THE EFFECT OF NONSPHERICAL Pores AND MULTIPLE SCATTERING ON THE
ULTRASONIC CHARACTERIZATION OF POROSITY

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INTRODUCTION

Recently, several papers have dealt with the use of the frequency dependent ultrasonic attenuation to characterize gas porosity in structural solids such as cast aluminum [1-4]. The methods proposed provide an estimate of the volume fraction of pores and the average pore size. The estimate of the volume fraction for laboratory samples has been sufficiently accurate that it encourages thoughts of routine industrial use. In the past year, we have considered several corrections which may be of use in transferring the laboratory methods to industrial samples. Three corrections have been developed. The first is a simple way of accounting for the effects of as-cast surface roughness on the inference of ultrasonic attenuation from phase-sensitive scattering measurements. This topic will be dealt with in the companion paper by Nagy et al. [5]. The second correction deals with the effects of nonspherical pores on the inference of the volume fraction. This effect can be very substantial (e.g., a distribution of microcracks may lead to a large ultrasonic attenuation while having zero volume fraction). Finally, the third correction deals with the effects of multiple scattering. That is, the methods used up until now have relied on the scattering being uncorrelated (which is reasonable for low density gas porosity, i.e., volume fractions <2%). However, at higher volume fractions this assumption begins to breakdown. We propose a strategy based on the Kramers-Kronig relations for including multiple scattering effects. The last two corrections are the subject of this paper.

Pore Shape Effects

The structure of this section is as follows. First, the effects of pore shape on the high frequency attenuation is briefly described. Next, the sensitivity of the methods given in Refs. [2-3] to nonsphericity is discussed for spheroidal pores. Finally, the results are summarized.

A plane wave pulse of longitudinally polarized sound is assumed to propagate in an infinite space filled with pores (voids). The attenuation of the coherent part of the amplitude, averaged over an ensemble of samples, is denoted by \( \alpha(k) \) where \( k \) is the wavevector. Finally, the pores are assumed to be distributed uniformly over space on the average. The basic theoretical approximation is that the porosity
is sufficiently dilute that each pore attenuates the beam independently. Consequently, the attenuation per pore is $1/2 \gamma$, where $\gamma$ is the total cross-section for longitudinal scattering. The total attenuation is then obtained from the sum of the total cross-sections.

In the high frequency limit, ray optics becomes valid and the total scattering cross-section approaches twice the geometrical cross-section. Consequently, the attenuation approaches the sum of the geometrical cross-sections of all the pores in a unit volume.

Below we imagine a space filled with nonspherical pores. One starts by defining a space of spherical pores of various sizes. These pores are then deformed in such a way that their shape remains convex and their initial volume is conserved. In the rest of this paragraph, we assume that these aspherical, deformed pores are randomly oriented. Van de Hulst [6] quotes a theorem that the average over angle of the geometrical cross-section of a convex body is equal to one-quarter of its surface area. It is well known that spheres have the lowest surface to volume ratio of any shape. Consequently, pores with any other convex shape will increase the attenuation in the high frequency limit; the increase being proportional to the ratio of the pore's surface area to that of an equal volume sphere. Since the attenuation is increased, one expects that techniques for determining the volume fraction which are based on the assumption of sphericity will tend to overestimate $c$.

For the purposes of nondestructive evaluation (NDE), this is a favorable result since it is generally considered more desirable to reject good parts unnecessarily than to accept bad parts. In Refs. [3] and [4] it was shown that in the dilute limit the volume fraction is given by

$$c = \frac{4}{3\pi A_2} \int_0^\infty \frac{\alpha(k)}{k^2} dk. \quad (1)$$

The constant $A_2$ is calculated for a distribution of arbitrarily shaped pores from the long wavelength limit of the forward scattering amplitude for each flaw. It can also be related to the long wavelength limit of the porosity induced shift, $\Delta v$, in the longitudinal sound velocity, $v_0$, by

$$A_2 = -2/3 \left( \frac{1}{c} \frac{\Delta v(k=0)}{v_0} \right). \quad (2)$$

The relationship between $\Delta v/v_0$ and $\alpha$ implied by Eqs. (1) and (2) stem from the Kramers-Kronig relations as discussed in the next section.

