USE OF THE CONNECTION MACHINE TO STUDY ULTRASONIC WAVE PROPAGATION IN MATERIALS

P. P. Delsanto

Dipartimento di Fisica
Politecnico di Torino
10129 Torino, Italy

T. Whitcombe, H. H. Chaskelis and R. B. Mignogna

Mechanics of Materials Branch
Naval Research Laboratory
Washington, DC 20375-5000

INTRODUCTION

The architecture of conventional (von Neumann) computers, with a single processor and millions of memory units, is inherently inefficient for most applications. In fact, while the processor is extremely busy all the time, only a very small portion of the memory is active. Larger computers are even less efficient, since the ratio of processing power to memory is even smaller and the length of computation is dominated by the ever increasing time required to move data between processor and memory. To overcome this so-called "von Neumann bottleneck," a new kind of computer, called the "Connection Machine" (CM) has been designed, with a larger number (thousands) of processors, connected in a programmable way, in the framework of a fixed physical wiring scheme [1]. This parallelism allows an opportunity to efficiently reformulate the problem to be studied and modify the approach [2-4]. Currently, the memory available is limited and requires some care in programming. This limitation should decrease with new CM-type machines.

In the present work we wish to demonstrate the applicability of parallel processing to the problem of studying the propagation of ultrasonic waves in media of arbitrary complexity. We present a computer simulation technique in which the differential equations are converted into difference equations by relating the processor network to the spatial mesh in a very natural way.

In our approach we divide the material along the wave propagation path into "cells," in a one-to-one correspondence with the CM processors. Then we input the material properties to each processor by assigning the corresponding values of the physical parameters that characterize the material via the front-end computer for the CM. These values may vary from processor

to processor, thus simulating the effect of possible spatial variations in the physical properties of the medium, such as those due to inhomogeneities, discontinuities, texture and/or stress gradients, layers of different materials, defects, etc.

Time, used as a variable, must also be properly discretized. At the initial times ($t = 0$ and $t = 1$) the initial conditions are imposed, i.e. all the displacements are set equal to zero, except for the processor(s) representing the material surface. The pulse (source wave) to be propagated through the material is input at these processors. The front end initialization is then completed by specifying the source wave. This is accomplished by inputting the values of the displacements of the source wave for the time duration of the pulse.

At this point the "rules of the game" governing the propagation of the disturbance from cell to cell are "taught" to each processor by means of an iteration equation. With this the CM can independently evaluate the displacement for each cell at the time $t + 1$ as a function of the displacements of that cell and neighboring ones at times t and $t - 1$.

Our approach differs substantially from finite element methods [5-13], inasmuch as we use a spatial mesh, not as an aid for the solution of the equations or the evaluation of integrals over complex geometries, but instead as a tool for computer simulation. Our approach is closer, in the formalism, to finite difference methods [14-15], except for the crucial difference on our part of using parallel processing.

Our treatment is readily applicable to the study of ultrasonic wave propagation in two and three dimensions. However, for illustrative purposes, we will limit our discussion to the one-dimensional case in the next section. Finally, we will present some numerical results obtained in the elementary case of a homogeneous material. Although, as mentioned earlier, our method may be applied to media of great complexity, these calculations were performed in order to check the validity of the treatment and of the approximations and assumptions involved through comparisons with well-known analytical results.

ONE-DIMENSIONAL CASE

We assume that an ultrasonic wave is transmitted through a material plate, normal to its surface. We set the x-axis in the direction of wave propagation with $x = 0$ and $x = h$ (plate thickness) representing two opposite surfaces of the plate. We also assume that all displacements are in the x-direction, so that the problem is truly one-dimensional.

In the general case, the elastodynamic wave equation for the particle displacements W_k is [16]:

$$\partial_l(S_{klmn}\partial_n W_m) = \rho\ddot{W}_k \tag{1}$$

where ρ is the density and S is the stiffness tensor. In the case of a homogeneous, unstressed and isotropic material

$$S_{klmn} = \lambda\delta_{kl}\delta_{mn} + \mu(\delta_{km}\delta_{\ln} + \delta_{kn}\delta_{lm}) \tag{2}$$

where λ and μ are the Lamé constants.

