetic electrons. Third, the photomultiplier does not produce output pulses of exactly equal amplitude for equal intensity light pulses. These last two effects are not only due to the statistical nature of the problem but also to the variation of light producing ability throughout the crystal and to the variation of photoelectric response of the photomultiplier photosurface. The net result of these effects is to produce a broadening by the instrument of any monoenergetic line. It is advantageous to make this effect as small as possible.

Thus in setting up equipment of this type it is well to have a large selection of photomultiplier tubes and scintillating crystals from which to choose, since the individual variation of the resolution (particularly in tubes) may be
the tube, and the motor drive of the oscilloscope recorded

support a batch of compressed air to blow the source down

synchrotron beam is turned off, a solenoid valve is actuated

...shutter of the steeplecog quadrant * at the time the

spontaneous reduces the case which must be taken with 

the pulsing frequency connected with the synchrotron. Thus

the irradiation on the x-ray beam is turned off, as are all

the irradiation is turned by a preset clock. At the end

the sample room rotated during irradiation.

where was made with an overall cross section. The above presented

accurately placed the source at the counting position. The

made faster or slower by varying the exit pressure. To add to

made in this tube is approximately 0.1 second, this can be

sample in this tube is approximately 0.1 second, this can be

the upper right of the x-ray rack. The beam is tuned for a

compressed air into the back synchrotron box (b) shown to

time. After irradiation they are short time in the tube by

long illumination tube (v) shown in the upper right of the -

long illumination tube (v) shown in the upper right of the -

observed the very short x-ray scattered. Some of these are in

Figure 1 shows a photograph of the system arranged for

section

the interpreted resolution will be discussed in the next

resolution. The distortion of the data which results from

were satisfactory. Several investigators have reported better

very great. Although the tube and crystals used in this work

11
Fig. 1. Experimental arrangement for studying short-lived activities.
camera is started. After sufficient data are collected the camera is turned off manually. The sample may be returned to the beam by operation of a solenoid valve in the vacuum line. After a time lapse sufficient to allow longer lived activities to die out, the cycle may be repeated.

Inside the spectrometer box, the source is located above the crystal as shown in Figure 2. The tube is surrounded by a Mumetal magnetic shield. The source holder consists of a wooden plug backed with leather which serves as the carrier for the source, and a 0.005 inch nickel foil into which a 7/8 inch circular hole is punched. The foil is secured to the wooden plug with nickel wire. The thin source is placed in the 7/8 inch hole.

Figure 3 shows a detail of the photomultiplier tube arrangement and also a block diagram of the complete system. An anthracene crystal (either one-half inch or one inch thick, depending upon the maximum energy anticipated) is mounted with Canada balsam on a Lucite light pipe as shown. The light pipe in turn is cemented to the face of the RCA 5819 photomultiplier, again using Canada balsam. The crystal, light pipe and top of the photomultiplier tube are then wrapped with an aluminum foil, 0.00025 inch thick, which serves as a light reflector.

In studying the spectral distribution of negatrons or positrons resulting from short-lived beta-decay it is
Fig. 2. Scintillation spectrometer box showing source, crystal, photomultiplier tube and magnetic shield.
Fig. 3. Block diagram of scintillation spectrometer.
necessary to have some type of many-channel pulse height sorter to eliminate the difficult task of making decay corrections. Also, if possible, it would be well to incorporate into the equipment some means of measuring the half-life with reasonable accuracy.

The method chosen to achieve this end was the photographic recording of pulse heights which were displayed on an oscilloscope screen. The photographic record, which was in the form of a continuous strip, could then be read both in an amplitude and a time sense to determine the spectrum and decay time of the activity.

In order to record the data in this way the output pulses from the photomultiplier were treated in the following manner. Cathode follower coupling was made to an amplifier whose gain is variable from zero to fifty. The circuit diagram of the photomultiplier tube, preamplifier and amplifier is shown in Figure 4 and that of the regulated power supply for the photomultiplier tube is shown in Figure 5. This power supply is of the same type as used by Palmer (13). The output of the amplifier is then sent to two pulse shaping circuits for which Figure 6 represents a composite diagram. These circuits comprise a modification of the Watkins circuit (14), which was originally used for a somewhat different application. The first circuit consists of a peak reading voltmeter whose output is delay line shorted three microseconds
Fig. 4. Circuit diagram of photomultiplier tube, preamplifier and amplifier.
Fig. 5. Circuit diagram of regulated power supply for photomultiplier tube.
Fig. 6. Schematic diagram of modified Watkins circuit.
after the incoming pulse has risen to its maximum value. This produces a flat-topped pulse whose amplitude is proportional to that of the incoming pulse. By use of a conventional Los Alamos Model 1000 amplifier this flat-topped pulse is applied to the vertical deflection plates of a 5CP11A cathode ray oscilloscope tube. The second shaping circuit produces a one-microsecond gating pulse one microsecond after the flat-topped pulse has risen. This gating pulse is applied to the cathode of the cathode ray tube. If the cathode ray tube is biased nearly to cutoff and if no sweep is applied, the result will be a dot whose height above the baseline will be proportional to the amplitude of an incoming signal.

