Economics of feed utilization with special emphasis on risk and uncertainty

Russell O. Olson
Iowa State College

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UMI
ECONOMICS OF FEED UTILIZATION
WITH SPECIAL EMPHASIS ON RISK AND UNCERTAINTY

by
Russell C. Olson

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
DOCTOR OF PHILOSOPHY

Major Subject: Agricultural Economics

Approved:

In Charge of Major Work

Head of Major Department

Dean of Graduate College

Iowa State College

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INTRODUCTION

The problem of feed utilization has come to the forefront in recent years in connection with the increased emphasis on soil conservation and the recognition that grasses and legumes play an important part in farm cropping systems as one means of conserving soil. Grasses and legumes in a cropping system may contribute to farm income by increasing or maintaining the yields of other crops through their beneficial effects on soil productivity and, more directly, by providing a product which can be used in livestock production. Thus the profitability of increasing forage acreage is dependent partially on its conservation value but to a large extent on its value in livestock feeding.

The United States Conservation Service has from its birth encouraged a shift in crop acreage from grain to forage crops. Other public agencies, numerous private organizations and individual conservation enthusiasts have in the past decade been pleading for increased attention to soil conservation, and especially through shifting land now in grain crops to hay and pasture production. More recently the United States Department of Agriculture and The Association of Land Grant Colleges have drawn up a joint resolution calling for increased efforts in promoting "grassland farming" (50).

These recommendations for increased forage imply the assumptions that a shift to more grass and legume acreage will be (a) profitable for the individual farmer and (b) beneficial to society. These assumptions are not arrived at altogether intuitively. Numerous testimonials and
many rotation experiments support the view that increased forage production is profitable. That more forage in the rotation would retard the rate of soil loss and thus leave society less impoverished with respect to soil resources in the future has been adequately demonstrated for most soil situations. Yet, there is not adequate evidence that individual farmers will profit from increased forage acreage. Nor can we say definitely that society is made better off by substituting forage for grain production.

The assumption that society stands to benefit from an increase in forage acreage seems to rest largely on the proposition that society looks favorably on any sacrifice of current consumption which contributes to the amount of goods available for posterity. Such an assumption is not entirely unrealistic; we do through state and federal legislation express a high regard for providing for future generations. On the other hand, the individuals who make up society also express their preferences through the market mechanism. In a society such as ours, where free consumer choice is permitted, prices are normally free to reflect the aggregative preferences of society. If the market is taken as an expression of society's interest in future vs. current agricultural production there is some doubt that a shift to forage crops is beneficial to society. Thus society has at least two ways of indicating its desires with regard to inter-temporal substitution of agricultural production, and these two indices may be contradictory. The exact nature of society's indifference map for consumption in different time periods defies
measurement⁶. But to say that society is made better off by postponement of consumption of agricultural resources implies considerable knowledge about such an indifference map. If only a relatively short period of time is taken into account, however, prices established in the market may be taken as society's criterion for allocating resources between soil conserving and soil depleting agricultural products. If so, it can be said that in a competitive economy the optimum position for an individual farmer is consistent with society's optimum position.

The assumption that it is profitable for farmers to increase forage acreage may have the following bases:

(a) The present average ratio of forage acreage to grain crops is low; when the ratio of forage to grain in a cropping system is low the response of yields to small increases in the ratio is generally large and, with so low a ratio, forage and grain production may even be complementary — i.e. an increase in the proportion of forage in the rotation may increase total grain output. Recommendations to increase forage acreage may then be rationalized on the basis that compliance would bring the aggregate ratio of forage to grain acreage nearer the level at which the value of added returns from forage just offsets the loss in returns from grain. One fallacy of the argument is that some individuals may already be operating beyond this level of forage intensity; compliance by them may place them in a position even farther from equilibrium.

(b) Total production of feed units is increased as forage is

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⁶For a discussion of inter-temporal welfare criteria see Heady (17, p. 399).
substituted for grain in the rotation and therefore the quantity of livestock product which can be produced from a given land area is increased by a shift to forage. This argument fails to take into account the differences in bulkiness and other features of feeds which cause them to substitute at diminishing rather than fixed rates in livestock production. Further, it does not take into account the inability of individual farmers to reorganize their resources to market a new combination of feeds. Grain crops may be sold directly, but forage crops must ordinarily be processed through livestock in order to provide a return. Inability to handle additional livestock (due to capital or labor limitations, for example) may preclude obtaining any returns from the feeds.

(c) Static analyses of costs and returns may indicate that many farmers can increase their forage acreage profitably. These analyses fail to take into account the effect of time in the production process. Livestock production processes take considerable time; consequently, the prices, costs and other factors which determine the net returns cannot be known with certainty at the time many of the decisions affecting production are made. The optimum position indicated by the static analysis may be entirely inappropriate for the situation involving uncertainty.

Despite the efforts being made in urging farmers to devote more of their land to forage production the percentage of land in grasses and legumes has actually declined in some of the major farm areas. In Iowa, for example, the percentage of all land in farms used for hay and pasture production averaged about forty per cent over the period 1930 to 1939.
The percentage increased to about forty-five per cent in 1940 under the
impetus of the Soil Conservation payments. Since then it has declined
until in 1950 only thirty-eight per cent of all farm land was used for
hay and pasture production. The failure of farmers to accept recommen­
dations for increasing forage production is not due to a failure to
recognize the importance of forage crops in building and maintaining soil
productivity. These benefits are generally conceded. But the profit­
ability of increasing forage acreage may also depend on making efficient
use of the added forage. The problem of forage utilization thus becomes
an important obstacle to the increase of forage acreage on many farms (9,

A tremendous amount of research funds and scientific effort has been
directed toward research in livestock feeding problems since the estab­
ishment of the land grant colleges and agricultural experiment stations.
That this research has contributed greatly to more efficient livestock
production is not questioned; but, in spite of the great amount of infor­
mation concerning animal feeding now assembled, there still remains con­
siderable doubt, confusion, and conflicting advice with respect to the
profitability of alternative ways of utilizing feeds in livestock pro­
duction.

Unambiguous recommendations concerning the profitability of increased
forage production requires an understanding of the technical relationships
between crops in crop production and between these crops as feeds in
livestock production. In addition, insight is needed into the economic
forces affecting returns from alternative feed combinations, the risk and
uncertainty surrounding alternative decisions and the effect of farmers' attitudes toward uncertainty.
Nature of the Problem

The discussion of the preceding section poses the problem of how much forage to produce. More specifically, answers are sought to the following questions: (a) What is the optimum forage acreage for an individual farmer to produce? (b) What is the optimum forage acreage from the standpoint of society? In this analysis an attempt is made to determine the optimum position for the individual farmer. This may or may not be the optimum forage output for society. As pointed out earlier, society may express its desires regarding the amount of forage or other product to be produced through the prices it establishes in a free market or through legislation. Farmers in pursuing their own self interests are guided by relative prices provided by consumers in the market. Allocation of resources by farmers in accordance with this joint expression of individual consumers may often be inconsistent with the longer term aims and objectives of society as expressed through various federal, state and local regulations, penalties and subsidies. However, to the extent that market price relationships truly reflect society's preferences regarding resource use, efficient allocation of resources within the individual farm firm is consistent with the goals of society.

Interrelated aspects of this problem

The most profitable forage acreage for any individual farmer is
dependent on (a) the relationship of forage to grain in crop production and (b) the relationship between forage and grain in livestock feeding. The optimum forage acreage for any farmer can be determined only as these two relationships are integrated.

A recent study by Heady and Jensen (15) draws on considerable experimental work on crop rotations to demonstrate the nature of the relationship between forage and grain in crop rotations. It shows that forage and grain in a rotation may be competitive or they may be complementary. The two are competitive whenever an increase in the production of one necessitates a reduction in the output of the other. They are complementary when an increase in the production of one is accompanied by an increase in the output of the other crop. On many soils a complementary relationship between forage and grain exists for the present levels of forage production. As more and more acreage is withdrawn from grain production and put into forage production the response in grain yields becomes less and less (forage substitutes for grain at an increasing rate) until the end of the complementary relationship is reached -- grain output becomes a maximum. Beyond this point any increase in forage acreage must come at the expense of a diminution of grain output -- forage becomes competitive with grain. Obviously, the gross returns from crops would always be increased by increasing forage acreage to the limit of the complementary relationship, even if none of the forage was sold or utilized. As long as the cost per acre of producing forage did not exceed the per acre cost of producing grain net income would also be increased. According to estimates made by Heady and Jensen (15, p. 444)
the production costs are no greater for forage than for grain. Since harvesting costs would be saved, the total cost involved in obtaining the maximum grain output would be less than for a smaller grain output obtained if more acres are devoted to grain production. The full response in yields to increased forage in the rotation is realized only over a number of years, of course. But for the individual who remains on a farm for a sufficient length of time that the rotation can be reflected in the yields, net income is increased by expanding forage production to the end of the complementary relationship -- the point where total grain output is a maximum.

The profitability of increasing forage acreage beyond the limit of the complementary relationship with grain depends upon the value of additional forage produced. Gross returns will be increased by any increase in forage acreage as long as the value of the forage added is worth more than the grain output sacrificed. Since forage is used almost exclusively for livestock feed, its value is determined by its productivity in terms of livestock and livestock products. Only as the forage has a value as a livestock feed is it profitable for a farmer to expand forage production (except in the special case where a ready cash market for forage exists) beyond the limit of the complementary relationship with grain. The extent to which it pays to expand forage acreage beyond that point depends on the rate at which forage replaces grain production in the crop rotation and the rate at which forage replaces grain in the livestock ration. Thus the optimum forage acreage for an individual farmer cannot be determined independently of his ability to utilize the
forage in livestock production.

Scope and Objectives of the Study

The relationship of forage to grain in crop production has been dealt with in considerable detail in other recent studies (15) and will not be considered further here. The focus of this investigation will be the problems of forage utilization in livestock feeding. The specific objectives of the study are:

(a) To indicate some of the alternative possibilities for increasing forage consumption by livestock

(b) To evaluate alternative feed utilization systems with respect to potential returns and variability of returns.

(c) To suggest criteria for determining the optimum forage-grain feed combinations in feeding livestock for individual farmers in different situations with special emphasis on the basis for selection in a setting of uncertainty of expectations.

Applications to be Made of Results

Attainment of the above objectives will provide a basis for recommendations regarding efficient utilization of feeds. Used in conjunction with information regarding forage-grain relationships in crop production intelligent recommendations can be made regarding the extent to which it pays farmers in particular situations to increase forage acreage. The analysis of the effect of uncertainty of expectations on production
plans should throw some light on the effectiveness of market prices in allocating resources for the most efficient production of livestock products.

The results of the study should be of particular value in formulating public policy with respect to soil conservation and pricing of agricultural products. Only as the level of conservation which is profitable and feasible for the individual farmer is determined can the need for public assistance in order to attain desired conservation goals be determined. Also, the effect of uncertainty of expectations on production plans indicates the cost of market instability in terms of less than optimum production plans being followed by farmers. The differences among individuals in their attitudes toward uncertainty and the subjective nature of uncertainty itself makes the analysis of uncertainty difficult and inconclusive. But if some knowledge is gained of the degree of uncertainty associated with alternative production plans and of the response of individuals in different circumstances to different degrees of uncertainty such information would be extremely useful in working out any program involving price ceilings or minimum prices for farm products.
THEORETICAL ANALYSIS

In the search for solutions to the problems posed in the preceding section a few technical relationships and generally accepted economic principles provide useful models. In this section these fundamental concepts are applied to obtain a theoretical solution to the problem of feed utilization.

The primary function of the theoretical analysis is to give direction to the empirical investigation of the problem to be analyzed. The theoretical models facilitate the empirical investigation by organizing and classifying the relevant data, indicating the types of data needed and their form, specifying the appropriate statistical techniques and tests to be employed and setting forth the criteria for determining the optimum position of individual producers.

The Firm in a Static Setting

Production is a dynamic process. Plans must be laid, investments made and costs incurred well in advance of any returns. Throughout the production period prices and cost change, and in ways which cannot be predicted accurately at the outset. It is therefore unrealistic to propose production plans to maximize net income which are based on perfect knowledge of production functions, costs and price relationships. Nevertheless, such assumptions are useful as a starting point. Economic concepts of the firm in a "timeless" situation provide a useful set of
analytical tools. The analysis can be extended later to take account of the complexities encountered in actual farm operation.

Any livestock producer has four types of decisions to make. He must decide: (a) what product or combination of products to produce, (b) the scale of operations, (c) the level of production per unit of livestock and (d) the combination of resources to use in producing that quantity of the selected product. In a static setting, with prices, costs and production responses known with certainty, the following equilibrium conditions (19, pp. 78-88) must be met if the firm is to maximize its net returns:

(a) The marginal rate of substitution between any two products is equal to the inverse ratio of their price ratios.

(b) The marginal rate of transformation of any factor into any product equals the ratio of their prices.

(c) The marginal rate of substitution between any two factors equals their inverse price ratios.

Also, in order for these points to be optimum, the following stability conditions corresponding to each of the above equilibrium conditions must be satisfied:

(a) The marginal rate of substitution between alternative products is increasing.

(b) The marginal rate of transformation of each factor into any product is decreasing.

(c) The marginal rate of substitution between factors is decreasing.

If these equilibrium and stability conditions hold no possibility
exists for improving the firm's position in respect to net income.

Combination of enterprises

Normally a livestock producer has an opportunity to produce several different kinds of livestock or combinations of livestock. Feed and labor may be used effectively with dairy cows, hogs, feeder cattle, sheep or other kinds of livestock. Similarly, more specialized resources such as buildings and equipment may often be used for any of several different kinds of livestock production. The optimum combination of enterprises is attained when the marginal rates of substitution between any two products is equal to the inverse ratio of their prices. Normally enterprise relationships are such that some combination of livestock enterprises satisfies this condition; however, in simplifying the following analysis, it is assumed (except as otherwise noted) that a single livestock product is being produced.

Scale of operation

The question arises as to how many units of livestock to produce -- that is, what scale of livestock operations to achieve. Not a great deal is known about the economies of scale in livestock production. While there are logical reasons for expecting constant returns to scale, farmers are seldom in a position to expand all services proportionately. Land area, management and often capital are limited resources which cannot be expanded at the will of the entrepreneur. Thus the problem of scale as
ordinarily considered is more nearly one of variable proportions and the principles determining the optimum level of production per unit of livestock apply as well in defining the optimum size of an enterprise.

Level of production

Given a single livestock product to be produced the question arises as to what level of output per unit of livestock is most profitable. The relationships relevant to the problem are: (a) the technical relationship of resource inputs to production response, and (b) the price of the product relative to the price of the productive factors. These relationships can be expressed in terms of cost and revenue curves as in Figure 1. Normally the nature of the production relationship is such that diminishing marginal productivity causes each additional unit of output to require a greater resource input than the preceding one. Thus as output is increased beyond some point (OA in Figure 1) total costs increase at an increasing rate. Eventually a limit is reached beyond which production cannot be increased regardless of the quantity of resources applied and the total cost curve becomes vertical (at output OC).

Assuming a purely competitive market for the product, total revenue is a linear function of the output and price of the product (Curve R). The optimum level of output is OB. At this output the net income is cd, a maximum. This corresponds to the condition that the marginal rate of transformation of any factor into a product is equal to their price ratio. It is apparent that any increase in the price of the product will increase the slope of the total revenue curve, pushing the optimum
Fig. 1 Optimum level of output.
level of production (the point at which cd is a maximum) to the right. A reduction in the cost of the factors of production would lower the slope of the total cost curve and have a similar effect on the optimum level of production.

**Substitution between factors**

Ordinarily several alternative production plans will yield the same output. In Figure 1 a single total cost curve was considered. Actually a farmer may choose one from among many possible cost functions. Total cost is a function of labor, equipment, management, protein, grain, forage, the price of each factor, and perhaps other less important variables. These inputs need not be combined in fixed proportions. On the contrary, for most kinds of livestock considerable substitution between factors is possible. Equipment may be substituted for labor. A particular output can be achieved with any of several feed combinations. The extent to which it is profitable to substitute one factor for another depends on (a) the relative prices of the various factors and (b) the marginal rates of substitution between these factors in producing a given product.

