RESIDUAL LEAKAGE FIELD MODELING

Y. S. Sun*, S. S. Udpa and W. Lord
Electrical Engineering Department
Colorado State University
Fort Collins, Colorado 80523

INTRODUCTION

Residual leakage fields are set up around defects in ferromagnetic materials after the active source of d.c. excitation has been removed [1]. Defect detection and characterization then occur by using any flux density sensitive transducer such as magnetic particles, magnetic tape, a moving coil or a Hall element [2]. Past analytical attempts at modeling these phenomena have largely been based on representing simple defect shapes by an equivalent dipole or magnetic charge distribution [3]. In order to examine the effects of realistic defect shapes and material B/H properties, finite element analysis techniques have been used [4,5] to model both active and residual leakage field effects. This paper discusses alternative B/H representations and their impact on the numerical modeling of defect residual leakage fields.

RESIDUAL LEAKAGE FIELD MODELS

Several numerical models, which can be classified on the basis of their definition of magnetic permeability \( \mu \) or alternatively the reluctivity \( \lambda \), have been developed [6].

1. Traditional \( \mu \) or \( \lambda \) [7]

Traditionally, the permeability or reluctance is defined as

\[ \mu = \frac{B}{H} \text{ and } \lambda = \frac{H}{B} \]

Geometrically, this can be interpreted as

\[ \mu = \tan \alpha \quad \lambda = \cot \alpha \]

where \( \alpha \) is an angle as shown in Fig. 1a.

It is obvious that both \( \mu \) and \( \lambda \) vary over a tremendous range from \( -\infty \) to \( -0 \) when the material working point moves along a demagnetization curve. The large range of \( \mu \) or \( \lambda \) causes a serious problem of convergence.

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Furthermore, using this model results in a homogeneous field equation, which is impossible to solve.

2. Apparent $\mu^*$ or $\nabla^*$ [8]

Binns proposes the use of apparent permeability $\mu^*$ or reluctivity $\nabla^*$ defined as follows (see Fig. 1b):

$$\mu^* = \frac{(B-B_0)}{H} \text{ and } \nabla^* = \frac{H}{(B-B_0)}$$

Geometrically, this can be interpreted as

$$\mu^* = \tan \alpha^* \quad \nabla^* = \cot \alpha^*$$

where $\alpha^*$ is an angle as shown in Fig. 1b. The range of variation is much smaller than the range of traditional $\mu$ or $\nabla$ contributing to a faster solution of the scalar potential field equation.

However, when this procedure is used in the solution of a vector potential field equation, it still takes a long time for the iterative procedure to converge. Furthermore, if some material working points are very close to the horizontal axis where $B$ is very close to $B_0$ and $H$ is very close to zero, problems of computer overflow occur.

3. Equivalent $\mu_H$ or $\nabla_H$ [6]

To solve the vector potential field equation we suggest a new model, in which an equivalent permeability $\mu_H$ or reluctivity $\nabla_H$ has been used. Their definition are as follows:

$$\mu_H = \frac{B}{(H_0+H)} \text{ and } \nabla_H = \frac{(H_0+H)}{B}$$

Geometrically, this can be interpreted as

$$\mu_H = \tan \alpha_H \quad \nabla_H = \cot \alpha_H$$

where $\alpha_H$ is an angle as shown in Fig. 1c, which has also a small range of variation within the second quadrant of the $B-H$ plane.

**Fig. 1. Different definitions of $\mu$ and $\nabla$.**
The convergence problem for a vector potential field equation is not an issue of concern, since a constant-value right-hand term occurs using this model.

All three models above have a similar formulation for their corresponding field equations. With a scalar potential the field equation has a form

\[
\text{div}(\mu \text{ grad } U) = -\rho_m
\]

with an interface condition

\[
B_{n1} - B_{n2} = \sigma_m
\]

or

\[
\mu_1 H_{n1} - \mu_2 H_{n2} = \sigma_m
\]

where \( \rho_m \) and \( \sigma_m \) represent volume and surface densities of the bounded magnetic charge respectively. \( \mu_1 \) and \( \mu_2 \) represent permeabilities of media 1 and 2 respectively. \( B_{n1} \) and \( B_{n2} \) are normal components of \( B \) in media 1 and 2 respectively. \( H_{n1} \) and \( H_{n2} \) are normal components of \( H \) in media 1 and 2 respectively. For a vector potential case the field equation has a form

\[
\text{Curl}(\text{curl} A) = J + J_m
\]

with an interface condition for two-dimensional and axisymmetric problems

\[
\nabla_1 B_{t1} - \nabla_2 B_{t2} = J_m
\]

where \( A \) is the vector potential, where \( J \) is the real macroscopic current density, \( J_m \) and \( J_m' \) are densities of the volume and surface bounded currents respectively. \( \nabla_1 \) and \( \nabla_2 \) are reluctivities of media 1 and 2 respectively. \( B_{t1} \) and \( B_{t2} \) tagential components of \( B \) in media 1 and 2 respectively.