The dots obtained from the data in the manner indicated in the preceding paragraph are then recorded photographically together with a neon bulb timing marker. A Dumont oscilloscope camera, Type 314A, is used. A sample of the film record thus obtained is shown in Figure 7. The base line is the solid line and the timing marker is the dashed line, while the data are in the form of dots. On projection in a Flofilm microfilm reading projector, the data may be read both as a function of amplitude and as a function of time. One may take as many energy intervals as the dot size permits. For the dots shown in the sample strip, approximately 25 intervals may be taken. It is possible to vary the dot size if necessary by adjustment of the cathode ray tube voltages.
Fig. 8. Spectrum of Ca$^{137}$ used in obtaining resolution of instrument.

Fig. 7. Sample of data on 35-mm film record.
Since 75 dots per running inch of film may easily be accommodated, a spectrum of 20,000 counts may be obtained on 25 feet of 35-mm film. The film cost is then not prohibitive. The total error in reproduction from oscilloscope screen to viewer screen is less than 2 per cent of full scale and is governed primarily by imperfect film tracking in the microfilm viewer.

C. Procedure

After preparation and irradiation of the samples, the procedure is essentially the same for all of the studies made. Hence these procedures will be discussed first. The long-lived emitters, Cs\(^{137}\), P\(^{32}\), Sr\(^{90}\)–Y\(^{90}\), used in the preliminary tests of the equipment were prepared as deposits on thin Zapon film in the manner customary in conventional beta-spectroscopy. These sources were observed by placing them 1/8 inch above the aluminum foil reflector covering the anthracene crystal.

Praseodymium in the form of powdered praseodymium oxide (Pr\(_{6}O_{11}\)) was irradiated for eight minutes in a test tube. It was left as a powder, sprinkled onto Scotch tape and inverted on the aluminum foil reflector for observation.

Iron was prepared as follows. Strips of spectroscopically pure iron, 0.001 inch thick, were irradiated eighteen minutes.
The strips of iron foil were placed directly on the aluminum reflector foil for counting.

The sodium activity was obtained by irradiating sodium iodide. The sodium iodide was ground to a fine powder and mixed with a small amount of Zapon which acted as a binder. The bottom of the 7/8 inch hole in the source holder was covered with strong paper, 0.00025 inch thick, and the mixture pressed into it. The source thickness in this and in all other "powder-Zapon" preparations was approximately 0.1 mm. This in general amounts to a weight of 20 to 30 mg per square cm. The source is then covered with Scotch tape to add strength. It should be noted that for transit times of one-tenth second for four feet of tubing rather large acceleration and deceleration forces on the sample are involved. For this reason the criteria for a "thin" source had to be relaxed somewhat in order to make the source strong enough for the transit.

In all short irradiations the background activity due to the source holder was at least a factor of 50 below the measured activity. This activation was almost entirely attributable to the two-minute oxygen which has an endpoint of 1.7 Mev. Thus, besides being of negligible quantity, the energy is considerably below the lowest endpoint investigated by the present technique. Apparently the carbon activations are not strong for these irradiation times. Nickel was
particularly chosen as the source holder material because it did not activate noticeably. The preparations of the other activities which required use of the fast pneumatic tube are listed in Table 1.

Table 1. Sample Preparations for Short-Lived Activities

<table>
<thead>
<tr>
<th>Compound</th>
<th>Preparation</th>
<th>Irradiation Time (seconds)</th>
<th>Expected Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>NaI</td>
<td>Powder-Zapon</td>
<td>20</td>
<td>Na$^{21}$</td>
</tr>
<tr>
<td>Mg-metal</td>
<td>foil</td>
<td>15</td>
<td>Mg$^{23}$</td>
</tr>
<tr>
<td>Al-metal</td>
<td>foil</td>
<td>20</td>
<td>Al$^{25-26}$</td>
</tr>
<tr>
<td>SiO$_2$</td>
<td>Powder-Zapon</td>
<td>5</td>
<td>Si$^{27}$</td>
</tr>
<tr>
<td>S</td>
<td>Powder-Zapon</td>
<td>5</td>
<td>S$^{31}$</td>
</tr>
<tr>
<td>NH$_4$Cl</td>
<td>Powder-Zapon</td>
<td>3</td>
<td>Cl$^{33}$</td>
</tr>
<tr>
<td>KI</td>
<td>Powder-Zapon</td>
<td>2</td>
<td>K$^{37}$</td>
</tr>
<tr>
<td>CaF</td>
<td>Powder-Zapon</td>
<td>1.5</td>
<td>Ca$^{39}$</td>
</tr>
</tbody>
</table>