In the one-dimensional case under consideration, S becomes a scalar function $s(x)$ and Eq. (1) becomes:

$$\partial_x[s(x)W'] = \rho(x)\ddot{W} \tag{3}$$

We now proceed to discretize time and space. We define an elementary time unit τ, so that the continuous time t becomes $t\tau$, with $t = 0,1,2,\ldots$ Then we divide the propagation path into $N - 1$ "cells" of length

$$\epsilon = h/(N - 1) \tag{4}$$

and label them with the index i, so that $i = 1$ and $i = N$ correspond to $x = 0$ and $x = h$, respectively. We reserve the "cell" $i = 0$ for the input of the source pulse

$$W_{0,t} \equiv W_0(t) = W^{\text{source}}(t) \tag{5}$$

In order to transform Eq. (3) into a difference equation we use, for both time and space derivatives, the expansion

$$f'(0) = \frac{f(x) - f(-x)}{2x} - \frac{1}{3!}\, x^2 f'''(0) + \cdots \tag{6}$$

Neglecting in Eq. (6) terms in ϵ^2 and higher, Eq. (3) becomes:

$$W_{i,t+1} = c_i^+ W_{i+1,t} + c_i^- W_{i-1,t} + c_i^o W_{i,t} - W_{i,t-1} \tag{7}$$

where

$$c_i^{\pm} = c_i \pm (c_{i+1} - c_{i-1})/4 \tag{8}$$

$$c_i^o = 2(1 - c_i) \tag{9}$$

$$c_i = \frac{\tau^2 s_i}{\epsilon^2 \rho_i} \tag{10}$$

In. Eq. (10) the mesh functions s_i and ρ_i have replaced $s(x)$ and $\rho(x)$.

In the particular case of a homogeneous material, Eq. (7) becomes:

$$W_{i,t+1} = c(W_{i+1,t} + W_{i-1,t}) + c^o W_{i,t} - W_{i,t-1} \tag{11}$$

where

$$c = \frac{\tau^2(\lambda + 2\mu)}{\epsilon^2 \rho} \tag{12}$$

$$c^o = 2(1 - c)$$

It is particularly convenient to choose ϵ and τ so that $c = 1$. In fact we know, from the theory of difference equations, that for $c = 1$ the region of determination for the difference equation coincides with the region of determination of the differential equation [17]. Then Eq. (11) becomes:

$$W_{i,t+1} = W_{i+1,t} + W_{i-1,t} - W_{i,t-1}$$

Eq. (12) may be rewritten as

$$c = \left(\frac{v_L}{\epsilon/\tau}\right)^2$$

where $v_L = \sqrt{(\lambda + 2\mu)/\rho}$ is the longitudinal velocity. The choice $c = 1$ then requires that v_L be equal to the "characteristic" velocity $v_c = \epsilon/\tau$.

Eq. (12) may also be rewritten as

$$c = (\alpha/\beta)^2$$

where $\alpha = vT/\epsilon$ and $\beta = T/\tau$ are the number of spatial steps (cells) in the wavelength vT (where v is the velocity) and of elementary intervals in the period T of the source wave, respectively. The condition $c = 1$ thus requires that $\alpha = \beta$. Numerical results are expected to improve if the common value of α and β increases, but α is proportional to the number of active processors and β to the amount of CM time required, so that some degree of optimization is necessary.

RESULTS AND DISCUSSION

An approach, based on the capabilities of the CM as described earlier, has been used to simulate the one-dimensional wave propagation of different source pulses in a homogeneous, isotropic unstressed material. The thickness of the "plate" was assumed to be $h = 12.7$ mm, the longitudinal wave velocity $v_L = 6.3$ km/s and its frequency $f = 10$ MHz.

The results for a square pulse and for a sine wave are presented in Figs. 1 and 2, respectively. In these two figures and all of the following figures, the upper line ($i = 0$) represents the source pulse and the next line ($i = 1$) and bottom line represent the top and bottom surfaces, respectively. Also, for all of the figures, the horizontal scale is in units of τ and the vertical scale is in units of ϵ. Results for other pulse shapes are reported in Ref. 18. In all cases we observe that the pulse is propagated and reflected as predicted by the theory, with negligible numerical errors.

Figures 2-4 show that best results are obtained for $c = (\alpha/\beta)^2 = 1$, as previously discussed. In fact, when c decreases from $c = 1$ in Fig. 2 to $c = 1/4$ in Fig. 3 to $c = 1/16$ in Fig. 4, numerical errors begin to accumulate and propagate. This can be clearly seen in the second round trip of the propagating wave: the source pulse is almost unchanged in Fig. 2, has acquired some "noise" in Fig. 3 and is severely altered in Fig. 4. For $c > 1$ we have observed divergence in the displacement amplitudes during the wave propagation, as predicted by the theory.