**Marginal rates of substitution.** This study is concerned with the extent to which it is economical to substitute forage for grain in a livestock ration. Thus the important relationships which need to be established empirically are the marginal rates of substitution of forage for grain in livestock production and the ratio of the price of grain to
the price of forage. With given technical conditions of production the output of a Product $Y$ depends on the amounts of the variable factors $X_1$, $X_2$, \ldots, $X_n$ used. The production function can be expressed as $y = f(x_1, x_2, \ldots, x_n)$, where $y$ is the output of product forthcoming from quantities $x_1$, $x_2$, \ldots, $x_n$ of the productive factors. Letting $Y$ represent a particular kind of livestock product and letting $X_1$ and $X_2$ represent forage and grain respectively, the production function for livestock production may be expressed as $y = f(x_1, x_2)$, where other factors are assumed fixed or unimportant. In the present analysis continuous divisibility of factors and continuous variability of the production process are assumed. Thus the production function is a continuous function of continuous variables. The production function for grain and forage in the production of a livestock product can be represented graphically by a production surface in which $OX_1$ and $OX_2$ are horizontal axes and $OY$ is the vertical axis, as in Figure 2. The contour of the production surface consists of a system of curves in the Plane $OX_1X_2$ which represent constant product contours and are defined by $f(x_1x_2) = \text{Constant}$. Curve $ab$, corresponding to a given value $c$ of the constant product, includes all points $(x_1, x_2)$ representing amounts of the factors giving a definite Product $c$. These points may be extended to the corresponding Curve $a'b'$ in the $OX_1X_2$ plane. The entire system of curves is continuous and non-intersecting, covering the positive quadrant of the $OX_1X_2$ plane in such a way that one, and only one, curve passes through each point. As the quantities of grain and forage are varied in any way the Points $x_1x_2$ move across the constant product contours in the Plane.
Fig. 2 Production surface involving two factors.
determining the resulting change in livestock product. Thus a system of constant product contours or iso-quants can be established and represented in a two dimensional diagram such as Figure 3. Curve $y_1$ in the figure represents the various input combinations of grain ($x_2$) and forage ($x_1$) which lead to output $y_1$. Similarly Curves $y_2$, $y_3$, and $y_4$ represent the various combinations of grain and forage which yield $y_2$, $y_3$, and $y_4$ quantities of livestock product, where $y_2$ is larger than $y_1$, $y_3$ is larger than $y_2$ and $y_4$ is larger than $y_3$.

The equation of the iso-quant (constant product contour) is obtained directly from the production function

$$y_0 = f(x_1, x_2)$$

where $y_0$ represents the output for each particular iso-quant. From the differential of this function is obtained

$$0 = \frac{\partial y}{\partial x_1} dx_1 + \frac{\partial y}{\partial x_2} dx_2, \quad \text{or} \quad \frac{dx_2}{dx_1} = -\frac{\frac{\partial y}{\partial x_1}}{\frac{\partial y}{\partial x_2}}$$

The slope of the iso-quant is given by $\frac{dx_2}{dx_1}$. The partial rate of change of output with respect to factor $x_1$ is the marginal productivity of $x_1$ for a particular value of $x_2$ and is designated by $\frac{\partial y}{\partial x_1}$. Similarly, the marginal productivity of factor $x_2$ is expressed as $\frac{\partial y}{\partial x_2}$. Thus the slope of the iso-quant at a particular point is equal to minus one times the ratio of the marginal productivities of the productive factors. This ratio may be termed the marginal rate of substitution of factor $x_2$ (grain) for factor $x_1$ (forage) in producing a constant output $y_0$ of livestock
Nature of the product contours. The nature of the iso-quants is a part of the technical data and, as shown in the previous section, is derived directly from the production function. Several types of iso-quants are possible. In certain types of production factors must be combined in a fixed proportion, with no substitution possible between factors. Thus the relevant portion of each iso-quant is a single point representing that required factor combination. A second possibility is that the two factors are perfect substitutes for each other. In this case a given output is obtainable from a series of combinations of the two factors and, further, a unit change in one of the factors requires a constant opposite change in the other factor to maintain that output. Since substitution is perfect the relationship between marginal productivities of the factors does not change as the factor combination changes. The marginal rate of substitution, and consequently the slope, of each iso-quant is constant. In other words, the iso-quants are straight lines parallel to each other.

Usually in production factor combinations are not technically fixed, nor are they often perfectly substitutable. In livestock production one would expect the factors grain and forage to be imperfect substitutes for each other. Logically the iso-quants will be downward sloping and convex to the origin at all points. As forage is substituted for grain each additional unit increase in forage would be expected to replace less and less grain in producing a particular output of livestock product —
i.e. diminishing marginal rates of substitution are expected. Curve $y_0$ of Figure 4 describes the general nature of the hypothetical iso-quant for various combinations of grain and forage in production of a livestock product.

**The iso-cost curve and optimum combination**

By combining into curves all factor combinations that have the same cost a system of iso-cost curves is obtained. In livestock production involving only two variable productive services (forage and grain) whose prices are fixed (in the sense that the prices are independent of the quantity used by the firm) the iso-cost curves are expressed by the equation

$$ P_1x_1 + P_2x_2 = \text{Constant}, $$

where the $p$'s and $x$'s stand for the prices and quantities of grain and forage. Differentiation of this equation gives the tangent to the iso-cost curve,

$$ \frac{dx_2}{dx_1} = -\frac{P_1}{P_2}. $$

The iso-cost curves are in this case straight lines with a downward slope equal to the (inverse) ratio of their prices.

If the grain and forage combination for a particular output are varied along the given iso-quant, the minimum cost combination is reached where the iso-quant is tangent to the lowest possible iso-cost curve. In
Fig. 3 Constant product contours.

Fig. 4 Optimum factor combination.
this case, where the prices of the factors are fixed and the iso-cost curves straight and parallel lines, the iso-quant will be tangent to only one such iso-cost curve. Moving from the point of tangency along the given iso-quant must lead to a higher iso-cost curve. The iso-cost curve which is tangent to the constant product contour $y_0$ is given in Figure 4 by the Line $P$. The two curves are tangent at Point $c$. It is here that the marginal rate of substitution of forage for grain in producing output $y_0$ of livestock product is equal to the inverse ratio of the prices of forage and grain. The feed combination which minimizes the feed cost of producing $y_0$ quantity of livestock product is evidently $Oa$ units of grain and $Ob$ units of forage.

Dynamic Concepts

The preceding analysis was based on the assumption of a static, or "timeless", situation. The actual situation in livestock production is more complex. The production process is spread over time. With the passage of time prices, costs, and the technical production relationships are subject to change. Production plans cannot be made on the basis of given costs, prices, and production responses but on the basis of expectations of what these will be. Thus before production plans are made expectation of future events must be formulated.

Subjective certainty

While in actual practice it is not possible to estimate exactly which
of the numerous possibilities with respect to prices, costs and production relationships will come about in any future time period, in order to keep the analysis manageable it is assumed for the present that the farmer thinks he knows exactly what will happen under each business plan he contemplates. Thus plans are made in a setting of subjective certainty. The business plan can now be viewed as a stream of investments or expenditures and a stream of returns through time. The optimum plan will be the one which maximizes the present value of the net receipts over time. The present value of receipts and expenditures anticipated in the future is determined by the expected interest rate. The present value of the stream of costs is given by the equation

$$C = \sum_{t=0}^{n} c_t (1 + i_t)^{-t}$$

where \( C \) is the present value of the stream of expenditures, \( c_t \) is the cost in each time period \((t = 0, 1, 2, \ldots n)\), and \( i_t \) is the expected interest rate in each time period. The present value of the receipts stream is given by the equation

$$R = \sum_{t=0}^{n} r_t (1 + i_t)^{-t}$$

where \( R \) is the present value of the stream of receipts and \( r_t \) is the receipts in each time period \((t = 0, 1, 2, \ldots n)\), and \( i_t \) is the interest rate in each period. The present value of the stream of net returns, \( NR \), is given by the equation
The optimum production plan for producing a particular livestock product, the plan which maximizes the capitalized value of net receipts, must meet these conditions: (a) the marginal rate of substitution between the products of any two time periods must equal the ratio of their discounted prices; (b) the marginal rate of transformation of any factor into a product must equal the ratio of their discounted prices; and (c) the marginal rate of substitution between any two inputs must equal the inverse ratio of their discounted prices. Condition (a) is analogous to the condition in the static situation for the optimum combination of products; here products of different points in time are treated as different products. The effect of condition (b) is to lower the optimum level of production from what it would be without the same prices and costs but without the time consideration. This is especially true in the case of livestock production, where much of the cost is incurred well in advance of production, since

\[
\frac{\text{discounted price of product}}{\text{discounted price of factor}} < \frac{\text{price of product}}{\text{price of factor}}
\]

\[\text{NR} = R - C.\quad ^{a}\]

\[^a\text{Lutz and Lutz (32, pp. 16-48) show that maximization of } R - C \text{ is only one of four logical criteria. Alternative criteria are } R/C, \text{ the} \]

"internal rate of return" \[\text{on the total capital invested, and the rate of return on his own capital. They show further that the four criteria coincide when the rate of interest equals the maximized average rate of return, which in turn equals the marginal internal rate of return. These conditions are not always met; when they are not met the four criteria may not lead to the same answers. They conclude that } R - C \text{ is the appropriate criteria to use. However, Hildreth (20, pp. 156-164) has suggested certain situations in which it is rational to maximize the internal rate of return rather than } R - C.\]
If costs for the substitutable factors, grain and forage, are incurred simultaneously the optimum combination of grain and forage in the ration is not affected by the discounting process —

\[
\frac{\text{discounted price of grain}}{\text{discounted price of forage}} = \frac{\text{price of grain}}{\text{price of forage}}.
\]

In that case, in a setting of subjective certainty, production plans for minimizing the feed cost of a particular output of livestock product are identical with plans in a static framework.

The analysis is made more complex if in substituting forage for grain the length of the production period is extended (or reduced). If, for example, the price of beef is different at different points in time it is not realistic to consider beef at different dates as identical products. The concept of an iso-product contour connecting forage-grain combinations which yield a given weight of beef, but at different dates, is then not a very useful one. We are then no longer interested only in minimizing the feed cost of a given livestock output, but also in finding the optimum feed investment period.

Uncertainty and production planning

The assumption of subjective certainty underlying the analysis of the previous section, while useful in examining the effect of the passage of time in the production process on the optimum production plan, is at variance with the facts. Farmers seldom have a single valued expectation about prices, costs, or yields. They recognize that their estimates are
subject to error; for any particular plan they set out to follow they realize that a whole set of outcomes is possible.

Anticipations about future events which are not known with certainty involve either risk or uncertainty. Knight (29) makes essentially this distinction between the two terms: in the former case the parameters of the probability distribution of probable outcomes are known a priori; whereas in the situation involving uncertainty the parameters of the probability distribution are not known — that is, we are faced with a probability distribution of probability distributions. Tintner (42) has introduced the term "subjective risk" to apply to the situation where the individual thinks he knows the parameters of the probability distribution of probable outcomes and uses the term "subjective uncertainty" to describe the situation where the individual views the probable outcomes in terms of a set of probability distributions corresponding to a set of uncertain contingencies.

Hicks (19, pp. 125-129) and Lange (31, pp. 29-34) use a device in handling uncertainty (in the sense in which Tintner uses the term subjective risk) which permits use of the same analysis as outlined above.

As Hart (12, p. 51) points out "The event viewed in isolation is always uncertain. But viewed as a member of a group of events so related that their joint outcome is more certain than the individual events in the group, it is a risk." Within a firm risk in this sense is encountered only where the firm carries on a large number of comparable operations whose results are independent, thus giving an "actuarial" basis for assigning probabilities. In the agricultural problem of feed combination with which this study is concerned the situation is more generally one of uncertainty.
for a situation of subjective certainty. Their procedure is essentially
the following: An entrepreneur faced with various possible prices (or
costs or transformation coefficients) may consider one of these the most
probable, or perhaps the mean, outcome. But individuals are not likely
to react in the same way to an uncertain as to a certain expectation of
a given magnitude. Rather, they will want to make some allowance for the
fact that the future event is uncertain. The allowance made will be
different for different individuals, depending not only on the degree of
uncertainty each attaches to the event but also on the aversion each has
to risk bearing.

It is assumed that each probability distribution (whether its
parameters are known with certainty or judged subjectively) of probable
outcomes has a corresponding unique outcome expectation — the "repre­
sentative expectation" — for each individual. That is, the individual
reacts in the same manner to the probability distribution as he would to
the representative expectation. Thus formally the analysis in a situation
of uncertainty involves only single valued expectations just as in planning
under subjective certainty.ª

Shackle (37, pp. 109-127) has been critical of the orthodox idea of
frequency-ratio probability in treating uncertainty and argues strongly
for his concept of "potential surprise". Briefly, Shackle's theory may
be sketched as follows: An entrepreneur looking out from the present
point in time (his "viewpoint") at the date when returns are expected to

ªStiendl (38, pp. 47-48) suggests that instead of adjusting the ex­
pected prices downward to compensate for the uncertainty individuals may
adjust the interest rate used in discounting future incomes upward —
thus decreasing the present value.
be forthcoming (his "image date") is uncertain as to the outcome. But
he visualizes a series of hypotheses regarding possible outcomes (yields,
for example) and assesses the surprise which the fulfillment of each of
these hypotheses would afford him. The "potential surprise function" is
defined by the degrees of surprise he considers each of the different
outcomes would present. While surprise may not be capable of numerical
measurement, the alternative hypotheses can be arranged in order of the
surprise the individual believes each would produce should it be ful-
filled. Thus the hypotheses may be arranged in order from "zero surprise"
to some maximum surprise (Shackle's $\bar{y}$) which would be associated with an
outcome considered impossible.

The enjoyment or distress obtained from anticipating a hypothesis
depends on (a) the profit or loss which it involves and (b) the potential
surprise associated with it. If an event is considered "perfectly
possible" the enjoyment afforded by its contemplation is derived entirely
from the outcome. If, on the other hand, some potential surprise is
associated with an event, the enjoyment is related to the outcome but is
reduced according to the amount of surprise the individual attaches to
its occurrence.

An individual, according to Shackle, relates his enjoyment (or
distress) to the alternative outcomes and their degrees of surprise by
means of a "stimulation function". Enjoyment and distress are ordered
and represented numerically on a scale whose zero represents complete
absence of stimulus. Any of the possible outcomes whose contemplation
affords any enjoyment or distress possesses some stimulus.
The various hypotheses visualized by the entrepreneur are mutually exclusive. The stimuli to which they give rise are not, therefore, additive but are entirely independent rivals. Shackle (36, p. 70) considers this a "central strand" of his theory, for it provides a measure of acceptance of a hypothesis which is independent of the degrees of acceptance attached at the same time to rival hypotheses. Since the degrees of potential surprise (or of stimulation) do not need to add to any number they need not be affected by the discovery of a new possible outcome or a change in the surprise attached to an alternative hypothesis.

The entrepreneur fixes his attention on only two of the hypotheses, according to Shackle, -- the one offering the greatest enjoyment and the one stimulating the most distress by anticipation. These are the "primary focus-outcomes". In making comparisons, these are replaced by "standardized focus-outcomes" -- outcomes which give the same stimuli but involve zero potential surprise. If on one of a pair of cartesian axes we measure focus-gains and on the other we measure focus-losses, any point on the plane will represent a plan involving the combination of a particular standardized focus-gain and a particular standardized focus-loss. All of the opportunities open to an individual may be represented by a series of points on this plane. Some of these points will be equally attractive to an individual. It is likely that all these points representing equally attractive opportunities will fall on a smooth and continuous curve sloping upward to the right. These curves are the "gambler indifference curves". The curves forming the gambler indifference map will, of course, be different for different individuals --
making inter-personal comparisons impossible. For each individual, however, the gambler indifferences map will consist of a non-intersecting family of curves, one through each point on the map. In deciding between two alternative plans an individual will prefer the plan represented by a point on the highest gambler indifference curve (the indifference curve in a position nearest the focus-gain axis).

Shackle's objections to the use of frequency-ratio probability may be summarized as follows:

1. For many important kinds of decisions it is impossible to find a sufficient number of past instances which occurred under the same conditions; that is, no "well founded" figures of probability of different outcomes can be established from experience.

2. Even if a probability is established, many kinds of decisions are virtually unique for each individual, in the sense that failure may prevent the entrepreneur from remaining in business.

3. The entrepreneur cannot, therefore, look upon the possible outcomes of a particular course of action as being related and occurring with given relative frequencies over time. Rather, they must be regarded as independent and mutually exclusive alternatives arising out of a single set of actions.

Shackle's objection that it is impossible to carry out a long series of decisions under identical conditions does not seem to be a serious one. A general knowledge gained from experiences involving several similar yet not identical situations may be the basis for forming a judgement of the probabilities of alternative outcomes from a given action.
The objection that many decisions facing an entrepreneur are of such a nature that he has only a single chance — that a failure prevents him from continuing in business — does not necessarily strike at the orthodox theory. The entrepreneur faced with such a situation might base his decision on the frequency ratio probability while limiting the size of his investment to take account of the prospect of failure.

Shackle's contention that the application of frequency-ratio probability to economic decisions is unrealistic appears well taken, however. It is unrealistic in the sense that the true probability distribution of future events contains elements completely unknown (such as whether or not there will be war or peace, or who will win the election next year). Any frequency-ratio will be based on past events; probabilities assigned to future events on the basis of these frequency ratios must be subjective.

Carter (4, p. 99) cites the following paragraph from John Venn (51, p. 158), one of the chief authors of the frequency-ratio concept of probability, to support his view that the probabilities on which entrepreneurial decisions are based lie outside the field of frequency-ratio theory:

In every case in which we extend our inferences by Induction or Analogy, or depend upon the witness of others, or trust to our own memory of the past, or come to a conclusion through conflicting arguments, or even make a long and complicated deduction by mathematics or logic, we have a result of which we can scarcely feel as certain as of the premisses from which it was obtained. In all these cases then we are conscious of varying quantities of belief, but are the laws according to which the belief is produced and varied the same? If they cannot be reduced to one harmonious scheme . . . then it is in vain to endeavor to force them into one science.

