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Nuclear disintegration studies with a beta-ray spectrometer

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NUCLEAR DISINTEGRATION STUDIES WITH A BETA-RAY SPECTROMETER

by

William W. Pratt

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
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I. INTRODUCTION

The use of the beta-ray spectrometer, in the analysis of nuclear decay schemes, has been discussed by several authors (1-3). The primary use of the instrument consists in the determination of energies of beta-rays, internal conversion electrons, and of gamma-radiation through the analysis of the secondary electrons produced in a radiating foil. In addition to the determination of energy, which is discussed in detail in references 1-3, it is possible to estimate the relative intensities of the various types of radiation studied. A further extension of the application of the spectrometer consists in the introduction of a secondary detector of radiation, and of the measurement of coincidences between the electrons selected by the spectrometer and the radiations detected by the secondary detector. The three types of measurement mentioned above, if carried out with sufficient precision, permit the decay scheme of the radioactive isotope under consideration to be determined. The work reported here will be concerned with three particular features of the application of the spectrometer to the determination of nuclear decay schemes. The instrument described in this work is a magnetic focusing spectrometer of the thin lens type. Its construction features have been described in detail by Jensen (3). In section III a method of improving the performance of the instrument, through the use of a new type of baffle system, will be described. In section IV the possibilities and limitations of the
determination of relative intensities will be considered. In section V the application of the spectrometer to the method of coincidence counting will be described.
II. REVIEW OF THE LITERATURE

The principles involved in the use of the magnetic lens spectrometer for the determination of electron energies have been discussed in detail in references 1-3. It is shown in these references that, through the use of an appropriate baffle system, and a suitable magnetic field, electrons with momenta in a small range of the order of 1-2% may be selected and "focused" on the aperture of a Geiger counter, or other suitable detector. Electrons of other momenta are caused to strike one of the baffles, and are thus prevented from reaching the counter. Both Siegbahn (1) and Deutsch, Elliott and Evans (2) have considered the relation between resolution of the instrument and transmitted electron intensity. Jensen (3) has considered the effect of the momentum loss in the source on the determination of the electron energy from the observed focusing current. Deutsch, Elliott and Evans (2) have proposed a semi-empirical method for the determination of the relative intensities of gamma-rays.

The existence of ring focusing (section III) in spectrometers of the solenoid type was first proposed by Witcher (4). Recently it was suggested by Frankel (5) that this type of focusing may also exist in spectrometers of the thin lens type. This problem has been considered theoretically by Keller, Keenigseberg and Faskin (6,7). They predicted an increase by a factor of two in intensity, for a given resolution,
The other work by other authors will be cited in the report of this

some transactions

these two groups to determine differences between

these groups. If it is possible to use a spectrometer separate steps

be correctly detected by the method of counterclockwise

several decay schemes of the method of counterclockwise

of Gamow-Gutenbergs, Sheehan and Johnson (12) have determined

and along the two lines or the trajectories of the electrons of the

and along the two lines or the trajectories of the electrons of the

counterclockwise in the process of making such studies.

counter deeply enough, and have indicated how it is possible to determine

have detected counterclockwise in the study of general part

(11,12) reports and others. In counterclockwise experiment,

has considered a possible approach to the statistical problem involved

(10) has considered a possible approach to the statistical problem involved

does not. (6) has been treated in a general way by Hille of

the application of counterclockwise counting in the analysis of neutron

stated counterclockwise.

the use of fine focus. Moreover, (9) has considered the same
The position of any aperture at the position of the 
object, as determined by the use of an appropriate aperture, is the position of the
image of the aperture. The image of the aperture is determined by observation of the
position of the aperture, which is made to coincide with the aperture in the
image of the aperture. The aperture is then moved in a plane between the source and
the aperture, which is then moved in a plane between the source and the aperture.

I. Introduction

II. PAGE NUMBER
Fig. 1. Spectrometer baffle system for axial focusing.
solid angle $\Lambda$, and these electrons are focused on the counter by the magnetic field. The counter window diameter is made equal to the diameter of the electron beam. To observe the existence of ring focusing, photographic films were placed in the spectrometer in a plane perpendicular to the axis, as shown by the dotted line C in Fig. 1. The baffle A was removed, and reinserted at the source end of the spectrometer, as shown by the dotted lines A'. The baffle D, which does not come in contact with the focused electron beam, but is for the purpose of eliminating scattering, was removed. The electron beam, defined by the trajectories a and b, was then limited by the baffles A' and B. A thin source of Cs$^{137}$ was inserted at the source end of the spectrometer, and the coil current necessary to focus the $K$ internal conversion line of the 0.66 MeV gamma-ray on an 0.82 cm diameter counter aperture was determined. The film C was then placed in position and exposed for 19 hours, with the above determined current through the coil. Exposures were made with films at various distances $z_2$ from the counter plane. It was found that with $z_2 = 20$ cm a ring image, corresponding to the focused $K$ line, was obtained which was narrower than the images for $z_2$ greater or less than 20 cm. It is estimated that the position $z_2$ corresponding to the narrowest ring can be found by this method to within $\pm 2$ cm. The ring obtained at 20 cm is shown in Fig. 2, which also shows a ring of larger diameter corresponding to the higher energy $L$ line. The inner and outer diameters of the $K$ ring are 5.2 cm and 6.1 cm respectively. In this experiment $z_1$ (Fig. 1) was taken as 30.27 cm, which was estimated to correspond
Fig. 2 Photographic image of electron beam from $^{137}$Cs source
to the same solid angle as had been used in most of the previous work with axial focusing.

G. Baffle Design

After thus locating the position, mean diameter, and width of the narrowest part of the focused electron beam, a baffle system was constructed to make use of this constriction in the beam. The baffle system used for ring focusing is shown in Fig. 3. The fixed baffle C has a central circular aperture of mean diameter 6.10 cm. The plane of the center of this baffle is placed 20 cm from the plane of the counter window. The movable baffle C also has a mean diameter of 6.10 cm, and is attached to an 0.25 inch rod, extending through the end of the spectrometer tube, by means of which the baffle may be adjusted axially along the tube. Similarly, baffle A' may be adjusted axially along the tube to limit the electron solid angle.

With this system of baffles it is possible to vary the location of A' and C as well as the counter window diameter, and in this way to vary the resolution of the instrument, and simultaneously the accepted solid angle. It is of course desirable to determine the optimum relation between the variable parameters, so that for a given resolution the largest possible solid angle may be obtained. This was done in the work described here by determining a large number of line shape curves with the spectrometer, corresponding to various combinations of the three variable parameters.
Fig. 3. Spectrometer baffle system for ring focusing.
Before considering the results of these measurements, it will be of interest to consider two problems concerning the design and insertion of the ring baffles C and C'.

The orbit calculations made by the theoretical group in this laboratory (7), which were completed before this baffle was designed, showed that in the plane of constriction of the beam the trajectory with the largest diameter corresponds to a particular angle of emergence of the electrons from the source with respect to the spectrometer axis. This angle is found to be approximately 8.5°. Since, in making the photographic record of the trajectories, the angles selected by the baffles A' and B were 7° and 10° respectively, this trajectory of greatest ring diameter was included in the photographic image. Thus the baffle C', with a diameter equal to the outer diameter of the photographic ring image, will not cut out part of the beam regardless of the setting of the solid angle baffle A'. It will, in fact, coincide with the outer edge of the beam whenever baffle A' subtends an angle at the source less than 8.5°.

Due to the small width of the electron beam in the region of constriction, and the consequently narrow annulus used between baffles C and C', it will be recognized that the alignment of C and C' with respect to the spectrometer axis is critical. If these baffles are not located centrally with respect to the spectrometer axis, then mono-energetic electrons, emitted at different azimuthal angles, will require different focusing currents. This will result in an increase in the half-width of the spectral line. In order to align these baffles
properly they were constructed on a common assembly which could be inserted into the spectrometer as a single unit. After inserting this unit, four line shape curves were obtained, using the photoelectron line from K conversion in lead due to the 1.3 Mev gamma-ray of Co$^{60}$. For each line shape curve, electrons entering only one quadrant of the focusing region were used; the other three quadrants were masked out by the use of a sector baffle inserted for this particular experiment. This baffle was made of a disc of 1.27 cm micarta with one quadrant cut out, and was inserted in a plane perpendicular to the spectrometer axis at a distance of 12.3 cm from the source. A failure to align the ring baffles C and C' in the center of the spectrometer will cause the spectral lines obtained with different azimuthal orientations of the sector baffle to occur at different values of coil current. The magnitude of this effect may be estimated by the use of the thin lens formula (reference 2). The result is

\[
\frac{\delta I}{I} = \frac{ba}{a(2f-b)}
\]

where \(\delta I/I\) is the fractional shift in the peak with opposite sectors open, \(a\) is the mean diameter of the ring baffles, \(b\) is the distance from the ring baffles to the counter position, \(f\) is the focal length of the magnetic lens, and \(e\) is the amount by which the center of the ring baffle is displaced from the spectrometer axis. In the present case, \(b = 20\) cm, \(a = 6.1\) cm and \(f = 25\) cm. Thus

\[e = 9.15 \ \delta I/I\ \text{cm.}\]
the center of particle c and the center of particle
time (reference ?) to be about 0.15. It is the distance in com between
in the position of the time particles have been determined by the coordinates-
which is defined by $r = (z - E_0)$. The mean square of the coordinates

to express the position of particle c in terms of an effective point
de-centered by the coordinates (reference ?). It will then be determined
as will also be determined by the integral of the effective point as a function of the integral
of the effective point to express the position of particle c in terms of the
source diameter of 0.5 cm was used throughout. It will be more
particle a point, particle b point, and particle effective diameter.

These curves correspond to different effective points of the three parameters.

The monochromatic electrons from the important conversion A time of the
aspects measured axis. So the shape curves were determined, using the
when the time particles have been adjusted with their centers on the

D. EXPERIMENTAL RESULTS

is of the order of 0.15

that the conversion is to the half-thickness of the spectrometer
ment of the peaks has been reduced to about 0.2%, and thus it is es-
attenuation of the time particle, It is seen that the attenuation depends
in $t = 0$, $t = 1/2$, $t = 1/2$, and $t = 1/2$, $t = 1/2$, $t = 1/2$. In the present case, with $t = 1/2$, the results
of the spectrometer time, due to parallel measurement, is estimated to be
of $r = 0.2"$ may be detected. The width of the half-thickness
of the limits of detection of 0.1% is of the order of 0.2% so that values

-17-
Fig. 4. Ring baffle alignment test. Peaks at the left were taken before centering the baffle and peaks at the right were taken after centering the baffle. The open sector is indicated for each peak by the letters N, S, E and W. The amount by which the baffle is estimated to be off center is indicated by the numbers beside each pair of peaks.
Fig. 5 shows the results obtained using a counter with a 2.7 cm diameter aperture, which was the largest counter aperture available. The half-width of the spectral line is plotted as a function of the maximum counting rate at the peak of the line shape curve. The intensity is plotted on an arbitrary scale, and includes corrections for source decay and for the relative strengths of the several ThB sources used. For each curve in Fig. 5 the intensity is varied by varying the ring width R. Each curve is for a different solid angle $\phi$ measured in per cent of $4\pi$. It is seen from Fig. 5 that for a given half-width the maximum intensity is obtained by a proper choice of both $\phi$ and R. Thus the optimum operating curve is determined by the envelope of the curves in Fig. 5; and the parameters $\phi$ and R are uniquely related to the desired half-width by this envelope. It is seen, however, that the 0.59% solid angle setting gives results which differ very little from the optimum conditions, for half-widths greater than about 1.6%. Since for half-widths below 1.6% a transmitted intensity is obtained which is usually too low to be of interest, in practice it is found convenient to leave the solid angle baffle A' fixed and to select the desired resolution by varying only the ring width R. It is evident from Fig. 5 that a larger solid angle than 0.59% would be expected to give somewhat better intensity at half-widths of the order of 2% or greater. However, since some further modifications of the instrument would be required to achieve this increased solid angle, and since in any event the improvement obtained would not be expected to be very great, this has not as yet been done.
Fig. 5. Transmission-resolution relation as a function of solid angle and ring width. The point A represents an axial focusing case.
Line shape curves using other counter aperture diameters showed the same general features as those represented in Fig. 5. In each case the solid angle setting of 0.59° gave the best intensity for a given half-width, except in the region of excessively low intensity. Fig. 6 shows the results for different counter aperture diameters. The solid angle baffle $A'$ is set in each case to give $\alpha = 0.59°$ and the variable parameter in each curve is the ring width $R$.

As in the previous curves, an optimum relation is indicated at the lower values of intensity; that is, for a given resolution, the maximum intensity is obtained by a proper choice of both ring width and counter aperture diameter. However, in the intensity region of primary interest, the 2.7 cm counter gives the best results. Additional measurements have shown that the counter aperture diameter may be reduced slightly below this value without appreciable effect. This indicates, in agreement with the calculations (reference 7) that the image of the source formed at the counter position is of the order of 2.7 cm diameter, and thus that the use of a larger counter would not give improved intensity.