The differences for the three models in their field equations are: a) different definitions for permeability or reluctivity, b) different right-hand terms, \( \rho_m \) and \( \sigma_m \) or \( J_m \) and \( J_m' \), as shown in Table 1.

<table>
<thead>
<tr>
<th></th>
<th>Scalar Potential Eq.</th>
<th>Vector Potential Eq.</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>( \rho_m )</td>
<td>( \sigma_m )</td>
</tr>
<tr>
<td>Original ( u ) or ( v )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Apparent ( u ) or ( v )</td>
<td>-\text{div}(\vec{n})</td>
<td>\vec{n} = 0</td>
</tr>
<tr>
<td>Equivalent ( u_h ) or ( v_h )</td>
<td>-\text{div}(\vec{n})</td>
<td>\vec{n} = 0</td>
</tr>
</tbody>
</table>

Table 1. Right-hand terms of field equation for different models.
It can be seen in Table 1 that the right-hand terms of Model 1 with a traditional \(\mu\) or \(\psi\) are all zeros. This implies that we have to solve a homogeneous equation. A direct solution of a homogeneous equation mathematically is impossible either analytically, or by using a numerical method. This is overcome by using a nonzero value of excitation current which is decreased until a selected point in the interior is close to the B axis in the B-H plane [5]. This approach involves the expenditure of a considerable amount of computer resources.

Alternative definitions for \(\mu\) or \(\psi\) in Models 2 and 3 make the field equations inhomogeneous and the constant-value right-hand terms in Model 2 for a scalar potential and in Model 3 for a vector potential offer improvement in convergence time.

MODEL IMPLEMENTATION IN THE FE METHOD

The use of the finite element method for solving problems in electromagnetic NDE has been widely reported [1] and is therefore not repeated here. Differences in implementation of this model from a solution of a conventional magnetostatic problem is the calculation of the right-hand term, i.e. the bounded current densities \(J_m\) and \(j_m\) since they are functions of the coercive force vector \(H_0\).

As \(H_0\) is a parameter of the demagnetization curve on which the specific part of the material is working, the first step in the determination of \(H_0\) is to solve the active field problem with the current density at which the specimen was magnetized. After determining the working point of each element \(P\) along the initial magnetization curve as shown in Fig. 2, we evaluate the magnetic density \(B_{st}\), and find \(H_0\) as the \(H\) value of a special point on the demagnetization curve (where \(B=0\)) starting from point \(P\). The angle of vector \(H_0\) which is the same as that of \(B_{st}\) as shown in Fig. 3 is then determined.

![Fig. 2. Determination of \(H_0\).](image)

![Fig. 3. Determination of \(H_0\) angle.](image)

The volume bounded current density \(J_m\) is always zero if we use first order triangle elements, since \(H_0\) is considered to be constant within each element, i.e.,
\[ \mathbf{J}_m = \text{Curl} \mathbf{H}_0 = 0 \]  \hspace{1cm} (5)

The surface bounded current density \( j_m \) is determined by

\[ \mathbf{J}_m = \mathbf{H}_0 \times \mathbf{n} \]  \hspace{1cm} (6)

This can be treated as a Neumann condition on each side of each ferromagnetic material element (see Eq. 4).

The whole procedure of calculation is summarized by the flow-chart in Fig. 4.

![Flow-chart of calculation](image)

**RESULTS**

In order to verify the performance of the model the field around a rectangular defect in a ferromagnetic circular pipe with a current carrying copper rod in the center of the pipe as shown in Fig. 5 was calculated. Figure 5 shows the normal component of the residual leakage field around the defect. Figure 6 shows the dependence of the peak-peak magnitude of the signal as a function of the initial excitation current. Figures 7 and 8 show the dependence of the peak-peak magnitude with the defect width and depth respectively. These results agree very well qualitatively with results obtained in previous experimental studies [5], [7].
Fig. 5. Plot of normal component of B of the residual leakage field as a function of initial excitation current.

Fig. 6. Peak-peak magnitude of signal as a function of initial excitation current.
Fig. 7. Dependence of the peak-peak magnitude of signal with defect width.

Fig. 8. Dependence of the peak-peak magnitude of signal with defect depth.
1. Defect width 1/16"   2. Defect width 1/8"
CONCLUSIONS

A new efficient model has been developed for calculation of the residual leakage field requiring significantly lower computer time. The results obtained show qualitative agreement with experimental results. The new model has greatly speeded up the calculation procedure. It takes only a few iterations to reach a relatively high precision in calculation of the examples as shown in Table 2.

Table 2. Model Computational Properties.

<table>
<thead>
<tr>
<th>relative precision required</th>
<th>iteration times</th>
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<tbody>
<tr>
<td>$10^{-4}$</td>
<td>5 - 7</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>10 - 13</td>
</tr>
</tbody>
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REFERENCES