Carter (4, p. 95) then argues as follows:

We are left, in fact, with no more than a set of entirely
subjective assessments of probability, capable of formal measurement on a numerical scale by a single individual, but not of comparison between individuals. The calculation of a mathematical expectation in such circumstances seems a formal and irrelevant exercise.

But if we reduce probability to this level, how does it differ from Mr. Shackle's concept of potential surprise? Clearly there is a one to one correspondence between the two; if the essence of our knowledge of probabilities is that we can place them in order, this order of 'more or less probable' corresponds to 'less or more surprising'. To the concept 'zero probability' corresponds 'maximum potential surprise'. But the other bound of the potential surprise scale, 'perfect possibility' corresponds to the highest probability in the series under consideration, whatever that may be.

Carter's conclusion, which seems reasonable to this writer, is that Shackle's theory of potential surprise is essentially equivalent to "the only kind of probability theory valid for economic hypothesis".

Professor Ingvar Svennilson (40, pp. 39-55) has explicitly assigned "marks" to the alternative hypothesis and treated them as mutually exclusive. Shackle (36, p. 73) raises no objection to the use of frequency-ratio probability in this sense. It seems, however, that this same ordering is implicit in the conventional use of the probability concept. Economist have, it seems, avoided its use for inter-personal comparisons.

As long as the limitation of the frequency-ratio probability concept pointed up by Shackle are recognized and the use of the concept confined to personal comparisons it appears that its use is a harmless convenience. The assignment of probabilities may be considered a useful way of ordering the degree of belief accorded the alternative hypothesis. We may then return to the less complex methods of Hicks and Lange in conducting the empirical analysis of uncertainty.

The conditions for equilibrium of the competitive firm producing a
single livestock product in the context of uncertainty may now be stated in the following terms: (a) the representative marginal rate of substitution between products of any two points in time must be equal to the inverse ratio of their discounted representative prices, (b) the representative marginal rate of transformation of each factor into a given livestock product must equal the ratio of their discounted representative prices, and (c) the representative marginal rates of substitution between grain and forage (or any other factor inputs) must equal the discounted representative prices of grain and forage (or other input factors). Where the farmer has the opportunity of producing several different livestock products a fourth condition, which is analogous to the first, is that the representative marginal rate of substitution between products be equal to the inverse ratio of their discounted representative prices.

Three types of uncertainty are important in agricultural production: technical uncertainty, technological uncertainty and market uncertainty. Technical uncertainty exists in the sense that the physical response in output to a particular combination of resource inputs is not known with certainty a priori. That is, the physical output from a particular plan may be viewed at the time plans are made as a probability distribution of possible results. In terms of the present problem of determining the optimum feed combination, technical uncertainty may be viewed as producing a whole set of possible product contours for each level of output. If the variation in production is independent of the composition of the ration the possible product contours may take the form of essentially
parallel curves. This is illustrated in Figure 5 where Curve B represents the mean contour for a given level of livestock product and Curves A and C represent the upper and lower limits of the forage-grain combinations which yield that output. The variation in feed requirements to produce a given output of livestock product may, on the other hand, be correlated (either positively or negatively) with the proportion of grain in the ration. For example, as the proportion of grain in the ration is increased dairy cows may be subjected to greater chance of udder difficulties; feeder cattle and sheep may be in greater danger of "going off feed"; or, increasing the proportion of forage in the ration may increase the chances of death loss from bloat. Where the variation in the production function is associated with the makeup of the ration the dispersal of the possible product contours for a given output may be represented by Figure 6, where Curve B is again the mean contour for a given level of output and Curves A and C represent the outer limits of the possible feed combination to give the same level of production.

Technological uncertainty is present in agricultural production in that innovations leading to new and more efficient production functions are not foreseeable. An innovation, even though it may merely result in the saving of resources for an individual firm, results in an increased output for the industry as a whole -- and consequently, a lower price per unit of output. Some innovations in livestock production may be such that an individual farmer cannot take advantage of them immediately. For example, investments in housing, equipment and breeding stock once made constitute part of the fixed costs of production for a considerable time.
Fig. 5 Technical variation in production independent of factor combination.

Fig. 6 Technical variation in production correlated with factor combination.
into the future. Improvements in building or equipment design or in animal breeding often cannot be adopted by a farmer economically except as old buildings and equipment wear out or as present herds can be replaced. Yet, the more efficient techniques encourage production and tend to depress product prices. Thus technological advances of this nature result in a decline in net income for the farmers who are not in a position to change immediately to the improved production function. The possibility of such innovations contributes to the uncertainty involved in laying production plans of a long term nature. Other technological advances have a similar effect in reducing the factor requirements for a given output of product and yet result in a diminution of uncertainty. For example, considerable advancement has been made in controlling livestock disease; recent development of antibiotic and other nutritional discoveries have the effect of reducing the grain and forage required to produce a given amount of livestock product. Adoption of these innovations add nothing to the fixed cost of livestock production. They do add to the variable costs, and consequently to the total costs. They also increase the total production and reduce the variability of production. The effect of such an innovation is illustrated in Diagrams I and II of Figure 7, where Diagram I shows the mean and range of possible product contours for a given livestock output before the innovation and Diagram II shows the mean and range of possible product contours for the same output following adoption of the innovation. Curve B', the mean product contour following the innovation, lies nearer the origin than Curve B, the mean pre-innovation contour. Curve C', the upper limit of
Fig. 7 Technological uncertainty and technical variability of resource requirement.

Fig. 8 Optimum factor combination under market uncertainty.
the product contour following the innovation, lies nearer the origin than Curve C; it also lies nearer the mean than does Curve C. Curve A', the lower limit of possible product contours for a given output of livestock product, will also lie nearer the mean product contour; it may or may not be nearer the origin than Curve A -- depending on the nature of the innovation. To illustrate, the innovation may be one such as vaccination of livestock. Vaccination may greatly reduce the risk of heavy death loss (and thus high feed requirements per pound of product) but even without vaccination livestock may avoid getting that disease and can make as thrifty gains as those which were vaccinated. In such cases it appears that Curves A and A' are identical. Other innovations may increase the production potential of the animal and thus cause Curve A' to lie nearer the origin than Curve A.

A third source of uncertainty in agricultural production is that involved in the purchase of factors and the sale of products. At the time plans are made the farmer does not know with certainty the prices he will have to pay for resources used throughout the production process, nor can he be certain what prices will be obtained at the time the products are sold. These uncertainties must be taken account of in deciding the combination of products to be produced, the level of production per unit of livestock, and the feed combination with which to produce that output. In each case a wide range of price ratios are possible. As pointed out in an earlier section some "representative" price ratio (different for different individuals) will determine the best decision in each case. Market uncertainty as it relates to the selection of the best feed combination in
producing a given output of livestock product is illustrated in Figure 8. Line E may be considered the mean ratio of the price of forage to the price of grain. Lines D and F may be considered the extremes of possible forage-grain price ratios. Curve Q is a given product contour for a particular livestock product. The feed combination which minimizes feed costs in years when the grain forage price ratio is average is at Point b. The least cost feed combination in any year may vary from that given by Point a to that given by Point c. The proper planned combination will vary for each individual depending on his judgement of the probability of each of the possible price ratios being obtained and his attitude toward risk bearing.

Measurement of uncertainty. In the above paragraphs it was suggested that individuals faced with uncertainty view the future event as a probability distribution of possible outcomes but that they plan on the basis of a single valued "representative" outcome. That is, the individual reacts in the same way to the probability distribution as he would to the representative expectation. The important elements which determine an individual's reaction to an uncertain event -- and thus determine his representative expectation -- are (a) the degree of uncertainty associated with the event and (b) the individual's "risk" aversion.

The degree of uncertainty associated with a venture is given by the various characteristics of the probability distribution of possible results; such as the variance, range, skewness and kurtosis. In a situation of risk these characteristics are known with certainty. In a situation
of true uncertainty these characteristics are not known; they can only be judged subjectively. How, then, can a notion be gained of the degree of uncertainty involved in alternative feed utilization systems? Perhaps some indication of the degree of uncertainty surrounding future prices and costs can be obtained from historical price and cost behavior. It may be assumed that certain characteristics of future prices will be a reflection of past prices. For example, the relative prices of various farm products in the future may be expected to be roughly similar to the relative prices of these products in the past (this is the assumption made in the "parity" concept); too, it may be assumed that if the prices of particular commodities exhibited a great deal of variation in the past the future prices of the commodity will also be characterized by considerable variability. Thus by measuring the distribution of price ratios of grain to forage over an historical period some notion of the variance, range, skewness, and kurtosis of the probability distribution of future grain-forage price ratios may be obtained.

Response to uncertainty. Where an event involves merely risk some individuals may be quite indifferent to the degree of risk involved in a venture. For instance, an individual in a strong financial position may be able to withstand severe losses in any one year; he may be willing to do this if he is assured that these losses will be offset in subsequent years by larger rewards. In other cases involving merely risk it is often possible to insure against the occurrence of an unfavorable outcome; thereby reducing risk to a known cost. Situations involving
uncertainty are not insurable; neither are they likely to be treated with indifference. Even though an individual has sufficient resources to withstand severe losses in any one year he is not likely to care to do so if he does not know what the chances are of being able to recoup that loss in following years.

The reaction of an individual to a situation involving a particular degree of uncertainty is conditioned by his financial position, his previous training and experience, and his peculiar personality traits — boldness, timidity, or love for adventure. In general, however, it is perhaps true that most people prefer a certain event to another less certain event of equal magnitude. Many farmers may be concerned with business survival; a severe loss in a single year may force them out of business. Such farmers may have a strong preference for safe ventures and may select business plans which promise considerably less returns over time than some other venture if the latter involves a greater degree of uncertainty.

Determination of the optimum plan

Any plan an entrepreneur may consider with respect to combinations of products to produce, scale of enterprise, level of production per unit, or combination of inputs for particular levels of output may be represented by points on an indifference map, once the pertinent parameters of the probability distribution of present value of net returns from each is known. If we can use a single measure of dispersion the indifference map of an individual may be represented by a two dimensional diagram such as
Figure 9, where the mean net returns are represented on the horizontal axis and the dispersion is shown on the vertical axis. The indifference curves connect the points (representing the various opportunities) which are equally attractive to the individual. Thus, the plan represented by Point p, which involves a dispersion of OB and a mean of OD, is as attractive as the plan represented by Point q, which involves a dispersion of OA and a mean of OC. Both of these plans are just as desirable as a plan represented by the Point r, which involves a mean of Or and zero dispersion. The value Or may be thought of as the representative net returns for any of the plans falling on the risk indifference curve I2. In choosing between alternative plans the producer will choose the one represented by the point on the highest indifference curve — the one furthest to the right. Of the possible plans which are represented by points on the map there may be several which have the same dispersion, but only the one which has the maximum mean value is important. By connecting all of the plans which have a maximum mean value for each level of dispersion a boundary line such as S is obtained. The optimum plan is represented by the point at which the Line S is tangent to a risk indifference curve (Point s in the diagram).

The same principles would apply if more than two parameters are involved, but in that case, of course, the presentation cannot be made in terms of a two dimensional diagram.

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*If the producer has a preference for risk the indifference curve will fall from left to right.*
Fig. 9 A risk indifference map.
Capital Rationing

The assumption has been implicit in much of the preceding discussion that factors were available to the firm in any quantity desired. Many farmers face a situation wherein they are unable to apply capital in the amount justified on the basis of profitability. This is capital rationing. Capital rationing is a consequence of uncertainty. Investors may be uncertain about the integrity of the borrower, his ability to repay the loan, or the profitability of the contemplated venture by the borrower. Because of these uncertainties lenders have come to put more emphasis on the borrower's equity in the investment and normally give little consideration to the prospective returns from additional applications of capital by the borrower. The effect is a limitation on the amount of capital available to the firm; and this limitation bears little or no relationship to the marginal productivity of capital.

The firm faced with capital rationing is unable to secure enough capital to equate marginal cost with marginal revenue.\textsuperscript{a} Equilibrium in this situation is achieved where the values of the marginal products of capital from each of its alternative uses are equated. These values will exceed the cost (interest rate) of capital.

\textsuperscript{a}The marginal cost curve may, of course, be viewed as becoming perfectly vertical at the point of discontinuity.
PRESENTATION OF RESULTS

A theoretical analysis of the feed utilization problem was presented in the preceding section. The empirical counterpart presented in this section is somewhat more limited in scope. Ideally, data would have been obtained to test all of the relationships represented in each of the models of the theoretical analysis. Unfortunately, some of the data needed for a precise calculation of the desired relationships must await further technical research. Also, inadequacy of statistical techniques limit the precision with which true relationships can be measured. Thus, while the following empirical analysis takes its direction from the theoretical analysis presented earlier, it should be clear that the data used and methods employed may not lead to the ideal solution. It is hoped, however, that the results of this study will give some insight into the problem of feed utilization and permit some inferences concerning profitable adjustments in forage-grain production and utilization.

Feed Substitution Relationships

Grain and forage make up nearly all of the feed cost (and a major part of the total cost) in producing most kinds of livestock products. The substitution relationship between forage and grain is therefore technical data which is needed for determining the feed combination which will produce a particular livestock output at a minimum cost. A recent Iowa study (16) provides estimates of substitution rates between forage
and grain in feeding dairy cows, beef cattle, hogs and sheep. These estimates and their applicability are discussed below:

**Forage-grain substitution in dairy production**

Estimates of forage-grain substitution relationships in dairy production were based on data pertaining to heavy breeds (Holsteins and Brown Swiss) from the Jensen-Woodward study (25). Production records from only those cows which received comparable feeds (legume hay, corn silage and grain) and with an expected production capacity of 300 to 400 pounds of butterfat (when fed the standard Haeker ration) were used in arriving at these estimates. Thus the relationships estimated are applicable over only a small range of output levels and a narrow selection of feeds. The milk production function estimated was

\[ Y = 3.56 \times 1.5035 \times \frac{X_1}{X_2}^{.4} \]

where \( Y \) is the pounds of 4 per cent fat corrected milk produced per cow, \( X_1 \) is the pounds of forage fed and \( X_2 \) is the pounds of grain fed. By setting \( Y \) at various levels the iso-product equation for each of those outputs is determined. Thus the iso-product equation for 8500 pounds of milk\(^a\) is given by

\[ X_2 = \left( \frac{8500}{3.56 \times 1.5035} \right)^{2.5} \]

\(^a\)Observations on production per cow, on which estimates were based, ranged from nearly 8000 pounds to slightly over 10000 pounds of 4 per cent milk.
From this the marginal rate of substitution of grain for forage is obtained as the first derivative:

\[
\frac{dX_2}{dX_1} = -\frac{350,640,222}{x_1^2.25875}
\]

**Forage-grain substitution in pork production**

Estimates of forage-grain substitution rates in hog production were based on experiments by the United States Department of Agriculture at Beltsville, Maryland.\(^a\) These experiments involved fall pigs fed different combinations of chopped legume hay and No. 2 yellow corn. All hogs in the experiment were raised to a weight of about 225 pounds. The data were inadequate for a determination of a significant portion of the production surface. It was therefore necessary to calculate the product contour directly as the least squares regression of grain \((X_2)\) on forage \((X_1)\). This procedure of arbitrarily considering pounds of grain the dependent variable is subject to some criticism. It is defended here on the grounds that a more precise estimate must await the results of additional research designed to give observations over a wider range of the production surface. The contour for one hundred pounds of pork was estimated to be

\(^a\)For details of this study see Ellis (7). The data used in arriving at these estimates included the initial weight of the pigs at weaning time (about 60 pounds), the final weight per pig at the end of the experiment (225 pounds) and the pounds of grain and of forage fed per pig from weaning until the end of the experiment. Thus the pounds of pork produced is about 165 pounds. The feed requirements per 100 pounds of pork produced reflect the average over the entire 165 pounds.
The marginal rate of substitution between forage and grain in producing one hundred pounds of pork is obtained as the first derivative of the above function with respect to $X_1$ and is

$$\frac{dX_2}{dX_1} = -0.5113 \times 0.0042X_1^2$$

Forage-grain substitution in beef production

Beef may be produced by any of a large number of systems. Estimates made in the Iowa study are for choice beef produced on yearling steers. The estimates were based on an experiment conducted at Page County, Iowa by the Agricultural Experiment Station (24). The experiment involved yearling steers purchased in the fall, wintered, and fed out the following fall. Four lots were fed out each year for five years (1946 to 1950). Rations for each of the four lots contained different proportions of forage and grain. In deriving the product contour, feed inputs were reduced to the basis of 100 pounds of gain. The one hundred pound beef contour equation was estimated directly as follows:

$$X_2 = 1111.15 - 0.4219X_1 + 0.0000686X_1^2$$

The equation for the marginal rates of substitution is derived as the first derivative of the above equation as follows:

$$\frac{dX_2}{dX_1} = -0.4219 + 0.000137X_1$$
Forage-grain substitution rates in lamb feeding

Feed substitution rates for fattening lambs were also derived. These estimates were based on an Iowa experiment (5) involving lambs fed six different proportions of chopped hay and corn. All lambs were finished to prime or choice grade. Since there was considerable variation in marketing weights of the lambs the data permitted an estimate of the production function. The following functional relationship was estimated:

\[ Y = -158.4345 + .7157X_1 - 2.3118X_2 - .001046X_1^2 - .0074X_2^2 - .0037X_1X_2 \]

where \( Y \) is the pounds of lamb produced and, as before, \( X_1 \) is the pounds of forage fed and \( X_2 \) is the pounds of grain fed. The product contour equation is obtained by setting \( Y \) at a particular value and expressing the above relationship in terms of \( X_2 \). The contour equation for 25 pounds of lamb may then be stated as

\[ X_2 = \frac{2.3118 - .0037X_1 - [(2.3118 - .0037X_1)^2 + .021175X_1 - .00031X_1^2 - 5.4267]}{.014792} \]

The marginal rates of substitution are derived from the contour equation as follows:

\[ \frac{\partial X_2}{\partial X_1} = -2.50676 \frac{5.210359 - .078568X_1}{[18.42072X_1 - .078568X_1^2 - 375.5419]^{1/2}} \]

Least Cost Feed Combinations

The criteria for minimizing feed costs of producing a particular
livestock output were set forth in an earlier section. It was shown that in a timeless situation the least cost feed combination is the one at which the marginal rate of substitution between feeds equals the inverse of their price ratios. In a non-static situation, but one involving subjective certainty, the optimum combination is attained where the marginal rate of substitution equals the inverse of the discounted prices. Where, as is generally true in livestock feeding, the forage and grain are fed simultaneously the price ratios are unaffected by the discounting process. In a setting of uncertainty, it was pointed out, the least cost feed combination is the one which equates the representative marginal rate of substitution with the inverse of the discounted representative price ratios.