To determine the improvement effected by using ring focusing rather than axial focusing, a photoelectron source of Co$^{60}$ was inserted into the modified spectrometer; and the peak due to the 1.3 Mev gamma-ray was determined. The baffles were adjusted to give the same half-width of 2.5° as had been obtained when this same source was measured under axial focusing conditions. After correcting for source decay, the ratio of peak intensity of the line using ring focusing to that using axial focusing was found to be 2.0. The point $A$ on Fig. 5 represents
Fig. 6. Transmission-Resolution relation as a function of counter diameter and ring width.
the experimental point for axial focusing. It appears therefore that an improvement of the order of a factor of two in the intensity, at a half width of 2.5%, may be obtained by using ring focusing rather than axial focusing in a thin lens spectrometer of the type discussed here.

A question arises as to the choice of the source size to be used. It will be expected in general that in order to obtain the maximum intensity for a given line half-width, there will be an optimum source size to be used, which will depend on the half-width desired. A simple analysis, based on the assumption of vectorial addition of half-widths due to independent effects (which is valid in the case of Gaussian functions), indicates that the source size of 0.5 cm diameter is approximately optimum in the region of 2% to 3% half-width; and that the ratio of intensity to half-width is not particularly sensitive to small deviations from the optimum value.
IV. INTENSITY MEASUREMENTS

A. Spectrometer Efficiency

It will be of interest to consider a quantity which may be designated as the efficiency of the spectrometer and will be denoted by the symbol \( e_s \). The efficiency will be defined as the ratio of the counting rate observed in the spectrometer counter to the rate of emission by the source of the particles, or quanta, investigated. Three cases will be considered: The detection of internal conversion electrons, the detection of beta-rays, and the detection of gamma-rays by means of photoelectric conversion in a radiator. In the cases of internal conversion electrons and of photoelectrons due to gamma-rays, where spectral peaks are to be expected, the spectrometer efficiency will be understood to refer to the counting rate at the peak of the spectral line. In the case of beta-rays, where no such spectral line appears, the efficiency will refer to any arbitrarily selected portion of the spectrum, and will thus be a function of the focusing current.

The following considerations will apply in each of these cases. Consider a source of monoenergetic electrons of momentum \( p \). The probability that one of these electrons will be focused on the counter aperture will in general depend upon the momentum \( p \), the focusing current \( I \), the angle of emission \( \psi \) (defined as the angle between the direction of emission of the electron and the spectrometer axis), the point on the source at which the electron originated, and the angular momentum of
the electron. In order to obtain an expression for the spectrometer efficiency, the following simplifying assumptions concerning the focusing action of the spectrometer will be made.

It will be assumed that the source is sufficiently small that it may be regarded as a point. It will furthermore be assumed that the baffle system is equivalent to one in which electrons in a small region of emission angles, defined by

$$\psi_1 < \psi < \psi_2$$

are selected sufficiently close to the source that the angles selected are not influenced by the magnetic field of the spectrometer; and it will be assumed that with the proper focusing current all of these electrons are focused on the counter. The work of Keller, Koenigsberg and Paskin (7) indicates that these approximations are reasonable for the purpose of a rough calculation.

If these approximations are used, and if it is recalled that for a magnetic field of a given form the trajectory of an electron with given initial conditions depends only on the ratio of momentum to field strength, then the probability that an electron with an emission angle \( \psi \) will strike the counter aperture may be written as:

$$F(\psi, p/l) = f(\psi)T(p/p_0),$$  \hspace{1cm} (1)

where

$$f(\psi) = \begin{cases} 1 & \psi_1 < \psi < \psi_2 \\ 0 & \text{otherwise} \end{cases}$$  \hspace{1cm} (2)

and

$$T(1) = 1,$$  \hspace{1cm} (3)

with

$$p_0 = EI,$$  \hspace{1cm} (4)
where \( X \) is a constant known as the calibration constant of the spectrometer.

The function \( T(p/p_0) \), will be called the transmission curve of the spectrometer. It is a function of \( p \) and \( I \) which designates the relative efficiency with which electrons of momentum \( p \) are focused, when the focusing current is such that the maximum counting efficiency is attained for electrons of momentum \( p_0 \). Fig. 7 shows the result of an experimental determination of the transmission curve of the spectrometer as used in the work described here. The width at half-maximum \( h \) is called the half-width of the transmission curve, and is commonly expressed in percent.

The shape of the transmission curve may be determined experimentally as follows. A source of monoenergetic electrons of momentum \( p \) is placed in the spectrometer. The counting rate is then determined as a function of \( I \). The counting rate to be expected at any current value \( I \) may be obtained by means of the function, \( F(\psi,p/I)\):

\[
N_0(I) = \int_{0}^{\pi} F(\psi,p/I)N_0(\psi)\,d\psi,
\]

where \( N_0(\psi)\,d\psi \) represents the rate of emission by the source of electrons with angles between \( \psi \) and \( \psi + d\psi \). In the case of a very thin source of monoenergetic electrons, such as that considered here, this function is given by:

\[1\text{For the purpose of simplifying the terminology, a distribution function of the form } N(x)dx \text{ will, in the future, be described as "the number of (particles) in the range } x, dx\text{".}\]
Fig. 7. Transmission curve of the beta-ray spectrometer. The triangular approximation is indicated by the dotted lines.
\[ N \psi (\psi) d\psi = \frac{1}{2} N \sin\psi d\psi, \quad (6) \]

where \( N \) is the rate of disintegration of the source. If Eqs. 1, 2 and 6 are used, Eq. 5 becomes:

\[
N_c(I) = \int_0^\pi f(\psi) T(p/p_0) 1/2N \sin\psi d\psi
= \frac{1}{2} NT(p/p_0) \int_{\psi_1}^{\psi_2} \sin\psi d\psi
= \frac{1}{2} NT(p/p_0)(\cos\psi_1 - \cos\psi_2),
\]

or

\[
N_c(I) = \omega NT(p/p_0), \quad (7)
\]

where

\[
\omega = \frac{1}{2}(\cos\psi_1 - \cos\psi_2) \quad (8)
\]

is the fractional solid angle over which electrons from the source are collected by the counter. Let the current corresponding to the maximum counting rate be given by \( I_0 \). Then from the definition of \( N_c \),

\[
p = NT_0. \quad (9)
\]

If Eqs. 4 and 9 are substituted in Eq. 7, the result is:

\[
N_c(I) = \omega NT(I_0/I). \quad (10)
\]

For the special case \( I = I_0 \), Eq. 10 becomes:

\[
N_c(I_0) = \omega NT(1) = \omega N. \quad (11)
\]
If Eq. 10 is combined with Eq. 11, the result is:

\[ T(I_o/I) = N_c(I)/N_c(I_o). \]  (12)

Thus if the ratio \( N_c(I)/N_c(I_o) \) is measured as a function of \( I_o/I \),
the value of the function \( T \) is obtained. It will be observed from
Eq. 12 that the observed spectral line, \( N_c(I) \) resembles a reflection of
the transmission curve, \( T(I_o/I) \). Thus values of \( I \) less than that of the
peak value \( I_o \) correspond to values of the argument of \( T \) greater than
the peak value of unity.

A special case of the above described experiment results when the
momentum of the electrons from the source is known. In this case,
Eq. 9 may be written

\[ K = p/I_o. \]  (13)

and thus the calibration constant \( K \) may be determined.

B. Efficiency for Detecting Internal Conversion Electrons

The spectrometer is used to measure the energy of internal conversion
electrons by means of the source assembly shown in Fig. 5a. The radio-
active source \( A \) is mounted, in the form of a thin film, on the source
holder \( B \). Ordinarily \( B \) is a cylindrical lucite shell, on the face of
which is mounted a thin plastic or aluminum foil. The source material
is deposited on this foil. Monoclonal internal conversion electrons
of momentum \( p' \), originating in the source film at a depth \( x \), will emerge
at an angle \( \psi \) with respect to the spectrometer axis, as shown in Fig. 5b.
Fig. 8. Source assembly for the measurement of internal conversion electron energies.
If scattering of the electrons in the film is neglected, the emerging electrons will have lost a momentum $\Delta p$, given by the expression:

$$\Delta p = \frac{1}{\cos \psi} \int_0^x \frac{dp}{dx'} dx' \quad (14)$$

where $\frac{dp}{dx'}$ represents the space rate of momentum loss in the source material.

The term $\frac{dp}{dx'}$ may be transformed as follows:

$$\frac{dp}{dx'} = \frac{dp}{dW} \frac{dW}{dx'} = \frac{1}{v} \frac{dW}{dx'} \quad (15)$$

where $v$ is the electron velocity and $W$ is the electron energy. Thus,

$$\frac{dp}{dx'} = \frac{\Lambda}{v} \quad (16)$$

where $\Lambda$ is the space rate of energy loss of an electron with momentum $p$.

It should be observed that it is not necessary to carry out the transformation, Eq. 15, in order to solve the problem considered. However, since data concerning the slowing of electrons in matter are customarily given in terms of energy loss rather than momentum loss, the transformation, Eq. 15, leads to results which are more readily adaptable to numerical calculations.

Eq. 14 may now be written:

$$\Delta p = \frac{1}{\cos \psi} \int_0^x \frac{\Lambda}{v} dx'.$$
The momentum of the emerging electrons is then:

\[ p = p' - \Delta p = p' - \frac{1}{\cos \psi} \int_0^x \frac{\hbar}{\Psi} \, dx'. \quad (18) \]

The use of Eq. 18 implies a unique relationship between the momentum of the electron and its position in the source film. Such a relationship does not actually exist, since the loss of energy of an electron in passing through matter is a statistical process, and may vary between wide limits in different cases under the same conditions. Thus to make use of the expression developed here, it will be necessary to use a suitable average value for the energy loss of an electron as a function of the electron energy and of the material through which it passes.

From Eq. 18, the momentum distribution of the electrons emerging from the face of the source film may be obtained. Electrons originating at a depth in the range \( x_0 \), \( dx \), with a momentum \( p' \), will emerge with momenta

\[ \text{moments} \]

It has been customary in this laboratory to use the average energy loss as given by Heitler (24). Horryak, Lauritsen and Rasmussen (25) have suggested that, since the average loss is appreciably affected by a relatively small number of electrons suffering extremely large losses, the most probable loss would be a more reasonable value to employ. Work in this laboratory (reference 26), however, has indicated that the use of the most probable loss does not give as good agreement with experiment as the use of the average loss. It is possible that the effect of scattering, which tends to lengthen the electron path in a foil, may be compensated for to some extent by the use of the average loss rather than of the most probable loss.
in the range \( dp \), where from Eq. 18,

\[
dp = \frac{1}{\cos \psi} \frac{\hbar}{v} \, dx. \tag{19}
\]

Now consider the electrons originating at a depth in the range \( dx \) with emission angles in the range \( \psi, d\psi \). These electrons will emerge with the distribution

\[
\frac{N}{x} \, dx \cdot \frac{1}{2} \sin \psi \, d\psi,
\]

where \( \frac{N}{x} \) is the source rate of emission of electrons per unit source thickness. If Eq. 19 is used, the distribution in momentum and emission angle of the emerging electrons becomes:

\[
N_{p,\psi} (p, \psi) dp \, d\psi = \frac{N}{x} \cos \psi \cdot \frac{\hbar}{\lambda} \, dp \cdot \frac{1}{2} \sin \psi \, d\psi, \tag{20}
\]

where \( N_{p,\psi} (p, \psi) dp \, d\psi \) is the rate of emergence of electrons with momenta in the range \( p \), \( dp \) and emission angles in the range \( \psi, d\psi \). The rate at which these electrons arrive at the counter is given by

\[
F(\psi, p/1) \, N_{p,\psi} (p, \psi) dp \, d\psi.
\]

If the efficiency with which these electrons are counted by the counter is designated by \( e_0(p) \), the counting rate is given by:

\[
N_{10}(1) = \int_{0}^{\pi} d\psi \int_{p_1}^{p_2} dp \, F(\psi, p/1) \, N_{p,\psi} (p, \psi) e_0(p). \tag{21}
\]

The integral over \( p \) is to be taken over all momenta represented in the distribution. The upper limit \( p_2 \) is represented by electrons emerging
from zero depth, \( x = 0 \). Thus:

\[ p_2 = p' . \tag{22} \]

The lower limit \( p_1 \) is represented by electrons emerging from the depth \( x = X \), where \( X \) is the thickness of the source film. From Eq. 18, \( p_1 \) is given by:

\[ p_1 = p' - \frac{1}{\cos \psi} \int_0^X \frac{\lambda}{v} dx' = p' - \delta, \tag{23} \]

where

\[ \delta = \frac{1}{\cos \psi} \int_0^X \frac{\lambda}{v} dx' . \tag{24} \]

If Eqs. 1, 20, 22 and 23 are used, Eq. 21 becomes:

\[ N_{10}(I) = \frac{1}{2} N_x \int_0^\pi d\psi \; f(\psi) \sin \psi \cos \psi \int_{p' - \delta}^{p'} \frac{dp'}{p^2} \frac{\gamma}{\lambda} e_0(p). \]

Due to Eq. 2, this may be written:

\[ N_{10}(I) = \frac{1}{2} N_x \int_0^\psi_2 d\psi \; \sin \psi \cos \psi \int_{\psi'}^{\psi_2} \frac{dp'}{p^2} \frac{\gamma}{\lambda} e_0(p). \tag{25} \]

Since \( \psi' \) and \( \psi_2 \) are ordinarily both of the order of 0.1 radian, there will be an error of the order of only 1% if \( \cos \psi \) is taken as unity.

