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Tree Height Determination From Aerial Photographs In Iowa

Allen W. Goodspeed

At least two projects in the Forestry Section of the Iowa State College Agricultural Experiment Station now use, or are planning to use, aerial photographs as a basis for classifying forest stands, and stand height is one of the factors used in the classification.

Available for the work are United States Department of Agriculture vertical photographs with an average scale of about 1:20,000 or 3.17 inches per mile. In general, stand classification has followed the pattern used by the Central States Forest Experiment Station in its work on the National Forest Survey.

The parallax method of stand height determination was selected for Iowa as best meeting the requirements of general applicability, speed, and accuracy sufficient for classification purposes. The custom of running many roads and fences along section lines makes most photographs easy to scale and the relatively flat nature of the terrain has permitted a modification of the parallax formula and its presentation in convenient alinement chart form without significant loss in accuracy over the elevation ranges commonly encountered.

![Diagram of geometric relations in vertical photographs](image)

Geometric relations in vertical photographs

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Photographs, a simple lens stereoscope, engineer's scale, a parallax wedge and the alinement chart are the interpreter's basic equipment and height results have been reported satisfactory as judged by subsequent field checks.

It is the purpose of this article to discuss the parallax formula and to present the alinement chart used, in the hope it may prove useful to others working under conditions similar to those found in Iowa.

The basic parallax formula, on which the determination of tree heights depends, can easily be derived by considering the geometry of Figure 1, representing two vertical photographs and the ground area common to each.

**Figure 1**

In Figure 1 O₁, and O₂, are the photo stations separated by the air base B. Their photographic equivalents are the principal points, o₁, o₂. H is the flight height above the base plane, h the height from the base plane to the top of the tree, and f the focal length of the lens used. The points t₁ and t₂ are the apparent positions of the tree top at the base level as seen by the camera from photo stations 1 and 2 respectively. Parallax difference, Pd, on the base plane has its photographic equivalent pd, the sum of d₁ and d₂ on the photographs.

From an examination of Figure 1 the following proportion can be established at once:

\[
\frac{h}{H-h} = \frac{B}{B}
\]

Rearrangement of the terms in \(-1\) leads to an equation for h as follows:

\[
h = H \cdot \frac{p}{B+pd}
\]

Replacement in \(-2\) of Pd and B by their photographic equivalents pd and p, o₁-o₂, leads to the usual form of the parallax formula as used to determine elevations:

\[
h = H \cdot \frac{pd}{p+pd}
\]

In the above equation h and H are in feet while p and pd, measured on the photographs, are in inches.

Inspection of the parallax formula reveals that h varies directly as H and that when pd is small as compared to p, the usual case in practice, h varies almost exactly as pd. This latter point is extremely important in the practical determination of tree heights.

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When the principal points are at the same elevation and $H$ is the flight height above this elevation the formula as it stands will give the correct height to any point above the base plane. Thus in Figure 1 if $H$ is 13,750 feet, $p$ is 2.80 inches, and the parallax difference, $pd$, between the top of the tree and a point on the base plane is .0205 inches, use of the formula will give a tree height of 100 feet.

When the bottom of a tree is not at the base level its height can be correctly determined as a difference in heights by making parallax difference measurements to its top, $pd_2$, and its base, $pd_1$, and by solving the parallax formula twice. If we denote $pd_2 - pd_1$, as $\Delta p$ the two separate solutions of the parallax formula can be combined into a single equation for tree height as follows:

$$h = \frac{H_p \cdot \Delta p}{(p + pd_2)(p + pd_1)}$$

The location of the principal points on the base plane has been assumed in the discussion so far. Under this condition the value of $p$ can easily be determined by measuring on either photograph the distance between its principal point and the transferred principal point of the adjacent photograph. In practice, unfortunately, adjacent principal points are seldom at the same elevation and in consequence the two values of $p$ so obtained will not agree. A working solution of this problem that has found considerable sanction (1), (2), (3) is to use the average value of $p$ as found by measurement on each photograph of a stereoscopic pair. The base plane so determined lies in elevation somewhere between the principal points.