The particular price ratio to equate with the marginal substitution rates determined in the preceding section depends, then, on the nature of the setting in which production decisions must be made. A setting of timelessness is inconceivable. A dynamic setting involving subjective certainty is conceivable but not realistic. In the present economic order decisions concerning the combination of feeds in feeding livestock are ordinarily made in an atmosphere of uncertainty.

Least cost feed combinations under uncertainty

This study is primarily concerned with market uncertainty; no attempt is made to take into account technical and technological variations, which also affect the decisions of producers. Market uncertainties are assumed to be independent of the physical uncertainties and may
therefore be studied in isolation. In a situation involving price un-
certainty (but with production coefficients known) the optimum (ex ante)
feed combination is obtained where the marginal rate of substitution
between feeds is equal to the inverse ratio of the discounted repre-
sentative prices. The optimum feed combination for each individual
producer depends on (a) his expectations regarding feed prices and
(b) his aversion or affinity for risk taking. Thus no unique solution
exists; at any point in time the representative prices of different
individuals may be quite different.

Analysis is simplified if the following assumptions are made: first,
forage and grain in the ration are fed simultaneously and thus their
price ratios are not affected by the discounting process. Second, the
degree of uncertainty involved in the purchase of a unit of grain is
equivalent to the degree of uncertainty attached to a unit of forage and
each individual makes the same proportional adjustment in his expectations
for grain prices as he does for forage prices. The effect of this latter
assumption is to eliminate the influence of risk aversion (or affinity)
on the decision of the producer. As a consequence of the two assumptions
the discounted representative price ratio is identical with the ratio of
the prices considered most probable. The problem is reduced to deter-
mining the most probable outcome. Assuming a normal distribution of the
anticipated price ratios we may consider, instead of the modal, the mean
of the probable prices. This mathematical expectation of the distribution
of anticipated price ratios we shall call the expected price ratio. The
condition for an optimum forage-grain combination may now be restated as
the equality of the marginal rate of substitution between feeds with the reciprocal of their expected price ratio.

Farmers expectations concerning future prices may be influenced by a variety of information. Past prices appear to play an important part. While future absolute prices may be expected to differ considerably from historical prices, past relative prices of substitutable feed crops might be expected to be similar to their future price ratios. The ratio of the average price per pound of corn in November to the average price per pound of alfalfa hay in December for each of thirty-two years (1917-1948) are presented in Table 1. During this period the price ratio ranged from as low as .61 in 1932 to a high of 3.9 in 1947. The mean price ratio over the period was 1.95, with a standard deviation of 67. On the basis of this sample we can be confident that the actual corn-hay price will deviate from the mean of 1.95 by less than .67 over 68 per cent of the time. It will deviate from the mean by less than 1.34 over 95 per cent of the time. Expressing it another way, in less than one year out of twenty would we expect the ratio of the price of corn to the price of hay to be less than .61 or more than 3.29. While the corn-hay price ratio realized may be quite different from the mean, under the assumptions made it is presumed that plans are made on the basis of the most probable outcome. The least cost combination of forage and grain in a livestock ration from the ex ante viewpoint is determined by setting the reciprocal of the mean price ratio (1/1.95, or .5128) equal to the marginal rate of substitution between forage and grain and solving. The resulting combinations are: (a) 8157 pounds of forage and 3320 pounds of grain in
Table 1. Ratios of November price of corn to December price of alfalfa hay, Iowa average, 1917-1948.

<table>
<thead>
<tr>
<th>Year</th>
<th>Price per lb. corn/price per lb. alfalfa hay</th>
<th>Year</th>
<th>Price per lb. corn/price per lb. alfalfa hay</th>
</tr>
</thead>
<tbody>
<tr>
<td>1917</td>
<td>2.62</td>
<td>1933</td>
<td>1.00</td>
</tr>
<tr>
<td>1918</td>
<td>2.01</td>
<td>1934</td>
<td>1.38</td>
</tr>
<tr>
<td>1919</td>
<td>2.01</td>
<td>1935</td>
<td>2.52</td>
</tr>
<tr>
<td>1920</td>
<td>1.47</td>
<td>1936</td>
<td>2.56</td>
</tr>
<tr>
<td>1921</td>
<td>1.05</td>
<td>1937</td>
<td>1.94</td>
</tr>
<tr>
<td>1922</td>
<td>1.33</td>
<td>1938</td>
<td>1.92</td>
</tr>
<tr>
<td>1923</td>
<td>1.61</td>
<td>1939</td>
<td>2.00</td>
</tr>
<tr>
<td>1924</td>
<td>2.40</td>
<td>1940</td>
<td>2.62</td>
</tr>
<tr>
<td>1925</td>
<td>1.43</td>
<td>1941</td>
<td>2.37</td>
</tr>
<tr>
<td>1926</td>
<td>1.16</td>
<td>1942</td>
<td>2.32</td>
</tr>
<tr>
<td>1927</td>
<td>1.65</td>
<td>1943</td>
<td>2.17</td>
</tr>
<tr>
<td>1928</td>
<td>1.61</td>
<td>1944</td>
<td>2.23</td>
</tr>
<tr>
<td>1929</td>
<td>1.94</td>
<td>1945</td>
<td>2.31</td>
</tr>
<tr>
<td>1930</td>
<td>1.69</td>
<td>1946</td>
<td>2.84</td>
</tr>
<tr>
<td>1931</td>
<td>.89</td>
<td>1947</td>
<td>3.90</td>
</tr>
<tr>
<td>1932</td>
<td>.61</td>
<td>1948</td>
<td>2.36</td>
</tr>
</tbody>
</table>
producing 8500 pounds of 4 per cent fat corrected milk, (b) 327.5 pounds of grain and no forage in the production of 100 pounds of pork, (c) no forage\(^a\) and 1111 pounds of grain in the production of 100 pounds of choice beef on yearling steers, and (d) 50 pounds of forage and 125 pounds of grain in the production of 25 pounds of prime or choice gain on feeder lambs.

**Limitations of analysis**

The actual outcome of price ratios will frequently be such that the above feed combinations do not minimize the feed cost of producing a given amount of product. But the question is: could the producer have made a wiser choice on the basis of the information at hand when the decision had to be made? It does not appear so if the assumptions stated earlier are accepted. We may well examine these assumptions further, however.

In comparing corn and hay prices it is found that corn prices exhibited a great deal more sensitivity to the movement of the general level of farm prices. As a result the ratio of corn prices to hay prices increased significantly with increases in farm prices. The regression of the ratio of the price of corn per pound to the price of alfalfa hay per pound (Y) on the index of prices received by farmers (X) for the thirty-two year period (1917-1948) is estimated as

\[^a\]This combination is outside the range of data. Projection of the estimated product contour would give a combination involving a negative quantity of forage. Since this is impossible the contour was extended only to the X\(_g\) (grain) intercept and that quantity of grain selected as the optimum combination.
Y = .91692 + .006818X

The correlation coefficient (r = .570) is significant at the one percent level of probability.

The apparent correlation between the level of farm prices and the ratio of the price of corn to the price of hay weakens the assumption that expectations of future corn-hay ratios are rationally based on the historical relationships since there is little logic for thinking that the level of farm prices in the past thirty-two years are a satisfactory guide to expectation concerning the level of prices in the future. Also, if corn prices are more responsive to price level changes than hay prices, the degree of uncertainty attached to grain prices may be greater than for forage prices, resulting in disproportionate adjustments of expected prices in establishing representative prices.

One further limitation of the above analysis is more serious. It concerns the failure to take the time variable fully into account. As forage is substituted for grain the length of the production period may be extended; if so, the validity of using iso-product contours at all is open to question. Strictly speaking, given quantities of product turned out at different points in time are not identical products — they do not command the same price. Unless time can be treated as a factor of production, coordinate with the forage and grain inputs, the solution cannot be achieved with the aid of constant product contours. It does not appear that time can ordinarily be so considered because time has the effect of changing the product (in the sense that 100 pounds of choice beef today is not the same product as 100 pounds of choice beef next month) rather than
of changing the quantity of product. It may be possible to transform a
given quantity of output at one point in time to its equivalent in another
time period but since this involves a consideration of the prices of the
product in each time period the simplicity of the product contour analysis
is soon lost. An alternative procedure is to compare costs and returns
which would be expected from each of several discrete feeding systems.
This is the approach taken in the following sections.

Optimum Net Income Under Uncertainty

The above analysis has been in terms of minimizing feed costs in produc­ing a given output of livestock product. This may be justified on the
basis that feed costs make up the largest single element of costs in live­
stock production, and minimizing feed costs for a given output would usu­
ally be consistent with minimizing total costs for that output. Other
costs are important too, however, and are often not independent of the
forage-grain combination. Also, as was pointed out in the preceding sec­
tion, different combinations of feeds may require different investment
periods, and specific quantities of output at different points in time may
have different values. Thus minimum feed costs for producing a given live­
stock output is not the only important consideration in selecting a feed
combination for livestock production in a situation involving a variable
feed investment period and uncertain price expectations. The following
analysis provides estimates of costs, returns and income variability as the
bases for choice from among a few of the possible feeding systems open to
a livestock producer.

Costs and returns for each of thirty-two years (1917-1948) were esti­
mated for (a) four different feed combinations for dairy cows; (b) one
system of handling feeder calves, one system of feeding two-year old steers and three feed combinations for yearling steers; and (c) three feed combinations for hogs on pasture and three feed combinations involving forage and grain fed to hogs in dry lot. All of these systems are representative of feeding systems which are either common in the corn belt or offer possibilities for forage utilization under corn belt conditions.

Net returns were calculated for each feeding system for each of the thirty-two years. Net returns for each year reflect prices and costs for that year but assume present coefficients of production. A detailed description of the data and procedure used in arriving at the estimates of costs and returns is provided in the Appendix.

The relative frequencies of occurrence of various values of income may be expressed in a probability density function, where the probability of a particular value occurring is expressed as the number of times it occurred in the thirty-two year period divided by the total number of years it could have occurred (thirty-two). The sum of these probabilities for all values is one. These probability distributions, or certain parameters of them, may serve as guides for assigning degrees of belief to the various hypothetical outcomes visualized for the future for each feeding system considered.

It need not be assumed that the hypotheses concerning outcomes at some future date reflect precisely the same probabilities of occurrence as the distribution functions for the historical period. It is only necessary that we assume that some parameters of the historical distri-
bution resemble the parameters of its future counterpart closely enough that the subjective ordering of degrees of belief (or assignment of probabilities) concerning the hypothetical outcomes is related to the probability distribution for the previous period. For example, while it is not inconceivable that some people have notions of a "normal" price based on an historical mean, it seems likely that most individuals would be influenced more in forming their expectations about the level of corn prices by the outlook for the general level of business activity, employment and national income. But, on the other hand, expectations regarding relative prices and relative incomes for rival plans might very well be based on their relative positions in the past. Since many of the rival products produced by farmers are substitutes for each other in consumption and compete for the same resources in production, their relative prices and costs might be expected to change only as people's tastes change or as innovations affect the costs of production of one relative to another.

Further, the relative amplitude of the variations in prices or returns for rival plans may be important in forming expectations about relative variability of returns from these alternatives in the future. Fluctuation in production, responses of consumers to changes in incomes and similar phenomena account for fluctuations in returns from farm products. All farm products do not exhibit the same stability of production from year to year; the demands for some products are more sensitive to changes in consumer incomes than are others. If we assume that such characteristics of the rival production plans do not change
much over time, we may expect the relative variability of returns from the alternatives in the past to be a good indication of their relative variability in the future.

In the pages that follow characteristics of the frequency distributions of returns from fifteen different livestock feeding systems for the thirty-two year historical period are compared. These comparisons are intended to indicate (a) the relative mean net income farmers might expect from alternative opportunities in the future and (b) the relative uncertainty which will be associated with the alternatives in planning future production. Thus it is hoped that these comparisons can be used in conjunction with the gambler indifference curves of any livestock producer in determining which of the alternative feed utilization systems will provide him with the greatest satisfaction in his particular situation.

In order to place returns from different classes of livestock on a comparable basis net returns are expressed in terms of returns per $100 of costs. Returns per $100 of all costs were computed by dividing the gross returns per unit of livestock by the total cost of producing one unit (including imputed costs for interest, depreciation and family labor) and multiplying the quotient by 100. Returns per $100 of feed and labor costs were computed in a similar way except that gross returns were divided by feed and labor cost only. Returns per $100 feed costs were calculated on the basis of feed cost only.

Average returns and the variability of returns per $100 all costs for the thirty-two year period of 1917 to 1948 are shown in Table 2.
Table 2. Variability of returns per $100 all costs for various

<table>
<thead>
<tr>
<th>Returns per $100</th>
<th>Dairy cows</th>
<th>Feeder cattle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High grain (a)</td>
<td>Medium-grain (b)</td>
</tr>
<tr>
<td></td>
<td>Feeder calves (a)</td>
<td>High</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>113</td>
</tr>
<tr>
<td>0-19</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>20-39</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>40-59</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>60-79</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>80-99</td>
<td>-</td>
<td>9</td>
</tr>
<tr>
<td>100-119</td>
<td>13</td>
<td>11</td>
</tr>
<tr>
<td>120-139</td>
<td>9</td>
<td>6</td>
</tr>
<tr>
<td>140-159</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>160-179</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>180-199</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>200-219</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>220-239</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Average returns</td>
<td>111</td>
<td>106</td>
</tr>
<tr>
<td>Variance</td>
<td>219</td>
<td>172</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>14.8</td>
<td>13.1</td>
</tr>
<tr>
<td>Coef. of var.</td>
<td>13.3</td>
<td>12.3</td>
</tr>
<tr>
<td>Range</td>
<td>57.5</td>
<td>45.6</td>
</tr>
<tr>
<td>( \bar{x} - 2\sigma )</td>
<td>81.40</td>
<td>79.80</td>
</tr>
<tr>
<td>( \bar{x} + 2\sigma )</td>
<td>140.60</td>
<td>132.20</td>
</tr>
<tr>
<td>Maximum loss</td>
<td>18.60</td>
<td>20.20</td>
</tr>
<tr>
<td>Maximum gain</td>
<td>40.60</td>
<td>32.20</td>
</tr>
</tbody>
</table>
per $100 all costs for various livestock feeding systems, 1917-1948.

<table>
<thead>
<tr>
<th>Feeder cattle</th>
<th>Hogs</th>
<th>Pasture hogs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yearling steers</td>
<td>Dry lot hogs</td>
<td></td>
</tr>
<tr>
<td>Feeder calves</td>
<td>(a)</td>
<td>(b)</td>
</tr>
<tr>
<td>High grain</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>Medium grain</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>High forage</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2-yr.-old steers</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>High grain</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Medium grain</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>High forage</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>-</td>
<td>3</td>
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</tr>
<tr>
<td>1</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>06</td>
<td>104</td>
<td>112</td>
</tr>
<tr>
<td>83</td>
<td>1388</td>
<td>1416</td>
</tr>
<tr>
<td>23.70</td>
<td>37.50</td>
<td>37.80</td>
</tr>
<tr>
<td>28.0</td>
<td>35.9</td>
<td>33.5</td>
</tr>
<tr>
<td>19.1</td>
<td>186.6</td>
<td>185.2</td>
</tr>
<tr>
<td>46.80</td>
<td>29.0</td>
<td>37.4</td>
</tr>
<tr>
<td>65.40</td>
<td>179.00</td>
<td>187.60</td>
</tr>
<tr>
<td>55.40</td>
<td>71.00</td>
<td>62.60</td>
</tr>
<tr>
<td>65.40</td>
<td>79.00</td>
<td>87.60</td>
</tr>
</tbody>
</table>
Variability of returns is expressed in terms of the variance, standard deviation, coefficient of variation and range. In addition, the frequency distribution, showing the number of years out of thirty-two in which returns per $100 of all costs fell in various intervals, gives an indication of the skewness and kurtosis of the distribution.

