ULTRASONIC STUDIES OF STRESSES AND PLASTIC DEFORMATION IN STEEL DURING TENSION AND COMPRESSION

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ABSTRACT

A steel bar subjected to four-point bending was instrumented so that various strains and sound velocities could be measured during elastic and plastic deformation on both the tension and compression side of the bar. During plastic deformation, the load was reduced several times before it was increased again. We present the acoustoelastic constants and the corresponding third-order elastic constants \( l, m, \) and \( n \) in both tension and compression in the "as-treated" specimen and after various amounts of plastic deformation. The changes in various sound velocities with plastic deformation are also discussed.

INTRODUCTION

Ultrasonic velocities have been used to study residual stresses in metals for many years. The linear relationship between these two quantities, the stress acoustic constant, can be expressed in terms of second- and third-order elastic constants. The problem is that the sound velocity of a metal is also affected by plastic deformation and crystalline orientation. If one deals with a polycrystalline material of sufficiently small grain size with random orientation, the velocity measurement averages the dependence on orientation. Thus, one is still left with the effects of plastic deformation.

Plastic deformation can change the sound velocity and the stress acoustic constant. In a practical situation, when one tries to measure residual stresses via shear waves, the effect of plastic deformation cannot be taken into account because the amount of plastic deformation is not known a priori, and also because the change of velocity with plastic deformation is not available. In addition, one does not know how these quantities are affected by plastic deformation in compression versus plastic deformation in tension.

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In this work, we have studied the acoustoelastic constants for shear and longitudinal waves obtained in tension and compression and the effect of plastic deformation in tension and compression on these constants. We have also investigated the effect of plastic deformation on the velocities. The specimen used was a bar of ASTM 4340 steel which had been hot-rolled. The bar was austenitized, quenched, and tempered to a hardness of Rc 31. The measurements were accomplished by continuous monitoring of ultrasonic time delays and of transverse and longitudinal strains during the four-point bending experiment through elastic and plastic regions of deformation.

THEORETICAL CONSIDERATIONS

Elastic and Plastic Deformation of Rectangular Bars

Assuming uniaxial stress \( \sigma_0 \) and ideal elastic-plastic behavior, the yield condition for a narrow curved beam is \( \sigma_0 = \pm \sigma_0 \), with \( \sigma_0 \) taken as the yield strength in compression and tension. Yielding begins at the extreme fibers \( y = \pm h/2 \), where \( y \) is the distance from the neutral axis, and \( h \) is the height of the beam. With increasing load, the elastic-plastic interface moves inward. The central portion of the beam \(-\eta \leq y \leq \eta\) remains in the elastic state with the stress given by

\[
\sigma_0 = \sigma_0 \frac{y}{\eta} = E\frac{y}{R_0} \quad (1)
\]

where \( E \) is the modulus of elasticity and \( R_0 \) is the radius of curvature of the beam. Stresses in the two plastic regions are given by \( \sigma_0 = \pm \sigma_0 \) and independent of the strain \( \varepsilon = y/R_0 \).

During unloading, the plastic region reverts to the elastic state and the radius of curvature increases to a value \( R \). The stresses are given by

\[
\sigma_0 = \frac{E y}{R} \quad (2)
\]

for \(-\eta \leq y \leq \eta\), and

\[
\sigma_0 = E[y/R - (y\bar{\eta})/R_0] \quad (3)
\]

where the \( \bar{\eta} \) sign pertains to the regions \( y > \eta \) and \( y < -\eta \), respectively.

The bending moment due to stress distributions of Eqs. (2) and (3) which must be balanced by the applied moment is given by

\[
M(R) = \int_{-h/2}^{h/2} \sigma_0 y \, dy = \frac{Ebh^3}{12} \left[ -\frac{1}{R} \left( \frac{\eta}{h} \right) + \frac{1}{R_0} \left( \frac{\eta}{h} \right)^3 \right] \quad (4)
\]

where \( b \) is the width of the beam. Equations (2) through (4) can also be used for the loading cycle by setting \( R = R_0 \) and for the purely elastic case by additionally setting \( \bar{\eta} = h/2 \).