Then from Eq. 24,

\[ \delta \approx \int_0^X \frac{\lambda}{v} dx'. \tag{26} \]
If this approximation is made in Eq. 25, then the integral over $p$ is independent of $\psi$ and the integrals may be iterated. The result is:

$$N(1) = N_n(\omega) \omega,$$

where

$$\omega = \frac{1}{2} \int \frac{\nu_2}{\nu_1} \sin \psi \quad \text{and} \quad \psi = \frac{1}{2} (\cos \psi_1 - \cos \psi_2),$$

and

$$G(1) = \int \frac{T(p/p_0)}{\nu} \frac{v}{\hbar} e_c(p) \, dp.$$

Contributions to the integral will occur only for values of $p$ where the integrals all differ from zero. The design of the spectrometer is such that $T(p/p_0)$ differs from zero only in the region where $|1 - p/p_0| \leq 0.05$. This corresponds to variations of $p$ of the order of a few percent. Since $v$, $\hbar$, and $e_c$ do not in general change rapidly over this range of momenta, the term

$$\frac{v e_c(p)}{\hbar}$$

may be considered constant:

$$\frac{v e_c(p)}{\hbar} \approx \frac{v(p') e_c(p')}{\hbar(p')}.$$

If it is understood that the integral $G$ is being evaluated in the region $p_0 \approx p'$. The result of this approximation is indicated by the dotted rectangle in Fig. 9. It has been found experimentally (Fig. 7)
Fig. 9. Momentum distribution and transmission curve for the case of internal conversion electrons. The dotted lines indicate approximations used in the evaluation of the spectrometer efficiency.
that the transmission curve $T(p/p_0)$ may be closely approximated by
an isosceles triangle, with half-width $\Delta (2p_0) = h$, where $h$ is the
experimentally determined half-width of the actual transmission curve.
The result of this approximation is indicated by the dotted triangles
in Figs. 7 and 9. If these approximations are made, Eqs. 29 and 26
become:

$$G(1) = \frac{v(p') e_0(p')}{\Lambda(p')} \int_{p' - \delta}^{p'} T(p/p_0) dp,$$  \hspace{1cm} (30)

and

$$\delta = \frac{\Lambda(p') X}{v(p')} \hspace{1cm} (31)$$

Jensen (3) has shown that, in this case, the value of $I$ corresponding
to the maximum of $G$ is given by:

$$I_0 \approx \frac{1}{K} (p' - \delta/2) \text{ or } p_0 = p' - \delta/2, \quad 2 h p' > \delta,$$
$$I_0 \approx \frac{1}{K} (p' - h p_0) \text{ or } p_0 = p'/(1 + h), \quad 2 h p' < \delta.$$ 

The corresponding value, $G(I_0)$, of $G$ is easily found by integrating
the triangle $T(p/p_0)$ between $p' - \delta$ and $p'$. The result is:

$$G(I_0) = \frac{h p' v e_0}{\Lambda} \hspace{1cm} \alpha > 1$$
$$= \left(1 - \frac{\alpha}{2}\right) e_0 X \hspace{1cm} \alpha < 1.$$
where

\[ \alpha = \frac{\hbar X}{2 h v p^t} = \frac{\delta}{2 h p^t} \]  \hspace{1cm} (33)

is the ratio of the momentum loss in the source, \( \delta \), to the base of the transmission curve, \( 2 h p^t \).

The counting rate at the peak of the spectral line may now be written:

\[ N_{10}(I_o) = N_x \omega G(I_o) \]

\[ = \frac{N_x \omega h p^t \nu \varepsilon_o}{\hbar} \hspace{1cm} \alpha > 1 \]

\[ = N_x \omega \varepsilon_o (1 - \frac{\alpha}{2}) \hspace{1cm} \alpha < 1. \] \hspace{1cm} (34)

Since the total source strength \( N \) is given by

\[ N = N_x X, \]

the efficiency,

\[ e_g(I_o) = \frac{N_{10}(I_o)}{N}, \]

becomes

\[ e_g(I_o) = \frac{\omega h p^t \nu \varepsilon_o}{\hbar X} \hspace{1cm} \alpha > 1 \]

\[ = \omega \varepsilon_o (1 - \frac{\alpha}{2}) \hspace{1cm} \alpha < 1, \] \hspace{1cm} (35)

where

\[ \alpha = \frac{\hbar X}{2 h v p^t}. \]
C. Efficiency for Detecting Beta-Rays

The continuous distribution of electrons, characteristic of beta-decay, is analysed with the same source assembly described in the case of internal conversion electrons. The parent distribution of electrons, however, instead of consisting of a group of monoenergetic electrons of momentum $p'$, is characterised by a distribution function

$$n(p')dp',$$

where $n(p')$ is the rate at which electrons in a unit momentum interval, centered about the momentum $p'$, are emitted by the source. The counting rate, at a given focusing current $I$, is obtained by integrating Eq. 27 over all momenta, $p'$. The source strength $N_x$ must in this case be multiplied by the factor

$$\frac{n(p')dp'}{N}$$

(which represents the fraction of the emitted electrons in the momentum range $p'$, $dp'$), where $N$ is the total rate of emission of beta-particles. The resulting counting rate is then:

$$N_\beta(I) = \frac{\omega N_x}{N} \int_0^\infty G(I)n(p')dp' = \frac{\omega}{X} \int_0^\infty G(I)n(p')dp'. \quad (36)$$

From Eq. 29, this becomes:

$$N_\beta(I) = \frac{\omega}{X} \int_0^\infty n(p') \int_{p' - \delta}^{p'} T(p/p_0) \sum \xi \sigma dp dp'.$$  \quad (37)
Since $T(p/p_0)$ differs from zero only for $p \approx p_0$, this may be written approximately:

$$N_p(I) = \frac{\omega}{x} \frac{\gamma(p \rho_0)}{h(p_0)} e_0(p_0) \int_0^{\infty} n(p') \int_{p'-\delta}^{p'} T(p/p_0) dp \, dp'. \quad (38)$$

Consider now the case in which

$$\delta(p' \approx p_0) \ll h p_0. \quad (38a)$$

This case is illustrated in Fig. 10. The integral over $p$ is approximately $\delta T(p'/p_0)$.

The integral over $p'$ is then

$$\int_0^{\infty} n(p') \delta T(p'/p_0) \, dp'.$$

Since $n(p')$ and $\delta$ are ordinarily slowly varying functions of $p'$ compared to $T(p'/p_0)$ (as long as the upper energy limit of the beta-spectrum is avoided), these terms may be removed from the integral. The result is

$$\delta n(p_0) \int_0^{\infty} T(p'/p_0) \, dp',$$

or

$$\delta n(p_0) p_0 \int_0^{\infty} T(x) \, dx.$$

where $\delta$ is to be evaluated for $p' \approx p_0$. 
Fig. 10. Momentum distribution and transmission curve for the case of beta rays.
If this expression is inserted in Eq. 38, there results:

\[
N_p(I) = \frac{\omega}{x} \frac{v(p_0)}{\Lambda(p_0)} e_o(p_0) \delta(p' \approx p_0) n(p_0) \int_0^\infty T(x)dx. \tag{39}
\]

The term \( \delta \) is given by Eq. 31:

\[
\delta(p' \approx p_0) = \frac{\Lambda(p_0) x}{v(p_0)}.
\]

The term \( \int_0^\infty T(x)dx \)

is simply the area under the transmission curve. This may be obtained by numerical integration of the experimentally determined transmission curve, or, if the triangular approximation is used, it may be taken as the half-width \( h \). Eq. 39 then becomes:

\[
N_p(I) = \omega e_o(p_0) n(p_0) p_0 h. \tag{40}
\]

Since \( p_0 = nI \), this may be written:

\[
\frac{N_p(I)}{I} = \omega e_o(p_0) n(p_0) h. \tag{41}
\]

Thus, if the counter efficiency \( e_o \) is independent of energy, Eq. 41 shows that the ratio

\[
N_p(I)/I
\]

is proportional to the momentum distribution function, \( n(p_0) \).
Consider now the case in which the inequality $\delta (p' \approx p_0) \ll P_0$ is satisfied, but the inequality $\delta (p' \approx p_0) \ll h P_0$ is not satisfied. This is the case in which the momentum loss in the source is much less than the focused momentum $p_0$, but is not much less than the width of the transmission curve. Under these conditions it is to be expected that electrons which originate with a momentum $p'$ in the deeper layers of the source and emerge with a momentum $p' - \Delta p$ will be partially compensated for by electrons which originate with a momentum $p' + \Delta p$ and emerge with a momentum $p'$. Thus for sources of moderate thickness ($\delta \ll P_0$) the observed counting rate will be expected to be approximately the same as for thin sources ($\delta \ll h P_0$) of the same total activity; and Eqs. 40 and 41 will still apply in these cases. In the case of the thin source previously considered, these considerations will not apply in the immediate neighborhood of the upper energy limit of the beta-spectrum.

The counting rate in the spectrometer, when it is focused on a region of a beta-spectrum, is thus given by Eq. 40:

$$H_\beta (1) = \omega \varepsilon_0 (p_0) \lambda (p_0) P_0 h,$$

with the condition $\delta (p' \approx p_0) \ll P_0$.

---

*Due to the slope of the parent beta-ray distribution, particles originating with a momentum $p' + \delta$ will differ in abundance from those originating with a momentum $p'$ by a term $(\delta / p') d n(p) / d (\ln p)$. Thus a term of this order of magnitude is neglected in the above discussion.
If the efficiency of the spectrometer is designated by:

\[ e_s(\beta) = \frac{N(\beta)}{N} , \]

then from Eq. 40, and since

\[ N = \int_0^{\infty} n(p_0) \, dp_0 , \]

the following result is obtained:

\[ e_s(\beta) = \omega e_s(p_0) \frac{h p_0 \, n(p_0)}{\int_0^{\infty} n(p_0) \, dp_0} . \]  \hspace{1cm} (42)

If a thin source only is considered (\( \alpha \ll 1 \)), it is seen by comparison with Eq. 35 that the spectrometer efficiency is reduced from that for counting internal conversion electrons by a factor:

\[ \frac{h p_0 \, n(p_0)}{\int_0^{\infty} n(p_0) \, dp_0} . \]  \hspace{1cm} (42a)

This is, of course, due to the fact that only a fraction of the order of \( h \) of the beta-particles are counted at any current setting. The reduction factor \( (42a) \) may be computed from the measured half-width, \( h \), of the transmission curve and from the measured shape of the beta-ray momentum distribution.
D. Efficiency for Detecting Gamma-Rays

A common method of gamma-ray analysis with the spectrometer involves the use of a source assembly such as that shown in Fig. 11. The source material A is deposited in a container B of low atomic number. On the face of B is attached a radiator foil C of high atomic number. The container B is made sufficiently thick to stop any electrons emitted by the source. Gamma-rays undergo the processes of Compton scattering and photoelectric absorption in B and in C. The container B is constructed of a light element to minimize the effect of photoelectric absorption, and the radiator C is ordinarily a heavy element to favor the photoelectric effect. The photoelectrons produced in C will have an energy

$$E = E_\gamma - E_B,$$

where $E_\gamma$ is the photon energy and $E_B$ is the binding energy of the emitted photoelectrons. By measuring the energy $E$, it is possible to deduce the gamma-ray energy $E_\gamma$.