Criteria for choice of feeding system

Before proceeding with an interpretation of the data in Table 2 the appropriateness and limitation of the various measures used in the table will be considered. The mean is used as the measure of central tendency rather than the mode. On theoretical grounds the mode may be preferred because it is the most typical value regardless of the symmetry of the distribution; the mean, on the other hand, is distorted by extreme values within the distribution and in the case of asymmetrical distributions is an unsatisfactory measure of central tendency. The mean has the advantage, however, of being more easily determined. Also, in the case of symmetrical distributions it has the same value as the mode. The apparent symmetry of the frequency distributions shown in Table 2 justifies the use of the mean rather than the mode in these distributions.

The range is one measure of the absolute dispersion of values within a distribution. Since its value is determined by the high and low extremes within an array, it is often distorted by an unusual event at either or both extremes.

The variance, a measure of the squared deviations from the mean, gives a good indication of the dispersion of a distribution based on all
the observations. The square root of the variance, the standard deviation, is a more convenient measure of the dispersion. 68.27 per cent of all randomly drawn observations from a symmetrical distribution will lie within one standard deviation above and below the mean. The mean plus and minus two standard deviations will include 95.45 per cent of all the observations. Thus the standard deviation provides an estimate of the probability of particular outcomes. If a distribution is skewed the percentages of observations falling within one or two standard deviations of the mean will be changed slightly.

Frequently it is desired to compare dispersions for different types of data where a common denominator is needed. Relative measures of variability are needed rather than absolute measures, such as the range or standard deviation. One measure which is often useful is the coefficient of variation, which is obtained by dividing the standard deviation by the mean and multiplying the quotient by 100. One difficulty with the use of the coefficient of variation is that unless it is accompanied by its mean and standard deviation it may be very misleading. It is misleading when abstracted from its mean and standard deviation because there is then no way of knowing whether differences in its value are due to differences in means or in standard deviations.\(^a\) Since returns from

\(^a\)The misleading nature of the coefficient of variation is illustrated in the diagram at the right. The coefficient of variation is the ratio of the standard deviation of a distribution to the mean expressed as a percentage. The coefficient of variation for each of the alternative opportunities open to an entrepreneur may be represented by points in a plane, where the horizontal coordinate is the mean returns and standard deviations are represented on the vertical axis. Then all plans having a common coefficient of variation must fall on a straight line going
various livestock systems have already been placed on a comparable basis by stating them in terms of returns per $100 of costs there is no apparent advantage in including a comparison of the coefficients of through the origin. Line CV₁ in the diagram connects all points representing plans with a ratio of standard deviations to mean returns equal to the slope of CV₁. Similarly, the ratio of standard deviations to mean returns for all distributions represented by Line CV₂ are equal to the slope of Line CV₂. If the coefficients of variation of alternative plans are considered apart from their means and standard deviations, all plans having identical coefficients of variation will be considered equally attractive. If we now consider the diagram at the right as a risk indifference map, similar to that in Figure 9, it is seen that the risk indifference curves are straight lines emanating from the origin, with the flattest such line, the horizontal axis, as the highest possible risk indifference curve. This implies a knowledge about the nature of the risk indifference maps of individuals which can hardly be verified. Moreover, it implies similar risk indifference curves for all individuals. It is easily shown that such use of the coefficient of variation is inconsistent with our earlier hypothesis of the nature of the risk indifference map. Curves I₁ and I₂ are two possible risk indifference curves for an individual. The individual is interested in finding the distribution having the combination of mean returns and standard deviation falling on the highest risk indifference curve. Of all the distributions having a coefficient of variation represented by Line CV₁, that distribution represented by Point P, and only that distribution, falls on indifference Curve I₂. Point Q, which represents a plan having the same coefficient of variation, falls on indifference Curve I₁ -- a lower indifference curve. Another individual having a stronger aversion to risk taking (a flatter indifference curve) might find Point Q on a higher indifference curve than any other point on Line CV₁. Thus the important difference between the above use of the coefficient of variation and our concept of the indifference map described earlier is that the former assumes a fixed reaction to income variation while the latter treats the attitude to variability as a subjective value, different for different individuals.
variation. The coefficients of variation are included in the table primarily to show how they compare with the other measures of dispersion.

Skewness and kurtosis are other characteristics of the distribution of possible returns with which we are concerned. Skewness, or the departure from symmetry, is important in this respect: if a distribution is skewed the most probable and the median outcomes may be quite different from the mean outcome. If a distribution is positively skewed the mean value will be larger than the median; less than 34.13 per cent of the values included in the distribution will be within one standard deviation above the mean, while more than 34.13 per cent will be within one standard deviation below the mean.

Kurtosis is characterized by the flatness or peakedness of the distribution near the mean. A positive kurtosis is characterized by an excess of values near the mean and more distant from the mean with a deficit in the intermediate areas. A negative kurtosis is characterized by a flat topped distribution -- one in which the probabilities of moderate deviations are very high.

Skewness and kurtosis may both be measured. A measure of skewness is provided by the third moment about the mean. The generally accepted measure of relative skewness is the ratio of the third moment about the mean to the cube of the standard deviation. Relative kurtosis is measured by the ratio of the fourth moment about the mean to the square of the variance.

A visual examination of the frequency distributions in Table 2 suggests that there is no marked skewness or kurtosis in any of the
distributions. This observation is verified by the measurements of these characteristics; they were found to be very small in each case. Thus each of the distributions can be treated as normally distributed. The means are then satisfactory measures of central tendency and the standard deviations may be used as measures of dispersion.

In considering only the mean returns and the standard deviation of returns when comparing the attractiveness of alternative plans an important feature of an individual's attitude toward uncertainty may be overlooked. It seems that a person's aversion to uncertainty is directed primarily at the prospects of loss resulting from an unfavorable outcome. To illustrate, suppose that an individual views the prospective outcomes from a particular plan as having a distribution such as A in Figure 10, and that he views the distribution for a rival plan as that of B in the figure. Distribution B exhibits considerably more dispersion than does Distribution A. It also has a larger mean. The misgivings which an individual may have about proceeding with either plan is due, we assume here, to the distress he feels in contemplating negative deviations from the mean, or most probable, outcome and not due to the prospects of positive dispersion in the distribution of outcomes. Thus in comparing two rival plans he might ask himself this question: "What is the most unfavorable outcome I can expect from this plan as compared with the least favorable outcome expected from the rival plan?" Suppose that he considers the outcome two standard deviations below the mean as the lowest outcome which he is likely to obtain -- he is 97.725 per cent confident that no lower value will be obtained. These limits may be labeled $L_{1A}$
and $L_{1B}$ for distributions A and B in Figure 10. The relative level of these limits together with the means for each distribution may be the criteria on which the choice between the alternative ventures is made.

These limits may be expressed in another way when considering returns per $100 of costs: taking a return of $100 as a position of zero gain and loss, the value of the Limit $L_1$ subtracted from $100 defines the magnitude of loss at Position $L_1$. This we shall refer to as the maximum loss associated with a plan. A new risk indifference map can be constructed in which the standard deviation is replaced by the maximum loss on the vertical axis. In Diagram I of Figure 11 the two rival plans, A and B, whose distributions are shown in Figure 10, are represented on an indifference map based on the standard deviation and mean returns. In Diagram II these same plans are represented on an indifference map where the coordinates are maximum loss and mean returns. The relative positions of the points representing the two plans are changed considerably in going from Diagram I to Diagram II. Curve I in Diagram I is the indifference curve of an individual who considers Plans A and B equally attractive from the standpoint of mean returns and standard deviation of returns. Any individual possessing a stronger aversion to risk taking (i.e., a risk indifference curve having less slope) prefers Plan A, while individuals who are more indifferent to uncertainty (expressed in terms of standard deviations) prefer Plan B. When the two plans are compared in Diagram II, however, it is apparent that the only individuals who prefer Plan A are the ones having a strong affinity for suffering losses. Rational individuals would not have
Fig. 10 Probability distribution B superimposed on distribution A.

Fig. 11 Risk indifference maps involving distribution A and B.
negatively sloping risk indifference curves based on these criteria since negative curves express a preference for high maximum losses as well as for low average returns (or most probable returns).

It is possible that many individuals consider the probability of obtaining very high gains as well as the chances of heavy losses in deciding between alternative plans. They may consider returns greater than two standard deviations above the mean so unlikely that they ignore the possibility of their occurrence -- they can be 97.725 per cent confident that larger returns will not occur. Thus the mean plus two standard deviations may be considered the effective upper limit of values for a distribution. These are labeled \( L_{2A} \) and \( L_{2B} \) for distributions A and B in Figure 10. Considering returns of \$100 per \$100 of costs as the position of zero gain or loss, maximum gain may be defined as the difference between \( L_2 \) and \$100. In Diagram III of Figure 11 the plans represented by Distributions A and B are indicated by points in a plane having as coordinates the maximum loss and maximum gain. Again risk aversion can be represented by positively sloping risk indifference curves. The vertical Curve I in Diagram III indicates complete indifference to the possibilities of loss (i.e. maximum gain is sole basis for choice between rival plans). A negatively sloping risk indifference curve appears completely foolhardy and need not be considered.\(^a\)

\(^a\)A negatively sloped risk indifference curve (when dealing with normally distributed populations, at least) means that a person prefers a situation involving the prospects of high losses or low gains to one of smaller losses and higher gains.
Diagrams I, II, and III illustrate three of the possible bases for choosing from among alternative plans. In many situations the solutions provided by each of these methods of comparison will be identical; this will ordinarily be true when dealing with normal distributions among which the differences in means are small relative to the differences in dispersion of outcomes. The possibility of contradictory conclusions on the basis of these three comparisons are clearly demonstrated in Figure 11.

It is possible that some individuals compare rival opportunities on the basis of the mean returns and some measure of the dispersion as in Diagram I. Others may consider the mean returns in relation to the magnitude of prospective losses as in Diagram II. Still others may, as we have suggested in Diagram III, consider the magnitude of possible losses in relation to the size of gains they consider possible. While the process of selection may not consciously involve any of these criteria, it appears that all are useful in indicating the relative attractiveness of alternative plans to individuals possessing different attitudes toward uncertainty. In the comparisons of the alternative feeding systems in the pages that follow all three bases for comparison are used.

Comparison on basis of all costs

The information in Table 2 provides two interesting kinds of comparisons: First, returns from alternative rations can be compared for each type of livestock. Second, returns for different kinds of livestock may be compared. In each case comparisons can be made in respect to
average returns over time and variability of returns. Differences in average returns and variability of returns per $100 of all costs for different rations fed each type of livestock are considered first.

**Optimum dairy system.** Average returns per $100 of all costs for the four dairy feeding systems differ considerably. The cows fed the high-grain rations returned an average of $111 for each $100 of costs over the entire period compared to only $99 returned per $100 of costs for the cows on the high-forage ration. At the same time, each of the measures of dispersion show that the variability of returns increased as the proportion of grain in the ration increased. The standard deviation of returns for the high-forage system was only 11.2 compared to 14.8 for the high-grain system. Thus in determining which is the optimum feeding system the higher mean returns for the high-grain system must be balanced against the lower variability of returns for the high-forage systems. Different individuals will balance these in different ways depending on their own attitudes toward uncertainty. In many cases where both mean returns and the variability of returns are larger for one plan than for its rival no unique "best" choice can be made. In Figure 12 each of the dairy systems are represented by points in a plane where the vertical coordinate measures the standard deviation of returns and the horizontal axis measures the mean return per $100 of all costs. Curve 1b represents an indifference curve for an individual who considers dairy system a (the high-grain system) equally as attractive as System c (the medium-forage system). Anyone possessing a more steeply sloping risk indifference
curve prefers System a. An individual having a risk indifference curve such as Ia considers Systems d and c equally attractive. Anyone having a stronger aversion to risk taking (a more gently sloping indifference curve) prefers System d.

Comparison of the returns from the four dairy rations, however, also shows that the mean returns for the high-grain system are sufficiently higher than for the other systems that, despite the larger variability of returns, the probability of large losses from that system is less than for the less variable systems. 97.725 per cent of the values of returns per $100 of all costs fall above the following levels for the four systems: $81.40 for System a (the high-grain system), $79.80 for System b (the medium-high grain ration), $79.00 for System c (the medium-high forage system) and $76.60 for System d (the high-forage system). Thus the maximum losses, as defined above, are $18.60, $20.20, $21.00 and $23.40 respectively.

The four dairy systems are compared on the basis of maximum losses and mean returns in Figure 13. The relative positions of the points representing the four rations are changed from what they were in Figure 12, where the comparison was on the basis of standard deviations of returns and mean returns. In Figure 13 Curve I' is the indifference curve of an individual who is completely indifferent to the magnitude of

---

a The Lines Ia and Ib may, as an alternative, be viewed as forming a boundary line or "opportunity" curve (dca) corresponding to Curve S in Figure 9. All plans which can be represented by points in the plane to the left of and above Curve dca must be on a lower indifference curve than some alternative plan.
Fig. 12 Standard deviations and mean returns for alternative dairy systems.

Fig. 13 Maximum loss and mean returns for alternative dairy systems.

Fig. 14 Maximum loss and maximum gain for alternative dairy systems.
loss possible. Only the unusual individual who enjoys dangerous investments would prefer any of the plans represented by points to the left of Curve I', since they involve greater likelihood of loss and less possibility of gain. Thus the comparisons in Figure 13 indicate that System a is the most attractive of the four dairy systems; it lies on a higher indifference curve (further to the right) than any of the other systems. This holds true regardless of the degree of positive slope an individual's risk indifference curve may have. The only individuals for whom it would not be true are those having negatively sloped indifference curves.

Maximum losses and maximum gains are used as criteria of choice in the comparison of the four dairy systems in Figure 14. The relative positions of the points representing the different rations are quite similar to those of Figure 13. Again any positively sloping risk indifference curves passing through the points representing Systems b, c, and d will lie to the left of one passing through the point representing System a. Since a negative sloping indifference curve is inconsistent with our assumption of rationality System a may be considered the most attractive of the four dairy systems.

Optimum cattle feeding system. Similar comparisons of the five feeder cattle systems are less conclusive. Again the mean returns per $100 of all costs as well as the variability of returns differ from one system to another. But which is the most attractive? First, consider the three feed combinations fed to yearling steers. The steers fed the largest amount of grain gave a mean return of $104 per $100 of all costs.
compared to $112 for those fed the moderate-forage ration and $113 return for those on the high-forage ration. The standard deviations of returns from each of these systems were $37.50, $37.80 and $43.20, respectively. The intervals including the mean returns plus and minus two standard deviations (including 95.45 per cent of the values in each distribution) have the following lower limits: $29.00 for the high-grain system, $37.40 for the moderate-forage system, and $28.60 for the high-forage system. The upper limits are $179.00, 187.60 and 199.40, respectively.

On the basis of the comparisons of standard deviations and mean returns in Diagram I of Figure 15 the high-grain yearling steer system is preferred over the other two yearling steer systems only by individuals having risk indifferences curves with less slope than that of dotted Line Ic. Curve Ib indicates indifference between the high-forage yearling steer system and the medium-grain yearling steer system.

According to the comparison in Diagram II of Figure 15, involving maximum losses and mean returns, and the comparison in Diagram III, where maximum losses and maximum gains are the criteria, the choice must be between the high-forage ration and medium-grain system, with Curves I''b of Diagram II and I''b of Diagram III indicating indifference between the two systems.

Comparisons of the yearling steer systems with the feeder calf and two year old steer systems are also included in Figure 15. The choice in each case is between the high-forage yearling steer system, the medium-grain yearling steer system and the feeder calf system, depending on the slope of the risk indifference curve. Regardless of the degree of positive
Fig. 15 Relative attractiveness of alternative cattle feeding systems.

Fig. 16 Relative attractiveness of alternative dry-lot hog systems.

Fig. 17 Relative attractiveness of alternative pasture hog systems.
slope, (linear) indifference curves passing through any of the points representing these three systems will lie to the right of (be greater than) those passing through the points representing the two year old steer system or the high-grain yearling steer system. On the basis of the criteria of choice employed no unique "most attractive" feeder cattle system can be determined since "most attractive" to each individual depends on his attitude toward uncertainty.

Optimum hog feeding system. The hog systems used in this analysis consist of (a) three systems based on an experiment by the United States Department of Agriculture involving different proportions of chopped legume hay and grain fed in dry lot and (b) three systems of handling hogs adapted from an Iowa experiment involving different proportions of forage and grain fed hogs on pasture. Because these two experiments were conducted under such different conditions each set of feeding systems is analyzed separately.