The term $pd$ also needs further explanation. As so far used it has been the difference in parallax between the point for which the elevation was sought and a point on the base plane. The heart of the parallax method lies in the fact that points of equal parallax difference with respect to the base plane have the same elevation above or below this plane.

In determining the height of a tree not on the base plane, what is required is the parallax difference between the tree’s tip and base. This is truly a parallax difference, but not one referred to the base plane, and to avoid any possible confusion it seems better to represent it by the symbol $\Delta p$ as in equation —4— for tree height. Writers frequently do not make this distinction although it is implicit in their descriptions of the parallax formula’s use.

While equation —4— is analytically correct, it is awkward in use, especially when the principal points are at different eleva-

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tions. Under such circumstances the location of the base plane on the photographs may not be exactly known and the measurement of \( p_d \) and \( p_d \) may in consequence prove troublesome. In addition formula \( -4 \) is slow so we seek a more rapid method for finding tree heights.

A practical answer to the problem of speed and ease of operation is the parallax wedge, the nature and use of which has been well explained by Spurr (3). This device is particularly suitable for the rapid measurement of parallax differences between points of different elevations on stereoscopic pairs of aerial photographs. With the wedge the value of \( \Delta p \) is observed between the top and bottom of a tree. The value of \( p \) is obtained by direct measurement. These two values together with flight height or photo scale lead to a speedy solution of the parallax formula for tree height. Theoretically, this solution is correct only when the base plane is at the same elevation as the bottom of the tree, but in comparatively flat areas the error incurred by the procedure outlined is not great.

Simple tables have been drawn up from curves prepared by the Northeastern Forest Experiment Station which show the change in elevation for .001 inches of parallax difference and requiring for entry only the value of \( p \) and the photo scale expressed as a representative fraction, RF. These tables have still further quickened the determination of tree heights.

Currently an alinement chart based on a slight modification of the parallax formula is being used for finding tree and stand heights in Experiment Station forestry projects in Iowa.

Reference to Figure I will show that when \( H \) and \( f \) are expressed in the same linear units the representative fraction for a photograph may be expressed as:

\[
RF = \frac{f}{H} \tag{5}
\]

By rearrangement of the terms in \( -5 \) an equation for \( H \) is obtained:

\[
H = \frac{f}{RF} \tag{6}
\]

If in the parallax equation \( -3 \) \( H \) is replaced by its equivalent, \( f/RF \), there results:

\[
h = \frac{f \cdot p_d}{RF (p + p_d)} \tag{7}
\]

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As already mentioned, Iowa has rather flat terrain conditions so $pd$ in the denominator is a relatively non critical value and may be omitted. In the numerator $pd$ is replaced by $\Delta p$ as it is tree height we are after. When these changes are made there results the parallax formula for tree height used in alinement chart construction.

$$h = \frac{f \cdot \Delta p}{RF \cdot p}$$

In constructing an alinement chart from the above formula the focal length in feet of the lens used replaces $f$ and a value of .001 inches is assigned to $\Delta p$. The formula can then be thrown into logarithmic form and an alinement chart prepared by the usual methods. The independent variables are $RF$ and $p$ and the dependent variable is the change in elevation, or tree height, in
feet for .001 inches of \( \Delta p \). Such a chart for a focal length of 8.25 inches is presented in Figure 2. In use the same factors are required for entry as with the Forest Service Tables. Tree height is obtained as the product of the observed \( \Delta p \) and the value in feet of .001 inches of \( \Delta p \) as read from the chart.

\[ \Delta h = .001 \cdot f \]

\[ \frac{RF \cdot p}{9} \]

A word of caution is necessary with respect to the use of the alinement chart when the bottom of a tree is considerably above or below the base plane. The alinement chart solves the formula:

\[ \Delta h = .001 \cdot f \]

\[ \frac{RF \cdot (p + pd)}{10} \]

The parallax difference, \( pd \), in the above expression is that between the base of the tree and a point on the base plane. As previously noted, it is ordinarily non critical under Iowa conditions but it can be allowed for if an estimate, even only approximate, of the elevation difference between the base plane and the bottom of the tree can be made. The procedure follows.

Table 1. 100 Foot Height Determinations from the Alinement Chart Parallax Formula.