After irradiation the various samples are shot through the pneumatic tube to the spectrometer and data collected, photographed, and subsequently read with a microfilm viewer as described in the preceding section. Two sets of data result. One gives the number of decays detected in each chosen interval of time and the other gives the amplitude spectrum of the photomultiplier pulses. Approximately 15,000 to 20,000 counts are obtained in each spectrum. The decay data may in general be plotted directly, no dead time
corrections being necessary. The half-life is determined by making a weighted least square determination, using the total number of counts in each time interval as a weight factor. Probable errors are calculated using the methods discussed by A. G. Worthing and J. Geffner (15).

The spectral distributions are first treated to take into account the finite resolution of the spectrometer. Calculations of this type of corrections have been made by Palmer and Laslett (16). End corrections are applicable to the high energy end of the spectrum and take into account the resolution effect, namely that more counts are recorded at and near the apparent endpoint than exist in the true spectrum. These corrections are shown in graphical form in Figure 9 which was taken from reference (16). The abscissa expresses the ratio of the distance below the apparent endpoint divided by the full width at half maximum of the resolution curve at the apparent endpoint. Thus \( \varepsilon = 0 \) at the apparent endpoint and \( \varepsilon = 1 \) at a distance \( R \) below the apparent endpoint. The ordinate is the ratio of true corrected counts divided by the number of counts obtained experimentally.

The body correction, which involves the rest of the spectrum, is given as a formula at the bottom of Figure 9. This correction is seen to take account of the slope and curvature of the spectrum. It is obtained by use of the first three terms of a Taylor's expansion in the solution of the
Results of resolution corrections of Palmer and Laslett.

\[ N_T(E) = \frac{1}{1-\alpha} \left[ N_E(V) - \kappa \kappa V N_E(V) - \left( \frac{2}{2} \right) V N_E''(V) \right] \]

**BODY CORRECTION FORMULA**

Fig. 9. Results of resolution corrections of Palmer and Laslett.
integral equation relating the experimental spectrum to the resolution or transmission of the instrument and the true spectrum. Use is made of the fact that the resolution goes inversely as the square root of the kinetic energy (11). In the formula given, K and k are proportionality constants where \( E = k V \) and \( k E_0 \) represents the second moment of the normalized pulse height distribution taken with respect to the mean. \( E \) is the energy in Mev and \( V \) is the pulse height in arbitrary units. \( E_0 \) is the endpoint energy. \( N_T(E) \) is the number of true corrected counts at energy \( E \) and \( N_E(V) \) is the number of counts obtained experimentally of pulse height \( V \). Primes denote differentiation with respect to the argument. The body corrections in general contribute little toward changing the spectrum or the subsequent Fermi plots.

In using these corrections, one must, of course, know the full width at half maximum of the resolution curve of the instrument. This was obtained in the present investigation by observing the Cs\(^{137}\) conversion line which corresponds to an electron energy of 0.624 Mev. Since the resolution is known to go inversely with the square root of energy, it may be calculated at any given energy.

Resolution corrections have been applied to all data reported here.

It should be noted that examples of the effect of applying these corrections to the spectra are shown as solid
dots on a number of the graphs appearing later. After these corrections have been applied a Fermi plot may be made to determine the maximum energy. This is done in the conventional manner using the Bethe approximation (36),

$$F(Z, p) = \frac{2 \pi \psi^2}{1 - e^{2 \pi y}} \left[ \frac{(1 + p^2)(1 + 4\gamma^2) - 1}{4} \right]^s,$$

for obtaining the Fermi function,

where $Z$ is the atomic number of the resultant nuclei

$p$ is the momentum of the emitted electrons in units of $mc$

$$y = \frac{\gamma(1 + p^2)^{1/2}}{p}$$

$$s = (1 - \gamma^2)^{1/2} - 1$$

$$\gamma = \begin{cases} + Z \propto = + Z/137 & \text{for negatron emission} \\ - Z \propto = - Z/137 & \text{for positron emission} \end{cases}$$

In contrast to data obtained with magnetic spectrometers, the counting rate in the present work is obtained directly as a function of the total electron energy, $W$. The number, $N(W)$, of electrons obtained per unit energy interval is related to the number $N(p)$ per unit momentum interval by the relation

$$N(W) = N(p) \frac{dp}{dW} = \frac{W}{p} N(p),$$

where the derivative $dp/dW$ is evaluated by differentiation of the identity $W^2 = p^2 + 1$. It is accordingly appropriate to construct a Fermi plot with

$$\frac{pN(W)}{F(Z, p)}$$

as ordinate and $W$ as abscissa.