It is of interest to determine the number of photoelectrons counted by the spectrometer counter due to a source emitting $N$ quanta of energy $E_\gamma$ per unit time. Consider a quantum emitted at an angle between $\theta$ and $\theta$ and $d\theta$ with respect to the spectrometer axis. The number of such quanta is given by:

$$P_1(\theta) d\theta = \frac{1}{2} N \sin \theta d\theta$$

(43)
Fig. 11. Source assembly for the measurement of gamma-ray energies.
The probability that this quantum will emit a photoelectron from the K shell of an atom of the radiator, at a depth $x$ in the radiator, is given by:

$$P_2(x, \theta) \ dx = \frac{1}{\cos \theta} \gamma_K \ dx,$$  \hspace{1cm} (44)

where $\gamma_K$ is the absorption coefficient of the radiator for photoelectron emission from the K shell. Let the probability that this electron be emitted at an angle between $\phi$ and $\phi + d\phi$ with respect to the direction of the photon be designated by

$$P_3(\phi) \ d\phi.$$  \hspace{1cm} (44a)

If scattering is neglected, the electron will emerge from the face of the radiator with its direction unchanged. A certain fraction of such electrons will emerge at an angle $\psi$ with respect to the spectrometer axis, such that

$$\psi_1 < \psi < \psi_2.$$  \hspace{1cm} (44b)

Let this fraction be designated by

$$P_4(\theta, \phi, \psi_1, \psi_2).$$  \hspace{1cm} (44c)

The fraction of the electrons satisfying the condition (44b), which reach the counter is given by the transmission curve of the spectrometer:

$$P_5(p) = T(p/p_0).$$  \hspace{1cm} (44d)

The fraction of these which is counted is given by the counter efficiency:
\[ P_6(p) = e_0(p), \quad (45) \]

The spectrometer counting rate is then given by forming the product

\[ P_1 P_2 \cdots P_6, \]

and integrating over all of the variables. The result is

\[
\gamma_\gamma = \int_0^\infty d\theta \int_0^\infty dx \frac{\gamma_{\text{K}}}{\cos \theta} \int_0^w d\phi f_3(\phi) f_4(\theta, \phi, \psi_1, \psi_2) T(\phi) e_0(p) \quad (46)
\]

Before attempting to evaluate the integral it is necessary to express the variable \( p \) in terms of the other variables. This is done in the same way as in the case of internal conversion electrons discussed above. The result is:

\[
\gamma_\gamma = \int_0^\infty d\theta \int_0^{p'/\delta} \frac{\gamma_{\text{K}}}{\cos \theta} \int_0^\infty d\phi f_3(\phi) f_4(\theta, \phi, \psi_1, \psi_2) T(p/p_0) e_0(p) \quad (47)
\]

\[
= N \frac{\gamma_{\text{K}}}{\cos \theta} \int_0^{p'/\delta} \frac{\gamma_{\text{K}}}{\cos \theta} T(p/p_0) e_0(p) \quad (47)
\]

where

\[
\phi = 1/2 \int_0^\pi \tan \theta \int_0^w f_3(\phi) f_4(\theta, \phi, \psi_1, \psi_2) d\phi \quad (48)
\]

and \( \theta_0 \) is the angle subtended by the radiator at the source. The integral
over p has been evaluated in section IV.B, and, if the maximum counting rate only is considered, is given by Eq. 32:

\[ G(I_o) = \frac{h p' v e_o}{\lambda} , \quad \alpha > 1 \]

\[ = e_o x(1 - \alpha/2) , \quad \alpha < 1. \]

Eq. 47 becomes:

\[ N_y(I_o) = N \gamma K \omega e G(I_o) , \]

or

\[ N_y(I_o) = \frac{N \gamma K \omega e h p' v e_o}{\lambda} , \quad \alpha > 1 \]

\[ = N \gamma K \omega e e_o x(1 - \alpha/2) , \quad \alpha < 1. \]

where as before:

\[ \alpha = \frac{\lambda x}{2 h v p'} . \]

The evaluation of the integral \( \omega e \) will be considered for two limiting cases.

Consider first the non-relativistic case,

\[ v/c \ll 1, \]

where \( v \) is the velocity of the photoelectron considered and \( c \) is the velocity of light. It is in this region that electron scattering will be of primary importance. Scattering will tend to destroy the angular
dependence of the electrons on the angles $\theta$ and $\phi$. Thus let it be assumed that the electrons emerging from the radiator have an isotropic distribution in angle. The integral

$$\int_0^\pi P_3(\phi)P_4(\theta, \phi, \psi_1, \psi_2) \, d\phi$$

is the probability that an electron emerge from the radiator with an emission angle $\psi$ such that

$$\psi_1 < \psi < \psi_2.$$ 

Since it is assumed that the angular distribution of the emerging electrons is isotropic, this term is simply the fractional solid angle subtended by $\psi_1$ and $\psi_2$. Thus

$$\int_0^\pi P_3(\phi)P_4(\theta, \phi, \psi_1, \psi_2) \, d\phi = \omega.$$ 

The term $\omega$ then becomes:

$$\omega_{\psi} = \frac{\omega}{2} \int_0^\theta \tan \theta \, d\theta = \frac{\omega}{2} \ln \sec \theta_0 .$$ \hspace{1cm} (53)

*It must also be noted that scattering will, in this case, have a considerable effect in increasing the electron path length, and will thus give rise to excessive energy losses in the radiator. This will undoubtedly destroy the rectangular momentum distribution which is being assumed. The results in this case will have a qualitative significance only.*
where \( \phi \) is defined such that

\[
(\phi, \phi) = 0 = (\phi, -\phi) \quad (\phi, \phi) \neq 0
\]

(93)

\[
\phi \approx (\phi)^2
\]

Consider next the relativistic case.

\[(94)\]

\[
\gamma \approx 1
\]

will be taken as unit.

Consider the effect of the factor \( \gamma \). For this case, therefore, the expression \( \gamma^2 \) approximately does not justify.

The form not been taken into account in the consideration of above is one in which scattering is important, and since the effect of scatter is small, the case where \( \gamma = 1/2 \) in see 8 is of the order of 0.17 to 0.2. Since this case

\[
\gamma \approx \gamma
\]

The term in see \( \gamma \) may be determined from the geometry of the source.
\[
\frac{d}{x^2} = a
\]

with

\[\frac{H}{10^x} = \frac{1}{\gamma}
\]

If \( N^x \) and \( \gamma \) are taken into account, \( N^x \) becomes:

\[(52) \quad c^n = (\frac{z}{x} \cos \frac{1}{\gamma} = \frac{z}{x} \cos \frac{1}{\gamma} = \frac{z}{x} \cos \frac{1}{\gamma} = \frac{z}{x} \cos \frac{1}{\gamma}
\]

and

\[\tan \theta \approx \tan \theta
\]

and since \( h \) and \( r_e \) are both of the order of \( 0.1\),

\[\tan h \approx \tan h
\]

\[= 0, \text{ otherwise}
\]

\[h > \theta > h
\]

But from the definition of \( h \)

\[(56) \quad \tan \theta \approx \tan \theta
\]

the result is:

\[\tan \theta \approx \tan \theta
\]

where \( \theta \) is an arbitrary function. If \( x^2 \) is inserted in \( y^2 
\]
and the spectrometer efficiency for the detection of the gamma-radiation due to photoelectric conversion in the K shell becomes:

\[ e_\alpha(\gamma) = \frac{\gamma K h p_1 v e_0 \omega}{\lambda}, \quad \alpha > 1 \]

\[ = \gamma K e_0 \omega x(1 - \alpha/2), \quad \alpha < 1. \]

A similar expression is obtained for the case of photoelectric conversion in the L shell, M shell, etc.

E. Summary

The Eqs. 35, 42 and 59 may be rewritten in the following forms:

\[ e_\alpha(10) = [\omega e_0] S(\alpha), \quad (60) \]

\[ e_\alpha(0) = [\omega e_0] [h p_0 n(p_0)/N], \quad (61) \]

\[ e_\alpha(\gamma) = [\omega e_0] S(\alpha) [\gamma x], \quad (62) \]

The notation used is as follows:

The term,

\[ \omega = 1/2 (\cos \psi_1 - \cos \psi_2) \]

is the fractional solid angle characteristic of the angle of acceptance of the spectrometer for electrons of momentum

\[ p_0 = K \lambda. \]

This solid angle is of the order of 1\%. 
The term $e_c$ is the intrinsic efficiency of the counter for counting electrons of momentum $p_0$. In the case of a Geiger counter, of not too low a pressure, it is characterized by a value close to unity for electron energies above a critical energy. Below this energy, which is typically of the order of 25 to 50 KeV, the transmission of the counter window decreases sharply with decreasing energy, with a corresponding decrease in $e_c$.

In Eq. 61, which refers to the detection of beta-rays, $n(p_0)$ is the rate of emission of electrons in a unit momentum interval centered about the momentum $p_0$.

$N$ is the total source strength, given by:

$$N = \int_0^\infty n(p_0) \, dp_0.$$

In Eq. 62, $\gamma$ is the absorption coefficient for the photoelectric effect from the orbital shell under consideration, and $X$ is the thickness of the radiator.

The term $\alpha$ is defined by:

$$\alpha = \frac{\hbar X}{2 \hbar v p_0} \approx \frac{\hbar X}{2 \hbar v p_0},$$

where $X$ is the thickness of the source in the case of internal conversion electrons, and of the radiator in the case of photoelectrons.

The function $S(\alpha)$ is obtained from Eq. 35 or Eq. 59, and is given by:
\[ S(\alpha) = (1 - \alpha/2), \quad \alpha < 1 \]

\[ = 1/2\alpha, \quad \alpha > 1. \]  

In each case the efficiency is seen to contain the factor \( \omega e_c \). This factor is the probability with which electrons from a monoenergetic source are counted if the focusing current is of such a value as to focus them most efficiently.

In the case of internal conversion electrons, the factor \( \omega e_c \) is multiplied by the function \( S(\alpha) \), which depends upon the electron momentum, the source thickness, and the source material. This function gives the effect of energy loss in the source. It is seen to reduce to unity for very thin sources.

In the case of beta-rays, the factor \( S(\alpha) \) is not involved in the order of approximation used here. The factor \( \omega e_c \) is multiplied by a factor involving the form of the beta-spectrum. This factor gives the fraction of the beta-spectrum which lies within the transmission curve of the spectrometer.

In the case of photoelectrons due to gamma-rays, the same factors are involved as in the case of internal conversion electrons, with the exception that the terms in \( S(\alpha) \) refer to the radiator material and thickness rather than those of the source. In addition there is a factor \( \gamma' x \), which is seen to be the probability with which a photoelectron is produced by a photon in passing through the radiator.

Due to the fact that the effect of electron scattering in the source material (or in the radiator) has been neglected throughout, it is not
suggested that the above expressions for the spectrometer efficiency will be useful for accurate calculations. They do, however, furnish an order of magnitude estimate of counting rates to be expected under given conditions. In addition they serve to indicate many of the qualitative features of intensity relations, such as the decrease of efficiency for internal conversion electrons with increasing source thickness. Lastly, these relations furnish interpolation formulae for the interpretation of experimental data. Thus if effective values of \( \alpha \) in Eqs. 60 and 62 are determined experimentally for various combinations of energy, source material and source thickness, it may be anticipated that these formulae will apply with some degree of rigor.
V. COINCIDENCE COUNTING

A. Introduction

The method of coincidence counting in the analysis of nuclear decay schemes has been discussed by many authors (9-18). The basic experimental arrangement is customarily as indicated in Fig. 12. A radioactive source is placed at some position such that its radiations may impinge upon the counters C_1 and C_2. Particles which are detected by these counters give rise to electrical pulses which are recorded by the registers R_1 and R_2. These pulses are also transmitted to the coincidence circuit R_c. This circuit is of such a nature that a pulse arriving from C_1 at a time t and a pulse arriving from C_2 at a time t ± Δt will cause the coincidence circuit to register if and only if Δt < τ_c, where τ_c is a characteristic of the coincidence circuit known as the resolving time. By means of this basic arrangement it is possible to determine whether two particles are emitted within a time τ_c, implying a cascade transition, or whether there is no observable time correlation, implying a parallel transition or a metastable state. Since the resolving time τ_c is customarily of the order of 10^{-6} sec., and since the half-life for allowed gamma-emission is of the order of 10^{-13} sec., the time between a beta- or a gamma-transition and a succeeding allowed gamma-transition is always much less than τ_c. When this is the case the particles are said to be "in coincidence" with each other.
Fig. 12. Block diagram of coincidence counting system.
Let \( N \) and \( M \) be the number of participants of type 1 and per unit time, let each experience session be dedicated to a particular
counter a source which decreases at the rate of \( \gamma \), and \( \gamma \) becomes

\[
\gamma = \alpha - \beta \gamma
\]

It must be observed that, even though there may be no participants in
Then:

\[ N_0 = N_c + N_a, \]  
\[ N_1 = N \sum_{i} n_i e_{1i}, \]  
\[ N_2 = N \sum_{j} n_j e_{2j}, \]  
\[ N_0 = N \sum_{i,j} a_{ij} e_{1i} e_{2j}. \]  

Also, from Eq. 65:

\[ N_a = 2 \gamma_c \frac{N_1 N_2}{N_0} \]

\[ = 2 \gamma_c \frac{N \sum_{i} n_i e_{1i} \sum_{j} n_j e_{2j}}{N \sum_{i} n_i e_{1i} e_{2j}}. \]  

(70)

The problem of determining the decay scheme of a particular isotope consists in determining the coefficients \( n_i \) and \( a_{ij} \). The coefficients \( n_i \) may, in principle, be determined by means of energy measurements in the beta-ray spectrometer, and through the use of intensity relations such as those discussed in section IV. To determine the coefficients \( a_{ij} \) it is necessary to resort to coincidence measurements. This may be done, in principle, by counting coincidences and varying the efficiencies \( e_{1i} \) and \( e_{2j} \) in Eqs. 67-69.