Finally in Figs. 5 and 6 we show, in detail, the mechanism of reflection at $x = h$. A comparison between the two figures shows that numerical errors are smaller for $\alpha = \beta = 100$ (Fig. 5), than for $\alpha = \beta = 20$ (Fig. 6). These errors will affect the reflected wave if only very slightly at later times. The choice $\alpha = \beta = 100$ results, in our case, in values of $\epsilon = 6.35\ \mu$ and $\tau = 1$ ns.

CONCLUSIONS

The computational results reported in the previous section and in Ref. 18 confirm the validity of the present approach and ascertain the range of the parameters α and β or, equivalently, ϵ and τ. The results are based on a very elementary case, but similar conclusions are expected to hold also for complex media, since the processors simulating the material "cells" are mutually independent. Calculations to prove the validity of the approach in layered and other complex media are in progress.

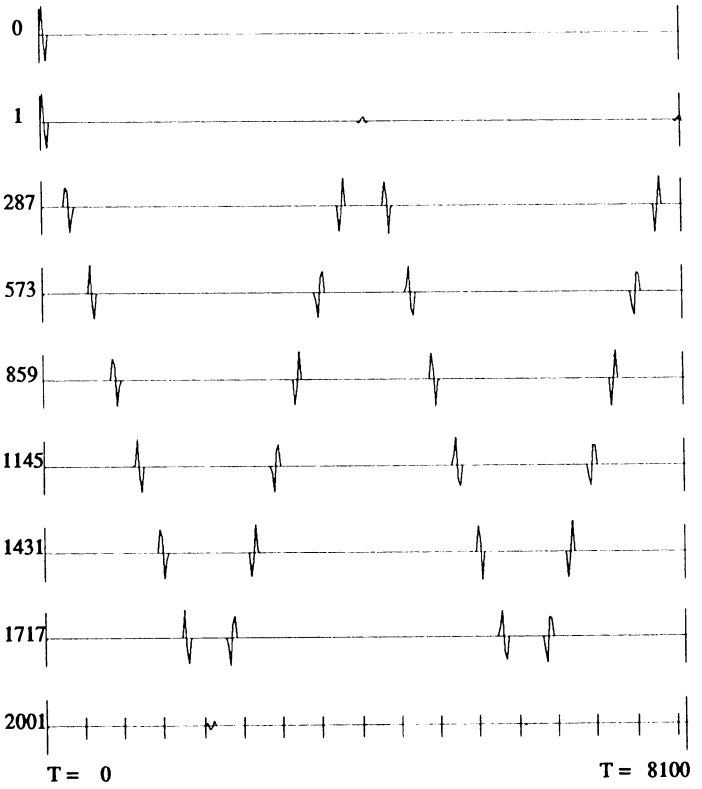

Fig. 2 — Propagation of a sine wave. The parameters used in this plot are $\alpha = \beta = 100$, $\tau = 1$ ns and $\epsilon = 6.35\ \mu$.

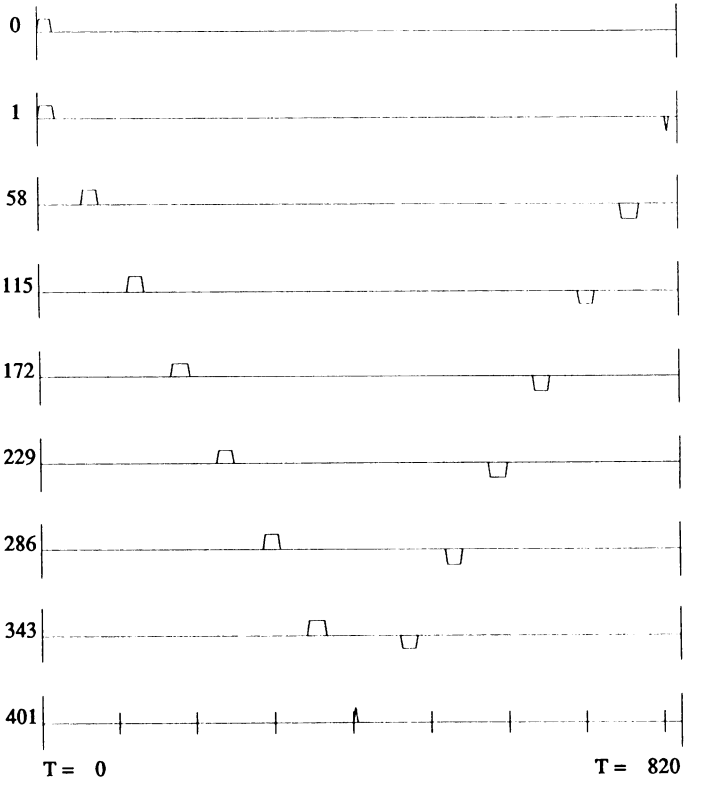

Fig. 1 — Propagation of a square pulse through a homogeneous material. The parameters used in this plot are $\alpha = \beta = 20$. The horizontal scale is in units of $\tau = 5$ ns. The vertical scale is in units of $\epsilon = 31.75\ \mu$.