The three feeding systems involving different proportions of chopped hay and grain fed in dry lot show remarkably small differences both as to mean returns per $100 of all costs and as to variability of returns. Returns for the hogs on the high-grain ration (System a) averaged $122 per $100 of all costs, for those on the medium-forage ration (System b) the returns averaged $121, and returns for those on the high-forage ration (System c) averaged $118 over the thirty-two year period. The standard deviation of returns were $27.10, $27.10, and $26.80 respectively. If, in choosing one from among these three hog feeding systems, only the
standard deviation and the mean returns of each are taken into account, System a is clearly more attractive than System b. This is true since the mean returns is larger for System a while the standard deviations of the two systems are identical. Whether or not an individual prefers System a to System c depends on his attitude toward uncertainty, since the standard deviation as well as the mean is smaller for System c. Diagram I of Figure 16 shows that an individual having a risk indifference curve such as la is indifferent as to whether he follows System a or System c. Anyone having a stronger aversion to risk taking prefers System c. Those less cautious prefer System a.

A comparison of the dry lot hog systems on the basis of minimum loss in relation to mean returns is made in Diagram II of Figure 16. The relative position of the points representing the three feeding systems indicates that any individual, unless he enjoys contemplating losses, prefers System a. In Diagram III of Figure 16 the three hog systems are compared on the basis of maximum loss in relation to maximum gain. Again any rational individual employing these criteria will choose System a since any positively sloping indifference curve passing through the point representing System a must lie to the right of the points representing Systems b and c.

The set of hog systems involving different proportions of pasture in the ration show considerable differences in both mean returns and variability of returns. The hogs on the high-grain ration (System a) gave an average return of $114 per $100 of all costs; those receiving a medium-forage ration (System b) returned an average of $122, and those
on the high-forage ration (System c) returned an average of $115 per
$100 of all costs. The standard deviations of returns were $25.70,
$32.90, and $31.60, respectively. While the expected returns from
System a are considerably lower than for System b, some individuals may
prefer System a because of its lower variability of returns. Anyone
having a stronger risk aversion than that represented by Curve Ia in
Diagram I of Figure 17 prefers System a.

If the decision in choosing between alternative systems involves
the maximum loss associated with each rather than the standard deviation
the analysis is not much different from that above. As shown in Diagram
II, System b is preferred by those who have less risk aversion than that
indicated by Curve I'a, while System a is preferred by those individuals
whose risk indifference curve is less steeply sloped than I'a. In
Diagram III of Figure 17 the comparison of the three hog systems on the
basis of maximum loss and maximum gain leads to a similar conclusion.
Either System a or System b will be preferred, depending on the slope of
the individual's risk indifference curve.

Optimum livestock system. In the above sections alternative forage
grain feed combinations have been compared with a view toward determining
the relative attractiveness of alternative feed combinations for a
particular type of livestock. Similar comparisons can be made to deter­
mine the relative attractiveness of different kinds of livestock. In
doing so two important limitations of such comparisons should be recog­
nized. First, these comparisons cannot take into account important
enterprise relationships. For most farm situations a single livestock enterprise cannot make efficient use of available resources; some combination of livestock enterprises will ordinarily constitute the optimum livestock program. In the second place, the data on which these comparisons are based do not take into account technical uncertainty. This may be very unimportant in comparing different rations for a particular kind of livestock as differences in technical uncertainty may then be assumed to be small or unrelated to the composition of the ration. In comparing different kinds of livestock such an assumption appears less valid. These limitations should be kept in mind in drawing inferences from the following comparisons.

If unique solutions had been obtained in determining the "best" feeding system for each kind of livestock these best systems could then be compared on a risk indifference map. But different individuals may consider different feeding systems most attractive depending on their attitudes toward risk taking. All of the feeding systems for the three types of livestock are therefore compared in determining the type of livestock to produce. These comparisons are made on the basis of mean returns per $100 of all costs and standard deviations of returns in Figure 18. Comparisons in Figure 19 are on the basis of mean returns and maximum loss. In Figure 20 the fifteen systems are compared on the basis of maximum loss and maximum gain.

When all fifteen systems are compared on a plane involving the standard deviations and mean returns six different systems may be considered the optimum choice, depending on the slope of the risk indifference
Fig. 18 Standard deviations and mean returns for alternative livestock systems.

Fig. 19 Maximum loss and mean returns for alternative livestock systems.
curve. An individual having a risk indifference curve of less positive slope than \( I_a \) in Figure 18 prefers dairy System \( d \) to all other livestock feeding systems considered. Anyone having risk indifference curve with a slope less than \( I_b \) but more than \( I_a \) prefers dairy System \( c \). An individual possessing more aversion to risk taking than that expressed by Curve \( I_c \) but more than that expressed by \( I_b \) prefers dairy System \( a \) to all the other livestock feeding systems. If an individual's risk indifference curve slopes more than \( I_c \) but less than \( I_d \) he prefers the dry lot hog System \( a \).

A negatively sloped indifference curve falling between \( I_d \) and \( I_e \) indicates a preference for pasture hog System \( b \). Only if his indifference curve was negatively sloped less than \( I_e \) would an individual prefer the yearling steer System \( c \). The remaining nine feeding systems would never fall on as high indifference curves as some of these six systems regardless of the slopes of indifference curves (i.e. unless the indifference curves were non-linear).

The comparison of all fifteen systems in Figure 19 on the basis of maximum loss and mean returns reduces the number of possible "best" choices to only four. Line \( I'a \) indicates indifference between dairy System \( a \) and dry lot hog System \( a \). More gently sloping positive curves indicate a stronger risk aversion and a preference for dairy System \( a \).

Line \( I'b \) indicates complete indifference to the magnitude of the maximum loss and indifference between pasture hog System \( b \) and dry lot hog System \( a \). Only if an individual had a strong affinity for risking large losses, characterized by a risk indifference curve having less negative slope than \( I'c \), would he prefer the yearling steer System \( c \).
The comparisons in Figure 20 are based on the maximum losses and maximum gain associated with each feeding system. Here again either of the same four systems may be considered most attractive depending on the degree of risk aversion a person has. Any linear indifference curve passing through the points representing these four feeding systems must lie to the right of indifference curves passing through points representing any of the other eleven systems.

These comparisons suggest that most individuals prefer hog feeding systems to other livestock systems. Only persons with a strong risk aversion prefer dairy cows, and only people who are very indifferent to uncertainty or are foolhardy have a preference for feeder cattle. The limitations pointed out earlier, however, need to be kept in mind. While no data were examined to determine the technical variability of production for the different kinds of livestock, a priori knowledge of technical relationships in livestock production is to the effect that physical hazards of production are considerably greater for hogs than for dairy or beef cattle. If this is taken into account the relative attractiveness of the hog feeding systems is diminished. Even so, many farmers may still consider hogs the most attractive type of livestock from the standpoint of probable returns relative to the uncertainty associated with it but still raise considerable numbers of other kinds of livestock; enterprise relationships may be such that a combination of hogs with beef and/or dairy cattle gives the most satisfactory combination of mean returns and variability of returns.
Comparisons on the basis of feed and labor costs

Net returns for the above comparisons were calculated in terms of returns per $100 of all costs. Expenses connected with buildings and equipment, including interest and depreciation, make up an important part of that total cost. However, buildings and equipment often do not involve any actual cost to the farm operator. For example, tenants are ordinarily provided buildings by the landlord. Also, on many farms buildings and equipment have been provided in a previous period; their present use for livestock may involve no more expense than if they were permitted to stand idle. Farmers in such situations are not concerned with building and equipment costs.

Feed and labor costs account for nearly all of the costs of livestock production aside from the costs associated with buildings and equipment needed for handling the livestock. Thus many farmers are primarily concerned with feed and labor costs. In this section the relative attractiveness of investments in alternative livestock feeding systems is based on the frequency distributions of the returns per $100 of feed and labor costs for each system.

The mean returns and the variability of returns per $100 of feed and labor costs are shown in Table 3 for each of the feeding systems. Again it appears that for each class of livestock the variability of returns is generally positively correlated with the mean returns. Both the

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a The rent paid may include a charge for the buildings, but most commonly rent is charged by the acre or on the basis of a share of the crop. In any event, the amount of rent paid is ordinarily not dependent on the use made of the buildings.
Table 3. Variability of returns per $100 feed and labor costs for various categories.

<table>
<thead>
<tr>
<th>Returns per $100 feed &amp; labor costs</th>
<th>Dairy cows</th>
<th>Yearling steers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High grain</td>
<td>Medium grain</td>
</tr>
<tr>
<td></td>
<td>(a)</td>
<td>(b)</td>
</tr>
<tr>
<td>0-19</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>20-39</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>40-59</td>
<td>-</td>
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</tr>
<tr>
<td>60-79</td>
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</tr>
<tr>
<td>80-99</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>100-119</td>
<td>11</td>
<td>13</td>
</tr>
<tr>
<td>120-139</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>140-159</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>160-179</td>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td>180-199</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>200-219</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>220-239</td>
<td>-</td>
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</tr>
<tr>
<td>240-259</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>250-269</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Average returns</td>
<td>128</td>
<td>122</td>
</tr>
<tr>
<td>Variance</td>
<td>410</td>
<td>300</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>20.25</td>
<td>17.32</td>
</tr>
<tr>
<td>Coef. of var.</td>
<td>15.88</td>
<td>14.19</td>
</tr>
<tr>
<td>Range</td>
<td>82.27</td>
<td>56.32</td>
</tr>
<tr>
<td>$\bar{x} - 2 \sigma$</td>
<td>87.50</td>
<td>87.36</td>
</tr>
<tr>
<td>$\bar{x} + 2 \sigma$</td>
<td>168.60</td>
<td>156.64</td>
</tr>
<tr>
<td>Maximum loss</td>
<td>12.50</td>
<td>12.64</td>
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<tr>
<td>Maximum gain</td>
<td>68.50</td>
<td>56.64</td>
</tr>
</tbody>
</table>
100 feed and labor costs for various livestock feeding systems, 1917-1948.

<table>
<thead>
<tr>
<th>Yearling steers</th>
<th>Feeder cattle</th>
<th>Dry lot hogs</th>
<th>Pasture hogs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feeder calves</td>
<td>High grain</td>
<td>Medium grain</td>
<td>High grain</td>
</tr>
<tr>
<td></td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
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<td>-</td>
<td>-</td>
</tr>
<tr>
<td>120</td>
<td>118</td>
<td>131</td>
<td>136</td>
</tr>
<tr>
<td>462</td>
<td>1914</td>
<td>2045</td>
<td>2766</td>
</tr>
<tr>
<td>38.24</td>
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<td>141.18</td>
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<td>8.18</td>
<td>10.50</td>
<td>7.96</td>
<td>9.04</td>
</tr>
<tr>
<td>7.84</td>
<td>7.38</td>
<td>8.56</td>
<td>7.24</td>
</tr>
</tbody>
</table>
means and the dispersions for each of the distributions are larger than their corresponding values in Table 2, where all costs were considered. But, just as in Table 2, the frequency distributions show remarkably symmetrical dispersion about the mean. Computed measures of skewness and kurtosis also indicate that the values are approximately normally distributed in each case.

The relative attractiveness of the alternative feeding systems may, as before, be compared on the basis of the following criteria: (a) the mean return relative to the standard deviation of returns, (b) the mean return relative to the maximum loss, and (c) the maximum gain relative to the maximum loss. These comparisons are shown graphically in Figures 21, 22 and 23. All fifteen systems are represented on each indifference map. The solid lines represent the possible highest indifference curves between different feeding systems for a given type of livestock. The broken lines represent the highest indifference curves between types of livestock.

The comparisons in Figure 21 are on the basis of mean returns and the standard deviations of returns per $100 feed and labor costs. The choice between alternative dairy feeding systems is not clear cut; it depends on the nature of an individual's indifference map. Those whose risk indifference may be represented by solid Line 1a are indifferent as to whether they follow dairy System a, System b, or System d; each of these are slightly preferred to System c. Those possessing indifference curves more gently sloping than 1a prefer System d; those whose indifference curves are characterized by a slope greater than that of 1a prefer
Fig. 20 Maximum loss and maximum gain for alternative livestock systems.

Fig. 21 Standard deviations and mean returns per $100 feed and labor for alternative livestock systems.
Of the five feeder cattle systems considered, it appears, according to Figure 21, that any of the systems except the yearling steer system involving a high-grain ration (System a) may be the optimum system, depending upon the slope of an individual's indifference curves. The feeder calf system appeals to those who have very strong aversion to risk (in terms of standard deviation), while the yearling steers fed the high-forage ration (System c) are most attractive to individuals who are primarily concerned with the mean returns and are quite indifferent to the amount of variation in returns associated with alternative investments.

The six hog systems are also compared in Figure 21 on the basis of mean returns and the standard deviation of returns. Of the three dry lot systems, System b (moderate forage) is preferred to System a (high grain) by all except those who are completely indifferent to the magnitude of the standard deviation. The mean returns are identical for the two systems but the standard deviation about the mean is slightly smaller for System b. Individuals possessing very gently sloping indifference curves (less than 1b) prefer dry lot System c to either System a or System b. Of the pasture hog systems, the choice is between System a and System b. When all six systems are considered together the optimum may be dry lot System b, dry lot System c, or pasture System b, depending on the slope of an individual's indifference curves.

The comparison of all feeding systems in Figure 21 shows that five of the fifteen systems are possible optima. Individuals possessing
indifference curves flatter than Line Ia prefer dairy System d to all other feeding systems considered. For individuals having indifference curves with slopes between that of Ia and Ih dairy System a is preferred. If the indifference curves fall between Ih and Ic dry lot hog System b appears most attractive. Pasture hog System b is the optimum for individuals whose indifference curves are steeper than Ic. Only if an individual's indifference curves are negatively sloped and flatter than If will he prefer yearling steer System c over alternative livestock systems. On the basis of these comparisons all of the other ten systems must lie on lower indifference curves than one or another of the above five systems, regardless of the slope of an individual's indifference curves.

The comparisons of feeding systems in Figures 22 and 23, on the basis of maximum loss relative to mean returns and on the basis of maximum loss relative to maximum gain, yields results only slightly different from that given by the comparisons in Figure 21. One important difference is found in the comparison of dairy systems. Where Figure 21 showed that some individuals might be indifferent as to whether they followed Systems a, b, or d while others might prefer either System d or System a to all others, Figures 22 and 23 show System a to be clearly more attractive than any of the other dairy systems.

The relative attractiveness of alternative feeder cattle systems does not appear much different in Figures 22 and 23 than in the previous comparison. Any one of the five systems except the yearling steers fed the high-grain ration is a possible optimum.
Fig. 22 Maximum loss and mean returns per $100 feed and labor for alternative livestock systems.
Fig. 23 Maximum loss and maximum gain per $100 feed and labor for alternative livestock systems.
A few differences from Figure 21 appear in the comparison of hog feeding systems. Of the dry lot hogs, Figures 22 and 23 show that System c cannot be an optimum. The comparisons in Figures 22 and 23 of pasture hogs shows that pasture hog System c may be an optimum only for individuals having negatively sloped indifference curves. Such curves are conceivable for Figure 22, where the criteria are mean returns and maximum loss, only if an individual associates the possibility of extremely high returns with the possibility of high losses. In the comparisons of Figure 23 a backward sloping indifference curve is inconceivable except for someone completely foolhardy. Where all six hog feeding systems are considered, either dry lot System b or pasture System b are preferred.

Only four of the fifteen feeding systems considered are possible optima according to the comparisons in Figures 22 and 23. The optimum may be, in order of decreasing risk aversion (increasing slope of indifference curve), as follows: (1) dairy System a, (2) dry lot System b, (3) pasture System b, and (4) yearling steer System c. At least one of the linear indifference curves passing through the points representing these four systems must lie to the right of the points in the planes representing the other eleven systems.

Comparison on the basis of feed costs only

In computing costs for the above comparisons the value of labor

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\(^{a}\)Since we are dealing with normally distributed populations, an individual who prefers a venture involving higher maximum losses and lower maximum gain to another venture promising lower maximum losses and higher maximum gains prefers, in effect, a lower to a higher income.
used was imputed at the going wage rate (average annual daily wage rate, without board). On many farms, however, labor costs do not represent an actual outlay of cash; often only family labor is used. Where no alternative employment opportunities exist the imputed value of labor may be considerably higher than the value placed on the labor by farm operators. Farmers in such situations may be willing to handle livestock even though the returns to labor are very low; they may be more concerned with returns to other resources. In this section alternative feeding systems are compared on the basis of characteristics of the frequency distributions of their returns per $100 of feed costs.

The mean returns per $100 of feed costs over the thirty-two year period and their dispersions are shown in Table 4 for each of the feeding systems being considered. The mean returns for each system are considerably higher than their corresponding values in Tables 2 and 3, with the largest increases in mean returns occurring in the case of the dairy systems. This is to be expected since labor requirements are relatively higher for dairy cows than for feeder cattle and hogs. The variability of returns is larger for each of the distributions than when all costs or feed and labor costs are considered. Again, the symmetry of each of the distributions is evident. Tests of kurtosis and skewness also indicate that each population is approximately normally distributed.

The relative attractiveness of the alternative feeding systems is compared on the basis of standard deviation of returns relative to mean returns per $100 of feed costs in Figure 24. In Figure 25 the comparisons are made on the basis of maximum loss relative to mean returns, and in
Table 4. Variability of returns per $100 feed for various livestock.