It will be of interest to consider a particular example of the application of Eqs. 66-70. Consider the known decay scheme (Deutsch, Elliott and Roberts (29) ) of \(^{60} Co\) as indicated in Fig. 13. A beta-ray transition is followed by two gamma-ray transitions. All of these processes occur in cascade in a time believed to be of the order of
Fig. 13. Decay scheme of $^{60}$Co.
\((12)\) 
\[ P = \lambda \in \lambda \in h + \lambda = e \in h \]

\((13)\) 
\[ N = 2 \]

\((14)\) 
\[ N = 0 \]

\((15)\) 
\[ N = 0 \]

\((16)\) 
\[ N = 0 \]

\((17)\) 
\[ N = 0 \]

\((18)\) 
\[ N = 0 \]

\((19)\) 
\[ N = 0 \]

\((20)\) 
\[ N = 0 \]

\((21)\) 
\[ N = 0 \]

\(\text{Theorem:} 67-69\) then becomes:

\[ \text{Suppose that } G \text{ is a game-counter which will count Game-1 and Game-2.} \]

\[ \text{Then } G \text{ must be a beta-counter which will count the approximate values.} \]

\[ \text{Let } G \text{ be a beta-counter which will count the approximate values.} \]

\[ \text{Let } G \text{ be a beta-counter which will count the approximate values.} \]

\[ \text{Suppose that } G \text{ is a game-counter which will count Game-1 and Game-2.} \]
From Eqs. 73-75 it is possible to determine the three quantities \( N_e \), \( a_T \), and \( a_B \) from measured values of \( N_1 \), \( N_2 \), and \( N_c \). Thus:

\[
N = \frac{N_1 N_2}{N_c} \quad (76)
\]

\[
a_B = \frac{N_c}{N_1} \quad (77)
\]

\[
a_T = \frac{N_c}{2N_2} \quad (78)
\]

It is of course necessary to know the decay scheme of Co\(^{60}\) in order to obtain Eqs. 73-75. It is possible, however, to determine simple decay schemes such as that of Co\(^{60}\) by means of a qualitative study of coincidence measurements such as the one described here. Then, having determined the counter efficiencies, more complicated decay schemes may be studied.

**B. Statistical Considerations**

Since in practice the rate at which coincidences are recorded is very low, it is of great importance to consider the effect of statistical fluctuations on the results obtained. The true coincidence rate may be deduced from the observed coincidence rate by means of Eqs. 66 and 70.

---

* See, for example, the series of articles by Roberts and others (11-16).

**The term "true coincidence rate" is customarily employed to designate coincidences due to particles which are actually emitted in cascade, as opposed to the accidental coincidence rate.*
In the equation (2) of the original and corrected (3) p. 206, the standard deviations from the measured quantities \( N \) and \( N^* \) are denoted by \( \sigma N \) and \( \sigma N^* \). It is essential to note that the true concentration rate is determined by the equation

\[
\frac{N}{N^*} = 1 - \frac{\sigma N}{\sigma N^*}
\]

or, writing \( x = \frac{N}{N^*} \), this becomes

\[
x - 1 = x = 0
\]

By using this equation it is shown that the true concentration rate is calculated for the concentration measurements in question, then, from a known concentration, by means of the experiment, the term \( x \) is determined. Hence

\[
\frac{1}{2} \frac{\sigma N}{\sigma N^*} = \frac{1}{2}
\]

and thus

\[
x = 0
\]

No true concentration

Since in this case there are no true concentrations, the expression (2) is not applicable. The true concentration rate is determined by the equation

\[
\frac{1}{2} \frac{\sigma N}{\sigma N^*} = \frac{1}{2}
\]

and thus

\[
x = 0
\]

If the concentration and the concentration rate are not determined experimentally, then the calculated concentration rate is

-60-
\[
\sigma^2(N_0) = \left( \frac{\partial N_0}{\partial N_c} \right)^2 \sigma^2(N_0) + \left( \frac{\partial N_0}{\partial \Gamma_c} \right)^2 \sigma^2(\Gamma_c) + \left( \frac{\partial N_1}{\partial N_1} \right)^2 \sigma^2(N_1) + \left( \frac{\partial N_2}{\partial N_2} \right)^2 \sigma^2(N_2).
\]

If Eq. 80 is used, Eq. 81 may be written in the form:

\[
\sigma^2(N_0) = \sigma^2(N_0) + (2 \Gamma_c N_1 N_2)^2 \left[ \frac{\sigma^2(\Gamma_c)}{\Gamma_c^2} + \frac{\sigma^2(N_1)}{N_1^2} + \frac{\sigma^2(N_2)}{N_2^2} \right].
\]

In a similar way the standard deviation in \( \Gamma_c \) may be found to be given by:

\[
\sigma^2(\Gamma_c) = \left( \frac{N_0'}{N_1 N_2} \right)^2 \left[ \frac{\sigma^2(N_0)}{N_0'^2} + \frac{\sigma^2(N_1)}{N_1'^2} + \frac{\sigma^2(N_2)}{N_2'^2} \right].
\]

The standard deviation in any measured counting rate, due to random events, is given by (Bunkerford, Chadwick and Ellis (32, p. 156)):

\[
\sigma(N_i) = \sqrt{\frac{N_i}{t_i}}
\]

where \( N_i \) is the counting rate concerned and \( t_i \) is the time of counting.

Since radioactive decay is such a process; it is seen that in any particular measurement of a counting rate due to a radioactive source, the fractional standard deviation,

\[
\frac{\sigma(N_i)}{N_i}
\]

is given by
\[
\frac{\sigma(N_1)}{N_1} = \frac{1}{\sqrt{N_1 \tau_1}}, \tag{85}
\]

and is thus inversely proportional to the square root of the counting rate. If this fact is used, together with the fact that in any coincidence measurement with counters of ordinary efficiency, which is much less than unity, the coincidence rate is much less than the individual counting rates \(N_1\) and \(N_2\); then it is seen that the only important terms in Eqs. 82 and 83 are the terms involving \(\sigma^2(N_0)\) and \(\sigma^2(N'_0)\).

These equations may thus be written to a good approximation:

\[
\sigma^2(N_0) = \sigma^2(N'_0) + (2 \tau_c N_1 N_2 \tau^2_0) \sigma^2(\tau_c), \tag{86}
\]

\[
\sigma^2(\tau_c) = \frac{\sigma^2(N'_0)}{(2 N'_1 N'_2)^2}. \tag{87}
\]

Two general procedures for determining \(\tau_c\) may be mentioned. The customary procedure is to carry out a determination before and after the measurement of true coincidences, by means of the method outlined above. Very often the separate sources used to actuate the counters \(C_1\) and \(C_2\) are of such a strength that the counting rates \(N'_1\) and \(N'_2\) are high, resulting in a small error \(\sigma(\tau_c)\). An alternative procedure is the continuous determination of \(\tau_c\) by alternating measurements of \(N'_0\) with measurements of \(N_0\). Furthermore, the separate sources used in the determination of \(N'_0\) may be adjusted to such a strength that
Consider a particular concentration measurement to be made with a source of strength \( \mu \), and decay constant \( \lambda \). From Eq. 66.

The method of determining the second

...
It is of interest to determine the optimum source strength to be selected and the optimum apportionment of counting time between the determination of \( N_0 \) and of \( \gamma \). Let:

\[
x = aN \quad , \tag{94}
\]

\[
c = b/a^2 \quad . \tag{95}
\]

Then Eqs. 88-90 become:

\[
N_c = x \quad , \tag{96}
\]

\[
N_a = c x^2 \quad , \tag{97}
\]

\[
N_0 = x + c x^2 \quad . \tag{98}
\]

The standard deviation in \( N_c \) is given by Eqs. 86 and 87:

\[
\sigma^2(N_c) = \sigma^2(N_0) + \frac{(2 \mu \sigma)^2}{(2N_1 N_2)^2} \sigma^2(N' \gamma) \ ,
\]

and if in the determination of \( \gamma \), \( N'_1 \approx N_1 \), and \( N'_2 \approx N_2 \), this becomes:

\[
\sigma^2(N_c) \approx \sigma^2(N_0) + \sigma^2(N' \gamma) .
\]

If Eq. 85 is used, this becomes:

\[
\sigma^2(N_0) = \frac{E_0}{t_o} + \frac{N'_0}{t_a} \ ,
\]

where \( t_o \) and \( t_a \) are the times of counting \( N_0 \) and \( N'_0 \). Since \( N'_0 \approx N_a \), this becomes:
\[ \sigma^2(N_c) = \frac{N_0}{t_o} + \frac{N_a}{t_a} \]

From Eqs. 96-98, this may be written:

\[ R^2 = \frac{1 + c\alpha}{t_o^2} + \frac{c}{t_a} \quad (99) \]

where

\[ R = \frac{\sigma(N_c)}{N_c} \]

is the fractional standard deviation in the true coincidence rate, \( N_c \).

If the total time of counting is \( T \), then Eq. 99 may be written:

\[ R^2 = \frac{1 + c\alpha}{t_o^2} + \frac{c}{T - t_o} \quad (99a) \]

The apportionment of the total time \( T \) between \( t_o \) and \( t_a \) which will give the lowest possible value for \( R \) is obtained by setting the derivative of \( R \) with respect to \( t_o \) equal to zero. This results in the relation:

\[ \frac{t_a}{t_o} = \sqrt{\frac{c\alpha}{1 + c\alpha}} = \sqrt{\frac{N_a}{N_c}} \quad (100) \]

This gives the relative amount of time to be taken in determining \( T_c \) and \( N_c \). It is of interest next to determine the proper source strength \( N \) to be employed. This is equivalent to determining the proper value of \( x \). It is seen by inspection of Eq. 99a that there is no minimum of \( R \) with respect to \( x \). The larger \( x \) is made, the smaller will \( R \) become.

If \( t_a \) and \( t_o \) are eliminated from Eq. 99 by means of Eq. 100, the result is:

\[ R = \frac{c}{T} F(y) \quad (101) \]
where

\[ y = \alpha x = N_a / N_c \]  \hspace{1cm} (102)

and

\[ F(y) = \left[ (1 + \sqrt{\frac{y}{1+y}} \left( \frac{1+y}{y} + \sqrt{\frac{1+y}{y}} \right) \right]^{1/2} \]  \hspace{1cm} (103)

The value of the function \( F(y) \) is tabulated in Table I for four points of interest.

<table>
<thead>
<tr>
<th>( y )</th>
<th>( F(y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( \infty )</td>
</tr>
<tr>
<td>1</td>
<td>2.42</td>
</tr>
<tr>
<td>2</td>
<td>2.22</td>
</tr>
<tr>
<td>( \infty )</td>
<td>2.00</td>
</tr>
</tbody>
</table>

It is seen that \( R \) is reduced by only about 20% or less by selecting \( y \) greater than unity. Thus a reasonable value for \( y \) is given by \( y \approx 1 \), or

\[ N_a = N_c \]  \hspace{1cm} (104)

Thus the statistical error in determining the coincidence rate \( N_c \) is given by

\[ \frac{\sigma(N_c)}{N_c} = 2.42 \sqrt{\frac{5}{T}} \]

\[ = 2.42 \sqrt{\frac{b}{aT}} \]  \hspace{1cm} (105)
with the source strength chosen such that \( N_c = N_a \), and with

\[
\frac{t_a}{t_o} = \sqrt{\frac{N_a}{N_o}} = 0.707.
\]

If the factors \( a \) and \( b \) are evaluated from Eqs. 92 and 93, then Eq. 105 becomes:

\[
\frac{c(N_o)}{N_c} = 3.42 \left[ \frac{\tau_c \sum_{i,j} n_i n_j \epsilon_{1j} \epsilon_{2j}}{T(\sum_{i,j} \epsilon_{1j} \epsilon_{2j})} \right]^{1/2}. \tag{106}
\]

Thus it is seen that the fractional error in \( N \) varies directly as the square root of \( \tau_c \) and inversely as the square root of the counter efficiencies \( \epsilon_{1j} \) and \( \epsilon_{2j} \). It is thus of importance to maintain \( \tau_c \) as small as possible without losing true coincidences, and to maintain the counter efficiencies as large as possible.

A limitation which has not been considered in the above analysis is the possibility of loss of counts in the individual counters due to the dead time of the counters. If it is desired to avoid excessive loss in the counters it may be necessary either to reduce the source strength below that given by Eq. 104, or to reduce the counter efficiencies \( \epsilon_{1j} \) and \( \epsilon_{2j} \) by decreasing the solid angles. The effect of either of these alternatives may be investigated in any particular case by use of Eq. 104. In general it is found to be preferable to reduce the source strength until the counting loss in the individual counters is not excessive.
For a description of these contact

intertwined and spaced (7x)

For a description of these contact

The contacts which reach the common contact case in O and O.  The

separate outputs which are connected to two separate types of contact

contacts on a mechanism register. In another notation this red to

puttees are fed to a master type contact case where the

standard secondary square wave outputs of 5 microseconds duration. These standard

comanate or the output case, where they are used to operate

that of hour (37). It is clear from 0 and 0 which are connected

shown in Fig. 1. The output case output to a modification of

separate auxiliary with the common case output and intertwined register as

secondary square which will be deactivated as 0, O and O are connected

as shown in Fig. 1. 7. Immediately bringing the source is inserted the

as shown in Fig. 1. 1. The output case output to the end opposite to the source

is inserted into the output case output to the end opposite to the source.

The output case output to the end opposite to the source.

A moment (7). A schematic diagram of the output case output to the source in Fig. 1.

The output case output to the source in Fig. 1. 7. The

output case output to the source in Fig. 1. 7.

caution: precautions with the both key outputs

consulted above.

sufficiently great that these outputs are not of interest in the problem

source strength need in the mechanism to be described are

of backfound due to coarse calibration and direct communication.

another effect which has not been treated above is the presence

-65-
coefficient was inserted a tuning fork 60-7000 vibrations per second which covered

the counter, the counter was made from a copper coil, the interior of the
are shown schematic drawings of the detector and the counter, counter assembly.

the outer wall of the counter and the inner wall of the source tube. In Fig. 1, the
counter with a vacuum seal to prevent leakage of air between the outer
- a roll was not necessary, and it was necessary to provide the bore-

the counter where Q is a counter tube. In the case where Q was a beta-counter, such
so that the counter could be removed without disturbing the vacuum

behind the source with a wall of phantom bronze (0.041"/2") total.