Using the measured photo scale, RF, and the average value of \( p \) determine from the alinement chart the height increment, \( \Delta h \), for .001 inches of \( \Delta p \). On dividing the estimated elevation difference by \( \Delta h \) an approximate value for \( pd \) will be obtained. If the bottom of the tree is above the base plane \( pd \) should be added to \( p \), if below it should be subtracted. The result is an estimate of the absolute stereoscopic parallax at the base of the tree, the implicit value of \( p \) in the correct application of the parallax formula. This value is then used to reenter the alinement chart for a new \( \Delta h \). Strictly speaking, the value of RF in the formula should also be corrected and \( \Delta p \) should be included in the denominator but these refinements have not been necessary under the usual Iowa conditions.

Table I represents the basic data and results, using the alinement chart, of 100 foot height determinations at successive 100 foot intervals above the base level. The starting information was a base \( p \) of 2.80 inches, a focal length of 8.25 inches and a scale of 1:20 000.
Table 1

<table>
<thead>
<tr>
<th>Elev. Ft.</th>
<th>( \Delta p'' ) for 100' interval</th>
<th>uncorrected ( \Delta h' ) per .001&quot; ( \Delta p ) interval</th>
<th>corrected ( \Delta h' ) per ( p + pd ) .001 ( \Delta p ) interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>0.0205</td>
<td>4.91 101</td>
<td>2.80 4.91 101</td>
</tr>
<tr>
<td>+100</td>
<td>0.0208</td>
<td>4.91 102</td>
<td>2.82 4.88 102</td>
</tr>
<tr>
<td>+200</td>
<td>0.0211</td>
<td>4.91 104</td>
<td>2.84 4.84 102</td>
</tr>
<tr>
<td>+300</td>
<td>0.0215</td>
<td>4.91 106</td>
<td>2.86 4.81 103</td>
</tr>
<tr>
<td>+400</td>
<td>0.0218</td>
<td>4.91 107</td>
<td>2.88 4.77 104</td>
</tr>
</tbody>
</table>

Column 2 of the table shows the actual values of \( \Delta p \) for the predetermined 100 foot intervals. The values are carried one decimal place farther than a parallax wedge can be read in order to show that parallax differences are nearly but not exactly the same for equal height increments at different elevations above the base.

Column 3 shows the value at the base level of \( \Delta h \) in feet per .001 inches of \( \Delta p \). A moment's reflection will show that an error of .002 inches in reading the wedge in this case will result in an error of nearly 10 feet in the estimate of tree height, an error larger than any incurred by using an uncorrected \( p \) over the range of heights exhibited, as may be seen from the height values so determined in column 4.

The values of \( p \) corrected in the manner previously explained are shown in column 5 and the corresponding values of \( \Delta h \) in column 6. Column 7 shows the heights obtained from the corrected data. The maximum error in height determination for a 100 foot tree the bottom of which is 400 feet above the base level is 4 feet, or less than the errors frequently inurred through inability to read the wedge as closely as .001 inches. In short, the alinement chart seems adequate for the parallax wedge with which it is designed to be used.

Values of \( \Delta h \) obtained from an alinement chart constructed for a given focal length are easily converted to corresponding values for other focal lengths by observing in formula \( -9- \) that \( \Delta h \) varies directly as \( f \). Thus to find the value of \( \Delta h \) correct for a focal length of 12 inches it is only necessary to multiply the reading from Figure 2 by 1.454. For a focal length of 6 inches the factor is .727.

Finally, it should be noted that the nature of the alinement chart permits a closer entry with \( p \) and RF than is possible with
certain forms of tables commonly used for parallax conversion purposes.

There is every reason to believe that aerial photographs will be increasingly used in forestry work throughout the country. Certainly the present trend is in that direction. In such work, the determination of tree and stand heights is frequently an important classification factor. With the advent of the parallax wedge the forester has been provided an inexpensive and useful tool for measuring parallax differences on aerial photographs and is in a position to determine tree heights by the parallax method.

In Iowa, with its comparatively flat terrain, the alinement chart solution of a modified parallax formula has provided a compact, speedy, and sufficiently accurate method of converting parallax differences into tree heights. Such charts may prove useful to others working under similar conditions.

REFERENCES