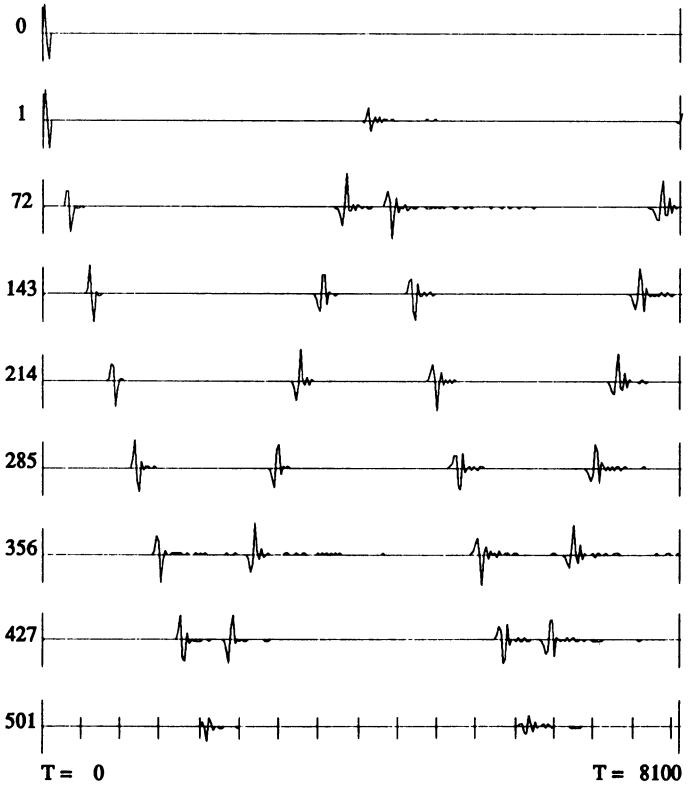

Fig. 4 — Propagation of a sine wave. The parameters used in this plot are $\alpha = 25$, $\beta = 100$, $\tau = 1$ ns and $\epsilon = 25.4\ \mu$.

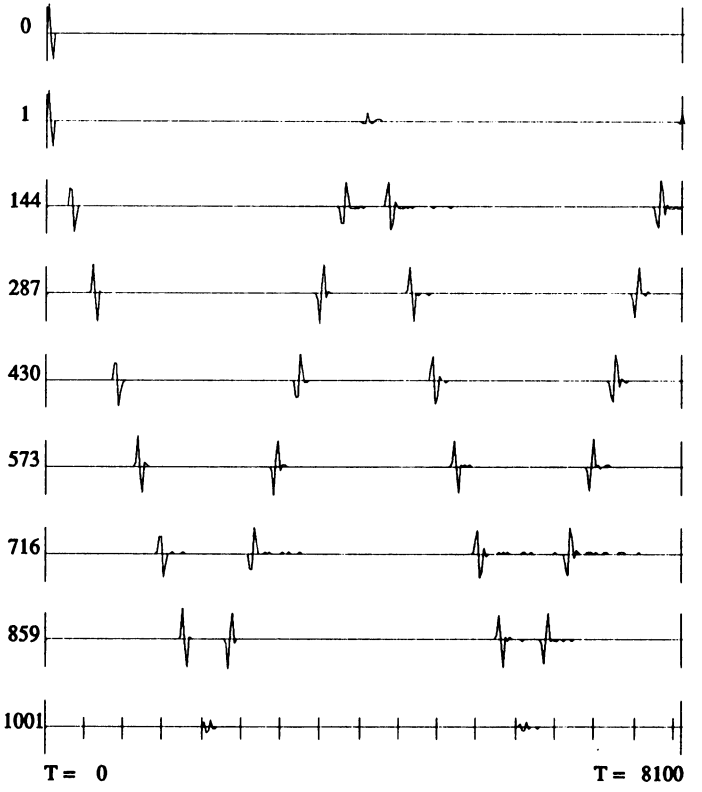

Fig. 3 — Propagation of a sine wave. The parameters used in this plot are $\alpha = 50$, $\beta = 100$, $\tau = 1$ ns and $\epsilon = 12.7\ \mu$.