In the case where Q was a counter tube, the source tube was needed

tronometer tube. Q was then inserted into the tube behind the source.
detector tubes when the tubes were mounted in the source end of the spectrometer.

which was then attached to the spectrometer was mounted on a joint tube. This water was then attached to the 2,4-

when countourneas were to be counted in the spectrometer, the source

below.

the amount needed in any of the experiment described

the sample less than one microsecond. For the reason, no resonance

to be lost. Due to losses in the time of flight of the counter, at

the experiment described below, introduced that countourneas began

in place of different sections of 0.5 with the other counters used in

countourneas to microsecond, a comparison of observed countourneas

may be verified in these from 0.5
Fig. 14. Gamma-counter and beta-counter assemblies for coincidence measurements in the spectrometer.
the wall and the face of the counter nearest to the source. The gamma-counter with the gauze lining was found to have an efficiency 1.4 times as great at 1.25 keV as a similar counter without the gauze, and 4.2 times as great at 0.4 keV. The counter was provided with a 2 mil brass (0.0422 g/cm²) window. The beta-counter was a commercial end-window counter with a steel wall and a 2.5 mg/cm² mica window. This counter was mounted within an aluminum cylinder with dimensions similar to those of the gamma-counter. The gamma-counter subtended a solid angle at the source of 10%, and the beta-counter subtended a solid angle of 2%.

It will be of interest to make some rough calculations to determine the feasibility of making coincidence measurements in the spectrometer with the arrangement described here.

Consider first a coincidence measurement in which two gamma-rays are in cascade. Let one of these gamma-rays be completely converted in the K shell, and let the other be converted only to a negligible degree. Let the focusing current of the spectrometer be adjusted to focus the conversion electrons. Then:

\[ N_1 = N e_e \]  
\[ N_2 = N e_\gamma \]  
\[ N_c = N e_e e_\gamma \]  
\[ N_a = 2 \gamma_c N_1 N_2 = 2 \gamma_c e_e e_\gamma N^2. \]

Since \( \gamma_c \) is to be as low as possible without losing true coincidences,
it may be taken as $10^{-6}$ sec. The efficiency of a Geiger counter for
gamma-radiation, including the effect of a $10^2$ solid angle, may be
taken as of the order of $10^{-3}$ (Von Droste (30)). The efficiency of
the spectrometer for internal conversion electrons is obtained from
Eq. 60:

$$e_s = \omega e_o S(\alpha).$$

If the source is sufficiently thin, and the electron energy sufficiently
high, then $S(\alpha)$ and $e_o$ may be taken as unity. Since $\omega$ is of the order
of $1/\mu$,

$$e_s \approx 10^{-2}.$$  

In order to minimize the statistical error it is desirable to choose
a source strength such that $N_o \approx N_a$ (Eq. 104). If this is done, and
if the proper apportionment of counting times, as given by Eq. 100, is
used, then from Eq. 105:

$$\frac{c(N_o)}{N_o} = 2.42 \sqrt{\frac{b}{a^2 T}}.$$  \hspace{1cm} (111)

Comparing Eqs. 109 and 110 with Eqs. 88 and 89, it is seen that:

$$a = e_s e_\gamma = 10^{-5}$$

$$b = 2 \gamma e_s e_\gamma = 2 \times 10^{-11}$$

Inserting these values in Eq. 111 leads to the result:

$$\frac{c(N_o)}{N_o} = \frac{1}{\sqrt{T \text{ (sec.)}}}.$$  \hspace{1cm} (112)
It will thus take a very short time to attain a low statistical error, and the experiment may be considered feasible. It must be observed that when the source strength is chosen such that \( N'_c = N'_a \), the counting rate \( N'_1 \) is found to be of the order of 500 counts per sec. With a counter dead time of the order of \( 2 \times 10^{-4} \) sec, this will result in a counting loss of \( 10^5 \) in \( C'_1 \). For this reason it would be desirable to choose a somewhat weaker source than that implied by Eq. 104.

As another example, consider the case where neither of the gamma-rays is converted. In order to perform a measurement in the spectrometer, it will then be necessary to use a radiating foil, and count Compton electrons or photoelectrons emitted from the foil by the gamma-ray. The efficiency of the spectrometer in the case of photoelectrons is given by Eq. 62:

\[
E = \omega E_c S(\alpha) T(x).
\]

Since \( T(x) \) is customarily of the order of \( 10^{-3} \), the term \( E \) is of the order of \( 10^{-5} \). Replacing \( E \) in the previous case by \( 10^{-5} \) leads to the result:

\[
\frac{\sigma(N'_c)}{N'_c} = \sqrt{\frac{10^{-3}}{T(\text{sec.})}}
\]

(113)

For an error in \( N'_c \) of \( 10^5 \), this requires a counting time of 25 hours. Such an experiment must be regarded as possible, but not particularly promising.

Other cases may be treated in a similar manner and need not be discussed here.
curve and the distribution constant $k$, using the trip - time and the
factor $f$ were evaluated by measuring the spectrometer transmission

\[ \int_{0}^{\infty} \frac{e^{\phi}}{e^{\phi} + 1} \, d\phi = \pi \]

where

\[ \int_{0}^{\infty} \frac{e^{\phi}}{e^{\phi} + 1} \, d\phi = (e) \]

The term $(e)$ is defined by $\phi = 0$.

\[ \text{(III)} \quad \left[ (\lambda \theta \varepsilon) + (\Phi \lambda \varepsilon) \right] \quad \text{(IV)} \quad n = \frac{\phi}{\pi} \]

\[ \text{(III)} \quad \left[ (\lambda \theta \varepsilon) + (\Phi \lambda \varepsilon) \right] \quad n = \frac{\phi}{\pi} \]

\[ \text{(III)} \quad \left[ (\lambda \theta \varepsilon) + (\Phi \lambda \varepsilon) \right] \quad n = \frac{\phi}{\pi} \]

The curves of $e^\phi$ were derived from the known decay
to be expected under these conditions and used in the
computer to provide synthetic aerosol extinction curves.

The computer output at 1.25 m/s was inserted in front of the
cooler and the spectrometer was inserted in the source position of the spectrometer. A press plate was inserted in the source of the spectrometer. A filter source of 60
cooler output at 1.25 m/s will be discussed. A filter source of 60
the determination of the spectrometer solid angle in the spectrometer.

D. Determination of counter efficiencies
shape of the beta-spectrum of Co\(^{60}\), extrapolating below the low energy cutoff of the spectrometer counter by means of the theoretical Fermi distribution as given by Feister (35). The result was

\[ A = 0.0485. \]

The half-width of the spectrometer transmission curve was found to be

\[ h = 0.049. \]

Since the efficiency of a Geiger counter for detecting gamma-radiation is a slowly varying function of energy, Eqs. 114-116 may be written approximately:

\[ N_1 = N A \omega = 0.0485 N \omega, \]

\[ N_2 = 2 N \epsilon_\gamma(1.25 \text{ Mev}), \]

\[ N_\epsilon = 2 N A \omega \epsilon_\gamma(1.25 \text{ Mev}) \]

\[ = 0.096 N \omega \epsilon_\gamma(1.25 \text{ Mev}). \]

The measured values, after correcting for accidental coincidences and backgrounds, were:

\[ N_1 = 531 \text{ counts per min}, \]

\[ N_2 = 4649 \text{ counts per min}, \]

\[ N_\epsilon = 0.815 \pm 0.079 \text{ counts per min}. \]

*Errors (\(\pm\)) given in this paper will refer to standard deviation.*
From these relations, and Eqs. 114-116, there results:

\[ e_\gamma(1.25 \text{ MeV}) = (0.767 \pm 0.074) \times 10^{-3} , \]
\[ \omega = (0.362 \pm 0.035) \times 10^{-2} , \]
\[ N = (3.03 \pm 0.29) \times 10^6 \text{ dis per min,} \]
\[ = 1.36 \pm 0.13 \text{ microcuries.} \]

A repetition of this experiment led to the results:

\[ e_\gamma(1.25 \text{ MeV}) = (0.688 \pm 0.069) \times 10^{-3} , \]
\[ \omega = (0.340 \pm 0.034) \times 10^{-2} , \]
\[ N = 1.52 \pm 0.15 \text{ microcuries.} \]

When these values are averaged, there results for the efficiency of the gamma-counter at 1.25 MeV, and for the efficiency of the spectrometer for monoenergetic electrons:

\[ e_\gamma = (0.728 \pm 0.051) \times 10^{-3} , \]
\[ \omega = (0.351 \pm 0.024) \times 10^{-2} . \]

This value for \( e_\gamma \) applies only when the gamma-counter is used with the 3.97 g/cm\(^2\) brass plug. It will be of more general interest to correct this value to refer to the extrapolated efficiency with no absorbing material between the source and the interior of the gamma-counter. It will thus be necessary to correct for absorption in the brass plug, in the brass counter window, and in the phosphor bronze foil attached to the end of the source holder. The absorption coefficient of brass is taken to be the same as that of copper. The absorption
From the data of Weather, Fyles and Preece (19) :

\[ x = 0.282 \]

From the data of Hartin and Whitten (37)

\[ x = 0.372 \]

corresponding to the same way may be determined.

The number of internal conversion \( \lambda \) in the conversion per disintegration \( Q \) begins to be introduced in 1956. From the data of these authors, the decay scheme is introduced in 1965. The decay scheme (97) and by Weather, Fyles and Preece (19) the decay is approximately known previously been studied by Hartin and Whitten (37) who have recently been studied by Hartin and Whitten (37).

By means of a simultaneous measurement method,

the efficiency of the gamma-counter at two other points may be evaluated.

Since the efficiency of a gamma-counter is in general a function

where the superscript \( g \) refers to the extrapolated efficiency

\[ \lambda_0 = 1.25 \text{ keV} \]

Where the experimental efficiency of the gamma-counter is

this correction is made, the result for the extrapolated efficiency

when compared with other \( 37 \) and comparison and addition (29) p. 600-606.

Absorption corrections are taken from data of Rafter (34), p. 160.

\( \lambda_0 \) in Handbook of Chemistry and Physics (36) p. 1201.

Correction of prochorop bromine is taken to be that of a mixture of 60%
Fig. 15. Decay scheme of ThB → ThC.
The average of these two values is:

\[ \alpha_K = 0.31. \]

If the number of L-converted electrons per disintegration (\( \alpha_L \)) is given by *:

\[ \frac{\alpha_L}{\alpha_K} = \frac{15}{115}, \quad \alpha_L = 0.04, \]

and if \( \alpha_M, \alpha_N, \) etc. are neglected, then the number of gamma-quanta per disintegration is given by

\[ \gamma = 0.88 - (0.31 + 0.04) = 0.53. \]

The electron spectrum of ThB is such that in the vicinity of the K conversion line (ThB - F line) the only significant contribution of the beta-spectrum is due to \( \beta_2 \), which is in coincidence with the gamma-ray. Now let the spectrometer be set to focus electrons due to the ThB - F line, which is superimposed on the spectrum of \( \beta_2 \). The spectrometer counting rate will be given by:

\[ N_1 = 0.31 N e_g + 0.88 A H \omega. \]

There will be coincidences between the internal conversion electrons of the F line and their associate 0.090 Mev X-rays, between the beta-rays and these X-rays, and between the beta-rays and the 0.1238 Mev gamma-rays. The resulting coincidence rate is then given by:

\[ N_c = 0.31 N e_\gamma(0.090 \text{ Mev}) + N A \omega[0.31 e_\gamma(0.090 \text{ Mev}) + 0.53 e_\gamma(0.238 \text{ Mev})], \]

* For information concerning conversion coefficients in the naturally radioactive elements, see Rosetti (140, p. 134).
The following experiment was conducted as described above to test the formation of a composite extensional split in the known center of gravity. The extensional split was not determined to be of the order of 0.9. Although the experiment was designed to determine the extensional split from the equation:

\[ \frac{0.05 \times 10^7}{(0.9)^2 + (0.5)^2} \]

and

\[ \frac{10^7}{(0.9)^2 + (0.5)^2} \]

then

\[ \frac{10^7}{(0.9)^2 + (0.5)^2} \]

becomes

so that the value is no longer meaningful. Now let the extensional center be determined strictly from the graph at x = 0, for

where \( c \) is the source of error in the experiment. As the source is very thin, and were
where the efficiencies extrapolated to zero thickness of material between source and counter interior are given. Using these values, together with the previously determined value at 1.25 Mev, a curve of \( e_x \) as a function of gamma-ray energy is plotted in Fig. 16. Also plotted in Fig. 16, is a curve indicating the variation of efficiency of a platinum gamma-counter as calculated by the method of Von Droste (30). The calculated efficiency curve is normalized arbitrarily, since the effective solid angle subtended by the counter is not known.