<table>
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<tr>
<th>Returns per $100 feed costs</th>
<th>Dairy cows</th>
<th>Feeder cattle</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Medium-</td>
<td>Medium-</td>
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<tr>
<td></td>
<td>High grain</td>
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<tr>
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<td>(d)</td>
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<td>300-319</td>
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<tr>
<td>Average returns</td>
<td>205</td>
<td>193</td>
</tr>
<tr>
<td>Variance</td>
<td>2406</td>
<td>1877</td>
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<tr>
<td>Standard deviation</td>
<td>49.05</td>
<td>45.33</td>
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<tr>
<td>Coef. of var.</td>
<td>25.88</td>
<td>22.40</td>
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<tr>
<td>Range</td>
<td>183.63</td>
<td>159.74</td>
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<td>$\bar{x} - 2\sigma$</td>
<td>106.90</td>
<td>106.34</td>
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<td>$\bar{x} + 2\sigma$</td>
<td>203.10</td>
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<tr>
<td>Maximum loss</td>
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<tr>
<td>Maximum gain</td>
<td>203.10</td>
<td>179.66</td>
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<th>Hogs</th>
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<td>70</td>
<td>2346</td>
<td>2611</td>
<td>3189</td>
<td>2729</td>
</tr>
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</table>

| 40.37           | 48.43            | 51.09       | 56.47 | 52.24       | 40.76       | 40.89     | 40.18       | 38.17       | 52.32       | 52.05       |
| 50.50           | 37.93            | 35.39       | 39.18 | 38.50       | 25.46       | 25.21     | 25.25       | 26.18       | 31.82       | 32.04       |
| 34.32           | 236.27           | 238.69      | 245.84 | 234.04     | 155.79      | 159.83    | 158.97      | 146.31      | 207.10      | 202.37      |
| 52.26           | 31.14            | 41.82       | 31.05 | 31.52       | 78.48       | 80.22     | 78.64       | 69.66       | 62.36       | 57.90       |
| 15.74           | 224.86           | 246.18      | 256.94 | 240.49     | 241.52      | 243.78    | 239.56      | 222.34      | 259.64      | 266.10      |
| 17.74           | 68.66            | 68.18       | 68.94 | 68.48       | 21.52       | 19.78     | 21.36       | 30.34       | 37.64       | 42.10       |
| 15.74           | 124.86           | 146.18      | 156.94 | 140.48     | 141.52      | 143.78    | 139.56      | 122.34      | 169.64      | 166.10      |

| Breaks per $100 feed for various livestock systems, 1917-1948.
Fig. 24 Standard deviation and mean returns per $100 feed costs for alternative livestock systems.

Fig. 25 Maximum loss and mean returns per $100 feed costs for alternative livestock systems.
Fig. 26 Maximum loss and maximum gain per $100 feed costs for alternative livestock systems.
Figure 26 the systems are compared on the basis of minimum losses relative to maximum gains.

By disregarding labor costs the relative attractiveness of the different feeding systems is altered considerably. One important consequence is the change in relative position of the alternative types of livestock. The dairy systems appear considerably more attractive than the other feeding systems when only feed costs are taken into account. This holds true even for individuals who are completely indifferent to the amount of uncertainty (in terms of standard deviations of returns or maximum losses). The only persons who consider feeder cattle or hogs more attractive than dairy cows are those having unusually high preferences for risk taking.

The positions of alternative feeding systems for a particular kind of livestock relative to each other are also affected by disregarding the cost of labor. In the case of dairy cows, the position of the high-forage ration (System d) is enhanced considerably. According to the comparisons in Figure 25 and 26 System d is considered superior to other dairy systems by livestock producers having less risk aversion than that indicated by Lines I'a or I"a. More cautious individuals prefer dairy System a.

The positions of the alternative feeder cattle systems to each other are changed only slightly by disregarding labor costs. The most significant change among the feeder cattle systems is the less favorable position given two year old steers. The possible optima choices among the five feeder cattle systems are reduced from four to three when the
comparison is made on the basis of mean returns relative to the standard deviation of returns; the possible optima systems are reduced from four to only two when the comparison is on the basis of maximum loss relative to mean returns or maximum loss relative to maximum gain.

Analysis of the relative attractiveness of investments in alternative hog feeding systems is unaffected by considering only feed costs rather than feed and labor costs or all costs. The optimum hog system remains either pasture System b or dry lot System b, depending on the individual's attitude toward income variability.

Limitations of analysis

In the above analysis the probability distributions of returns from alternative feeding systems have been compared in an attempt to determine which feeding system offers the most desirable investment opportunity. Certain limitations of this analysis which need to be emphasized are: (a) the inadequacy of historical probability distributions in assessing the uncertainty of income from alternative ventures, and (b) the inappropriateness of the assumption that a particular system is followed consistently year after year.

The frequency distributions for each feeding system used in the analysis were for an historical period. In making plans for future production these can only serve as rough guides of the future outcomes. The probability distributions viewed by a producer must necessarily be subjectively determined. The influence of past relationships in establishing the probability distributions viewed by individual producers in a
situation of true uncertainty is not known and is likely to be quite
different for different individuals. Use of the historical population
in determining optima feeding systems was based on the assumption that,
while the level of returns expected in the future might be quite
different from the average over a previous period, the ordering of mean
returns and of measures of dispersion will bear a very close resemblance
to their past relative values. In some of the comparisons, it may have
been noted, the differences in mean values or in measures of dispersion
among the feeding systems were slight. Bearing in mind that these values
are merely bases for subjective evaluation of the relative positions of
the alternative plans for the future, it is doubtful that the probability
distributions actually visualized by the producer in formulating future
plans would in each case carry over with precision the same ordering of
mean incomes and variability. Thus the historical probabilities must be
viewed as only crude indices of the relative values of the various
characteristics of the probability distributions upon which plans for
future production are based. One must be cautious, therefore, in con­
cluding that a particular system is considered more attractive than
another by an individual, especially when the calculated values of the
historical populations being compared differ only slightly.

The assumption that a particular feeding system is to be followed
consistently through time was useful in simplifying the comparisons but
is not altogether realistic. Farmers need not follow the same system
year after year. They may alter the proportion of forage in the ration
from time to time. Also, there is often some opportunity to shift from
one kind of livestock to another from one year to the next.

Adjustments in the ration fed to a particular type of livestock are usually quite easily accomplished. The dairy ration, for example, can be changed frequently during the year in accord with changes in relative prices of feeds. In cattle fattening somewhat less flexibility may exist, depending to some extent upon the system of handling and type and grade of feeder. High quality cattle being fed on a high grain ration offer little flexibility; it will seldom be profitable to shift such cattle to a ration containing more forage, and the time at which to sell the cattle is determined, within a rather narrow range, once the cattle are put on full feed. Calves or young steers on a ration containing considerable forage, on the other hand, permit more turning points. Decisions as to when to finish these for market may be delayed awaiting more certain expectations; thus several changes in the production plan can be made during a single production period.

Changes in the feeding system for a particular kind of livestock from year to year are usually very easily accomplished. The size and grade of cattle purchased, the ration fed, and length of time kept on hand can be varied from year to year without important changes in facilities. Similarly, changes in methods of handling hogs and milk cows are very easily changed from year to year.

Changes in the type of livestock fed are ordinarily less easily accomplished. Specialized facilities and special skills are often required for each type of livestock. Once investments have been made in a dairy herd and dairy buildings and equipment, for example, these
investments cannot be recouped except as consumed in dairy production, since these facilities may not be suitable for feeder cattle or hog production. It is possible, of course, to plan facilities to permit greater flexibility between enterprises. Ordinarily, the more suited facilities are for a particular type of livestock, the less flexible is their use. Flexibility will often be achieved at the expense of somewhat less efficient production for any one enterprise (39, pp. 14, 168, 240-257).

Thus, while decisions concerning the type of livestock to feed involve expectations over considerable lengths of time, decisions as to the feed combination to feed a particular kind of livestock need be made to cover only a relatively short period of time, and plans can be revised frequently to take account of new information regarding prices and costs. As the period of time involved (between the time a decision is put into effect and when results of that decision are realized) is shortened, the degree of uncertainty (in terms of the deviation of actual outcomes from the expected) will ordinarily be reduced. If this is true, the degree of uncertainty assigned to each feeding system in the comparisons in Tables 2, 3 and 4 exaggerate the degree of uncertainty actually associated with alternative ventures. However, unless the amount of flexibility inherent in each of the feeding systems is different, the ordering of the "degrees of belief" concerning the possible outcomes may still be no different than that indicated by the above analysis of the historical data.
Effect of Capital Limitations

Up to this point in the analysis nothing has been said about the availability of capital. The assumption was implicit that adequate capital was available to invest in any of the fifteen rival feeding systems. Actually, many farmers are faced with a shortage of savings and inability to borrow as much capital as they would like. In this section some of the effects of limited capital on the optimum livestock system are examined.

As a farmer's borrowed capital increases relative to his assets, lenders tend to view additional loans to him as increasingly "risky". The uncertainty as to returns forthcoming from the use of additional capital as well as the uncertainty as to the responsibility and integrity of the borrower take on more importance as the possibility of collecting the entire amount of the obligation from the collateral seems less certain. One possible way for the lender to deal with this situation is to increase the rate of interest charged. This should have the dual effect of discouraging further borrowing and compensating for the added risk. If the lender deals with the situation in this way the farmer may still borrow additional money as long as the marginal value productivity of the capital exceeds the marginal cost. No capital rationing is involved in such a case.

Another manner in which the lender may react to the more unfavorable position of the borrower is to refuse to make additional loans to him. This is frequently done. Many farmers find themselves in a position where it is impossible for them to obtain additional capital. Farmers
in this position may make an optimum selection of livestock systems quite different from that of farmers having adequate capital.

A livestock producer having a large accumulation of savings or having access to an unlimited amount of capital at a given rate of interest makes the optimum allocation of capital by investing in each enterprise up to the point where the marginal returns equal the interest rate, providing he has perfect expectations. In the usual case, where uncertainty is involved, such an individual would stop short of the point where marginal cost equates marginal revenue; how far short of that equilibrium depending upon his attitude toward uncertainty and the uncertainty he associated with the various ventures. That is, a livestock producer may impose capital rationing on himself (in the sense of restricting capital investments below their ex post equilibria) because of uncertainty. In general, the more precarious his financial position (the greater the proportion of capital is borrowed capital) the more severe will the self imposed rationing be. That is to say, the lower his equity, the stronger his risk aversion.

The farmer faced with a severe shortage of capital and inability to borrow additional funds must necessarily restrict his investments in various livestock enterprises to less than the expected equilibrium. In allocating the scarce capital among alternative opportunities he is still guided to some extent by the uncertainty he associates with each rival venture and also by the expected returns he visualizes from investments in each.

In the absence of uncertainty the optimum adjustment for a farmer
faced with capital rationing is to equate the marginal returns to capital from all investment opportunities open to him (this would be at a rate somewhere above the interest rate). The presence of uncertainty may not cause such a farmer to reduce his total investment, as would be expected for the farmer having adequate capital; rather, the adjustment made to uncertainty is likely to involve a reduction in investments in ventures considered more uncertain and an increase in the investments in the more certain ventures.
SUMMARY AND CONCLUSIONS

Farmers, and others interested in agricultural programs and policies, are concerned with the question of what is the most profitable forage acreage to produce. The most profitable forage acreage for any individual farmer is dependent on (a) the relationship of forage to grain in crop production and (b) the relationship between forage and grain in livestock feeding. This investigation focuses on the relationship between forage and grain and the problems involved in forage utilization through livestock feeding.

The specific objectives of the study are: (a) to indicate some of the alternative possibilities for increasing forage consumption by livestock, (b) to evaluate alternative feed utilization systems with respect to potential returns and variability of returns, and (c) to suggest criteria for determining the optimum forage-grain feed combinations in feeding livestock for individual farmers in different situations, with special emphasis on the basis for choice in a setting of uncertain market expectations.

In a static setting the criteria of choice between alternative forage-grain feed combinations is that the marginal rates of substitution between forage and grain equal the inverse of their price ratios; or, where feeds produced on a farm are used entirely for livestock production, the least cost combination is that which equates the marginal rate of substitution of forage for grain in livestock feeding with the marginal rate of substitution of the two feeds in crop production.
Previous empirical research, as well as production economic logic, indicates a diminishing marginal rate of substitution between forage and grain in livestock production. The following substitution relationships between forage and grain have been found for various classes of livestock:

a. Dairy cows producing 8500 pounds of 4 per cent fat corrected milk were found to substitute forage and grain according to the following production contour:

$$x_2 = \left( \frac{8500}{3.56x_1} \right)^{2.5}$$

where $x_1$ is the pounds of forage fed and $x_2$ is the pounds of grain fed per cow to achieve an annual production of 8500 pounds of milk.

b. Good to choice feeder steers fed to a good to choice finish were found to produce one hundred pounds of gain with various combinations of forage and grain indicated by the following iso-quant:

$$x_2 = 1111.15 - .4219x_1 + .0000686x_1^2.$$  

c. The product contour for one hundred pounds of pork production was estimated to be:

$$x_2 = 327.5 - .5113x_1 + .00423x_1^2.$$  

d. The product contour for production of one hundred pounds of prime or choice lamb on feeder lambs was estimated as:

$$x_2 = \frac{2.3118 - .0037x_1 - [(2.3118 - .0037x_1)^2 - .021175x_1 - .000031x_1^2 - 5.4267]}{.014782}.$$
The least cost feed combination is easily found by equating the inverse ratio of forage prices to grain prices with the tangent to each of the above iso-quants.

The above analysis fails to take into account the time variable. As forage is substituted for grain the length of the feeding period required to obtain a given livestock output may be lengthened. In extending the analysis to include the effect of timing of production, costs and returns from several discrete livestock feeding systems were derived by budgeting technique and compared.

Costs and returns for each of thirty-two years (1917-1948) were estimated for (a) four different feed combinations for dairy cows, (b) five systems of handling feeder cattle, and (c) six feed combinations for hogs. All systems are representative of feeding systems which are either common in the corn belt or offer possibilities for forage utilization under corn belt conditions.

In order to simplify comparisons between classes of livestock, returns were measured in terms of returns per $100 of costs. Computations were made on the basis of (a) returns per $100 all costs, (b) returns per $100 feed and labor costs, and (c) returns per $100 feed costs. Comparisons on the basis of feed and labor costs only are applicable in the many situations in which buildings and equipment are provided at no cost to the farm operator. Comparisons on the basis of returns per $100 of feed costs only are appropriate where labor has no alternative profitable employment opportunities.

In choosing between alternative feeding systems it is assumed that
livestock producers are guided by (a) their expectations regarding the probability distribution of future returns from each system and (b) their attitudes toward risk taking. While expectations regarding uncertain events must be subjectively determined, it is assumed here that various characteristics of the historical frequency distributions of returns from alternative ventures are helpful in ordering the relative attractiveness of the alternatives.

First, alternative plans are compared on the basis of mean returns over the thirty-two year period and the standard deviation of returns. Generally, the higher the mean returns for a feeding system the higher the variability of returns. Where this is true no unique "best" system can be determined; the system appearing most attractive to a particular individual depends on the intensity of his aversion to risk taking (i.e. the nature of his indifference map between standard deviation of returns and mean returns). In general, rational individuals will prefer a plan with a low variability (standard deviation) to a rival plan offering the same mean returns but with greater variability. But the extent to which individuals are willing to sacrifice mean returns (or total returns over time) in order to secure less variability of returns is different for different individuals, depending on such things as previous experience, educational background, financial position, and personality traits.

An alternative criterion of choice between rival feeding systems is the maximum loss relative to mean returns associated with the alternatives. Maximum loss is defined as the level of net loss given by the mean return
minus two standard deviations. Again, no unique solutions are found unless a single feeding system has higher mean returns as well as a lower maximum loss associated with it.

A third criterion of choice is the maximum loss relative to the maximum gain associated with alternative plans, where maximum gain is defined as the level of returns two standard deviations above the mean.

The employment of the three criteria of selection, while it does not lead to determination of unique best feeding systems, narrows down the number of systems which might be optimum. The best choice for any one individual can be determined only as the nature of his risk preference is known.
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The pursuit of this study would not have been possible without the generous support of the Iowa Agricultural Experiment Station and the Bureau of Agricultural Economics of the United States Department of Agriculture.
APPENDIX
Estimates of physical data for computing costs and returns from alternative livestock systems were obtained from published and unpublished results of studies conducted at the Iowa Experiment Station, the U. S. Department of Agriculture and several other agricultural experiment stations. In some cases the various sources differed considerably in their estimates of input requirements; the estimates used were the ones which in the judgment of the authors most nearly represented requirements under present corn belt conditions. The data used and the method of computation followed in deriving the annual cost and returns estimates for each livestock system are described in the following sections.

Costs and Returns from Alternative Dairy Cow Feeding Systems

Each of the four dairy cow feeding systems considered in this study are based on the study by Einar Jensen and others (25, p. 80) of the United States Department of Agriculture. System (a) corresponds to level of feeding 13, System (b) corresponds to level of feeding 9, System (c) corresponds to level of feeding 5, and System (d) is representative of level of feeding 1, as discussed in Table 27 of Jensen's publication.