Norris [7] has calculated $\Delta v/v_0$ for spheroidal voids in an elastic solid assuming that the axes of the pores are randomly distributed. He found that the velocity shift is always smallest for spherical pores. Consequently, $A_2$ becomes larger as the aspect ratio of the spheroids increases.

In this paper, Eshelby's [8] solutions for the static stress on an ellipsoidal flaw due to a static uniform applied strain are used to compute $A_2$. Both oriented and random distributions are considered. The pores are all assumed to have the same shape in a given calculation,
although their sizes may vary. Finally, the results reported depend on the material properties of the host only through $\eta$, the ratio of the shear to longitudinal velocity.

In a given experiment, the degree of asphericity of the pores will often be unknown. Suppose an estimate for the volume fraction is nonetheless required. Then the evaluation of Eq. (1), assuming the pores are spherical, is one possible step. This procedure will lead to errors whose sizes are discussed below. Aluminum A357 alloy is of particular interest to us and hence results are reported for its velocity ratio, $\eta = 0.479$.

Consider a mixture of spheroids, all with the same aspect ratio, which are randomly oriented. For materials with $0.40 < \eta < 0.60$ Eq. (1), evaluated under the assumption that the pores are spherical, overestimates the volume fraction as follows: 2:1 prolates, 9%; 3:1 prolates 16-17%; 2:1 oblates 16-19%; and 3:1 oblate 55-57%.

If the pores are oriented, underestimates may occur as well. From an NDE point of view, this is a more serious problem than overestimating $c$. For prolate spheroids, the underestimate occurs when the axes of symmetry of all the pores are oriented parallel to the direction of incidence. For oblates it occurs when one of the major axes is oriented along the direction of incidence. Table I shows the underestimate for a collection of oriented spheroids with the given aspect ratio in comparison to a mixture of equal volume spherical pores.

The underestimate is seen to increase for larger aspect ratio prolates and for larger values of $\eta$. It is encouraging that for Al A357 the maximum value of the underestimate is 30%. Any degree of randomness in the orientation of the axes will reduce the underestimate.

Equation (1), evaluated assuming the pores are spherical, also leads to large overestimates for oriented nonspherical pores. The overestimates increase for larger values of $\eta$ and are largest for oblate spheroids whose axes of symmetry are along the direction of incidence. For prolate spheroids the largest overestimates occur when one of the minor axes is oriented along the direction of incidence. For both cases the overestimate correlates with (but is not proportional to) the geometrical cross-section. The calculated maximum overestimates for Al A357 alloy are for: 2:1 prolates, 25%, 3:1 prolates, 38%, 2:1 oblates, 78% and 3:1 oblates, 168%. Again, any randomness in the orientation decreases the overestimate.

An estimate for the mean size of spherical pores was given in Refs. [3,4], $a = \langle a^3 \rangle / \langle a^2 \rangle$. Here $\langle \ldots \rangle$ denotes an expectation value over the pore size distribution. Using these results one obtains

<table>
<thead>
<tr>
<th>Spheroid</th>
<th>$\eta = 0.40$</th>
<th>$\eta = 0.479$</th>
<th>$\eta = 0.59$</th>
<th>$\eta = 0.60$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2:1 prolate</td>
<td>12%</td>
<td>25%</td>
<td>30%</td>
<td>50%</td>
</tr>
<tr>
<td>3:1 prolate</td>
<td>12%</td>
<td>30%</td>
<td>35%</td>
<td>63%</td>
</tr>
<tr>
<td>2:1 oblate</td>
<td>0%</td>
<td>11%</td>
<td>15%</td>
<td>53%</td>
</tr>
<tr>
<td>3:1 oblate</td>
<td>-17%</td>
<td>0%</td>
<td>7%</td>
<td>43%</td>
</tr>
</tbody>
</table>
Below it is assumed that the aspect ratio of the pores is unknown. Consequently, Eq. (3) is evaluated using \( A_2 \) computed for spheres.