Equation (4) predicts a linear dependence of \( 1/R \) on the applied load during unloading. This result is predicated on the assumption that the stress-strain relations of Eqs. (2) and (3) are linear \((E = \text{constant})\). Thus, experimental verification Eq. (4) can serve as a check to which extent this assumption is warranted.
In analyzing ultrasonic velocity changes, knowledge of all three principal stresses is required. However, stresses in the direction perpendicular to the uniaxial stress can be neglected here since they are expected to be small. Boundary conditions require these stresses to be zero on the respective perpendicular surfaces. Furthermore, the condition of plane stress, while not strictly applicable, leads to the compatibility equation

\[ \sigma_\theta = \sigma_r + r \frac{d\sigma_r}{dr} = \sigma_r + (R+y) \frac{d\sigma_r}{dy} \]  

(5)

Since \( \sigma_r \) disappears for \( y = \pm h/2 \), it follows from Eq. (5) that \( |\sigma_r/\sigma_\theta| \leq h/2R = 10^{-2} \) for the conditions of the present experiment.

**Velocity Changes Due to Elastic and Plastic Deformation**

Hughes and Kelly [1] have derived expressions for the velocities of ultrasonic waves in elastically stressed solids using Murnaghan's theory of finite deformations. The velocities depend on the triaxial finite strains \( \epsilon_i \) through the Lamé or second-order elastic constants \( \lambda \) and \( \mu \), and the third-order constants \( l, m, \) and \( n \).

For propagation along the 1-axis, the three solutions corresponding to longitudinal waves and transverse waves polarized along the 2- and 3-axis, respectively, are

\[ P_0 v_{11}^2 = \lambda + 2\mu + (2l+\lambda)\theta + (4m+4\lambda+10\mu)\epsilon_1 \]  

(6)

\[ P_0 v_{12}^2 = \mu + (\lambda+m)\theta + 4\mu\epsilon_1 + 2\mu\epsilon_2 - \frac{1}{2} n\epsilon_3 \]  

(7)

\[ P_0 v_{13}^2 = \mu + (\lambda+m)\theta + 4\mu\epsilon_1 + 2\mu\epsilon_3 - \frac{1}{2} n\epsilon_2 \]  

(8)

Here, \( P_0 \) is the initial density and \( \theta = \epsilon_1 + \epsilon_2 + \epsilon_3 \).

Johnson [2,3], has generalized these results to include the effects of plastic deformation using two approaches. His first approach [2] leads to the addition of a term which depends on the plastic deformations, \( \epsilon_i^P \), to each of the Eqs. (6) through (8). The resulting velocities may be written as

\[ P_0 v_{11,\rho}^2 = (\lambda+2\mu)4\epsilon_1^P + P_0 v_{11}^2 \]  

(9)

\[ P_0 v_{12,\rho}^2 = \mu(2\epsilon_1^P+2\epsilon_2^P) + P_0 v_{12}^2 \]  

(10)

\[ P_0 v_{13,\rho}^2 = \mu(2\epsilon_1^P+2\epsilon_3^P) + P_0 v_{13}^2 \]  

(11)

Johnson's second approach [3] leads to a set of equations which are formally identical to Eqs. (6) through (8), but with the additional proviso that all coefficients in these equations are now functions of the plastic deformation. This approach then predicts that velocity changes observed during plastic deformation are correlated with changes in the second- and third-order elastic constants. In both approaches, the \( \epsilon_i \) still represent the elastic strains.
If we make the additional assumption of plastic incompressibility, Eqs. (6) through (11) can be rewritten in terms of relative velocity changes. Specializing for the case of uniaxial strain along the 2-axis and setting \( \varepsilon_{1,3} = -\nu \varepsilon_2 = -[\lambda/(2\lambda+2\mu)] \varepsilon_2 \), we can combine the various theories into the following set of equations

\[
\Delta v_{11} = a_{11} + \frac{\mu}{(\lambda+2\mu)(\lambda+\mu)} (\lambda-2\lambda-\mu) \frac{m\lambda}{\mu} \Delta \varepsilon_2 \tag{12}
\]

\[
\Delta v_{12} = a_{12} + \frac{1}{2(\lambda+\mu)} \frac{n\lambda}{4\mu} \Delta \varepsilon_2 \tag{13}
\]

\[
\Delta v_{13} = a_{13} + \frac{1}{2(\lambda+\mu)} \frac{(\lambda-2\lambda-\mu)(\lambda+\mu)}{2\mu} \Delta \varepsilon_2 \tag{14}
\]

Here, \( v_{01} \) and \( v_{02} \) are the longitudinal and shear velocity in the absence of finite stresses. The coefficients \( a_{ij} \) are zero during elastic loading and constant for elastic unloading after plastic deformation. In Johnson's first approach [2], we have \( a_{11} = a_{13} = -\varepsilon_2^2 \), \( a_{12} = \varepsilon_2^2/2 \) and all the elastic constants remain unchanged. In Johnson's second approach [3], \( 2a_{11} = (\lambda+2\mu)/(\lambda+2\mu) \) and \( 2a_{12} = 2a_{13} = \Delta \mu/\mu \). \( 1, m, \) and \( n \) can also change during plastic deformation.