E. Decay Scheme of \(^{181}\)Hf

The 46 day activity due to \(^{181}\)Hf has been studied by many investigators (41-50, 52). The decay scheme most generally accepted until recently was that originally proposed by Chu and Wiedenbeck (41) and indicated in Fig. 17. It has recently been suggested by Deutsch and Hedgran (52) that this decay scheme cannot be correct, since these authors find the transition corresponding to \( \gamma_3 \) to be associated with a longer lived activity than \(^{181}\)Hf. It will be the purpose of the investigation described here to use the coincidence method to analyze the decay scheme of \(^{181}\)Hf.

Analysis of the coincidence data assuming the decay scheme of Chu and Wiedenbeck: Let it be assumed first that the decay scheme of Chu and Wiedenbeck is correct, with the exception of possible rearrangements of the order of the various transitions.

In Table II are given the conversion probabilities of internal
Fig. 16. Efficiency of platinum grid gamma-counter.
Fig. 17. Decay scheme of Hf$^{181}$ proposed by Chu and Wiedenbeck.
The number of each type of transition was found to be in narrow limits. Let

\[ I, \] 

from the above information, it is possible to decide the transition

of the quantum intensities of \( \nu \) to \( \nu' \) \( < \nu, \) if the result for the ratio of the average of these two values will be used. The ratio of the present information gives a ratio of 1.2. For the purpose of the present investigation, a comparison of the transition peaks

\[ \nu = \nu' \] 

for the comparison of the components of the transition gives a ratio of 1.2. A comparison of the transition peaks obtained both components of transition and projection spectra shows that these two methods are a good test for a peak-except-exception (I) have measured the ratio of \( \nu' \) to \( \nu \) in addition to the conversion products listed in Table II.

<table>
<thead>
<tr>
<th>( \text{H} ) &amp;</th>
<th>( \text{H} ) &amp;</th>
<th>( \text{H} ) &amp;</th>
<th>( \text{H} )</th>
</tr>
</thead>
<tbody>
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<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
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<td>1.050</td>
<td>950</td>
<td>950</td>
</tr>
<tr>
<td>0.8</td>
<td>1.250</td>
<td>220</td>
<td>220</td>
</tr>
<tr>
<td>0.7</td>
<td>1.350</td>
<td>960</td>
<td>960</td>
</tr>
</tbody>
</table>

Table II

The conversion products

H to \( \text{H} \)
represent the number of quanta of type \( i \) per decaying atom of \(^{151}\text{Hf} \), and let \( T_i \) represent the total number of transitions of type \( i \) per decaying atom. Thus

\[
T_i = \gamma_i + a_{1L} + a_{1M} + a_{1L} + \ldots
\]

The following relations may then be deduced:

\[
\begin{align*}
T_1 &= 1, \quad T_2 = T_3 = 0.306, \quad T_4 = 0.694 \\
\gamma_1 &= a_{2L} + a_{2M} = 0.074 \\
\gamma_2 &= a_{1L} + a_{1M} = 0.428 \\
\gamma_3 &= 0.232 \quad \gamma_4 = 0.666 \\
a_{1L} + a_{2L} &= 0.425 \\
a_{1M} + a_{2M} &= 0.089
\end{align*}
\]

The only undetermined quantities are \( a_{1L}, a_{2L}, a_{1M}, a_{2M} \), and consequently \( \gamma_1 \) and \( \gamma_2 \). Although there is not sufficient information available to determine these quantities uniquely, it is possible to obtain approximate values by considering two limiting cases:

**Case I:** The probability \( a_{1L} \) is as large as possible. There results:

\[
\begin{align*}
a_{1L} &= 0.425 \\
a_{2L} &= 0 \\
a_{1M} &\approx 0.009 \\
a_{2M} &\approx 0.08
\end{align*}
\]

\[
\begin{align*}
\gamma_1 &\approx 0.006 \\
\gamma_2 &\approx 0.006
\end{align*}
\]

**Case II:** The probability \( a_{2L} \) is as large as possible. There results:
\[ \alpha_{1L} = 0.339 \quad \alpha_{2L} = 0.086 \]
\[ \alpha_{1K} = 0.089 \quad \alpha_{2K} = 0 \]
\[ \gamma_1 = 0.012 \quad \gamma_2 = 0 \]

These two extreme cases do not lead to very different results when used to predict the data to be expected from spectrometer coincidence measurements. For the purpose of the present investigation, Case I will be assumed. The decay scheme indicated in Fig. 15 results from Case I.

The results of the first set of coincidence measurements are indicated in Table III. These measurements were made with the spectrometer operating at a half-width of \(4.9\%\) and a solid angle \(\omega\) of \(0.35\%\). Due to the thickness of the \(^{181}\text{Hf}\) source used, the spectrometer efficiency was reduced by a factor of approximately 3 when focusing on the conversion line \(\alpha_{1L}\). Thus \(S(\alpha) \approx 1/3\) (Eq. 64). Furthermore since a large window (2.2 cm diam) counter was used as the spectrometer counter, and a correspondingly thick window was required, the counter efficiency \(e_c\) was considerably reduced for energies corresponding to the conversion line \(\alpha_{1K}\). For this reason, no coincidence measurements were made with the spectrometer focused on the line \(\alpha_{1K}\). The platinum grid gamma-counter was used as \(C_2\). The total thickness of material between the source and the gamma-counter interior was 0.241 g/cm\(^2\) of phosphor bronze and 0.0422 g/cm\(^2\) of brass.

Column 1 in Table III indicates the code number of the experiment.
Fig. 13. Proposed decay scheme of Hf$^{181}$ with conversion probabilities and gamma-ray intensities deduced from the data of Chu and Wiedenbeck.
<table>
<thead>
<tr>
<th>Expt. No.</th>
<th>Spectral Component</th>
<th>( N_0 ) ( \times 10^3 )</th>
<th>( N_0/N_2 ) ( \times 10^3 )</th>
<th>( N_0/N_1 ) ( \times 10^3 )</th>
<th>( N_0/N_1 ) (Calc) ( \times 10^3 )</th>
<th>( \tau_c )</th>
<th>Delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \alpha_{1L} )</td>
<td>± 0.14</td>
<td>± 0.022</td>
<td>± 0.080</td>
<td>0.62</td>
<td>1.5</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>( \alpha_{4K} )</td>
<td>± 0.192</td>
<td>± 0.038</td>
<td>± 0.83</td>
<td>0.44</td>
<td>1.5</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>( \alpha_{4K} )</td>
<td>± 0.030</td>
<td>± 0.005</td>
<td>± 0.30</td>
<td>0.30</td>
<td>1.5</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>( \beta )</td>
<td>± 0.164</td>
<td>± 0.010</td>
<td>± 0.20</td>
<td>0.10</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>( \alpha_{1L} )</td>
<td>± 0.140</td>
<td>± 0.023</td>
<td>± 0.085</td>
<td>0</td>
<td>3</td>
<td>6 ( \mu s ) in ( C_1 )</td>
</tr>
<tr>
<td>6</td>
<td>( \beta )</td>
<td>± 0.062</td>
<td>± 0.011</td>
<td>± 0.089</td>
<td>0.17</td>
<td>3</td>
<td>6 ( \mu s ) in ( C_1 )</td>
</tr>
<tr>
<td>7</td>
<td>( \alpha_{1L} )</td>
<td>± 0.12</td>
<td>± 0.02</td>
<td>± 0.07</td>
<td>0</td>
<td>3</td>
<td>6 ( \mu s ) in ( C_2 )</td>
</tr>
</tbody>
</table>
Column 2 indicates the part of the $^{151}$Hf electron spectrum on which the spectrometer was focused. Column 3 indicates the coincidence counting rate, corrected for coincidences due to the beta-spectrum in cases where the spectrometer was focused on a conversion line superimposed on this spectrum. Columns 4 and 5 indicate the ratio of the coincidence rate to the counting rates in the gamma-counter and in the spectrometer respectively. The counting rate in the spectrometer, $N_1$, was corrected where necessary for contributions due to the beta-spectrum on which a focused conversion line was superimposed. Column 5 indicates the ratio $N_0/N_1$ calculated on the basis of the decay scheme of Fig. 15. These calculations were performed in the same manner as in the determinations of counter efficiency previously discussed. The gamma-counter efficiency, as determined from Fig. 16, with corrections for absorption, was used in these calculations. Due to the fact that the ratio $N_0/N_1$ does not depend on the spectrometer efficiency $e_s$, it was unnecessary to evaluate $e_s$. Columns 6 and 7 indicate respectively the resolving time $\tau_0$ and any delay introduced into either counting circuit.

Consider first the experiments in which the spectrometer was focused on the conversion line $\alpha_{1L}$. It is seen that in experiment 1 the agreement between the calculated and observed ratio $N_0/N_1$ is fairly good. In experiments 5 and 7, delayed coincidences between $\gamma_1$ and the other gamma-radiation were sought. It is seen that no such coincidences were observed. Since it appears certain that the 20 microsecond metastable state is the initial state of the transition represented by $\gamma_1$, the above
results rule out the possibility that this transition is preceded by \( \gamma_2 \), \( \gamma_3 \) and \( \gamma_4 \).

Further verification of this is furnished by the measurement in which the spectrometer was focused on the beta-spectrum alone. Measurements were made both with no delay and with a 6 microsecond delay in the spectrometer counter circuit. The observed ratios were:

\[
\frac{N_0}{N_1} = (0.24 \pm 0.06) \times 10^{-3}, \text{ no delay}
\]

\[
= (0.09 \pm 0.02) \times 10^{-3}, \text{ with delay.}
\]

If the decay scheme of Fig. 15 is assumed, the calculated ratios are:

\[
\frac{N_0}{N_1} = 0.10, \text{ no delay}
\]

\[
= 0.17, \text{ with delay.}
\]

On the other hand, if the metastable state is preceded by \( \gamma_2 \), \( \gamma_3 \) and \( \gamma_4 \), then the calculated ratios are:

\[
\frac{N_0}{N_1} = 1.0, \text{ no delay}
\]

\[
= 0, \text{ with delay.}
\]

Although the observed coincidence rate was so low that the statistics are poor, the measurements are in better agreement with the decay scheme shown in Fig. 15 than with the alternative possibility.

When the spectrometer was focused on the conversion lines \( \alpha_{3K} \) and \( \alpha_{4K} \), no interpretable results were obtained. The results from experiments 2 and 3 were:

\[\alpha_{3K}: \quad \frac{N_0}{N_1} (\text{observed}) = (0.83 \pm 0.19) \times 10^{-3}\]

\[\frac{N_0}{N_1} (\text{calculated}) = 0.4 \times 10^{-3}\]

\[\alpha_{4K}: \quad \frac{N_0}{N_1} (\text{observed}) = (0.71 \pm 0.11) \times 10^{-3}\]

\[\frac{N_0}{N_1} (\text{calculated}) = 0.3 \times 10^{-3}\].
The results of an experiment on measurement are shown in Fig. 19.

The resistance of a conductor that appears to indicate a low resistance is not considered but does appear to indicate a low resistance in the experiment. The experimental data were obtained from the source, from which the comparison was made by subtracting the source from the current-comparator by means of an observed resistance. The experimental data of this comparison were inserted, and such considerations should be neglected completely to the extent that on the present situation of the condition it is obvious that the comparison has been considered. Comparison based on the present situation is not considered. Comparison based on several due to the present situation has been considered. Comparison based on several due to the present situation has been considered. Comparison based on several due to the present situation has been considered. Comparison based on several due to the present situation has been considered. Comparison based on several due to the present situation has been considered. Comparison based on several due to the present situation has been considered.
Fig. 19. Electron-gamma coincidence absorption measurement with Hf-181. The curves a, b and c represent calculated absorption curves.
\( \frac{N_0}{N_1} \), normalized to unity at zero absorber thickness is indicated in Fig. 19, as a function of the absorber thickness. Also indicated in this figure are curves representing the behavior to be expected if \( \gamma_1 \) is in coincidence with (a) both \( \gamma_3 \) and \( \gamma_4 \), (b) \( \gamma_4 \) only, and (c) \( \gamma_3 \) only. The latter possibility is clearly ruled out by the observed data. Although the data are not sufficiently precise to distinguish definitely between the first two cases, the agreement with case (a) is somewhat better than with case (b).