Energy calibration of the equipment was achieved by observing the endpoints of $Y^{90}$ and $Cl^{37}$. Yttrium$^{90}$ in
equilibrium with strontium$^{90}$ was used for the $2.25 \pm 0.03$ Mev (17) endpoint. The Cl$^{34}$ was prepared by irradiating a test tube full of finely ground sodium chloride for twenty minutes and placing a thin layer of the powder on 0.00025 inch paper. The twenty-three-second sodium$^{21}$ activity was allowed to die out during a ten-minute period following the irradiation. The $4.45 \pm 0.11$ Mev endpoint (18) of Cl$^{34}$ was then observed.

D. Results

1. Instrumental resolution

An essential piece of information to be obtained concerning the spectrometer was a measure of its resolution, R is the full width at half maximum of the resolution curve. As was indicated in Section C, this was obtained by use of the conversion line Cs$^{137}$. The spectrum obtained for a $\frac{1}{2}$ inch anthracene crystal is shown in Figure 3. This yields a value of $2\mu$ per cent for R at the energy of this conversion line. A value of $4\mu$ per cent was similarly obtained for the 1 inch crystal used with the higher energy studies. All data throughout this investigation have been corrected for resolution.
2. Results with known beta-spectra

Check runs were made using the effectively long-lived activities of P$^{32}$ and Sr$^{90}$-Y$^{90}$. The spectrum and Fermi plot of the P$^{32}$ are shown in Figure 10 and Figure 11, respectively. In general, probable errors are shown for all data given where the error exceeds the dot size. Errors are not given in Fermi plots for points which are breaking away from the straight line at lower energies. By use of the Cs$^{137}$ conversion line as a calibration a value for the endpoint of 1.78\(\pm\)0.07 Mev was obtained. This is in satisfactory agreement with magnetic spectrometer results which give 1.72 Mev (19, 20). A conspicuous feature of the Fermi plot shown in Figure 11 is the manner in which it breaks away from the straight line at 50 to 60 per cent of the endpoint energy. This is believed to be attributable to the over-abundance of low energy counts resulting from the scattering out process discussed in Section B.

The spectrum of Sr$^{90}$-Y$^{90}$ is given in Figure 12. The conventional Fermi plot which is given by the solid circles in Figure 13 is obviously somewhat S-shaped about the straight line and does not break away from the straight line in the neighborhood of 50 to 60 per cent of the endpoint. This is presumably due to the first-forbidden character of the Y$^{90}$ decay, it being known from magnetic spectrometer work (17) that these nuclei do give an S-shaped character to the Fermi
Fig. 10. Negatron spectrum of phosphorus$^{32}$. 
Fig. 11. Fermi plot of phosphorus$^{32}$. 

Fermi Plot of Phosphorus$^{32}$
Fig. 12. Negatron spectrum of strontium$^{90}$-yttrium$^{90}$. 
Fig. 13. Conventional and a-corrected Fermi plots of strontium$^{90}$-yttrium$^{90}$. 
plot. The S-shape is apparently strong enough to overshadow even the over-supply of lower energy events. Application of the \( a = \sqrt{(W_0 - W)^2 + p^2} \) correction factor due to Konopinski (21) is known to produce a straight-line modified Fermi plot from the S-shaped first-forbidden plot of \( Y^{90} \). When this correction is applied, the open circles result. Near the endpoint these lie quite well on a straight line and break away at the usual 50 to 60 per cent. Thus, it is possible to detect first forbiddenness with this scintillation spectrometer. On the basis of the \( Cs^{137} \) calibration, the endpoint was found at \( 2.27 \pm 0.06 \) Mev, which is again in satisfactory agreement with magnetic spectrometer work (17).

3. Results with \( Pr^{140} \) and \( Fe^{53} \)

Having obtained reasonable results for the "well-known" spectra, it was felt that a somewhat less completely known activity could be investigated. Praseodymium\(^{140} \), which was known to activate strongly, was chosen. The decay curve and positron spectrum are shown in Figure 14 and Figure 15. The praseodymium was irradiated as praseodymium oxide (\( Pr_6O_{11} \)) for eight minutes. The half-lives observed were \( 3.4 \pm 0.1 \) minutes and \( 15 \pm 1 \) minute. The 3.4-minute activity (22, 23, 24) is undoubtedly due to \( Pr^{140} \). The 15-minute activity is unexplained. No energy spectrum was obtained for this 15-minute activity. No evidence for the presence of the expected
Fig. 14. Decay curve of Pr$_{6}O_{11}$ after 8 minutes irradiation.
Positron Spectrum of Praseodymium$^{140}$

Fig. 15. Positron spectrum of praseodymium$^{140}$. 
two-minute oxygen activity was observed. It is apparently masked by the strong praseodymium activation. The Fermi plot for Pr$^{140}$ is shown in Figure 16. An endpoint of $2.35 \pm 0.10$ Mev is obtained on the basis of the Cs$^{137}$ conversion line and the Y$^{90}$ endpoint. Three other groups of investigators (22, 23, 24), by use of absorption and cloud chamber measurements, give 2.4 Mev as the endpoint. The agreement with these previous results is satisfactory.