By following the velocity changes through the elastic-plastic and then the plastic-elastic transition, the elastic constants \( 1, m, \) and \( n \) can be determined before and after plastic deformation and their changes can be correlated via the elastic strain \( \varepsilon_2 \) to observed velocity changes during plastic deformation. Thus, some definite statements about the various theories may be possible.

**EXPERIMENTAL DETAILS**

The specimen used was a 36-inch long rectangular bar of width \( b = 0.9077 \) inch and height \( h = 1.95 \) inches, made from ASTM 4340 steel which had been hot-rolled. The bar was austenitized, quenched, and tempered to a hardness of Rc 31. The bar was subjected to four-point bending. The support and the load were supplied vertically through roller bearings, 30-1/4 and 9-9/16 inches apart, respectively. Longitudinal strain was measured at the top and bottom face of the bar and also at five positions evenly spaced across the height of the bar. Transverse strain was measured at the top and bottom face of the bar. Four 1/4-inch diameter shear transducers were mounted with their centers 3/16-inch from the upper and lower edge of the bar, with polarization along the length and the height of the bar. In addition, two 3/8-inch diameter longitudinal transducers were placed with their centers 1/4-inch from the top and bottom edge. However, only one of the longitudinal transducers retained its bond throughout the entire experiment. All transducers operated at 5 MHz.

The load was typically varied in 1250 lb. steps, up to a maximum value of 21,250 lbs. At each load, strain gage readings were taken and corrected for transverse sensitivity. Ultrasonic time increments relative to the unstressed state were measured by observing the time shift of the third transverse and the sixth longitudinal echo on an oscilloscope using the 20 nsec/cm sweeprate.
RESULTS AND DISCUSSION

Figure 1 shows the load versus the inverse of the radius of curvature as determined from a linear fit to the longitudinal strain gage readings. The modulus of elasticity derived from the linear region during loading up to 10,000 lbs. is $E = 29.99 \times 10^6$ lb./in.$^2 = 20.68 \times 10^6$ bar. This is in close agreement with the value $E = \mu(3\lambda+2\mu)/(\lambda+\mu) = 20.61 \times 10^6$ bar obtained using $\lambda = 11.03 \times 10^5$ bar and $\mu = 7.99 \times 10^5$ bar from an earlier experiment on a similar type steel [4]. Thus, we have used these values for $\lambda$ and $\mu$ in the analysis below.

As can also be seen in Figure 1, the transition from elastic to plastic deformation starts at about 15,000 lbs. During unloading, the bar promptly reverts to the elastic state with approximately the same modulus of elasticity. However, deviations from linearity are apparent at small loads, indicating that there is reverse yielding of favorably oriented grains. Consequently, we have restricted the data analysis at present to the upper part of the unloading curve.

The load at minimum radius of curvature can be used to determine the location of the elastic-plastic interface from Eq. (4). We find $|\eta| = 0.40$ in good agreement with residual stress measurements across the bent bar.

Fig. 1. Load versus radius of curvature of the bent bar. Filled-in and open symbols correspond to loading and unloading cycles, respectively.
Relative velocity changes were determined in the usual manner from the relative time increments and the transverse strain, calculated at the detector position from the strain gage readings. They are shown in Figure 2 as a function of longitudinal strain. The result for the shear wave polarized along the principal stress direction (bottom) is particularly noteworthy. It shows a nearly linear dependence on strain during elastic loading. The corresponding stress scale on the right-hand side is obtained using the value of $E$ given above. At 17,500 lbs., a sharp, unusually well-defined change in slope to nearly constant velocity occurs, indicating a drastic change in material behavior, presumably the elastic-plastic transition. During partial unloading, the material reverts promptly to the elastic state.

Fig. 2. Velocity change ($\Delta v_{ij}/v_{01,s}$) versus strain for longitudinal (top) and transverse waves polarized perpendicular (middle) and along (bottom) the direction of the stress.
From the data given in Figure 2 we have extracted acoustoelastic constants and the third-order elastic constants 1, m, and n with the help of Eqs. (12) through (14), using values for \( \lambda \) and \( \mu \) given above [4]. In deriving these constants, we have restricted ourselves to the central region of the elastic loading and the top part of the elastic unloading curve. Results are collected in Table 1. Inasmuch as systematic errors tend to be self-compensating in the present experiment, the differences in the values for compression and tension, and before and after plastic deformation are believed to be significant. Stress acoustic constants can be obtained from these acoustoelastic constants by dividing by \( E \).