For \( 0.40 < \eta < 0.60 \) \( \bar{a} \) is changed as follows: +25% for 3:1 oblates, +9% for 2:1 oblates, -2% for 3:1 prolates, and 0% for 2:1 prolates. Thus, estimates for \( \bar{a} \) are less sensitively affected by asphericity than estimates for the volume fraction. The evaluation of \( \bar{a} \) was also carried out for oriented distributions of pores. The sensitivity to orientation is generally considerably less than the corresponding estimate of \( c \).

In summary, nonsphericity increases the attenuation for randomly oriented pores at high frequencies. Consequently, a tendency to overestimate the volume fraction appears. The size of the overestimate is found to depend strongly on flaw shape, but very weakly on material properties for \( 0.40 < \eta < 0.60 \). For oriented distributions of pores errors correlate with the geometrical cross-section of the pores normal to the direction of incidence. Orientation effects were least for smaller values of \( \eta \) and increased monotonically over the range studied.

**Multiple Scattering Corrections**

Below we present a strategy for including the effects of multiple scattering on the inference of \( c \) from \( \alpha(k) \). First, as is widely known, the volume fraction of porosity can be inferred from the porosity induced shift in the long wavelength sound velocity, \( \Delta v(k=0) \). Further, the effects of multiple scattering on the inference of \( c \) from \( \Delta v(k=0) \) have been widely discussed. Second, we note that the Kramers-Kronig (K.K.) relation allows us to compute \( \Delta v(k=0)/v_0 \) exactly if the attenuation is known at all frequencies. Hence, a viable strategy for including multiple scattering effects on the inference of \( c \) from \( \alpha(k) \) proceeds schematically as follows

\[
\text{K.K.} \quad \alpha(k) \rightarrow \Delta v(k=0) \rightarrow c. \tag{4}
\]

Here the various multiple scattering corrections for going from \( \Delta v(k=0) \) to \( c \) are used in the second part of the inference.

The Kramers-Kronig relations connect \( \Delta v(k) \) and \( \alpha(k) \). The relations express causality in the frequency domain for a propagating ultrasonic pulse. An elementary derivation is given in Ref. [9]. The basic result we need is

\[
\frac{v_\infty}{v(k)} - 1 = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{dk' \alpha(k')}{k' \left(k' - k\right)} \tag{5}
\]

Here \( v_\infty \) is the velocity of propagation at infinite frequency. We assume that it is equal to the velocity of propagation in the pore free host \( v_\infty = v_0 \). Using this result, the definition \( \delta v = v(k) - v_0 \) and taking the limit \( k \rightarrow 0 \) we rewrite Eq. (5) as
\[
\frac{\Delta v(k=0)}{v_0} = -\frac{2}{\pi} \int_0^\infty \frac{dk' \alpha(k')}{k'^2} \left[ 1 + \frac{2}{\pi} \int_0^\infty \frac{dk' \alpha(k')}{k'^2} \right].
\]  

(6)

This provides \( \Delta v \) from \( \alpha(k) \) the first step in the inference (4).

In the limit that \( c=0 \) and the scattering is uncorrelated, one finds that \( c \) is related to \( \Delta v(k=0)/v_0 \) by rewriting Eq. (2)

\[
c = -\frac{2}{3} A_2 \frac{\Delta v(k=0)}{v_0}.
\]  

(7)

This simple linear approximation fails at higher volume fractions where multiple (correlated) scattering becomes important. Several authors have commented theoretically on the inclusion of multiple scattering corrections in the inference of \( c \) from \( \Delta v(k=0) \). These include Waterman and Truell [10], Sayers and Smith [11], Varadan, Ma, and Varadan [12], and Brauner and Beltzer [13]. In addition, Thompson, Spitzig, and Gray [14] have provided some experimental comparisons for pores in iron compacts.

It seems that at present there is significant controversy among the above named authors as to the correct form of the multiple scattering corrections for \( \Delta v(k=0) \). However, we have pointed out, once this question has been adequately resolved, the same corrections can be applied to the inference of \( c \) from \( \alpha(k) \).

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