The decay curve, spectrum and Fermi plot of Fe$^{53}$ are shown in Figure 17, Figure 18 and Figure 19, respectively. The anticipated reactions of gamma -n on Fe$^{54}$ to give Fe$^{53}$ and gamma -p on Fe$^{57}$ to give the 2.59 hour Mn$^{56}$ were observed in decay studies. The Mn$^{56}$ appeared to be about 4 per cent of the total activity at the time the spectrum was taken. No other activity was found. The half-life of Fe$^{53}$ is given as 10 \pm 1 seconds from this study. The half-life has been studied by other investigators (25, 26) with 8.9 minutes reported. In noting the Fermi plot of Fe$^{53}$, it is seen to be straight line and curves upward at about 60 per cent of the endpoint. An endpoint of $2.6 \pm 0.1$ Mev is obtained. In this case, the energy calibration was made by utilizing the known endpoint of yttrium$^{90}$. Nelson and Pool (27) gave the endpoint as $2.8 \pm 0.1$ Mev.
Fig. 16. Fermi plot of praseodymium$^{140}$. 

Fermi Plot of Praseodymium$^{140}$

Kinetic Energy in Mev.

$\frac{\sqrt{N_{TP}}}{WF}$

2.35 Mev.
Fig. 17. Decay curve of iron after 18 minutes irradiation.
Fig. 18. Positron spectrum of iron$^{53}$. 

Positron Spectrum of Iron$^{53}$.
Fig. 19. Fermi plot of iron$^{53}$. 

FERMI PLOT OF 

Fe$^{53}$

\( \sqrt{ \frac{N_n}{e_F} } \)

KINETIC ENERGY IN MEV.
4. Results with nuclei of the $Z - N = 1$ type

In stating the results obtained from the remaining activities which are all of the $Z - N = 1$ group, only two need be given special consideration apart from the others. After sodium iodide was irradiated it gave the spectrum shown in Figure 20. The shape is obviously quite different from those obtained earlier from other decays. Sodium was also irradiated as a nitrate but with the same spectrum resulting. It was suspected that besides the lower energy Na$^{21}$ activity anticipated, there might have been produced a small amount of some other isotope yielding higher energy beta-particles. That this was the case becomes clear by a study of the decay curves shown in Figure 21. Here the upper data were obtained by taking only those pulses below an energy of 2.7 Mev, while the lower one resulted from taking only counts lying above 2.7 Mev. The Na$^{21}$ endpoint is expected to lie between 2.6 and 2.7 Mev. There were undoubtedly two activities involved - one with a half-life of $27 \pm 4$ seconds and the other of $9 \pm 3$ seconds. Fermi plots of these activities are shown in Figure 22. The endpoint of the shorter-lived, higher-energy activity is found to be $4.9 \pm 0.3$ Mev, while by subtraction that of the longer-lived is $2.5 \pm 0.3$ Mev. It is only possible to obtain a reliable subtraction of these activities because the higher energy group is activated so much less strongly
Fig. 20. Energy spectrum observed from sodium iodide after 23-second irradiation.
Sodium Iodide Decay
(Irradiated 23 Seconds)

Low energy component
$T_{1/2} = 27 \pm 4$ seconds

High energy component
$T_{1/2} = 9 \pm 3$ seconds

Fig. 21. Decay curves of sodium iodide after 23 seconds irradiation.
Fig. 22. Fermi plots of activities resulting from irradiation of sodium iodide.
than the lower energy group. If the two groups had been activated nearly equally, the subtraction would not have been valid since the breaking away from the straight line Fermi plot due to the scattering out process could easily have obscured the true endpoint of the lower energy group.

Fluorine$^{20}$ has a reported half-life of 12 seconds and an endpoint of 5.0 Mev (37, 38). On the basis of energy and half-life it seems reasonable to assign the short period to F$^{20}$ and the longer one to Na$^{21}$.