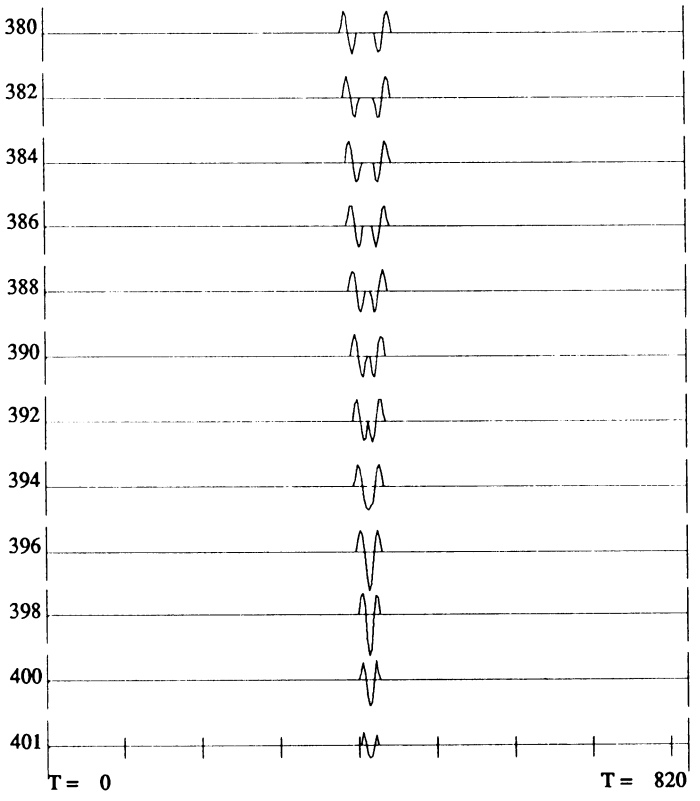

Fig. 6 — Detailed representation of the reflection of a sine wave by the bottom surface of the material. The parameters used in this plot are $\alpha = \beta = 20$, $\tau = 5$ ns and $\epsilon = 31.75\ \mu$.

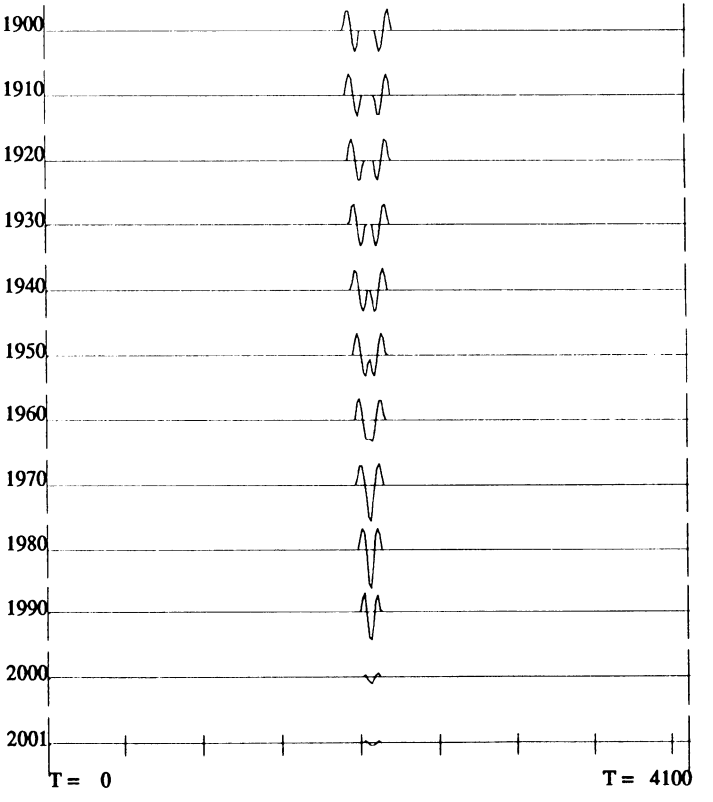

Fig. 5 — Detailed representation of the reflection of a sine wave by the bottom surface of the material. The parameters used in this plot are $\alpha = \beta = 100$, $\tau = 1$ ns and $\epsilon = 6.35\ \mu$.

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