Costs of production per cow were considered to be the same for each system of handling except in respect to labor and feed costs. A summary of the costs included in the computations follows. Miscellaneous costs
(including grinding, veterinary expenses, cow testing association dues, supplies, and repairs), were based on estimates given in an Iowa study (2). The figure $6.93 was used as the miscellaneous cost per cow for the year 1948. This was adjusted for the other years (1917-1947) by the index of prices paid by farmers for supplies. The investment in silo, fences, and buildings per cow were also based on Iowa Experiment Station Research Bulletin 278. The figure $234.14 was used for 1948. This was adjusted by the index of building costs for each of the other years. Interest on this investment was computed on the basis of 4 per cent. Depreciation was figured at 3 per cent.

Investment requirements per cow in dairy equipment (including milking machine, separator, and miscellaneous equipment) was figured at $12.09 for 1948 and adjusted by the index of farm machinery costs for the preceding thirty-one years. Interest and depreciation on dairy equipment were each computed at 6 per cent.

The annual Iowa average price of good milk cows was used as the investment per cow in dairy cows each year. Replacement stock was figured on the basis of one-third of a calf and one-third of a yearling per cow, with a total value of 20 per cent the value of a dairy cow. The value of the bull per cow was computed at 10 per cent of the value of the cow (2).

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a Index of prices paid by farmers for equipment and supplies used in production, United States (47).

b Index of prices paid by farmers for building materials other than for houses, United States (47).

c Index of prices paid by farmers for farm machinery, United States (47).
Interest on the investment in cow, bull, and replacement stock per cow was figured at 6 per cent.

Labor requirements varied with the feeding system on the basis of milk production (2). The amount of labor required for handling the bull, replacement stock, and calves sold were figured at 6 days per cow for each system (6). The total days of labor required for each system (see Table 5) were multiplied by the Iowa annual average daily wage rate (without board) (8) to get labor costs per cow.

Feed inputs also varied with different systems, as shown in Table 5. Grain costs were computed by multiplying grain fed per cow under each system by the price of corn the preceding October (8). Hay costs were found by multiplying the tons of hay fed by the price of alfalfa hay the preceding November (8). An annual pasture charge per acre was computed on the basis of annual cash rents and crop yields. These pasture charges were then applied to the acres of pasture used per cow. Costs for protein supplement were based on the price of cottonseed meal the preceding December (8).

Gross returns from dairy cows include returns from butterfat produced, the value of skim milk produced, the value of calves sold, and the gain in value of replacement stock. The gain in value of the replacement stock per cow for each system was estimated as 25 per cent of the value of one dairy cow. Beef produced (calves sold) was estimated on the basis of 200 pounds of beef sold annually per cow for each system. The Iowa farm price of medium grade feeders in October was used to get the annual value

---

8 Monthly average at Chicago adjusted for transportation and commission costs (48).
### Table 5. Quantities of various resources used per cow under alternative feeding systems.\(^a\)

<table>
<thead>
<tr>
<th>Item</th>
<th>Dairy cow feeding system</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a)</td>
</tr>
<tr>
<td>Labor, days</td>
<td>20.0</td>
</tr>
<tr>
<td>Grain, bu.(^b)</td>
<td>105.1</td>
</tr>
<tr>
<td>Hay, tons(^b)</td>
<td>1.1</td>
</tr>
<tr>
<td>Pasture, acres</td>
<td>.44</td>
</tr>
<tr>
<td>Protein, supplement cwt.</td>
<td>3.66</td>
</tr>
</tbody>
</table>

\(^a\)Feed requirements are based on Jensen (25, p. 80). System a corresponds to level of feeding 13, System b corresponds to level 9, System c corresponds to level 5, and System d corresponds to level 1.

\(^b\)Includes cow feed plus bull and young stock feed per cow.

### Table 6. Production of butterfat and skim milk per cow for specific dairy cow feeding systems.\(^a\)

<table>
<thead>
<tr>
<th>Product</th>
<th>Dairy cow feeding system</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a)</td>
</tr>
<tr>
<td>Butterfat, lbs.</td>
<td>399</td>
</tr>
<tr>
<td>Skim milk, cwt.</td>
<td>99.8</td>
</tr>
</tbody>
</table>

\(^a\)Source of data was study by Jensen (25).
of beef produced for sale. The annual production of butterfat and skim milk per cow was different for each system (see Table 6). The pounds of butterfat per cow was multiplied by the annual average price of butterfat in Iowa (8). The value of skim milk was figured on the basis of one hundred pounds of skim milk being worth 12 per cent of the price of a bushel of corn.

Costs and Returns from Alternative Feeder Cattle Systems

Costs and returns for five distinct feeder cattle systems were computed. One of these systems involved the purchase of good-choice calves weighing about 440 pounds in October, wintering them, and then feeding them out in dry lot for sale as choice cattle in August at a weight of 1000 pounds. Feed requirements for this system were based on a study by Beresford (1).

Another system involved the purchase of choice two-year-old steers weighing 800 pounds in August. These were pastured about a month in the fall, then put in dry lot and finished to choice cattle weighing 1150 pounds in January. Feed requirements for this system were also adapted from Beresford's (1) study.

Three systems of handling yearling steers were considered. These systems were based on five years of experiments by the Iowa Agricultural Experiment Station in Page County, Iowa (24). Choice feeders weighing an average of 610 pounds were purchased in November. All were wintered on the same ration to gain about one pound per day. In May they were separated into four lots. One lot (System a) was placed in dry lot and
fed to a choice finish in October at a weight of 1060 pounds. A second lot (System b) was pastured 60 days, placed on full feed on pasture for an additional 90 days, then finished in dry lot for sale as choice cattle weighing 1120 pounds in October. A third group (System c) was put on alfalfa brome pasture in May and grazed continuously for about 130 days without any grain feeding. The pastures were subdivided into three parts and the cattle rotated on the three areas at two to three week intervals. After 130 days on pasture they were placed in dry lot, brought to full feed, and finished to choice cattle weighing 1135 pounds in December. The fourth group, which was not considered in this study, was handled in a manner similar to System c except that the cattle were not rotated on pasture.

In computing annual costs and returns from the different feeder cattle systems the following procedures were used: The initial cost of the livestock sold in a particular year was computed by multiplying the purchase weight by the average Chicago price of the particular grade of feeder cattle in the appropriate month of the preceding year (48), adjusted for transportation and commission costs. The value of steers at the end of the feeding period was based on sale weight and the average Chicago price (48), adjusted for transportation and commission, for the appropriate grade and month of sale.

Investment in buildings and equipment per head were computed for 1948 on the basis of current costs of building materials and labor required to provide the minimum housing for each system of handling the feeders. These figures were adjusted for the other years by the index
of building costs (47). Interest on the investment in livestock was figured at 6 per cent per year of the purchase value adjusted for the length of time the livestock were on the farm. Taxes were computed at 1.1 per cent of purchase value. Insurance was figured at .4 per cent of purchase value.

Feed requirements per steer under each system of feeding are shown in Table 7. The value of feeds per head was computed on the basis of the average Iowa price of corn the preceding October, the price of alfalfa hay the preceding November, and the price of cottonseed meal the preceding December (8). Pasture was evaluated on the basis of the current annual value of pasture per acre.

The value of labor per head was computed by multiplying the Iowa average daily wage rate each year (without board) (8) by the days of labor required under each system.

Costs and Returns from Beef Cows

Two systems of handling beef cows were considered. The differences in the two systems are in the disposition of the calf drop. Under System a the calves were sold each fall at a weight of 400 pounds as good-choice feeder calves. Under System b the calves were wintered through the first winter, pastured the following summer and fall, wintered through the second winter, and grazed through part of the following summer. They were then fed out in dry lot from July to October and sold as good grade cattle weighing about 1200 pounds. This latter system of handling the calves follows System V described in a Missouri study (46).
Table 7. Feed, labor, and certain miscellaneous requirements per steer for specific feeder cattle systems.

<table>
<thead>
<tr>
<th>Item</th>
<th>Choice calves (a)</th>
<th>Yearling steers (b)</th>
<th>2 yr. old steers (c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corn, bu. (^\text{a})</td>
<td>63.00</td>
<td>53.71</td>
<td>47.46</td>
</tr>
<tr>
<td>Hay, tons (^\text{a})</td>
<td>.70</td>
<td>1.50</td>
<td>1.24</td>
</tr>
<tr>
<td>Pasture, acres (^\text{a})</td>
<td>.06</td>
<td>.11</td>
<td>.90</td>
</tr>
<tr>
<td>Protein supplement, cwt. (^\text{a})</td>
<td>2.6</td>
<td>1.48</td>
<td>.33</td>
</tr>
<tr>
<td>Labor, days (^\text{b})</td>
<td>1.74</td>
<td>1.53</td>
<td>1.30</td>
</tr>
<tr>
<td>Veterinary (^\text{c})</td>
<td>.18</td>
<td>.18</td>
<td>.18</td>
</tr>
<tr>
<td>Value tractor &amp; horse labor (^\text{d})</td>
<td>7.79</td>
<td>3.23</td>
<td>5.70</td>
</tr>
<tr>
<td>Investment in bldg. &amp; equip. (^\text{e})</td>
<td>115.00</td>
<td>115.00</td>
<td>115.00</td>
</tr>
<tr>
<td>Annual bldg. &amp; equip. costs (^\text{f})</td>
<td>3.33</td>
<td>3.33</td>
<td>3.33</td>
</tr>
<tr>
<td>Death loss, (^%)</td>
<td>2.5</td>
<td>1.5</td>
<td>1.5</td>
</tr>
</tbody>
</table>

\(^\text{a}\)Based on an Iowa Agricultural Experiment Station study (24).

\(^\text{b}\)Based mostly on a study by Wilcox and others (52).

\(^\text{c}\)Figures shown are for 1948. Previous years adjusted by index of daily wage rate (w/o board) (47).

\(^\text{d}\)Figures shown are for 1948. Previous years adjusted by index of machinery costs (47).

\(^\text{e}\)Figures shown are for 1948. Previous years adjusted by index of building costs (47).
Costs and returns per cow under System (a) are considered first. The investment per cow in beef cows was found by multiplying the average annual Chicago price per hundred pounds of good grade cows, less transportation and commission costs, by 1100 pounds. Figuring one calf retained for replacement for each eight cows and assuming one bull for twenty cows (35), the value of bulls and replacement stock per cow was computed at 13.3 per cent of the value of the cow. Interest on investment in cattle was calculated at 6 per cent. Taxes and insurance per cow was computed at 1.5 per cent of the livestock investment. The investment in building and equipment per beef cow was estimated at $125 for 1948 based on current costs of building materials and labor and adjusted by the index of building costs for other years. Interest on investment in buildings and equipment was figured at 4 per cent; depreciation was figured at 3 per cent.

Miscellaneous cash expenses (including veterinary, salt, supplies, etc.) per cow were estimated at 47 cents for 1948 (35). This figure was adjusted by the index of prices paid by farmers for equipment and supplies for previous years. The cost of tractor and horse power was estimated as $1.44 (35) per cow in 1948. This was adjusted by the index of prices paid for farm machinery for other years.

Days of labor per cow were estimated at 1.2 days per year for the beef cows and .3 day per cow for replacement stock and bull, making a total of 1.5 days labor annually per beef cow. The value of labor was calculated on the basis of the annual daily wage rate, without board.

Feed requirements per cow, including replacement stock and bull,
were estimated at 6.7 bushel of corn, 1.15 tons hay, and 1.8 acres pasture (21). The values of these feeds were computed on the basis of the average price of corn the preceding October, the Iowa average price of alfalfa hay the preceding November, and the current annual pasture charge. Gross product per cow from the beef herd included 150 pounds of beef from cull cows (on the basis of a 90 per cent calf crop and replacement every 8 years) and 310 pounds of feeder calves sold per cow annually. The 150 pounds of beef from cull cows was evaluated on the basis of the annual average price of good cows at Chicago, less freight and commission costs. The value of the feeder calves was based on the October price for good-choice calves at Chicago, less transportation and commission costs.

Costs per cow under beef System b included the costs of maintaining the beef herd and the costs of raising the calves to finished cattle. Costs for maintaining the herd were identical with System a in all respects except that annual costs for a particular year were based on prices in the second preceding year. Costs of raising the calves for sale as finished cattle were calculated on the basis of prices in each of the three years covered by the production process. Thus the cost and returns figures for a particular year represent costs incurred over a three year period and returns in the year of sale. Taxes, insurance and miscellaneous costs were calculated at the same rates as used for the other feeder systems. Total feed requirements per feeder was estimated at 18.75 bushels of corn, 2.16 tons hay, 1.88 acres of pasture, and 105 pounds of protein supplement (46). Assuming a 90 per cent calf crop, 3 per cent
death loss, and one-eighth of the calf crop retained for replacements, only .775 head of finished cattle were marketed per beef cow. Thus feed requirements per cow, in addition to requirements for maintenance of the herd, were estimated as 77.5 per cent of the above figures. Labor requirements per cow, in addition to that required for maintenance of the herd were estimated at 1.8 days. Building and equipment investments per cow were estimated at $115 for 1948 and adjusted for previous years by the index of building costs. Returns per cow were calculated on the basis of 921 pounds of (1189 x .775) good grade cattle sold in October.

Costs and Returns for Hog Feeding Systems

Seven systems of feeding swine were considered. Input requirements for one of these, System I, were based on the 1945 Iowa Capacity Studies (49). Systems IIa, IIb, and IIc, are representative of systems followed in an experiment conducted at the Beltsville Experiment Station by the U. S. D. A. (7). Hogs fed under System IIa received no forage; hogs fed under System IIb received 10 per cent of their feed in the form of chopped legume hay; those fed according to System IIc obtained 20 per cent of their feed as chopped legume hay. All three groups were fed in dry lot. Systems IIIa, IIIb, and IIIc were adapted from pasture studies carried out at the Iowa Agricultural Experiment Station (23). Each of these three systems (a, b, and c) used a different proportion of forage (pasture) in the ration. Hogs fed according to System (a) received no pasture; those following System (b) were fed grain equal to 3 per cent of their body weight while on good pasture; and those following System (c)
were limited to grain equal to 1 per cent of their body weight while on pasture.

Costs for all seven systems were considered to be the same except with respect to feed, labor, and tractor and horse power costs. Miscellaneous costs, including insurance, taxes, veterinary, and other miscellaneous supplies, were estimated at $1.45 per pig for 1948 (23). This was adjusted by the index of prices paid by farmers for supplies for other years. Investment per pig in buildings and equipment was estimated to be $8.17 for 1948 (23) on the basis of building material and labor costs for providing the minimum buildings and equipment under corn belt conditions. Annual building and equipment costs were figured at $1.00 per pig for 1948 (23) and adjusted by the index of building costs for other years.

The annual investment in breeding stock per pig was calculated by multiplying 250 pounds (the average weight of the brood sow) by the average price of hogs in the preceding October and dividing the product by 6 (the assumed number of pigs saved per litter). The interest on investment in breeding stock per pig was calculated at the rate of 6 per cent per year and adjusted to a 9 month basis -- the length of time the sow would normally be kept. No depreciation was assumed on the breeding stock. Estimates of the days of labor and quantities of feed required per pig are given in Table 8. The value of labor was calculated by multiplying the days of labor required per pig by the Iowa annual average daily wage rate (without board). Corn was evaluated on the basis of the price of corn the preceding October. The value of hay required was
figured on the basis of the average Iowa price of alfalfa hay the preceding November. The value of protein supplement was based on the price of soybean oil meal the previous December.

Table 8. Estimated feed, labor, and power requirements per pig for specific hog feeding systems.

<table>
<thead>
<tr>
<th>Item</th>
<th>Iowa average</th>
<th>Dry lot, System II</th>
<th>Pasture, System III</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>System I</td>
<td>(a)</td>
<td>(b)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(d)</td>
<td>(e)</td>
</tr>
<tr>
<td>Corn, bushel</td>
<td>13.5</td>
<td>10.8</td>
<td>9.8</td>
</tr>
<tr>
<td>Soybean oil meal</td>
<td>.39</td>
<td>.91</td>
<td>.91</td>
</tr>
<tr>
<td>Tankage</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Hay, tons</td>
<td>-</td>
<td>-</td>
<td>.035</td>
</tr>
<tr>
<td>Pasture, acres</td>
<td>.004</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Labor, days</td>
<td>.59</td>
<td>.59</td>
<td>.65</td>
</tr>
<tr>
<td>Tractor &amp; horse power</td>
<td>$1.33</td>
<td>$1.33</td>
<td>$1.46</td>
</tr>
</tbody>
</table>

*aBased on unpublished data from Iowa Capacity Studies (49, p. 35).

*bBased on study by Ellis and others in the U. S. Department of Agriculture (7).


The hogs in each of the systems were considered sold at a weight of 225 pounds. Gross returns per pig from each feeding system was calculated by multiplying the average Iowa price of butcher hogs in the month of sale by 225 pounds. The hogs fed according to Systems IIa, IIb, IIc, and IIIa were considered sold in September; those fed according to Systems I and IIIb were treated as sold in October; and those fed according to System IIIc were considered sold in November.