Aluminum should also be considered separately. When aluminum undergoes irradiation both gamma -n and gamma -2n reactions probably occur. Since the resultant nuclei Al$^{25}$ and Al$^{26}$ have comparable half-lives (7.3 and 6.3 seconds respectively) and endpoints (approximately 3 Mev), it is nearly impossible to separate accurate information concerning either from gamma irradiation of aluminum. The decay curve and spectrum of the aluminum activities are shown in Figure 23 and Figure 24, respectively. A half-life of 7.0 ± 1.0 seconds and an endpoint of 3.03 ± 0.10 Mev were obtained. It is safe to say that these are probably due to a composite of Al$^{25}$ and Al$^{26}$, but beyond this nothing further may be concluded from this investigation.

The results obtained from all the other of these $Z - N = 1$ nuclides are quite straightforward and these results are given in Table 2. Calibration was achieved by use of the
Fig. 23. Decay curve of aluminum foil after 20 seconds irradiation.

Positron Spectrum of $\text{Al}^{25}$ plus $\text{Al}^{26}$

Fig. 24. Positron spectrum of aluminum$^{25-26}$. 
endpoints of $^{90}Y$ and $^{34}Cl$. The probable error quoted for the endpoint determination could be decreased considerably if the endpoint of $^{34}Cl$ were known to greater accuracy. Spectra and Fermi plots of these calibration activities are given in Figure 25 and Figure 26.

Table 2. Endpoints and Half-Lives of Positron Emitters in the $Z - N = 1$ Class

<table>
<thead>
<tr>
<th>Emitter</th>
<th>Half-life (sec.)</th>
<th>Endpoint (MeV)</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{21}Na$</td>
<td>$27 \pm 4$</td>
<td>$2.5 \pm 0.3$</td>
<td>$20, 21, 22$</td>
</tr>
<tr>
<td>$^{23}Mg$</td>
<td>$12.3 \pm 0.4$</td>
<td>$2.99 \pm 0.10$</td>
<td>$27, 28, 29$</td>
</tr>
<tr>
<td>$^{25-26}Al$</td>
<td>$7.0 \pm 1.0$</td>
<td>$3.03 \pm 0.10$</td>
<td>$23, 24$</td>
</tr>
<tr>
<td>$^{27}Si$</td>
<td>$5.4 \pm 0.4$</td>
<td>$3.48 \pm 0.10$</td>
<td>$30, 31, 32$</td>
</tr>
<tr>
<td>$^{31}S$</td>
<td>$3.2 \pm 0.3$</td>
<td>$4.06 \pm 0.20$</td>
<td>$33, 34, 35$</td>
</tr>
<tr>
<td>$^{33}Cl$</td>
<td>$1.8 \pm 0.1$</td>
<td>$4.13 \pm 0.20$</td>
<td>$36, 37, 38$</td>
</tr>
<tr>
<td>$^{37}K$</td>
<td>$1.2 \pm 0.2$</td>
<td>$4.57 \pm 0.20$</td>
<td>$39, 40, 41$</td>
</tr>
<tr>
<td>$^{39}Ca$</td>
<td>$1.1 \pm 0.2$</td>
<td>$5.13 \pm 0.20$</td>
<td>$42, 43, 44$</td>
</tr>
</tbody>
</table>

*Obtained by subtraction.

**Inconclusive (mixed activities).
Fig. 25. Spectra of Cl$_{34}^+$ and $\gamma^{90}$ for energy calibration of instrument.
Fig. 26. Fermi plots of Cl$^{34}$ and Y$^{90}$ for energy calibration of instrument.
Magnesium Decay

$T_{1/2} = 12.3 \pm 0.4$ seconds

Fig. 27. Decay curve of magnesium after 15 seconds irradiation.
Positron Spectrum of Magnesium$^{23}$

Solid dots indicate result of resolution correction

Fig. 28. Positron spectrum of magnesium$^{23}$. 
Fermi Plot of Magnesium$^{23}$

Fig. 29. Fermi plot of magnesium$^{23}$. 
Silicon Dioxide Decay

Time in Seconds

Counts per Second

T = 5.4 ± 0.4 seconds

Fig. 30. Decay curve of silicon dioxide after 5 seconds irradiation.
Positron Spectrum of Silicon$^{27}$

Solid dots indicate result of resolution correction.

Fig. 31. Positron spectrum of silicon$^{27}$. 
Fig. 32. Fermi plot of silicon$^{27}$. 

Fermi Plot of Silicon$^{27}$
Fig. 33. Decay curve of sulfur after 5 seconds irradiation.
Solid dots indicate result of resolution correction

Fig. 34. Positron spectrum of sulfur$^{31}$. 
Fig. 35. Fermi plot of sulfur$^{31}$. 
Fig. 36. Decay curve of ammonium chloride after 3 seconds irradiation.
Position Spectrum of Chlorine$^{33}$

Solid dots indicate result and resolution correction.