Table 1. Acoustoelastic Constants \( d(\Delta v_{ij}/\nu_{ij,s})/d\epsilon_2 \) and Third-Order Elastic Constants in \( 10^5 \) Bar.

<table>
<thead>
<tr>
<th>( ij )</th>
<th>Load</th>
<th>Unload</th>
<th>Load</th>
<th>Unload</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>-1.641</td>
<td>-1.444</td>
<td>-1.753</td>
<td>-1.815</td>
</tr>
<tr>
<td>13</td>
<td>0.128</td>
<td>0.238</td>
<td>0.000</td>
<td>-0.102</td>
</tr>
<tr>
<td>12-13</td>
<td>-1.769</td>
<td>-1.682</td>
<td>-1.753</td>
<td>-1.713</td>
</tr>
<tr>
<td>11</td>
<td>0.210</td>
<td>0.305</td>
<td>0.140</td>
<td>0.140</td>
</tr>
<tr>
<td>n</td>
<td>-75.8</td>
<td>-73.6</td>
<td>-75.4</td>
<td>-74.4</td>
</tr>
<tr>
<td>m</td>
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<td>-56.5</td>
<td>-67.7</td>
<td>-70.4</td>
</tr>
<tr>
<td>l</td>
<td>-36.6</td>
<td>-36.5</td>
<td>-36.5</td>
<td>-50.9</td>
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</table>

Table 1 depicts the acoustoelastic constants and 1, m, and n in tension and compression as well as changes in these constants with plastic deformation. The least variability is observed in n which is the only third-order constant that enters into the often used birefringent expression, Eq. (13) minus Eq. (14). The differences in the corresponding acoustoelastic constant (Table 1) are also rather minimal.

In the region of plastic deformation, all velocities appear to decrease with the amount of plastic deformation. From the intercepts of straight line sections fitted to the data points in Figure 2, relative velocity changes in the stressed state can be estimated. Using Eq. (3) in conjunction with the value \( |\eta| = 0.40 \) inch for the elastic-plastic interface, the strain for zero stress at the transducer position can also be estimated and the corresponding relative velocity changes read off from Figure 2. Results are given in Table 2. The observations are in disagreement, in magnitude as well as in relative signs, with the predictions of Johnson's first approach [2] as discussed in conjunction with Eqs. (12) through (14). Furthermore, they cannot be explained solely on the basis of changes in texture since for any texture \( \rho_o (v_{11}^2+v_{12}^2+v_{13}^2) \) has to remain constant and decreases in some velocities must be compensated for by increases in others [5].

In Johnson's second approach [3], relative velocity changes during plastic deformation at zero stress are due to changes in \( \lambda \) and \( \mu \), and the differences between the stressed and the zero stress values are caused by changes in acoustoelastic response, i.e., changes in 1, m, and n. Table 2 also lists values for these differences calculated from Eqs. (12) through (14) with the acoustoelastic constants from Table 1.
Table 2. Relative Velocity Changes ($\Delta v_{ij}/v_{os,1} \times 10^3$) Due to Plastic Deformation. Experiment, Under Stress (1); Experiment, Zero Stress (2); Difference (3) = (1) - (2); Calculated Difference (4). See Text for Details.

<table>
<thead>
<tr>
<th>ij</th>
<th>Compression</th>
<th>Tension</th>
</tr>
</thead>
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<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>(12)</td>
<td>-1.52</td>
<td>-0.17</td>
</tr>
<tr>
<td>(13)</td>
<td>-0.65</td>
<td>-0.51</td>
</tr>
<tr>
<td>(12)-(13)</td>
<td>-0.87</td>
<td>0.34</td>
</tr>
<tr>
<td>(11)</td>
<td>-1.23</td>
<td>-0.98</td>
</tr>
</tbody>
</table>

Although the agreement with the experimental differences is not perfect, the numbers are remarkably close suggesting that there is some basis for Johnson's second approach [3]. It must also be realized that errors of the experimental differences may be fairly large and that residual texture induced velocity changes may also be present. Further work, experimental as well as theoretical, would clearly be desirable.

CONCLUSION

The present experiment demonstrates that four-point bending of a rectangular beam can be used to determine the acoustoelastic response of ASTM 4340 steel during elastic and plastic deformation in both tension and compression. Third-order elastic constants $l$, $m$, and $n$ derived from these measurements show differences between tension and compression and changes with plastic deformation. Least affected is the constant $n$ which determines the acoustoelastic constant for birefringence measurements. The measured velocities decreased with plastic deformation. Measured relative velocity changes due to plastic deformation have been compared with predictions of various theories.

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REFERENCES