The results of the next set of coincidence measurements are indicated in Table IV. The spectrometer was again adjusted for a half-width of 4.5% and a solid angle of 0.35%. The end-window beta-counter was used as \( O_2 \). The mica window thickness of the counter and the mica backing of the source resulted in a total thickness of mica of 5 mg/cm² between the source and the interior of the beta-counter, \( O_2 \). Column 1 in Table IV indicates the code number of the experiment, and column 2 indicates the region of the \(^{181}\text{Hf} \) electron spectrum on which the spectrometer was focused. Column 3 indicates the observed coincidence rate, and columns 4 and 5 indicate the observed and calculated ratios \( \frac{N_0}{N_1} \). Both \( N_0 \) and \( N_1 \) were corrected for contributions due to the beta-spectrum on which the internal conversion lines are superimposed. Column 6 indicates the geometric solid angle subtended by \( O_2 \); column 7 indicates the resolving time \( T_c \), and column 8 indicates the thickness of any absorbing material between the source and the interior of \( O_2 \). Column 9 indicates any delay introduced into either counter circuit. The calculated ratios \( \frac{N_0}{N_1} \) were computed in the same way as for the case of electron-gamma-ray
<table>
<thead>
<tr>
<th>Expt. Spectral No.</th>
<th>N&lt;sub&gt;c&lt;/sub&gt; (c/m)</th>
<th>N&lt;sub&gt;c&lt;/sub&gt;/N&lt;sub&gt;1&lt;/sub&gt; x 10&lt;sup&gt;-3&lt;/sup&gt;</th>
<th>N&lt;sub&gt;c&lt;/sub&gt;/N&lt;sub&gt;1&lt;/sub&gt; (Calc) x 10&lt;sup&gt;-3&lt;/sup&gt;</th>
<th>Solid Angle for C&lt;sub&gt;2&lt;/sub&gt; (%)</th>
<th>T&lt;sub&gt;c&lt;/sub&gt; (μs)</th>
<th>Abs (mg/cm&lt;sup&gt;2&lt;/sup&gt;)</th>
<th>Delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 a&lt;sub&gt;1L&lt;/sub&gt;</td>
<td>± 0.11 ± 0.22</td>
<td>2.61</td>
<td>2.34</td>
<td>2</td>
<td>1.0</td>
<td>5 (mica)</td>
<td>0</td>
</tr>
<tr>
<td>2 a&lt;sub&gt;1L&lt;/sub&gt;</td>
<td>± 0.04 ± 0.09</td>
<td>0.63</td>
<td>0.52</td>
<td>2</td>
<td>1.6</td>
<td>21.6 (Al)</td>
<td>0</td>
</tr>
<tr>
<td>3 a&lt;sub&gt;1L&lt;/sub&gt;</td>
<td>± 0.13 ± 0.30</td>
<td>2.38</td>
<td>2.84</td>
<td>2</td>
<td>2.9</td>
<td>5 (mica)</td>
<td>0</td>
</tr>
<tr>
<td>4 a&lt;sub&gt;1L&lt;/sub&gt;</td>
<td>± 0.55 ± 1.28</td>
<td>0.30</td>
<td>1.25</td>
<td>2</td>
<td>2.9</td>
<td>5 (mica)</td>
<td>6 μs</td>
</tr>
<tr>
<td>5 a&lt;sub&gt;1L&lt;/sub&gt;</td>
<td>± 0.12 ± 0.27</td>
<td>0.02</td>
<td>0.02</td>
<td>2</td>
<td>2.9</td>
<td>5 (mica)</td>
<td>6 μs</td>
</tr>
<tr>
<td>6 a&lt;sub&gt;3K&lt;/sub&gt;</td>
<td>± 0.23 ± 0.56</td>
<td>2.13</td>
<td>8.54</td>
<td>2</td>
<td>1.1</td>
<td>5 (mica)</td>
<td>0</td>
</tr>
<tr>
<td>7 a&lt;sub&gt;3K&lt;/sub&gt;</td>
<td>± 0.04 ± 0.41</td>
<td>0</td>
<td>0.41</td>
<td>5</td>
<td>1.2</td>
<td>21.6 (Al)</td>
<td>0</td>
</tr>
<tr>
<td>8 a&lt;sub&gt;4K&lt;/sub&gt;</td>
<td>± 0.04 ± 0.6</td>
<td>4.41</td>
<td>4.95</td>
<td>2</td>
<td>1.2</td>
<td>5 (mica)</td>
<td>0</td>
</tr>
<tr>
<td>9 a&lt;sub&gt;4K&lt;/sub&gt;</td>
<td>± 0.053 ± 0.16</td>
<td>0.8</td>
<td>0.16</td>
<td>5</td>
<td>1.3</td>
<td>21.6 (Al)</td>
<td>0</td>
</tr>
</tbody>
</table>
Table IV (cont'd)

<table>
<thead>
<tr>
<th>Expt. No.</th>
<th>Spectral Component</th>
<th>$N_e$ (e/m)</th>
<th>$N_e/10^3$ x</th>
<th>$N_e/10^3$ (Calc)</th>
<th>Solid Angle for C₂ (%°)</th>
<th>$\gamma_0$ (µs)</th>
<th>Abs (mg/cm²)</th>
<th>Delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>β</td>
<td>± 0.042</td>
<td>± 0.22</td>
<td>0.34</td>
<td>2</td>
<td>1.5</td>
<td>5</td>
<td>(mica) 0</td>
</tr>
<tr>
<td>11</td>
<td>β</td>
<td>± 0.058</td>
<td>± 0.35</td>
<td>0.05</td>
<td>5</td>
<td>1.2</td>
<td>21.6</td>
<td>(Al) 0</td>
</tr>
<tr>
<td>12</td>
<td>β</td>
<td>± 0.148</td>
<td>± 0.86</td>
<td>0.67</td>
<td>2</td>
<td>2.8</td>
<td>5</td>
<td>(mica) 0</td>
</tr>
<tr>
<td>13</td>
<td>β</td>
<td>± 0.059</td>
<td>± 0.60</td>
<td>1.12</td>
<td>2</td>
<td>2.8</td>
<td>5</td>
<td>(mica) 6 µs</td>
</tr>
</tbody>
</table>
coincidences. In this case, however, it was necessary to know the efficiency of the beta-counter in place of that of the gamma-counter used previously. This efficiency was assumed to be given by the product of the solid angle subtended at the source and of the transmission of electrons in the material between the source and the interior of the beta-counter, \( C_2 \). The transmission was obtained by measuring the spectrum of Compton electrons from \( \text{Co}^{60} \) in the spectrometer, using various thicknesses of window material in front of the spectrometer counter. The absorption correction curves obtained this way are indicated in Fig. 20. In determining the efficiency of \( C_2 \) for the beta-spectrum, it was necessary to integrate the product of the theoretical allowed beta-spectrum of \( \text{Hf}^{181} \) (Feister (35)) and of the counter window transmission factor.

Consider first the coincidence rates obtained when the spectrometer was focused on \( \alpha_{1L} \). Instantaneous coincidences in this case were to be expected with the conversion electrons due to the transitions corresponding to \( \gamma_2, \gamma_3 \) and \( \gamma_4 \), and delayed coincidences with the beta-spectrum. Experiments 1 and 2 show good agreement between observed and calculated values using two different absorber thicknesses between the source and \( C_2 \). Experiments 3, 4 and 5 indicate the results of introducing delays into each of the counter circuits. In experiment 4, in which a delay was introduced in the circuit of \( C_2 \), coincidences were to be expected between the beta-rays, and the conversion electrons represented by \( \alpha_{1L} \). It is seen that there is good agreement between the calculated and observed values. This rules out the possibility that the metastable state occurs after the transition \( \gamma_1 \). This conclusion is also verified by experiment 5,
Fig. 20. Transmission of mica and aluminum films for electrons of different momenta.
which indicated no delayed coincidences between $\gamma_1$ and any subsequent transitions.

In the case where the spectrometer was focused on $\alpha_{3K}$ or on $\alpha_{4K}$, it is seen from experiments 6-9 that the coincidence rates were reduced by a large factor when a 21.6 mg/cm$^2$ aluminum absorber (which is of a thickness approximately equal to the range of the conversion electrons corresponding to the transition $\gamma_1$) was introduced between the source and $C_2$. This is in agreement with the decay scheme of Fig. 18, which indicates that $\gamma_3$ and $\gamma_4$ are in coincidence with $\gamma_1$.

In experiments 10-13, the spectrometer was focused on the beta-spectrum alone. Although the errors are large, an order of magnitude agreement with the decay scheme of Fig. 18 is indicated by these results.

Analysis of the coincidence data assuming the modification of Deutsch and Hedgran: Consider now the modification of the decay scheme of Hf$^{181}$ proposed by Deutsch and Hedgran (52). These authors suggest that $\gamma_3$ is not included in this decay scheme, since they have found the relative intensities of $\gamma_3$ and $\gamma_4$ to differ in sources of different age. A slight increase with time in the ratio of $\alpha_{3K}$ to $\alpha_{4K}$ was also observed in the course of the present work. However, over a period of 140 days, this ratio increased by only a factor of 1.24 whereas an increase by a factor of 2 would be expected if these peaks were due to Hf$^{175}$ and Hf$^{181}$ respectively. Since the source was of a moderate thickness and was deposited somewhat loosely, it is not impossible that some change in the observed ratio $\alpha_{3K}/\alpha_{4K}$ could be caused by movement of the source deposit with a corresponding change
Table V
Summary of Coincidence Measurements with Hf$^{181}$

<table>
<thead>
<tr>
<th>Expt. Spectral No. Component</th>
<th>$N_c/N_1$ $x 10^3$</th>
<th>$N_c/N_1$ $x 10^3$</th>
<th>$N_c/N_1$ $x 10^3$</th>
<th>$N_c/N_1$ $x 10^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 $\alpha_{1L}$ $\pm$ 0.080</td>
<td>0.88</td>
<td>0.90</td>
<td>0.86</td>
<td></td>
</tr>
<tr>
<td>2 $\alpha_{3L}$ $\pm$ 0.73</td>
<td>0.44</td>
<td>0.38</td>
<td>0.52</td>
<td></td>
</tr>
<tr>
<td>3 $\alpha_{4L}$ $\pm$ 0.31</td>
<td>0.30</td>
<td>0.34</td>
<td>0.30</td>
<td></td>
</tr>
<tr>
<td>4 $\beta$ $\pm$ 0.08</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>5 $\alpha_{1L}$ $\pm$ 0.066</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>6 $\beta$ $\pm$ 0.079</td>
<td>0.17</td>
<td>0.17</td>
<td>0.16</td>
<td></td>
</tr>
<tr>
<td>7 $\alpha_{1L}$ $\pm$ 0.07</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Gamma - electron measurements

<table>
<thead>
<tr>
<th>Expt. Spectral No. Component</th>
<th>$N_c/N_1$ $x 10^3$</th>
<th>$N_c/N_1$ $x 10^3$</th>
<th>$N_c/N_1$ $x 10^3$</th>
<th>$N_c/N_1$ $x 10^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 $\alpha_{1L}$ $\pm$ 2.61</td>
<td>2.34</td>
<td>1.9</td>
<td>0.82</td>
<td></td>
</tr>
<tr>
<td>2 $\alpha_{1L}$ $\pm$ 0.30</td>
<td>0.52</td>
<td>0.39</td>
<td>0.36</td>
<td></td>
</tr>
<tr>
<td>3 $\alpha_{1L}$ $\pm$ 2.38</td>
<td>2.64</td>
<td>2.41</td>
<td>1.32</td>
<td></td>
</tr>
<tr>
<td>4 $\alpha_{1L}$ $\pm$ 1.28</td>
<td>1.25</td>
<td>1.25</td>
<td>1.25</td>
<td></td>
</tr>
<tr>
<td>5 $\alpha_{1L}$ $\pm$ 0.27</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Electron - electron measurements
**Table V (cont'd)**

<table>
<thead>
<tr>
<th>Expt. No.</th>
<th>Spectral Component</th>
<th>$\frac{N_0}{N_1}$ obs</th>
<th>$\frac{N_0}{N_1}$ C and D</th>
<th>$\frac{N_0}{N_1}$ D and $\text{H}(1)$</th>
<th>$\frac{N_0}{N_1}$ D and $\text{H}(II)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>$\alpha_{3K}$</td>
<td>$\pm 0.56$</td>
<td>8.54</td>
<td>0.2</td>
<td>0.7</td>
</tr>
<tr>
<td>7</td>
<td>$\alpha_{3K}$</td>
<td>$\pm 0.41$</td>
<td>0.41</td>
<td>0.5</td>
<td>0.8</td>
</tr>
<tr>
<td>8</td>
<td>$\alpha_{4K}$</td>
<td>$\pm 0.6$</td>
<td>4.95</td>
<td>6.1</td>
<td>4.8</td>
</tr>
<tr>
<td>9</td>
<td>$\alpha_{4K}$</td>
<td>$\pm 0.3$</td>
<td>0.16</td>
<td>0.23</td>
<td>0.15</td>
</tr>
<tr>
<td>10</td>
<td>$\beta$</td>
<td>$\pm 0.15$</td>
<td>0.34</td>
<td>0.33</td>
<td>0.27</td>
</tr>
<tr>
<td>11</td>
<td>$\beta$</td>
<td>$\pm 0.35$</td>
<td>0.05</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>12</td>
<td>$\beta$</td>
<td>$\pm 0.32$</td>
<td>0.67</td>
<td>0.61</td>
<td>0.51</td>
</tr>
<tr>
<td>13</td>
<td>$\beta$</td>
<td>$\pm 0.34$</td>
<td>1.12</td>
<td>1.0</td>
<td>0.8</td>
</tr>
</tbody>
</table>
These measurements do not serve to distinguish between the complete
decay scheme of Chu and Wiedenbeck and the modification proposed by
Deutsch and Hedgran. With a few discrepancies, for which no reasonable
explanation has been found, both the electron-electron measurements
and the gamma-electron measurements are in fair agreement with either
alternative.
VI. LITERATURE CITED


VII. ACKNOWLEDGEMENTS

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