Fig. 37. Positron spectrum of chlorine$^{33}$. 
Fig. 38. Fermi plot of chlorine$^{33}$. 
Potassium Iodide Decay

Fig. 39. Decay curve of potassium iodide after 2 seconds irradiation.
POSITRON SPECTRUM OF POTASSIUM$^{37}$

Solid dots indicate result of resolution correction.

Fig. 40. Positron spectrum of potassium$^{37}$. 
Fig. 41. Fermi plot of potassium$^{37}$. 
Calcium Fluoride Decay

$T_{1/2} = 1.1 \pm 0.2$ seconds

Fig. 42. Decay curve of calcium fluoride after 1.5 seconds irradiation.
Fig. 13. Positron spectrum of calcium$^{39}$. 

Positron Spectrum of Calcium$^{39}$

Solid dots indicate result of resolution correction.
Fermi Plot of Calcium$^{39}$

Fig. 14. Fermi plot of calcium$^{39}$. 

5.13 Mev.
III. DISCUSSION

No further discussion will be given the $^{32}$P, $^{90}$Y, Fe$^{53}$ and Fe$^{140}$ activities since the data concerning these were of a preliminary nature and were discussed at some length in the previous section.

Table 3 and Table 4 compare the values obtained for half-lives and endpoints in this investigation with those previously known.

Table 3. Comparison of Present Results on $Z - N = 1$ Activities with Previously Known Values of Half-Life

<table>
<thead>
<tr>
<th>Nuclei</th>
<th>This Investigation $T_{1/2}$ (sec.)</th>
<th>Previous Investigations $T_{1/2}$ (sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Na$^{21}$</td>
<td>$27 \pm 4$</td>
<td>$23 \pm 2$</td>
</tr>
<tr>
<td>Mg$^{23}$</td>
<td>$12.3 \pm 0.4$</td>
<td>$11.6 \pm 0.5$</td>
</tr>
<tr>
<td>Si$^{27}$</td>
<td>$5.4 \pm 0.4$</td>
<td>$4.9 \pm 1$</td>
</tr>
<tr>
<td>S$^{31}$</td>
<td>$3.2 \pm 0.3$</td>
<td>$3.2 \pm 0.2$</td>
</tr>
<tr>
<td>Cl$^{33}$</td>
<td>$1.8 \pm 0.1$</td>
<td>$2.4 \pm 0.2$</td>
</tr>
<tr>
<td>K$^{37}$</td>
<td>$1.2 \pm 0.2$</td>
<td>$1.3 \pm 0.1$</td>
</tr>
<tr>
<td>Ca$^{39}$</td>
<td>$1.1 \pm 0.2$</td>
<td>$1.06 \pm 0.03$</td>
</tr>
</tbody>
</table>
Table 4. Comparison of Present Results on $Z - N = 1$ Activities with Previously Known Values of Endpoint

<table>
<thead>
<tr>
<th>Nuclei</th>
<th>This Investigation</th>
<th>Previous Investigations</th>
<th>Calc.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_{\text{max}}$ (Mev)</td>
<td>$E_{\text{max}}$ (Mev)</td>
<td>$E_{\text{max}}$ (Mev)</td>
</tr>
<tr>
<td>Na$^{21}$</td>
<td>$2.53 \pm 0.15$</td>
<td>None</td>
<td>2.63</td>
</tr>
<tr>
<td>Mg$^{23}$</td>
<td>$2.99 \pm 0.10$</td>
<td>2.82</td>
<td>(7)</td>
</tr>
<tr>
<td>Si$^{27}$</td>
<td>$3.48 \pm 0.10$</td>
<td>$3.74$</td>
<td>(31)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$3.54 \pm 0.1$</td>
<td>(28)</td>
</tr>
<tr>
<td>S$^{31}$</td>
<td>$4.06 \pm 0.20$</td>
<td>$3.85 \pm 0.07$</td>
<td>(8)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$3.87 \pm 0.15$</td>
<td>(9)</td>
</tr>
<tr>
<td>Cl$^{33}$</td>
<td>$4.43 \pm 0.20$</td>
<td>$4.13 \pm 0.07$</td>
<td>(8)</td>
</tr>
<tr>
<td>K$^{37}$</td>
<td>$4.57 \pm 0.20$</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>Ca$^{39}$</td>
<td>$5.13 \pm 0.20$</td>
<td>None</td>
<td>None</td>
</tr>
</tbody>
</table>

Numbers in parentheses indicate references to literature.

The calculated values of $E_{\text{max}}$ were determined according to the equation

$$E_{\text{max}} = C \left[ (A - 1)/A^{1/2} \right] + 1.80 \text{ Mev}$$

which was discussed earlier. The constant $C$ was determined as $0.61 \pm 0.03$ Mev by plotting the value of $E_{\text{max}}$ obtained experimentally (both in this and in other work) vs. $(A - 1)/A^{1/2}$ and finding the slope of the best fitting straight line with an intercept of 1.80 Mev (See Figure 45). The value of $0.61 \pm 0.03$ is comparable to the 0.60 found by one other group of investigators (8) and the 0.592 used by another group (9). The latter determination was made on the basis of only one
Fig. 45. Graph of maximum positron energy ($E_{\text{max}}$) vs. $(A - 1)/A^{1/3}$, where $A$ is the atomic mass number.
endpoint, however. A value of $1.40 \times 10^{-13}$ cm. is indicated for $r_0$ by the 0.61 value of $C$.

As a whole, the data obtained here agree fairly well with the work of others. Again it should be pointed out that the large probable errors are to a considerable extent due to the uncertainty of the $\alpha_\alpha^{34}$ endpoint. Over-emphasis must not be placed on the agreement between calculated and experimental values in this case since $C$ is determined by best fit. Reference to Figure 45 will illustrate the difficulty in obtaining a precise value of $C$ from the experimental results (in order to calculate a value for $E_{\text{max}}$ for comparison with the original experimental results).

Earlier it was mentioned that the fifth root of the half-life should be inversely proportion to the maximum energy. A plot of $(T_3^{1/5})$ vs $W_0$ is shown in Figure 46. The data from this experiment indicate that the rule may be roughly obeyed. A value for the constant of proportionality of 10.9 is obtained.

Any theory of beta-decay predicts that all of these $Z - N = 1$ activities should have fairly constant $ft$ values. $ft$ values have been calculated independently and using the Feenberg and Trigg curves (35) for all activities investigated here and the results are given in Table 5. The $ft$ value depends essentially on the fifth power of $E_{\text{max}}$. 
Fig. 46: Graph of inverse of fifth root of half-life vs. maximum positron energy ($E_{\text{max}}$).
Table 5. ft Values Found in This Investigation of $Z = N = 1$

<table>
<thead>
<tr>
<th>Nuclei</th>
<th>ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Na$^{21}$</td>
<td>1460 ± 1300</td>
</tr>
<tr>
<td>Mg$^{23}$</td>
<td>4670 ± 900</td>
</tr>
<tr>
<td>Si$^{27}$</td>
<td>3740 ± 700</td>
</tr>
<tr>
<td>S$^{31}$</td>
<td>4320 ± 1200</td>
</tr>
<tr>
<td>Cl$^{33}$</td>
<td>3440 ± 900</td>
</tr>
<tr>
<td>K$^{37}$</td>
<td>2700 ± 1000</td>
</tr>
<tr>
<td>Ca$^{39}$</td>
<td>3990 ± 1300</td>
</tr>
</tbody>
</table>

and linearly on the half-life. Thus small errors of either $E_{\text{max}}$ or $T_1$ could change ft considerably. The ft value of K$^{37}$ does appear to be somewhat low.
IV. CONCLUSIONS

1. Conclusions regarding observed endpoints, half-lives and f values are given in the last chapter (Tables 3, 4 and 5). In addition, an endpoint for Fe$^{53}$ was obtained as $2.6 \pm 0.1$ Mev.

2. The Wigner theory of the $Z - N = 1$ nuclei is confirmed in these studies with a proportionality constant in the equation $E_{\text{max}} = C \left[ (A - 1)/A^{1/2} \right] - 1.80$ of $C = 0.61 \pm 0.03$ Mev resulting.

3. f values for these nuclei appear to be reasonably constant. Only $K^{39}$ seems to depart appreciably from the others.

4. An attempt should again be made in the future to observe gamma rays from the $Z - N = 1$ nuclei. This can conveniently be done by preparing thicker sources to give more counts and then interposing absorbers between the sources and a NaI (TlI) scintillation crystal sufficient to stop the emitted positrons. This type of crystal is quite efficient for stopping gamma rays. One should then observe the annihilation radiation of the positron plus any other gamma rays present.
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VI. ACKNOWLEDGMENTS

The author wishes to express his deepest gratitude to Prof. L. Jackson Laslett for many helpful discussions and help with calculations; to Dr. D. J. Zaffarano for many hours spent in helping with the experiment and in counting data; to Messrs. A. A. Reed, Clarence Harper and W. Reinhardt of the electronics shop for help in setting up the equipment; to Mr. E. H. Dewell for preparation of samples; and to the entire synchrotron group for their great assistance with this work.

It is also a pleasure to acknowledge the support given this work by the Socony-Vacuum Corporation in the form of a research fellowship. Equipment for this work was provided by the Ames laboratory of the AEC.